NDA MATHS MOCK TEST - 186 (SOLUTION)

1. (D) $\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A}$

$\Rightarrow \frac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A}$

$+ \frac{\cos^3 A - 4\cos^3 A + 3\cos A}{\cos A}$

$\Rightarrow \frac{3\sin A - 3\sin^3 A}{\sin A} + \frac{-3\cos^3 A + 3\cos A}{\cos A}$

$= (3 - 3\sin^2 A) + (-3\cos^2 A + 3)$

$= 6 - 3(\sin^2 A + \cos^2 A) = 6 - 3(1) = 3$

2. (B) $\sin^2 66\frac{1^\circ}{2} - \sin^2 23\frac{1^\circ}{2}$

$\Rightarrow \left[\sin\left(90^\circ - 23\frac{1^\circ}{2}\right)\right]^2 - \sin^2 23\frac{1^\circ}{2}$

$\Rightarrow \cos^2 23\frac{1^\circ}{2} - \sin^2 23\frac{1^\circ}{2}$

$\Rightarrow \cos 2\left(23\frac{1^\circ}{2}\right)$

$[\because \cos 2A = \cos^2 A - \sin^2 A]$

$\Rightarrow \cos\left[2\times\left(\frac{47}{2}\right)^\circ\right] = \cos 47^\circ$

3. (D) Given that, $\tan A = x+1$ and $\tan B = x-1$

Now, $x^2 \tan(A-B) \Rightarrow x^2\left(\frac{\tan A - \tan B}{1+\tan A.\tan B}\right)$

$\Rightarrow x^2\left\{\frac{(x+1)-(x-1)}{1+(x+1).(x-1)}\right\}$

$\Rightarrow x^2\left\{\frac{2}{1+x^2-1}\right\} \Rightarrow x^2.\frac{2}{x^2} = 2$

4. (D) $(\sin^4\theta - \cos^4\theta + 1).\text{cosec}^2\theta$

$\Rightarrow \{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) + 1\}.\text{cosec}^2\theta$

$\Rightarrow \{(\sin^2\theta - \cos^2\theta).1 + 1\}.\text{cosec}^2\theta$

$\Rightarrow \{(\sin^2\theta - \cos^2\theta)+1\}.\text{cosec}^2\theta$

$\Rightarrow (2\sin^2\theta).\frac{1}{\sin^2\theta} = 2$

5. (C) $\because \sec\alpha = \frac{13}{5} \Rightarrow \cos\alpha = \frac{5}{13}$

Now, $\sin\alpha \Rightarrow \sqrt{1-\cos^2\alpha} \Rightarrow \sqrt{1-\frac{25}{169}}$

$\Rightarrow \sqrt{\frac{144}{169}} = -\frac{12}{13}$ $[\because 270 < \alpha < 360°]$

6. (B) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{4} = \frac{b}{5} = \frac{c}{6} = K$

$\Rightarrow a = 4K,\ b = 5K,\ c = 6K$

Now, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \cos A = \frac{25K^2+36K^2-16K^2}{60K^2} = \frac{3}{4}$

and $\cos B = \frac{a^2+c^2-b^2}{2ab}$

$\Rightarrow \cos B = \frac{15K^2+36K^2-25K^2}{48K^2} = \frac{9}{16}$

and $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos C = \frac{16K^2+25K^2-36K^2}{40K^2} = \frac{1}{8}$

$\therefore \cos A : \cos B : \cos C = \frac{3}{4} : \frac{9}{16} : \frac{1}{8}$

$= 12 : 9 : 2$

7. (A) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (true)

$\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ (given)

$\therefore \tan A = \tan B = \tan C$

$\therefore \Delta ABC$ is equilateral and so each of its angles is 60°.

$\therefore \Delta = \frac{1}{2} a \times a \times \sin 60^\circ$

$\Rightarrow \Delta = \frac{1}{2}\times 2\times 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

8. (A) $r = 4R \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Here $A = B = C = 60^\circ$

$\therefore r = 4R \sin 30^\circ \times \sin 30^\circ \times \sin 30^\circ$

$r = \frac{R}{2}$

9. (A) $a\cos A = b\cos B$

$\Rightarrow$ K sinA cosA = K sinB cosB

$\Rightarrow$ 2 sinA cosA = 2 sinB cosB

$\Rightarrow$ sin2A − sin2B = 0

$\Rightarrow$ 2 cos(A + B) sin(A – B) = 0

$\Rightarrow$ cos(A + B) = 0 or sin(A – B) = 0

$\Rightarrow$ A + B = 90° or A – B = 0°

$\Rightarrow \angle C = 90°$ or A = B

$\Rightarrow \Delta ABC$ is either right angled or isosceles.

10. (A) $2\tan^{-1}x = \cos^{-1}\left[\dfrac{1-x^2}{1+x^2}\right]$

$$\tan^{-1}x = \frac{1}{2}\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right] \quad \ldots\text{(ii)}$$

Let $\dfrac{1-x^2}{1+x^2} = \dfrac{\sqrt{2}}{3}$

by componendo and dividendo Rule

$$\Rightarrow \frac{2}{2x^2} = \frac{3+\sqrt{2}}{3-\sqrt{2}}$$

$$\Rightarrow x^2 = \frac{(3-\sqrt{2})}{(3+\sqrt{2})}\times\frac{(3-\sqrt{2})}{(3-\sqrt{2})} = \frac{(3-\sqrt{2})^2}{7}$$

$$\Rightarrow x = \frac{3-\sqrt{2}}{\sqrt{7}}$$

from eq(i)

$$\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{2}}{3}\right) = \tan(\tan^{-1}x)$$

$$\Rightarrow x = \frac{(3-\sqrt{2})}{\sqrt{7}}$$

11. (B) $\tan^{-1}\left(\dfrac{1-\cos x}{\sin x}\right)$

$$= \tan^{-1}\left\{\frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right\}$$

$$= \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$$

12. (C) $\tan^{-1}3 + \tan^{-1}x = \tan^{-1}8$

$$\Rightarrow \tan^{-1}\left(\frac{3+x}{1-3x}\right) = \tan^{-1}8$$

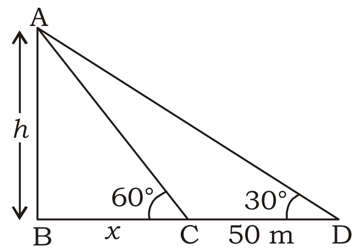
$$\Rightarrow \frac{3+x}{1-3x} = 8 \Rightarrow x = \frac{1}{5}$$

13. (C) $\sin^{-1}x + \sin^{-1}y = \dfrac{2\pi}{3}$

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1}x\right) + \left(\frac{\pi}{2} - \cos^{-1}y\right) = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}x + \cos^{-1}y = \left(\pi - \frac{2\pi}{3}\right) = \frac{\pi}{3}$$

14. (B)



Let BC = x, AB = h

In ΔACB :-

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}\,m \quad \ldots\text{(i)}$$

In ΔADB :-

$$\tan 30° = \frac{h}{x+50}$$

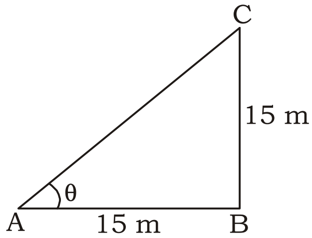
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+50} \Rightarrow \sqrt{3}\,h = x + 50$$

$$\Rightarrow \sqrt{3}\,h = \frac{h}{\sqrt{3}} + 50 \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = 50 \Rightarrow 2h = 50\sqrt{3}$$

$$\Rightarrow h = 25\sqrt{3}\,m$$

15. (C) Let the angle of elevation = θ



In ΔBAC,

$$\tan\theta = \frac{BC}{AB} = \frac{15}{15} = 1$$

$$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$$

16. (C) Given that, α and β are the roots of the equation $4x^2 + 3x + 7 = 0$.

$\therefore \alpha + \beta = -\dfrac{3}{4}$ and $\alpha\beta = \dfrac{7}{4}$

Now, $\alpha^{-2} + \beta^{-2} \Rightarrow \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} \Rightarrow \dfrac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$\Rightarrow \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} \Rightarrow \dfrac{\frac{9}{16} - \frac{7}{2}}{\frac{49}{16}}$

$\Rightarrow \dfrac{\frac{9-56}{16}}{\frac{49}{16}} \Rightarrow \dfrac{-47}{16} \cdot \dfrac{16}{49} = -\dfrac{47}{40}$

17. (D) Given that equations
$x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$
have real roots.

$\therefore k^2 \geq 4 \times 64$ $\quad [\because B^2 - 4AC \geq 0]$

$\Rightarrow k^2 \geq 16$ ...(i)

and $64 \geq 4k \Rightarrow k \leq 16$ ...(ii)

From eq(i) and (ii),
$k = 16$

18. (B) Given that, α and β are the roots of $x^2 - 2x + 4 = 0$

$\therefore$ Sum of roots $= \alpha + \beta = 2$ and product of roots $= \alpha\beta = 4$

Now, $\alpha^3 + \beta^3 \Rightarrow (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$\Rightarrow 2^3 - 3 \times 4 \times 2$

$\Rightarrow 8 - 24 = -16$

19. (B) Given $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

Here, $|A| = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$

$\text{adj}(A) = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{adj(A)}{|A|} = -1\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

Here, $|B| = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 0 - (-1) = 1$

$\text{adj}(B) = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$

$\therefore B^{-1} = \dfrac{adj(B)}{|B|} = 1.\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow B^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$

$\therefore B^{-1}A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} -2+1 & 4-1 \\ 1+0 & -2+0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

20. (B) $\therefore A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha \times \alpha & 0 \\ 1 \times \alpha + 1 & 0 + 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$

But $A^2 = B$

$\Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

On comparing, $\alpha^2 = 1$ and $\alpha + 1 = 2$

$\therefore \alpha = 1$

21. (C) $(\text{adj}A^T) = (\text{adj}A)^T \Rightarrow (\text{adj}A^T) - (\text{adj}A)^T$ = null matrix

22. (B) $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$

$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$

$\Rightarrow (3a-x)(4x^2) = 0$

$\therefore x = 3a, x = 0$

$\therefore$ Solution set $= \{3a, 0\}$

23. (C) $\begin{vmatrix} 3 & \omega & \omega^2 \\ \omega & 2+\omega^2 & 1 \\ \omega^2 & 1 & 2+\omega \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 2+(1+\omega+\omega^2) & \omega & \omega^2 \\ 2+(1+\omega+\omega^2) & 2+\omega^2 & 1 \\ 2+(1+\omega+\omega^2) & 1 & 2+\omega \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2 & \omega & \omega^2 \\ 2 & 2+\omega^2 & 1 \\ 2 & 1 & 2+\omega \end{vmatrix}$ $[\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow 2\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & 2+\omega^2 & 1 \\ 1 & 1 & 2+\omega \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow 2\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & 2+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & 2+\omega-\omega^2 \end{vmatrix}$

$\Rightarrow 2\begin{vmatrix} 1 & \omega & \omega^2 \\ 0 & 1-2\omega & 1-\omega^2 \\ 0 & 1-\omega & 1-2\omega^2 \end{vmatrix}$

$\Rightarrow 2[1.\{(1-2\omega)(1-2\omega^2) - (1-\omega)(1-\omega^2)\}]$
$\Rightarrow 2[(1 - 2\omega - 2\omega^2 + 4) - (1 - \omega - \omega^2 + 1)]$
$\Rightarrow 2[5 + 2(1) - (2 + 1)] = 8$

24. (A) $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$

$\Rightarrow \begin{bmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{bmatrix} + \begin{bmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{bmatrix} = 0$

$\Rightarrow (abc)\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} + \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = 0$

$\Rightarrow (1 + abc).\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow (1 + abc).\begin{bmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{bmatrix} = 0$

$\Rightarrow (1 + abc)(b-a)(c-a).\begin{bmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{bmatrix} = 0$

$\Rightarrow (1 + abc)(b - a)(c - a)(c - b) = 0$
$\Rightarrow abc = -1 \ (\because a \neq b \neq c)$

25. (A) Given that a, b, c and d are in AP.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ and $\frac{1}{d}$ are in HP.

$\Rightarrow$ bcd, acd, abd and abc are in HP.
Hence, abc, abd, acd and bcd are in HP.

26. (C) Given that, $\frac{1}{4}, \frac{1}{x}$ and $\frac{1}{10}$ are in HP.
4, x and 10 are in AP.

$\therefore$ Arithmetic mean, $x = \frac{4+10}{2} = \frac{14}{2} = 7$

27. (D) $S = \frac{a}{1-r}$, where $r < 1$

$\therefore \frac{a}{1-r} = 6 \Rightarrow a = 6(1 - r)$ ...(i)

and $a + ar = \frac{9}{2}$ [given]

$\Rightarrow 6(1 - r) + 6r(1 - r) = \frac{9}{2}$

$\Rightarrow 12 - 12r + 12r - 12r^2 = 9$

$\Rightarrow r^2 = \frac{3}{12} = \frac{1}{4} \Rightarrow r = \frac{1}{2}$ or $-\frac{1}{2} \Rightarrow a = 3$ or 9

28. (C) $\log_y(x)^5.\log_x(y)^2.\log_z(z)^3$
$\Rightarrow 5\log_y x.2\log_x y.3\log_z z$
$[\because \log_a b^n = n \log_a b]$
$\Rightarrow 5\log_y x.2\log_x y.3.1$ $[\because \log_a a = 1]$

$\Rightarrow 5.\frac{\log x}{\log y}.2.\frac{\log y}{\log x}.3$ $\left[\because \log_a b = \frac{\log b}{\log a}\right]$

$\Rightarrow 5.2.3 = 30$

29. (D) $\log_9 x - \log_9\left(\frac{x}{10} + \frac{1}{9}\right) = \log_9 9$

$\Rightarrow \log_9 \frac{x}{\left(\frac{x}{10} + \frac{1}{9}\right)} = \log_9 9$

$\Rightarrow \frac{x}{\left(\frac{x}{10} + \frac{1}{9}\right)} = 9 \Rightarrow x = \frac{9x}{10} + 1 \Rightarrow x = 10$

30. (D) $(a^4 - 2a^2b^2 + b^4)^{x-1} = (a-b)^{2x}.(a+b)^{-2}$

$\Rightarrow \left[(a^2 - b^2)^2\right]^{(x-1)} = (a-b)^{2x}.(a+b)^{-2}$

$\Rightarrow (a^2 - b^2)^{2(x-1)} = (a-b)^{2x}.(a+b)^{-2}$

$\Rightarrow (a-b)^{(2x-2)}(a+b)^{(2x-2)} = (a-b)^{2x}.(a+b)^{-2}$

$\Rightarrow \dfrac{(a-b)^{(2x-2)}}{(a-b)^{2x}} \cdot \dfrac{(a+b)^{(2x-2)}}{(a+b)^{-2}} = 1$

$\Rightarrow (a-b)^{-2}(a+b)^{+2x} = 1$

$\Rightarrow -2\log(a-b) + 2x.\log(a+b) = \log 1$

$\Rightarrow 2x\log(a+b) = 2\log(a-b)$

$\Rightarrow x = \dfrac{\log(a-b)}{\log(a+b)}$

31. (D) $\lim\limits_{x\to 2}\dfrac{\sin(e^{x-2}-1)}{\ln(x-1)}$ $\left[\dfrac{0}{0}\text{form}\right]$

by L' Hospital's rule

$\Rightarrow \lim\limits_{x\to 2}\dfrac{\cos(e^{x-2}-1).e^{x-2}}{\dfrac{1}{x-1}} = \dfrac{1\times 1}{1} = 1$

32. (C) Given that, $f(9) = 9$ and $f'(9) = 4$

Now $\lim\limits_{x\to 9}\dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ $\left[\dfrac{0}{0}\text{form}\right]$

by L' Hospital's rule

$\Rightarrow \lim\limits_{x\to 9}\dfrac{\dfrac{1}{2\sqrt{f(x)}}.f'(x)}{\dfrac{1}{2\sqrt{x}}.1}$

$\Rightarrow \lim\limits_{x\to 9}\dfrac{f'(x)\times\sqrt{x}}{\sqrt{f(x)}} = \dfrac{f'(9)\times\sqrt{9}}{\sqrt{f(9)}}$

$\Rightarrow \dfrac{4\times 3}{\sqrt{9}} \Rightarrow \dfrac{4\times 3}{3} = 4$

33. (D) Now, $\lim\limits_{x\to\pi/2}\left(\dfrac{1-\sin x}{(\pi-2x)^2}\right)$ $\left[\dfrac{0}{0}\text{form}\right]$

by L'Hospital's rule

$\Rightarrow \lim\limits_{x\to\pi/2}\dfrac{-\cos x}{2(\pi-2x)(-2)}$

$\Rightarrow \lim\limits_{x\to\pi/2}\dfrac{\cos x}{4(\pi-2x)}$ $\left[\dfrac{0}{0}\text{form}\right]$

Again, by L'Hospital's rule

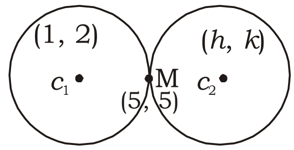
$\Rightarrow \lim\limits_{x\to\pi/2}\dfrac{-\sin x}{4(-2)} \Rightarrow \lim\limits_{x\to\pi/2}\dfrac{\sin x}{8}$

$\Rightarrow \dfrac{1}{8}.\sin\dfrac{\pi}{2} \Rightarrow \dfrac{1}{8}\times 1 = \dfrac{1}{8}$

34. (A) Let $c_2 = (h, k)$

$x^2 + y^2 - 2x - 4y - 20 = 0$ ...(i)

centre $c_1(1, 2)$



radius $= \sqrt{(1)^2 + (2)^2 - (-20)} = \sqrt{1+4+20} = 5$

It is clear that the point M is the mid point of c_1 and c_2.

$\therefore\ 5 = \dfrac{h+1}{2}$, $5 = \dfrac{k+2}{2}$

$\Rightarrow h = 9,\ k = 8$

Hence the equation of required circle

$(x-9)_2 + (y-8)^2 = 25$

35. (B) Line $px + qy + r = 0$ ...(i)

circle $x^2 + y^2 = a^2$ ...(ii)

If equation (i) is the tangent of circle (ii) then perpendicular distance from centre (0, 0) to the straight line (i) is equal to radius.

i.e. $\dfrac{r}{\sqrt{p^2+q^2}} = a \Rightarrow r^2 = a^2(p^2 + q^2)$

36. (C) Let the equation of ellipse be

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ...(i)

The coordinates of the foci are $(0, \pm 4)$

$\therefore\ be = 4$ and $e = \dfrac{4}{5}$

$\Rightarrow b\left(\dfrac{4}{5}\right) = 4 \Rightarrow b = 5$

Now, $a^2 = b^2(1 - e^2)$

$\Rightarrow a^2 = (5)^2\left(1 - \dfrac{16}{25}\right)$

$\Rightarrow a^2 = 25\times\dfrac{9}{25} \Rightarrow a^2 = 9$

from eq(i)

ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

37. (B) The equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ ...(i)

The equation of line $lx + my + n = 0$

$my = -lx - n$

$\Rightarrow y = \left(-\frac{l}{m}\right)+\left(-\frac{n}{m}\right)$

Now we know that the line $y = mx + c$ touches the ellipse (i)

If $c^2 = a^2m^2 + b^2$

So line (ii) touches the ellipse (i) when

$\left(-\frac{n}{m}\right)^2 = a^2\left(\frac{l}{m}\right)^2 + b^2$

$n^2 = a^2l^2 + b^2m^2$

38. (A) The equation of ellipse $9x^2 + 16y^2 = 144$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9} = 1 \Rightarrow \frac{x^2}{(4)^2}+\frac{y^2}{(3)^2} = 1$

The equation of line $y = x + \lambda$

Now we know that line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$

if $c^2 = a^2m^2 + b^2$

$\Rightarrow \lambda^2 = (4)^2(1)^2 + (3)^2 \Rightarrow \lambda^2 = 16 + 9$

$\Rightarrow \lambda^2 = 25 \Rightarrow \lambda = \pm 5$

39. (A) Slope of line $x - y + 4 = 0$ is 1

So slope of perpendicular to it = -1 = slope of tangent

$\Rightarrow m = -1$

given Hyperbola equation

$x^2 - 4y^2 = 36$

$\Rightarrow \frac{x^2}{36}-\frac{y^2}{9} = 1$

equation of tangent

$y = mx \pm \sqrt{a^2m^2 - b^2} \Rightarrow y = -x \pm \sqrt{36-9}$

$\Rightarrow y = -x \pm \sqrt{27} \Rightarrow y = -x \pm 3\sqrt{3}$

40. (A) The hyperbola equation

$\frac{x^2}{100}-\frac{y^2}{49} = 1$ ...(i)

and line equation $y = mx + 6$ ...(ii)

We know that the line $y = mx + c$ touches the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{49} = 1$

If $c^2 = a^2m^2 - b^2$

Here, $c = 6$, $a = 10$, $b = 7$

$\Rightarrow (6)^2 = (10)^2m^2 - (7)^2$

$\Rightarrow 100m^2 = 85$

$\Rightarrow m^2 = \frac{85}{100} \Rightarrow m = \sqrt{\frac{17}{20}}$

41. (A) Let I = $\int e^{x\log a}.e^x dx$

I = $\int e^{\log a^x}.e^x dx$

I = $\int a^x.e^x dx = \int (ae)^x.dx = \frac{(ae)^x}{\log(ae)}$ + C

42. (B) I = $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

$\Rightarrow$ I = $\int \frac{dx}{x^2.(x^4)^{3/4}\left(1+\frac{1}{x^4}\right)^{3/4}}$

$\Rightarrow \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$

Let $1+\frac{1}{x^4} = t \Rightarrow \frac{-4}{x^5}dx = dt \Rightarrow \frac{dx}{x^5} = -\frac{1}{4}dt$

$\Rightarrow$ I = $\int \frac{-dt}{4t^{3/4}} \Rightarrow -\frac{1}{4}\int t^{-3/4}dt$

$\Rightarrow -\frac{1}{4}\left[\frac{t^{\frac{-3}{4}+1}}{\left(-\frac{3}{4}+1\right)}\right]+C \Rightarrow t^{1/4}$ + C

$\Rightarrow -\left(1+\frac{1}{x^4}\right)^{1/4}$ + C

43. (A) Let I = $\int \cos^3 x.e^{\log(\sin x)}.dx$

I = $\int \cos^3 x.\sin x\, dx$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

I = $\int -t^3\, dt$

I = $\frac{-t^4}{4}$ + C = $-\frac{1}{4}\cos^4 x$ + C

44. (B) $(x - 1)^3 + 8 = 0$

$\Rightarrow (x - 1)^3 = -8 = (-2)^3$

$\Rightarrow x - 1 = -2$

or -2ω or $-2\omega^2$

$\Rightarrow x = -1,\ 1 - 2\omega,\ 1 - 2\omega^2$

45. (D) $\frac{1+i}{1-i} \Rightarrow \frac{(1+i)^2}{(1-i)(1+i)} \Rightarrow \frac{1-1+2i}{2} \Rightarrow i$

Now, $i^n = 1$

$\Rightarrow$ the smallest positive integral value of n should be 4.

46. (A) A.T.Q,

$|x + iy - 5i| = |x + iy + 5i|$

$\Rightarrow |x + (y - 5)i| = |x + (y + 5)i|$

$\Rightarrow x^2 + (y - 5)^2 = x^2 + (y + 5)^2$

$\Rightarrow x^2 + y^2 - 10y + 25 = x^2 + y^2 + 10y + 25$

$\Rightarrow 20y = 0 \Rightarrow y = 0$ i.e. x-axis

47. (C) Probability = $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

48. (B) Possible outcomes n(S) = 6

There are 2 odd numbers less than 5,

So n(E) = 2

Hence P(E) = $\frac{2}{6} = \frac{1}{3}$

49. (C) Possibilities of sum of the dice is more than 10{(5, 6), (6, 5), (6, 6)} = 3

Possibilities of sum of the dice is divisible by 3{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6) (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)} = 12

Required ratio = $\frac{3}{36} : \frac{12}{36} = 1 : 4$

50. (C) The equation of normal of parabola $y^2 = 4ax$ in terms of slope is given as $y = mx - 2am - am^3$ ...(i)

Now for given curve $y^2 = x$, we have $4a = 1$

$\Rightarrow a = \frac{1}{4}$

So from equation (i), we can write $y = mx - \frac{1}{2}m - \frac{1}{4}m^3$

Since it passes through $(c, 0)$, so we have

$0 = mc - \frac{1}{2}m - \frac{1}{4}m^3$

$\Rightarrow m\left(c - \frac{1}{2} - \frac{1}{4}m^2\right) = 0 \Rightarrow m = 0$ or $c - \frac{1}{2} - \frac{1}{4}m^2 = 0$

Now let us consider $c - \frac{1}{2} - \frac{1}{4}m^2 = 0$

$\Rightarrow \frac{1}{4}m^2 = c - \frac{1}{2} \Rightarrow m = 2\sqrt{c - \frac{1}{2}}$

Now for three normals m should be real, therefore $c > \frac{1}{2}$

51. (B) The equation of normal of $y^2 = 4x$ at point $(m^2, -2m)$ is $y = mx - 2m - m^3$

If the normal makes equal angles with the coordinate axes, then $m = \tan\frac{\pi}{4} = 1$

$\therefore$ The required point $(m^2, -2m)$ i.e. $(1, -2)$

52. (A) Parabola equation $x^2 + 2y = 8x - 7$

$\Rightarrow x^2 - 8x = -2y - 7$

$\Rightarrow x^2 - 8x + 16 = -2y + 9$

$\Rightarrow (x - 4)^2 = -2\left(y - \frac{9}{2}\right)$

$\therefore$ coordinate of vertex = $\left(4, \frac{9}{2}\right)$

53. (A) Curve $y = 2x^2 - x + 1$ ...(i)

$\frac{dy}{dx} = 4x - 1$

line equation $y = 3x + 9$ ...(ii)

Slope = 3

If tangent is parallel to the line (ii) then slope is equal i.e.

$4x - 1 = 3$

$x = 1$

The value put in equation of curve (i)

$y = 2 - 1 + 1 = 2$

Hence the required point (1, 2)

54. (B) Curve $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$

At point $(2, -1)$ we have $2 = t^2 + 3t - 8$

$\Rightarrow t^2 + 3t - 10 = 0 \Rightarrow (t - 2)(t + 5) = 0$

$\Rightarrow t = 2, -5$

and $-1 = 2t^2 - 2t - 5$

$\Rightarrow 2t^2 - 2t - 4 = 0$

$\Rightarrow 2(t - 2)(t + 1) = 0 \Rightarrow t = 2, -1$

So $t = 2$ for point $(2, -1)$

Now $\frac{dx}{dt} = 2t + 3$ and $\frac{dy}{dt} = 4t - 2$

Slope, $\frac{dy}{dt} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3} = \frac{6}{7}$

55. (A) Curve $x^2 + y^2 - 2x - 3 = 0$ ...(i)
diff. w.r.t x

$$\Rightarrow 2x + 2y.\frac{dy}{dx} - 2 = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

Since tangent is parallel to x-axis so $\frac{dy}{dx} = 0 \Rightarrow \frac{1-x}{y} = 0 \Rightarrow x = 1$

from eq(i)
$1 + y^2 - 2 - 3 = 0$
$\Rightarrow y^2 = 4 \Rightarrow y = \pm 2$
So required $(1, \pm 2)$

56. (A) Equation $(b-c)x^2 + (c-a)x + (a-b) = 0$
one root = 1
Let other root = α
A.T.Q,

$$1 + \alpha = \frac{-(c-a)}{b-c}$$

and $1.\alpha = \frac{a-b}{b-c}$

$$\Rightarrow \alpha = \frac{a-b}{b-c}$$

57. (C) Equation $x^2 + 3x + 2 = 0$
Roots are $\alpha = -1$, $\beta = -2$ [$\therefore \alpha > \beta$]

Now, $\begin{bmatrix} 1 & \alpha \\ \beta & \beta \end{bmatrix}\begin{bmatrix} \alpha & \alpha \\ 1 & \beta \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1\times(-1)+(-1)\times 1 & 1\times(-1)+(-1)\times(-2) \\ (-2)\times(-1)+(-2)\times 1 & (-2)\times(-1)+(-2)\times(-2) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 0 & 6 \end{bmatrix}$$

58. (D) Work done $\vec{W} = \vec{F}.(\overrightarrow{AB}) = \vec{F}.(\overrightarrow{OB} - \overrightarrow{OA})$

$\vec{W} = (2\hat{i} + 4\hat{j} + 5\hat{k}).(-5\hat{i} + \hat{j} + 2\hat{k})$

$\vec{W} = 2\times(-5) + 4\times 1 + 5\times 2$

$\vec{W} = -10 + 4 + 10 = 4$ units

59. (B) We know that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

Statement 1
$\Rightarrow 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma = 2$
$\Rightarrow 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 = 2 - 3$
$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
Statement 1 is correct.

Statement 2
from eq(i)
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow \cos^2\alpha + \cos^2\beta = 1 - \cos^2\gamma$
$\Rightarrow \cos^2\alpha + \cos^2\beta = \sin^2\gamma$
Statement 2 is correct.

Statement 3
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
Statement 3 is incorrect.
$\therefore$ Statements 1 and 2 are correct.

60. (B) Ellipse $3x^2 + 4y^2 = 54$
Now, $3(3)^2 + 4(-2)^2$
$\Rightarrow 27 + 16 = 43 < 54$
$\therefore$ point $(3, -2)$ ellipse inside the ellipse but not at the focus.

61. (C) $y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \operatorname{cosec}^{-1}\left(\frac{x+1}{x-1}\right)$

$$\Rightarrow y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$$

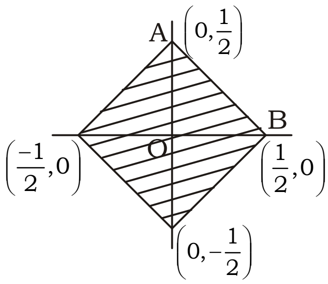
$$\Rightarrow y = \frac{\pi}{2} \qquad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

On differentiating both sides

$$\Rightarrow \frac{dy}{dx} = 0$$

62. (D)

63. (C)



Area of $\Delta AOB = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

The required area = $4 \times \frac{1}{8} = \frac{1}{2}$ sq. unit

64. (C) Given that $f(x) = \frac{2x+x^2}{1+2x^3}$ and $g(x) = \ln\left(\frac{1+x}{1-x}\right)$

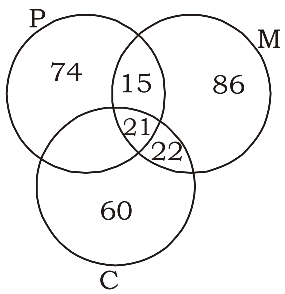
Now, $fog\left(\frac{e-1}{e+1}\right) \Rightarrow f\left[g\left(\frac{e-1}{e+1}\right)\right]$

$\Rightarrow f\left[\ln\left(\frac{1+\frac{e-1}{e+1}}{1-\frac{e-1}{e+1}}\right)\right] \Rightarrow f\,[\ln e]$

$\Rightarrow f(1) = \frac{2\times1+1^2}{1+2\times1^3} = 1$

65. (C) No. of two-digit numbers = 5×4 = 20
No. of three-digit numbers = 5×4×3 = 60
The required numbers = 20 + 60 = 80

(66-68)



Total students = 300

66. (B) No. of students who are good in Physics and Mathematics but not in Chemistry = 15

67. (C) No. of students who are in either Mathematics or Chemistry but not in Physics = 86 + 22 + 60 = 168

68. (C) No. of students who are good in Physics and Chemistry but not in Mathematics
= 300 – (74 + 15 + 86 + 21 + 22 + 60)
= 300 – 278 = 22

69. (B) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left[\frac{(1+x)+(1-x)}{1-(1+x)(1-x)}\right] = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left[\frac{2}{1-(1-x^2)}\right] = \frac{\pi}{6}$

$\Rightarrow \frac{2}{x^2} = \tan\frac{\pi}{6}$

$\Rightarrow \frac{2}{x^2} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}$

70. (C) $(1 + x + x^2 + x^3 + \ldots + \infty)^2$

$\Rightarrow \left(\frac{1}{1-x}\right)^2 = (1-x)^{-2} \quad \left(\because S_\infty = \frac{a}{1-r}\right)$

$\Rightarrow 1 + 2x + 3x^2 + \ldots + (n+1)x^n + \ldots\infty$

Hence coefficient of $x^n = (n+1)$

71. (A) $(998)^{1/3} \Rightarrow (1000-2)^{1/3}$

$\Rightarrow (1000)^{1/3}\left[1-\frac{2}{1000}\right]^{1/3}$

$\Rightarrow 10\left[1-\frac{2}{1000}\right]^{1/3}$

$\Rightarrow 10\left[1-\frac{1}{3(500)}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{500}\right)^2+\ldots\right]$

$\Rightarrow 10\left[1-\frac{1}{1500}-\frac{1}{9\times250000}\right]$

$\Rightarrow 10\left[\frac{2250000-1500-1}{2250000}\right]$

$\Rightarrow \frac{22484990}{2250000} = 9.99$

72. (C) $\because r_{xy} = \frac{\text{cov}(x,y)}{\sigma_x.\sigma_y} \Rightarrow 0.6 = \frac{16}{4.\sigma_y}$

$\Rightarrow \sigma_y = \frac{16}{4\times0.6} = \frac{20}{3}$

73. (B) Equation is $ax^2 - 12x + 15 = 0$
One root is $2 + i$, then other root is $2 - i$.

Now, $2 + i + 2 - i = \frac{12}{a}$

$\Rightarrow 4 = \frac{12}{a} \Rightarrow a = 3$

74. (A) If a, b, c, d are in HP, then

$b = \frac{2ac}{a+c}$ and $c = \frac{2bd}{b+d}$

Now, $bc = \frac{4abcd}{(a+c)(b+d)}$

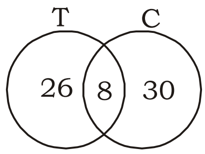
$\Rightarrow bc = \frac{4abcd}{ab+ad+bc+cd}$

$\Rightarrow ab + ad + bc + cd = 4ad$

$\Rightarrow ab + bc + cd = 3ad$

75. (A) The shaded region is $(A \cap B) \cup (A \cap C)$

76. (A)



$n(T \cup C) = 64$, $n(T - C) = 26$, $n(T) = 34$

Now, $n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow 34 = 26 + n(T \cap C) \Rightarrow n(T \cap C) = 8$

Again, we have

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$\Rightarrow 64 = 34 + n(C) - 8$

$\Rightarrow 64 = 26 + n(C) \Rightarrow n(C) = 38$

Now, $n(C) = n(C - T) + n(T \cap C)$

$\Rightarrow 38 = n(C - T) + 8 \Rightarrow n(C - T) = 30$

77. (C) $f(x) = x^3 + 3x^2 - 4$

$f'(x) = 3x^2 + 6x$

For increasing function $f'(x) > 0$

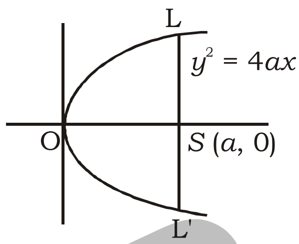
$\Rightarrow 3x^2 + 6x > 0$

$\Rightarrow 3x(x + 2) > 0$

$\Rightarrow x < -2$ or $x > 0$

So, $f(x)$ is increasing at $x > 0$ or $x < -2$.

78. (B)



Required area = Area LOL'

Area = 2 × (Area of LOS)

Area = $2 \times \int_0^a y\,dx$

Area = $2 \times \int_0^a \sqrt{4ax}\,dx$

Area = $2 \times 2\sqrt{a} \int_0^a \sqrt{x}\,dx$

Area = $4\sqrt{a} \left(\frac{x^{3/2}}{3/2}\right)_0^a$

Area = $4\sqrt{a} \times \frac{2}{3}[a^{3/2} - 0]$

Area = $\frac{8}{3}\sqrt{a} \times (a)^{3/2} = \frac{8}{3}a^2$

79. (B) $\sin^{-1}\cos(\sin^{-1}x) + \cos^{-1}\sin(\cos^{-1}x)$

$\Rightarrow \sin^{-1}\cos\{\cos^{-1}\sqrt{1-x^2}\} + \cos^{-1}\sin\{\sin^{-1}\sqrt{1-x^2}\}$

$\Rightarrow \sin^{-1}\sqrt{1-x^2} + \cos^{-1}\sqrt{1-x^2} = \frac{\pi}{2}$

$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

80. (C) In the parabola $y^2 = 4ax$, the smallest focal chord is $4a$.

81. (B) $|\vec{a} \times \vec{b}| - \sqrt{3}\,|\vec{a}\cdot\vec{b}| = 0$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta - \sqrt{3}|\vec{a}||\vec{b}|\cos\theta = 0$

$\Rightarrow |\vec{a}||\vec{b}|[\sin\theta - \sqrt{3}\cos\theta] = 0$

$\Rightarrow |\vec{a}||\vec{b}| \neq 0$, so $\sin\theta = \sqrt{3}\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

82. (B) $f(x) = \begin{cases} x+2, & \text{when } x \le 1 \\ 4x-1, & \text{when } x > 1 \end{cases}$

L.H.L. $= \lim_{x\to1^-} f(x) = \lim_{h\to0} f(1-h)$

$= \lim_{h\to0}(1 - h + 2)$

$= 3 - h = 3$

R.H.L. $= \lim_{x\to1^+} f(x) = \lim_{h\to0} f(1+h)$

$= \lim_{h\to0} 4(1+h) - 1$

$= 3$

So, $\lim_{x\to1} f(x) = 3$

83. (D) $y = f(x) = \left(\frac{1}{x}\right)^{2x}$ ...(i)

On taking log

$\Rightarrow \log y = 2x.\log\left(\frac{1}{x}\right)$

$\Rightarrow \log y = -2x.\log x$

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -2\left[x \times \frac{1}{x} + \log x \times 1\right]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -2[1 + \log x]$

$\Rightarrow \frac{dy}{dx} = -2y(1 + \log x) = -2\left(\frac{1}{x}\right)^{2x}[1 + \log x]$

Again, differentiating

$\frac{d^2y}{dx^2} = -2\left[\frac{dy}{dx}(1 + \log x) + y \times \frac{1}{x}\right]$

$\frac{d^2y}{dx^2} = -2\left[-2\left(\frac{1}{x}\right)^{2x}(1 + \log x)^2 + \left(\frac{1}{x}\right)^{2x} \times \frac{1}{x}\right]$

for maxima and minima

$\frac{dy}{dx} = 0$

$\Rightarrow -2\left(\frac{1}{x}\right)^{2x}[1 + \log x] = 0$

$\Rightarrow 1 + \log x = 0 \Rightarrow x = \frac{1}{e}$

Now, $\frac{d^2y}{dx^2}\left(\text{at } x = \frac{1}{e}\right)$

$\Rightarrow -2e^{2/e}\left[-2\left(1+\log\frac{1}{e}\right)^2 + e\right]$

$\Rightarrow -2e^{2/e}[-2(1 - \log e)^2 + e]$

$\Rightarrow -2e \times e^{2/e}$ (maxima)

Maximum value $= f(1/e) = e^{2/e}$

84. (A) Word "MOTHER"

The required arrangements $= {}^5C_3 \times 4!$

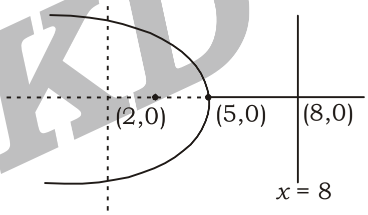
$= \frac{5!}{2!3!} \times 4! = \frac{5 \times 4 \times 24}{2} = 240$

85. (A) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin x}{1+\cos x}\, dx = 0$

We know that

$$\int_{-a}^{a} f(x)\, dx = \begin{cases} 0, \text{function is odd} \\ 2\int_0^a f(x)\, dx, \text{function is even} \end{cases}$$

86. (B)



equation of directix

$x = 8$

87. (C) Direction ratios of lines are (–1, 2,–4) and (–2, x, –3).

A.T.Q.,

$\cos\frac{\pi}{2} = \frac{-1\times(-2)+2\times x+(-4)\times(-3)}{\sqrt{(-1)^2+2^2+(-4)^2}\sqrt{(-2)^2+x^2+(-3)^2}}$

$\Rightarrow 0 = \frac{2+2x+12}{\sqrt{21}\sqrt{x^2+13}}$

$\Rightarrow 0 = 2x + 14 \Rightarrow x = -7$

88. (B) $y = \tan^{-1}\left[\frac{x^{1/2}(x^{1/2}-1)}{1+x^{3/2}}\right]$

$y = \tan^{-1}\left[\frac{x - x^{1/2}}{1 + x.x^{1/2}}\right]$

Let $x = \tan A$ and $x^{1/2} = \tan B$

$y = \tan^{-1}\left[\frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$

$y = \tan^{-1}[\tan(A - B)]$

$y = A - B$

$y = \tan^{-1}x - \tan^{-1}(x^{1/2})$

On differetiating both side w.r.t. 'x'

$\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{1+x} \times \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{2\sqrt{x}(1+x)}$

89. (A) We know that

A.M. $\geq$ G.M. $\geq$ H.M.

$\Rightarrow \frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$

$\Rightarrow \frac{2ab}{a+b} \leq \sqrt{ab} \leq \frac{a+b}{2}$

90. (C) A.T.Q.,

Mean $= \frac{a^{n-9}+b^{n-9}}{a^{n-10}+b^{n-10}}$

$\Rightarrow \frac{a+b}{2} = \frac{a^{n-9}+b^{n-9}}{a^{n-10}+b^{n-10}}$

On comparing

$n - 9 = 1 \Rightarrow n = 10$

91. (B) The required remainder = 4

92. (D) S = 3 + 6 + 9 + ...+ 99

S = 3(1 + 2 + 3 + ...+ 33)

$S = 3 \times \frac{33 \times 34}{2}$

S = 33 × 51 = 1683

93. (C) Let $a + ib = \sqrt{1+2\sqrt{2}i}$

On squaring both sides

$\Rightarrow (a^2 - b^2)^2 + 2abi = 1 + 2\sqrt{2}\, i$

On comparing

$\Rightarrow a^2 - b^2 = 1$ and $2ab = 2\sqrt{2}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 1 + 8$

$\Rightarrow a^2 + b^2 = 3$ ...(ii)

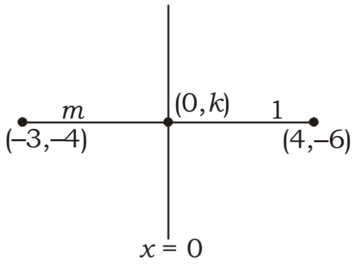
from eq(i) and eq(ii)

$\Rightarrow 2a^2 = 4$ and $2b^2 = 2$

$\Rightarrow a = \pm\sqrt{2}$, $b = \pm 1$

Hence $\sqrt{1+2\sqrt{2}i} = \pm\left(\sqrt{2}+i\right)$

94. (B)



Let the $x = 0$ divides the line joining the points (3, –4) and (4, –6) in the ratio $m : 1$,

then $\frac{4m-3}{m+1} = 0 \Rightarrow m = \frac{3}{4}$

The required ratio = 3 : 4

95. (C) $(3x + 4y - 5) + \lambda(5x - y + 11) = 0$

$\Rightarrow (3 + 5\lambda)x + (4 - \lambda)y - 5 + 11\lambda = 0$

$\Rightarrow y = \frac{-(3+5\lambda)}{(4-\lambda)}x + \frac{5-11\lambda}{4-\lambda}$

Slope $m = \frac{-(3+5\lambda)}{4-\lambda}$

Given straight line parallel to x-axis i.e $\theta = 0 \Rightarrow m = 0$

then $\frac{-(3+5\lambda)}{4-\lambda} = 0 \Rightarrow \lambda = \frac{-3}{5}$

96. (A) $A = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 20 & 24 \\ 24 & 32 \end{bmatrix}$

From option A

$A^2 - 6A - 8I = \begin{bmatrix} 20 & 24 \\ 24 & 32 \end{bmatrix} - 6\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 - 6A - 8I = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$A^2 - 6A - 8I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 - 6A - 8I = 0$

97. (D) $\begin{vmatrix} 1 & \omega & 3\omega^2 \\ 3 & 3\omega^2 & 9\omega^3 \\ 2 & 2\omega^3 & 6\omega^4 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & \omega & 3\omega^2 \\ 3 & 3\omega^2 & 9 \\ 2 & 2 & 6\omega \end{vmatrix}$

$\Rightarrow 1(18\omega^3 - 18) - \omega(18\omega - 18) + 3\omega^2(6 - 6\omega^2)$

$\Rightarrow 1(18 - 18) - 18\omega^2 + 18\omega + 18\omega^2 - 18\omega^4$

$\Rightarrow 0 + 18\omega - 18\omega = 0$

98. (B) $z = 1 + \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$

$z = 2\cos^2\frac{\pi}{24} + i \times 2\sin\frac{\pi}{24} \times \cos\frac{\pi}{24}$

$z = 2\cos\frac{\pi}{24}\left[\cos\frac{\pi}{24} + i\sin\frac{\pi}{24}\right]$

Hence $|z| = 2\cos\frac{\pi}{24}$

99. (C) $\left[\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^3 \Rightarrow \left[\frac{(1+\sqrt{3}i)(1+\sqrt{3}i)}{(1-\sqrt{3}i)(1+\sqrt{3}i)}\right]^3$

$\Rightarrow \left[\frac{1+3i^2+2\sqrt{3}i}{1-3i^2}\right]^3 \Rightarrow \left[\frac{-2+2\sqrt{3}i}{4}\right]^3$

$\Rightarrow \left[\frac{-1+\sqrt{3}i}{2}\right]^3 \Rightarrow \omega^3 = 1$

100. (B) Sum of n terms

$S_n = n^2 + 3n$ ...(i)

and $S_{n-1} = (n-1)^2 + 3(n-1)$

$\Rightarrow S_{n-1} = n^2 + n - 2$ ...(ii)

n^{th} term of the series

$T_n = S_n - S_{n-1}$

$\Rightarrow T_n = (n^2 + 3n) - (n^2 + n - 2)$

$\Rightarrow T_n = 2n + 2$

101. (B) $\sqrt{(4-\sqrt{5})} \Rightarrow \sqrt{\frac{8-2\sqrt{15}}{2}} \Rightarrow \sqrt{\frac{(\sqrt{5}-\sqrt{3})^2}{2}}$

$\Rightarrow \frac{\sqrt{5}-\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \frac{\sqrt{10}-\sqrt{6}}{2}$

102. (D) $\lim_{x\to 0}\frac{\sin x + \cos x - 1}{\tan x}$ $\left[\frac{0}{0}\right]$ from by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{\cos x - \sin x}{\sec^2 x} \Rightarrow \frac{\cos 0 - \sin 0}{\sec^2 0}$

$\Rightarrow \frac{1-0}{1} = 1$

103. (C) Ratio of angles = 8 : 5 : 2

Let Angles = $8x, 5x, 2x$

Now, $8x + 5x + 2x = 180$

$\Rightarrow 15x = 180 \Rightarrow x = 12$

Angles = 96, 60, 24

Now, cos96 + cos60 + cos24

$\Rightarrow$ cos96 + cos24 + cos60

$\Rightarrow 2\cos\frac{96+24}{2}.\cos\frac{96-24}{2} + \frac{1}{2}$

$\Rightarrow 2\cos 60.\cos 36 + \frac{1}{2}$

$\Rightarrow 2 \times \frac{1}{2}\cos 36 + \frac{1}{2}$

$\Rightarrow \frac{\sqrt{5}+1}{4} + \frac{1}{2} = \frac{\sqrt{5}+3}{4}$

104. (C) $\frac{\cot\theta}{1+\sin\theta}-\frac{\tan\theta}{1+\cos\theta}$

$\Rightarrow \frac{\cot\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}-\frac{\tan\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\Rightarrow \frac{\cos\theta(1-\sin\theta)}{\sin\theta\times\cos^2\theta}-\frac{\sin\theta(1-\cos\theta)}{\cos\theta\times\sin^2\theta}$

$\Rightarrow \frac{1-\sin\theta}{\sin\theta.\cos\theta}-\frac{1-\cos\theta}{\sin\theta.\cos^2\theta}$

$\Rightarrow \frac{1-\sin\theta-1+\cos\theta}{\sin\theta.\cos\theta}$

$\Rightarrow \frac{\cos\theta-\sin\theta}{\sin\theta.\cos\theta}$ = cosecθ – secθ

105. (C) Equations $2x + y + 2z = 4$, $4x + y + 2z = 6$ and $5x - 3y - z = 11$

Using elementary method

$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 4 & 1 & 2 & 6 \\ 5 & -3 & -1 & 11 \end{array}\right]$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - \frac{5}{2}R_1$

$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & \frac{-11}{2} & -6 & 1 \end{array}\right]$

$R_3 \to 2R_3$

$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & -11 & -12 & 2 \end{array}\right]$

$R_3 \to R_3 - 11R_2$

$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 10 & 24 \end{array}\right]$

Rank A = 3 and Rank [A/B] = 3

Hence solution is consistent with an unique solution.

106. (B) Number of elements in set B = 4

Number of subsets of a set B = 2^4 = 16

Number of subsets of set A = 16 + 48 = 64 = 2^6

Hence no. of elements in set A = 6

107. (A) **Statement I**

In a leap year = 366 days = 52 weeks and 2 days

The probability = $\frac{2}{7}$

In a normal year = 365 days = 52 weeks and 1 days

The probability = $\frac{1}{7}$

Statement I is correct.

Statement II

In month of October = 31 days = 28 + 3

The probability = $\frac{3}{7}$

In month of September = 30 days = 28 + 2

The probability = $\frac{2}{7}$

Statement II is incorrect.

108. (C) $4\sin x.\sin\left(\frac{\pi}{3}+x\right).\sin\left(\frac{\pi}{3}-x\right)$

$\Rightarrow 2\sin x.\left[2\sin\left(\frac{\pi}{3}+x\right).\sin\left(\frac{\pi}{3}-x\right)\right]$

$\Rightarrow 2\sin x\left[\cos\left(\frac{\pi}{3}+x-\frac{\pi}{3}+x\right).\cos\left(\frac{\pi}{3}+x+\frac{\pi}{3}-x\right)\right]$

$\Rightarrow 2\sin x\left[\cos 2x-\cos\frac{2\pi}{3}\right]$

$\Rightarrow 2\sin x.\cos 2x - 2\sin x.\cos\frac{2\pi}{3}$

$\Rightarrow \sin(x + 2x) + \sin(x - 2x) - 2\sin x\left(\frac{-1}{2}\right)$

$\Rightarrow \sin 3x - \sin x + \sin x = \sin 3x$

109. (D) $I = \int \frac{\sin x}{\cos(x+a)}\,dx$

Let $x + a = t \Rightarrow x = t - a \Rightarrow dx = dt$

$I = \int \frac{\sin(t-a)}{\cos t}\,dt$

$I = \int \frac{\sin t.\cos a - \cos t.\sin a}{\cos t}\,dt$

$I = \cos a\int \tan t\,dt - \sin a\int 1\,dt$

$I = \cos a.\log\sec(x + a) - \sin a.(x + a) + c$

$I = \cos a.\ \log\sec(x + a) - x\sin a - a.\sin a + c$

$I = \cos a.\log\sec(x + a) - x\sin a + c$

110. (A) $A = \{x \in R : x^2 + 4x + 3 < 0\}$

$A = \{x \in R : -3 < x < -1\}$

$B = \{x \in R : x^2 - 7x + 12 > 0\}$

$B = \{x \in R : -\infty < x < 3 \text{ and } 4 < x < \infty\}$

Statement 1

$A \cap B = \{x \in R : -3 < x < -1\}$

Statement 1 is correct.

Statement 2

$A - B = \{\phi\}$

Statement 2 is incorrect.

111. (B) $\sqrt{\dfrac{\omega}{1+\omega^2}} \Rightarrow \sqrt{\dfrac{\omega}{-\omega}}$ $[\because 1+\omega+\omega^2 = 0]$

$\Rightarrow \sqrt{\dfrac{1}{-1}} = \sqrt{-1} = i$

112. (D) We know that

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

On putting $A = 22\frac{1}{2}$

$\Rightarrow \tan 45 = \dfrac{2\tan 22\frac{1}{2}}{1-\tan^2 22\frac{1}{2}}$

$\Rightarrow 1 = \dfrac{2\tan 22\frac{1}{2}}{1-\tan^2 22\frac{1}{2}}$

$\Rightarrow \tan^2 22\frac{1}{2} + 2\tan 22\frac{1}{2} - 1 = 0$

$\Rightarrow \tan 22\frac{1}{2} = \dfrac{-2 \pm \sqrt{(-2)^2 - 4\times 1\times(-1)}}{2\times 1}$

$\Rightarrow \tan 22\frac{1}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$\Rightarrow \tan 22\frac{1}{2} = -1 \pm \sqrt{2}$

Hence $\tan 22\frac{1}{2} = \sqrt{2} - 1$

113. (B) $\dfrac{dy}{dx} + y.\tan x = \sec x$

On comparing with general equation

$P = \tan x$ and $Q = \sec x$

$\text{I.F.} = e^{\int P\,dx}$

$\text{I.F.} = e^{\int \tan x\,dx}$

$\text{I.F.} = e^{\log \sec x} = \sec x$

Solution of the differential equation

$y \times \text{I.F.} = Q \times \text{I.F.}\ dx$

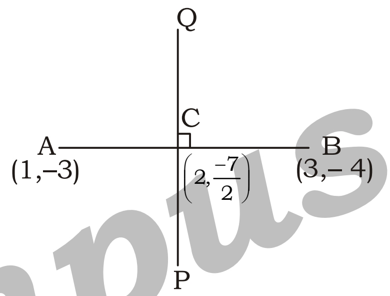
$\Rightarrow y \times \sec x = \int \sec x.\sec x\,dx$

$\Rightarrow y \sec x = \tan x + c$

$\Rightarrow \dfrac{y}{\cos x} = \dfrac{\sin x}{\cos x} + c$

$\Rightarrow y = \sin x + c.\cos x$

114. (C)



Mid-point of line joining the points

$= \left(\dfrac{1+3}{2}, \dfrac{-3-4}{2}\right) = \left(2, \dfrac{-7}{2}\right)$

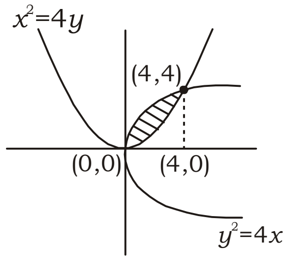
Slope of line AB $(m_1) = \dfrac{-4+3}{3-1} = \dfrac{-1}{2}$

Slope of line PQ $(m_2) = \dfrac{-1}{\frac{-1}{2}} = 2$

equation of line PQ

$y + \dfrac{7}{2} = 2(x-2) \Rightarrow 4x - 2y = 11$

115. (C)



Curve

$y_1 \Rightarrow y = 2\sqrt{x}$, $y_2 \Rightarrow y = \dfrac{x^2}{4}$

The required Area (A) = $\int_0^4 (y_1 - y_2)dx$

$\Rightarrow \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$

$\Rightarrow \left[2 \times \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{4 \times 3}\right]_0^4$

$\Rightarrow \left[\frac{4}{3} \times (4)^{3/2} - \frac{1}{12}(4)^3 - 0 - 0\right]$

$\Rightarrow \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ sq. unit

116. (A) $\int \frac{1}{\sqrt{1-\sin x}}\, dx \Rightarrow \frac{1}{\sqrt{1-\cos\left(\frac{\pi}{2} - x\right)}}\, dx$

$\Rightarrow \frac{1}{\sqrt{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}}\, dx$

$\Rightarrow \frac{1}{\sqrt{2}} \int \text{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) dx$

$\Rightarrow \frac{1}{\sqrt{2}} \frac{\log\left|\text{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) - \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|}{\frac{-1}{2}} + c$

$\Rightarrow \sqrt{2} \log\left|\text{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) + \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right| + c$

117. (B)

Class	x	f	$f \times x$	$d = \|x - A\|$	$f \times d$
0-10	5	11	55	30	330
10-20	15	14	210	20	250
20-30	25	15	375	5	75
30-40	35	16	560	5	80
40-50	45	12	540	20	240
50-60	55	32	1760	30	960
		$\sum f = 100$	$\sum f \times x = 3500$		$\sum f \times d = 1965$

Mean A = $\frac{\sum f \times x}{\sum f} \Rightarrow \frac{3500}{100} = 35$

Mean-Deviation = $\frac{\sum f \times d}{\sum f}$

$\Rightarrow \frac{1965}{100} = 19.65$

118. (A) In the expansion of $\left(4\sqrt{x} + \frac{1}{2x}\right)^9$

$T_{r+1} = {}^9C_r\, (4\sqrt{x})^{9-r}\left(\frac{1}{2x}\right)^r$

$T_{r+1} = {}^9C_r 2^{18-3r}\, x^{\frac{9-3r}{2}}$

Here, $\frac{9-3r}{2} = 0 \Rightarrow r = 3$

The value of constant term = ${}^9C_3 \times 2^9$

119. (C) $\int \frac{2^x}{\sqrt{4^x - 1}}\, dx \Rightarrow \int \frac{2^x}{\sqrt{(2^x)^2 - 1}}\, dx$

Let $2^x = t$

$2^x \log 2\, dx = dt \Rightarrow 2^x\, dx = \frac{1}{\log 2}\, dt$

$\Rightarrow \frac{1}{\log 2} \int \frac{dt}{\sqrt{t^2 - 1}}$

$\Rightarrow \frac{1}{\log 2} \log\left[t + \sqrt{t^2 - 1}\right] + c$

$\Rightarrow \frac{1}{\log 2} \log\left[2^x + \sqrt{4^x - 1}\right] + c$

120. (B) **Statement I**

$\tan^{-1}1 + \tan^{-1}(2 + \sqrt{3}) = \tan^{-1}\left(\frac{1 + 2 + \sqrt{3}}{1 - 1(2 + \sqrt{3})}\right)$

$\Rightarrow \tan^{-1}1 + \tan^{-1}(2 + \sqrt{3}) = \tan^{-1}\left(\frac{3 + \sqrt{3}}{-1 - \sqrt{3}}\right)$

$\Rightarrow \tan^{-1}1 + \tan^{-1}(2 + \sqrt{3}) = \tan^{-1}\left(\frac{\sqrt{3}(\sqrt{3} + 1)}{-1(\sqrt{3} + 1)}\right)$

$\Rightarrow \tan^{-1}1 + \tan^{-1}(2 + \sqrt{3}) = \tan^{-1}(-\sqrt{3})$

Statement I is incorrect.

Statement II

$\sin^{-1}\frac{7}{25} + \sin^{-1}\frac{24}{25} = \sin^{-1}\frac{7}{25} + \cos^{-1}\frac{7}{25}$

$\Rightarrow \sin^{-1}\frac{7}{25} + \sin^{-1}\frac{24}{25} = \frac{\pi}{2}$

Statement II is correct.

NDA (MATHS) MOCK TEST - 186 (Answer Key)

1.	(D)	21.	(C)	41.	(A)	61.	(C)	81.	(B)	101.	(B)
2.	(B)	22.	(B)	42.	(B)	62.	(D)	82.	(B)	102.	(D)
3.	(D)	23.	(C)	43.	(A)	63.	(C)	83.	(D)	103.	(C)
4.	(D)	24.	(A)	44.	(B)	64.	(C)	84.	(A)	104.	(C)
5.	(C)	25.	(A)	45.	(D)	65.	(C)	85.	(A)	105.	(C)
6.	(B)	26.	(C)	46.	(A)	66.	(B)	86.	(B)	106.	(B)
7.	(A)	27.	(D)	47.	(C)	67.	(C)	87.	(C)	107.	(A)
8.	(A)	28.	(C)	48.	(B)	68.	(C)	88.	(B)	108.	(C)
9.	(A)	29.	(D)	49.	(C)	69.	(B)	89.	(A)	109.	(D)
10.	(A)	30.	(D)	50.	(C)	70.	(C)	90.	(C)	110.	(A)
11.	(B)	31.	(D)	51.	(B)	71.	(A)	91.	(B)	111.	(B)
12.	(C)	32.	(C)	52.	(A)	72.	(C)	92.	(D)	112.	(D)
13.	(C)	33.	(D)	53.	(A)	73.	(B)	93.	(C)	113.	(B)
14.	(B)	34.	(A)	54.	(B)	74.	(A)	94.	(B)	114.	(C)
15.	(C)	35.	(B)	55.	(A)	75.	(A)	95.	(C)	115.	(C)
16.	(C)	36.	(C)	56.	(A)	76.	(A)	96.	(A)	116.	(A)
17.	(D)	37.	(B)	57.	(C)	77.	(C)	97.	(D)	117.	(B)
18.	(B)	38.	(A)	58.	(D)	78.	(B)	98.	(B)	118.	(A)
19.	(B)	39.	(A)	59.	(B)	79.	(B)	99.	(C)	119.	(C)
20.	(B)	40.	(A)	60.	(B)	80.	(C)	100.	(B)	120.	(B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777