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NDA MATHS MOCK TEST - 108 (SOLUTION)

1. (A) $\Rightarrow (35x-38)^5 = {}^5C_0(35x)^5 + {}^5C_1(35x)^4.38 + {}^5C_2(35x)^3.38^2 + \ldots\ldots\ldots\ldots + {}^5C_5 38^5$

$\Rightarrow (35x-38)^5 = {}^5C_0 35^5 + {}^5C_2 35^4.38^1 + {}^5C_3 35^4.38^1\ldots\ldots\ldots {}^5C_5 38^5$

$\Rightarrow (-3)^5$ = sum of the coeffi. of all terms

$\Rightarrow$ sum of the coefficients of all terms $= -243$

2. (B) given that A.M. $= \frac{a+b}{2} = 14 \Rightarrow a+b = 28$

G.M. $= \sqrt{ab} = 7 \Rightarrow ab = 49$

Now, H.M. $= \frac{2ab}{a+b}$

H.M. $= \frac{2\times 49}{28} = \frac{7}{2}$

3. (C) I $= \int \cot^{-1}(\operatorname{cosec}2x - \cot 2x)$

I $= \int \cot^{-1}\left(\frac{1-\cos 2x}{\sin 2x}\right)dx$

I $= \int \cot^{-1}(\tan x)$

I $= \int \cot^{-1}\left[\cot\left(\frac{\pi}{2}-x\right)\right]dx$

I $= \int \left(\frac{\pi}{2}-x\right)dx$

I $= \frac{\pi}{2}x - \frac{x^2}{2} + c$

4. (D) given that $y = \omega - \omega^2 + 3$

$\Rightarrow y - 3 = \omega - \omega^2$

$\Rightarrow (y-3)^2 = (\omega - \omega^2)^2$

$\Rightarrow y^2 + 9 - 6y = \omega^2 + \omega^4 - 3\omega^3$

$\Rightarrow y^2 - 6y + 9 = \omega^2 + \omega - 3$

$\Rightarrow y^2 - 6y + 9 = -1 - 3$

$\Rightarrow y^2 - 6y + 11 = -13 + 11$

$\Rightarrow y^2 - 6y + 11 = -2$

5. (B) 110 .11

$0\times 2^0 = 0$ $\quad \frac{1}{2} = 1\times 2^{-1}$

$1\times 2^1 = 2$

$1\times 2^2 = 4$ $\quad \frac{1}{4} = 1\times 2^{-2}$

$\underline{6}$ $\quad \frac{1}{2}+\frac{1}{4} = \frac{3}{4} = 0.75$

$(110)_2 = (6)_{10}$, $\quad (.11)_2 = (0.75)_{10}$

Hence $(110.11)_2 = (6.75)_{10}$

6. (C) A.T.Q

$a + ar = 16$

$a(1+r) = 16$ ....(ii)

and $a + ar + ar^2 + ar^3 = 160$

$16 + ar^2 + ar^3 = 160$ from eq (i)

$ar^2(1+r) = 144$ (ii)

from eq (i) and eq (ii)—

$\frac{a(1+r)}{ar^2(1+r)} = \frac{16}{144}$

$\frac{1}{r^2} = \frac{1}{9} \Rightarrow r = 3$

from eq (ii)—

$a(1+3) = 16 \Rightarrow a = 4$

Now, $T_7 = ar^{n-1}$

$T_7 = 4\times(3)^{7-1} = 2916$

7. (C) $\sqrt{\frac{2+\sqrt{5}i}{2-\sqrt{5}i}} = \sqrt{\frac{(2+\sqrt{5}i)(2+\sqrt{5}i)}{(2-\sqrt{5}i)(2+\sqrt{5}i)}}$

$\sqrt{\frac{2+\sqrt{5}i}{2-\sqrt{5}i}} = \frac{2+\sqrt{5}i}{4-5i^2} = \frac{2+\sqrt{5}i}{9}$

8. (A) A.M $\geq$ G.M $\geq$ H.M.

9. (B) $\lim_{x\to\pi/4} \frac{(1-\tan x)(1+\sin 2x)}{(1+\tan x)(\pi-4x)} \left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to\pi/4} \frac{(1-\tan x)(2\cos 2x)+(1+\sin 2x)(-\sec^2 x)}{(1+\tan x)(-4)+(\pi-4x)(\sec^2 x)}$

$\Rightarrow \frac{\left(1-\tan\frac{\pi}{4}\right)\left(2\cos\frac{\pi}{2}\right)+\left(1+\sin\frac{\pi}{2}\right)\left(-\sec^2\frac{\pi}{4}\right)}{\left(1+\tan\frac{\pi}{4}\right)(-4)+(\pi-\pi)\sec^2\frac{\pi}{4}}$

$\Rightarrow \frac{0+2(-2)}{2(-4)+0} = \frac{-4}{-8} = \frac{1}{2}$

10. (D) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega^4 \\ \omega^7 & \omega^8 & 1 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & 1 & \omega \\ 1+\omega+\omega^2 & \omega^2 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & 1 & \omega \\ 0 & \omega^2 & 1 \end{vmatrix} = 0$$

11. (C) $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$

$y = \cos x$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = -\sin x$$

12. (A) given that $f(x) = x^2 - 2x + 2$

$a = 2,\ b = \frac{5}{2} \Rightarrow f(a) = 2,\ f(b) = \frac{13}{4}$

$f'(x) = 2x - 2$

$f'(c) = 2c - 2$

Now, $f'(c) = \frac{f(b) - f(a)}{b - a}$

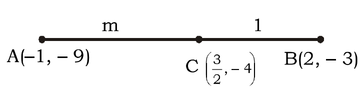
$$2c - 2 = \frac{\frac{13}{4} - 2}{\frac{5}{2} - 2}$$

$$2c - 2 = \frac{\frac{5}{4}}{\frac{1}{2}}$$

$$2c - 2 = \frac{5}{2}$$

$$2c = \frac{9}{2} \Rightarrow c = \frac{9}{4}$$

13. (D) Let ratio = m : 1



$\frac{m \times 2 - 1}{m+1} = \frac{3}{2}$, $\quad \frac{-3m - 9}{m+1} = -4$

$4m - 2 = 3m + 3$, $\quad -3m - 9 = -4m - 4$

$m = 5$, $\quad m = 5$

Hence ratio = 5 : 1

14. (B) $i^{n+3} + i^{n+4} + i^{n+5} + i^{n+6} + i^{n+7}$

$\Rightarrow i^{n+3}(1 + i + i^2 + i^3 + i^4)$

$\Rightarrow i^{n+3}(1 + i - 1 - i + 1) = i^{n+3}$

15. (B) Given that

$S_n = 4n^2 - 3$

$S_{n-1} = 4(n-1)^2 - 3$

$S_{n-1} = 4n^2 - 8n + 1$

Now, $T_n = S_n - S_{n-1}$

$T_n = 4n^2 - 3 - 4n^2 + 8n - 1$

$T_n = 8n - 4$

$T_8 = 8 \times 8 - 4 = 60$

16. (D)

17. (D) A × (B + C) = A × B + A × C

= A × B – C × A

18. (A) Sphere $x^2 + y^2 + z^2 - 16x + 18y - 24z + 33 = 0$

$u = -8,\ v = 9,\ z = -12,\ d = 33$

radius of sphere = $\sqrt{u^2 + v^2 + z^2 - d}$

$r = \sqrt{64 + 81 + 144 - 33}$

$r = \sqrt{256} = 16$

Diameter of sphere = $2r = 2 \times 16 = 32$ unit

19. (A) $z = 1 - \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$$z = 2\sin^2\frac{\pi}{6} - i.\ 2\sin\frac{\pi}{6}.\cos\frac{\pi}{6}$$

$$z = 2\sin\frac{\pi}{6}\left[\sin\frac{\pi}{6} - i\cos\frac{\pi}{6}\right]$$

$$z = 2 \times \frac{1}{2}\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$

$$z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

Now, $\arg(z) = \tan^{-1}\left(\frac{-\sqrt{3}/2}{1/2}\right)$

$= \tan^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

$= \tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right] = \frac{-\pi}{3}$

20. (C) $y = e^{ln(x.\sin x^2)}$ and $z = x^2 \Rightarrow \frac{dz}{dx} = 2x$

$y = x.\sin x^2$

$\frac{dy}{dx} = x.\cos x^2.(2x) + \sin x^2$

$\frac{dy}{dx} = 2x^2.\cos x^2 + \sin x^2$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\frac{dy}{dz} = \frac{2x^2\cos x^2 + \sin x^2}{2x}$

$\frac{dy}{dz} = x\cos x^2 + \frac{1}{2}\frac{\sin x^2}{x}$

21. (B) $I = \int_0^5 |x-3|\,dx$

$I = \int_0^3 -(x-3)\,dx + \int_3^5 (x-3)\,dx$

$I = -\left[\frac{x^2}{2} - 3x\right]_0^3 + \left[\frac{x^2}{2} - 3x\right]_3^5$

$I = -\frac{3}{2} + \left[\frac{25}{2} - 15 - \frac{9}{2} + 9\right]$

$I = \frac{-9}{2} + 9 + \frac{25}{2} - 15 - \frac{9}{2} + 9$

$I = 3 + \frac{7}{2} = \frac{13}{2}$

22. (C) $\vec{a} = \hat{i} - 4\hat{j} + 2\hat{k}$, and $\vec{b} = 8\hat{i} - 9\hat{j} + 12\hat{k}$

projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{8+36+24}{\sqrt{64+81+144}}$

$= \frac{68}{17} = 4$

23. (B) $I = \int_0^{2\pi} \frac{\tan\frac{x}{4}}{\tan\frac{x}{4} + \cot\frac{x}{4}}\,dx$ ....(i)

$I = \int_0^{2\pi} \frac{\tan\frac{2\pi - x}{4}}{\tan\frac{2\pi-x}{4} + \cot\frac{2\pi-x}{4}}\,dx$

$I = \int_0^{2\pi} \frac{\cot\frac{x}{4}}{\cot\frac{x}{4} + \tan\frac{x}{4}}\,dx$ ....(ii)

from eq. (i) and eq (ii)

$2I = \int_0^{2\pi} \frac{\tan\frac{x}{4} + \cot\frac{x}{4}}{\tan\frac{x}{4} + \cot\frac{x}{4}}\,dx$

$2I = \int_0^{2\pi} 1.dx \Rightarrow 2I = [x]_0^{2\pi}$

$\Rightarrow 2I = 2\pi \Rightarrow I = \pi$

24. (B) two circles

$x^2 + y^2 + 2x + 4y + 7 = 0$ and

$x^2 + y^2 + 6x - 4y + \lambda = 0$

condition of orthogonality

$2gg' + 2ff' = c + c'$

$2 \times 1 \times 3 + 2\times2 \times (-2) = 7 + \lambda$

$6 - 8 = 7 + \lambda \Rightarrow \lambda = -9$

25. (C) Let angel of a triangle = x, $2x$, x

Now, $x + 2x + x = 180$

$4x = 180 \Rightarrow x = 45°$

angles = 45°, 90°, 45°

By Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 45°} = \frac{b}{\sin 90°} = \frac{c}{\sin 45°}$

$\frac{a \times \sqrt{2}}{1} = \frac{b}{1} = \frac{c \times \sqrt{2}}{1} \Rightarrow \frac{a}{1} = \frac{b}{\sqrt{2}} = \frac{c}{1}$

Hence $a : b : c = 1 : \sqrt{2} : 1$

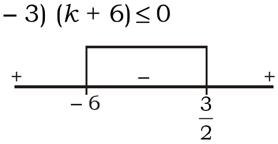
26. (A) $\int_0^3 \{k^2 + 3kx - 3x^2\}\,dx \le 0$

$\left[k^2x + \frac{3kx^2}{2} - \frac{3x^3}{3}\right]_0^3 \le 0$

$3k^2 + \frac{27k}{2} - 27 \le 0$

$2k^2 + 9k - 18 \le 0$

$(2k - 3)(k + 6) \le 0$



$-6 \le k \le \frac{3}{2}$

27. (B) equation $ax^2 + bx + c = 0$

$$\alpha+\beta = \frac{-b}{a} \text{ and } \alpha.\beta = \frac{c}{a}$$

Now, $(\alpha - 2)(\beta - 2) = \alpha\beta - 2\beta - 2\alpha + 4$

$$(\alpha-2)(\beta-2) = \frac{c}{a} - 2\left(\frac{-b}{a}\right) + 4$$

$$(\alpha-2)(\beta-2) = \frac{c+2b+4a}{a}$$

$$(\alpha-2)(\beta-2) = \frac{4a+2b+c}{a}$$

28. (B) | 4 | 5 | 4 | 3 | 2 | 1 | $= 4\times5\times4\times3\times2\times1 = 480$

29. (C) $I = \int e^x\left(1 - \frac{\sin 2x}{2}\right)\text{cosec}^2 x\, dx$

$$I = \int e^x.\text{cosec}^2 x\, dx - \int \frac{e^x.\sin 2x}{2}.\text{cosec}^2 x\, dx$$

$$I = \int e^x.\text{cosec}^2 x\, dx - \int e^x \frac{2\sin x.\cos x}{2.\sin^2 x}\, dx$$

$$I = \int e^x.\text{cosec}^2 x\, dx - \int e^x.\cot x\, dx$$

$$I = -\int e^x(\cot x \neg \cos ec^2 x)\, dx]$$

$I = -e^x.\cot x + c$

$$\left[\because \int e^x\left(f(x) + f'(x)\right)dx = e^x.f(x) + c\right]$$

30. (D) $x^m - y^m = 1$

On differentiating both side w.r.t. 'x'

$$mx^{m-1} - my^{m-1}\frac{dy}{dx} = 0$$

$$x^{m-1} = y^{m-1}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \left(\frac{x}{y}\right)^{m-1}$$

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1-m}$$

given that $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/4}$

then, $1 - m = \frac{1}{4} \Rightarrow m = \frac{3}{4}$

31. (C) Given that

$\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$

Now, $\left[(\vec{a}-\vec{b})\times(\vec{b}+2\vec{a})\right].\vec{b}$

$= [\vec{a}\times\vec{b} - \vec{b}\times\vec{b} + \vec{a} \times 2\vec{a} - \vec{b}\times 2\vec{a}].\vec{b}$

$= [\vec{a}\times\vec{b} + 2\vec{a}\times\vec{b}].\vec{b}$

$= 3[\vec{a}\times\vec{b}].\vec{b} = 0$

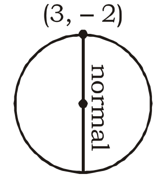
32. (B) $\lim_{x\to a}\frac{x^{-2} - a^{-2}}{x - a}$

$$\Rightarrow \lim_{x\to a}\frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a}$$

$$\Rightarrow \lim_{x\to a}\frac{(a^2 - x^2)}{x^2a^2(x-a)}$$

$$\Rightarrow \lim_{x\to a}\frac{-(x-a)(a+x)}{x^2a^2(x-a)} \Rightarrow \frac{-2a}{a^4} = \frac{-2}{a^3}$$

33. (C)



Equation of circle

$(x - 3)(x + 1) + (y + 2)(y - 4) = 0$

$x^2 - 2x - 3 + y^2 - 2y - 8 = 0$

$x^2 + y^2 - 2x - 2y - 11 = 0$

34. (A) $y = c.e^{\tan^{-1}x}$ ...(i)

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = c.e^{\tan^{-1}x}.\frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{y}{1+x^2} \qquad \text{[from eq (i)]}$$

$$(1 + x^2)\frac{dy}{dx} = y$$

35. (B) $\frac{d}{dx}(\coth x) = \text{cosech}^2 x$

36. (C) $f(x) = \left[\log_{10}\left(\frac{6x - x^2}{5}\right)\right]^{1/2}$

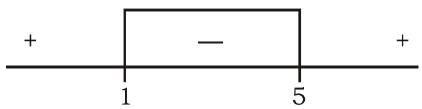
Now, $\log_{10}\left(\frac{6x - x^2}{5}\right) \geq 0$

$$\frac{6x - x^2}{5} \geq 1$$

$$6x - x^2 \geq 5$$

$$x^2 - 6x + 5 \leq 0$$

$(x-5)\,(x-1) \le 0$



$x \in [1, 5]$

37. (A) equations $4x + ky = 6$ and $6x - 15y = 9$ will have infinitely many solutions,

then, $\frac{4}{6} = \frac{k}{-15} \Rightarrow = -10$

38. (C) determinant $\begin{vmatrix} 2 & 3 & 4 & 5 \\ -1 & -2 & 3 & 0 \\ 3 & 7 & -2 & 1 \\ 5 & 9 & -3 & 0 \end{vmatrix}$

minor of the element 7 = $M_{32} = \begin{vmatrix} 3 & 4 & 5 \\ -1 & 3 & 0 \\ 5 & -3 & 0 \end{vmatrix}$

$= 3(0) - 4(0) + 5\,(3 - 15)$
$= -60$

39. (A) $x + iy = \dfrac{1}{3 - \cos\theta - i\sin\theta}$

$x + iy = \dfrac{3 - \cos\theta + i\sin\theta}{(3-\cos\theta)^2 + \sin^2\theta}$

$x + iy = \dfrac{3 - \cos\theta + i\sin\theta}{10 - 6\cos\theta}$

On Comparing

$x = \dfrac{3-\cos\theta}{10-6\cos\theta}$ and $y = \dfrac{\sin\theta}{10-6\cos\theta}$

Now, $(2x - 1)(4x - 1)$

$\Rightarrow \left[2 \times \dfrac{3-\cos\theta}{10-6\cos\theta} - 1\right]\left[4 \times \dfrac{3-\cos\theta}{10-6\cos\theta} - 1\right]$

$\Rightarrow \left[\dfrac{3-\cos\theta}{5-3\cos\theta} - 1\right]\left[\dfrac{6-2\cos\theta}{5-3\cos\theta} - 1\right]$

$\Rightarrow \dfrac{-2(1+\cos\theta)}{5-3\cos\theta} \times \dfrac{1+\cos\theta}{5-3\cos\theta}$

$\Rightarrow \dfrac{-2\sin^2\theta}{(5-3\cos\theta)^2}$

$\Rightarrow \dfrac{-2 \times 4\sin^2\theta}{(10-6\cos\theta)^2} \Rightarrow -8y^2$

40. (D)

41. (C) $I = \int_0^1 \dfrac{x^2(1-x)^2}{1+x^2}\,dx$

$I = \int_0^1 \dfrac{x^2(1+x^2-2x)}{1+x^2}\,dx$

$I = \int_0^1 x^2\,dx - \int_0^1 \dfrac{2x^3}{1+x^2}\,dx$

$I = \left[\dfrac{x^3}{3}\right]_0^1 - 2\int_0^1 \left(x - \dfrac{x}{1+x^2}\right)dx$

$I = \dfrac{1}{3} - 2\int_0^1 x.\,dx + \int_0^1 \dfrac{2x}{1+x^2}\,dx$

$I = \dfrac{1}{3} - 2\left[\dfrac{x^2}{2}\right]_0^1 + \left[\log(1+x^2)\right]_0^1$

$I = \dfrac{1}{3} - 2 \times \dfrac{1}{2} + [\log 2 - \log 1]$

$I = \dfrac{1}{3} - 1 + \log 2 = \log 2 - \dfrac{2}{3}$

42. (C) $f(x) = \begin{cases} 4x^2 + 9x - 1, & 0 \le x \le 1 \\ 30 - x \quad , & 1 < x \le 2 \end{cases}$

(a) $f(x) = 13 - x$ on $[1, 2]$
$1 < 2$
but $f(1) > f(2)$
$f(x)$ is decreaseing on [1, 2].

(b) L.H.L. $= \lim\limits_{x \to 1} f(x)$

$= \lim\limits_{x \to 1} (4x^2 + 9x - 1) = 12$

R.H.L. $= \lim\limits_{x \to 1} f(x)$

$= \lim\limits_{x \to 1} 13 - x = 12$

L.H.L. = R.H.L.
$f(x)$ is continuous on [0, 2].

(c) $f(x)$ is increasing on [0, 1]
and $f(x)$ is decreasing on [1, 2]
Hence **$f(x)$ is maximum at $x = 1$.**

(d) L.H.D. $= \lim\limits_{h \to 0} \dfrac{f(1-h) - f(1)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{4(1-h)^2 + 9(1-h) - 1 - 12}{-h}$

$= \lim\limits_{h \to 0} \dfrac{4h^2 - 8h - 9h}{-h}$

$= \lim\limits_{h \to 0} -4h + 8 + 9 = 17$

R.H.D. = $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

= $\lim_{h \to 0} \frac{13 - (1+h) - 12}{h}$

= $\lim_{h \to 0} \frac{-h}{h} = -1$

L.H.D. $\neq$ R.H.D.

Hence $f'(1) \neq 1$

43. (A) **Short Method:-**

$$\begin{vmatrix} x & \frac{y^2+z^2}{x} & x \\ y & y & \frac{x^2+y^2}{y} \\ \frac{x^2+y^2}{z} & z & z \end{vmatrix} = \frac{xyz}{k}$$

x, y, z take any number except '0'

Let $x = y = z = 1$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \frac{1}{k}$$

$\Rightarrow 1\,(1-2) - 2(1-4) + 1(1-2) = \frac{1}{k}$

$\Rightarrow -1 + 6 - 1 = \frac{1}{k} \Rightarrow 4 = \frac{1}{k} = k = \frac{1}{4}$

44. (B) In the expansion of $\left(\frac{3}{x} - \frac{xy}{2}\right)^6$

Middle term = $T_4 = T_{3+1} = {}^6C_3 \left(\frac{3}{x}\right)^3 \left(-\frac{xy}{2}\right)^3$

$= -20 \times \frac{27}{x^3} \times \frac{x^3y^3}{8}$

$= \frac{-135}{2}\ y^3$

45. (B) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{4\pi}{3}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{4\pi}{3}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos\frac{4\pi}{3}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{2\pi}{3}}}}$

$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2\frac{\pi}{3}}}$

$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2\frac{\pi}{3}}}$

$\Rightarrow \sqrt{2+2\cos\frac{\pi}{3}}$

$\Rightarrow \sqrt{2+2\cos^2\frac{\pi}{6}}$

$\Rightarrow 2\cos\frac{\pi}{6} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

46. (C) $\lim_{x \to 8} \frac{\sqrt{4x+4}-6}{\sqrt{x+1}-3}$ $\left[\frac{0}{0}\right]$ form

$\Rightarrow \lim_{x \to 8} \frac{2\sqrt{x+1}-6}{\sqrt{x+1}-3}$

$\Rightarrow \lim_{x \to 8} \frac{2\left(\sqrt{x+1}-3\right)}{\sqrt{x+1}-3} = 2$

47. (C) $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$

$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^{n-2}\ .\cot^2 x\, dx$

$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^{n-2}\ (\text{cosec}^2\, x - 1)\ dx$

$I_n = -\int_{\pi/4}^{\pi/2} (\cot x)^{n-2}\ (-\cos ec^2 x)\ dx$

$- \int_{\pi/4}^{\pi/2} (\cot x)^{n-2}$

$I_n = -\left[\frac{(\cos x)^{n-2+1}}{n-2+1}\right]_{\pi/4}^{\pi/2} - I_{n-2}$

$I_n + I_{n-2} = -\left[\frac{\left(\cos\frac{\pi}{2}\right)^{n-1} - \left(\cot\frac{\pi}{4}\right)^{n-1}}{n-1}\right]$

$I_n + I_{n-2} = -\left[\frac{0-1}{n-1}\right]$

$I_n + I_{n-2} = \frac{1}{n-1}$

48. (D) $\sqrt{\frac{\sqrt{3}}{\sqrt{2}\cos 345^\circ}+\frac{1}{\sqrt{2}\sin 285^\circ}}$

$\Rightarrow \sqrt{\frac{\sqrt{3}}{\sqrt{2}\cos(360-15)}+\frac{1}{\sqrt{2}\sin(270+15)}}$

$\Rightarrow \sqrt{\frac{\sqrt{3}}{\sqrt{2}\cos 15}-\frac{1}{\sqrt{2}\cos 15}}$

$\Rightarrow \sqrt{\frac{(\sqrt{3}-1)}{\sqrt{2}\cos 15}}$

$\Rightarrow \sqrt{\frac{(\sqrt{3}-1)2\sqrt{2}}{\sqrt{2}(\sqrt{3}+1)}}$

$\Rightarrow \sqrt{\frac{2(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}}$

$\Rightarrow \sqrt{\frac{2(\sqrt{3}-1)^2}{2}} = \sqrt{3}-1$

49. (A) $I = \int_0^1 \frac{x^8}{\sqrt{1-x^6}}\,dx$

$I = \int_0^1 \frac{x^6 . x^2}{\sqrt{1-(x^3)^2}}\,dx$

Let $x^3 = \sin\theta$ when $x \to 0, \theta = 0$

$3x^2.dx = \cos\theta\,.d\theta$ $x \to 1, \theta = \frac{\pi}{2}$

$x^2.dx = \frac{1}{3}\cos\theta\,.d\theta$

$I = \int_0^{\pi/2} \frac{1}{3}\frac{\sin^2\theta.\cos\theta.d\theta}{\sqrt{1-\sin^2\theta}}$

$I = \frac{1}{3}\int_0^{\pi/2}\sin^2\theta.d\theta$

$I = \frac{1}{3}\int_0^{\pi/2}\frac{1-\cos 2\theta}{2}\,d\theta$

$I = \frac{1}{6}\left[\theta-\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$I = \frac{1}{6}\left[\frac{\pi}{2}-0\right) = \frac{\pi}{12}$

50. (C) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \cos^{-1}\left(-\cos\frac{\pi}{4}\right)$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \cos^{-1}\left[\cos\left(\pi-\frac{\pi}{4}\right)\right]$

$\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \cos^{-1}\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

51. (A) word "COMMISSION"

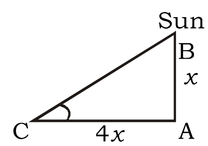
No. of Permutation = $\frac{10!}{2!2!2!2!} = 226800$

52. (C) cos (375°) = cos (360 + 15)

$= \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

53. (B) Let AB = x m

AC = 4 x m



ΔABC

$\tan\theta = \frac{AB}{AC}$

$\tan\theta = \frac{x}{4x} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{4}\right)$

54. (A) $I = \int\sqrt{2-2\cos x}\,dx$

$I = \int\sqrt{2\times 2\sin^2\frac{x}{2}}\,dx$

$I = \int 2.\sin\frac{x}{2}\,dx$

$I = -2\,\frac{\cos\frac{x}{2}}{\frac{1}{2}} + c$

$I = -4\cos\frac{x}{2} + c$

55. (B) The Probability = $\frac{{}^7C_2\times{}^5C_1+{}^7C_3\times{}^5C_0}{{}^{12}C_3}$

$= \frac{21\times5+35\times1}{220}$

$= \frac{140}{220} = \frac{7}{11}$

56. (A) $y = (\cot x)^{(\cot x)^{(\cot x)\ldots\ldots\infty}}$

$\Rightarrow y = (\cot x)^y$

taking log both side

$\Rightarrow \log y = y \log(\cot x)$

On differentiating both side

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y\,\frac{1}{\cot x}(-\operatorname{cosec}^2 x) + \log(\cot x).\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - \log(\cot x)\frac{dy}{dx} = \frac{-2y}{\sin 2x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log\cot x}{y}\right) = \frac{-2y}{\sin 2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2y^2}{\sin 2x(1 - y\log\cot x)}$$

57. (B)

2	71	1
2	35	1
2	17	1
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(71)_{10} = (1000111)_2$

58. (C) plane $24x + 6y - 8z + 17 = 0$ and point $(-1, 2, -4)$

$$\text{Distance} = \frac{24(-1) + 6\times 2 - 8\times(-4)}{\sqrt{(24)^2 + 6^2 + (-8)^2}}$$

$$= \frac{-24 + 12 + 32}{\sqrt{576 + 36 + 64}}$$

$$= \frac{20}{\sqrt{676}} = \frac{20}{26} = \frac{10}{13}$$

59. (A) $\dfrac{1 - \tan 32^\circ.\tan 205^\circ}{\tan 212^\circ - \cot 115^\circ}$

$$\Rightarrow \frac{1 - \tan 32^\circ.\tan(180 + 25^\circ)}{\tan(180 + 32^\circ) - \cot(90 + 25^\circ)}$$

$$\Rightarrow \frac{1 - \tan 32^\circ.\tan 25^\circ}{\tan 32^\circ + \tan 25^\circ}$$

$$\Rightarrow \frac{1}{\tan(32 + 25^\circ)} = \cot 57 = \tan 33^\circ$$

60. (A) We know that

$(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2\, x^2 + \ldots\ldots\ldots + {}^nC_n\, x^n$

On differentiating both side w.r.t. 'x'

$n(1+x)^{n-1} = 0 + {}^nC_1 + C_2\,(2x) + \ldots\ldots {}^nC_n\,(nx^{n-1})$

On putting $x = 1$

$n.\ 2^{n-1} = {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 \ldots n\,{}^nC_n$

Hence $C_1 + C_2 + 3\,C_3 + \ldots\ldots\ldots\ldots nC_n = n.\ 2^{n-1}$

61. (C) Area $= \int_0^2 \left(e^x - e^{-x}\right)dx$

$$= \left[e^x + e^{-x}\right]_0^2$$

$$= e^2 + e^{-2} - e^0 - e^0$$

$$= e^2 + \frac{1}{e^2} - 2$$

$$= \left(e - \frac{1}{e}\right)^2 \text{ sq. unit}$$

62. (B) When $\theta = 180$

$M = \dfrac{60}{11}(H \pm 6)$ where $+ \to H < 6$

$- \to H > 6$

H = 4 (between 4 and 5 O' clock)

$$M = \frac{60}{11}(4 + 6)$$

$$= \frac{60}{11}\times 10 = 54\frac{6}{11}$$

Hence time $= 4 : 54\dfrac{6}{11}$

63. (C) A = {1,2,3,4,6,8,9,0}, $n = 8$

No. of proper subsets of A $= 2^n - 1 = 2^8 - 1$
$= 255$

64. (A) Degree = 3

65. (B) given that circle $x^2 + y^2 + x - y + 7 = 0$ and point $(2, -7)$

Let required equation of circle

$x^2 + y^2 + x - y + c = 0$ ....(i)

it passes through the point $(2, -7)$

$4 + 49 + 2 + 7 + c = 0 \Rightarrow c = -62$

from eq. (i)

$x^2 + y^2 + x - y - 62 = 0$

66. (C) Let $y = 3^{31}$

taking log both side

$\log_{10} y = 31 \log_{10} 3$

$\log_{10} y = 31 \times 0.4771$

$\log_{10} y = 14.7901$

The no. of digits = 14 + 1 = 15

67. (B) differential equation

$$x^2\frac{dy}{dx} - y = 1$$

$$\frac{dy}{dx} - \frac{1}{x^2}\ y = \frac{1}{x^2}$$

On comparing with general equation

$$P = \frac{-1}{x^2}, \qquad Q = \frac{1}{x^2}$$

$$\text{I.F.} = e^{\int P\,dx}$$

$$\text{I.F.} = e^{\int \frac{-1}{x^2}dx}$$

$$\text{I.F.} = e^{1/x}$$

Solution of differential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx$$

$$y \times e^{1/x} = -\int\left(\frac{-1}{x^2}\right) \times e^{1/x}\,dx$$

$$y \times e^{1/x} = -e^{1/x} + c$$

$$y = c.\,e^{-1/x} - 1$$

68. (A) $\sin\frac{\pi}{12} < \tan\frac{\pi}{12} < \cos\frac{\pi}{12}$

69. (B) $\lim\limits_{x \to 1} \frac{\log_5(2-x)}{1-x}$ $\left[\frac{0}{0}\right]$ form

by L- Hospital's Rule

$$\Rightarrow \lim_{x\to 1}\frac{\frac{-1}{2-x}\log_5 e}{-1}$$

$$\Rightarrow \frac{\log_5 e}{2-1} = \log_5 e$$

70. (A) Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -1 & 1 \\ 3 & 2 & 4 \end{bmatrix}$

Co-factor of A—

$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 2 & 4 \end{vmatrix}$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}$, $C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$

= – 6 = – 5 = – 7

$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$, $C_{22} = (-1)^{2+2}\begin{vmatrix} 0 & 2 \\ 3 & 4 \end{vmatrix}$, $C_{23} = (-1)^{2+3}\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix}$

= – 0 = – 6 = 3

$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix}$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix}$, $C_{33} = (-1)^{3+3}\begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}$

= 3 = 4 = – 2

$$C = \begin{bmatrix} -6 & -5 & 7 \\ 0 & -6 & 3 \\ 3 & 4 & -2 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} -6 & 0 & 3 \\ -5 & -6 & 4 \\ 7 & 3 & -2 \end{bmatrix}$$

71. (B) Conic $4x^2 - 6y^2 = 48$

$$\frac{x^2}{12} - \frac{y^2}{8} = 1$$

Now, eccentricity $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$e = \sqrt{1 + \frac{8}{12}}$$

$$e = \sqrt{\frac{20}{12}} = \sqrt{\frac{5}{3}}$$

72. (B) $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

73. (C) Let $I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$ ...(i)

Prop. IV $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)} + \sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$$

$I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}$ ...(ii)

from eq (i) and eq (ii)

$$2I = \int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx$$

$$2I = \int_{\pi/6}^{\pi/3} 1.dx$$

$$2I = [x]_{\pi/6}^{\pi/3}$$

$$2I = \frac{\pi}{3} - \frac{\pi}{6}$$

$$2I = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

74. (C) equation $ax^2 + bx + c = 0$

one root = $3 - 4i$, other root = $3 + 4i$

Now, $3 - 4i + 3 + 4i = \frac{-b}{a}$

$6 = \frac{-b}{a} \Rightarrow a = \frac{-b}{6}$

and $(3 - 4i).(3 + 4i) = \frac{c}{a}$

$9 + 16 = \frac{c}{a} \Rightarrow a = \frac{c}{25}$

then $\frac{-b}{6} = \frac{c}{25}$

$25b + 6c = 0$

75. (A) maximum value of $(24\sin\theta + 7\cos\theta)$

$= \sqrt{(24)^2 + (7)^2}$

$= \sqrt{576 + 49} = 25$

76. (D) $\frac{1+\sin\theta}{1-\sin\theta} = 3$

$1 + \sin\theta = 3 - 3\sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\sin\theta = \sin\frac{\pi}{6}$

$\theta = n\pi + (-1)^n\frac{\pi}{6}$

77. (C) $3\sin110°.\sin130°.\sin150°.\ \sin170°$

$\Rightarrow 3\cos20°.\ \cos40°.\ \cos60°.\ \cos80°$

$\Rightarrow 3 \times \cos60°\{\cos20°.\cos40°.\cos80°\}$

$\Rightarrow 3 \times \frac{1}{2} \times \frac{1}{4}\cos(3 \times 20)°$

$\Rightarrow 3 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{2} = \frac{3}{16}$

78. (A) $I = \int_0^2 \frac{dx}{\sqrt{x^2 - 4x + 13}}$

$I = \int_0^2 \frac{dx}{\sqrt{(x-2)^2 + (3)^2}}$

$I = \left[\log\left|x - 2 + \sqrt{(x-2)^2 + (3)^2}\right|\right]_0^2$

$I = \log\left|0 + \sqrt{0+9}\right| - \log\left|-2 + \sqrt{4+9}\right|$

$I = \log|3| - \log\left|\sqrt{13} - 2\right|$

$I = \log\left|\frac{3}{\sqrt{13} - 2}\right|$

$I = \log\left|\frac{3\left(\sqrt{13}+2\right)}{13-4}\right|$

$I = \log\left|\frac{\sqrt{13}+2}{3}\right|$

79. (B) $\sin^{-1}\frac{8}{17} + \tan^{-1}\frac{3}{4}$

$\Rightarrow \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}}\right]$

$\Rightarrow \tan^{-1}\left[\frac{32+45}{60-24}\right] = \tan^{-1}\left(\frac{77}{36}\right)$

80. (A)

81. (A) $S = \sqrt{2} + \sqrt{8} + \sqrt{32} + \sqrt{128}$ ....

$S = \sqrt{2} + 2\sqrt{2} + 4\sqrt{2} + 8\sqrt{2} +$....................

We know that

$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$

$S_{10} = \frac{\sqrt{2}\left(2^{10} - 1\right)}{2 - 1} \Rightarrow S_{10} = \sqrt{2}\ (2^{10} - 1)$

82. (C) $n(S) = 6 \times 6 = 36$

$E = \begin{Bmatrix} (6,4),(4,6),(5,5) & \text{for sum} = 10 \\ (6,5),(5,6) & \text{for sum} = 11 \\ (6,6) & \text{for sum} = 12 \end{Bmatrix}$

$n(E) = 6$

The required Probability $= \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

83. (D) given that $\theta = \frac{15\pi}{4}$

Now, $\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2-\theta}$

$\Rightarrow \left(1-2\sin^2\frac{\theta}{2}\right)$

$\Rightarrow \frac{3(-1)-(-1)^3}{1-3(-1)^2} \Rightarrow \frac{-3+1}{1-3} = \frac{-2}{-2} = 1$

84. (C) $S = 21^3 + 22^3 + 23^3 + \ldots\ldots\ldots\ldots + 30^3$

$S = [1^3 + 2^3 + \ldots. + 20^3 + 21^3 + \ldots.. + 30^3]$

$- [1^3 + 2^3 + 3^3 + \ldots\ldots + 20^3]$

$S = \left[\frac{30(30+1)}{2}\right]^2 - \left[\frac{20(20+1)}{2}\right]^2$

$S = (15 \times 31)^2 - (10 \times 21)^2$

$S = 216225 - 44100 = 172125$

85. (A) $(3+3\omega^2-2\omega)^{31}$

$\Rightarrow \left[3(1+\omega^2)-2\omega\right]^{31}$

$\Rightarrow \left[3(-\omega)-2\omega\right]^{31}$

$\Rightarrow [-5\omega]^{31}$

$\Rightarrow -5^{31}\omega^{31} = -5^{31}\omega \quad [\because \omega^2 = 1]$

86. (A) Differential equation

$3(1+e^{2x})\, y\, dy = e^x\, dx$

$\Rightarrow 3y\, dy = \frac{e^x}{1+e^{2x}}\, dx$

Let $e^x = t \Rightarrow e^x\, dx = dt$

$\Rightarrow 3y\, dy = \frac{dt}{1+t^2}$

On integrating

$\Rightarrow \frac{3y^2}{2} = \tan^{-1}t + c$

$\Rightarrow 3y^2 = 2\tan^{-1}(e^x) + c$

87. (C) $I = \int_{0.1}^{2.5} [x]\, dx$

$I = \int_{0.1}^{1} [x]\, dx + \int_{1}^{2} [x]\, dx + \int_{2}^{2.5} [x]\, dx$

$I = \int_{0.1}^{1} 0.\, dx + \int_{1}^{2} 1.\, dx + \int_{2}^{2.5} 2.\, dx$

$I = 0 + [x]_1^2 + 2\,[x]_2^{2.5}$

$I = 2 - 1 + 2\,(2.5 - 2)$

$I = 1 + 2 \times \frac{1}{2} = 2$

88. (A) $\cos\left(\sin^{-1}\left(\cos\frac{2\pi}{3}\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(\cos\left(\pi - \frac{\pi}{3}\right)\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(-\cos\frac{\pi}{3}\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(\sin\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)\right)\right)$

$\Rightarrow \cos\left(\frac{3\pi}{2} - \frac{\pi}{3}\right)$

$\Rightarrow -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

89. (D) given that $A = B \cap C$

Now, (U– (U– (U– (U– (U – A)))))

$\Rightarrow$ (U– (U– (U– (U – A'))))

$\Rightarrow$ (U– (U– (U – A)))

$\Rightarrow$ (U– (U – A'))

$\Rightarrow (U - A) = A' = (B \cap C)' = (B' \cup C')$

90. (B) equation $(x + a)(x + b) + c = 0$

$x + (a + b)\, x + ab + c = 0$

Now, $\alpha + \beta = -(a + b), \alpha\beta = ab + c$ …(i)

again, another equation

$(x+\alpha)(x+\beta) = c$

$x^2 + (\alpha+\beta)x + \alpha\beta = c$

from eq (i)

$x^2 - (a + b)x + ab + c = c$

$x^2 - (a + b)x + ab = 0$

Hence roots are (a, b).

91. (C) $\begin{vmatrix} x^{\omega} & x^{3\omega}-1 & x^{2\omega} \\ x & x^3-1 & x^2 \\ x^{\omega^2} & x^{3\omega^2}-1 & x^{2\omega^2} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} x^{\omega} & x^{3\omega} & x^{2\omega} \\ x & x^3 & x^2 \\ x^{\omega^2} & x^{3\omega^2} & x^{2\omega^2} \end{vmatrix} - \begin{vmatrix} x^{\omega} & 1 & x^{2\omega} \\ x & 1 & x^2 \\ x^{\omega^2} & 1 & x^{2\omega^2} \end{vmatrix}$

$$\Rightarrow x^{\omega}.x.x^{\omega^2}\begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix} - \begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix}$$

$$\Rightarrow x^{1+\omega+\omega^2}\begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix} - \begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix}$$

$$\Rightarrow 1.\begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix} - \begin{vmatrix}1 & x^{2\omega} & x^{\omega}\\ 1 & x^2 & x\\ 1 & x^{2\omega^2} & x^{\omega^2}\end{vmatrix} = 0$$

92. (C)
```
  1 x 0 0 1
+ 1 0 1 y 1
-----------
1 0 1 z 0 0
```
$y = 1,\ z = 0,\ x = 0$

93. (A) given that $n = 147!$

Now,

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+....+\frac{1}{\log_{147} n}$$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 + ...\ \log_{147} n$

$\Rightarrow \log_n (2 \times 3 \times 4 \times ......147)$

$\Rightarrow \log_{147!}(147!) = 1$

94. (B) $\sin\frac{\theta}{2} = \frac{1}{2}\left(x+\frac{1}{x}\right)$

$$2\sin\frac{\theta}{2} = x+\frac{1}{x}$$

On Squaring both side

$$4\sin^2\frac{\theta}{2} = x^2+\frac{1}{x^2}+2$$

$$x^2+\frac{1}{x^2} = 4\sin^2\frac{\theta}{2}-2$$

$$x^2+\frac{1}{x^2} = -2\left(1-2\sin^2\frac{\theta}{2}\right)$$

$$x^2+\frac{1}{x^2} = -2\cos\theta$$

95. (D) Given that

ratio of sides $= \left(\sqrt{3}-1\right):\sqrt{6}:\left(\sqrt{3}+1\right)$

Sine Rule

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

sinA : sinB : sinC = $a : b : c$

sinA : sinB : sinC = $\left(\sqrt{3}-1\right):\sqrt{6}:\left(\sqrt{3}+1\right)$

sinA : sinB : sinC = $\frac{\sqrt{3}-1}{2\sqrt{2}}:\frac{\sqrt{6}}{2\sqrt{2}}:\frac{\sqrt{3}+1}{2\sqrt{2}}$

sinA : sinB : sinC = $\frac{\sqrt{3}-1}{2\sqrt{2}}:\frac{\sqrt{3}}{2}:\frac{\sqrt{3}+1}{2\sqrt{2}}$

sinA : sinB : sinC = sin15° : sin60° : sin105°

A : B : C = 15° : 60° : 105°

Hence greatest angle = 105°

96. (B) given that $\sin\alpha = \frac{8}{17}$ and $\cos\beta = \frac{5}{13}$

$$\cos\alpha = \frac{15}{17},\qquad \sin\beta = \frac{12}{13}$$

Now, $\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

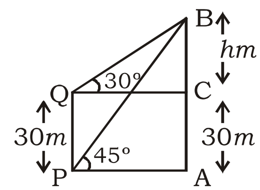
$$\cos(\alpha+\beta) = \frac{15}{17}\times\frac{5}{13}-\frac{8}{17}\times\frac{12}{13}$$

$$2\cos^2\left(\frac{\alpha+\beta}{2}\right)-1 = \frac{-21}{221}$$

$$2\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{-21}{221}+1$$

$$2\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{200}{221} \Rightarrow \cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{100}{221}$$

97. (C) Let BC = h m



In ΔABP

$$\tan 45 = \frac{AB}{AP}$$

$$1 = \frac{30+h}{AP} \Rightarrow AP = 30+h = QC$$

In ΔQCB

$$\tan 30° = \frac{BC}{QC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{30+h} \Rightarrow h = 15\left(\sqrt{3}+1\right)$$

Height of the tower = $30 + h$

$= 30 + 15\sqrt{3} + 15$

$= 15\left(3+\sqrt{3}\right)m$

98. (A) tanA + 2tan2A + 4tan4A + 8 cot8A

$\Rightarrow$ cotA – (cotA–tanA) + 2tan2A+ 4tan4A + 8 cot 8A

$\Rightarrow$ cotA– 2cot2A + 2tan2A+ 4tan4A + 8 cot 8A

$\Rightarrow$ cotA–2(cot2A–tan2A)+4tan4A + 8cot 8A

$\Rightarrow$ cotA – 2× 2 cot 4A + 4 tan4A + 8 cot 8A

$\Rightarrow$ cotA– 4 (cot4A – tan4A) + 8 cot8A

$\Rightarrow$ cotA– 4 × 2 cot 8A + 8 cot 8A

$\Rightarrow$ cotA

99. (D) Given line

$\frac{x}{2} + \frac{y}{5} = 1$

$5x + 2y = 10$

Slope $m_1 = \frac{-5}{2}$

Slope of perpendicular line $m_2 = \frac{-1}{m_1} = \frac{2}{5}$

100. (A) Tangents of a circle

$8x - 6y + 30 = 0 \Rightarrow 4x - 3y + 15 = 0$

and $-12x + 9y + 15 = 0 \Rightarrow 4x - 3y - 5 = 0$

Both lines are parallel.

Now, radius of circle $= \frac{1}{2}\left|\frac{15+5}{\sqrt{(4)^2+(-3)^2}}\right|$

$= \frac{20}{2\times 5} = 2$ unit

101. (C) circle $x^2 + y^2 + 2x - 4y - k = 0$

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{1^2 + (-2)^2 + k}$

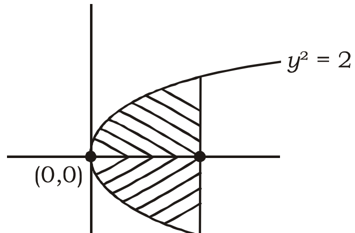
$r = \sqrt{5 + k}$

Area of circle = πr^2

$28\pi = \pi(5 + k) \Rightarrow k = 23$

102. (A) Curve $y = \sqrt{2}\sqrt{x}$

and line $x = 2$



Area $= 2\int_0^2 y.dx$

Area $= 2\int_0^2 \sqrt{2}.\sqrt{x}\, dx$

Area $= 2\sqrt{2}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^2$

Area $= 2 \times \frac{2\sqrt{2}}{3}\left[2^{3/2}\right] = \frac{16}{3}$ sq. unit

103. (C) parabola $y^2 + 14y + 3x + 39 = 0$

$(y + 7)^2 - 49 + 3x + 39 = 0$

$(y + 7)^2 = -3x + 10$

$(y + 7)^2 = -3\left(x - \frac{10}{3}\right)$

$Y^2 = -3X$ when $Y = y + 7$

$X = x - \frac{10}{3}$

equation of latus-rectum

$X = -a$

$x - \frac{10}{3} = \frac{-3}{4} \Rightarrow x = \frac{31}{12}$

104. (C) Vectors $2\hat{i} + \hat{j} - 2\hat{k}$, $\lambda\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ are coplaner, then

$$\begin{vmatrix} 2 & 1 & -2 \\ \lambda & -1 & 2 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow 2(1 + 2) - 1(-\lambda - 2) - 2(-\lambda + 1) = 0$

$\Rightarrow 6 + \lambda + 2 + 2\lambda - 2 = 0$

$\Rightarrow 3\lambda + 6 = 0 \Rightarrow \lambda = -2$

105. (B) $\sum_{r=1}^{3} C(20 + r, 3) + C(21, 4)$

$\Rightarrow {}^{21}C_3 + {}^{22}C_3 + {}^{23}C_3 + {}^{21}C_4$

$\Rightarrow {}^{21}C_3 + {}^{21}C_4 + {}^{22}C_3 + {}^{23}C_3$

$\Rightarrow {}^{22}C_4 + {}^{22}C_3 + {}^{23}C_3$

$\Rightarrow {}^{23}C_4 + {}^{23}C_3 \Rightarrow {}^{24}C_4$

106. (A) The required Probability $= \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{12} = \frac{1}{13}$

107. (B) Vectors $-\hat{i}+\hat{j}+\lambda\hat{k}$ and $\hat{i}-2\hat{j}+\hat{k}$

and $\theta = \frac{\pi}{2}$

$$\cos\theta = \frac{-1\times1+1\times(-2)+\lambda\times1}{\sqrt{(-1)^2+1^2+\lambda^2}\sqrt{1^2+(-2)^2+1^2}}$$

$$\cos\frac{\pi}{2} = \frac{-1-2+\lambda}{\sqrt{\lambda^2+2}\sqrt{6}}$$

$$0 = \frac{\lambda-3}{\sqrt{\lambda+2}\sqrt{6}} \Rightarrow \lambda = 3$$

108. (C) The required Probability $= \frac{1}{7}$

109. (A) Given that $2(l+b) = 80 \Rightarrow l+b = 40$

Now, Area (A) = lb

$A = l(40-l)$

$A = 40l - l^2$

$$\frac{dA}{dl} = 40 - 2l$$

$$\frac{d^2A}{dl^2} = -2 \ (maxima)$$

for maxima and minima

$$\frac{dA}{dl} = 0 \Rightarrow 40 - 2l = 0 \Rightarrow l = 20$$

maxi. Area of a rectangle = lb

= 20 × 20 = 400 sq.cm

110. (A) Given data 7, 8, 12, 15, 25, 35, 40, 32, 35, 21

$\sum x_i$ = 7 + 8 + 12+ 15 +25 +35 +40 +32 + 35 + 21 = 230

$\sum x_i^2$ = $7^2 + 8^2 + 12^2 + 15^2 + 25^2 + 40^2 + 32^2 + 35^2 + 35^2 + 21^2 = 6622$

$$\text{S.D.} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$\text{S.D.} = \sqrt{\frac{6622}{10} - \left(\frac{230}{10}\right)^2}$$

$$\text{S.D.} = \sqrt{\frac{66220-52900}{100}} = \sqrt{\frac{13320}{100}}$$

$$\text{Variance} = (\text{S.D.})^2 = \left(\frac{\sqrt{13320}}{100}\right)^2 = 133.2$$

111. (C)

class	x	f	$f\times x$
0 – 5	2.5	5	12.5
5 – 10	7.5	6	45.0
10 – 15	12.5	12	150.0
15 – 20	17.5	18	315.0
20 – 25	22.5	12	270.0
25 – 30	27.5	17	467.5
30 – 35	32.5	19	617.5
35 – 40	37.5	11	412.5
		$\sum f = 100$	$\sum f\times x = 2290$

$$\text{Mean} = \frac{\sum f\times x}{\sum f} = \frac{2290}{100} = 22.9$$

112. (B) lines $\frac{x-1}{-2} = \frac{y+1}{2} = \frac{z-5}{-1}$ and

$$\frac{x+2}{-8} = \frac{y-3}{24} = \frac{z+6}{6}$$

angle b/w lines

$$\cos\theta = \frac{-2\times(-8)+2\times24+(-1)\times6}{\sqrt{(-2)^2+2^2+(-1)^2}\sqrt{(-8)^2+24^2+6^2}}$$

$$\cos\theta = \frac{16+48-6}{3\times26}$$

$$\cos\theta = \frac{58}{3\times26}$$

$$\cos\theta = \frac{29}{39} \Rightarrow \theta = \cos^{-1}\left(\frac{29}{39}\right)$$

113. (C) 2tan32° + tan328° + tan148°

$\Rightarrow$ 2tan32° + tan(360 – 32) + tan(180 – 32°)

$\Rightarrow$ 2tan32° – tan32° – tan32° = 0

114. (D) $\lim_{x\to2} \frac{\sqrt{2}-\sqrt{x}}{x^2-4}$ $\left[\frac{0}{0}\right]$ form

by L- Hospital's Rule

$$\Rightarrow \lim_{x\to2} \frac{\frac{-1}{2\sqrt{x}}}{2x}$$

$$\Rightarrow \lim_{x\to2} \frac{-1}{4x\sqrt{x}}$$

$$\Rightarrow \frac{-1}{4\times2\sqrt{2}} = \frac{-1}{8\sqrt{2}}$$

115. (B) Curve $3x^2 + y^2 = 16$

On differentiating both side w.r.t. 'x'

$$3 \times 2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-3x}{y}$$

Slope of tangent $\left(\frac{dy}{dx}\right)_{\text{at}(-2,\ 2)} = 3$

Slope of normal $= \frac{-1}{3}$

Equation of normal at (–2, 2)

$$y - 2 = \frac{-1}{3}(x + 2)$$

$$\Rightarrow x + 3y - 4 = 0$$

On Comparing with $ax + cy + b = 0$

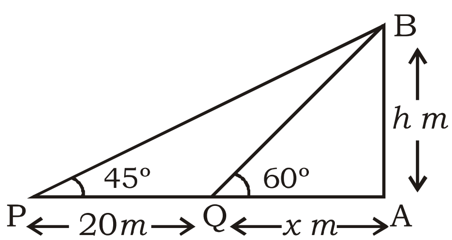
$a = 1,\ b = -4,\ c = 3$

116. (C)

117. (C)

Let height of tower = h m

AQ = x m



In ΔABQ

$$\tan 60° = \frac{AB}{AQ}$$

$$\sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$$

In ΔABP

$$\tan 45° = \frac{AB}{AP}$$

$$\Rightarrow \quad 1 = \frac{h}{20 + x}$$

$$\Rightarrow 20 + x = h$$

$$\Rightarrow 20 + \frac{h}{\sqrt{3}} = h \Rightarrow h = 10\sqrt{3}\,(\sqrt{3} + 1)m$$

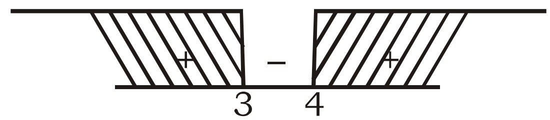
118. (B) $f(x) = \dfrac{1}{\sqrt{\log_e(x^2 - 7x + 13)}}$

$\log_e(x^2 - 7x + 13) > 0$

$x^2 - 7x + 13 > 1$

$x^2 - 7x + 12 > 0$

$(x - 4)(x - 3) > 0$



$x \in (-\infty, 3) \cup (4, \infty)$

119. (A) $\dfrac{\cos 4x - 2\cos 3x + \cos 2x}{\sin 4x - \sin 2x}$

$$\Rightarrow \frac{\cos 4x + \cos 2x - 2\cos 3x}{\sin 4x - \sin 2x}$$

$$\Rightarrow \frac{2\cos 3x.\cos x - 2\cos 3x}{2\cos 3x.\sin x}$$

$$\Rightarrow \frac{-2\cos 3x(1 - \cos x)}{2\cos 3x.\sin x}$$

$$\Rightarrow \frac{-2\sin^2\frac{x}{2}}{2.\sin\frac{x}{2}.\cos\frac{x}{2}} = -\tan\frac{x}{2}$$

120. (D) Let $a - ib = \sqrt{7 - 24i}$

On squaring both side

$(a^2 - b^2) - 2abi = 7 - 24i$

On comparing

$a^2 - b^2 = 7$ and $2ab = 24$ .....(i)

Now, $(a^2 + b^2) = (a^2 - b^2) + (2ab)^2$

$(a^2 + b^2) = 7^2 + 24^2$

$(a^2 + b^2)^2 = 25^2$

$a^2 + b^2 = 25$ .....(ii)

from eq. (i) and eq. (ii)

$a = \pm 4$ and $b = \pm 3$

Hence $\sqrt{7 - 24i} = \pm(4 - 3i)$

NDA (MATHS) MOCK TEST - 108 (Answer Key)

1. (A)	21. (B)	41. (C)	61. (C)	81. (A)	101. (C)
2. (B)	22. (C)	42. (C)	62. (B)	82. (C)	102. (A)
3. (C)	23. (B)	43. (A)	63. (C)	83. (D)	103. (C)
4. (D)	24. (B)	44. (B)	64. (A)	84. (C)	104. (C)
5. (B)	25. (C)	45. (B)	65. (B)	85. (A)	105. (B)
6. (C)	26. (A)	46. (C)	66. (C)	86. (A)	106. (A)
7. (C)	27. (B)	47. (C)	67. (B)	87. (C)	107. (B)
8. (A)	28. (B)	48. (D)	68. (A)	88. (A)	108. (C)
9. (B)	29. (C)	49. (A)	69. (B)	89. (D)	109. (A)
10. (D)	30. (D)	50. (C)	70. (A)	90. (B)	110. (A)
11. (C)	31. (C)	51. (A)	71. (B)	91. (C)	111. (C)
12. (A)	32. (B)	52. (C)	72. (B)	92. (C)	112. (B)
13. (D)	33. (C)	53. (B)	73. (C)	93. (A)	113. (C)
14. (B)	34. (A)	54. (A)	74. (C)	94. (B)	114. (D)
15. (B)	35. (B)	55. (B)	75. (A)	95. (D)	115. (B)
16. (D)	36. (C)	56. (A)	76. (D)	96. (B)	116. (C)
17. (D)	37. (A)	57. (B)	77. (C)	97. (C)	117. (C)
18. (A)	38. (C)	58. (C)	78. (A)	98. (A)	118. (B)
19. (A)	39. (A)	59. (A)	79. (B)	99. (D)	119. (A)
20. (C)	40. (D)	60. (A)	80. (A)	100. (A)	120. (D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777