NDA MATHS MOCK TEST - 112 (SOLUTION)

1. (B) $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$\Rightarrow y = \coth x$

On differentiating both side w.r.t. 'x'

$\Rightarrow \dfrac{dy}{dx} = \text{cosech}^2 x$

$\Rightarrow \dfrac{dy}{dx} = \left(\dfrac{2}{e^x - e^{-x}}\right)^2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{4}{e^{2x} + e^{-2x} - 2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{4e^{2x}}{e^{4x} + 1 - 2e^{2x}}$

2. (C) The required no. of hand-shakes in party $= 12 \times (12 - 1) = 132$

3. (A) $y = \log_{\cos x} \sin x$

$\Rightarrow y = \dfrac{\log \sin x}{\log \cos x}$

On differentiating both side w.r.t. 'x'

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\log \cos x . \dfrac{\cos x}{\sin x} - \log \sin x . \dfrac{(-\sin x)}{\cos x}}{(\log \cos x)^2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\cot x . \log \cos x + \tan x . \log \sin x}{(\log \cos x)^2}$

4. (D) Given that $x + \dfrac{1}{x} = 2\sin\dfrac{\pi}{12}$

On squaring both side

$\Rightarrow x^2 + \dfrac{1}{x^2} + 2 = 4\sin^2\dfrac{\pi}{12}$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 4\sin^2\dfrac{\pi}{12} - 2$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2\left[1 - 2\sin^2\dfrac{\pi}{12}\right]$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2\cos\left(2 \times \dfrac{\pi}{12}\right)$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2 \times \dfrac{\sqrt{3}}{2} = -\sqrt{3}$

5. (A) Equations $x - y - 3z = -2$, $x + y - z = 3$, $2x + y - 3z = 4$, then

$$D = \begin{vmatrix} 1 & -1 & -3 \\ 1 & 1 & -1 \\ 2 & 1 & -3 \end{vmatrix}$$

$D = 1(-3 + 1) + 1(-3 + 2) - 3(1 - 2) = 0$

$$D_1 = \begin{vmatrix} -2 & -1 & -3 \\ 3 & 1 & -1 \\ 4 & 1 & -3 \end{vmatrix}$$

$= -2(-3 + 1) + 1(-9 + 4) - 3(3 - 4) \neq 0$

Hence equation has no solution.

6. (B) $\lim\limits_{x \to \pi/2} (1 + \cos x)^{\tan x}$ $\quad [1^\infty \text{ form}]$

$\Rightarrow \lim\limits_{x \to \pi/2} \left[(1 + \cos x)^{\frac{1}{\cos x}}\right]^{\sin x}$

$\Rightarrow e^{\lim_{x \to \pi/2} \sin x}$

$\Rightarrow e^{\sin \pi/2} = e$

7. (C) $x^2 + y^2 = 27$

Let $A = xy^2$

$A = x(27 - x^2)$

$A = 27x - x^3$ ....(i)

On differentiating both side w.r.t. 'x'

$\dfrac{dA}{dx} = 27 - 3x^2$ ....(ii)

Again, differentiating

$\dfrac{d^2A}{dx^2} = -6x$ ....(iii)

for maxima and minima

$\dfrac{dA}{dx} = 0$

$27 - 3x^2 = 0$

$x = 3, -3$

from eq. (ii)

$\left(\dfrac{d^2A}{dx^2}\right)_{\text{at } x = 3} = -6 \times 3 = -18$ (maxima)

$\left(\dfrac{d^2A}{dx^2}\right)_{\text{at } x = -3} = -6 \times (-3) = 18$ (minima)

Hence maximum value of $xy^2 = 3 \times 18 = 54$

8. (C) $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$

$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^{n-2} . \cot^2 x \, dx$

$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^{n-2} (\text{cosec}^2 x - 1) \, dx$

$I_n = \int_{\pi/4}^{\pi/2} (\cot x)^{n-2} . \text{cosec}^2 x \, dx - \int_{\pi/4}^{\pi/2} \cot^{n-2} x \, dx$

$I_n = -\left[\frac{(\cot x)^{n-2+1}}{n-2+1}\right]_{\pi/4}^{\pi/2} - I_{n-2}$

$I_n + I_{n-2} = \frac{-1}{n-1}[0-1] = \frac{1}{n-1}$

9. (A) Given equations $x - 2y + 3z = 0$, $2x - y + 2z = 4$ and $3x + y - z = 5$

$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$

by elementary Row method

$[A/B] = \left[\begin{array}{ccc|c} \mathbf{1} & -2 & 3 & 0 \\ 2 & -1 & 2 & 4 \\ 3 & 1 & -1 & 5 \end{array}\right]$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$[A/B] = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & \mathbf{3} & -4 & 4 \\ 0 & 7 & -10 & 5 \end{array}\right]$

$R_3 \to R_3 - \frac{7}{3}R_2$

$[A/B] = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 3 & -4 & 4 \\ 0 & 0 & -2/3 & \frac{-13}{3} \end{array}\right]$

$x - 2y + 3z = 0$ ....(i)

$3y - 4z = 4$ ....(ii)

$\frac{-2}{3}z = \frac{-13}{3}$ ....(iii)

On solving eq (i), (ii) and (iii)

$x = \frac{1}{2}$, $y = 10$, $z = \frac{13}{2}$

10. (C) Let $I = \int_1^2 [x^2]^2 \, dx$

$I = \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 4.dx + \int_{\sqrt{3}}^{2} 9.dx$

$I = [x]_1^{\sqrt{2}} + 4[x]_{\sqrt{2}}^{\sqrt{3}} + 9[x]_{\sqrt{3}}^{2}$

$I = \sqrt{2} - 1 + 4(\sqrt{3} - \sqrt{2}) + 9(2 - \sqrt{3})$

$I = 17 - 3\sqrt{2} - 5\sqrt{3}$

11. (C) Differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} = \frac{3}{\frac{d^2y}{dx^2}}$

$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{d^2y}{dx^2}\right)\frac{dy}{dx} = 3$

Order = 2 and Degree = 3

12. (B) $x^x . y^y = 1$

taking log both side

$\Rightarrow x \log x + y \log y = 0$

On differentiating both side w.r.t. ' x '

$\Rightarrow x \times \frac{1}{x} + \log x + y \times \frac{1}{y}\frac{dy}{dx} + \log y \times \frac{dy}{dx} = 0$

$\Rightarrow (1 + \log x) + (1 + \log y)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{1 + \log x}{1 + \log y}$

13. (C)

14. (B) $x = a^{y + a^{y + a^{y + \cdots\cdots\infty}}}$

$\Rightarrow x = a^{y + x}$

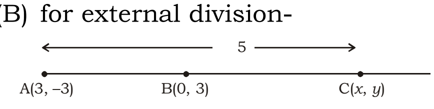
On differentiating both side w.r.t. 'x'

$\Rightarrow 1 = a^{y+x} \log a \left(\frac{dy}{dx} + 1\right)$

$\Rightarrow 1 = x \log a \left(\frac{dy}{dx} + 1\right)$

$\Rightarrow \frac{1}{x \log a} - 1 = \frac{dy}{dx} \Rightarrow \frac{1 - x\log a}{x \log a}$

15. (B) for external division-



$x = \frac{5 \times 0 - 3 \times 2}{5 - 2} = -2$

and $y = \frac{5 \times 3 - 2(-3)}{5 - 2} = 7$

Co-ordinate of C = (–2 , 7)

16. (A) Let $z = \frac{1+i}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$$z = \frac{\left(1+\sqrt{3}\right)+\left(1-\sqrt{3}\right)i}{4}$$

amplitude of $z = \tan^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right)$

$$= \tan^{-1}\left[-\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{12}\right)$$

$$= \tan^{-1}\left[\tan\left(-\frac{\pi}{12}\right)\right] = \frac{-\pi}{12}$$

17. (C) roots of the quadratic equation $2\lambda x^2 + (\lambda-2)x - 1 = 0$ are equal in magnitude but opposite in sign,

Let roots are $\alpha, -\alpha$.

$$\alpha - \alpha = -\frac{\lambda-2}{2\lambda}$$

$$0 = -\frac{\lambda-2}{2\lambda} \Rightarrow \lambda = 2$$

18. (C)

$$\begin{array}{r} 10101001.01 \\ -1001.11 \\ \hline 10011111.10 \end{array}$$

1 0 0 1 1 1 1 1

$1\times2^0=1$

$1\times2^1=2$

$1\times2^2=4$

$1\times2^3=8$

$1\times2^4=16$

$0\times2^5=0$

$0\times2^6=0$

$1\times2^7=128$

159

0.10

$\frac{1}{2} = 1 \times 2^{-1}$

$0 = 1 \times 2^{-2}$

$\frac{1}{2} = 0.5$

Hence $(10011111.10)_2 = (159.5)_{10}$

19. (A) $4.\ {}^7P_r = {}^8C_{r+1}$

$$\Rightarrow 4 \times \frac{7!}{(7-r)!} = \frac{8!}{(r+1)!(7-r)!}$$

$$\Rightarrow \frac{4\times7!}{(7-r)!} = \frac{8\times7!}{(r+1)!(7-r)!}$$

$\Rightarrow (r + 1)! = 2$

$\Rightarrow (r + 1)! = 2!$

$\Rightarrow r + 1 = 2 \Rightarrow r = 1$

20. (B) $I = \int e^a dx$

$I = e^a.x + c$

21. (D) $\begin{bmatrix}11 & x\end{bmatrix}\begin{bmatrix}1 & 0\\3 & -4\end{bmatrix}\begin{bmatrix}-3\\-5\end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix}11 & x\end{bmatrix}\begin{bmatrix}1\times(-3)+0\times(-5)\\3\times(-3)+(-4)+(-5)\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}11 & x\end{bmatrix}\begin{bmatrix}-3\\11\end{bmatrix} = 0$$

$\Rightarrow 11 \times (-3) + x \times 11 = 0 \Rightarrow x = 3$

22. (B) two-digit even numbers

10, 12, 14,............,98

$T_n = a + (n-1)\,d$

$98 = 10 + (n-1) \times 2$

$88 = (n-1) \times 2$

$n - 1 = 44 \Rightarrow n = 45$

Sum of all numbers

$$S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$S_{45} = \frac{45}{2}(2 \times 10 + 44 \times 2)$$

$S_{45} = 45 \times 54 = 2430$

23. (C) given numbers 1, 3, 5..........,39; $n = 20$

$$\text{mean } \bar{x} = \frac{1+3+5+.........+39}{20}$$

$$= \frac{400}{20} = 20$$

$$\sum\left(x-\bar{x}\right)^2 = (1-20)^2 + (3-20)^2 + ... + (39-20)^2$$

$= 361 + 289 + ...+ 289 + 361$

$= 2660$

$$\text{S.D.} = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n}}$$

$$\text{S.D.} = \sqrt{\frac{2660}{20}}$$

$$\text{Variance} = (\text{S.D})^2 = \frac{2660}{20} = 133$$

24. (D) $i^{n+1} + i^{n+3} + i^{n+5} + i^{n+7} + i^{n+9}$

$\Rightarrow i^{n+1}(1 + i^2 + i^4 + i^6 + i^8)$

$\Rightarrow i^{n+1}(1 - 1 + 1 - 1 + 1) = i^{n+1}$

25. (B) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \frac{3\pi}{2}$

$\cos^{-1}x = \frac{\pi}{2}, \quad \cos^{-1}y = \frac{\pi}{2}, \quad \cos^{-1}z = \frac{\pi}{2}$

$\Rightarrow x = \cos\frac{\pi}{2} = 0,\ y = \cos\frac{\pi}{2} = 0,\ z = \cos\frac{\pi}{2} = 0$

Now, $x^{2002} + y^{2002} + z^{2002} = 0 + 0 + 0 = 0$

26. (B) $I = \int_0^{\pi/4} \tan^{-1} x\, dx$

$I = \tan^{-1}x \int 1.dx - \int\left\{\frac{d}{dx}(\tan^{-1}x)\int 1.dx\right\}dx + C$

$I = x.\tan^{-1}x - \int \frac{1}{1+x^2} \times x\, dx + C$

$I = x.\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx + C$

$I = x.\tan^{-1}x - \frac{1}{2}\log(1 + x^2) + C$

27. (A) $I = \int_0^4 \sqrt{x}.e^{\sqrt{x}}\, dx$

Let $\sqrt{x} = t$ when $x \to 0,\ t \to 0$

$\frac{1}{2\sqrt{x}}\, dx = dt$ $x \to 4,\ t \to 2$

$dx = 2t\, dt$

$I = \int_0^2 2e^t.t\, dt$

$I = 2\left[t\int e^t dt - \int\left\{\frac{d}{dt}(t).\int e^t dt\right\}\right]_0^2$

$I = 2\left[t.e^t - \int 1.e^t dt\right]_0^2$

$I = 2\left[t.e^t - e^t\right]_0^2$

$I = 2\left[2.e^2 - e^2 - 0 + e^0\right] = 2\left[e^2 + 1\right]$

28. (B) $\begin{vmatrix} 1 & 1 & 1+z \\ 1+x & 1 & 1 \\ 1 & 1+y & 1 \end{vmatrix}$

$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} 0 & -y & z \\ x & -y & 0 \\ 1 & 1+y & 1 \end{vmatrix}$

$\Rightarrow y(x) + z(x + xy + y)$

$\Rightarrow xy + xz + xyz + yz$

$\Rightarrow xyz + xy + yz + zx$

29. (B) In the expansion of $\left(\frac{2}{3}x + \frac{3}{2x}\right)^8$

$T_{r+1} = {}^8C_r\left(\frac{2}{3}x\right)^{8-r}\left(\frac{3}{2x}\right)^r$

$= {}^8C_r\left(\frac{2}{3}\right)^{8-r}\left(\frac{3}{2}\right)^r . x^{8-2r}$

Now, $8 - 2r = 2 \Rightarrow r = 3$

Coefficient of $x^2 = {}^8C_3\left(\frac{2}{3}\right)^5\left(\frac{3}{2}\right)^3$

$= \frac{8!}{3!5!} \times \frac{2^5}{3^5} \times \frac{3^3}{2^3}$

$= 56 \times \frac{2^2}{3^2} = \frac{224}{9}$

30. (B) Given that mean = 23 and median = 17

We know that

Mode = 3 median – 2 mean

Mode = 3 × 17 – 2 × 23

Mode = 51– 46 = 5

(31-33). Given that

$C(n, r-1) : C(n, r) = 2 : 3$

$\Rightarrow \dfrac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{3}$

$\Rightarrow \frac{r!(n-r)!}{(r-1)!(n-r+1)!} = \frac{2}{3}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{3}$

$\Rightarrow 5r - 2n = 2$ .....(i)

and $C(n+1, r) : C(n+1, r+1) = 3 : 4$

$\Rightarrow \dfrac{\frac{(n+1)!}{r!(n+1-r)!}}{\frac{(n+1)!}{(r+1)!(n-r)!}} = \frac{3}{4}$

$\Rightarrow \frac{(r+1)!(n-r)!}{r!(n+1-r)!} = \frac{3}{4}$

$\Rightarrow \frac{r+1}{n+1-r} = \frac{3}{4}$

$7r - 3n = -1$ ....(ii)

On solving eq. (i) and eq. (ii)

$n = 19,\ r = 8$

31. (B) $n = 19$

32. (C) $r = 8$

33. (A) $\frac{P(n+1,r)}{P(n,r)} = \frac{P(20,8)}{P(19,8)}$

$$= \frac{\frac{20!}{12!}}{\frac{19!}{11!}} = \frac{5}{3}$$

Hence $P(n+1, r) : P(n, r) = 5 : 3$

34. (C) Given that $A + B + C = \pi$

Now, $\sin(A + C) - \sin B$

$\Rightarrow \sin(\pi - B) - \sin B$

$\Rightarrow \sin B - \sin B = 0$

35. (B)

36. (C) Equation of straight line which makes equal intercept on the co-ordinate areas, then $x + y = a$ .....(i)

it passes through the point $(-4, 1)$

$-4 + 1 = a \Rightarrow a = -3$

from eq. (i)

$x + y = -3 \Rightarrow x + y + 3 = 0$

37. (D) $\tan\left(2\tan^{-1}\frac{3}{4} - \frac{\pi}{4}\right)$

$$\Rightarrow \tan\left[\tan^{-1}\frac{24}{7} - \tan^{-1}1\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{24}{7}-1}{1+\frac{24}{7}\times 1}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{17}{7}}{\frac{31}{7}}\right)\right] = \frac{17}{31}$$

38. (B) $\frac{\cos 3A + 3\cos A}{\cos A} - \frac{\sin 3A + 3\sin A}{\sin A}$

$$\Rightarrow \frac{4\cos^3 A - 3\cos A + 3\cos A}{\cos A} - \frac{3\sin A - 4\sin^3 A - 3\sin A}{\sin A}$$

$\Rightarrow 4\cos^2 A + 4\sin^2 A = 4$

39. (A) Length of diagonal = $\sqrt{(24)^2 + (7)^2}$

$$a\sqrt{2} = \sqrt{576 + 49}$$

$$a\sqrt{2} = 25 \Rightarrow a = \frac{25}{\sqrt{2}}$$

Now, Area of square $= a^2$

$$= \frac{25}{\sqrt{2}} \times \frac{25}{\sqrt{2}}$$

$= 312.5$ sq. unit

40. (A) $y = \ln(x - \cos x)$, $z = x + \sin x$

$\frac{dy}{dx} = \frac{1+\sin x}{x - \cos x}$, $\frac{dz}{dx} = 1 + \cos x$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$$\frac{dy}{dz} = \frac{1+\sin x}{x-\cos x} \times \frac{1}{1+\cos x}$$

$$\frac{dy}{dz} = \frac{1+\sin x}{(x-\cos x)(1+\cos x)}$$

41. (C) $s = \sqrt{t^2 - 1}$ .....(i)

On differentiating both side w.r.t. 't'

$$\Rightarrow \frac{ds}{dt} = \frac{1 \times 2t}{2\sqrt{t^2-1}}$$

$$\Rightarrow \frac{ds}{dt} = \frac{t}{\sqrt{t^2-1}}$$

$$\Rightarrow \frac{d^2s}{dt^2} = \frac{\sqrt{t^2-1}.1 - t.\frac{1\times 2t}{2\sqrt{t^2-1}}}{\left(\sqrt{t^2-1}\right)^2}$$

$$\Rightarrow \frac{d^2s}{dt^2} = \frac{t^2 - 1 - t^2}{\sqrt{t^2-1}.(t^2-1)}$$

$$\Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{(t^2-1)^{3/2}}$$

$$\Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{s^3}$$

42. (B) Let locus of point = (h, k)

Now, $\frac{4h + 3k - 5}{\sqrt{4^2+3^2}} = \frac{12h + 5k + 6}{\sqrt{12^2 + 5^2}}$

$$\Rightarrow \frac{4h+3k-5}{5} = \frac{12h+5k+6}{13}$$

On solving

$\Rightarrow 8h - 14h + 95 = 0$

locus of a point

$8x - 14y + 95 = 0$

43. (A) Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $z = \cot^{-1}x$

$x = \cot z$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cot^2 z}-1}{\cot z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\operatorname{cosec} z - 1}{\cot z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1-\sin z}{\cos z}\right)$

$\Rightarrow y = \tan^{-1}\left[\frac{1-\cos\left(\frac{\pi}{2}-z\right)}{\sin\left(\frac{\pi}{2}-z\right)}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4}-\frac{z}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{z}{2}\right).\cos\left(\frac{\pi}{4}-\frac{z}{2}\right)}\right]$

$\Rightarrow y = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{z}{2}\right)\right]$

$\Rightarrow y = \frac{\pi}{4}-\frac{z}{2}$

On differentiating both side w.r.t. 'z'

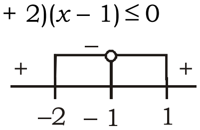
$\Rightarrow \frac{dy}{dz} = -\frac{1}{2}$

44. (C) $\frac{\sqrt{2-x-x^2}}{x^2+2x+1}$

Now, $2 - x - x^2 \geq 0$ and $x^2 + 2x + 1 \neq 0$

$x^2 + x - 2 \leq 0$ $(x + 1)^2 \neq 0$

$(x + 2)(x - 1) \leq 0$ $x \neq -1$



domain = $[-2, 1] - \{-1\}$

45. (D) Vectors $\vec{x} = \hat{i} - 2a\hat{j} + 4\hat{k}$, $\vec{y} = b\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{z} = 6\hat{i} - 3\hat{j} + c\hat{k}$ are perpendicular to each other,

then, $\vec{x}.\vec{y} = 0$

$b - 6a + 4 = 0$ .....(i)

$\vec{y}.\vec{z} = 0$

$6b - 9 + c = 0$ ....(ii)

and $\vec{x}\cdot\vec{z} = 0$

$6 + 6a + 4c = 0$ .....(iii)

On solving eq(i), (ii) and (iii)

$a = 1$, $b = 2$, $c = -3$

46. (C) $\lim_{x \to 8} \frac{x-8}{64-x^2}$ $\left[\frac{0}{0}\right]$ form

by L- Hospital's Rule

$\Rightarrow \lim_{x \to 8} \frac{1}{-2x} = \frac{1}{-2\times 8} = \frac{-1}{16}$

47. (B) Given data 8, 16, 32, 64, 128, 256, 512, 1024, $n = 8$

$\Sigma x = 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 = 2040$

$\text{mean}\left(\bar{x}\right) = \frac{\Sigma x}{n}$

$\left(\bar{x}\right) = \frac{2040}{8} = 255$

$\Sigma\left|x-\bar{x}\right| = |8-255| + |16-255| + |32-255| + |64-255| + |128-255| + |256-255| + |512-255| + |1024-255|$

= 247 + 239 + 223 + 191 + 127 + 1 + 257 + 769

= 2054

$\text{Mean - deviation} = \frac{\Sigma\left|x-\bar{x}\right|}{n}$

$= \frac{2054}{8} = 256.75$

48. (B) In the expansion of $\left(\frac{3}{2x}-\frac{x}{3}\right)^6$

$T_{r+1} = {}^6C_r\left(\frac{3}{2x}\right)^{6-r}\left(-\frac{x}{3}\right)^r$

$= {}^6C_r\left(\frac{3}{2}\right)^{6-r}\left(\frac{-1}{3}\right)^r x^{2r-6}$

Now, $2r - 6 = 0 \Rightarrow r = 3$

The required term = 3 + 1 = 4^{th}

49. (D) In ΔABC, $a = \sqrt{6}$, $c = 2$ and B = 75°

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \cos 75 = \frac{6 + 4 - b^2}{2 \times \sqrt{6} \times 2}$$

$$\Rightarrow \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{10 - b^2}{4\sqrt{6}}$$

$\Rightarrow b^2 = 4 + 2\sqrt{3} \Rightarrow b = \sqrt{3} + 1$

by Sine Rule -

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{\sqrt{6}} = \frac{\sin 75}{\sqrt{3} + 1}$$

$$\frac{\sin A}{\sqrt{6}} = \frac{\sqrt{3} + 1}{2\sqrt{2}\left(\sqrt{3} + 1\right)}$$

$$\sin A = \frac{\sqrt{3}}{2} \Rightarrow A = 60°$$

and C = 180 – 60 – 75 $\Rightarrow$ C = 45°

50. (C) 1 1 1 0

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$1 \times 2^2 = 4$
$1 \times 2^3 = 8$
14

0.1101

$\frac{1}{2} = 1 \times 2^{-1}$
$\frac{1}{4} = 1 \times 2^{-2}$
$0 = 0 \times 2^{-3}$
$\frac{1}{16} = 1 \times 2^{-4}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{16} = 0.8125$

$(1110)_2 = (14)_{10}$, $(0.1101)_2 = 0.8125$

Hence $(1110.1101)_2 = (14.8125)_{10}$

51. (C) digits 1, 0, 2, 4, 9, 8,7

6	6	5	4

= 6 × 6 × 5 × 4 = 720

'0' can not put here.

52. (C) $I = \int_0^{\pi/4} \tan^4 x \, dx$

$$I = \int_0^{\pi/4} \tan^2 x . \tan^2 x \, dx$$

$$I = \int_0^{\pi/4} \tan^2 x (\sec^2 x - 1) dx$$

$$I = \int_0^{\pi/4} (\tan x)^2 . \sec^2 x \, dx - \int_0^{\pi/4} \tan^2 x \, dx$$

$$I = \int_0^{\pi/4} (\tan x)^2 . \sec^2 x \, dx - \int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$I = \left[\frac{(\tan x)^3}{3} - \tan x + x\right]_0^{\pi/4}$$

$$I = \left[\left(\frac{1}{3} - 1 + \frac{\pi}{4}\right) - 0\right] = \frac{3\pi - 8}{12} \text{ sq. unit}$$

53. (A) $\omega^{1000} + \omega^{2000} + \omega^{3000}$

$\Rightarrow \omega^{3\times333+1} + \omega^{3\times666+2} + \omega^{3\times1000+0}$

$\Rightarrow \omega + \omega^2 + \omega^0 = 0 \quad [\because 1 + \omega + \omega^2 = 0]$

54. (B) Series 1.2 + 2.3 + 3. 4 +..........+ $n(n + 1)$

$T_n = n(n + 1)$

Now $S_n = \Sigma T_n$

$S_n = \Sigma n(n + 1)$

$S_n = \Sigma (n^2 + n)$

$S_n = \Sigma n^2 + \Sigma n$

$$S_n = \frac{n}{6}(n + 1)(2n + 1) + \frac{n(n+1)}{2}$$

$$S_n = \frac{n(n+1)}{6}(2n + 1 + 3)$$

$$S_n = \frac{n(n+1)}{6} \times (2n + 4)$$

$$S_n = \frac{n(n+1)(n+2)}{3}$$

55. (D) $I = \int e^x \left[(x+1)^2 \tan^{-1} x + 1\right] dx$

$$I = \int e^x \left[(x^2 + 1 + 2x)\tan^{-1} x + 1\right] dx$$

$$I = \int e^x \left[\left\{(1 + x^2)\tan^{-1} x\right\} + \left\{1 + 2x \tan^{-1} x\right\}\right] dx$$

$I = e^x (1 + x^2) \tan^{-1} x + c$

$$\left[\because \int e^x \left[f(x) + f'(x)\right] dx = e^x . f(x) + c\right]$$

56. (B) $\left(1 - \cos\frac{\pi}{4}\right)\left(1 - \cos\frac{3\pi}{4}\right)\left(1 - \cos\frac{5\pi}{4}\right)\left(1 - \cos\frac{7\pi}{4}\right)$

$$\Rightarrow \left(1 - \cos\frac{\pi}{4}\right)\left(1 + \cos\frac{\pi}{4}\right)\left(1 + \cos\frac{\pi}{4}\right)\left(1 - \cos\frac{\pi}{4}\right)$$

$$\Rightarrow \left(1 - \cos^2\frac{\pi}{4}\right)\left(1 - \cos^2\frac{\pi}{4}\right)$$

$$\Rightarrow \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{2}\right) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

57. (B) Let $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = 4\hat{j} + 3\hat{k}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\cdot\vec{b}|} = \left|\frac{3-16+3}{\sqrt{3^2+(-4)^2}\sqrt{4^2+3^2}}\right|$$

$$\cos\theta = \left|\frac{-10}{5\times 5}\right|$$

$$\cos\theta = \frac{2}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{5}\right)$$

58. (B) $\begin{bmatrix} a & b & c \\ m & n & o \end{bmatrix}_{2\times 3} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}_{3\times 1} \quad [p \quad q]_{1\times 2}$

Hence order = 2 × 2

59. (A) differential equation

$$\frac{dy}{dx} + y.\tan x = \sec x$$

here P = $\tan x$ and Q = $\sec x$

I.F. = $e^{\int P.dx}$

$= e^{\int \tan x\, dx}$

$= e^{\int \log\sec x} = \sec x$

Solution of differential equation

$y \times$ I.F. = $\int Q \times I.F.dx$

$\Rightarrow y \times \sec x = \int \sec x.\sec x\, dx$

$\Rightarrow y \times \sec x = \tan x + c$

$\Rightarrow y = \sin x + c.\cos x$

60. (B) The required Probability $= \frac{1+1}{7} = \frac{2}{7}$

61. (B) Given that $U = \{1,2,3,4,5,6,7,8,9\}$,

$A = \{7,8,3\}$, $B = \{3,8,9\}$ and $C = \{9,3,4\}$

Now, $(A \cup B) = \{3,7,8,9\}$, $(B \cap C) = \{3\}$

and $(A \cap C) = \{3\}$

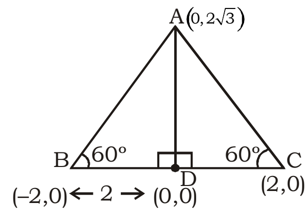
$\{(A \cup B) - (B \cap C)\} \times (A \cap C)$

$= [\{3,7,8,9\} - \{3\}] \times \{3\}$

$= \{7,8,9\} \times \{3\}$

$= \{(7,3),(8,3),(9,3)\}$

62. (C)

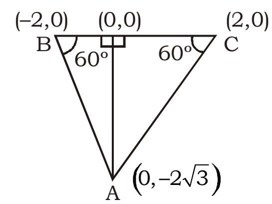


In ΔABD

$$\tan 60^\circ = \frac{AD}{BD}$$

$$\sqrt{3} = \frac{AD}{2} \Rightarrow AD = 2\sqrt{3}$$

Similarly



Hence A = $(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$

63. (C) Given that $S_n = n^2 + 3n - 2$

$S_{n-1} = (n-1)^2 + 3(n-1) - 2$

$S_{n-1} = n^2 + n - 4$

Now, $T_n = S_n - S_{n-1}$

$T_n = n^2 + 3n - 2 - n^2 - n + 4$

$T_n = 2n + 2$

$T_7 = 2 \times 7 + 2 = 16$

64. (A) $= \begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}\begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9+\alpha^2 & 3\alpha+3\alpha \\ 3\alpha+3\alpha & \alpha^2+9 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9+\alpha^2 & 6\alpha \\ 6\alpha & \alpha^2+9 \end{bmatrix}$$

Given that det (A²) = 0

$$\Rightarrow \begin{bmatrix} 9+\alpha^2 & 6\alpha \\ 6\alpha & \alpha^2+9 \end{bmatrix} = 0$$

$\Rightarrow (9+\alpha^2)^2 - 36\alpha^2 = 0$

$\Rightarrow 81 + \alpha^4 + 18\alpha^2 - 36\alpha^2 = 0$

$\Rightarrow \alpha^4 - 18\alpha^2 + 81 = 0$

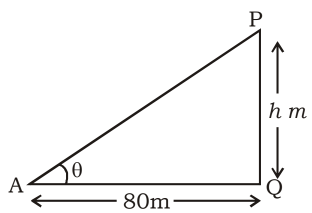
$\Rightarrow (\alpha^2 - 9)^2 = 0$

$\Rightarrow \alpha^2 = 9 \Rightarrow \alpha \pm 3$

65. (C) Let $\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\theta = \frac{3}{5} \Rightarrow \tan\theta = \frac{3}{4}$

Let height of light-house = h m



In ΔAPQ

$\tan\theta = \frac{PQ}{AQ}$

$\Rightarrow \frac{3}{4} = \frac{h}{80} \Rightarrow h = 60$

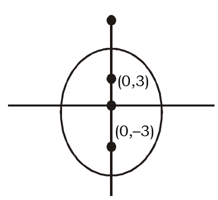
Hence height of light-house = 60 m

66. (C) Given that

$e = \frac{1}{2}$

and foci = (0, ± 3)

$be = 3 \Rightarrow b \times \frac{1}{2} = 3 \Rightarrow b = 6$



Now, $e^2 = 1 - \frac{a^2}{b^2}$

$\Rightarrow \frac{1}{4} = 1 - \frac{a^2}{36} \Rightarrow \frac{a^2}{36} = \frac{3}{4} \Rightarrow a^2 = 27$

equation of ellipse

$\frac{x^2}{27} + \frac{y^2}{36} = 1$

$\Rightarrow 4x^2 + 3y^2 = 108$

67. (A) S = 0.2 + 0.22 + 0.222 +...............n terms

$S = \frac{2}{9}[0.9 + 0.99 + 0.999 + ...n \text{ terms}]$

$S = \frac{2}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...n \text{ terms}\right]$

$S = \frac{2}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + ...n \text{ terms}\right]$

$S = \frac{2}{9}\left[(1 + 1 + ...n \text{ terms}) - \left(\frac{1}{10} + \frac{1}{100} + ...n \text{ terms}\right)\right]$

$S = \frac{2}{9}\left[n - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}\right]$

$S = \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

$S = \frac{2}{81}\left[9n - 1 + \frac{1}{10^n}\right]$

68. (B) Let know that

$(1 + x)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

$n = 8, \quad x = 1$

$(1 + 1)^8 = {}^8C_0 + {}^8C_1 + {}^8C_2 + ...{}^8C_8$

Hence ${}^8C_0 + {}^8C_1 + {}^8C_2 + ...{}^8C_8 = 2^8 = 256$

69. (C) $a + (m - 1)d = n$ ....(i)

and $a + (n - 1)d = m$....(ii)

On solving eq(i) and eq(ii)

$a = m + n - 1, \quad d = -1$

Now $T_{mn} = a + (mn - 1)d$

$T_{mn} = m + n - 1 + (mn - 1) \times (-1)$

$T_{mn} = m + n - mn$

70. (C) $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

On squaring both side

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow 1 - \cos^2\theta + 1 - \sin^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow 2 - 2\cos^2\theta = \cos^2\theta + \sin^2\theta - 2\sin\theta.\cos\theta$

$\Rightarrow 2\sin^2\theta = (\sin\theta - \cos\theta)^2$

$\Rightarrow \sin\theta - \cos\theta = \sqrt{2}\sin\theta$

71. (D) $(x^3y + xy - y)\, dx = xdy$

$\Rightarrow (x^3y + xy)\, dx = xdy + y\, dx$

$\Rightarrow xy\,(x^2 + 1)\, dx = xdy + y\, dx$

$\Rightarrow (x^2 + 1)\, dx = \frac{xdy + ydx}{xy}$

$\Rightarrow (x^2 + 1)\, dx = d[\log(xy)]$

On integrating

$\frac{x^3}{3} + x = \log xy + \log c$

$\frac{x^3 + 3x}{3} = \log(cxy)$

$cxy = e^{\frac{x^2+3x}{3}}$

72. (D)

73. (C) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$A^2 = A^2.A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Now, $A^3 - 6A^2 + 7A$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= -2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -2I$$

74. (D) cot85° + tan40° + tan40°.cot85°

$\Rightarrow$ tan5° + tan 40° + tan40°.tan5°

We know that

$$\tan(5 + 40) = \frac{\tan 5 + \tan 40}{1 - \tan 5.\tan 40}$$

$$1 = \frac{\tan 5 + \tan 40}{1 - \tan 5.\tan 40}$$

$\Rightarrow$ 1 – tan5.tan40 + tan40.tan5 = 1

75. (B) We know that

$\cos^2 A - \cos^2 B = \sin(A + B).\ \sin(B - A)$

Now, $\cos^2 6 - \cos^2 24 = \sin(6 + 24).\ \sin(24 - 6)$

= sin30. sin18

$$= \frac{1}{2} \times \frac{\sqrt{5} - 1}{4} = \frac{\sqrt{5} - 1}{8}$$

76. (D) $\sin\theta = \frac{8}{17}$, $\sin\phi = \frac{15}{17}$

$\cos\theta = \frac{15}{17}$, $\cos\phi = \frac{8}{17}$

$\cos(\theta - \phi) = \cos\theta.\cos\phi + \sin\theta.\sin\phi$

$$= \frac{15}{17} \times \frac{8}{17} + \frac{8}{17} \times \frac{15}{17} = \frac{240}{289}$$

Now, $\sin\left(\frac{\theta - \phi}{2}\right) = \sqrt{\frac{1 - \cos(\theta - \phi)}{2}}$

$$= \sqrt{\frac{1 - \frac{240}{289}}{2}}$$

$$= \sqrt{\frac{49}{2 \times 289}} = \frac{7}{17\sqrt{2}}$$

77. (C) Let know that

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\ldots\ldots\ldots$$

put $x = 0.4$

$$\log(1 + 0.4) = 0.4 - \frac{(0.4)^2}{2} + \frac{(0.4)^3}{3} - \ldots\ldots$$

$$\log(1.4) = 0.4 - \frac{(0.4)^2}{2} + \frac{(0.4)^3}{3} - \ldots\ldots\ldots$$

$$0.4 - \frac{(0.4)^2}{2} + \frac{(0.4)^3}{3} - \ldots\ldots\ldots\ldots = \log\left(\frac{7}{5}\right)$$

78. (A) Equation $px^2 + qx + r = 0$

root are α, 4α.

then $\alpha + 4\alpha = \frac{-q}{p}$

$$5\alpha = \frac{-q}{p} \Rightarrow \alpha = \frac{-q}{5p}$$

and $\alpha.4\alpha = \frac{r}{p}$

$$4\alpha^2 = \frac{r}{p}$$

$$4 \times \left(\frac{-q}{5p}\right)^2 = \frac{r}{p}$$

$$\frac{4q^2}{25p^2} = \frac{r}{p}$$

$4q^2 = 25pr$

79. (B) $\lim_{x \to 0} \frac{\sin x}{x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x \to 0} \frac{\cos x}{1}$

$\Rightarrow \cos 0 = 1$

80. (C) $(1 - \sin x + \cos x)^2 = 1 + \sin^2 x + \cos^2 x - 2\sin x - 2\sin x.\cos x + 2\cos x$

$= 1 + 1 - 2\sin x - 2\sin x.\cos x + 2\cos x$

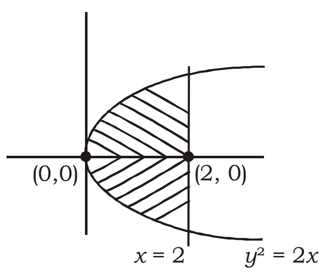
$= 2(1 - \sin x) + 2\cos x(1 - \sin x)$

$= (1 - \sin x)(2 + 2\cos x)$

$= 2(1 - \sin x)(1 + \cos x)$

81. (C) Conic

$y^2 = 2x \Rightarrow y = \sqrt{2}\sqrt{x}$ and line $x = 2$



Area $= 2\int_0^2 y\,dx$

$= 2\int_0^2 \sqrt{2}\sqrt{x}\,dx$

$= 2\sqrt{2}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^2$

$= 2\sqrt{2} \times \frac{2}{3}\left[2^{3/2} - 0\right] = \frac{16}{3}$ sq. unit

82. (B) (a) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right) = \tan^{-1}\left(\tan\left(\frac{-\pi}{4}\right)\right) = \frac{\pi}{4}$

(b) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\pi - \frac{\pi}{3}\right)$

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

(c) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right) = \cos^{-1}\left(\cos\left(\pi + \frac{\pi}{3}\right)\right)$

$= \cos^{-1}\left(-\cos\frac{\pi}{3}\right) = \cos^{-1}\left(\cos\left(\pi - \frac{\pi}{3}\right)\right)$

$= \cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

(d) $\cot^{-1}\left(\cot\frac{5\pi}{4}\right) = \cot^{-1}\left[\cot\left(\pi + \frac{\pi}{4}\right)\right]$

$= \cot^{-1}\left(\cot\frac{\pi}{4}\right) = \frac{\pi}{4}$

83. (C) $\tan\left(2\tan^{-1}\frac{2}{7} + \tan^{-1}\frac{3}{7}\right)$

$\Rightarrow \left[\tan^{-1}\frac{28}{45} + \tan^{-1}\frac{3}{7}\right]$

$\Rightarrow \left[\tan^{-1}\left(\frac{331}{231}\right)\right] = \frac{331}{231}$

84. (A) In ΔABC, A(3, –2), B(–3, 4) and C(–1, 0)

Co-ordinate of centroid

$\bar{x} = \frac{3 - 3 - 1}{3} = \frac{-1}{3}$, $\bar{y} = \frac{-2 + 4 + 0}{3} = \frac{2}{3}$

Co-ordinate of centroid $= \left(\frac{-1}{3}, \frac{2}{3}\right)$

85. (C) Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{5}$

$P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{3}{5}$

The Probability $= P\left(A \cap \bar{B}\right) + P\left(\bar{A} \cap B\right)$

$= P(A).P(\bar{B}) + P(\bar{A}).P(B)$

$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{5} = \frac{5}{10} = \frac{1}{2}$

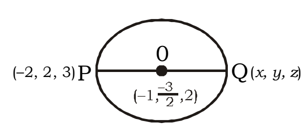
86. (B) Equation of sphere

$x^2 + y^2 + z^2 + 2x + 3y - 4z = 15$

$u = 1$, $v = \frac{3}{2}$, $w = -2$

centre $\left(-1, \frac{-3}{2}, 2\right)$

Let co-ordinate of Q = (x, y, z)



$\frac{x - 2}{2} = -1$, $\frac{y + 2}{2} = \frac{-3}{2}$, $\frac{z + 3}{2} = 2$

$x = 0$, $y = -5$, $z = 1$

Hence end point of a diameter = (0, – 5, 1)

87. (B) $f(x) = \begin{cases} 3 - x^2, & 0 \le x < 1 \\ 2\lambda + x, & 1 \le x < 2 \end{cases}$ is continuous at $x = 1$, then

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$3 - 1 = 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$

88. (B) $\begin{vmatrix} \frac{1}{x} & x^2 & yz \\ \frac{1}{y} & y^2 & xz \\ \frac{1}{z} & z^2 & xy \end{vmatrix}$

$\Rightarrow \frac{1}{xyz}\begin{vmatrix} 1 & x^3 & xyz \\ 1 & y^3 & xyz \\ 1 & z^3 & xyz \end{vmatrix}$

$\Rightarrow \frac{xyz}{xyz}\begin{vmatrix} 1 & x^3 & 1 \\ 1 & y^3 & 1 \\ 1 & z^3 & 1 \end{vmatrix} = 0$

$[\because \text{Two columns are identical.}]$

89. (D) Let A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$|A| = 1(1 - 0) - 2(3 - 1) + 1(0 - 1)$

$= 1 - 4 - 1 = -4 \neq 1$

Determinent of A is not equal to 1.

Hence A is not an elementary matrix.

90. (B) Given that $z = 1 + i$

$f(z) = \frac{6 - z^2}{1 - z}$

$f(z) = \frac{6 - (1+i)^2}{1 - 1 - i}$

$f(z) = \frac{6 - 2i}{-i} \times \frac{i}{i}$

$f(z) = 2 + 6i$

$|f(z)| = \sqrt{2^2 + 6^2}$

$|f(z)| = \sqrt{4 + 36} = 2\sqrt{10}$

91. (C) The total no. of arrangement $= \frac{9!}{2!2!} = \frac{9!}{4}$

No. of arrangement when I's come together $= \frac{8!}{2!} = \frac{8!}{2}$

No. of arrangement when I's don't come together $= \frac{9!}{4} - \frac{8!}{2} = \frac{7 \times 8!}{4}$

The required Probability $= \dfrac{\frac{7 \times 8!}{4}}{\frac{9!}{4}} = \frac{7}{9}$

92. (B) $y = \sec(\tan^{-1} x)$ ....(i)

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \sec(\tan^{-1}x).\ \tan(\tan^{-1}x).\ \frac{1}{1+x^2}$

$\frac{dy}{dx} = \frac{y.x}{1+x^2}$ [from eq (i)]

$(1 + x^2)\frac{dy}{dx} = xy$

93. (D) $(A \cap B) \cup (B \cap C)$

94. (A) Given series

$1.2 + 2.2^2 + 3.2^3 + \ldots\ldots 10.2^{10}$ is an A.G.P. , then

$a = 1,\ b = 2,\ r = 2,\ d = 1,\ n = 10$

We know that

$S_n = b\left[\frac{a - [a + (n-1)d]r^n}{1 - r} + \frac{dr(1 - r^{n-1})}{(1-r)^2}\right]$

$S_{10} = 2\left[\frac{1 - [1 + (10-1) \times 1]2^{10}}{1 - 2} + \frac{1 \times 2(1 - 2^{10-1})}{(1-2)^2}\right]$

$S_{10} = 2\left[\frac{1 - 10 \times 2^{10}}{-1} + \frac{2(1 - 2^9)}{(-1)^2}\right]$

$S_{10} = 2\left[-1 + 10.2^{10} + 2 - 2^{10}\right]$

$S_{10} = 2\left[1 + 9.2^{10}\right]$

95. (B)

96. (A) $z = \frac{2 - i}{(1 - 2i)^2} = \frac{2 - i}{-3 - 4i}$

$z = \frac{i - 2}{3 + 4i} \times \frac{3 - 4i}{3 - 4i} \Rightarrow z = \frac{-2 + 11i}{13}$

conjugate of $z = \frac{-2 - 11i}{13}$

97. (D) $\begin{bmatrix} 2x & 3 \\ -5 & 3x \end{bmatrix} = \begin{bmatrix} 4y+8 & y+6 \\ 2y+1 & y-3 \end{bmatrix}$

On comparing

$2x = 4y + 8,\quad 3 = y + 6$

$-5 = 2y + 1,\quad 3x = y - 3$

On solving

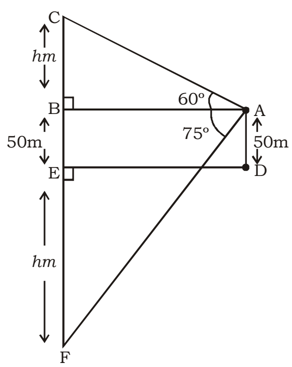
$y = -3$ and $x = -2$

98. (C) Equation $\lambda x^2 + 3x + (\lambda - 1) = 0$

product of roots = – 2

$$\frac{\lambda - 1}{\lambda} = -2 \Rightarrow \lambda = \frac{1}{3}$$

99. (C) Let BC = h m



In ΔABC

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{h}{AB} \quad \text{.............(i)}$$

In ΔABF

$$\tan 75° = \frac{BF}{AB}$$

$$2 + \sqrt{3} = \frac{h+50}{h/\sqrt{3}} \quad \text{............(ii)}$$

$2h + h\sqrt{3} = h\sqrt{3} + 50\sqrt{3} \Rightarrow h = 25\sqrt{3}$

height of the aeroplane above the lake level = $h + 50$

$= 25\sqrt{3} + 50 = 25(2 + \sqrt{3})$m

100. (D) According to question

$$\theta \times \theta \times \frac{\pi}{180} = \frac{125\pi}{4}$$

$$\theta^2 = \frac{180 \times 125}{4} \Rightarrow \theta = 75°$$

101. (B) Given that $\int x^2.\ln x\,dx = \frac{x^3}{a}.\ln x + \frac{x^3}{b} + C$

$$\int x^2.\ln x\,dx = \ln x \int x^2\,dx - \int\left\{\frac{d}{dx}(\ln x).\int x^2 dx\right\} + C$$

$$\int x^2.\ln x\,dx = \frac{x^3}{3}.\ln x - \int \frac{1}{x} \times \frac{x^3}{3}dx + C$$

$$\int x^2.\ln x\,dx = \frac{x^3}{3}.\ln x - \frac{1}{3}.\frac{x^3}{3} + C$$

$$\int x^2.\ln x\,dx = \frac{x^3}{3}.\ln x - \frac{x^3}{9} + C$$

Compare with eq (i)

$a = 3,\quad b = -9$

102. (C) Let $a - ib = \sqrt{-7 - 24i}$

On squaring both side

$(a^2 - b^2) - 2abi = -7 - 24i$

$a^2 - b^2 = -7$ and $2ab = 24$ ....(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = (-7)^2 + (24)^2$

$(a^2 + b^2)^2 = (25)^2$

$a^2 + b^2 = 25$ ......(ii)

from eq(i) and eq(i)

$2a^2 = 18,\quad 2b^2 = 32$

$a = \pm 3,\quad b = \pm 4$

Hence $\sqrt{-7 - 24i} = \pm (3 - 4i)$

103. (B) $(\log_3 x)(\log_{x^2} 4) = \log_3 y^2$

$$\Rightarrow \frac{\log x}{\log 3} \times \frac{\log 4}{\log x^2} = \frac{\log y^2}{\log 3}$$

$$\Rightarrow \frac{\log x}{\log 3} \times \frac{\log 4}{2\log x} = \frac{\log y^2}{\log 3}$$

$$\Rightarrow \frac{2\log 2}{2} = \log y^2$$

$\Rightarrow y^2 = 2 \Rightarrow y = \sqrt{2}$

104. (B) We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$

$\Rightarrow 3 + \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

105. (D) $n(S) = {}^{10}C_3 = 120$

$n(E) = {}^4C_1 \times {}^4C_1 \times {}^2C_1 + {}^4C_2 \times {}^4C_1 \times {}^2C_0 + {}^4C_2 \times {}^4C_0 \times {}^2C_1 + {}^4C_3 \times {}^4C_0 \times {}^2C_0$

$= 4 \times 4 \times 2 + 6 \times 4 \times 1 + 6 \times 1 \times 2 + 4 \times 1 \times 1 = 72$

The required Probability $= \frac{n(E)}{n(S)} = \frac{72}{120} = \frac{3}{5}$

106. (C) $\begin{vmatrix} 8! & 9! & 7! \\ 9! & 10! & 8! \\ 10! & 11! & 9! \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 8\times7! & 9\times8\times7! & 7! \\ 9\times8! & 10\times9\times8! & 8! \\ 10\times9! & 11\times10\times9! & 9! \end{vmatrix}$

$\Rightarrow 7! \times 8! \times 9! \begin{vmatrix} 8 & 72 & 1 \\ 9 & 90 & 1 \\ 10 & 110 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow 7! \times 8! \times 9! \begin{vmatrix} 8 & 72 & 1 \\ 1 & 18 & 0 \\ 2 & 38 & 0 \end{vmatrix}$

$\Rightarrow 7! \times 8! \times 9!\ [0 - 0 + 1\ (38 - 36)]$

$\Rightarrow 2 \times 7! \times 8! \times 9!$

107. (B) Equation

$x^3 + 4x^2 - 9x - 36 = 0$

Let roots are $\alpha, -\alpha, \beta$

$\alpha - \alpha + \beta = -4 \Rightarrow \beta = -4$

and $\alpha(-\alpha)\beta = -(-36)$

$-\alpha^2(-4) = 36$

$\alpha = -3, 3$

Hence roots are – 3, 3, – 4.

108. (C) $\cot\theta + \cos\theta = x$

$\cot^2\theta + \cos^2\theta + 2\cot\theta . \cos\theta = x^2$

and $\cot\theta . \cos\theta = y$

$\cot^2\theta + \cos^2\theta - 2\cot\theta . \cos\theta = y^2$

$x^2 - y^2 = 4\cot\theta . \cos\theta$ and $xy = \cot^2\theta - \cos^2\theta$

$x^2 - y^2 = 4\dfrac{\cos\theta}{\sin\theta}.\cos\theta$, $\quad xy = \dfrac{\cos^4\theta}{\sin^2\theta}$

$x^2 - y^2 = 4\dfrac{\cos^2\theta}{\sin\theta}$, $\quad \sqrt{xy} = \dfrac{\cos^2\theta}{\sin\theta}$

$x^2 - y^2 = 4\sqrt{xy}$

109. (C) cos6.cos42.cos66.cos78

$\Rightarrow \dfrac{1}{4}$[2cos6.cos66][2cos42.cos78]

$\Rightarrow \dfrac{1}{4}$[cos(6+66) + cos(66 – 6)]

[cos(42 + 78) + cos(78 – 42)]

$\Rightarrow \dfrac{1}{4}$[cos72 + cos60] [cos120 + cos36]

$\Rightarrow \dfrac{1}{4}$[sin18 + cos60] [– sin30 + cos 36]

$\Rightarrow \dfrac{1}{4}\left[\dfrac{\sqrt{5}-1}{4} + \dfrac{1}{2}\right]\left[-\dfrac{1}{2} + \dfrac{\sqrt{5}+1}{4}\right]$

$\Rightarrow \dfrac{1}{4}\left[\dfrac{\sqrt{5}+1}{4}\right]\left[\dfrac{\sqrt{5}-1}{4}\right]$

$\Rightarrow \dfrac{1}{4} \times \dfrac{5-1}{16} = \dfrac{1}{16}$

110. (A) $\sin x + \sin y = 2(\cos y - \cos x)$ .....(i)

x replace by $-x$ and y replace by $-y$

$\Rightarrow \sin(-x) + \sin(-y) = 2[\cos(-y) - \cos(-x)]$

$\Rightarrow -\sin x - \sin y = 2[\cos y - \cos x]$

$\Rightarrow -\sin x - \sin y = \sin x + \sin y$ [from eq (i)]

$\Rightarrow 2(\sin x + \sin y) = 0$

$\Rightarrow \sin x = -\sin y$

$\Rightarrow \sin x = \sin(-y) \Rightarrow x = -y$

Now, $\dfrac{\cos 2x}{\cos 2y} = \dfrac{\cos 2(-y)}{\cos 2y}$

$\dfrac{\cos 2x}{\cos 2y} = \dfrac{\cos 2y}{\cos 2y} = 1$

111. (D) Given that $\theta = 140°$

Now, $x = \sin\theta + \cos\theta$

x = sin140 + cos 140

x = sin140 + cos(90 + 50)

x = sin140 – sin50

We know that

140 > 50

sin140 > sin 50

Hence $x > 0$

112. (C) $\dfrac{x}{b}\sin\theta - \dfrac{y}{a}\cos\theta = 2$

$\Rightarrow \dfrac{x^2}{b^2}\sin^2\theta + \dfrac{y^2}{a^2}\cos^2\theta - \dfrac{2xy}{ab}\sin\theta.\cos\theta = 4$ ..(i)

and $\dfrac{y}{a}\sin\theta + \dfrac{x}{b}\cos\theta = 2$

$\Rightarrow \dfrac{y^2}{a^2}\sin^2\theta + \dfrac{x^2}{b^2}\cos^2\theta + \dfrac{2xy}{ab}\sin\theta.\cos\theta = 4$ ..(ii)

from eq(i) and eq (ii)

$\left(\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2}\right)\sin^2\theta + \left(\dfrac{y^2}{a^2} + \dfrac{x^2}{b^2}\right)\cos^2\theta = 8$

$\Rightarrow \left(\frac{x^2}{b^2}+\frac{y^2}{a^2}\right)(\sin^2\theta + \cos^2\theta) = 8$

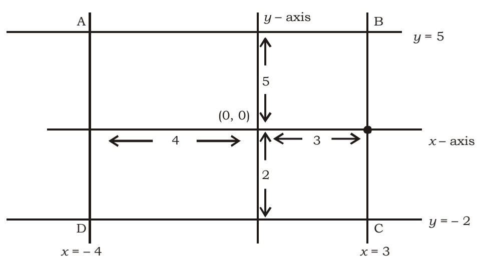
$\Rightarrow \frac{x^2}{b^2}+\frac{y^2}{a^2} = 8$

113. (C) ${}^nC_{r-1} + 2\,{}^nC_r + {}^nC_{r+1}$

$\Rightarrow {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$

$\Rightarrow {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

114. (B)



Area of ABCD = AB × BC

= 7 × 7 = 49 sq. unit

115. (B)

116. (A) In ΔABC, cotA, CotB and cotC are in A.P., then

2cotB = cotA + cotC

$\frac{2\cos B}{\sin B} = \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}$

$\frac{2\cos B}{\sin B} = \frac{\cos A.\sin C + \sin A.\cos C}{\sin A.\sin C}$

$\frac{2\cos B}{\sin B} = \frac{\sin(A + C)}{\sin A.\sin C}$

$\frac{2\cos B}{\sin B} = \frac{\sin B}{\sin A.\sin C}$

$2\frac{\frac{a^2+c^2-b^2}{2ac}}{bk} = \frac{bk}{ak.ck}$

$\frac{a^2+c^2-b^2}{abc} = \frac{b}{ac}$

$a^2 + c^2 - b^2 = b^2$

$2b^2 = a^2 + c^2$

a^2, b^2 and c^2 are in A.P.

117. (B)

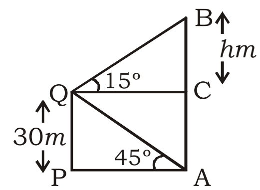
118. (D) A' = co-factor of A

$|A'| = |\text{co} - \text{factor of A}|$

$|A'| = (A)^{5-1}$ $[\because \text{order} = 5]$

$|A'| = A^4$

119. (C) Let BC = h m



In Δ APQ

$\tan 45° = \frac{PQ}{AP}$

$1 = \frac{30}{AP} \Rightarrow AP = 30 \text{ m} = QC$

In Δ BCQ

$\tan 15° = \frac{BC}{QC}$

$2 - \sqrt{3} = \frac{h}{30} \Rightarrow h = 60 - 30\sqrt{3}$

Hence height of tower = h + 30

$= 60 - 30\sqrt{3} + 30$

$= 90 - 30\sqrt{3}$

$= 30\sqrt{3}(\sqrt{3} - 1)$m

120. (D) Given data

12, 8, 14, 6, 17, 8, 19, 15

arrange in ascending both

6, 8, 8, 12, 14, 15, 17, 19

middle terms = 12 and 14

Median = $\frac{12+14}{2}$ = 13

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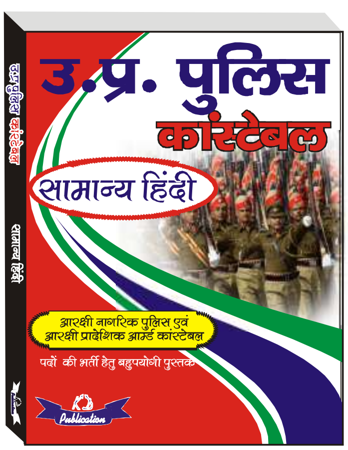
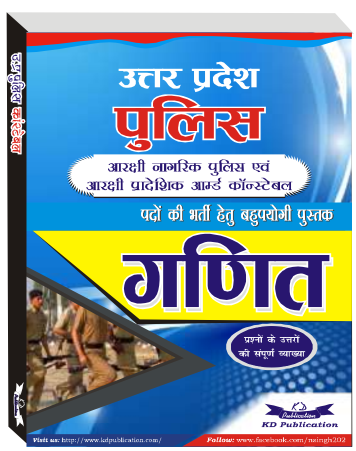
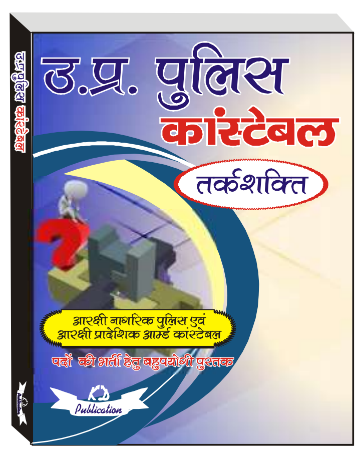
NDA (MATHS) MOCK TEST - 112 (Answer Key)

1.	(B)	21.	(D)	41.	(C)	61.	(B)	81.	(C)	101.	(B)
2.	(C)	22.	(B)	42.	(B)	62.	(C)	82.	(B)	102.	(C)
3.	(A)	23.	(C)	43.	(A)	63.	(C)	83.	(C)	103.	(B)
4.	(D)	24.	(D)	44.	(C)	64.	(A)	84.	(A)	104.	(B)
5.	(A)	25.	(B)	45.	(D)	65.	(C)	85.	(C)	105.	(D)
6.	(B)	26.	(B)	46.	(C)	66.	(C)	86.	(B)	106.	(C)
7.	(C)	27.	(A)	47.	(B)	67.	(A)	87.	(B)	107.	(B)
8.	(C)	28.	(B)	48.	(B)	68.	(B)	88.	(B)	108.	(C)
9.	(A)	29.	(B)	49.	(D)	69.	(C)	89.	(D)	109.	(C)
10.	(C)	30.	(B)	50.	(C)	70.	(C)	90.	(B)	110.	(A)
11.	(C)	31.	(B)	51.	(C)	71.	(D)	91.	(C)	111.	(D)
12.	(B)	32.	(C)	52.	(C)	72.	(D)	92.	(B)	112.	(C)
13.	(C)	33.	(A)	53.	(A)	73.	(C)	93.	(D)	113.	(C)
14.	(B)	34.	(C)	54.	(B)	74.	(D)	94.	(A)	114.	(B)
15.	(B)	35.	(B)	55.	(D)	75.	(B)	95.	(B)	115.	(B)
16.	(A)	36.	(C)	56.	(B)	76.	(D)	96.	(A)	116.	(A)
17.	(C)	37.	(D)	57.	(B)	77.	(C)	97.	(D)	117.	(B)
18.	(C)	38.	(B)	58.	(B)	78.	(A)	98.	(C)	118.	(D)
19.	(A)	39.	(A)	59.	(A)	79.	(B)	99.	(C)	119.	(C)
20.	(B)	40.	(A)	60.	(B)	80.	(C)	100.	(D)	120.	(D)







Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777