NDA MATHS MOCK TEST - 116 (SOLUTION)

1. (D) Given that n = 1908!

Now, $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+.........+\frac{1}{\log_{1908} n}$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 + ...+ \log_n 1908$

$\Rightarrow \log_n(2 \times 3 \times 4 \times 5 \times.....1908)$

$\Rightarrow \log_n 1908! = \log_n n = 1$

2. (C) two circles $x^2 + y^2 + 4x + 2y - 9 = 0$ and $x^2 + y^2 - x + 5y + k = 0$

Condition of orthogonality

$2gg' + 2ff' = C + C'$

$\Rightarrow 2 \times 2 \times \left(\frac{-1}{2}\right) + 2 \times 1 \times \frac{5}{2} = -9 + k$

$\Rightarrow -2 + 5 = -9 + k \Rightarrow k = 12$

3. (B) A = {0, 1, 3, 5, 8, 7, 9}; n = 7

The required no. of subsets of A = 2^n

$= 2^7 = 128$

4. (B) Circle $x^2 + y^2 + x + 5y + 9 = 0$ ...(i)

Let equation of circle concentric with eq.(i)

$x^2 + y^2 + x + 5y + \lambda = 0$ ...(ii)

its passes through the point (–3, 0)

$\Rightarrow 9 + 0 - 3 + 0 + \lambda = 0 \Rightarrow \lambda = -6$

from eq(i)

$x^2 + y^2 + x + 5y - 6 = 0$

$x^2 + y^2 + x + 5y = 6$

5. (A)

6. (B) S = 0.7 + 0.77 + 0.777 + ...upto n terms

S = 7(0.1 + 0.11 + 0.111 + ... upto n terms)

$S = \frac{7}{9}(0.9 + 0.99 + 0.999 + ... + \text{upto } n \text{ terms})$

$S = \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+.....n \text{ terms}\right]$

$S = \frac{7}{9}(1+1+1+.....n \text{ terms})$

$-\frac{7}{9}\left[\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+....\text{n terms}\right]$

$S = \frac{7}{9}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$S = \frac{7}{9}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

7. (A) $I = \int \frac{2^x}{2^x - 1} dx$

Let $2^x - 2 = t$

$2^x \log 2 \, dx = dt \Rightarrow 2x\, dx = \frac{1}{\log 2} dt$

$I = \int \frac{dt}{t} \frac{1}{\log 2}$

$I = \frac{\log t}{\log 2} + C$

$I = \log_2(2^x - 1) + C$

8. (B) Differential equation

$\frac{dy}{dx} + y \tan x = \cos x$

here P = $\tan x$ and Q = $\cos x$

I.F. = $e^{\int \text{P}.dx}$

$= e^{\int \tan x.dx}$

$= e^{\log \sec x} = \sec x$

Solution of differential equation

$y \times \text{I.F.} = \int \text{Q} \times \text{I.F.}\, dx$

$y \times \sec x = \int \cos x \times \sec x\, dx$

$y \times \sec x = \int 1.dx$

$y \times \sec x = x + C$

9. (B) $\lim_{x \to 0} \frac{\tan 2x^2}{\sin 3x^2}$

$\Rightarrow \lim_{x \to 0} \frac{\tan 2x^2}{2x^2} \times \frac{3x^2}{\sin 3x^2} \times \frac{2x^2}{3x^2}$

$\Rightarrow \frac{2}{3}$ $\left[\because \lim_{x\to 0}\frac{\tan x}{x} = \lim_{x \to 0}\frac{\sin x}{x} = 1\right]$

10. (A) $i^{1001} + i^{1002} + i^{1003} + i^{1004} + i^{1005}$

$\Rightarrow i^{1001}(i^0 + i^1 + i^2 + i^3 + i^4)$

$\Rightarrow i^{3\times333+2}(1 + i - 1 - i + 1)$

$\Rightarrow i^2 (1) = -1$

11. (D) $f(x) = \frac{[x-2]}{[x]}$

for x = 2

L.H.L. = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2 - h)$

$= \lim_{h\to 0} \frac{[2-h-2]}{[2-h]}$

$= \lim_{h\to 0} \frac{[0-h]}{[2-h]}$

$= \frac{-1}{1} = 1$

R.H.L. = $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} \frac{[2+h-2]}{[2+h]}$

$= \lim_{h\to 0} \frac{[0+h]}{[2+h]}$

$= \frac{0}{2} = 0$

L.H.L. $\neq$ R.H.L.

Hence $f(x)$ is not continuous at $x = 2$.

for $x = 1$

L.H.L. = $\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0} \frac{[1-h-2]}{[1-h]}$

$= \lim_{h\to 0} \frac{[-1-h]}{[1-h]}$

$= \frac{-2}{0} = \infty$

R.H.L. = $\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} \frac{[1+h-2]}{[1+h]}$

$= \lim_{h\to 0} \frac{[-1+h]}{[1+h]}$

$= \frac{-1}{1} = -1$

L.H.L. $\neq$ R.H.L.

$f(x)$ is not continuous at $x = 1$.

12. (B) $z = \frac{(i-1)^2}{1+2i}$

$z = \frac{-2i}{1+2i} \times \frac{1-2i}{1-2i}$

$z = \frac{-2i+4i^2}{1-4i^2}$

$z = \frac{-4-2i}{5}$

conjugate of $z = \frac{-4+2i}{5}$

13. (D)

14. (B)

		1	x	1	1	1
+		1	0	1	y	1
	1	0	z	1	0	0

$x = 0,\ y = 0,\ z = 1$

15. (C) $4\sin^2\theta + 7\cos^2\theta = 4$

$\Rightarrow 4\sin^2\theta + 7(1-\sin^2\theta) = 4$

$\Rightarrow 4\sin^2\theta + 7 - 7\sin^2\theta = 4$

$\Rightarrow 3\sin^2\theta = 3$

$\Rightarrow \sin^2\theta = \sin^2\frac{\pi}{2}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$

16. (b) roots of the equation

$(a-b)x^2 + 2(b-c)x + (c-a) = 0$

are equal, then

$B^2 = 4AC$

$\Rightarrow [2(b-c)]^2 = 4(a-b)(c-a)$

$\Rightarrow 4(b^2 + c^2 - 2bc) = 4ac - 4bc - 4a^2 + 4ab$

$\Rightarrow b^2 + c^2 - 2bc = ac - bc - a^2 + ab$

$\Rightarrow a^2 + b^2 + c^2 = ab + bc + ca$

17. (a) $y = (\tan x)^{\cos x}$

taking log both side

$\Rightarrow \log y = \cos x.\ \log\tan x$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x.\ \frac{\sec^2 x}{\tan x} + \log\tan x.\ (-\sin x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \text{cosec}x - \sin x\ .\log\tan x$

$\Rightarrow \frac{dy}{dx} = y\ (\text{cosec}x - \sin x.\ \log\tan x)$

18. (C) ${}^{34}C_6 + \sum_{r=1}^{3} {}^{33+r}C_5$

$\Rightarrow {}^{34}C_6 + {}^{34}C_5 + {}^{35}C_5 + {}^{36}C_5$

We know that

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{35}C_6 + {}^{35}C_5 + {}^{36}C_5$

$\Rightarrow {}^{36}C_6 + {}^{36}C_5 = {}^{37}C_6$

19. (D) $\frac{\cosh x + \cosh y}{\sinh x - \sinh y}$

$$\Rightarrow \frac{2\cosh\frac{x+y}{2}.\cosh\frac{x-y}{2}}{2\cosh\frac{x+y}{2}.\sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$$

20. (C) $I = \int_0^{\pi/2} \frac{\Psi(x)}{\Psi(x)+\Psi\left(\frac{\pi}{2}-x\right)}dx$ ....(i)

Prop. IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\Psi\left(\frac{\pi}{2}-x\right)}{\Psi\left(\frac{\pi}{2}-x\right)+\Psi(x)}dx$ ....(ii)

from eq(i) and eq(ii)

$$2I = \int_0^{\pi/2} \frac{\Psi(x)+\Psi\left(\frac{\pi}{2}-x\right)}{\Psi(x)+\Psi\left(\frac{\pi}{2}-x\right)}dx$$

$$2I = \int_0^{\pi/2} 1.dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

21. (B) In ΔABC, $2s = a + b + c$

Now, $\frac{s(s-c)}{ab} - \frac{(s-a)(s-b)}{ab}$

$$\Rightarrow \frac{s^2 - sc - s^2 + sa + sb - ab}{ab}$$

$$\Rightarrow \frac{2s(a+b-c)-2ab}{2ab}$$

$$\Rightarrow \frac{(a+b+c)(a+b-c)-2ab}{2ab}$$

$$\Rightarrow \frac{(a+b)^2 - c^2 - 2ab}{2ab}$$

$$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \cos C$$

22. (A) $\tan(2505) = \tan(360 \times 7 - 15)$
$= -\tan 15$

$$= -\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-\sqrt{3}}{\sqrt{3}+1}$$

23. (C) $\tan x$, $\cos x$ and $\sin x$ are in G.P., then $(\cos x)^2 = \tan x . \sin x$

$\Rightarrow \cos^2 x = \frac{\sin x}{\cos x} \times \sin x$

$\Rightarrow \cos^3 x = \sin^2 x$

$\Rightarrow \cos^3 x = 1 - \cos^2 x$

$\Rightarrow \cos^3 x + \cos^2 x = 1$

24. (A) Given lines $2x + y = 5$ ....(i)

$x - y = 7$ ....(ii)

$3x - 7y = 9$ ....(iii)

from eq(i) and eq(ii)

$x = 4,\ y = -3$

intersecting point $(4, -3)$

equation of line which is parallel to eq(iii)

$3x - 7y = C$ ....(iv)

it passes through the point $(4, -3)$

$3 \times 4 - 7 \times (-3) = C \Rightarrow C = 33$

from eq(iv)

the required equation

$3x - 7y = 33$

25. (C) Lines $9x - 12y = 13$

$3x - 4y = \frac{13}{3}$ ....(i)

and $4y - 3x = 11$

$3x - 4y = -11$ ....(ii)

$$\text{Distance} = \frac{\frac{13}{3}+11}{\sqrt{3^2+(-4)^2}}$$

$$= \frac{46}{3\times 5} = \frac{46}{15}$$

26. (C) Let $a + ib = \sqrt{1+\sqrt{3}i}$

On squaring both side

$(a^2 - b^2) + 2abi = 1 + \sqrt{3}i$

On comparing

$a^2 - b^2 = 1$ and $2ab = \sqrt{3}$ ....(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2) + (2ab)^2$

$\Rightarrow (a^2 + b^2) = 1 + 3$

$\Rightarrow (a^2 + b^2)^2 = 4$

$\Rightarrow a^2 + b^2 = 2$ ....(ii)

from eq(i) and eq(ii)

$2a^2 = 3$ and $2b^2 = 1$

$a = \pm\frac{\sqrt{3}}{2}$, $b = \pm\frac{1}{\sqrt{2}}$

Hence, $\sqrt{1+\sqrt{3}i} = \pm\left(\frac{\sqrt{3}+i}{\sqrt{2}}\right)$

27. (A) In ΔABC,

$a(b \cos C - c \cos B)$

$\Rightarrow a\left(b \times \frac{a^2+b^2-c^2}{2ab} - c \times \frac{a^2+c^2-b^2}{2ac}\right)$

$\Rightarrow \frac{a^2+b^2-c^2}{2} - \frac{a^2+c^2-b^2}{2}$

$\Rightarrow b^2 - c^2$

28. (B) Given that Mode = 17 and Median = 27

We know that

Mode = 3 Median – 2 Mean

$\Rightarrow$ 17 = 3 × 27 – 2 Mean

$\Rightarrow$ 2 Mean = 81 – 17

$\Rightarrow$ 2 Mean = 64 $\Rightarrow$ Mean = 31

29. (C) Given line $\frac{x-2}{2} = \frac{y+3}{2} = \frac{z-1}{-1}$

and $y - z = 8$

Angle between line and plane

$\sin\theta = \frac{2\times0+2\times1+(-1)(-1)}{\sqrt{2^2+2^2+(-1)^2}\sqrt{(1)^2+(-1)^2}}$

$\sin\theta = \frac{2+1}{3\times\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$

30. (B)

31. (D) $n(S) = 6 \times 6 = 36$

E = {(6,3), (3, 6), (5, 4), (4, 5)}; $n(E) = 4$

Probability = $\frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

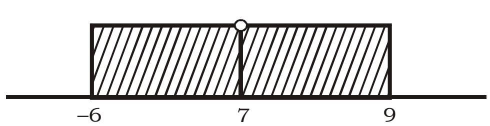
32. (B) function $f(x) = \frac{\sqrt{\log_e(55+3x-x^2)}}{x-7}$

$\log_e(55 + 3x - x^2) \geq 0, \quad x - 7 \neq 0$

$\Rightarrow 55 + 3x - x^2 \geq 1, \quad x \neq 7$

$\Rightarrow x^2 - 3x - 54 \leq 0$

$\Rightarrow (x + 6)(x - 9) \leq 0$



$x \in [-6, 9] - \{7\}$

33. (C) $\lim_{x\to2} \frac{\sqrt{x-1}-1}{\sqrt{8-x}-\sqrt{6}}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to2} \frac{\frac{1}{2\sqrt{x-1}} - 0}{\frac{1\times(-1)}{2\sqrt{8-x}} - 0}$

$\Rightarrow \lim_{x\to2} \frac{-\sqrt{8-x}}{\sqrt{x-1}}$

$\Rightarrow \frac{\sqrt{6}}{1} = -\sqrt{6}$

34. (A)

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	23	25	24	12	16

Modal class = (10 – 20)

$l_1 = 10$, $l_2 = 20$, $f = 25$, $f_0 = 23$, $f_1 = 24$

Mode = $l_1 + \left(\frac{f - f_0}{2f - f_0 - f_1}\right) \times (l_2 - l_1)$

$= 10 + \frac{25-23}{2\times25-23-24} \times (20 - 10)$

$= 10 + \frac{2}{3} \times 10 = 16\frac{2}{3}$

35. (D) Remainder = $\frac{11^{23}+7^{23}}{9}$

$= (2)^{23} + (-2)^{23}$

$= 2^{23} - 2^{23} = 0$

The number $11^{23} + 7^{23}$ will be divisible by 9.

36. (B) The required no. of triangles = ${}^9C_3 - {}^4C_3$

$= 84 - 4 = 80$

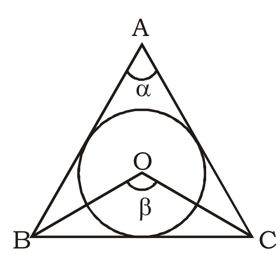
37. (B) $\cosh x + \sinh x = \frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2} = e^x$

38. (D)

5	5	4	3

= 5 × 5 × 4 × 3 = 300

39. (B)



We know that

$\angle BOC = 90 + \frac{\angle BAC}{2}$

$\Rightarrow \beta = 90 + \frac{\alpha}{2}$

$\Rightarrow \sin\beta = \sin\left(90+\frac{\alpha}{2}\right)$

$\Rightarrow \sin\beta = \cos\frac{\alpha}{2}$

$\Rightarrow \frac{2\tan\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}} = \cos\frac{\alpha}{2}$

40. (C) Let $y = \sin\left(x-\frac{\pi}{6}\right) + \cos\left(x-\frac{\pi}{6}\right)$

$\Rightarrow \frac{dy}{dx} = \cos\left(x-\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right)$

for maximum and minima

$\cos\left(x-\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = 0$

$\Rightarrow \cos\left(x-\frac{\pi}{6}\right) = \sin\left(x-\frac{\pi}{6}\right)$

$\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{2} - x + \frac{\pi}{6} \Rightarrow x = \frac{5\pi}{12}$

41. (C) Given that
cosA + cosB = 0 sinA + sinB
where $0 < A < B < 2\pi$
Now, cosA + cosB = 0
$\Rightarrow$ cosA = – cosB
$\Rightarrow$ – cosA = cosB]
$\Rightarrow$ cos (π + A) = cosB
$\Rightarrow$ B = π + A [$\because$ A < B]

42. (D) $I = \int_1^e x^2.lnx\ dx$

$I = \left[lnx.\int x^2dx - \int\left\{\frac{d}{dx}(lnx).\int x^2.dx\right\}dx\right]_1^e$

$I = \left[(lnx).\frac{x^3}{3} - \int\frac{1}{x}.\frac{x^3}{3}dx\right]_1^e$

$I = \left[\frac{x^3}{3}.lnx - \frac{1}{3}\int x^2dx\right]_1^e$

$I = \left[\frac{x^3}{3}.lnx - \frac{1}{3}.\frac{x^3}{3}\right]_1^e$

$I = \frac{e^3}{3}.lne - \frac{1}{9}e^3 - \frac{1}{3}ln1 + \frac{1}{9}$

$I = \frac{e^3}{3} - \frac{e^3}{9} + \frac{1}{9}$

$I = \frac{3e^3 - e^3 + 1}{9} = \frac{2e^3+1}{9}$

43. (C) $f(x) = \begin{cases} 5x^2 - 7 & 1 \le x < 3 \\ 2x+\lambda & 3 \le x < 6 \end{cases}$ is continuous

at $x = 3$,then

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\Rightarrow \lim_{x\to 3} 5x^2 - 7 = \lim_{x\to 3} 2x + \lambda$

$\Rightarrow 5 \times 9 - 7 = 2 \times 3 + \lambda$

$\Rightarrow 38 = 6 + \lambda \Rightarrow \lambda = 32$

44. (B) 1 0 1 0 0.0 1 1

$0 \times 2^0 = 2$ $0 = 0 \times 2^{-1}$

$1 \times 2^1 = 2$ $\frac{1}{4} = 1 \times 2^{-2}$

$0 \times 2^2 = 0$ $\frac{1}{8} = 1 \times 2^{-3}$

$1 \times 2^3 = \frac{8}{10}$ $\frac{1}{4} + \frac{1}{8} = 0.375$

Hence $(1010.011)_2 = (10.375)_{10}$

45. (C) $I = \int\sqrt{2-2\sin 2x}\ dx$

$I = \int\sqrt{2\left[1-\cos\left(\frac{\pi}{2}-2x\right)\right]}\ dx$

$I = \int\sqrt{2\times 2\sin^2\left(\frac{\pi}{4}-x\right)}\ dx$

$I = \int 2\sin\left(\frac{\pi}{4}-x\right)\ dx$

$I = \frac{-2\cos\left(\frac{\pi}{4}-x\right)}{-1}\ dx$

$I = 2\ \cos\left(\frac{\pi}{4}-x\right) + C$

$I = 2\left[\cos\frac{\pi}{4}.\cos x + \sin\frac{\pi}{4}.\sin x\right] + C$

$I = 2\left[\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right] + C$

$I = \sqrt{2}(\cos x + \sin x) + C$

46. (B) Given that

$\int x^2.e^{3x}\ dx = ax^2.e^{3x} + bx.e^{3x} + c.e^{3x} + k$ ...eq.(i)

Let I = $\int x^2.e^{3x}\,dx$

I = $x^2.\int e^{3x}dx - \int\left\{\frac{d}{dx}(x^2).\int e^{3x}dx\right\}dx + k$

I = $x^2.\frac{e^{3x}}{3} - \int 2x.\frac{e^{3x}}{3}dx + k$

I = $\frac{1}{3}x^2.e^{3x} - \frac{2}{3}\left[x.\int e^{3x}dx - \int\left\{\frac{d}{dx}(x).\int e^{3x}dx\right\}dx\right] + k$

I = $\frac{1}{3}x^2.e^{3x} - \frac{2}{3}\left[x.\frac{e^{3x}}{3} - \int 1.\frac{e^{3x}}{3}dx\right] + k$

I= $\frac{1}{3}x^2.e^{3x}$ - $\frac{2}{3}\left[\frac{x}{3}.e^{3x} - \frac{1}{3}\frac{e^{3x}}{3}\right]$ + k

I = $\frac{1}{3}x^2.e^{3x}$ - $\frac{2}{9}x.e^{3x} + \frac{2}{27}e^{3x}$ + k

On comparing eq(i)

$a = \frac{1}{3}$, $b = \frac{-2}{9}$, $c = \frac{2}{27}$

47. (B) $(A\cap B) \cup (B\cap C) \cup (C\cap A) \cup (A\cap B\cap C)$

48. (A) Given that

$2a^2 = b^2 + c^2$

$a = b + c$

then A = B = C = 60^0

Now, $\frac{\sin 3x}{\sin x} = \frac{\sin 3\times 60}{\sin 60}$

$= \frac{\sin 180}{\sin 60} = \frac{0}{\sqrt{3}/2} = 0$

49. (C) Given that

$a = 4\sqrt{3}$ cm, b = 7 cm, c = $\sqrt{13}$ cm

Now, cosC = $\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow$ cosC = $\frac{48+49-13}{2\times 4\sqrt{3}\times 7}$

cosC = $\frac{84}{2\times 4\sqrt{3}\times 7}$

$\Rightarrow$ cosC = $\frac{\sqrt{3}}{2}$ $\Rightarrow$ C = 30^0

50. (C) S = $\sqrt{3} + \sqrt{12} + \sqrt{27} + \sqrt{48}$ +......10 terms

S= $\sqrt{3} + 2\sqrt{3} + 3\sqrt{3} + 4\sqrt{3}$ +......10 terms

S = $\frac{n}{2}\left[2a + (n-1)d\right]$

S = $\frac{10}{2}\left[2\sqrt{3} + 9\times\sqrt{3}\right] = 55\sqrt{3}$

51. (C) Digits 0, 1, 2, 3, 4, 5,6, 7, 8, 9

9	8	7	6	1

= 9×8×7×6×1 = 3024

52. (A) I = $\int_0^4 \frac{1}{\sqrt{x^2-4x}}\,dx$

I = $\int_0^4 \frac{1}{\sqrt{(x-2)^2-4}}\,dx$

I = $\left[\log\left|(x-2)+\sqrt{(x-2)^2-4}\right|\right]_0^4$

I = $\left[\log|2-0| - \log|-2|\right]$

I = $\left[\log 2 - \log 2\right] = 0$

53. (B) 5 cos20°.cos40°.cos60°.cos80°

$\Rightarrow$ 5cos60°[cos20°.cos40°.cos80°]

We know that

$\cos\theta.\cos(60-\theta).\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\Rightarrow 5\times\frac{1}{2}\times\frac{1}{4}\cos(3\times 20)$

$\Rightarrow 5\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{2} = \frac{5}{16}$

54. (B) Maximum value of ($40\sin\theta + 9\cos\theta$)

$= \sqrt{(40)^2 + 9^2}$

$= \sqrt{1600+81}$

$= \sqrt{1681} = 41$

55. (C) A = $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -4 \\ 5 & 6 & -1 \end{bmatrix}$

Co-factors of A -

$C_{11}=(-1)^{1+1}\begin{vmatrix} 1 & -4 \\ 6 & -1 \end{vmatrix}$, $C_{12}=(-1)^{1+2}\begin{vmatrix} 2 & -4 \\ 5 & -1 \end{vmatrix}$, $C_{13}=(-1)^{1+3}\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix}$

= 23   = -18   = 7

$C_{21}=(-1)^{2+1}\begin{vmatrix} 0 & 2 \\ 6 & -1 \end{vmatrix}$, $C_{22}=(-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 5 & -1 \end{vmatrix}$, $C_{23}=(-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 5 & 6 \end{vmatrix}$

= 12   = -11   = -6

$C_{31}=(-1)^{3+1}\begin{vmatrix} 0 & 2 \\ 1 & -4 \end{vmatrix}$, $C_{32}=(-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 2 & -4 \end{vmatrix}$, $C_{33}=(-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

= -2   = 8   = 1

C = $\begin{bmatrix} 23 & -18 & 7 \\ 12 & -11 & -6 \\ -2 & 8 & 1 \end{bmatrix}$

AdjA = $C^T = \begin{bmatrix} 23 & 12 & -2 \\ -18 & -11 & 8 \\ 7 & -6 & 1 \end{bmatrix}$

56. (D) $\cos\frac{5\pi}{12} < \sin\frac{5\pi}{12} < \tan\frac{5\pi}{12}$

57. (C) Conic $3x^2 + 7y^2 = 42$

$\Rightarrow \frac{x^2}{14} + \frac{y^2}{6} = 1,\ a > b$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{6}{14}} \Rightarrow e = \frac{2}{\sqrt{7}}$

58. (C) Equation $bx^2 + cx + a = 0$
its one root is $5 - 2i$,
then other root = $5 + 2i$

Sum of the roots = $-\frac{c}{b}$

$\Rightarrow 5 + 2i + 5 - 2i = -\frac{c}{b}$

$\Rightarrow 10 = -\frac{c}{b} \Rightarrow 10b + c = 0$

59. (C) $\lim_{x\to 2}\frac{\log_3(3-x)}{x^2+x-6}$ $\left[\frac{0}{0}\right]$ form
by L - Hospital's Rule

$\Rightarrow \lim_{x\to 2}\frac{\frac{-1}{3-x}\log 3}{2x+1}$

$\Rightarrow \lim_{x\to 2}\frac{-\log 3}{(3-x)(2x+1)}$

$\Rightarrow -\frac{\log 3}{1\times 5} = -\frac{1}{5}\log 3$

60. (B) We know that

$\cos 36^0 = \frac{\sqrt{5}+1}{4}$

61. (B) $\frac{1+\cot 126.\cot 96}{\tan 144 - \tan 174}$

$\Rightarrow \frac{1+\cot(90+36).\cot(90+6)}{\tan(180-36)-\tan(180-6)}$

$\Rightarrow \frac{1+\tan 36.\tan 6}{-\tan 36 + \tan 6}$

$\Rightarrow -\left[\frac{1+\tan 36.\tan 6}{\tan 36 - \tan 6}\right]$

$\Rightarrow \frac{-1}{\tan 30} = -\sqrt{3}$

62. (C) $(1+\omega)^7 = a + b\,\omega^2$

$\Rightarrow (-\omega^2)^7 = a + b\,\omega^2$ $[1 + \omega + \omega^2 = 0\ ]$
$\Rightarrow -\omega^{14} = a + b\,\omega^2$
$\Rightarrow -\omega^2 = a + b\,\omega^2$
On comparing
$a = 0,\ b = -1$
Hence $(a, b) = (0, -1)$

63. (B) Vectors $\vec{a} = 3\hat{i} + (\lambda - 2)\hat{j} + 2\hat{k}$ and
$\vec{b} = (3 - \lambda)\hat{i} + 2\hat{j} - \hat{k}$ are perpendicular,
then $\vec{a}.\vec{b} = 0$
$\Rightarrow 3(3 - \lambda) + (\lambda - 2) \times 2 + 2\times(-1) = 0$
$\Rightarrow 9 - 3\lambda + 2\lambda - 4 - 2 = 0$
$\Rightarrow 3 - \lambda = 0 \Rightarrow \lambda = 3$

64. (A) P (0, -1, -2), Q(4, 1, 2), R(5, -1, 3)
$\overrightarrow{PQ}$ = (4- 0, 1+1, 2+ 2) = (4, 2, 4)
$\overrightarrow{PR}$ = (5 - 0, -1+1, 3 + 2) = (5 , 0, 5)

$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 4 \\ 5 & 0 & 5 \end{vmatrix}$

$= \hat{i}\,(10 - 0) - \hat{j}\,(20 - 20) + \hat{k}\,(0 - 10)$

$= 10\hat{i} - 10\hat{k}$
$a = 10,\ b = 0,\ c = -10$
Equation of plane
$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
$\Rightarrow 10(x - 0) + 0\,(y + 1) - 10(z + 2) = 0$
$\Rightarrow 10x - 10z - 20 = 0 \Rightarrow x - z = 2$

65. (B) $\begin{vmatrix} \sec\theta & \tan\theta \\ -\tan\theta & -\sec\theta \end{vmatrix}$

$\Rightarrow -\sec^2\theta + \tan^2\theta$
$\Rightarrow -(\sec^2\theta - \tan^2\theta) = -1$

66. (B) Let $y = 5^{21}$
taking log both side
$\Rightarrow \log_{10}y = 21\log_{10}5$
$\Rightarrow \log_{10}y = 21\times 0.699$
$\Rightarrow \log_{10}y = 14.679$
The required no. of digits = 14 +1 = 15

67. (B) $x + \log_6(2^x - 1) = x\log_6 3 + \log_6 12$

$\Rightarrow x = \log_6\left(\frac{3^x \times 12}{2^x - 1}\right)$

$\Rightarrow 6^x = \dfrac{3^x \times 12}{2^x - 1}$

$\Rightarrow 2^x \times 3^x = \dfrac{3^x \times 12}{2^x - 1}$

$\Rightarrow (2^x)^2 - 2^x = 12$

$\Rightarrow (2^x - 4)(2^x + 3) = 0$

$\Rightarrow 2^x - 4 = 0, \quad 2^x + 3 \neq 0$

$\Rightarrow 2^x = 2^2 \Rightarrow x = 2$

68. (B) $n(S) = 2^5 = 32$

$n(E) = {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3$

$n(E) = 1 + 5 + 10 + 10 = 26$

Probability $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{26}{32} = \dfrac{13}{16}$

69. (C) $a + 3d$, $a + 6d$ and $a + 8d$ are in G.P.

Now,

$(a + 6d)^2 = (a + 3d)(a + 8d)$

$\Rightarrow a^2 + 36d^2 + 12ad = a^2 + 3ad + 8ad + 24d^2$

$\Rightarrow 12d^2 + ad = 0$

$\Rightarrow (12d + a)d = 0$

$\Rightarrow 12d + a = 0$

$\Rightarrow a = -12d, \ d \neq 0$

Common Ratio $= \dfrac{a + 6d}{a + 3d}$

$= \dfrac{-12d + 6d}{-12d + 3d}$

$= \dfrac{-6d}{-9d} = \dfrac{2}{3}$

70. (C) $f(x) = \sin^{-1}[\log_9 2x]$ exist,

if $-1 \le \log_9 2x \le 1$

$\Rightarrow 9^{-1} \le 2x \le 9^1$

$\Rightarrow \dfrac{1}{9} \le 2x \le 9$

$\Rightarrow \dfrac{1}{18} \le x \le \dfrac{9}{2}$

Hence domain $= \left[\dfrac{1}{18}, \dfrac{9}{2}\right]$

71. (C) $x = \dfrac{2at}{1 - t^2}$ ...(i)

$\Rightarrow \dfrac{dx}{dt} = \dfrac{(1 - t^2)2a - 2at(-2t)}{(1 - t^2)^2}$

$\Rightarrow \dfrac{dx}{dt} = 2a\left[\dfrac{1 - t^2 + 2t^2}{(1 - t^2)^2}\right]$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{2a(1 + t^2)}{(1 - t^2)^2}$

and $y = \dfrac{a(1 + t^2)}{(1 - t^2)}$ ...(ii)

$\Rightarrow \dfrac{dy}{dt} = a\left[\dfrac{(1 - t^2)2t - (1 + t^2)(-2t)}{(1 - t^2)^2}\right]$

$\Rightarrow \dfrac{dy}{dt} = a\left[\dfrac{2t - 2t^3 + 2t + 2t^3}{(1 - t^2)^2}\right]$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{4at}{(1 - t^2)^2}$

Now,

$\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{4at}{(1 - t^2)^2} \times \dfrac{(1 - t^2)^2}{2a(1 + t^2)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2t}{1 + t^2}$ ...(iii)

from eq.(i) and eq.(ii)

$\dfrac{x}{y} = \dfrac{2at}{1 - t^2} \times \dfrac{1 - t^2}{a(1 + t^2)}$

$\Rightarrow \dfrac{x}{y} = \dfrac{2t}{1 + t^2}$

from eq.(iii)

$\dfrac{dy}{dx} = \dfrac{2t}{1 + t^2} = \dfrac{x}{y}$

72. (B) Series

$\dfrac{1^2}{1} + \dfrac{1^2 + 2^2}{1 + 2} + \dfrac{1^2 + 2^2 + 3^2}{1 + 2 + 3} + \ldots\ldots\ldots$

$T_n = \dfrac{1^2 + 2^2 + 3^2 + \ldots\ldots. n^2}{1 + 2 + 3 + \ldots\ldots. n}$

$T_n = \dfrac{\frac{n}{6}(n + 1)(2n + 1)}{\frac{n(n + 1)}{2}} = \dfrac{2n + 1}{3}$

73. (A) $f'(x) = x^3 + \dfrac{3}{2x^4}$

On integrating both side

$\Rightarrow f(x) = \dfrac{x^4}{4} + \dfrac{3}{2}\dfrac{x^{-4+1}}{-4+1} + C$

$\Rightarrow f(x) = \frac{x^4}{4} - \frac{1}{2x^3} + C$ ...(i)

Given that $f(-2) = 7$

$\Rightarrow 7 = \frac{(-2)^4}{4} - \frac{1}{2(-2)^3} + C$

$\Rightarrow 7 = 4 + \frac{1}{16} + C \Rightarrow C = \frac{47}{16}$

from eq(i)

$f(x) = \frac{x^4}{4} - \frac{1}{2x^3} + \frac{47}{16}$

74. (C) Median

75. (B) $\frac{\left[1+\left(i^7\right)^{4n+1}\right]^{4n-1}}{\left[1+\left(i^7\right)^{4n-1}\right]^{4n+1}}$

$\Rightarrow \frac{\left[1+(-i)^{4n+1}\right]^{4n-1}}{\left[1+(-i)^{4n-1}\right]^{4n+1}}$

$\Rightarrow \frac{\left[1+(-i)^{1}\right]^{4n-1}}{\left[1+(-i)^{-1}\right]^{4n+1}}$

$\Rightarrow \frac{[1-i]^{4n-1}}{[1+i]^{4n+1}}$

$\Rightarrow \left[\frac{1-i}{1+i}\right]^{4n} \frac{[1-i]^{-1}}{[1+i]^{1}}$

$\Rightarrow (-i)^{4n} \frac{1}{(1-i)(1+i)}$

$\Rightarrow \frac{1}{1-i^2} = \frac{1}{1+1} = \frac{1}{2}$

76. (C) Differential equation

$\frac{d^2y}{dx^2} = x.e^{-2x}$

On integrating

$\frac{dy}{dx} = \int x.e^{-2x}\,dx$

$\frac{dy}{dx} = x.\int e^{-2x}dx - \int\left\{\frac{d}{dx}(x)\int e^{-2x}dx\right\}dx$

$\frac{dy}{dx} = x.\frac{e^{-2x}}{-2} - \int 1.\frac{e^{-2x}}{-2}\,dx + c$

$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx + c$

$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{2}\frac{e^{-2x}}{-2} + c$

$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{4}e^{-2x} + c$

Again, integrating

$y = \frac{-1}{2}\int x.e^{-2x}\,dx - \frac{1}{4}.\int e^{-2x}dx + c\int 1.dx + d$

$y = -\frac{1}{2}\left[\frac{-x}{2}e^{-2x} - \frac{1}{4}e^{-2x}\right] - \frac{1}{4}\times\frac{e^{-2x}}{-2} + cx + d$

$y = \frac{1}{4}x.e^{-2x} + \frac{1}{8}.e^{-2x} + \frac{1}{8}.e^{-2x} + cx + d$

$y = \frac{1}{4}x.e^{-2x} + \frac{1}{4}.e^{-2x} + cx + d$

77. (B) $\lim_{x\to\infty}\left[\frac{x^2+3x+2}{x^2+x+2}\right]^x$

$\Rightarrow \lim_{x\to\infty}\left[\frac{1+\frac{3}{x}+\frac{2}{x^2}}{x^2+\frac{1}{x}+\frac{2}{x^2}}\right]^x$ $\left[1^\infty\right]$ form

We know that

$\lim_{x\to\infty}\left[f(x)\right]^{g(x)} = e^{\lim_{x\to\infty} g(x)[f(x)-1]}$

Now,

$\lim_{x\to\infty}\left[\frac{x^2+3x+2}{x^2+x+2}\right]^x = e^{\lim_{x\to\infty} x.\left[\frac{x^2+3x+2}{x^2+x+2}-1\right]}$

$= e^{\lim_{x\to\infty} x.\left[\frac{2x}{x^2+x+2}\right]}$

$= e^{\lim_{x\to\infty}\left[\frac{2}{1+\frac{1}{x}+\frac{2}{x^2}}\right]}$

$= e^{\left(\frac{2}{1+0}\right)} = e^2$

78. (D) Integers from 1 to 100 divisible by 3

3, 6, 9, ...99

We know that

$\Rightarrow l = a + (n-1)d$

$\Rightarrow 99 = 3 + (n-1)\times 3 \Rightarrow n = 33$

Sum of Integers $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{33}{2}[2\times 3 + 32\times 3]$

$= \frac{33}{2}\times 102 = 1683$

79. (B) Let $y = \sin(\tan x^2)$ and $z = x^2$

$\Rightarrow \ y = \sin(\tan z)$

On differentiating both side w.r.t. 'z'

$\Rightarrow \frac{dy}{dz} = \cos(\tan z).\sec^2 z$

$\Rightarrow \frac{dy}{dz} = \cos(\tan x^2).\sec^2 x^2$

80. (B) **Statement I**

$x = \cot\theta$

Now, $x - \frac{1}{x} = \cot\theta - \tan\theta$

$x - \frac{1}{x} = \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$

$x - \frac{1}{x} = 2\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta.\cos\theta}$

$x - \frac{1}{x} = 2\frac{\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta$

Statement I is incorrect.

Statement II

$x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\times x\times\frac{1}{x}\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta)$

$\Rightarrow x^3 + \frac{1}{x^3} = 2\cos 3\theta$

Statement II is correct.

Statement III

maximum value of $24\sin\theta + 10\cos\theta$

$= \sqrt{24^2 + 10^2}$

$= \sqrt{576 + 100}$

$= \sqrt{676} = 26$

Statement III is incorrect.

81. (A) $I = \int_{\pi/4}^{\pi/2} e^x\left(\frac{\sin 2x - 2}{\sin^2 x}\right)dx$

$I = \int_{\pi/4}^{\pi/2} e^x\left(\frac{2\sin x.\cos x - 2}{\sin^2 x}\right)dx$

$I = \int_{\pi/4}^{\pi/2} e^x\left(2\cot x - 2\operatorname{cosec}^2 x\right)dx$

$I = 2\int_{\pi/4}^{\pi/2} e^x\left[\cot x + \left(-\operatorname{cosec}^2 x\right)\right]dx$

We know that

$\int e^x[f(x) + f'(x)]\,dx = e^x.f(x) + c$

$I = 2\left[e^x.\cot x\right]_{\pi/4}^{\pi/2}$

$I = 2\left[e^{\pi/2}.\cot\frac{\pi}{2} - e^{\pi/4}.\cot\frac{\pi}{4}\right]$

$I = 2\left[0 - e^{\pi/4}.1\right] = -2e^{\pi/4}$

82. (C) Given that $f(x) = 4[x] - 2$ and $g(x) = x + 1$

Now, $fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f[x + 1]$

$\Rightarrow fog(x) = 4[x + 1] - 2$

$\Rightarrow fog(x) = 4[2.5 + 1] - 2$

$\Rightarrow fog(x) = 4[3.5] - 2$

$\Rightarrow fog(x) = 4\times 3 - 2 = 10$

83. (D) Given that $f(x) = \frac{1}{g(x)}$, $g(x) = \frac{1}{x}$

then $f(x) = x$

L.H.S. $= f(f(f(f(f(g(x))))))$

$= f\left(f\left(f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)\right)\right)$

$= f\left(f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)\right)$

$= f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)$

$= f\left(f\left(\frac{1}{x}\right)\right)$

$= f\left(\frac{1}{x}\right) = \frac{1}{x}$

R.H.S. = $g(g(g(g(g(f(x))))))$

$= g(g(g(g(g(x)))))$

$= g\left(g\left(g\left(g\left(\frac{1}{x}\right)\right)\right)\right)$

$= g(g(g(x)))$

$= g\left(g\left(\frac{1}{x}\right)\right)$

$= g(x) = \frac{1}{x}$

L.H.S. = R.H.S

Hence option(D) is correct.

84. (B) Plane $-3x + 2y + 6z = 10$ and point (2, -1,4)

Distance = $\frac{-3\times2+2\times(-1)+6\times4-10}{\sqrt{(-3)^2+2^2+6^2}}$

$= \frac{-6-2+24-10}{\sqrt{9+4+36}} = \frac{6}{7}$

85. (A) Matrix $A_{x\times(x-7)}$ and matrix $B_{y\times(15-y)}$

Both AB and BA exist, then

$x - 7 = y \Rightarrow x - y = 7$ ...(i)

and $x = 15 - y \Rightarrow x + y = 15$ ...(ii)

On solving eq(i) and eq(ii)

$x = 11,\ y = 4$

86. (C) $I = \int_{-2.6}^{1.6} [x]\, dx$

$I = \int_{-2.6}^{-2}[x]\ dx + \int_{-2}^{-1}[x]\ dx + \int_{-1}^{0}[x]\,dx + \int_{0}^{1}[x]\,dx + \int_{1}^{1.6}[x]$

$I = \int_{-2.6}^{-2}(-3)\ dx + \int_{-2}^{-1}(-2)\ dx + \int_{-1}^{0}(-1)\ dx + \int_{0}^{1}0\ dx + \int_{1}^{1.6}1\ dx$

$I = -3[x]_{-2.6}^{-2} - 2[x]_{-2}^{-1} - 1[x]_{-1}^{0} + 0 + [x]_{1}^{1.6}$

$I = -3[-2+2.6] - 2[-1+2] - 1(0+1) + [1.6-1]$

$I = -3\times0.6 - 2\times1 - 1 + 0.6$

$I = -1.8 - 2 - 1 + 0.6 = -4.2$

87. (D) Equation $ax^2 + cx + b = 0$

$\alpha + \frac{1}{\beta} = -\frac{c}{b}$ ...(i)

$\alpha\beta = \frac{b}{a}$ ...(ii)

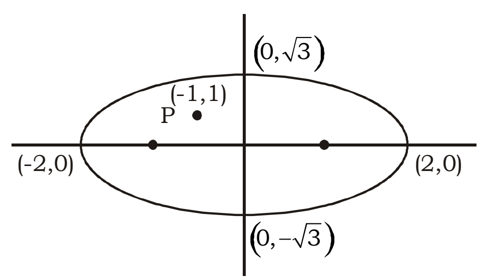
Now, $\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta-\alpha}{\alpha\beta}$

$= \frac{\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}}{\alpha\beta}$

$= \frac{\sqrt{\frac{c^2}{a^2} - \frac{4b}{a}}}{\frac{b}{a}}$

$= \frac{\sqrt{\frac{c^2-4ab}{a^2}}}{\frac{b}{a}} = \frac{\sqrt{c^2-4ab}}{b}$

88. (D)



An ellipse

$3x^2 + 4y^2 = 12$

$\frac{x^2}{4} + \frac{y^2}{3} = 1$

$a^2 = 4,\ b^2 = 3$ and point P (-1, 1)

Hence point P (-1, 1) is inside the ellipse but not at focus.

89. (A) $S = 21^2 + 22^2 + 23^2 + \ldots\ldots\ldots\ldots + 30^2$

$S = (1^2 + 2^2 + 3^2 + \ldots + 20^2 + 21^2 + 22^2 + \ldots + 30^2) - (1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + 20^2)$

We know that

$\Sigma n^2 = \frac{n}{6}(n+1)(2n+1)$

$S = \frac{30}{6}\times31\times61 - \frac{20}{6}\times21\times41$

$S = 9455 - 2870 = 6585$

90. (C) Given that, $\bar{x} = 20$, $\bar{y} = 20$, $\sigma_x = 4, \sigma_y = 2$ and $r_{xy} = 0.6$

regression equation of x on y -

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$\Rightarrow x - 20 = 0.6 \times \frac{4}{2}(y - 80)$

$\Rightarrow x - 20 = 1.2(y - 80)$

$\Rightarrow x - 20 = 1.2y - 96$

$\Rightarrow x = 1.2y - 76$

91. (A) $I = \int_0^1 \frac{x^5}{\sqrt{1-x^4}}\, dx$

Let $x^2 = \sin\theta$ When $x \to 0, \theta \to 0$

$2x = \cos\theta\, d\theta$ $x \to 1, \theta \to \frac{\pi}{2}$

$x = \frac{1}{2}\cos\theta\, d\theta$

$I = \int_0^{\pi/2} \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\, \frac{1}{2}\cos\theta\, d\theta$

$I = \frac{1}{2}\int_0^{\pi/2} \frac{\sin^2\theta}{\cos\theta}\cos\theta\, d\theta$

$I = \frac{1}{2}\int_0^{\pi/2} \frac{1-\cos 2\theta}{2}\, d\theta$

$I = \frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$I = \frac{1}{4}\left[\frac{\pi}{2} - \frac{\sin\pi}{2} - 0 - 0\right]$

$I = \frac{1}{4} \times \frac{\pi}{2} = \frac{\pi}{8}$

92. (C) $n(S) = {}^9C_3 = 84$

$n(E) = {}^4C_1 \times {}^5C_2 + {}^4C_2 \times {}^5C_1 \times {}^4C_3 \times {}^5C_0$

$n(E) = 4 \times 10 + 6 \times 5 + 4 \times 1 = 74$

Probability $P(E) = \frac{n(E)}{n(S)} = \frac{74}{84} = \frac{37}{42}$

93. (C) $\tan^{-1}\left(-\sqrt{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

$= \tan^{-1}\left[\tan\left(\frac{-\pi}{3}\right)\right] = \frac{-\pi}{3}$

Hence principal value of $\tan^{-1}\left(-\sqrt{3}\right) = \frac{-\pi}{3}$

94. (C) In the expansion of $\left(\frac{2}{x} - \frac{x^2}{4}\right)^8$

Middle term = $\left(\frac{8}{2}+1\right)^{th} = 5^{th}$

$T_5 = T_{4+1} = {}^8C_4\left(\frac{2}{x}\right)^{8-4}\left(\frac{-x^2}{4}\right)^4$

$= 70 \times \frac{2^4}{x^4} \times \frac{x^8}{4^4}$

$= 70 \times \frac{2^4}{2^4 \times 2^4} \times x^4 = \frac{35}{8}x^4$

95. (D) Determinant $\begin{vmatrix} 1 & 2 & -3 & -1 \\ 0 & 4 & 2 & 11 \\ 13 & 0 & -5 & 6 \\ 9 & -8 & -3 & 2 \end{vmatrix}$

Minor of 4 = $\begin{vmatrix} 1 & -3 & -1 \\ 13 & -5 & 6 \\ 9 & -3 & 2 \end{vmatrix}$

$= 1(-10 + 18) + 3(26 - 54) - 1(-39 + 45)$

$= 8 + 3(-28) - 1 \times 6 = -82$

96. (A) $\sqrt{2+\sqrt{2+\sqrt{2+2\cos\frac{4\pi}{3}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\left(1+\cos\frac{4\pi}{3}\right)}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{2\pi}{3}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+2\cos\frac{2\pi}{3}}}$

$\Rightarrow \sqrt{2+\sqrt{2\left(1+\cos\frac{2\pi}{3}\right)}}$

$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2\frac{\pi}{3}}}$

$\Rightarrow \sqrt{2+2\cos\frac{\pi}{3}}$

$\Rightarrow \sqrt{2\left(1+\cos\frac{\pi}{3}\right)}$

$\Rightarrow \sqrt{2\times 2\cos^2\frac{\pi}{6}}$

$\Rightarrow 2\cos\frac{\pi}{6} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

97. (B)

98. (C) Equations $-2x + ky = 5$ and $6x + 12y = 7$ has no solution,

then $\frac{-2}{6} = \frac{k}{12} \Rightarrow k = -4$

99. (B) $x + iy = \frac{1}{1-\cos\theta - i\sin\theta}\times\frac{1-\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}$

$x + iy = \frac{1-\cos\theta+i\sin\theta}{(1-\cos\theta)^2 - i^2\sin^2\theta}$

$x + iy = \frac{1-\cos\theta+i\sin\theta}{1+\cos^2\theta-2\cos\theta+\sin^2\theta}$

$x + iy = \frac{1-\cos\theta+i\sin\theta}{2-2\cos\theta}$

$x + iy = \frac{1}{2}\left[1+\frac{i\sin\theta}{1-\cos\theta}\right]$

$x + iy = \frac{1}{2}\left[1+i\frac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}\right]$

$x + iy = \frac{1}{2}\left[1+i\cot\frac{\theta}{2}\right]$

On comparing

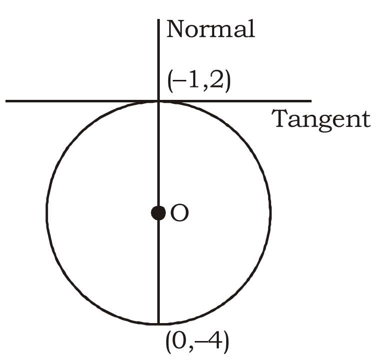
$x = \frac{1}{2}$, $y = \frac{1}{2}\cot\frac{\theta}{2}$

100. (C) Let $y = \text{cosech}x$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \text{cosech}x.\coth x$

101. (B)



Equation of circle

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow (x + 1)(x - 0) + (y - 2)(y + 4) = 0$

$\Rightarrow x^2 + x + y^2 - 2y + 4y - 8 = 0$

$\Rightarrow x^2 + y^2 + x + 2y = 8$

102. (C) $\int_0^2\{k^2+4kx-9x^2\}\,dx \le 0$

$\Rightarrow \left[k^2x+4k\frac{x^2}{2}-\frac{9x^3}{3}\right]_0^2 dx \le 0$

$\Rightarrow 2k^2 + 4k\times\frac{4}{2} - 3\times 8 - 0 \le 0$

$\Rightarrow 2k^2 + 8k - 24 \le 0$

$\Rightarrow k^2 + 4k - 12 \le 0$

$\Rightarrow (k+6)(k-2) \le 0$

Hence $-6 \le k \le 2$

103. (D) $I = \int e^x\left(1+x-\frac{1}{x}+\frac{1}{x^2}\right)dx$

$I = \int e^x\left[\left(x-\frac{1}{x}\right)+\left(1+\frac{1}{x^2}\right)\right]dx$

We know that

$\int e^x[f(x)+f'(x)]\,dx = e^x.f(x) + C$

$I = e^x\left(x-\frac{1}{x}\right) + C$

104. (B) Sphere $x^2 + y^2 + z^2 - 8x + 10y - 6z + 1 = 0$

$u = -4, v = 5, w = -3, d = 1$

radius of sphere $r = \sqrt{u^2+v^2+w^2-d}$

$r = \sqrt{(-4)^2+5^2+(-3)^2-1}$

$r = \sqrt{49} = 7$

Hence diameter of sphere = $2r$ = 14 unit

105. (C) Normal vector = $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & -5 & 7\\ -3 & -4 & 2\end{vmatrix}$

Normal vector = $18\hat{i} - 17\hat{j} - 7\hat{k}$

Equation of the plane passing through the point (1, - 2, 7)

$18(x-1) - 17(y+2) - 7(z-7) = 0$

$\Rightarrow 18x - 18 - 17y - 34 - 7z + 49 = 0$

$\Rightarrow 18x - 17y - 7z = 3$

106. (A) Vertices are $(-k, 2k)$, $(3, k)$ and $(k, 4)$

Area of a triangle = $\frac{1}{2}\begin{vmatrix} -k & 2k & 1 \\ 3 & k & 1 \\ k & 4 & 1 \end{vmatrix}$

$\Rightarrow 20 = \frac{1}{2}\left[-k(k-4) - 2k(3-k) + 1(12-k^2)\right]$

$\Rightarrow 40 = -k^2 + 4k - 6k + 2k^2 + 12 - k^2$

$\Rightarrow 40 = -2k + 12$

$\Rightarrow 2k = -28 \Rightarrow k = -14$

Vertices are (14, -28),(3, -14),(-14, 4).

Centroid = $\left(\frac{14+3-14}{3}, \frac{-28-14+4}{3}\right)$

$= \left(1, \frac{-38}{3}\right)$

107. (C) $\lim_{x\to 0}(1+\cos x) \times \frac{\sin x}{x} \times \frac{2x}{\tan 2x} \times \frac{1}{2}$

$\Rightarrow \lim_{x\to 0}\frac{1+\cos x}{2}\left[\because \lim_{x\to 0}\frac{\sin x}{x} = \lim_{x\to 0}\frac{\tan x}{x} = 1\right]$

$\Rightarrow \frac{1+\cos 0}{2} = \frac{2}{2} = 1$

108. (D) Given that

$(10)^5 + 5(10)^4(6)^1 + 10(10)^3(6)^2 + 10(10)^2(6)^3 + 5(10)^1(6)^4 = k \times 4^8$ ....(i)

We know that

$(x+a)^n = {}^nC_0\, x^n + {}^nC_0\, x^{n-1}a^1 + \ldots\ldots\ldots + {}^nC_n\, a^n$

put $x = 10$, $a = 6$, $n = 5$

$\Rightarrow (10+6)^5 = {}^5C_0\,(10)^5 + {}^5C_1\,(10)^4(6)^1 + {}^5C_2\,(10)^3(6)^2 + \ldots\ldots + {}^5C_2\,(10)^5$

$\Rightarrow (16)^5 = (10)^5 + 5(10)^4(6)^1 + 10(10)^3(6)^2 + 10(10)^2(6)^3 + 5(10)^1(6)^4$

On comparing with eq(i)

$(16)^5 = k \times 4^8$

$\Rightarrow 4^{10} = k \times 4^8 \Rightarrow k = 4^2 = 16$

109. (C) Ellipse $9x^2 + 4y^2 = 18$

$\frac{x^2}{2} + \frac{y^2}{9/2} = 1$

$a = \sqrt{2}$, $b = \frac{3}{\sqrt{2}}$

Area of an ellipse = πab

$= \pi \times \sqrt{2} \times \frac{3}{\sqrt{2}}$

$= 3\pi$ sq. unit

110. (B) Differential equation

$\frac{dy}{dx} + \frac{y}{\sqrt{x^2-4}} = \frac{1}{x+\sqrt{x^2-4}}$

here P = $\frac{1}{\sqrt{x^2-4}}$, Q = $\frac{1}{x+\sqrt{x^2-4}}$

I.F. = $e^{\int P.dx}$

$= e^{\int \frac{1}{\sqrt{x^2-4}}dx}$

$= e^{\log\left|x+\sqrt{x^2-4}\right|} = x + \sqrt{x^2-4}$

Solution of differential equation

$y \times \text{I.F.} = \int Q \times \text{I.F.}\, dx$

$\Rightarrow y \times \left(x + \sqrt{x^2-4}\right) = \int \frac{x+\sqrt{x^2-4}}{x+\sqrt{x^2-4}}dx$

$\Rightarrow y \times \left(x + \sqrt{x^2-4}\right) = \int 1.dx$

$\Rightarrow y \times \left(x + \sqrt{x^2-4}\right) = x + c$

111. (D) P(36, 13) = x

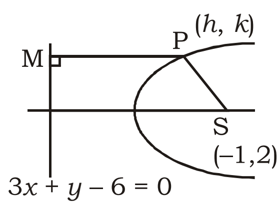
$\Rightarrow \frac{36!}{(36-13)!} = x \Rightarrow x = \frac{36!}{23!}$

and C(36, 23) = y

$\Rightarrow \frac{36!}{23!(36-23)!} = y$

$\Rightarrow x \times \frac{1}{13!} = y \Rightarrow x = y \times 13!$

112. (C)



Condition of parabola

$PS^2 = PM^2$

$\Rightarrow \left[\sqrt{(h+1)^2 + (k-2)^2}\right]^2 = \left[\frac{3h+k-6}{\sqrt{3^2+1^2}}\right]^2$

On solving

$\Rightarrow h^2 + 9k^2 - 6hk + 56h - 28k + 14 = 0$

Hence the equation of parabola

$x^2 + 9y^2 - 6xy + 56x - 28y + 14 = 0$

113. (A) (A - B) × C = A × C - B × C

114. (C) Given that, $a + 3d = 16$

$2a + 6d = 32$ ....(i)

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

put $n = 7$

$\Rightarrow S_7 = \frac{7}{2}[2a + 6d]$

$\Rightarrow S_7 = \frac{7}{2} \times 32 = 112$

115. (A) $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ....$

$\Rightarrow y = \sin x$ .....(i)

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = \cos x$

Again, differentiating

$\Rightarrow \frac{d^2y}{dx^2} = -\sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -y$ from eq(i)

116. (B) $\frac{2+\cos\theta}{2-\cos\theta} = 3$

$\Rightarrow 2 + \cos\theta = 6 - 3\cos\theta$

$\Rightarrow 4\cos\theta = 4$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 2n\pi \pm 0 \Rightarrow \theta = 2n\pi$

117. (B) $n(S) = 6 \times 6 = 36$

$$E = \begin{cases} (6,3),(3,6),(5,4),(4,5) \text{ for sum} = 9 \\ (6,4),(4,6),(5,5) \text{ for sum} = 10 \\ (6,5),(5,6) \text{ for sum} = 11 \\ (6,6) \text{ for sum} = 12 \end{cases}$$

$n(E) = 10$

The Probability that the sum of faces is more than 8 = $\frac{10}{36} = \frac{5}{18}$

The Probability that the sum of faces equals or less than 8 = $1 - \frac{5}{18} = \frac{13}{18}$

118. (C) Let intercept on positive x-axis = a,

intercept on positive y-axis = $3a$

equation of line

$\frac{x}{a} + \frac{y}{3a} = 1$ .....(i)

it passes through the point (- 3, 1)

$\frac{-3}{a} + \frac{1}{3a} = 1$

$\Rightarrow \frac{-9+1}{3a} = 1$

$\Rightarrow 3a = -8 \Rightarrow a = \frac{-8}{3}$

from eq(i)

$\frac{3x}{-8} + \frac{y}{-8} = 1 \Rightarrow 3x + y + 8 = 0$

119. (C) $\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

Now, $[(\vec{a}+\vec{b}) \times (\vec{a} - \vec{b})].\vec{a}$

$\Rightarrow [\vec{a}\times\vec{a} + \vec{b}\times\vec{a} - \vec{a}\times\vec{b} - \vec{b}\times\vec{b}].\vec{a}$

$\Rightarrow [\vec{b}\times\vec{a} + \vec{b}\times\vec{a}].\vec{a}$

$\Rightarrow (\vec{b}\times\vec{a}).\vec{a} + (\vec{b}\times\vec{a}).\vec{a} = 0$

120. (A) Given that, $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{1/3}$ .....(i)

Now, $x^n + y^n = 4$

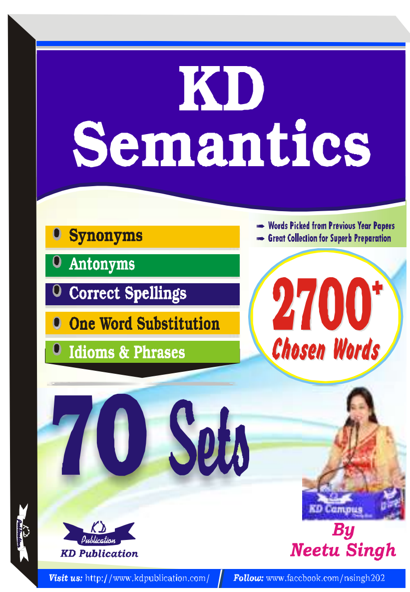
On differentiating both side w.r.t. 'x'

$\Rightarrow nx^{n-1} + ny^{n-1}\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1}$

On comparing with eq(i)

$n - 1 = \frac{1}{3} \Rightarrow n = \frac{4}{3}$



KD Campus

KD Campus Pvt. Ltd

2007, OUTRAM LINES, 1ST FLOOR, OPPOSITE MUKHERJEE NAGAR POLICE STATION, DELHI-110009

NDA (MATHS) MOCK TEST - 116 (Answer Key)

1.	(D)	21.	(B)	41.	(C)	61.	(B)	81.	(A)	101.	(B)
2.	(C)	22.	(A)	42.	(D)	62.	(C)	82.	(C)	102.	(C)
3.	(B)	23.	(C)	43.	(C)	63.	(B)	83.	(D)	103.	(D)
4.	(B)	24.	(A)	44.	(B)	64.	(A)	84.	(B)	104.	(B)
5.	(A)	25.	(C)	45.	(C)	65.	(B)	85.	(A)	105.	(C)
6.	(B)	26.	(C)	46.	(B)	66.	(B)	86.	(C)	106.	(A)
7.	(A)	27.	(A)	47.	(B)	67.	(B)	87.	(D)	107.	(C)
8.	(B)	28.	(B)	48.	(A)	68.	(B)	88.	(D)	108.	(D)
9.	(B)	29.	(C)	49.	(C)	69.	(C)	89.	(A)	109.	(C)
10.	(A)	30.	(B)	50.	(C)	70.	(C)	90.	(C)	110.	(B)
11.	(D)	31.	(D)	51.	(C)	71.	(C)	91.	(A)	111.	(D)
12.	(B)	32.	(B)	52.	(A)	72.	(B)	92.	(C)	112.	(C)
13.	(D)	33.	(C)	53.	(B)	73.	(A)	93.	(C)	113.	(A)
14.	(B)	34.	(A)	54.	(B)	74.	(C)	94.	(C)	114.	(C)
15.	(C)	35.	(D)	55.	(C)	75.	(B)	95.	(D)	115.	(A)
16.	(B)	36.	(B)	56.	(D)	76.	(C)	96.	(A)	116.	(B)
17.	(A)	37.	(B)	57.	(C)	77.	(B)	97.	(B)	117.	(B)
18.	(C)	38.	(D)	58.	(C)	78.	(D)	98.	(C)	118.	(C)
19.	(D)	39.	(B)	59.	(C)	79.	(B)	99.	(B)	119.	(C)
20.	(C)	40.	(C)	60.	(B)	80.	(B)	100.	(C)	120.	(A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777