NDA MATHS MOCK TEST - 118 (SOLUTION)

1. (B) 1 0 0 1 1 . 11

$1\times2^0 = 1$
$1\times2^1 = 2$
$0\times2^2 = 0$
$0\times2^3 = 0$
$1\times2^4 = 16$
$= 19$

$\frac{1}{2} = 1\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
$\frac{3}{4} = 0.75$

Hence $(10011.11)_2 = (19.75)_{10}$

2. (C) Let point (h, k)
According to question

$$\frac{3h+4k-11}{\sqrt{3^2+4^2}} = \frac{20h-21k-15}{\sqrt{20^2+(-21)^2}}$$

$$\Rightarrow \frac{3h+4k-11}{5} = \frac{20h-21k-15}{29}$$

On solving
$43h - 221k + 244 = 0$
Hence locus of point
$43x - 221y + 244 = 0$

3. (D) $I = \int \frac{x}{a}\, da$
$I = x \log a + c$

4. (A) A.M. ≥ G.M. ≥ H.M.

5. (C) $S = \frac{1}{2.3} + \frac{2}{3.5} + \frac{3}{5.8} + \ldots\ldots$ upto 10 terms

$$S = \frac{1}{2.3} + \frac{2}{3.5} + \frac{3}{5.8} + \ldots\ldots + \frac{10}{47.57}$$

$$S = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \ldots\ldots + \left(\frac{1}{47} - \frac{1}{57}\right)$$

$$S = \frac{1}{2} - \frac{1}{57} = \frac{55}{114}$$

6. (B) $\begin{bmatrix} x & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} -8 \\ 4 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} x & -2 \end{bmatrix} \begin{bmatrix} -8 \\ 12 \end{bmatrix} = 0$$

$\Rightarrow -8x - 24 = 0$
$\Rightarrow -8x = 24 \Rightarrow x = -3$

7. (C) Word "INTEGRATION"
The required Permutation

$$= \frac{11!}{2!2!2!} = \frac{39916800}{8} = 4989600$$

8. (B) $s = \frac{\sqrt{1+t^2}}{t}$
On differentiating both side w.r.t 't'

$$\frac{ds}{dt} = \frac{t.\frac{1\times 2t}{2\sqrt{1+t^2}} - \sqrt{1+t^2}.1}{t^2}$$

$$\frac{ds}{dt} = \frac{t^2-1-t^2}{t^2\sqrt{1+t^2}}$$

$$\frac{ds}{dt} = \frac{-1}{t^2\sqrt{1+t^2}}$$

9. (A) $y = x^x$
taking log both side
$\Rightarrow \log y = x \log x$
On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + 1.\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = y(1+\log x) = x^x(1+\log x)$$

10. (C) $\cos(60 - x) + \sin(30 - x)$
$\Rightarrow \cos 60.\cos x + \sin 60.\sin x + \sin 30.\cos x - \cos 30.\sin x$

$$\Rightarrow \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$

$$\Rightarrow \frac{1}{2}\cos x + \frac{1}{2}\cos x = \cos x$$

11. (C) We know that

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$$\frac{r.{}^nC_r}{{}^nC_{r-1}} = n-r+1$$

Now, $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \ldots\ldots\ldots + \frac{nC_n}{C_{n-1}}$

$\Rightarrow (n-1+1) + (n-2+1) + (n-3+1) + \ldots + (n-n+1)$
$\Rightarrow n + (n-1) + (n-2) + \ldots\ldots\ldots\ldots + 1$

$$\Rightarrow \frac{n(n+1)}{2}$$

12. (C) Let first four terms of an A.P. are
$a,\ a+d,\ a+2d,\ a+3d$
According to question
$a + a + d + a + 2d = 42$
$\Rightarrow 3a + 3d = 42$
$\Rightarrow a + d = 14$ …(i)

and $a + a + d + a + 2d + a + 3d = 86$

$\Rightarrow 4a + 6d = 86$

$\Rightarrow 2a + 3d = 43$ ...(ii)

On solving eq (i) and (ii)

$a = -1,\ d = 15$

Now, $T_{11} = a + 10d$

$T_{11} = -1 + 10 \times 15 = 149$

13. (C) In the expansion of $(1+x)^{31}$

$T_{r+7} = T_{(r+6)+1} = {}^{31}C_{r+6}$

$T_{2r-1} = T_{(2r-2)+1} = {}^{31}C_{2r-2}$

According to question

${}^{31}C_{r+6} = {}^{31}C_{2r-2}$

Now, $r + 6 + 2r - 2 = 31 \Rightarrow r = 9$

14. (B) $\lim_{x\to\infty}\left(\frac{x+7}{x+5}\right)^x$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{2}{x+5}\right)^x$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{2}{x+5}\right)^{\frac{x+5}{2}\times\frac{2x}{x+5}}$

$\Rightarrow e^{\lim_{x\to\infty}\frac{2x}{x+5}}$

$\Rightarrow \frac{\sqrt[3]{2}}{3} = e^2$

15. (D) $\sin(1305) = \sin(360 \times 4 - 135)$

$= -\sin 135$

$= -\sin(90 + 45)$

$= -\cos 45 = -\frac{1}{\sqrt{2}}$

16. (A) 1 1 0 0

$0\times2^0 = 0$

$0\times2^1 = 0$

$1\times2^2 = 4$

$1\times2^3 = 8$

Total: 12

0.101

$\frac{1}{2} = 1\times2^{-1}$

$0 = 0\times2^{-2}$

$\frac{1}{8} = 0\times2^{-2}$

$\frac{5}{8} = 0.625$

Hence $(1100.101)_2 = (12.625)_{10}$

17. (B) Given that $n = 21$

No. of diagonals $= \frac{n(n-3)}{2}$

$= \frac{21(21-3)}{2}$

$= \frac{21\times18}{2} = 189$

18. (C) Differential equation

$\sin^{-1}\left(\frac{dy}{dx}\right) = 2(x + y)$

$\Rightarrow$ Let $x + y = t$

$1+\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\Rightarrow \frac{dt}{dx} = 1 + \sin 2t$

$\Rightarrow \frac{dt}{1+\sin 2t} = dx$

$\Rightarrow \frac{dt}{1+\cos\left(\frac{\pi}{2}-2t\right)} = dx$

$\Rightarrow \frac{dt}{2\cos^2\left(\frac{\pi}{2}-2t\right)} = dx$

$\Rightarrow \frac{1}{2}\sec^2\left(\frac{\pi}{4}-t\right) = dx$

On integrating

$\Rightarrow -\frac{1}{2}\tan\left(\frac{\pi}{4}-t\right) = x + c$

$\Rightarrow -\tan\left(\frac{\pi}{4}-x-y\right) = 2x + c$

$\Rightarrow 2x + \tan\left(\frac{\pi}{4}-x-y\right) = c$

19. (C) Distance between circumcentre and incentre $= \sqrt{R(R-2r)}$

20. (B) $\lim_{x\to\infty}\frac{2x^3+5x^2-7}{3x^2-5x^3+3x-2}$

$\Rightarrow \lim_{x\to\infty}\frac{x^3\left(2+\frac{5}{x}-\frac{7}{x^3}\right)}{x^3\left(-5+\frac{3}{x}+\frac{3}{x^2}-\frac{2}{x^3}\right)}$

$\Rightarrow \frac{2+0+0}{-5+0+0} = \frac{-2}{5}$

21. (A) Short method :-

$\lim_{x\to0}\frac{\sqrt[m]{a+x^n}-\sqrt[m]{a-x^n}}{x^n} = \frac{2}{ma^{1-\frac{1}{m}}}$

Now, $\frac{\sqrt[3]{2+x^4}-\sqrt[3]{2-x^4}}{x^4} = \frac{2}{3\times2^{1-\frac{1}{3}}}$

$[\because m = 3,\ a = 2]$

$= \frac{2}{3\times2^{\frac{2}{3}}} = \frac{\sqrt[3]{2}}{3}$

22. (A) $\lim_{x\to\infty}\left(5^x+4^x\right)^{\frac{1}{x}}$

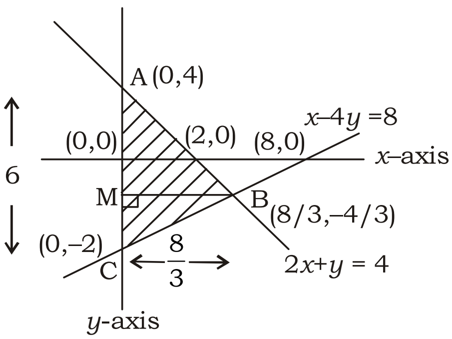
$\Rightarrow \lim_{x\to\infty}5\left(1+\left(\frac{4}{5}\right)^x\right)^{\frac{1}{x}}$

$\Rightarrow \lim_{x\to\infty}5\left(1+\frac{1}{\left(\frac{5}{4}\right)^x}\right)^{\frac{1}{x}}$

$\Rightarrow 5\left(1+\frac{1}{\infty}\right)^0$

$\Rightarrow 5(1)^0 = 5$

23. (C)



Lines

$2x + y = 4$ ...(i)

and $x - 4y = 8$ ...(ii)

On solving eq(i) and eq(ii)

$x = \frac{8}{3}$ and $y = -\frac{4}{3}$

Area of $\Delta ABC = \frac{1}{2}\times AC\times BM$

$= \frac{1}{2}\times 6\times\frac{8}{3} = 8$ sq. unit

24. (C) $z = \frac{3+4i}{(1+i)^2}$

Conjugate of $z = \frac{3-4i}{(1-i)^2}$

$= \frac{3-4i}{1+i^2-2i}$

$= \frac{3-4i}{-2i}\times\frac{i}{i}$

$= \frac{3i-4i^2}{-2i^2} = \frac{3i+4}{2}$

25. (D) Equation

$(1+\lambda)x^2+3x+(3-\lambda)=0$

Now, product of roots = $\frac{3-\lambda}{1+\lambda}$

$-5 = \frac{3-\lambda}{1+\lambda}$

$-5-5\lambda = 3-\lambda \Rightarrow \lambda = -2$

26. (A) $f(x)=\begin{cases}2x^2-5, & -1<x\le 3\\ x-\lambda, & 3<x\le 7\end{cases}$ is

continuous at $x = 3$, then

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\Rightarrow 2\times 3^2 - 5 = 3 - \lambda \Rightarrow \lambda = -10$

27. (B) $y = e^{\tan x}.\cos^2 x$

On differentiating both side w.r.t 'x'

$\Rightarrow \frac{dy}{dx} = e^{\tan x}.\sec^2 x.\cos^2 x$

$+ e^{\tan x}.2\cos x.(-\sin x)$

$\Rightarrow \frac{dy}{dx} = e^{\tan x} - e^{\tan x}.\sin 2x$

$\Rightarrow \frac{dy}{dx} = e^{\tan x}(1-\sin 2x)$

28. (A) A = $\{x\in R,\ x^2+3x-28\le 0\}$

$x^2+3x-28 \le 0$

$(x+7)(x-4)\le 0$

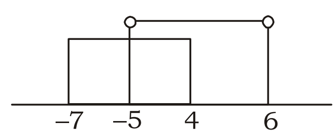
$-7\le x\le 4$

and B = $\{x\in R,\ x^2-x-30<0\}$

$x^2-x-30<0$

$(x-6)(x+5)<0$

$-5<x<6$



Statement I

$(A\cup B) = \{x\in R, -7\le x<6\}$

Statement I is correct.

Statement II

$(A\cup B) = \{x\in R, -5<x\le 4\}$

Statement II is incorrect.

29. (B) $\begin{bmatrix}a & b & c\end{bmatrix}\begin{bmatrix}p & f & g\\ f & q & h\\ g & h & r\end{bmatrix}$

$\Rightarrow \begin{bmatrix}ap+bf+cg & af+bq+ch & ag+bh+cr\end{bmatrix}$

30. (C) $\cos(2\sin^{-1}0.6)$

$$\Rightarrow \cos\left(2\sin^{-1}\frac{3}{5}\right)$$

$$\Rightarrow \cos\left(2\tan^{-1}\frac{3}{4}\right)\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \cos\left(\tan^{-1}\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}\right)\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right)$$

$$\Rightarrow \cos\left(\cos^{-1}\frac{7}{25}\right)\quad\left[\because \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$$

$$\Rightarrow \frac{7}{25}$$

31. (B) Series $1.2 + 2.3 + 3.4 + \ldots\ldots\ldots\ldots + n(n+1)$

$T_n = n(n+1)$

$$S_n = \sum T_n$$

$$S_n = \sum n(n+1)$$

$$S_n = \sum n^2 + \sum n$$

$$S_n = \frac{n}{6}(n+1)(2n+1) + \frac{n(n+1)}{2}$$

$$S_n = \frac{n(n+1)(n+2)}{3}$$

32. (C) Given that $T_n = \frac{n^2 - n + 1}{4}$

$$S_n = \sum T_n$$

$$S_n = \sum \frac{n^2 - n + 1}{4}$$

$$S_n = \frac{1}{4}\left[\sum n^2 - \sum n + \sum 1\right]$$

$$S_n = \frac{1}{4}\left[\frac{n}{6}(n+1)(2n+1) - \frac{n(n+1)}{2} + n\right]$$

$$S_n = \frac{n}{4}\left[\frac{2n^2+3n+1}{6} - \frac{n+1}{2} + 1\right]$$

$$S_n = \frac{n}{4}\times\frac{n^2+2}{3}$$

When, $n = 25$

$$S_{25} = \frac{25}{4}\times\frac{625+2}{3}$$

$$S_{25} = \frac{25}{4}\times 209 = \frac{5225}{4}$$

33. (B) <u>Short Method</u> :-

$T_p = q$ and $T_q = p$

$T_r = p + q - r$

Now, $T_{57} = 97$, $T_{97} = 57$

then, $T_{114} = 57 + 97 - 114$

$= 154 - 114 = 40$

34. (C) We know that

if $S_m = n$, $S_n = m$

then $S_{m+n} = -(m+n)$

now, $S_{27} = 53$ and $S_{53} = 27$

then, $S_{(27+53)} = S_{80} = -(27+53) = -80$

35. (B) $f(x) = \begin{cases} \frac{1-\cos 6x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{3\sqrt{x}}{\sqrt{9+\sqrt{x}}-3}, & x > 0 \end{cases}$ is continuous

at $x = 0$, then

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos 6x}{x^2} = \lim_{x\to 0}\frac{3\sqrt{x}}{\sqrt{9+\sqrt{x}}-3} = a$$

$$\Rightarrow \lim_{x\to 0}\frac{2\sin^2 3x}{9x^2}\times 9$$

$$= \lim_{x\to 0}\frac{3\sqrt{x}\left(\sqrt{9+\sqrt{x}}+3\right)}{9+\sqrt{x}-9} = a$$

$$\Rightarrow \lim_{x\to 0} 18\times\left(\frac{\sin 3x}{3x}\right)^2 = \lim_{x\to 0}\frac{3\sqrt{x}\left(\sqrt{9+\sqrt{x}}+3\right)}{\sqrt{x}} = a$$

$\Rightarrow 18 = 3\times(3+3) = a$

$\Rightarrow a = 18$

36. (D) $f(x) = \begin{cases} 2ax+b, & x < 2 \\ 10, & x = 2 \\ ax-2b, & x > 2 \end{cases}$ is continuous at

$x = 2$, then

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$

$$\lim_{x\to 2} 2ax + b = \lim_{x\to 2} ax - 2b = 10$$

$\Rightarrow 4a + b = 2a - 2b = 10$

$\Rightarrow 4a + b = 10$ ...(i)

$\Rightarrow 2a - 2b = 10$ ...(ii)

On solving eq(i) and eq(ii)

$a = 3$, $b = -2$

37. (A) Curve $y^2 = 2x$

$$2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y}$$

$$m_1 = \left(\frac{dy}{dx}\right)_{at(2,2)} = \frac{1}{2}$$

$$m_2 = \left(\frac{dy}{dx}\right)_{at\left(\frac{9}{2},3\right)} = \frac{1}{3}$$

equation of tangent which passes through the point (2,2)

$$y - 2 = m_1(x-2)$$

$$\Rightarrow y - 2 = \frac{1}{2}(x-2)$$

$$\Rightarrow x - 2y + 2 = 0 \quad ...(i)$$

equation of tangent which passes through the point $\left(\frac{9}{2},3\right)$

$$y - 3 = m_2\left(x - \frac{1}{3}\right)$$

$$\Rightarrow y - 3 = \frac{1}{3}\left(x - \frac{9}{2}\right)$$

$$\Rightarrow 2x - 6y + 9 = 0 \quad ...(ii)$$

On solving eq(i) and eq(ii)

$$x = 3 \text{ and } y = \frac{5}{2}$$

The required intersection point = $\left(3,\frac{5}{2}\right)$

38. (B) Conic

$9x^2 - 16y^2 + 12x - 24y + 59 = 0$

$\Rightarrow (9x^2 + 12x + 4) - (16y^2 + 24y + 9) + 64 = 0$

$\Rightarrow (3x+2)^2 - (4y+3)^2 + 64 = 0$

$$\Rightarrow 9\left(x + \frac{2}{3}\right)^2 - 16\left(y + \frac{3}{4}\right)^2 = -64$$

$$\frac{\left(x+\frac{2}{3}\right)^2}{\frac{64}{9}} - \frac{\left(y+\frac{3}{4}\right)^2}{4} = -1$$

let $x + \frac{2}{3} = X$, $y + \frac{3}{4} = Y$

$$\Rightarrow \frac{X^2}{\frac{64}{9}} - \frac{Y^2}{4} = -1$$

$$a^2 = \frac{64}{9},\ b^2 = 4$$

Now, $e = \sqrt{1 + \frac{a^2}{b^2}}$

$$e = \sqrt{1 + \frac{64}{9 \times 4}} \Rightarrow e = \frac{5}{3}$$

foci(X,Y) = $(0, \pm be)$

X = 0, Y = $\pm be$

$$x + \frac{2}{3} = 0, \quad y + \frac{3}{4} = \pm 2 \times \frac{5}{3}$$

$$x = -\frac{2}{3},\ y + \frac{3}{4} = \frac{10}{3} \text{ and } y + \frac{3}{4} = -\frac{10}{3}$$

$$y = \frac{31}{12} \text{ and } y = -\frac{49}{12}$$

foci are $\left(\frac{-2}{3},\frac{31}{12}\right)$ and $\left(\frac{-2}{3},\frac{-49}{12}\right)$

39. (B) Equation $x^2 + ax + b = 0$

let roots α, $k\alpha$

$$\alpha + k\alpha = -a \Rightarrow \alpha(1+k) = -a \quad ...(i)$$

$$\alpha.k\alpha = b \Rightarrow \alpha^2 k = b \quad ...(ii)$$

and equation

$x^2 + mx + n = 0$

let roots β, $k\beta$

$$\beta + k\beta = -a \Rightarrow \beta(1+k) = -m \quad ...(iii)$$

$$\beta.k\beta = h \Rightarrow \beta^2 k = n \quad ...(iv)$$

from eq(i) and eq(ii)

$$\frac{\alpha}{\beta} = \frac{a}{m} \quad ...(v)$$

from eq(ii) and eq(iv)

$$\frac{\alpha^2}{\beta^2} = \frac{b}{n}$$

$$\left(\frac{a}{m}\right)^2 = \frac{b}{n} \quad \text{[from eq (v)]}$$

$a^2 n = m^2 b$

40. (C) $\left(\frac{-1+\sqrt{3}i}{2}\right)^{10} + \left(\frac{-1-\sqrt{3}i}{2}\right)^{10}$

We know that

$$\omega = \frac{-1+\sqrt{3}i}{2} \text{ and } \omega^2 = \frac{-1-\sqrt{3}i}{2}$$

$$\Rightarrow \omega^{10} + (\omega^2)^{10}$$

$$\Rightarrow \omega + \omega^2 = -1 \quad [\because 1 + \omega + \omega^2 = 0]$$

41. (B) S = $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) + ...$

S = $(1^2 + 2^2 + 3^2 + ...) + (1+2+3+.........)$

$S = \frac{n}{6}(n+1)(2n+1) + \frac{n(n+1)}{2}$

$S = \frac{n(n+1)}{6} \times (2n+4)$

$S = \frac{n(n+1)(n+2)}{3}$

42. (B) According to question

$\frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2a'+(n-1)d')} = \frac{3n+1}{7n-3}$

Now, put $\frac{n-1}{2} = 12 \Rightarrow n = 25$

$\frac{T_{13}}{T'_{13}} = \frac{a+12d}{a'+12d'} = \frac{3\times25+1}{7\times25-3}$

$= \frac{76}{172} = \frac{19}{43}$

43. (C) $S_n = nA + \frac{n(n+1)B}{2}$

$S_{n-1} = (n-1)A + \frac{n(n-1)B}{2}$

$T_n = S_n - S_{n-1}$

$T_n = nA + \frac{n(n+1)B}{2} - (n-1)A - \frac{n(n-1)B}{2}$

$T_n = A + Bn$

$T_{n-1} = A + B(n-1)$

Now, common difference $d = T_n - T_{n-1}$

$\Rightarrow d = A + Bn - A - B(n-1) \Rightarrow d = B$

44. (B) $S = 7 \times 7^{\frac{1}{2}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{8}} \times \ldots\ldots\ldots\ldots\infty$

$S = 7^{\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\ldots\ldots\infty\right)}$

$S = 7^{\frac{1}{1-\frac{1}{2}}} \Rightarrow S = 7^2 = 49$

45. (B) One year = 365 days

= 52 weeks and 1 day

The required Probability $= \frac{1}{7}$

46. (B) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 5 & 2 \\ 3 & -2 & 4 \end{bmatrix}$

Co-factors of A -

$C_{11} = (-1)^{1+1}\begin{vmatrix} 5 & 2 \\ -2 & 4 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 2 \\ 3 & 4 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 5 \\ 3 & -2 \end{vmatrix}$

= 24 = 6 = –15

$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & -1 \\ -2 & 4 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$

= – 6 = 7 = 8

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix}$

= 9 = – 2 = – 5

$C = \begin{bmatrix} 24 & 6 & -15 \\ -6 & 7 & 8 \\ 9 & -2 & 5 \end{bmatrix}$

$\text{Adj } A = C^T = \begin{bmatrix} 24 & -6 & 9 \\ 6 & 7 & -2 \\ -15 & 8 & 5 \end{bmatrix}$

47. (A) Given that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$

$P(A \cup B) = \frac{4}{9}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{4}{9} = \frac{5}{36}$

Now, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{36}}{\frac{1}{4}} = \frac{5}{9}$

48. (A) We know that

$\sin\theta.\sin(60 - \theta).\sin(60 + \theta) = \frac{\sin 3\theta}{4}$

49. (A) S = {1, 2, 3, ...,29, 30}; $n(S) = 30$

X= {4, 8, 12, 16, 20, 24, 28}; $n(X) = 7$

Y = { 2, 4, 6, 8, ...,28, 30}; $n(Y) = 15$

1. $P(X) = \frac{n(X)}{n(S)} = \frac{7}{30}$

Statement 1 is correct.

2. $P(Y) = \frac{n(Y)}{n(S)} = \frac{15}{30} = \frac{1}{2}$

Statement 2 is incorrect.

50. (B) Series $S = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots + \frac{1}{n(n+1)}$

$S = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots\ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S = 1 - \frac{1}{n+1}$

$S = \frac{n+1-1}{n+1} = \frac{n}{n+1}$

51. (B) Let $z = x + iy$

Now, Re $(z^2 - 2i) = 4$

$\Rightarrow$ Re $[(x + iy)^2 - 2i] = 4$

$\Rightarrow$ Re $[x^2 - y^2 + 2xyi - 2i] = 4$

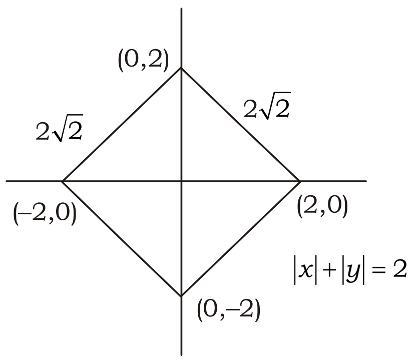
$\Rightarrow x^2 - y^2 = 4$

It is a rectangular hyperbola.

52. (C) $\lim\limits_{x\to\infty} \frac{\sin x}{x} = \frac{1}{\infty} \times (-1 \text{ to } 1) = 0$

$[\because -1 \le \sin x \le 1]$

53. (A)



Area = $2\sqrt{2} \times 2\sqrt{2}$

= 8 square unit

54. (B) $1^c = \left(1 \times \frac{180 \times 7}{22}\right)^o$

$\simeq 57^o\ 16'22''$

55. (A) Let $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta + \frac{4\pi}{3}\right)} = \frac{z}{\cos\left(\theta - \frac{4\pi}{3}\right)} = k$

$x = k\cos\theta,\ y = k\cos\left(\theta + \frac{4\pi}{3}\right),$

$z = k\cos\left(\theta - \frac{4\pi}{3}\right)$

Now, $x + y + z$

$= k\left[\cos\theta + \cos\left(\theta + \frac{4\pi}{3}\right) + \cos\left(\theta - \frac{4\pi}{3}\right)\right]$

$= k\left(\cos\theta + 2\cos\theta.\cos\frac{4\pi}{3}\right)$

$= k\left[\cos\theta + 2\cos\theta \times \left(-\frac{1}{2}\right)\right]$

$= k(\cos\theta - \cos\theta) = 0$

56. (B) $1 - \frac{\sin^2 x}{1 + \cos x} - \frac{\sin y}{1 - \cos y} + \frac{1 + \cos y}{\sin y}$

$\Rightarrow \frac{1 + \cos x - \sin^2 x}{1 + \cos x} + \frac{-\sin^2 y + (1 + \cos y)(1 - \cos y)}{\sin y(1 - \cos y)}$

$\Rightarrow \frac{\cos^2 x + \cos x}{1 + \cos x} + \frac{-\sin^2 y + 1 - \cos^2 y}{\sin y(1 - \cos y)}$

$\Rightarrow \frac{\cos x(1 + \cos x)}{1 + \cos x} + \frac{\cos^2 y - \cos^2 y}{\sin y(1 - \cos y)}$

$\Rightarrow \cos x + 0 = \cos x$

57. (C) $\frac{1}{\log_2 e} + \frac{1}{\log_2 e^3} + \frac{1}{\log_2 e^9} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{1}{\log_2 e} + \frac{1}{3\log_2 e} + \frac{1}{9\log_2 e} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{1}{\log_2 e}\left(1 + \frac{1}{3} + \frac{1}{9} + \ldots\ldots\ldots\infty\right)$

$\Rightarrow \log_e 2 \times \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}\log_e 2$

58. (B) $A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 \\ 3 & -4 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 3 & -4 \end{bmatrix}$

$AB = \begin{bmatrix} 1\times(-1) + 2\times3 & 1\times1 + 2\times(-4) \\ -2\times(-1) + (-3)\times3 & -2\times1 + (-3)\times(-4) \end{bmatrix}$

$AB = \begin{bmatrix} 5 & -7 \\ -7 & 10 \end{bmatrix}$; $|AB| = 50 - 49 = 1$

Co-factors of AB -

$C_{11} = (-1)^{1+1}(10) = 10,\ C_{12} = (-1)^{1+2}(-7) = 7$

$C_{21} = (-1)^{2+1}(-7) = 7,\ C_{22} = (-1)^{2+2}(5) = 5$

$C = \begin{bmatrix} 10 & 7 \\ 7 & 5 \end{bmatrix}$, $AdjAB = \begin{bmatrix} 10 & 7 \\ 7 & 5 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1} = \frac{Adj\ AB}{|AB|} = \begin{bmatrix} 10 & 7 \\ 7 & 5 \end{bmatrix}$

59. (A) $A + iB = \frac{3 - 7i}{1 - i} \times \frac{1 + i}{1 + i}$

$A + iB = \frac{3 - 7i + 3i - 7i^2}{1 - i^2}$

$A + iB = \frac{3 - 4i + 7}{1 + 1}$

$A + iB = \frac{10 - 4i}{2}$

$A + iB = 5 - 2i$

On comparing

A = 5, B = - 2

60. (D) $n = 14$

The required no. of hand-shakes in the party = 14 ×13 = 182

61. (B) Three digit-numbers formed from 0, 1, 2 is 102, 120, 201, 210

Sum of such three-digit numbers

= 102 + 120 + 201 + 210 = 633

62. (C) Time is 4 : 40

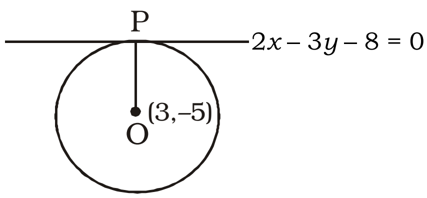
$$\theta = \frac{11M - 60H}{2}$$

$$\theta = \frac{11 \times 40 - 60 \times 4}{2}$$

$$\theta = \frac{440 - 240}{2}$$

$$\theta = \frac{200}{2} = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}$$

63. (A) Given line $2x - 3y - 8 = 0$



Perpendicular distance from point

(3, –5) to the given line $= \frac{2 \times 3 - 3(-5) - 8}{\sqrt{2^2 + (-3)^2}}$

$$r = \frac{6 + 15 - 8}{\sqrt{13}}$$

$$r = \frac{13}{\sqrt{13}} = \sqrt{13}$$

Area of circle $= \pi r^2$

$$= \pi \times \left(\sqrt{13}\right)^2 = 13\pi \text{ sq. unit}$$

64. (C) $y = e^x\,(b\sin x + a\cos x)$ ...(i)

On differentiating both side w. r. t. 'x'

$$\Rightarrow \frac{dy}{dx} = e^x(b\cos x - a\sin x) + e^x(b\sin x + a\cos x)$$

$$\Rightarrow \frac{dy}{dx} = e^x(b\cos x - a\sin x) + y \quad \text{...(ii)}$$

Again, differentiating

$$\Rightarrow \frac{d^2y}{dx^2} = e^x(-b\sin x - a\cos x) + e^x(b\cos x - a\sin x) + \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -e^x(b\sin x + a\cos x) + \frac{dy}{dx} - y + \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y + 2\frac{dy}{dx} - y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

65. (A) Mean of first 10 natural no. $= \frac{10 \times 11}{2 \times 10} = 5.5$

$\sum\left(x - \bar{x}\right)^2 = (1-5.5)^2 + (2-5.5)^2 + (3-5.5)^2 + (4-5.5)^2 + (5-5.5)^2 + (6-5.5)^2 + (7-5.5)^2 + (8-5.5)^2 + (9-5.5)^2 + (10-5.5)^2$

$\sum\left(x - \bar{x}\right)^2 = 20.25 + 12.25 + 6.25 + 2.25 + 0.25 + 0.25 + 2.25 + 6.25 + 12.25 + 20.25$

$\sum\left(x - \bar{x}\right)^2 = 82.5$

$$\text{S.D.} = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{n}}$$

$$= \sqrt{\frac{82.5}{10}} = \frac{\sqrt{33}}{2}$$

66. (B) $e^{-\sin x}\left[\frac{d}{dx}e^{\sin x}\right]$

$\Rightarrow e^{-\sin x}.e^{\sin x}.\cos x = \cos x$

67. (B) $z = \frac{1+i}{1+2i} + \frac{2-2i}{3-i}$

$$z = \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(2-2i)(3+i)}{(3-i)(3+i)}$$

$$z = \frac{3-i}{5} + \frac{8-4i}{10} = \frac{7-3i}{5}$$

$$\bar{z} = \frac{7+3i}{5}$$

Now, $\left(z^2 - z\bar{z}\right) = z\left(z - \bar{z}\right)$

$$\Rightarrow \left(z^2 - z\bar{z}\right) = \frac{7-3i}{5}\left(\frac{7-3i}{5} - \frac{7+3i}{5}\right)$$

$$\Rightarrow \left(z^2 - z\bar{z}\right) = \frac{7-3i}{5}\left(\frac{-6i}{5}\right)$$

$$\Rightarrow \left(z^2 - z\bar{z}\right) = \frac{-42i + 18i^2}{25}$$

$$\Rightarrow \left(z^2 - z\bar{z}\right) = \frac{-42i - 18}{25} = -\frac{18 + 42i}{25}$$

68. (D) Let $y = \sqrt{14 + 5\sqrt{14 + 5\sqrt{14 + \ldots\ldots\infty}}}$

$\Rightarrow y = \sqrt{14 + 5y}$

On squaring both side

$\Rightarrow y^2 = 14 + 5y$

$\Rightarrow y^2 - 5y - 14 = 0$

$\Rightarrow (y - 7)(y + 2) = 0$

$y = 7, -2$

Hence $\sqrt{14 + 5\sqrt{14 + 5\sqrt{14 + \ldots\infty}}} = 7$

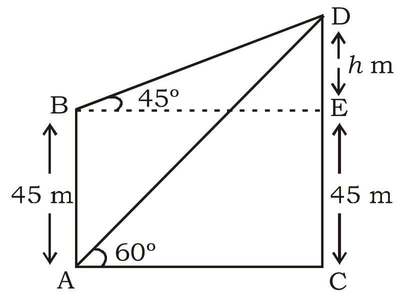
69. (C) In ΔABC, a = 20cm, b = 21cm, c =29cm

$s = \dfrac{20+21+29}{2} = 35$

$\Delta = \dfrac{1}{2} \times 20 \times 21 = 210$

Now, $r = \dfrac{\Delta}{s} = \dfrac{210}{35} = 6$ cm

70. (B)



Let DE = h m

In Δ BDE :–

$\tan 45° = \dfrac{DE}{BE}$

$\Rightarrow 1 = \dfrac{h}{BE} \Rightarrow BE = h = AC$

In ACD :–

$\tan 60° = \dfrac{CD}{AC}$

$\Rightarrow \sqrt{3} = \dfrac{45+h}{h}$

$\Rightarrow \sqrt{3}\, h - h = 45$

$\Rightarrow h = \dfrac{45}{\sqrt{3}-1} = \dfrac{45}{2}\left(\sqrt{3}+1\right)$ m

Height of the tower $= 45 + \dfrac{45}{2}\left(\sqrt{3}+1\right)$

$= \dfrac{45\left(3+\sqrt{3}\right)}{2}$ m

71. (B) Given that $\vec{a} = 2\hat{i} + 6\hat{j} - 3\hat{k}$ and

$\vec{b} = 8\hat{i} - 12\hat{j} + 9\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a}.\vec{b}}{\left|\vec{b}\right|}$

$= \left|\dfrac{2\times8+6\times(-12)-3\times9}{\sqrt{8^2+(-12)^2+9^2}}\right|$

$= \left|\dfrac{16-72-27}{17}\right| = \dfrac{83}{17}$

72. (C) n(S) = 6 × 6

$E = \begin{cases} (6,1),(1,6),(5,2),(2,5) \text{ for sum} = 7 \\ (6,4),(4,6),(5,5) \text{ for sum} = 10 \end{cases}$

n(E) = 7

Probability that the sum is 7 or 10

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{7}{36}$

Probability that the sum is neither 7 or 10 $= P\left(\overline{E}\right) = 1 - P(E)$

$P\left(\overline{E}\right) = 1 - \dfrac{7}{36} = \dfrac{29}{36}$

73. (C)

74. (C)

76. (B) $\begin{bmatrix} 1 & 2 & -4 \\ 2 & -1 & 0 \\ -3 & -4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 16 \end{bmatrix}$

Using elementary method

$\left[A/B\right] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 2 & -1 & 0 & 7 \\ -3 & -4 & -2 & 16 \end{array}\right]$

$R_2 \rightarrow R_2 - 2R_1$, R_3 R_3 + $3R_1$

$\left[A/B\right] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 2 & -14 & 64 \end{array}\right]$

$R_3 \rightarrow R_3 + \dfrac{2}{5}R_2$

$\left[A/B\right] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 0 & \frac{-54}{5} & 54 \end{array}\right]$

Now, $x + 2y - 4z = 16$ .....(i)

$5y + 8z = 25$ ....(ii)

$-\dfrac{54}{5}z = 54$ ...(iii)

On solving

$x = 2,\ y = -3,\ z = -5$

Hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$

76. (D) Differential equation

$\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + y = \dfrac{1}{\left(\dfrac{d^2y}{dx^2}\right)^2}$

$\left(\dfrac{d^2y}{dx^2}\right)^4 + \left(\dfrac{dy}{dx}\right)^3\left(\dfrac{d^2y}{dx^2}\right)^2 + y\left(\dfrac{d^2y}{dx^2}\right)^2 = 1$

Degree = 4

77. (C) $\sin(60 - x) + \sin(60 + x)$

We know that

$\sin C + \sin D = 2\sin\frac{C+D}{2} . \cos\frac{C-D}{2}$

$\Rightarrow 2\sin\frac{60-x+60+x}{2} . \cos\frac{60-x-60-x}{2}$

$\Rightarrow 2\sin 60 . \cos(-x)$

$\Rightarrow 2\times\frac{\sqrt{3}}{2}\cos x = \sqrt{3}\cos x$

78. (A) tan 2475 + sin 2475

$\Rightarrow \tan(360 \times 7 - 45) + \sin(360 \times 7 - 45)$

$\Rightarrow -\tan 45 - \sin 45$

$\Rightarrow -1 - \frac{1}{\sqrt{2}} = -\frac{\sqrt{2}+1}{\sqrt{2}}$

79. (C) $(\log_2 x)(\log_3 9) = \log_5 y$

$(\log_2 x)(\log_3 3^2) = \log_5 y$

$2(\log_2 x)(\log_3 3) = \log_5 y$

$(\log_2 x^2) = \log_5 y$ or $(\log_2 x)(\log_3 3) = \frac{1}{2}\log_5 y$

$x^2 = 2$ and $y = 5$ or $(\log_2 x) = \log_5\sqrt{y}$

$x = \sqrt{2}$ and $y = 5$ or $x = 2$ and $\sqrt{y} = 5$

$x = \sqrt{2}$ and $y = 5$ or $x = 2$ and $y = 25$

80. (C) $y = a^{x+a^{x+a^{x+.....\infty}}}$

$\Rightarrow y = a^{x+y}$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = a^{x+y}\log_e a\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(1-a^{x+y}\log_e a\right) = a^{x+y}\log_e a$

$\Rightarrow \frac{dy}{dx} = \frac{a^{x+y}\log_e a}{1-a^{x+y}\log_e a}$

81. (A) $\sin^2 5 + \sin^2 10 + \sin^2 15 + ...+ \sin^2 90$

$\Rightarrow \sin^2 5 + \sin^2 10 + ...+ \sin^2 40 + \sin^2 45 + \sin^2 50 + ...+\sin^2 80 + \sin^2 85 + 1$

$\Rightarrow (\sin^2 5 + \sin^2 85)(\sin^2 10 + \sin^2 80)+.... ....+(\sin^2 40 + \cos^2 50) + \sin^2 45 + 1$

$\Rightarrow (\sin^2 5 + \cos^2 5) + (\sin^2 10 + \cos^2 10) + ...+ (\sin^2 40 + \cos^2 40) + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$\Rightarrow 1 + 1 + ...$ 8 times $+ \frac{1}{2}+1$

$\Rightarrow 8 + \frac{1}{2}+1 = 9+\frac{1}{2} = 9\frac{1}{2}$

82. (A) $x = a\sec\alpha.\cos\beta$, $y = b\tan\alpha$, $z = c\sec\alpha. \sin\beta$

Now, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}$

$\Rightarrow \frac{a^2\sec^2\alpha - \cos^2\beta}{a^2} - \frac{b^2\tan^2\alpha}{b^2} + \frac{c^2\sec^2\alpha.\sin^2\beta}{c^2}$

$\Rightarrow \sec^2\alpha.\cos^2\beta - \tan^2\alpha + \sec^2\alpha. \sin^2\beta$

$\Rightarrow \sec^2\alpha. \cos^2\beta + \sec^2\alpha. \sin^2\beta - \tan^2\alpha$

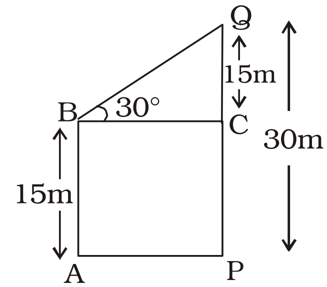
$\Rightarrow \sec^2\alpha(\cos^2\beta + \sin^2\beta) - \tan^2\alpha$

$\left[\because \sin^2 A + \cos^2 A = 1\right]$

$\Rightarrow \sec^2\alpha - \tan^2\alpha$ $\left[\because \sec^2 A - \tan^2 A = 1\right]$

$\Rightarrow 1$

83. (B)



In Δ BCQ :-

$\tan 30° = \frac{QC}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{BC} \Rightarrow BC = 15\sqrt{3}$

Distance between the poles $= 15\sqrt{3}$ m

84. (A) The required Probability $= \frac{2}{6}\times\frac{4}{9}+\frac{4}{6}\times\frac{5}{9}$

$= \frac{8}{54}+\frac{20}{54}$

$= \frac{28}{54} = \frac{14}{27}$

85. (B) $\frac{1+\cos(B-C)\cos A}{1+\cos(B-A)\cos C}$

$\Rightarrow \frac{1+\cos(B-C).\cos[180-(B+C)]}{1+\cos(B-A).\cos[180-(B+A)]}$

$\Rightarrow \frac{1-\cos(B-C).\cos(B+C)}{1-\cos(B-A).\cos(B+A)}$

We know that

$\cos(\alpha + \beta).\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$

$\Rightarrow \frac{1-\cos^2 B+\sin^2 C}{1-\cos^2 B+\sin^2 A}$

$\Rightarrow \frac{\sin^2 B+\sin^2 C}{\sin^2 B+\sin^2 A}$

$\Rightarrow \frac{b^2+c^2}{b^2+a^2}$ [by Sine Rule]

86.(C) Equations

$x - y + 2z = 4$

$2x + y - 3z = 5$

$x + y + z = 13$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 4 \\ 5 \\ 13 \end{bmatrix}$$

Using elementary method

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & -3 & 5 \\ 1 & 1 & -1 & 13 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & -3 & 5 \\ 1 & 1 & -1 & 13 \end{array}\right]$$

$$R_3 \to R_3 - \frac{2}{3} R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 3 & -7 & -3 \\ 0 & 0 & \frac{11}{3} & 11 \end{array}\right]$$

$x - y + 2z = 4$ ....(i)

$3y - 7z = -3$ ...(ii)

$\frac{11}{3} z = 11$ ...(iii)

On solving

$x = 4,\ y = 6,\ z = 3$

87. (A) $I = \int_0^a x(a-x)^6\, dx$

We know that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^a (a-x)x^6 dx$$

$$I = \int_0^a (ax^6 - x^7)dx$$

$$I = \left[\frac{ax^7}{7} - \frac{x^8}{8}\right]_0^a$$

$$I = \frac{a \times a^7}{7} - \frac{a^8}{8} = \frac{a^8}{56}$$

88. (C) $f(x) = ax + \frac{\sqrt{a}}{x}$

On differentiating both side w.r.t. 'x'

$$f'(x) = a - \frac{\sqrt{a}}{x^2}$$

$$f'(a) = a - \frac{\sqrt{a}}{a^2}$$

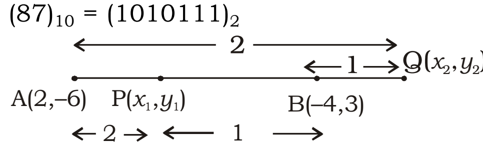
$$f(a) = a - \frac{1}{a^{3/2}} = \frac{a^{5/2} - 1}{a^{3/2}}$$

89. (B)

2	87	1
2	43	1
2	21	1
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$(87)_{10} = (1010111)_2$

90. (B)



$$x_1 = \frac{2\times(-4)+1\times2}{2+1},\ y_1 = \frac{2\times3+1\times(-6)}{2+1}$$

$x_1 = -2,$ $y_1 = 0$

$P(x_1, y_1) = (-2, 0)$

$$x_2 = \frac{2\times(-4)-1\times2}{2-1},\ y_2 = \frac{2\times3-1\times(-6)}{2-1}$$

$x_2 = -10$ $y_2 = 12$

$Q(x_2, y_2) = (-10, 12)$

Now, $PQ = \sqrt{(-2+10)^2+(0-12)^2}$

$\Rightarrow PQ = \sqrt{64+144}$

$\Rightarrow PQ = \sqrt{208} = 4\sqrt{13}$

91. (A) Given that $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$

and $e = \frac{3}{\sqrt{5}}$

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{3}{\sqrt{5}}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{9}{5}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{4}{5}$$

$$\Rightarrow \frac{2a}{a^2} = \frac{4}{5} \Rightarrow a = \frac{5}{2}$$

$$b^2 = 2 \times \frac{5}{2} \Rightarrow b^2 = 5$$

equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{4x^2}{25} - \frac{y^2}{5} = 1 \Rightarrow 4x^2 - 5y^2 = 25$$

92. (C) Let $y = 3^{41}$
taking log both side
$\log_{10}y = 41 \log_{10}3$
$\log_{10}y = 41 \times 0.4771 = 19.56$
The required no. of digits = 19 + 1 = 20

93. (B) $\tan\frac{\theta}{2} + 2\tan\theta + 4\tan 2\theta + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - \cot\frac{\theta}{2} + \tan\frac{\theta}{2} + 2\tan\theta + 4\tan 2\theta + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - \left(\cot\frac{\theta}{2} - \tan\frac{\theta}{2}\right) + 2\tan\theta + 4\tan 2\theta + 8\cot 4\theta$

We know that
cot A – tanA = 2 cot 2A

$\Rightarrow \cot\frac{\theta}{2} - 2\cot\theta + 2\tan\theta + 4\tan 2\theta + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - 2(\cot\theta - \tan\theta) + 4\tan 2\theta + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - 2 \times 2\cot 2\theta + 4\tan 2\theta + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - 4(\cot 2\theta - \tan 2\theta) + 8\cot 4\theta$

$\Rightarrow \cot\frac{\theta}{2} - 4 \times 2\cot 4\theta + 8\cot 4\theta \Rightarrow \cot\frac{\theta}{2}$

94. (B) A = $\{1,2,3,4,5,6,7,8\}$; $n = 8$
Number of subsets of A = $2^n = 2^8 = 256$

95. (A) $z = \left(\sqrt{3} + i\right)$

$\arg(z) = \tan\left(\frac{1}{\sqrt{3}}\right)$

$\arg(z) = \frac{\pi}{6}$

96. (C) $I = \int \frac{x^4 - 1}{x^2\sqrt{x^4 + x^2 + 1}}dx$

$I = \int \frac{x^2 - \frac{1}{x^2}}{x\sqrt{x^2 + 1 + \frac{1}{x^2}}}dx$

$I = \int \frac{x - \frac{1}{x^3}}{\sqrt{x^2 + \frac{1}{x^2} + 1}}dx$

Let $x^2 + \frac{1}{x^2} + 1 = t$

$\left(2x - \frac{2}{x^3}\right)dx = dt$

$\left(x - \frac{1}{x^3}\right)dx = \frac{1}{2}dt$

$I = \frac{1}{2}\int \frac{1}{\sqrt{t}}dt$

$I = \frac{1}{2}\ \frac{t^{1/2}}{1/2} + C$

$I = \sqrt{t} + C \Rightarrow I = \sqrt{x^2 + \frac{1}{x^2} + 1} + C$

97. (C) $I = \int e^x\left(\frac{\sin x - \cos x}{\sin^2 x}\right)dx$

$I = \int e^x(\text{cosec}x - \text{cosec}x.\cot x)dx$

We know that

$\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

$I = e^x.\ \text{cosec}x + c$

98. (B) $\sqrt{\frac{1+\omega}{1+\omega^2}} = \sqrt{\frac{-\omega^2}{-\omega}}$ $\quad [\because 1 + \omega + \omega^2 = 0]$

$= \sqrt{\omega}$

99. (C) We know that

$-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2 + b^2}$

Now,

$-\sqrt{9^2 + 12^2} \le 9\sin\theta + 12\cos\theta \le \sqrt{9^2 + 12^2}$

$\Rightarrow -15 \le 9\sin\theta + 12\cos\theta \le 15$

$\Rightarrow -30 \le 9\sin\theta + 12\cos\theta - 15 \le 0$

Max. value of $(9\sin\theta + 12\cos\theta - 15) = 0$

100. (A) Given that $m = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $n = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$|m\cos\theta - n\sin\theta| = \left|\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\cos\theta - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\sin\theta\right|$

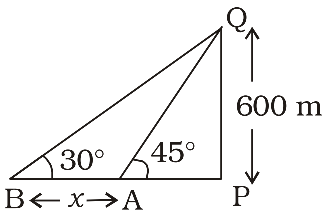
$= \left|\begin{bmatrix} 2\cos\theta & \cos\theta \\ \cos\theta & 2\cos\theta \end{bmatrix} - \begin{bmatrix} \sin\theta & 2\sin\theta \\ 2\sin\theta & \sin\theta \end{bmatrix}\right|$

$= \begin{vmatrix} 2\cos\theta & \cos\theta \\ \cos\theta & 2\cos\theta \end{vmatrix} - \begin{vmatrix} \sin\theta & 2\sin\theta \\ 2\sin\theta & \sin\theta \end{vmatrix}$

$= 4\cos^2\theta - \cos^2\theta - (\sin^2\theta - 4\sin^2\theta)$

$= 3\cos^2\theta + 3\sin^2\theta = 3$

101. (C)



Let AB = x m

In Δ APQ :-

$\tan 45° = \frac{PQ}{AP}$

$\Rightarrow 1 = \frac{600}{AP} \Rightarrow AP = 600$

In ΔBPQ :-

$\tan 30° = \frac{PQ}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{600}{x+600}$

$\Rightarrow x + 600 = 600\sqrt{3}$

$\Rightarrow x = 600\left(\sqrt{3}-1\right)$

Speed of the boat = $\frac{600\left(\sqrt{3}-1\right)}{5\times 60}$

$= 2\left(\sqrt{3}-1\right)$ m/sec

102. (C) Remainder = $\frac{3^{17}+7^{17}}{5}$

$= (-2)^{17} + 2^{17}$

$= -2^{17} + 2^{17} = 0$

Hence the given number is divisible by 5.

103. (D) $\frac{\log_2 8\times\log_{27}9}{\log_3 9\times\log_{16}64}$

$\Rightarrow \frac{3\log_2 2\times 2\log_{27}3}{2\log_3 3\times 6\log_{16}2}$

$\Rightarrow \frac{3\times 2\times\frac{\log 3}{\log 27}}{2\times 6\times\frac{\log 2}{\log 16}}$

$\Rightarrow \frac{\frac{\log 3}{3\log 3}}{2\times\frac{\log 2}{4\log 2}} = \frac{2}{3}$

104. (A) $I_n = \int_0^{\pi/4}\tan^n x\,dx$

$I_n = \int_0^{\pi/4}(\tan x)^{n-2}\tan^2 x\,dx$

$I_n = \int_0^{\pi/4}(\tan x)^{n-2}\left(\sec^2 x-1\right)dx$

$I_n = \int_o^{\pi/4}(\tan x)^{n-2}\sec^2 x\,dx - \int_o^{\pi/4}(\tan x)^{n-2}\,dx$

$I_n = \left[\frac{(\tan x)^{n-2+1}}{n-2+1}\right]_0^{\pi/4} - I_{n-2}$

$I_n + I_{n-2} = \left[\frac{(1)^{n-1}}{n-1} - 0\right]$

$I_n + I_{n-2} = \frac{1}{n-1}$

105. (D) Given that

$\left|\vec{a}\right| = \frac{7}{4}$, $\left|\vec{b}\right| = 8$ and $\vec{a}\times\vec{b} = 5\hat{i} - 3\hat{j} + 8\hat{k}$

Now,

$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$

$\Rightarrow \sqrt{5^2+(-3)^2+8^2} = \frac{7}{4}\times 8\times\sin\theta$

$\Rightarrow \sqrt{98} = 7\times 2\sin\theta$

$\Rightarrow 7\sqrt{2} = 7\times 2\sin\theta$

$\Rightarrow \frac{1}{\sqrt{2}} = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$

106. (B)

age	x	f	$f\times x$
20 – 30	25	5	125
30 – 40	35	6	210
40 – 50	45	8	360
50 – 60	55	9	495
60 – 70	65	2	130
		$\sum f = 30$	1320

Mean age = $\frac{\sum f\times x}{\sum f}$

Mean age = $\frac{1320}{30} = 44$

107. (C) Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $z = \cot^{-1}x$

$x = \cot z$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cot^2 z}-1}{\cot z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\operatorname{cosec} z-1}{\cot z}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{1-\sin z}{\cos z}\right)$

$\Rightarrow y = \tan^{-1}\left[\frac{1-\cos\left(\frac{\pi}{2}-z\right)}{\sin\left(\frac{\pi}{2}-z\right)}\right]$

$\Rightarrow y = \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4}-\frac{z}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{z}{2}\right).\cos\left(\frac{\pi}{4}-\frac{z}{2}\right)}\right]$

$\Rightarrow y = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{z}{2}\right)\right]$

$\Rightarrow y = \frac{\pi}{4}-\frac{z}{2}$

On differentiating both side w.r.t.'z'

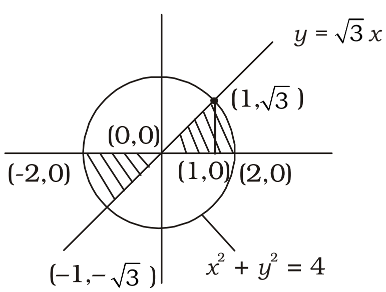
$\Rightarrow \frac{dy}{dz} = -\frac{1}{2}$

108. (D) Circles $x^2 + y^2 + 8x - 6y + 17 = 0$ and $x^2 + y^2 + 4x + 3y + \lambda = 0$
condition of orthogonality
$2gg' + 2ff' = c + c'$

$\Rightarrow 2 \times 4 \times 2 + 2 \times (-3) \times \frac{3}{2} = 17 + \lambda$

$\Rightarrow 16 - 9 = 17 + \lambda \Rightarrow \lambda = -10$

109. (D)



circle $y_1 \Rightarrow y = \sqrt{4-x^2}$

and line $y_2 \Rightarrow y = \sqrt{3}\,x$

Intersecting points of the circle and line

$= \left(1,\sqrt{3}\right),\ \left(-1,-\sqrt{3}\right)$

Area $= 2\left[\int_o^1 y_2 dx + \int_1^2 y_1 dx\right]$

$= 2\left[\int_o^1 \sqrt{3}x\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right]$

$= 2\left[\left(\sqrt{3}\frac{x^2}{2}\right)_0^1 + \left(\frac{1}{2}x\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right)_1^2\right]$

$= 2\frac{\sqrt{3}}{2} + 2\left[0 + 2\sin^{-1}1 - \frac{1}{2}\times\sqrt{3} - 2\sin^{-1}\frac{1}{2}\right]$

$= \sqrt{3} + 2\left[2\times\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\times\frac{\pi}{6}\right]$

$= \sqrt{3} + 2\pi - \sqrt{3} - \frac{2\pi}{3} = \frac{4\pi}{3}$ Sq. unit

110. (C) In the expansion of $\left(\frac{x}{5}-\frac{5}{x}\right)^{10}$

Middle term $= \left(\frac{10}{2}+1\right)^{th} = 6^{th}$

$T_6 = T_{5+1} = {}^{10}C_5\left(\frac{x}{5}\right)^5\left(-\frac{5}{x}\right)^5 = -252$

The required coefficient = – 252

111. (C) Digits (0,1,2,3,4)
Number of three-digit numbers divisible by 4
(i) when last two digits '04' is = 3
(ii) when last two digits '12' is = 2
(iii) when last two digits '20' is = 3
(iv) when last two digits '24' is = 2
(v) when last two digits '32' is = 2
(vi) when last two digits '40' is = 3
Hence the required numbers = 3 + 2 + 3 + 2 + 2 + 3 = 15

112. (C) $\begin{vmatrix} x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \\ 1 & 1 & 1 \end{vmatrix}$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} x^2 & y^2-x^2 & z^2-x^2 \\ x^3 & y^3-x^3 & z^3-x^3 \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow (y-x)(z-x)\begin{vmatrix} x^2 & y+x & z+y \\ x^3 & y^2+x^2+xy & z^2+x^2+xz \\ 1 & 0 & 0 \end{vmatrix}$

$C_3 \to C_3 - C_2$

$\Rightarrow (y-x)(z-x)$

$\Rightarrow (y-x)(z-y)\begin{vmatrix} x^2 & y+x & (z-y) \\ x^3 & y^2+x^2+xy & (z-y)(x+y+z) \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow (y-x)(z-x)(z-y)\begin{vmatrix} x^2 & y+x & 1 \\ x^3 & y^2+x^2+xy & x+y+z \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow (x-y)(y-z)(z-x)$

$\left[(y+x)(x+y+z) - x^2 - y^2 - xy\right]$

$\Rightarrow (x-y)(y-z)(z-x)(xy + yz + zx)$

113. (B) $\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{2\pi}{3}\right)\left(1+\cos\frac{4\pi}{3}\right)\left(1+\cos\frac{5\pi}{3}\right)$

$= 2\cos^2\frac{\pi}{6} \times 2\cos^2\frac{\pi}{3} \times 2\cos^2\frac{2\pi}{3} \times 2\cos^2\frac{5\pi}{6}$

$= 16 \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(-\frac{1}{2}\right)^2 \times \left(-\frac{\sqrt{3}}{2}\right)^2$

$= 16 \times \frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{16}$

114. (C) $x^2 + y^2 = 12$

Let $A = xy^2$

$A = x(12 - x^2)$

$A = 12x - x^3$ ...(i)

On differentiating both side w.r.t.'x'

$\frac{dA}{dx} = 12 - 3x^2$

Again, differentiating

$\frac{d^2A}{dx^2} = -6x$ ...(ii)

for maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 12 - 3x^2 = 0 \Rightarrow x = 2, -2$

from eq. (ii)

$\left(\frac{d^2A}{dx^2}\right)_{at\ x=2} = -6 \times 2 = -12$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{at\ x=-2} = -6 \times (-2) = 12$ (Minima)

Function attains minimum value at $x = -2$, $y = \pm 2\sqrt{2}$

Hence max. value of $xy^2 = -2 \times (2\sqrt{2})^2$

$= -2\times4\times2 = -16$

115. (A) We know that

$e^{ix} = \cos x + i\sin x$

i replace by $-i$

$e^{-ix} = \cos x - i\sin x$

116. (D) $I = \int_0^{\pi/4} \frac{\sqrt{\sin 2x}}{\sqrt{\sin 2x} + \sqrt{\cos 2x}}dx$ ...(i)

We know that

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/4} \frac{\sqrt{\sin 2\left(\frac{\pi}{4} - x\right)}}{\sqrt{\sin 2\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos 2\left(\frac{\pi}{4} - x\right)}}$

$I = \int_0^{\pi/4} \frac{\sqrt{\sin\left(\frac{\pi}{2} - 2x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - 2x\right)} + \sqrt{\cos 2\left(\frac{\pi}{2} - 2x\right)}}$

$I = \int_0^{\pi/4} \frac{\sqrt{\cos 2x}}{\sqrt{\cos 2x} + \sqrt{\sin 2x}}$ ...(ii)

From eq (i) and eq (ii)

$2I = \int_0^{\pi/4} \frac{\sqrt{\sin 2x} + \sqrt{\cos 2x}}{\sqrt{\sin 2x} + \sqrt{\cos 2x}}dx$

$2I = \int_0^{\pi/4} 1.dx$

$2I = [x]_0^{\pi/4} \Rightarrow 2I = \frac{\pi}{4} - 0 \Rightarrow I = \frac{\pi}{8}$

117. (A) $I = \int \frac{1 + \ln x}{\cos^2(x\ln x)}dx$

Let $x\ln x = t$

$(1+\ln x)\,dx = dt$

$I = \int \frac{dt}{\cos^2 t}$

$I = \int \sec^2 dt$

$I = \tan t + c \Rightarrow I = \tan(x\ln x) + c$

118. (B) 1 1 0 1 . 0 1

$1\times2^0 = 1$

$0\times2^1 = 0$

$1\times2^2 = 4$

$1\times2^3 = 8$

Total $= 13$

$0 = 0\times2^{-1}$

$\frac{1}{4} = 1\times2^{-2}$

Total $\frac{1}{4} = 0.25$

Hence $(1101.01)_2 = (13.25)_{10}$

119. (A) Given that $f(x) = \frac{1}{\sqrt{\log_e(43 + x - x^2)}}$

$\log_e(43 + x - x^2) > 0$

$\Rightarrow 43 + x - x^2 > 1$

$\Rightarrow x^2 - x - 42 < 0$

$\Rightarrow (x - 7)(x + 6) < 0$

+ (−6) − (7) +

$x \in (-6, 7)$

120. (D) $\lim_{x\to0} \frac{\sqrt{x+2} - \sqrt{2}}{\sqrt{x+4} - 2}$ $\left[\frac{0}{0}\right]$ Form

by L - Hospital's Rule

$\Rightarrow \lim_{x\to0} \frac{\frac{1}{2\sqrt{x+2}} - 0}{\frac{1}{2\sqrt{x+4}}}$

$\Rightarrow \lim_{x\to0} \frac{\sqrt{x+4}}{\sqrt{x+2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

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NDA (MATHS) MOCK TEST - 118 (Answer Key)

1.	(B)	21.	(A)	41.	(B)	61.	(B)	81.	(A)	101.	(C)
2.	(C)	22.	(A)	42.	(B)	62.	(C)	82.	(B)	102.	(C)
3.	(D)	23.	(C)	43.	(C)	63.	(A)	83.	(B)	103.	(D)
4.	(A)	24.	(C)	44.	(B)	64.	(C)	84.	(A)	104.	(A)
5.	(C)	25.	(D)	45.	(B)	65.	(A)	85.	(B)	105.	(D)
6.	(B)	26.	(A)	46.	(B)	66.	(B)	86.	(C)	106.	(B)
7.	(C)	27.	(B)	47.	(A)	67.	(B)	87.	(A)	107.	(C)
8.	(B)	28.	(A)	48.	(A)	68.	(D)	88.	(C)	108.	(D)
9.	(A)	29.	(B)	49.	(A)	69.	(C)	89.	(B)	109.	(D)
10.	(C)	30.	(C)	50.	(B)	70.	(B)	90.	(B)	110.	(C)
11.	(C)	31.	(C)	51.	(B)	71.	(B)	91.	(A)	111.	(C)
12.	(C)	32.	(C)	52.	(C)	72.	(C)	92.	(C)	112.	(C)
13.	(C)	33.	(B)	53.	(A)	73.	(C)	93.	(B)	113.	(B)
14.	(B)	34.	(C)	54.	(B)	74.	(C)	94.	(B)	114.	(C)
15.	(D)	35.	(B)	55.	(A)	75.	(B)	95.	(A)	115.	(A)
16.	(A)	36.	(D)	56.	(B)	76.	(D)	96.	(C)	116.	(D)
17.	(B)	37.	(A)	57.	(C)	77.	(C)	97.	(C)	117.	(A)
18.	(C)	38.	(B)	58.	(B)	78.	(A)	98.	(B)	118.	(B)
19.	(C)	39.	(B)	59.	(A)	79.	(C)	99.	(C)	119.	(A)
20.	(B)	40.	(C)	60.	(D)	80.	(C)	100.	(A)	120.	(D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777