NDA MATHS MOCK TEST - 122 (SOLUTION)

1. (B) Word "SUCCESS"

The required permutaion = $\frac{7!}{3!2!}$ = 420

2. (C) $I = \int \frac{\sin x}{\sin(x-a)}\, dx$

Let $x - a = t \Rightarrow x = a + t$
$dx = dt$

$I = \int \frac{\sin(a+t)}{\sin t}\, dt$

$I = \int \frac{\sin a.\cos t + \cos a.\sin t}{\sin t}\, dt$

$I = \sin a.\int \cot t\, dt + \cos a \int 1.dt$

$I = \sin a.\log\sin t + \cos a.t + k$

$I = \sin a.\log\sin(x - a) + (x - a)\cos a + k$

$I = \sin a.\log\sin(x - a) + x\cos a + c$

3. (A) In the explansion of $(1 + x)^{38}$

$T_{r+9} = T_{(r+8)+1} = {}^{38}C_{r+8}\ x^{r+8}$

and $T_{3r-5} = T_{(3r-6)+1} = {}^{38}C_{3r-6}\ x^{3r-6}$

According to question

${}^{38}C_{r+8} = {}^{38}C_{3r-6}$

Now, $r + 8 + 3r - 6 = 38$

$\Rightarrow 4r + 2 = 38$

$\Rightarrow 4r = 36 \Rightarrow r = 9$

4. (A) Differential equation

$\frac{dy}{dx} = \frac{y}{x} + \cot\frac{y}{x}$

Let $y = xt \Rightarrow t = \frac{y}{x}$

$\frac{dy}{dx} = x\frac{dt}{dx} + t$

$\Rightarrow x\frac{dt}{dx} + t = t + \cot t$

$\Rightarrow x\frac{dt}{dx} = \cot t$

$\Rightarrow \tan t\, dt = \frac{dx}{x}$

On integrating

$\Rightarrow \log\sec t = \log x + \log c$

$\Rightarrow \sec t = xc$

$\Rightarrow \frac{1}{x}\sec\left(\frac{y}{x}\right) = c$

5. (D) Let first four terms of an A.P. are $a - d$, a, $a + d$ and $a + 2d$

According to question

$a - d + a + a + d = 57$

$\Rightarrow 3a = 57 \Rightarrow a = 19$

and $a - d + a + a + d + a + 2d = 92$

$\Rightarrow 4a + 2d = 92$

$\Rightarrow 4 \times 19 + 2d = 92$

$\Rightarrow 76 + 2d = 92 \Rightarrow d = 8$

Now, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 19 + 9 \times 8]$

$\Rightarrow S_{10} = 10\times(19 + 36) = 550$

6. (B) $\lim_{x\to 0} x\sin\frac{1}{x} = 0 \times [-1 \text{ to } 1] = 0$

$[\because -1 \le \sin\theta \le 1]$

7. (C) Let $y = e^{\sqrt{x}}\tan^{-1}\sqrt{x}$ and $z = \sqrt{x}$

$y = e^z.\tan^{-1} z$

On differentiating both side w.r.t.'z'

$\frac{dy}{dz} = e^z.\frac{1}{1+z^2} + \tan^{-1}z.\ e^z$

$\frac{dy}{dz} = \frac{e^z}{1+z^2}\left[1 + (1+z^2)\tan^{-1} z\right]$

$\frac{dy}{dz} = \frac{e^{\sqrt{x}}}{1+x}[1+(1 + x)\tan^{-1}\sqrt{x}\,]$

8. (D) Quadratic equation

$x^2 + 2x + 3 = 0$

$\alpha + \beta = -2$ and $\alpha.\beta = 3$

Now, $\frac{\alpha^6+\beta^6}{\alpha^{-6}+\beta^{-6}} = \frac{(\alpha^6+\beta^6)(\alpha.\beta)^6}{\alpha^6+\beta^6}$

$= (\alpha.\beta)^6$

$= 3^6 = 729$

9. (A) $\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} a+b+c & 2b & 2c \\ a+b+c & b-c-a & 2c \\ a+b+c & 2b & c-a-b \end{vmatrix}$

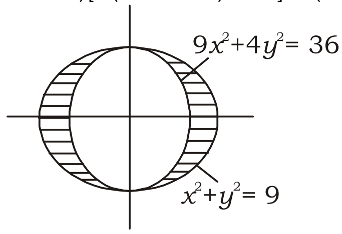
$\Rightarrow (a + b + c)\begin{vmatrix} 1 & 2b & 2c \\ 1 & b-c-a & 2c \\ 1 & 2b & c-a-b \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 2b & 2c \\ 0 & -a-b-c & 0 \\ 0 & 0 & -c-a-b \end{vmatrix}$$

$$\Rightarrow (a+b+c)[1(a+b+c)^2 - 0] = (a+b+c)^3$$

10. (B)



An ellipse

$9x^2 + 4y^2 = 36 \Rightarrow \dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

and circle

$x^2 + y^2 = 9$

Area of ellipse = πab

$= \pi \times 2 \times 3 = 6\pi$

Area of circle = πr^2

$= \pi \times (3)^2 = 9\pi$

The required Area = $9\pi - 6\pi = 3\pi$ sq.unit

11. (D) Let $f(x) = \dfrac{e^{1/x}}{e^{1/x}+1}$

L.H.L. = $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(0-h)$

$= \lim\limits_{h\to 0} \dfrac{e^{1/-h}}{e^{1/-h}+1}$

$= \dfrac{e^{-\infty}}{e^{-\infty}+1} = 0$

R.H.L. = $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{h\to 0} f(0+h)$

$= \lim\limits_{h\to 0} \dfrac{e^{1/h}}{e^{1/h}+1} \left[\dfrac{\infty}{\infty}\right]$ from

$= \lim\limits_{h\to 0} \dfrac{e^{1/h}}{e^{1/h}\left(1+\dfrac{1}{e^{1/h}}\right)}$

$= \lim\limits_{h\to 0} 1+\dfrac{1}{e^{1/h}}$

$= 1+\dfrac{1}{\infty} = 1$

L.H.L. $\neq$ R.H.L.

Hence Limit does not exist.

12. (B) We know that

$$\lim_{x\to 0} \frac{\cos ax - \cos bx}{x^2} = \frac{b^2 - a^2}{2}$$

Now, $\lim\limits_{x\to 0} \dfrac{\cos 6x - \cos 8x}{x^2} = \dfrac{8^2 - 6^2}{2}$

$= \dfrac{28}{2} = 14$

13. (C) $f(x) = \begin{Bmatrix} \dfrac{|x|}{\cos x}, & x \neq 0 \\ \lambda, & x = 0 \end{Bmatrix}$ is continuous

at $x = 0$, then

L.H.L. = R.H.L. = $f(0)$

$\lim\limits_{x\to 0^-} f(x) = \lambda$

$\Rightarrow \lim\limits_{h\to 0} f(0-h) = \lambda$

$\Rightarrow \lim\limits_{h\to 0} \dfrac{(0-h)}{\cos(0-h)} = \lambda$

$\Rightarrow \lim\limits_{h\to 0} \dfrac{h}{\cos h} = \lambda$

$\Rightarrow \dfrac{0}{1} = \lambda \Rightarrow \lambda = 0$

14. (A) **Short method:-**

$$\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx = \left(\frac{ac+bd}{c^2+d^2}\right)x$$

$$+ \left(\frac{ad-bc}{c^2+d^2}\right)\log|c\cos x + d\sin x| + c$$

Now, $\displaystyle\int \frac{3\cos x + \sin x}{2\cos x - \sin x}\,dx = \left[\frac{3\times 2 + 1(-1)}{2^2 + (-1)^2}\right]x$

$+ \dfrac{3(-1) - 1\times 2}{2^2+(-1)^2}\log|2\cos x - \sin x| + c$

$\Rightarrow \displaystyle\int \frac{3\cos x + \sin x}{2\cos x - \sin x}\,dx = \frac{5}{5}x$

$+ \dfrac{-5}{5}\log|2\cos x - \sin x| + c$

$\Rightarrow \displaystyle\int \frac{3\cos x + \sin x}{2\cos x - \sin x}\,dx = x - \log|2\cos x - \sin x| + c$

15. (C) Short method :-

$$\frac{d}{dx}\left[\left((f(x)^{g(x)}\right)\right] = (f(x))^{g(x)}$$

$$\left[\frac{g(x)}{f(x)}.f'(x) + \log(f(x)).g'(x)\right]$$

Now, $\dfrac{d}{dx}[(\log x)^x] = (\log x)^x$

$$\left[\frac{x}{\log x}.\frac{1}{x} + \log(\log x).1\right]$$

$\Rightarrow \dfrac{d}{dx}[(\log x)^x] = (\log x)^x\left[\dfrac{x}{\log x} + \log(\log x)\right]$

16. (A) Standand differential equation

$$\frac{dy}{dx} = \frac{a\,f(x,y)+b}{c\,f(x,y)+d}$$

where $f(x,y)$ is linear equation in x and y.
Solution of differential equation

$$\frac{\begin{vmatrix} x & y \\ c & a \end{vmatrix}}{\begin{vmatrix} c & a \\ a & b \end{vmatrix}} f(c,a) + \log|f(x,y).f(c,a)+f(d,b)| = c$$

Now, Given differential equation

$$\frac{dy}{dx} = \frac{-(x-y)+1}{2(x-y)-1}$$

Compare with standand diff. equation
$f(x,y) = x - y$, $a = -1$, $b = 1$, $c = 2$, $d = -1$
$f(2,-1) = 2 + 1 = 3$ and $f(-1,1) = -1-1 = -2$
Solution of differential equation

$$\frac{\begin{vmatrix} x & y \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}} f(2,-1) + \log|f(x,y).f(2,-1)+f(-1,1)| = c$$

$$\Rightarrow \frac{-x-2y}{2-1}(3) + \log|(x-y)\times 3 - 2| = c$$

$\Rightarrow -3x - 6y + \log|3x - 3y - 2| = c$
$\Rightarrow 3x + 6y + c = \log|3x - 3y - 2|$

17. (D) If $y = \sqrt{f(x)+\sqrt{f(x)+\ldots\ldots\infty}}$,

then $\dfrac{dy}{dx} = \dfrac{f'(x)}{2y-1}$

Now, $y = \sqrt{\tan x + \sqrt{\tan x + \ldots\ldots\infty}}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{d}{dx}(\tan x)}{2y-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$$

18. (C) $\log_2(x^2 - 5x + 28) < 6$
$\Rightarrow x^2 - 5x + 28 < 2^6$
$\Rightarrow x^2 - 5x + 28 < 64$
$\Rightarrow x^2 - 5x - 36 < 0$
$\Rightarrow (x-9)(x+4) < 0$
$x \in (-4, 9)$

19. (B) **Short method :-**

$$\lim_{x\to 0} \frac{\sqrt[m]{a+x^n} - \sqrt[m]{a-x^n}}{x^n} = \frac{2}{m\,a^{1-\frac{1}{m}}}$$

Now, $\dfrac{\sqrt[4]{3+x^5} - \sqrt[4]{3-x^5}}{x^5} = \dfrac{2}{4.3^{1-\frac{1}{4}}}$

$$= \frac{1}{2.3^{\frac{3}{4}}} = \frac{1}{2.\sqrt[4]{27}}$$

20. (C) $\displaystyle\lim_{x\to\infty} \frac{5x^2+6x+8}{-3x^2+6x^3-5x}$ $\left[\frac{\infty}{\infty}\right]$ form

$$\Rightarrow \lim_{x\to\infty} \frac{x^2\left[5+\frac{6}{x}+\frac{8}{x^2}\right]}{x^3\left[-\frac{3}{x}+6-\frac{5}{x^2}\right]}$$

$$\Rightarrow \lim_{x\to\infty} \frac{\left[5+\frac{6}{x}+\frac{8}{x^2}\right]}{x\left(\frac{-3}{x}+6-\frac{5}{x^2}\right)}$$

$$\Rightarrow \frac{5+0}{\infty(0+6)} = 0$$

21. (C) We know that
If $S_p = q$, $S_q = p$, then $S_{p+q} = -(p+q)$
Now, $S_{32} = 64$, $S_{64} = 32$
then $S_{(32+64)} = -(32+64) \Rightarrow S_{96} = -96$

22. (A)

$$\begin{array}{r} 1\,1\,x\,1\,0\,1 \\ +\;\;1\,1\,0\,1\,y\,1 \\ \hline 1\,1\,0\,1\,z\,1\,0 \\ \hline \end{array}$$

$x = 0$, $y = 0$, $z = 0$

23. (D) $f(x) = \dfrac{[x-2]}{[x-1]}$

At $x = 1$

L.H.L. $= \displaystyle\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$$= \lim_{h\to 0} \frac{[1-h-2]}{[1-h-1]}$$

$$= \lim_{h\to 0} \frac{[-1-h]}{[0-h]}$$

$$= \frac{-2}{-1} = 2$$

R.H.L. $= \displaystyle\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

$$= \lim_{h\to 0} \frac{[1+h-2]}{[1+h-1]}$$

$$= \lim_{h\to 0} \frac{[-1+h]}{[0+h]}$$

$$= \frac{-1}{0} = \infty$$

L.H.L $\neq$ R.H.L.
$f(x)$ is not continous at $x = 1$.

At $x = 2$

L.H.L. $= \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} \frac{[2-h-2]}{[2-h-1]}$

$= \lim_{h\to 0} \frac{[0-h]}{[1-h]}$

$= \frac{-1}{0} = \infty$

R.H.L. $= \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} \frac{[2+h-2]}{[2+h-1]}$

$= \lim_{h\to 0} \frac{[0+h]}{[1+h]}$

$= \frac{0}{1} = 0$

L.H.L. $\neq$ R.H.L.

$f(x)$ is not continuous at $x = 2$.

24. (A) Let A $= \begin{bmatrix} 1 & 0 & -1 \\ 3 & -2 & 4 \\ 5 & -1 & 3 \end{bmatrix}$

Co-factors of A –

$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 4 \\ -1 & 3 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -2 \\ 5 & -1 \end{vmatrix}$

$= -2 \qquad = 11 \qquad = 7$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ -1 & 3 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 5 & 3 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix}$

$= 1 \qquad = 8 \qquad = 1$

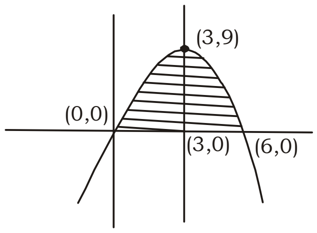
$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -1 \\ -2 & 4 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix}$

$= -2 \qquad = -7 \qquad = -2$

C $= \begin{bmatrix} -2 & 11 & 7 \\ 1 & 8 & 1 \\ -2 & -7 & -2 \end{bmatrix}$

Adj A $= C^T = \begin{bmatrix} -2 & 1 & -2 \\ 11 & 8 & -7 \\ 7 & 1 & -2 \end{bmatrix}$

25. (B)



Curve $y = 6x - x^2$

Area $= \int_0^6 y\,dx$

$= \int_0^6 (6x - x^2)\,dx$

$= \left[6 \times \frac{x^2}{2} - \frac{x^3}{3}\right]_0^6$

$= \left[6 \times \frac{6^2}{2} - \frac{6^3}{3} - 0\right]$

$= 108 - 72 = 36$ sq.unit

26. (C) Hyperbola

$16x^2 - 4y^2 = 1 \Rightarrow \frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{1}{4}} = 1$

$a^2 = \frac{1}{16}, b^2 = \frac{1}{4}$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 + \frac{\frac{1}{4}}{\frac{1}{16}}}$

$\Rightarrow e = \sqrt{1+4} = \sqrt{5}$

27. (C) $\log\left(a + \sqrt{a^2 + x^2}\right) + \log\left[\frac{1}{a + \sqrt{a^2 + x^2}}\right]$

$\Rightarrow \log\left(a + \sqrt{a^2 + x^2}\right)\left(\frac{1}{a + \sqrt{a^2 + x^2}}\right)$

$[\because \log m + \log n = \log mn]$

$\Rightarrow \log 1 = 0$

28. (B) $(3\sin\theta + 4)(\sqrt{2}\sin\theta + 1) = 0$

$\sin\theta \neq \frac{-4}{3}, \quad \sin\theta = \frac{-1}{\sqrt{2}}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

29. (C) $y = ax\sin\left(\frac{1}{x} + b\right)$

On differentiating both side w.r.t.'x'

$\Rightarrow y_1 = ax.\cos\left(\frac{1}{x} + b\right)\left(\frac{-1}{x^2}\right) + a\sin\left(\frac{1}{x} + b\right).1$

$\Rightarrow y_1 = \frac{-a}{x}\cos\left(\frac{1}{x} + b\right) + a\sin\left(\frac{1}{x} + b\right)$

$\Rightarrow xy_1 = -a\cos\left(\frac{1}{x}+b\right) + ax\sin\left(\frac{1}{x}+b\right)$

$\Rightarrow xy_1 = -a\cos\left(\frac{1}{x}+b\right) + y$

Again, differentiating

$\Rightarrow xy_2 + y_1 = -a(-1)\sin\left(\frac{1}{x}+b\right)\left(\frac{-1}{x^2}\right) + y_1$

$\Rightarrow xy_2 + y_1 = \frac{-a}{x^2}\sin\left(\frac{1}{x}+b\right) + y_1$

$\Rightarrow x^4y_2 + x^3y_1 = -ax\sin\left(\frac{1}{x}+b\right) + x^3y_1$

$\Rightarrow x^4y_2 = -y \Rightarrow x^4y_2 + y_1 = 0$

30. (B) $\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{12}{13}$

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12}$ $\left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow \tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right)$

$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$\Rightarrow \tan^{-1}\left(\frac{63}{16}\right)$

$\Rightarrow \cos^{-1}\frac{16}{65}$ $\left[\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

31. (A) Line $3x - 4y - 7$

$m_1 = \frac{3}{4}$

and line $3x + 5y = 9$

$m_2 = \frac{-3}{5}$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{3}{4}+\frac{3}{5}}{1+\frac{3}{4}\left(\frac{-3}{5}\right)}\right|$

$\Rightarrow \tan\theta = \left(\frac{\frac{27}{20}}{\frac{11}{20}}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{27}{11}\right)$

32. (B) The required probability P(E) = 0

33. (C)

2	49	1
2	24	0
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(49)_{10} = (110001)_2$

34. (D) In the expansion of $\left(\frac{x^2}{7} - \frac{7}{x}\right)^{10}$

Middle term = $\left(\frac{10}{2}+1\right)^{th} = 6^{th}$

$T_6 = T_{5+1} = {}^{10}C_5\left(\frac{x^2}{7}\right)^5\left(\frac{-7}{x}\right)^5$

$= 252 \times \frac{x^{10}}{7^5} \times (-1) \times \frac{7^5}{x^5}$

$= -252\,x^5$

The required coefficient = – 252

35. (A) Given that $\theta = 30°$

$m = \tan 30° = \frac{1}{\sqrt{3}}$

equation of line

$y = mx + c$

$y = \frac{1}{\sqrt{3}}x + c$ ....(i)

its passes through the point (–3,1)

$1 = \frac{1}{\sqrt{3}} \times (-3) + c \Rightarrow c = \sqrt{3} + 1$

from eq(i)

$y = \frac{1}{\sqrt{3}}x + \sqrt{3} + 1$

$\Rightarrow \sqrt{3}\,y - x = 3 + \sqrt{3}$

36. (C) S = 0.2 + 0.22 + 0.222+.......n terms

$S = \frac{2}{9}(0.9 + 0.99 + 0.999 + \ldots\ldots n \text{ terms})$

$S = \frac{2}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\ldots\ldots n \text{ terms}\right]$

$S = \frac{2}{9}(1 + 1 + \ldots\ldots n\text{ terms})$

$-\frac{2}{9}\left[\frac{1}{10}+\frac{1}{100}+\ldots\ldots n \text{ terms}\right]$

$S = \frac{2}{9}n - \frac{2}{9} \times \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}$

$$S = \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

37. (B) $3x - 16y = 4$

$3x - 16y = \frac{-5}{2}$ $\quad \updownarrow 13$

$3x - 16y = -9$

The required line

$$3x - 16y = \frac{-5}{2} \Rightarrow 6x - 32y + 5 = 0$$

38. (B) Differential equation

$$\frac{dy}{dx} + yx = e^{\frac{-x^2}{2}}$$

On comparing with standand linear differential equation

$$P = x,\ Q = e^{\frac{-x^2}{2}}$$

$$\text{I.F.} = e^{\int P.dx} = e^{\int x\,dx} = e^{\frac{x^2}{2}}$$

Solution of diffterential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx$$

$$\Rightarrow y \times e^{\frac{x^2}{2}} = \int e^{\frac{-x^2}{2}} \times e^{\frac{x^2}{2}}\,dx$$

$$\Rightarrow y \times e^{\frac{x^2}{2}} = \int 1\,dx$$

$$\Rightarrow y \times e^{\frac{x^2}{2}} = x + c$$

39. (C) $\overrightarrow{OA} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\overrightarrow{OB} = -3\hat{i} + 2\hat{j} - 4\hat{k}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\overrightarrow{AB} = -3\hat{i} + 2\hat{j} - 4\hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} - 7\hat{k}$$

Direction Cosine of line $\overrightarrow{AB} = < \frac{-4}{9}, \frac{4}{9}, \frac{-7}{9} >$

40. (B) $A = \begin{bmatrix} 0 & -1 & 1 \\ 3 & -2 & 0 \\ 1 & -4 & 3 \end{bmatrix}$

$|A| = 0 + 1(9 - 0) + 1(-12 + 2) = -1$

Short method :-

	C_1	C_2	C_3	C_1	C_2
R_1	0	−1	1	0	−1
R_2	3	−2	0	3	−2
R_3	1	−4	3	1	−4
R_1	0	−1	1	0	−1
R_2	3	−2	0	3	−2

$$C = \begin{bmatrix} -2\times3 - 0(-4) & 0\times1 - 3\times3 & 3(-4) - (-2)\times1 \\ -4\times1 - 3(-1) & 3\times0 - 1\times1 & 1(-1) - (-4)\times0 \\ -1\times0 - 1\times(-2) & 1\times3 - 0\times0 & 0(-2) - (-1)\times3 \end{bmatrix}$$

$$C = \begin{bmatrix} -6 & -9 & -10 \\ -1 & -1 & -1 \\ 2 & 3 & 3 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} -6 & -1 & 2 \\ -9 & -1 & 3 \\ -10 & -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= -\begin{bmatrix} -6 & -1 & 2 \\ -9 & -1 & 3 \\ -10 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -2 \\ 9 & 1 & -3 \\ 10 & 1 & -3 \end{bmatrix}$$

41. (A) In ΔABC, A(–4,2), B(–2,9) and C(0,–2) co-ordinates of centroid

$$= \left(\frac{-4-2+0}{3}, \frac{2+9-2}{3}\right) = (-2, 3)$$

42. (A) We know that

Plane $ax + by + cz + d = 0$

and point (x_1, y_1, z_1)

Image of the point-

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Now, Plane $2x + 4y - z = 7$ and point (–3, –3, – 4)

then image of the piont :-

$$\frac{x+3}{2} = \frac{y+3}{4} = \frac{z+4}{-1}$$

$$= \frac{-2[2\times(-3) + 4(-3) - 1(-4) - 7]}{2^2 + 4^2 + (-1)^2}$$

$$\Rightarrow \frac{x+3}{2} = \frac{y+3}{4} = \frac{z+4}{-1} = \frac{-2(-21)}{21}$$

$$\Rightarrow \frac{x+3}{2} = \frac{y+3}{4} = \frac{z+4}{-1} = 2$$

Now, $\frac{x+3}{2} = 2 \Rightarrow x = 1$

$$\frac{y-3}{4} = 2 \Rightarrow y = 5$$

$$\frac{z+4}{-1} = 2 \Rightarrow z = -6$$

Image of the point = (1, 5, –6)

43. (C) $(1 + \sin x - \cos x)^2$
$\Rightarrow 1 + \sin^2 x + \cos^2 x + 2\sin x - 2\sin x.\cos x - 2\cos x$
$\Rightarrow 1 + 1 + 2\sin x - 2\sin x.\cos x - 2\cos x$
$\Rightarrow 2(1 + \sin x) - 2\cos x(1 + \sin x)$
$\Rightarrow 2(1 + \sin x)(1 - \cos x)$

44. (A) We know that
$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots + C_n x^n$
On differentiating both side w.r.t.'x'
$\Rightarrow n(1 + x)^{n-1} = 0 + C_1 + 2C_2 x + \ldots + nC_n x^{n-1}$
on putting $x = 1$
$\Rightarrow n.2^{n-1} = C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n$
$\Rightarrow n.2^{n-1} + C_0 = C_0 + C_1 + 2C_2 + 3C_3 + \ldots + nC_n$
$\Rightarrow n.2^{n-1} + 1 = C_0 + C_1 + 2C_2 + 3C_3 + \ldots + nC_n$

45. (D) Given that $e = \frac{1}{3}$

and $ae = 2 \Rightarrow a \times \frac{1}{3} = 2 \Rightarrow a = 6$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \frac{1}{3} = \sqrt{1 - \frac{b^2}{36}}$

$\Rightarrow \frac{1}{9} = 1 - \frac{b^2}{36}$

$\Rightarrow \frac{b^2}{36} = \frac{8}{9} \Rightarrow b^2 = 32$

equation of an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{36} + \frac{y^2}{32} = 1 \Rightarrow 8x^2 + 9y^2 = 288$

46. (C) $\sin^{-1}\left[\sin\left(\frac{\pi}{4}\right)\right] = \sin^{-1}\left[\sin\left(2\pi + \frac{\pi}{4}\right)\right]$

$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$

47. (C) Median

48. (B) Differential equation

$\frac{dy}{dx} = x^2.e^{-x}$

$\Rightarrow \int dy = \int x^2.e^{-x}\,dx$

$\Rightarrow y = x^2\int e^{-x}dx - \int\left\{\frac{d}{dx}x^2.\int e^{-x}dx\right\}dx + c$

$\Rightarrow y = -x^2.e^{-x} - \int 2x(-e^{-x})\,dx + c$

$\Rightarrow y = -x^2.e^{-x} + 2\left[x.\int e^{-x}dx - \int\left\{\frac{d}{dx}(x).\int e^{-x}dx\right\}dx + c\right]$

$\Rightarrow y = -x^2.e^{-x} + 2\left[-x.e^{-x} - \int 1.(-e^{-x}dx\right] + c$

$\Rightarrow y = -x^2.e^{-x} - 2x.e^{-x} + 2\int e^{-x}dx + c$

$\Rightarrow y = -x^2.e^{-x} - 2x.e^{-x} - 2e^{-x} + c$

49. (A) $I = \int e^x(\sin x + 2\cos x).\sin x\,dx$

$I = \int e^x(\sin^2 x + 2\sin x.\cos x)dx$

We know that

$\int_0^a e^x[f(x) + f'(x)]\,dx = e^x.f(x) + c$

$I = e^x.\sin^2 x + c$

50. (B) Sphere $x^2 + y^2 + z^2 - 6x + 10y - 16z - 2 = 0$
$u = -3,\ v = 5,\ w = -8$

radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{(-3)^2 + (5)^2 + (-8)^2 + 2}$

$= \sqrt{9 + 25 + 64 + 2} = 10$

Dianeter of the sphere $= 2r$
$= 2 \times 10 = 20$ unit

51. (C) Let $y = \log(\log(\log x))$
On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = \frac{1}{\log(\log x)}\ \frac{1}{\log x}\ \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x\log x.\log(\log x)}$

52. (B) $P(23, 12) = x$

$\Rightarrow \frac{23!}{(23-12)!} = x \Rightarrow x = \frac{23!}{11!}$

and $3! \times (23,11) = y$

$\Rightarrow 6 \times \frac{23!}{11!\,12!} = y$

$\Rightarrow 6 \times \frac{x}{12!} = y$

$\Rightarrow \frac{x}{2 \times 11!} = y \Rightarrow x = 2y \times 11!$

53. (C) Let $y = 5^x + x^5$
On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = 5^x\log 5 + 5x^4$

54. (C) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha.\beta = \frac{c}{a}$

Now, $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2+2\alpha\beta}{\alpha\beta}$

$= \frac{\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}}{\frac{c}{a}}$

$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} = \frac{b^2-2ac}{ac}$

55. (B) $\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}$

$\Rightarrow \sqrt{2\cos^2\frac{\theta}{2}} - \sqrt{2\sin^2\frac{\theta}{2}}$

$\Rightarrow \sqrt{2}\cos\frac{\theta}{2} - \sqrt{2}\sin\frac{\theta}{2}$

56. (C) $\lim_{x\to 0} x\sin\frac{1}{x} = p$ $\left[\because -1 \le \sin\frac{1}{x} \le 1\right]$

$\Rightarrow 0 \times (-1 \text{ to } 1) = p \Rightarrow p = 0$

and $\lim_{x\to 0} \frac{\tan x}{x} = q$

$\Rightarrow 1 = q$

Hence $p = 0$, $q = 1$

57. (C) In ΔABC,

$4bc \sin\left(\frac{A-B-C}{2}\right)$

$\Rightarrow 4bc\sin\left[\frac{A-(\pi-A)}{2}\right]$ $[\because A+B+C=\pi]$

$\Rightarrow 4bc \sin\left[\frac{2A-\pi}{2}\right]$

$\Rightarrow 4bc \sin\left[A-\frac{\pi}{2}\right]$

$\Rightarrow -4bc\sin\left(\frac{\pi}{2}-A\right)$

$\Rightarrow -4bc\cos A$

$\Rightarrow -4bc \times \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow -2(b^2+c^2-a^2) = 2(a^2-b^2-c^2)$

58. (A) $\frac{\cos 15 + \sin 15}{\cos 15 - \sin 15}$

$\Rightarrow \frac{\frac{\cos 15}{\cos 15} + \frac{\sin 15}{\sin 15}}{\frac{\cos 15}{\cos 15} - \frac{\sin 15}{\cos 15}}$

$\Rightarrow \frac{1+\tan 15}{1-\tan 15}$

$\Rightarrow \frac{\tan 45 + \tan 15}{1 - \tan 45.\tan 15}$

$\Rightarrow \tan(45+15) = \tan 60° = \sqrt{3}$

59. (D) $n(S) = {}^{13}C_4 = 715$

$n(E) = {}^6C_1 \times {}^7C_3 + {}^6C_2 \times {}^7C_2 + {}^6C_3 \times {}^7C_1$

$= 6 \times 35 + 15 \times 21 + 20 \times 7 = 665$

The required probability $P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{665}{715} = \frac{133}{143}$

60. (C) $\tan\left(\tan^{-1}\frac{3}{7} + \frac{\pi}{4}\right)$

$\Rightarrow \tan\left[\tan^{-1}\frac{3}{7} + \tan^{-1}(1)\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{3}{7}+1}{1-\frac{3}{7}\times 1}\right)\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{10}{7}}{\frac{4}{7}}\right)\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{5}{2}\right)\right] = \frac{5}{2}$

61. (B) $\frac{\left[1+(i^5)^{8n-1}\right]^{4n+1}}{\left[1+(i^5)^{8n+1}\right]^{4n-1}}$

$\Rightarrow \frac{\left[1+(i)^{8n-1}\right]^{4n+1}}{\left[1+(i)^{8n+1}\right]^{4n-1}}$

$\Rightarrow \frac{\left[1+i^{8n}.i^{-1}\right]^{4n+1}}{\left[1+i^{8n}.i^{1}\right]^{4n-1}}$

$\Rightarrow \frac{(1+i)^{4n+1}}{i^{4n+1}\left[1+i\right]^{4n-1}}$

$\Rightarrow \frac{(1+i)^2}{i^{4n}.i^1}$

$\Rightarrow \frac{1+i^2+2i}{i}$

$\Rightarrow \frac{1-1+2i}{i} = 2$

62. (A) **Statement I**

$\tan\theta = x$ and $\frac{1}{x} = \cot\theta$

Now, $x + \frac{1}{x} = \tan\theta + \cot\theta$

$\Rightarrow x + \frac{1}{x} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{2(\sin^2\theta + \cos^2\theta)}{2\cos\theta.\cos\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{2}{\sin 2\theta}$

$\Rightarrow x + \frac{1}{x} = 2\ \text{cosec}2\theta$

Statement I is correct.

Statement II

$x - \frac{1}{x} = 2\tan\theta$

On squaring both side

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 4\tan^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\tan^2\theta + 2\tan^2\theta + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\tan^2\theta + 2\sec^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(\tan^2\theta + \sec^2\theta)$

Statement II is incorrect.

63. (D) Given that $f(x) = \frac{[x]-6}{[x-7]}$

At $x = 7$

L.H.L. $= \lim_{x\to 7^-} f(x) = \lim_{x\to 0} f(7-h)$

$= \lim_{x\to 0} \frac{[7-h]-6}{[7-h-7]}$

$= \lim_{x\to 0} \frac{6-6}{[0-h]}$

$= \frac{0}{-1} = 0$

R.H.L. $= \lim_{x\to 7^+} f(x) = \lim_{x\to 0} f(7+h)$

$= \lim_{x\to 0} \frac{[7+h]-6}{[7+h-7]}$

$= \lim_{x\to 0} \frac{7-6}{[0+h]} = \frac{1}{0} = \infty$

L.H.L. $\neq$ R.H.L.

$f(x)$ is not continuous at $x = 7$.

At $x = 6$

L.H.L.$= \lim_{x\to 6^-} f(x) = \lim_{x\to 0} f(6-h)$

$= \lim_{x\to 0} \frac{[6-h]-6}{[6-h-7]}$

$= \lim_{x\to 0} \frac{5-6}{[-1-h]}$

$= \frac{-1}{-2} = \frac{1}{2}$

R.H.L.$= \lim_{x\to 6^+} f(x) = \lim_{x\to 0} f(6+h)$

$= \lim_{x\to 0} \frac{[6+h]-6}{[6+h-7]}$

$= \lim_{x\to 0} \frac{6-6}{[-1+h]} = \frac{0}{-1} = 0$

L.H.L. $\neq$ R.H.L.

$f(x)$ is not continuous at $x = 6$.

64. (A) $A = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

Now, $A^2 = A.A$

$\Rightarrow A^2 = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix}$

$\Rightarrow A^2 = -6\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

$\Rightarrow A^2 = -6A$

65. (C) $\lim_{x\to\infty} \frac{3x^2-4x+1}{2x-4x^2-5}$ $\left[\frac{\infty}{\infty}\right]$ Form

$\Rightarrow \lim_{x\to\infty} \frac{x^2\left(3-\frac{4}{x}+\frac{1}{x^2}\right)}{x^2\left(-4+\frac{2}{x}-\frac{5}{x}\right)}$

$\Rightarrow \lim_{x\to\infty} \frac{3-0+0}{-4+0-0} = \frac{-3}{4}$

66. (B) $I = \int_0^{\pi/4} e^x\left(\frac{\sin 2x+2}{\cos^2 x}\right)dx$

$I = \int_0^{\pi/4} e^x\left(\frac{2\sin x.\cos x+2}{\cos^2 x}\right)dx$

$I = \int_0^{\pi/4} e^x\left(2\tan x+2\sec^2 x\right)dx$

$I = 2\int_0^{\pi/4} e^x\left(\tan x+\sec^2 x\right)dx$

We know that

$\int e^x [f(x)+f'(x)]\,dx = e^x.\,f(x) + c$

$I = 2\left[e^x \tan x\right]_0^{\pi/4}$

$I = 2\left[e^{\pi/4}.\tan\frac{\pi}{4} - e^0 \tan 0\right]$

$I = 2.e^{\pi/4}$

67. (A) $e = \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{1-\frac{a^2}{b^2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow 1 - \frac{a^2}{b^2} = \frac{1}{2}$

$\Rightarrow \frac{a^2}{b^2} = \frac{1}{2} \Rightarrow 2a^2 = b^2$ ....(i)

and $\frac{2a^2}{b} = \frac{3}{2}$

$\Rightarrow \frac{b^2}{b} = \frac{3}{2} \Rightarrow b = \frac{3}{2}$

from eq (i)

$2a^2 = \frac{9}{4} \Rightarrow a^2 = \frac{9}{8}$

equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{8x^2}{9} + \frac{4y^2}{9} = 1$

$\Rightarrow 8x^2 + 4y^2 = 9$

68. (D) $\frac{\tan 176.\tan 64 - 1}{\tan 176 + \tan 64}$

$\Rightarrow \frac{\tan(90+86).\tan(90-26)-1}{\tan(90+86)+\tan(90-26)}$

$\Rightarrow \frac{-\cot 86.\cot 26 - 1}{-\cot 86 + \cot 26}$

$\Rightarrow \frac{\cot 26.\cot 86 + 1}{\cot 86 - \cot 26}$

$\Rightarrow \cot(26-86)$ $\left[\because \cot(A-B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}\right]$

$\Rightarrow \cot(-60) = -\cot 60 = -\frac{1}{\sqrt{3}}$

69. (A) $a + 2d$, $a + 4d$ and $a + 7d$ are in G.P., then $(a + 4d)^2 = (a + 2d)(a + 7d)$

$\Rightarrow a^2 + 16d^2 + 8ad = a^2 + 2ad + 7ad + 14d^2$

$\Rightarrow 16d^2 + 8ad - 9ad - 14d^2 = 0$

$\Rightarrow 2d^2 - ad = 0$

$\Rightarrow d(2d - a) = 0$

$d \neq 0$, $2d = a$

Now,

Common ratio $= \frac{a+4d}{a+2d}$

$= \frac{2d+4d}{2d+2d}$

$= \frac{6d}{4d} = \frac{3}{2}$

70. (C) $n(S) = 2^4 = 16$

$n(E) = {}^4C_0 + {}^4C_1 + {}^4C_2$

$= 1 + 4 + 6 = 11$

The required Probability $= \frac{n(E)}{n(S)} = \frac{11}{16}$

71. (A) Function $\cos^{-1}[\log_7 3x]$

Now, $-1 \le \log_7 3x \le 1$

$\Rightarrow 7^{-1} \le 3x \le 7^1$

$\Rightarrow \frac{1}{7} \le 3x \le 7$

$\Rightarrow \frac{1}{21} \le x \le \frac{7}{3}$

72. (B) $x = \frac{at}{1-t^2}$ ....(i)

On differentiating both side w.r.t.'t'

$\Rightarrow \frac{dx}{dt} = \frac{a(1-t^2).1 - at(-2t)}{(1-t^2)^2}$

$\Rightarrow \frac{dx}{dt} = \frac{a(1+t^2)}{(1-t^2)^2}$

and $y = \frac{2a(1+t^2)}{1-t^2}$ ....(i)

On differentiating both side w.r.t.'t'

$\Rightarrow \frac{dy}{dt} = \frac{2a(1-t^2).2t - 2a(1+t^2)(-2t)}{(1-t^2)^2}$

$\Rightarrow \frac{dy}{dt} = \frac{8at}{(1-t^2)^2}$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{8at}{(1-t^2)^2} \times \frac{(1-t^2)^2}{a(1+t^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{8t}{1+t^2}$ ....(iii)

from eq(i) and (ii)

$\frac{x}{y} = \frac{at}{1-t^2} \times \frac{1-t^2}{2a(1+t^2)}$

$\Rightarrow \frac{x}{y} = \frac{t}{2(1+t^2)} \Rightarrow \frac{t}{1+t^2} = \frac{2x}{y}$ ...(iv)

from eq(iii) and eq(iv)

$\frac{dy}{dx} = 8 \times \frac{2x}{y}$ ....(v)

Again, differentiating

$\Rightarrow \frac{d^2y}{dx^2} = 16\left[\frac{y.1 - x.\frac{dy}{dx}}{y^2}\right]$

$\Rightarrow \frac{d^2y}{dx^2} = 16\left[\frac{y - x \times \frac{16x}{y}}{y^2}\right]$

$\Rightarrow \frac{d^2y}{dx^2} = 16\left[\frac{y^2 - 16x^2}{y^3}\right]$

73. (C) A.T.Q.

$a + 4d = 41$

$\Rightarrow 2a + 8d = 82$ ....(i)

We know that

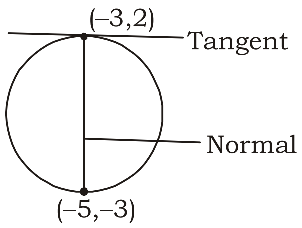
$S_n = \frac{n}{2}[2a + (n-1)d]$

Now, $S_9 = \frac{9}{2}[2a + (9-1)d]$

$S_9 = \frac{9}{2}[2a + 8d]$

$S_9 = \frac{9}{2} \times 82 = 369$ [from eq(i)]

74. (A)



When end points of a diameter are given, then

equation of circle

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow (x + 3)(x + 5) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 + 8x + 15 + y^2 + y - 6 = 0$

$\Rightarrow x^2 + y^2 + 8x + y + 9 = 0$

75. (D) $\Delta \neq 0$, $a = b$, $h = 0$

76. (C) $\lim_{x\to 0} \frac{3^x - 5^x}{x(3^x + 5^x)}$ $\left[\frac{0}{0}\right]$form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{3^x \log 3 - 5^x \log 5}{x(3^x \log 3 + 5^x \log 5) + (3^x + 5^x).1}$

$\Rightarrow \frac{3^0 \log 3 - 5^0 \log 5}{0 + 3^0 + 5^0}$

$\Rightarrow \frac{\log 3 - \log 5}{2} = \frac{1}{2} \log \frac{3}{5}$

77. (C) $I = \int_0^{\pi/2} \frac{\phi(x)}{\phi\left(\frac{x}{2} - x\right) + \phi(x)} dx$ ...(i)

Prop.IV $\int_0^a f(x) = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi/2} \frac{\phi\left(\frac{\pi}{2} - x\right)}{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)} dx$ ...(ii)

from eq(i) and (ii)

$2I = \int_0^{\pi/2} \frac{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)}{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)} dx$

$2I = \int_0^{\pi/2} 1\,dx$

$2I = [x]_0^{\pi/2} \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

78. (C) A is $(x - 2) \times (x - 4)$ matrix and B is $(y + 1) \times (9 - y)$ matrix

Both AB and BA exist, then

$x - 4 = y + 1 \Rightarrow x - y = 5$ ...(i)

and $x - 2 = 9 - y \Rightarrow x + y = 11$ ...(ii)

On solving

$x = 8$, $y = 3$

79. (A) Plane $2x - 5y - 14z = 16$ and point $(-1,2,-3)$

Distance $= \frac{2(-1) - 5(2) - 14(-3) - 16}{\sqrt{2^2 + (5)^2 + (-14)^2}}$

$= \frac{-2 - 10 + 42 - 16}{\sqrt{4 + 25 + 196}} = \frac{14}{15}$

80. (B) U = {1,2,3,4,5,6,7,8,9}

A = {4,6,7}, B = {4,3,9}, C = {4,6,9}

$(A \cap C) = \{4,6,7\} \cap \{4,6,9\} = \{4,6\}$

$(B \cap C) = \{4,3,9\} \cap \{4,6,9\} = \{4,9\}$

Now, $(A \cap C) - (B \cap C) = \{4,6\} - \{4,9\}$

$= \{6\}$

81. (A) $A = \begin{bmatrix} 1 & \alpha \\ \alpha & 2 \end{bmatrix}$

$A^2 = A.A$

$A^2 = \begin{bmatrix} 1 & \alpha \\ \alpha & 2 \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ \alpha & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+\alpha^2 & \alpha+2\alpha \\ \alpha+2\alpha & \alpha^2+4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1+\alpha^2 & 3\alpha \\ 3\alpha & \alpha^2+4 \end{bmatrix}$

Now, $\det(A^2) = 0$

$\Rightarrow \begin{vmatrix} 1+\alpha^2 & 3\alpha \\ 3 & \alpha^2+4 \end{vmatrix} = 0$

$\Rightarrow (1 + \alpha^2)(\alpha^2 + 4) - 9\alpha^2 = 0$

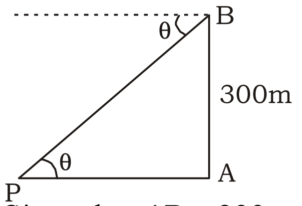
$\Rightarrow \alpha^2 + \alpha^4 + 4 + 4\alpha^2 - 9\alpha^2 = 0$

$\Rightarrow \alpha^4 - 4\alpha^2 + 4 = 0$

$\Rightarrow (\alpha^2 - 2)^2 = 0$

$\Rightarrow \alpha^2 = 2 \Rightarrow \alpha = \pm\sqrt{2}$

82. (C)



Given that AB = 300 m

Let $\theta = \sin^{-1}\left(\frac{5}{13}\right) \Rightarrow \sin\theta = \frac{5}{13}$

and $\tan\theta = \frac{5}{12}$

In ΔABP :-

$\tan\theta = \frac{AB}{AP}$

$\Rightarrow \frac{5}{12} = \frac{300}{AP} \Rightarrow AP = 720$ m

Hence Distance between boat and the lighthouse AP = 720 m

83. (D) $(1 + \omega^2)^{14} = a + b\omega^2$

$\Rightarrow (-\omega)^{14} = a + b\omega^2 \quad [\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow \omega^{14} = a + b\omega^2$

$\Rightarrow \omega^{3\times4+2} = a + b\omega^2$

$\Rightarrow \omega^2 = a + b\omega^2$

On comparing

$\Rightarrow a = 0,\ b = 1$

Hence $(a,b) = (0,1)$

84. (D) $\sin\theta = \frac{7}{25}$ and $\sin\phi = \frac{3}{5}$

$\cos\theta = \frac{24}{25}, \qquad \cos\phi = \frac{4}{5}$

$\cos(\theta + \phi) = \cos\theta.\cos\phi - \sin\theta.\sin\phi$

$\cos(\theta + \phi) = \frac{24}{25}\times\frac{4}{5} - \frac{7}{25}\times\frac{3}{5}$

$\cos(\theta + \phi) = \frac{96-21}{125} = \frac{75}{125} = \frac{3}{5}$

Now, $\cos\left(\frac{\theta+\phi}{2}\right) = \sqrt{\frac{1+\cos(\theta+\phi)}{2}}$

$= \sqrt{\frac{1+\frac{3}{5}}{2}}$

$= \sqrt{\frac{8}{5\times2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

85. (D) equation $ax^2 + bx + c = 0$

One root is $3 - 7i$ and other root is $3 + 7i$.

Now, Sum of roots $= \frac{-b}{a}$

$\Rightarrow 3 - 7i + 3 + 7i = \frac{-b}{a}$

$\Rightarrow 6 = \frac{-b}{a} \Rightarrow 6a + b = 0$

86. (B) $\sin 18 = \frac{\sqrt{5}-1}{4}$

87. (A) Vectors $\vec{a} = 2\vec{i} + (1 - 2\lambda)\vec{j} + 3\vec{k}$ and $\vec{b} = (2+\lambda)\vec{i} + 2\vec{j} - 4\vec{k}$ are perpendicular, then $\vec{a}.\vec{b} = 0$

$\Rightarrow 2\times(2+\lambda) + (1 - 2\lambda)\times2 + 3(-4) = 0$

$\Rightarrow 4 + 2\lambda + 2 - 4\lambda - 12 = 0$

$\Rightarrow -6 - 2\lambda = 0 \Rightarrow \lambda = -3$

88. (A) $\sin\frac{\pi}{12} < \tan\frac{\pi}{12} < \cos\frac{\pi}{12}$

89. (D) $2 \sin10.\sin30.\sin50.\sin70$

$\Rightarrow 2\sin10.\sin50.\sin70 \times \frac{1}{2}$

$\Rightarrow 2 \times \frac{1}{4}\sin(3\times10) \times \frac{1}{2}$

$\left[\because \sin\theta.\sin(60-\theta).\sin(60+\theta) = \frac{1}{4}\sin3\theta\right]$

$\Rightarrow 2 \times \frac{1}{4} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

90. (C) Maximum value of $21\sin\theta + 20\cos\theta$

$= \sqrt{21^2 + 20^2}$

$= \sqrt{441+400} = \sqrt{841} = 29$

91. (B) $\frac{\theta^\circ}{\theta^c} = \frac{180}{\pi}$ ...(i)

and $\theta^\circ \times \theta^c = 80\pi$

from eq(i)

$\theta^\circ \times \theta^\circ \times \frac{\pi}{180} = 80\pi$

$\Rightarrow (\theta^\circ)^2 = 180 \times 80$

$\Rightarrow (\theta^\circ)^2 = 14400 \Rightarrow \theta^\circ = 120$

92. (C) A line makes the angles α, β and γ with the axis, then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

93. (A) Line $\frac{x-2}{2} = \frac{y-3}{5} = \frac{z+5}{14}$

and plane $2x - y + 2z = 4$

Let angle between line and plane is θ, then

$$\sin\theta = \frac{2\times 2 + 5(-1) + 14\times 2}{\sqrt{2^2+5^2+14^2}\sqrt{2^2+(-1)^2+2^2}}$$

$$\Rightarrow \sin\theta = \frac{27}{15\times 3}$$

$$\Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{5}\right)$$

94. (D) $\begin{vmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 5! & 6\times 5! & 7\times 6\times 5! \\ 6! & 7\times 6! & 8\times 7\times 6! \\ 7! & 8\times 7! & 9\times 8\times 7! \end{vmatrix}$$

$$\Rightarrow 5! \times 6! \times 7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow 5! \times 6! \times 7! \begin{vmatrix} 1 & 6 & 42 \\ 0 & 1 & 14 \\ 0 & 2 & 30 \end{vmatrix}$$

$\Rightarrow 5! \times 6! \times 7!\ [1(30 - 28) - 6(0) + 42\ (0)]$

$\Rightarrow 5! \times 6! \times 7!\ (2) = 2 \times 5! \times 6! \times 7!$

95. (A)

96. (B) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos\frac{16\pi}{3}}}}}$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\left(1+\cos\frac{16\pi}{3}\right)}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{8\pi}{3}}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+2\cos\frac{8\pi}{3}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\left(1+\cos\frac{8\pi}{3}\right)}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2\frac{4\pi}{3}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+2\cos\frac{4\pi}{3}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2\left(1+\cos\frac{4\pi}{3}\right)}}$$

$$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2\frac{2\pi}{3}}}$$

$$\Rightarrow \sqrt{2+2\cos\frac{2\pi}{3}}$$

$$\Rightarrow \sqrt{2\left(1+\cos\frac{2\pi}{3}\right)}$$

$$\Rightarrow \sqrt{2\times 2\cos^2\frac{\pi}{3}} = 2\cos\frac{\pi}{3} = 2 \times \frac{1}{2} = 1$$

97. (D) Equation of straight line which makes equal intercept on both axes

$\frac{x}{a} + \frac{y}{a} = 1$

$x + y = a$ ....(i)

its passes through the point (–1,2)

$\Rightarrow -1 + 2 = a \Rightarrow a = 1$

From eq(i)

$x + y = 1$

98. (B) Given that $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/4}$ ...(i)

and $x^m - y^m = 9$

On differentiating both side w.r.t. 'x'

$mx^{m-1} - my^{m-1}\frac{dy}{dx} = 0$

$\Rightarrow my^{m-1}\frac{dy}{dx} = mx^{m-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{x}{y}\right)^{m-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right)^{1-m}$

On comparing with eq(i)

$\frac{1}{4} = 1 - m \Rightarrow m = \frac{3}{4}$

99. (C) Determinant $\begin{vmatrix} 1 & -2 & 4 & 4 \\ 2 & -4 & 3 & 5 \\ 12 & 14 & 16 & -1 \\ 6 & -7 & 3 & -5 \end{vmatrix}$

minor of element 6 = $\begin{vmatrix} -2 & 4 & 4 \\ -4 & 3 & 5 \\ 14 & 16 & -1 \end{vmatrix}$

$= -2(-3 - 80) - 4(4 - 70) + 4(-64 - 42)$

$= 166 + 264 - 424$

$= 430 - 424 = 6$

100. (A) Lines $9x - 40y = 19$
and $9x - 40y = -22$

Distance between lines = $\dfrac{19+22}{\sqrt{9^2+40^2}}$

$= \dfrac{41}{41} = 1$

101. (C) Ratio of angles = 1 : 7 : 2
Let angles = x, $7x$, $2x$
Now, $x + 7x + 2x = 180$
$\Rightarrow 10x = 180 \Rightarrow x = 18$
Angles = 18, 126, 36

Now, $\dfrac{a}{\sin A} = \dfrac{b}{\sin C} = \dfrac{c}{\sin C}$

$\dfrac{\text{largest side}}{\text{smallest side}}\left(\dfrac{b}{a}\right) = \dfrac{\sin B}{\sin A}$

$= \dfrac{\sin 126}{\sin 18}$

$= \dfrac{\cos 36}{\sin 18}$

$= \dfrac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}-1}{4}} = \dfrac{\sqrt{5}+1}{\sqrt{5}-1}$

102. (A) $(A \cup B \cup C) - (A \cap C) - (A \cap B) - (B \cap C)$

103. (D) $f(z) = \dfrac{4-z^2}{1+z}$
put $z = 1 + i$

$f(z) = \dfrac{4-(1+i)^2}{1+1+i}$

$f(z) = \dfrac{4-2i}{2+i} \times \dfrac{2-i}{2-i}$

$f(z) = \dfrac{6-8i}{5}$

Now, $|f(z)| = \dfrac{\sqrt{6^2+8^2}}{5} = \dfrac{10}{5} = 2$

104. (C) $S = \dfrac{3}{2} + \dfrac{5}{4} + \dfrac{9}{8} + \dfrac{17}{16} + \ldots\ldots\ldots n$ terms

$S = \left(1+\dfrac{1}{2}\right) + \left(1+\dfrac{1}{4}\right) + \left(1+\dfrac{1}{8}\right) + \ldots.\ n$ terms

S = (1 + 1 + 1 + …n terms)

$+ \left(\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8} + \ldots..n \text{ terms}\right)$

$S = n + \dfrac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$

$S = n + 1 - \dfrac{1}{2^n}$

$S = n + 1 - 2^{-n}$

105. (C) $f(x) = \begin{cases} 5x - x^2 + 1, & 2 \le x < 3 \\ -3x + \lambda, & 3 \le x < 4 \end{cases}$
is continuous at $x = 3$,

then $\lim\limits_{x \to 3^-} f(x) = f(3)$

$\Rightarrow 5 \times 3 - 3^2 + 1 = -3 \times 3 + \lambda$
$\Rightarrow 7 = -9 + \lambda = \lambda = 16$

106. (B) Let $y = \sin\left(x - \dfrac{\pi}{3}\right) + \cos\left(x - \dfrac{\pi}{3}\right)$

$\dfrac{dy}{dx} = \cos\left(x - \dfrac{\pi}{3}\right) - \sin\left(x - \dfrac{\pi}{3}\right)$

for maxima and minima

$\dfrac{dy}{dx} = 0$

$\Rightarrow \cos\left(x - \dfrac{\pi}{3}\right) - \sin\left(x - \dfrac{\pi}{3}\right) = 0$

$\Rightarrow \sin\left(\dfrac{\pi}{2} - x + \dfrac{\pi}{3}\right) = \sin\left(x - \dfrac{\pi}{3}\right)$

$\Rightarrow \dfrac{\pi}{2} - x + \dfrac{\pi}{3} = x - \dfrac{\pi}{3}$

$\Rightarrow 2x = \dfrac{\pi}{2} + \dfrac{2\pi}{3} = x = \dfrac{7\pi}{12}$

107. (B) $I = \int \sqrt{1 - \cos 4x}\, dx$

$I = \int \sqrt{2\sin^2 2x}\, dx \quad \left[1 - \cos\theta = 2\sin^2\dfrac{\theta}{2}\right]$

$I = \sqrt{2} \int \sin 2x\, dx$

$I = -\sqrt{2}\,\dfrac{\cos 2x}{2} + c$

$I = \dfrac{-1}{\sqrt{2}} \cos 2x + c$

108. (D) The required no. of triangles = ${}^{11}C_3 - {}^{6}C_3$
= 165 – 20 = 145

109. (A)

Class	f	c
0 - 4	13	13
4 - 8	18	31
8 -12	20	51
12-16	23	74
16-20	26	100

Median class (8 -12)

$N = 100, \dfrac{N}{2} = 50$

$l_1 = 8,\ l_2 = 12,\ f = 20,\ c = 31$

Median $= l_1 + \dfrac{\frac{N}{2} - C}{f} \times (l_2 - l_1)$

Median = $8 + \frac{50-31}{20} \times (12-8)$

Median = $8 + \frac{19}{20} \times 4 = 11.8$

110. (C) $u^2 + v^2 + w^2 - d > 0$

111. (A) In ΔABC,
sinA, sinB and sinC are in A.P., then
2 sinB = sinA + sinC
Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

$\Rightarrow 2bk = ak + ck$

$\Rightarrow 2b = a + c$

Hence a, b and c are in A.P.

112. (C)

113. (D) $x = \tan\theta + \sin\theta$ and $y = \tan\theta - \sin\theta$
Now $x^2 - y^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$\Rightarrow x^2 - y^2 = \tan^2\theta + \sin^2\theta + 2\tan\theta.\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta.\sin\theta$

$\Rightarrow x^2 - y^2 = 4\tan\theta.\sin\theta$ ...(i)

and $\sqrt{xy} = \sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$

$\Rightarrow \sqrt{xy} = \sqrt{\tan^2\theta - \sin^2\theta}$

$\Rightarrow \sqrt{xy} = \sqrt{\tan^2\theta(1-\cos^2\theta)}$

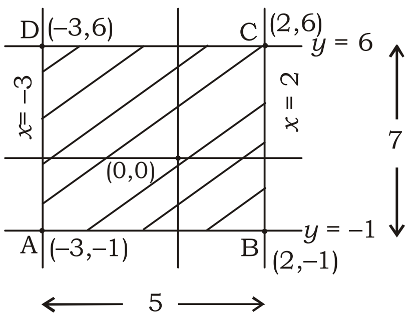
$\Rightarrow \sqrt{xy} = \sqrt{\tan^2\theta.\sin^2\theta}$

$\Rightarrow \sqrt{xy} = \tan\theta.\sin\theta$ ...(ii)

from eq(i) and eq(ii)

$x^2 - y^2 = 4\sqrt{xy}$

114. (D)



Area of a rectangle = AB × BC
= 5 × 7
= 35 sq.unit

115. (A) 32

116. (B) Given that $\begin{vmatrix} p & q & r \\ l & m & n \\ a & b & c \end{vmatrix} = 8$ ...(i)

Now, $\begin{vmatrix} a & -2b & c \\ 2p & -4q & 2r \\ 3l & -6m & 3n \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2p & -4q & 2r \\ 3l & -6m & 3n \\ a & -2b & c \end{vmatrix}$ $[R_1 \leftrightarrow R_2, R_2 \leftrightarrow R_3]$

$\Rightarrow 2\times3\begin{vmatrix} p & -2q & r \\ l & -2m & n \\ a & -2b & c \end{vmatrix}$ [2 from R_1, 3 fromR_2]

$\Rightarrow 6 \times (-2)\begin{vmatrix} p & q & r \\ l & m & n \\ a & b & c \end{vmatrix}$ [–2 from C_2]

$\Rightarrow -12 \times 8$ [from eq(i)]

$\Rightarrow -96$

117. (C) In Binomial expansion

$(23x - 26)^5 = {}^5C_0\,(23x)^5 + {}^5C_1\,(23x)^4(-26)^1 + {}^5C_2\,(23x)^3(-26)^2 + \ldots + {}^5C_5\,(-26)^5$

put $x = 1$

$\Rightarrow (23 - 26)^5 = {}^5C_0\,(23)^5 + {}^5C_1\,(23)^4(-26)^4 + {}^5C_2\,(23)^3(-26)^2 + \ldots + {}^5C_5\,(-26)^5$

$(-3)^5$ = Sum of all coefficients

Hence Sum of all coefficients = –243

118. (C) $\frac{\log_2 4 \times \log_9 3}{\log_{36}\sqrt{6}} = \frac{2\log_2 2 \times \frac{\log 3}{\log 9}}{\frac{\log\sqrt{6}}{\log 36}}$

$\frac{\log_2 4 \times \log_9 3}{\log_{36}\sqrt{6}} = \frac{2 \times \frac{\log 3}{2\log 3}}{\frac{\frac{1}{2}\log 6}{2\log 6}}$

$\frac{\log_2 4 \times \log_9 3}{\log_{36}\sqrt{6}} = \frac{2 \times \frac{1}{2}}{\frac{1}{2\times2}} = 4$

119. (A) Differential equation

$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = \frac{y}{\frac{d^2y}{dx^2}}$

$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^2 \frac{d^2y}{dx^2} = y$

Order = 2, Degree = 4

120. (B) $I = \int_{-\pi/4}^{\pi/4} \sin x\,dx$

$I = 0$ [$\because$ function is odd.]

NDA (MATHS) MOCK TEST - 122 (Answer Key)

1. (B)	21. (C)	41. (A)	61. (B)	81. (A)	101. (C)
2. (C)	22. (A)	42. (A)	62. (A)	82. (C)	102. (A)
3. (A)	23. (D)	43. (C)	63. (D)	83. (D)	103. (D)
4. (A)	24. (A)	44. (A)	64. (A)	84. (D)	104. (C)
5. (D)	25. (B)	45. (D)	65. (C)	85. (D)	105. (C)
6. (B)	26. (C)	46. (C)	66. (B)	86. (B)	106. (B)
7. (C)	27. (C)	47. (C)	67. (A)	87. (A)	107. (B)
8. (D)	28. (B)	48. (B)	68. (D)	88. (A)	108. (D)
9. (A)	29. (C)	49. (A)	69. (A)	89. (D)	109. (A)
10. (B)	30. (B)	50. (B)	70. (C)	90. (C)	110. (C)
11. (D)	31. (A)	51. (C)	71. (A)	91. (B)	111. (A)
12. (B)	32. (B)	52. (B)	72. (B)	92. (C)	112. (C)
13. (C)	33. (C)	53. (C)	73. (C)	93. (A)	113. (D)
14. (A)	34. (D)	54. (C)	74. (A)	94. (D)	114. (D)
15. (C)	35. (A)	55. (B)	75. (D)	95. (A)	115. (A)
16. (A)	36. (C)	56. (C)	76. (C)	96. (B)	116. (B)
17. (D)	37. (B)	57. (C)	77. (C)	97. (D)	117. (C)
18. (C)	38. (B)	58. (A)	78. (C)	98. (B)	118. (C)
19. (B)	39. (C)	59. (D)	79. (A)	99. (C)	119. (A)
20. (C)	40. (B)	60. (C)	80. (B)	100. (A)	120. (B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777