NDA MATHS MOCK TEST - 124 (SOLUTION)

1. (C) Let $y = e^{\sin x}.\cos x$

On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{dy}{dx} = e^{\sin x}.\cos x.\cos x + e^{\sin x}(-\sin x)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin x}[\cos^2 x - \sin x]$$

and $z = \tan x$

On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{dz}{dx} = \sec^2 x$$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$$\Rightarrow \frac{dy}{dz} = e^{\sin x}[\cos^2 x - \sin x] \times \frac{1}{\sec^2 x}$$

$$\Rightarrow \frac{dy}{dz} = e^{\sin x}[\cos^4 x - \sin x.\cos^2 x]$$

2. (B) Given that $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = 6$ ...(i)

Now, $\begin{vmatrix} 2a & b & 4c \\ -6m & -3n & -12p \\ 2x & y & 4z \end{vmatrix}$

$$\Rightarrow 2 \times 4 \begin{vmatrix} a & b & c \\ -3m & -3n & -3p \\ x & y & z \end{vmatrix}$$

[2 from C_1 and 4 from C_3]

$$\Rightarrow 8 \times (-3) \begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix}$$ [–3 from R_2]

$\Rightarrow -24 \times 6$ [from eq(i)]

$\Rightarrow -144$

3. (C) $I = \int a^x.\sin x\, dx$ ...(i)

$$I = a^x \int \sin x\, dx - \int \left\{ \frac{d}{dx}(a^x).\int \sin x.dx \right\} dx$$

$$I = -a^x \cos x - \int a^x \log a.(-\cos x)\, dx$$

$$I = -a^x.\cos x + \log a \int a^x \cos x\, dx$$

$I = -a^x.\cos x + \log a$

$$\left[a^x \int \cos x\, dx - \int \left\{ \frac{d}{dx}(a^x) \int \cos x \right\} dx \right]$$

$$\cos x + \log a \left[a^x.\sin x - \int a^x.\log a.\sin dx \right]$$

$$I = -a^x \cos x + a^x.\sin x.\log a$$

$$-(\log a)^2 \int a^x \sin x\, dx$$

$I = -a^x \cos x + a^x.\sin x.\log a - (\log a)^2 I$ [from eq(i)]

$$I + (\log a)^2 I = -a^x \cos x + a^x.\sin x.\log a$$

$$I = \frac{-a^x \cos x + \log a.a^x \sin x}{1 + (\log a)^2}$$

4. (B) In the expansion of $(1 + x)^{36}$

$T_{r+6} = T_{(r+5)+1} = {}^{36}C_{r+5}$

and $T_{2r-1} = T_{(2r-2)+1} = {}^{36}C_{2r-2}$

according to question

${}^{36}C_{r+5} = {}^{36}C_{2r-2}$

Now, $r + 5 + 2r - 2 = 36$

$\Rightarrow 3r + 3 = 36 \Rightarrow r = 11$

5. (C) $S = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \text{.........} + \frac{1}{10.11.12}$

$$S = \frac{1}{2}\left(\frac{1}{1.2} - \frac{1}{2.3}\right) + \frac{1}{2}\left(\frac{1}{2.3} - \frac{1}{3.4}\right) + \text{......}$$

$$\text{........} + \frac{1}{2}\left(\frac{1}{10.11} - \frac{1}{11.12}\right)$$

$$S = \frac{1}{2}\left[\frac{1}{1.2} - \frac{1}{11.12}\right]$$

$$S = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{132}\right]$$

$$S = \frac{1}{2} \times \frac{66-1}{132} = \frac{65}{264}$$

6. (A) **Short method :-**

$$\int_0^\pi \sin ax.\cos bx\, dx = \begin{cases} 0, & \text{if } a - b = \text{even} \\ \frac{2a}{a^2 - b^2}, & \text{if } a - b = \text{odd} \end{cases}$$

Now, $I = \int_0^\pi \sin x.\cos 2x\, dx$

On comparing $a = 1$, $b = 2$

and $a - b = 1 - 2 = -1$ (odd)

then, $I = \frac{2 \times 1}{1^2 - 2^2}$

$$\Rightarrow I = \frac{2}{1-4} = \frac{-2}{3}$$

7. (C) $\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{x}$ $\left[\frac{0}{0}\right]$ Form

$\Rightarrow \lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\times\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$

$\Rightarrow \lim_{x\to 0}\frac{a+x-a+x}{x\left(\sqrt{a+x}+\sqrt{a-x}\right)}$

$\Rightarrow \lim_{x\to 0}\frac{2x}{x\left(\sqrt{a+x}+\sqrt{a-x}\right)}$

$\Rightarrow \lim_{x\to 0}\frac{2}{\left(\sqrt{a+x}+\sqrt{a-x}\right)}=\frac{2}{\sqrt{a}+\sqrt{a}}=\frac{1}{\sqrt{a}}$

8. (B) Differential equation

$\frac{dy}{dx}+\tan x.y=\sin x$

On comparing with linear equation

P = $\tan x$, Q = $\sin x$

I.F. = $e^{\int P\,dx}$

= $e^{\int \tan x\,dx}$

= $e^{\log\sec x}=\sec x$

Solution of differential equation

$\Rightarrow y\times \text{I.F.}=\int \text{Q}\times\text{I.F.}\,dx$

$\Rightarrow y\times\sec x=\int \sin x.\sec x\,dx$

$\Rightarrow y\sec x=\int \tan x\,dx$

$\Rightarrow y\sec x=\log\sec x+c$

9. (A) Given points (3, 2, –5) and (–4, 1, 0)

Direction Ratios = <– 4 –3, 1–2, 0 – (–5)>

= <–7, –1, 5>

10. (C) $\begin{vmatrix}x+y & k & x^2+y^2\\ y+z & k & y^2+z^2\\ z+x & k & z^2+x^2\end{vmatrix}=(x-y)(y-z)(z-x)$

$R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$\Rightarrow\begin{vmatrix}x+y & k & x^2+y^2\\ z-x & 0 & z^2-x^2\\ z-y & 0 & z^2-y^2\end{vmatrix}=(x-y)(y-z)(z-x)$

$\Rightarrow(z-y)(z-x)\begin{vmatrix}x+y & k & x^2+y^2\\ 1 & 0 & z+x\\ 1 & 0 & z+y\end{vmatrix}=(x-y)(y-z)(z-x)$

$\Rightarrow-(y-z)\begin{vmatrix}x+y & k & x^2+y^2\\ 1 & 0 & z+x\\ 1 & 0 & z+y\end{vmatrix}=(x-y)(y-z)$

$R_3\to R_3-R_2$

$\Rightarrow-\begin{vmatrix}x+y & k & x^2+y^2\\ 1 & 0 & z+x\\ 0 & 0 & y-x\end{vmatrix}=x-y$

$\Rightarrow-(y-x)\begin{vmatrix}x+y & k & x^2+y^2\\ 1 & 0 & z+x\\ 0 & 0 & 1\end{vmatrix}=x-y$

$\Rightarrow(x-y)\begin{vmatrix}x+y & k & x^2+y^2\\ 1 & 0 & z+x\\ 0 & 0 & 1\end{vmatrix}=x-y$

$\Rightarrow(x+y)\times 0-k(1-0)+(x^2+y^2)\times 0=1$

$\Rightarrow -k=1\Rightarrow k=-1$

11. (C) We know that

$(1+x)^n={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots\ldots+{}^nC_nx^n$

multiply by x^2

$\Rightarrow x^2(1+x)^n={}^nC_0x^2+{}^nC_1x^3+{}^nC_2x^4+\ldots.$

$\ldots.+{}^nC_nx^{n+2}$

On differentiating both side w.r.t.'x'

$\Rightarrow x^2.n(1+x)^{n-1}+(1+x)^n.2x=2\,{}^nC_0x+$

$3\,{}^nC_1x^2+\ldots\ldots\ldots+(n+2)\,{}^nC_nx^{n+1}$

On putting $x=1$

$\Rightarrow n.2^{n-1}+2^n.2=2\,{}^nC_0+3\,{}^nC_1+\ldots\ldots\ldots\ldots$

$+(n+2)\,{}^nC_n$

$\Rightarrow 2^{n-1}(n+4)=2C_0+3C_1+\ldots\ldots+(n+2)C_n$

12. (A) Let $y=3^x$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx}=3^x\log 3$

and $z=x^3$

On differentiating both side w.r.t.'x'

$\Rightarrow\frac{dz}{dx}=3x^2$

Now, $\frac{dy}{dz}=\frac{dy}{dx}\times\frac{dx}{dz}$

$\Rightarrow\frac{dy}{dz}=3^x\log 3\times\frac{1}{3x^2}$

$\Rightarrow\frac{dy}{dz}=\frac{3^{x-1}\log 3}{x^2}$

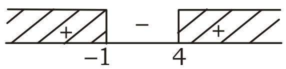
13. (C) $f(x) = \dfrac{1}{\sqrt{\log_3(x^2 - 3x - 3)}}$

Now, $\log_3(x^2 - 3x - 3) > 0$

$\Rightarrow x^2 - 3x - 3 > 3^0$

$\Rightarrow x^2 - 3x - 4 > 0$

$\Rightarrow (x - 4)(x + 1) > 0$

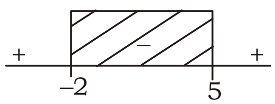


Domain = $(-\infty, -1) \cup (4, \infty)$

14. (B) $f(x) = \begin{cases} x^2 - 3x - 10, & -1 \le x < 3 \\ -13 + x, & 3 \le x \le 5 \end{cases}$

Statement I

$f(x) = x^2 - 3x - 10 = (x + 2)(x - 5)$



Function $f(x)$ is decreasing in interval (–2,5).
Hence function $f(x)$ will be decreasing in interval (–1, 3).
Statement I is incorrect.

Statement II

$f(x) = -13 + x$

$f(x) = -13 + 3 = -10$

$f(x) = -13 + 5 = -8$

$f(x)$ is increasing in interval [3, 5].
Statement II is correct.

15. (A) $f(x) = \begin{cases} x^2 - 3x - 10, & -1 \le x < 3 \\ -13 + x, & 3 \le x \le 5 \end{cases}$

Statement I

L.H.L. = $\lim_{x \to 3^-} f(x) = \lim_{h \to 0} f(3 - h)$

$= \lim_{h \to 0} (3-h)^2 - 3(3-h) - 10$

$= 9 - 9 - 10 = -10$

R.H.L. = $\lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3 + h)$

$= \lim_{h \to 0} -13 + (3 + h) = -10$

L.H.L. = R.H.L.
Hence $f(x)$ is continuous at $x = 3$.
Statement I is correct.

Statement II

L.H.D. = $\lim_{h \to 0} \dfrac{f(3-h) - f(3)}{-h}$

$= \lim_{h \to 0} \dfrac{(3-h)^2 - 3(3-h) - 10 + 10}{-h}$

$= \lim_{h \to 0} \dfrac{9 + h^2 - 6h - 9 + 3h}{-h}$

$= \lim_{h \to 0} \dfrac{h^2 - 3h}{-h}$

$= \lim_{h \to 0} -h + 3 = 3$

R.H.D. = $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$

$= \lim_{h \to 0} \dfrac{-13 + (3+h) + 10}{h}$

$= \lim_{h \to 0} \dfrac{h}{h} = 1$

L.H.D. ≠ R.H.D.
$f(x)$ is not differentiable at $x = 3$.
Statement II is incorrect.

16. (D) Quadratic equation

$x^2 + 9x + 11 = 0$

$\alpha + \beta = -9$ and $\alpha.\beta = 11$

For new quadratic equation

Sum of roots $= \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \dfrac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha.\beta}$

$= \dfrac{81 - 22}{11} = \dfrac{59}{11}$

Product of the roots $= \dfrac{\alpha}{\beta} \times \dfrac{\beta}{\alpha} = 1$

The required equation

$x^2 - \dfrac{59}{11}x + 1 = 0$

$\Rightarrow 11x^2 - 59x + 11 = 0$

17. (C) Given line $3x - 7y = 9$ ...(i)

$4x + y = 12$ ...(ii)

$5x + 6y = 23$ ...(iii)

from eq(i) and eq(ii)

$x = 3, y = 0$

intersecting point of line (i) and (ii) =(3,0)

equation of line which is parallel to eq.(iii)

$5x + 6y = \lambda$ ...(i)

its passes through the point (3,0)

$5 \times 3 + 6 \times 0 = \lambda \Rightarrow \lambda = 15$

The required equation

$5x + 6y = 15$

18. (C) Given that $b_{xy} = 1.2$ and $b_{yx} = 2.7$

Now, $r = \sqrt{b_{xy} \times b_{yx}}$

$\Rightarrow r = \sqrt{1.2 \times 2.7}$

$\Rightarrow r = \sqrt{3.24} = 1.8$

19. (D) equation

$(b-c)x^2 + (c-a)x + (a-b) = 0$

$$x = \frac{-(c-a) \pm \sqrt{(c-a)^2 - 4(b-c)(a-b)}}{2\times(b-c)}$$

$$x = \frac{-(c-a) \pm \sqrt{c^2 + a^2 - 2ca - 4(ab - ac - b^2 + bc)}}{2\times(b-c)}$$

$$x = \frac{-(c-a) \pm \sqrt{c^2 + a^2 - 4b^2 - 2ac - 4ab - 4bc}}{2\times(b-c)}$$

$$x = \frac{-(c-a) \pm (c+a-2b)}{2(b-c)}$$

$$x = \frac{-(c-a) + (c+a-2b)}{2(b-c)}, \frac{-(c-a) - (c+a-2b)}{2(b-c)}$$

$$x = \frac{a-b}{b-c}, 1$$

20. (C) $xdy + ydx = x^2dx$

$\Rightarrow d(xy) = x^2dx$

On integrating

$\Rightarrow \int d(xy) = \int x^2 dx$

$\Rightarrow xy = \frac{x^3}{3} + c$ ...(i)

Given that $y(3) = -2$

$\Rightarrow 3(-2) = \frac{27}{3} + c \Rightarrow c = -15$

from eq(i)

$\Rightarrow xy = \frac{x^3}{3} - 15$

$\Rightarrow 3xy = x^3 - 45$

put $x = -6$

$\Rightarrow 3(-6)y = (-6)^3 - 45$

$\Rightarrow -18y = -216 - 45$

$\Rightarrow -18y = -261 \Rightarrow y = \frac{29}{2}$

21. (A) $I = \int \cot^3 x\, dx$

$I = \int \cot^2 x .\cot x\, dx$

$I = \int (\text{cosec}^2 x - 1)\cot x\, dx$

$I = \int \text{cosec}^2 x.\cot x\, dx - \int \cot x\, dx$

$I = -\frac{\cot^2 x}{2} - \log\sin x + c$

$I = -\frac{\cot^2 x}{2} + \log \text{cosec}x + c$

22. (B) $f(x) = \begin{vmatrix} 0 & 0 & 1 \\ \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \end{vmatrix}$

$\Rightarrow f(x) = 1(\cos^2 x + \sin^2 x)$

$\Rightarrow f(x) = 1$

On differentiating both side w.r.t.'x'

$\Rightarrow f'(x) = 0$

23. (C) 1. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(1-0) = 1$

2. $\begin{vmatrix} 0 & 2 & 3 \\ 1 & 2 & 4 \\ 0 & 2 & 3 \end{vmatrix} = -2(3) + 3(2) = 0$

3. $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 3 \\ 2 & 4 & 0 \end{vmatrix} = 1(-12) - 2(-6) = 0$

24. (D) $g(x) = \frac{f(x)}{x} = \frac{[x]}{x}$

L.H.L. $= \lim_{x\to1^-} g(x) = \lim_{h\to0} g(1-h)$

$= \lim_{h\to0} \frac{[1-h]}{1-h}$

$= \lim_{h\to0} \frac{0}{1} = 0$

R.H.L. $= \lim_{x\to1^+} g(x) = \lim_{h\to0} g(1+h)$

$= \lim_{h\to0} \frac{[1+h]}{1+h}$

$= \lim_{h\to0} \frac{1}{1+h}$

$= \frac{1}{1+0} = 1$

L.H.L $\neq$ R.H.L.

Hence limit does not exist.

25. (B) $I = \int_2^4 x f(x)\, dx$

$I = \int_2^4 x[x]\, dx$

$I = \int_2^3 x[x]\, dx + \int_3^4 x[x]\, dx$

$I = \int_2^3 x \times 2\, dx + \int_3^4 x \times 3\, dx$

$I = 2\left[\frac{x^2}{2}\right]_2^3 + 3\left[\frac{x^2}{2}\right]_3^4$

$I = 2\left[\frac{9}{2} - \frac{4}{2}\right] + 3\left[\frac{16}{2} - \frac{9}{2}\right]$

$I = 5 + 3 \times \frac{7}{2} = \frac{31}{2}$

26. (B) The required probability = $\frac{1}{52}$

27. (C) Digits 0,1, 2, 3, 4, 5, 6, 7, 8, 9

4	9	8

= 4 × 9 × 8 = 288

(1, 2, 3, 4)

28. (B) $5^{2-2\log_5 4+3\log_5 2}$

$\Rightarrow 5^2 \times 5^{-2\log_5 4} \times 5^{3\log_5 2}$

$\Rightarrow 25 \times 5^{\log_5 (4)^{-2}} \times 5^{\log_5 (2)^3}$

$\Rightarrow 25 \times (4)^{-2} \times (2)^3$

$\Rightarrow 25 \times \frac{1}{16} \times 8 = \frac{25}{2}$

29. (D) $4f(x-2) + f\left(\frac{1}{x-2}\right) = x^2$ ...(i)

On putting $x = 4$

$4f(2) + f\left(\frac{1}{2}\right) = 16$ ...(ii)

On putting $x = \frac{5}{2}$ in eq(i)

$4f\left(\frac{1}{2}\right) + f(2) = \frac{25}{4}$

On solving eq(i) and eq(ii)

$f(2) = \frac{77}{20}$

30. (A)

31. (C) Angles of a triangle = 5 : 2 : 3

Let angles = $5x$, $2x$, $3x$

Now, $5x + 2x + 3x = 180$

$10x = 180 \Rightarrow x = 18$

Angles are 90, 36, 54.

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 90} = \frac{b}{\sin 36} = \frac{c}{\sin 54}$

$\frac{a}{1} = \frac{b}{\frac{\sqrt{10-2\sqrt{5}}}{4}} = \frac{c}{\frac{\sqrt{5}+1}{4}}$

$\frac{a}{4} = \frac{b}{\sqrt{10-2\sqrt{5}}} = \frac{c}{\sqrt{5}+1}$

Hence $a : b : c = 4 : \sqrt{10-2\sqrt{5}} : (\sqrt{5}+1)$

32. (B) $\lim_{x\to 0}\left[\frac{1+\sin x}{1+\tan x}\right]^{\cot x}$

$\Rightarrow \lim_{x\to 0}\frac{[1+\sin x]^{\cot x}}{[1+\tan x]^{\cot x}}$

$\Rightarrow \lim_{x\to 0}\frac{\left\{[1+\sin x]^{\frac{1}{\sin x}}\right\}^{\cos x}}{[1+\tan x]^{\frac{1}{\tan x}}}$ $\left[\because \lim_{x\to 0}(1+x)^{1/x} = e\right]$

$\Rightarrow \frac{e^{\lim_{x\to 0}\cos x}}{e^1} = \frac{e^1}{e} = 1$

33. (C) Time = 10 : 30

$\theta = \left|\frac{11M - 60H}{2}\right|$

$\theta = \left|\frac{11\times 30 - 60\times 10}{2}\right|$

$\theta = \frac{270}{2} = 135°$

34. (B) Given that no. of diagonals = 54

$\Rightarrow \frac{n(n-3)}{2} = 54$

$\Rightarrow n^2 - 3n - 108 = 0$

$\Rightarrow (n-12)(n+9) = 0$

n 12, $n \neq -9$

Hence no. of Sides = 12

35. (D) Line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z+1}{6}$

and plane $2x + 3y + 6z = 4$

Let angle between line and plane = θ

$\sin\theta = \frac{2\times 2 + 3\times 3 + 6\times 6}{\sqrt{2^2+3^2+6^2}\sqrt{2^2+3^2+6^2}}$

$\sin\theta = \frac{49}{7\times 7}$

$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$

36. (B) $m = \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2$

$n = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1 - 1 = -2$

Now, $m\cos^2\theta - n\sin^2\theta = 2\cos^2\theta - (-2)\sin^2\theta$

$= 2\cos^2\theta + 2\sin^2\theta$

$= 2(\cos^2\theta + \sin^2\theta) = 2$

37. (D) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$

$\Rightarrow i^n(1 + i + i^2 + i^3)$

$\Rightarrow i^n(1 + i - 1 - i) = 0$

38. (A) $S_p = q$, $S_q = p$
then $S_{(p+q)} = -(p+q)$

39. (C) 1 0 1 1 0 1 0

$0\times2^0 = 0$
$1\times2^1 = 2$
$0\times2^2 = 0$
$1\times2^3 = 8$
$1\times2^4 = 16$
$0\times2^5 = 0$
$1\times2^6 = 64$
Total $= 90$

1 1 1

$1\times2^0 = 1$
$1\times2^1 = 2$
$1\times2^2 = 4$
Total $= 7$

$90 \div 7 \Rightarrow$ 12 quotient and 6 remainder

2	12	0
2	6	0
2	3	1
2	1	1
	0	

2	6	0
2	3	1
2	1	1
	0	

$(12)_{10} = (1100)_2$, $(6)_{10} = (110)_2$
remainder $= (110)_2$ and quotient $= (1100)_2$

40. (D) Given that $A = \begin{bmatrix} 3i+5 & 2i \\ 6i & 1+2i \end{bmatrix}$ and $\lambda = \frac{1}{i}$

Now, $\lambda A = \frac{1}{i}\begin{bmatrix} 3i+5 & 2i \\ 6i & 1+2i \end{bmatrix}$

$= -i\begin{bmatrix} 3i+5 & 2i \\ 6i & 1+2i \end{bmatrix}$

$= \begin{bmatrix} -3i^2-5i & -2i^2 \\ -6i^2 & -i-2i^2 \end{bmatrix}$

$= \begin{bmatrix} 3-5i & 2 \\ 6 & 2-i \end{bmatrix}$

41. (A) $(1+\omega)(1+\omega+\omega^3)(1+\omega+\omega^2)(1+\omega^4) = 0$
$[\because 1+\omega+\omega^2 = 0]$

42. (B) $\lim_{x\to0} \frac{\tan x - \sin x}{x^2}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to0} \frac{\sec^2 x - \cos x}{2x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to0} \frac{2\sec x.\sec x.\tan x + \sin x}{2}$

$\Rightarrow \frac{2\sec 0.\sec 0.\tan 0 + \sin 0}{2} = \frac{0+0}{2} = 0$

43. (B) $\cos^{-1}\left(\frac{12}{13}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$\Rightarrow \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\times\frac{3}{4}}\right] \Rightarrow \tan^{-1}\left[\frac{56}{33}\right]$

44. (C) $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots\ldots\ldots \infty$

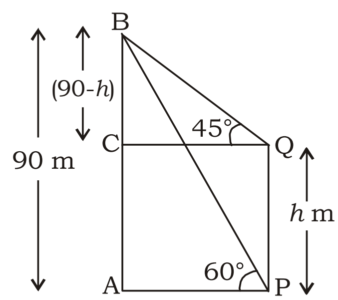
$\Rightarrow \frac{1}{\log_3 e} + \frac{1}{2\log_3 e} + \frac{1}{4\log_3 e} + \ldots\ldots\ldots \infty$

$\Rightarrow \log_e 3 + \frac{1}{2}\log_e 3 + \frac{1}{4}\log_e 3 + \ldots\ldots\ldots\ldots \infty$

$\Rightarrow \log_e 3\left[1+\frac{1}{2}+\frac{1}{4}\ldots\ldots\infty\right]$

$\Rightarrow \log_e 3\left[\frac{1}{1-\frac{1}{2}}\right] = 2\log_e 3$

45. (A)



Let PQ = h m

In ΔBCQ:-

$\tan 45° = \frac{BC}{CQ}$

$\Rightarrow 1 = \frac{90-h}{CQ} \Rightarrow CQ = 90 - h = AP$

In ΔABP:-

$\tan 60° = \frac{AB}{AP}$

$\Rightarrow \sqrt{3} = \frac{90}{90-h}$

$\Rightarrow 90\sqrt{3} - h\sqrt{3} = 90$

$\Rightarrow h\sqrt{3} = 90\sqrt{3} - 90 \Rightarrow h = 30(3-\sqrt{3})$m

Hence height of the pole $= 30(3-\sqrt{3})$m

46. (B) $I = \int\sqrt{1+\cos x}\, dx$

$I = \int\sqrt{1+\sin\left(\frac{\pi}{2}-x\right)}\, dx$

$I = \int\sqrt{\left[\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]^2}\,dx$

$I = \int\left[\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]dx$

$I = \dfrac{\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\frac{-1}{2}}-\dfrac{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\frac{-1}{2}}+c$

$I = -2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+2\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+c$

$I = 2\left[\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]+c$

47. (A) Given that $A = B\cap C$

Now, $U-(U-(U-(U-(U-A))))$

$\Rightarrow U-(U-(U-(U-A')))$ $[\because A' = U-A]$

$\Rightarrow U-(U-(U-A))$

$\Rightarrow U-(U-A')$

$\Rightarrow U-A$

$\Rightarrow A' = (B\cap C)' = B'\cup C'$

48. (C) $x\cos\theta + y\sin\theta = z$

On squaring both side w.r.t.'x'

$\Rightarrow x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta.\cos\theta = z^2$

$\Rightarrow x^2(1-\sin^2\theta)+y^2(1-\cos^2\theta)+2xy\sin\theta.\cos\theta = z^2$

$\Rightarrow x^2+y^2-z^2=x^2\sin^2\theta+y^2\cos^2\theta-2xy\sin\theta.\cos\theta$

$\Rightarrow x^2+y^2-z^2 = (x\sin\theta - y\cos\theta)^2$

$\Rightarrow (x\sin - y\cos\theta) = \sqrt{x^2+y^2-z^2}$

49. (B) In the expansion of $\left(3\sqrt{x}-\frac{1}{2x^2}\right)^{15}$

$T_{r+1} = {}^{15}C_r\left(3\sqrt{x}\right)^{15-r}\left(\frac{-1}{2x^2}\right)^r$

$= {}^{15}C_r\,3^{15-r}\,x^{\frac{15-5r}{2}}\left(\frac{-1}{2}\right)^r$

Now, $\frac{15-5r}{2} = 0 \Rightarrow r = 3$

4th terms will be the independent of x.

50. (A) $I = \int e^x\left(\frac{2x-1}{x^{3/2}}\right)dx$

$I = \int e^x\left(\frac{2}{x^{1/2}}-\frac{1}{x^{3/2}}\right)dx$

We know that

$\int e^x\left[f(x)+f'(x)\right]dx = e^x.f(x)+c$

$I = e^x\times\frac{2}{x^{1/2}}+c \Rightarrow I = \frac{2e^x}{\sqrt{x}}+c$

51. (C) we know that

$\cos 2A = 2\cos^2 A - 1$

put $A = 22\frac{1}{2}$

$\Rightarrow \cos 45 = 2\cos^2\left(22\frac{1}{2}\right)^\circ - 1$

$\Rightarrow \frac{1}{\sqrt{2}} = 2\cos^2\left(22\frac{1}{2}\right)^\circ - 1$

$\Rightarrow 2\cos^2\left(22\frac{1}{2}\right)^\circ = \frac{1+\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \cos^2\left(22\frac{1}{2}\right)^\circ = \frac{1+\sqrt{2}}{2\sqrt{2}}$

52. (D) $y = x\ln x + \sin x$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = x.\frac{1}{x} + \ln x.1 + \cos x$

$\Rightarrow \frac{dy}{dx} = 1 + \ln x + \cos x$

Again, differentiating both side w.r.t.'x'

$\Rightarrow \frac{d^2y}{dx^2} = 0 + \frac{1}{x} - \sin x$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1-x\sin x}{x}$

53. (B) In DABC,

$c = 6$ cm, $\angle A = 30°$, $\angle C = 45°$

then $\angle B = 180 - 30 - 45 = 105°$

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Now, $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{b}{\sin 105} = \frac{6}{\sin 45}$

$\Rightarrow b = \frac{6\times\sin(90+15)}{\sin 45}$

$\Rightarrow b = \frac{6\times\cos 15}{\sin 45}$

$\Rightarrow b = 6\times\dfrac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{1}{\sqrt{2}}} \Rightarrow b = 3(\sqrt{3}+1)$ cm

54. (C) $\begin{vmatrix} b-c & b^2-ab & c^2-ac \\ c-a & bc-ac & ac-a^2 \\ a-b & ab-a^2 & bc-ab \end{vmatrix}$

$\Rightarrow \begin{vmatrix} b-c & b(b-a) & c(c-a) \\ c-a & c(b-a) & a(c-a) \\ a-b & a(b-a) & b(c-a) \end{vmatrix}$

$\Rightarrow (b-a)(c-a) \begin{vmatrix} b-c & b & c \\ c-a & c & a \\ a-b & a & b \end{vmatrix}$

$C_1 \rightarrow C_1 - C_2$

$\Rightarrow (b-a)(c-a) \begin{vmatrix} -c & b & c \\ -a & c & a \\ b & a & b \end{vmatrix}$

$\Rightarrow -(b-a)(c-a) \begin{vmatrix} c & b & c \\ a & c & a \\ b & a & b \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two columns are identical.]

55. (C) $a + ib = \dfrac{13+i}{i-2}$

$\Rightarrow a + ib = \dfrac{13+i}{i-2} \times \dfrac{i+2}{i+2}$

$\Rightarrow a + ib = \dfrac{13i+i^2+26+2i}{i^2-4}$

$\Rightarrow a + ib = \dfrac{15i+25}{-5}$

$\Rightarrow a + ib = -3i - 5$

On comparing

$a = -5,\ b = -3$

56. (B) $f(x) = x^3 - 2x^2 + x - 5$ ...(i)

$f'(x) = 3x^2 - 4x + 1$

$f''(x) = 6x - 4$ ...(ii)

for maxima and minima

$f'(x) = 0$

$3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = 1,\ \dfrac{1}{3}$

from eq(ii)

$f''(1) = 6 \times 1 - 4 = 2$ (minima)

$f''\left(\dfrac{1}{3}\right) = 6 \times \dfrac{1}{3} - 4 = -2$ (maxima)

maximum value of the function

$f\left(\dfrac{1}{3}\right) = \left(\dfrac{1}{3}\right)^3 - 2\left(\dfrac{1}{3}\right)^2 + \dfrac{1}{3} - 5$

$f\left(\dfrac{1}{3}\right) = \dfrac{1}{27} - \dfrac{2}{9} + \dfrac{1}{3} - 5 = \dfrac{-131}{27}$

57. (B) $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$

Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \dfrac{3\times4+(-2)(-1)+6\times8}{\sqrt{4^2+(-1)^2+8^2}}$

$= \dfrac{12+2+48}{9}$

$= \dfrac{72}{9} = 8$

58. (B) Series $\dfrac{1.2^2}{3} + \dfrac{2.2^3}{4} + \dfrac{3.2^4}{5} + ...$

$T_n = \dfrac{n.2^{n+1}}{n+2}$

59. (C) Mode = 31

60. (B) Let $y = 11^{71}$

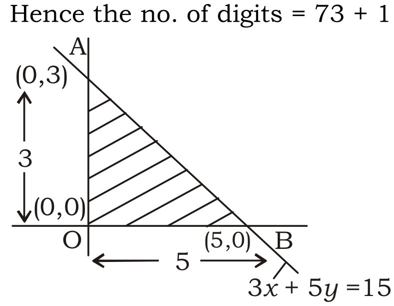
$\log_{10}y = 71 \log_{10}11$

$\log_{10}y = 71 \times 1.0414$

$\log_{10}y = 73.9394$

Hence the no. of digits = 73 + 1 = 74

61. (A)



Area = $\dfrac{1}{2} \times AO \times OB$

$= \dfrac{1}{2} \times 3 \times 5$

$= \dfrac{15}{2} = 7.5$ sq unit

62. (C)

63. (D) We know that

$(x+a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}a^1 + {}^nC_2\, x^{n-2}a^2 + ...$

$.... + {}^nC_n\, x^0 a^n$

Now,

$(13x-15)^7 = {}^7C_0\,(13x)^7 + {}^7C_1\,(13x)^6(-15)^1$

$+ {}^7C_2\,(13x)^5(-15)^2 + ... + {}^7C_7\,(-15)^7$

on putting $x = 1$

$\Rightarrow (13 \ \ -15)^7 = {}^7C_0\,(13)^7 + {}^7C_1\,13^6(-15)$

$+ {}^7C_2\,13^5(-15)^2 + ... + {}^7C_7\,(-15)^7$

$\Rightarrow (-2)^7$ = Sum of all coefficients

$\Rightarrow -128$ = Sum of all coefficients

64. (A) $\begin{vmatrix} 1-x & x^2 & x-x^2 \\ 1-y & y^2 & y-y^2 \\ 1-z & z^2 & z-z^2 \end{vmatrix}$

$C_3 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1-x & x^2 & 1 \\ 1-y & y^2 & 1 \\ 1-z & z^2 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1-x & x^2 & 1 \\ x-y & y^2-x^2 & 0 \\ x-z & z^2-x^2 & 0 \end{vmatrix}$

$\Rightarrow (y-x)(z-x) \begin{vmatrix} 1-x & x^2 & 1 \\ -1 & y+x & 0 \\ -1 & z+x & 0 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow (y-x)(z-x) \begin{vmatrix} 1-x & x^2 & 1 \\ -1 & y+x & 0 \\ 0 & z-y & 0 \end{vmatrix}$

$\Rightarrow (y-x)(z-x)(z-y) \begin{vmatrix} 1-x & x^2 & 1 \\ -1 & y+x & 0 \\ 0 & 1 & 0 \end{vmatrix}$

$\Rightarrow (y-x)(z-x)(z-y)\ [(1-x)\times 0 - x^2\times 0 + 1(-1)]$

$\Rightarrow -(y-x)(z-x)(z-y)$

$\Rightarrow -(x-y)(y-z)(z-x)$

65. (C) $f(x) = \begin{cases} x^2+3x, & 1 \le x < 2 \\ 2x+\lambda, & 2 \le x < 3 \end{cases}$ is continuous at $x = 2$, then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\Rightarrow 2^2 + 3 \times 2 = 2 \times 2 + \lambda$

$\Rightarrow 10 = 4 + \lambda \Rightarrow \lambda = 6$

66. (B) $\lim_{x\to\infty} \dfrac{3x^3+4x^2-5x+1}{5x^2-3x^3+4x+2}$ $\left[\dfrac{\infty}{\infty}\right]$ Form

$\Rightarrow \lim_{x\to\infty} \dfrac{x^3\left(3+\dfrac{4}{x}-\dfrac{5}{x^2}+\dfrac{1}{x^3}\right)}{x^3\left(\dfrac{5}{x}-3+\dfrac{4}{x^2}+\dfrac{2}{x^3}\right)}$

$\Rightarrow \lim_{x\to\infty} \dfrac{3+\dfrac{4}{x}-\dfrac{5}{x^2}+\dfrac{1}{x^3}}{\dfrac{5}{x}-3+\dfrac{4}{x^2}+\dfrac{2}{x^3}}$

$\Rightarrow \dfrac{3+0-0-0}{0-3+0+0} = \dfrac{3}{-3} = -1$

67. (C) Word "TEXTURE"

The total no.of arrangement $= \dfrac{7!}{2!2!}$

$= 1260$

68. (D) $\sin(1035) = \sin(360 \times 3 - 45)$

$= -\sin 45 = \dfrac{-1}{\sqrt{2}}$

69. (A) $I = \int_0^{\pi/2} \dfrac{\sqrt{\sec x}}{\sqrt{\sec x}+\sqrt{\text{cosec}\, x}}\, dx$ ...(i)

Prop $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi/2} \dfrac{\sqrt{\sec\left(\dfrac{\pi}{2}-x\right)}}{\sqrt{\sec\left(\dfrac{\pi}{2}-x\right)}+\sqrt{\text{cosec}\left(\dfrac{\pi}{2}-x\right)}}\, dx$

$I = \int_0^{\pi/2} \dfrac{\sqrt{\text{cosec}x}}{\sqrt{\text{cosec}x}+\sqrt{\sec x}}\, dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \dfrac{\sqrt{\sec x}+\sqrt{\text{cosec}x}}{\sqrt{\sec x}+\sqrt{\text{cosec}x}}\, dx$

$2I = \int_0^{\pi/2} 1\, dx$

$2I = [x]_0^{\pi/2}$

$2I = \dfrac{\pi}{2} - 0 \Rightarrow I = \dfrac{\pi}{4}$

70. (C) $A = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 5 & 6 \\ -1 & 2 & 3 \end{bmatrix}$

	C_1	C_2	C_3	C_1	C_2
R_1	1	0	3	1	0
R_2	4	5	6	4	5
R_3	–1	2	3	–1	2
R_4	1	0	3	1	0
R_5	4	5	6	4	5

$C = \begin{bmatrix} 15-12 & -6-12 & 8+5 \\ 6-0 & 3+3 & 0-2 \\ 0-15 & 12-6 & 5-0 \end{bmatrix}$

$C = \begin{bmatrix} 3 & -18 & 13 \\ 6 & 6 & -2 \\ -15 & 6 & 5 \end{bmatrix}$

$C^T = \text{Adj } A = \begin{vmatrix} 3 & 6 & -15 \\ -18 & 6 & 6 \\ 13 & -2 & 5 \end{vmatrix}$

71. (B) $n(S) = {}^{13}C_9 = {}^{7}C_5$

$n(E) = {}^{6}C_4 \times {}^{7}C_5 \times {}^{6}C_5 \times {}^{7}C_4 \times {}^{6}C_6 \times {}^{7}C_3$

$= 15 \times 21 + 6 \times 35 + 1 \times 35$

$= 315 + 210 + 35 = 560$

The required probability $P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{560}{715} = \frac{112}{143}$

72. (B) We know that

$\tan\theta.\tan(60 - \theta).\tan(60 + \theta) = \tan 3\theta$

Now, $\tan 20.\tan 40.\tan 80 = \tan(3 \times 20)$

$= \tan 60 = \sqrt{3}$

73. (A) An ellipse

$8x^2 + 9y^2 = 36$

$\frac{x^2}{9/2} + \frac{y^2}{4} = 1$

$a^2 = \frac{9}{2} \Rightarrow a = \frac{3}{\sqrt{2}}$, $b^2 = 4 \Rightarrow b = 2$

Area of an ellipse = πab

$= \pi \times \frac{3}{\sqrt{2}} \times 2 = 3\sqrt{2}\,\pi$ sq. unit

74. (B) Data 4, 6, 7, 6, 4, 2, 3, 5, 3, 6

Mean $x = \frac{4+6+7+6+4+2+3+5+3+6}{10}$

$\Rightarrow x = \frac{46}{10} = 4.6$

Mode $y = 6$

Arrange the given data in ascending order

2, 3, 3, 4, 4, 5, 6, 6, 6, 7

Median $z = \frac{\left(\frac{10}{2}\right)^{th}\text{ term} + \left(\frac{10}{2}+1\right)^{th}\text{ term}}{2}$

$\Rightarrow z = \frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2}$

$\Rightarrow z = \frac{4+5}{2} = 4.5$

Hence $y > x > z$

75. (C)

2	13	1
2	6	0
2	3	1
2	1	1
	0	

$(13)_2 = (1101)_2$

$0.125 \times 2 = \boxed{0}.250$

$0.250 \times 2 = \boxed{0}.500$

$0.500 \times 2 = \boxed{1}.000$

$(0.125)_{10} = (0.001)_2$

Hence $(13.125)_{10} = (1101.001)_2$

76. (B) $I = \int_0^{\pi} |\cos x|\,dx$

$I = 2\int_0^{\pi/2} \cos x\,dx$

$I = 2[\sin x]_0^{\pi/2}$

$I = 2\left[\sin\frac{\pi}{2} - \sin 0\right]$

$I = 2 \times 1 = 2$

(77-78) Equation

$ax^2 + bx + c = 0$

Roots are $\tan\alpha$ and $\tan\beta$, then

$\tan\alpha + \tan\beta = \frac{-b}{a}$

$\tan\alpha.\tan\beta = \frac{c}{a}$

77. (B) $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$\tan(\alpha + \beta) = \frac{\frac{-b}{a}}{1 - \frac{c}{a}}$

$\tan(\alpha + \beta) = \frac{-b}{a-c} = \frac{b}{c-a}$

78. (C) $\sin(\alpha + \beta).\sec\alpha.\sec\beta$

$\Rightarrow \frac{\sin\alpha.\cos\beta + \cos\alpha.\sin\beta}{\cos\alpha.\cos\beta}$

$\Rightarrow \tan\alpha + \tan\beta = \frac{-b}{a}$

79. (C) ${}^{16}C_r + {}^{16}C_3 = {}^{17}C_r$

We know that

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

On comparing $r = 4$

80. (A) We know that

If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then

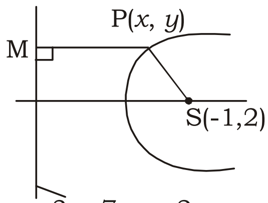
Area bounded by the curve $= \frac{a^2}{6}$

Now, curve $\sqrt{x} + \sqrt{y} = \sqrt{3}$

Area bounded by the curve $= \frac{3^2}{6}$

$= \frac{3}{2}$ sq. unit

81. (C)



$3x - 7y = -2$

Now, PS = PM

$$\Rightarrow \sqrt{(x-1)^2 + (y+2)^2} = \frac{3x - 7y + 2}{\sqrt{3^2 + (-7)^2}}$$

On squaring

$$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y = \frac{(3x - 7y + 2)^2}{9 + 49}$$

On solving

$49x^2 + 9y^2 + 42xy + 104x - 204y + 286 = 0$

82. (C) Let $a + ib = \sqrt{2i}$

On squaring both side w.r.t.'x'

$(a^2 - b^2) + 2abi = 2i$

On comparing

$a^2 - b^2 = 0,\ 2ab = 2$

$a = b, \quad 2 \times a^2 = 2 \Rightarrow a = \pm 1, b = \pm 1$

Hence $\sqrt{2i} = \pm(1 + i)$

83. (D) $\frac{b^2 - c^2}{a^2} = \frac{k^2 \sin^2 B - k^2 \sin^2 C}{k^2 \sin^2 A}$ (by Sine Rule)

$$\Rightarrow \frac{b^2 - c^2}{a^2} = \frac{\sin^2 B - \sin^2 C}{\sin^2 A}$$

We know that

$\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

$$\Rightarrow \frac{b^2 - c^2}{a^2} = \frac{\sin(B + C).\sin(B - C)}{\sin^2 A}$$

$$\Rightarrow \frac{b^2 - c^2}{a^2} = \frac{\sin(180 - A).\sin(B - C)}{\sin^2 A}$$

$$\Rightarrow \frac{b^2 - c^2}{a^2} = \frac{\sin A \sin(B - C)}{\sin^2 A}$$

$$\Rightarrow \frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin A}$$

84. (C) Let $y = \log\left(x + \sqrt{x^2 + 1}\right)$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}}\left[1 + \frac{1 \times 2x}{2\sqrt{x^2 + 1}}\right]$$

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2 + 1}} \times \frac{\sqrt{x^2 + 1} + x}{2\sqrt{x^2 + 1}}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2 + 1}}$$

85. (C) $I = \int x^3 \cos x\, dx$

D → $\overset{+}{x^3}$ $\quad \overset{-}{3x^2}$ $\quad \overset{+}{6x}$ $\quad \overset{-}{6}$ $\quad \overset{+}{0}$

I → $\sin x \quad -\cos x \quad -\sin x \quad \cos x \quad \sin x$

$I = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + c$

86. (C) $\sin^2 10 + \sin^2 20 + \sin^2 30 + ... + \sin^2 90$

$\Rightarrow \sin^2 10 + \sin^2 20 + ... + \sin^2 40 + \sin^2 50 + ... + \sin^2 80 + 1$

$\Rightarrow \sin^2 10 + \sin^2 20 + ... + \sin^2 40 + \cos^2 40 + ... + \cos^2 10 + 1$

$\Rightarrow (\sin^2 10 + \cos^2 10) + (\sin^2 20 + \cos^2 20) + ... + (\sin^2 40 + \cos^2 40) + 1$

$\Rightarrow 1 + 1 + 1 + 1 + 1 = 5$

87. (B) $\begin{bmatrix} a + b & c + 2d \\ c - d & a - 2b \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -2 & 5 \end{bmatrix}$

On comparing

$a + b = 2$ ...(i)

$c + 2d = 4$ ...(ii)

$c - d = -2$ ...(iii)

$a - 2b = 5$ ...(iv)

from eq(i) and eq(iv)

$a = 3,\ b = -1$

from eq(ii) and eq(iii)

$c = 0,\ d = 2$

Hence a, b, c and d are 3, –1, 0, 2.

88. (C) $\log_8 \frac{1}{2} + \log_m 16 = 1$

$\Rightarrow -\log_{2^3} 2 + \log_m 16 = 1$

$\Rightarrow \frac{-1}{3}\log_2 2 + \log_m 16 = 1$

$\Rightarrow \log_m 16 = 1 + \frac{1}{3}$

$\Rightarrow \log_m 16 = \frac{4}{3}$

$\Rightarrow m^{4/3} = 16$

$\Rightarrow m = (16)^{3/4} \Rightarrow m = 8$

89. (C) $f(x) = y = \frac{3^x + 3^{-x}}{3^x - 3^{-x}}$

by Componendo and Dividendo Rule

$$\Rightarrow \frac{y + 1}{y - 1} = \frac{2.3^x}{2.3^{-x}}$$

$$\Rightarrow \frac{y + 1}{y - 1} = 3^{2x}$$

$$\Rightarrow 2x = \log_3\left(\frac{y + 1}{y - 1}\right)$$

$\Rightarrow x = \frac{1}{2}\log_3\left(\frac{y+1}{y-1}\right)$

$\Rightarrow f^{-1}(y) = \frac{1}{2}\log_3\left(\frac{y+1}{y-1}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log_3\left(\frac{x+1}{x-1}\right)$

90. (D) $f(x) = \cos^{-1}(\log_3 x)$

Now, $-1 \le \log_3 x \le 1$

$\Rightarrow 3^{-1} \le x \le 3^1$

$\Rightarrow \frac{1}{3} \le x \le 3$

Domain of $f(x) = \left[\frac{1}{3}, 3\right]$

91. (B) $f(x) = \frac{1}{\sqrt{x+\sqrt{3x-1}}} + \frac{1}{\sqrt{x-\sqrt{3x-1}}}$

$f(3) = \frac{1}{\sqrt{3+2\sqrt{2}}} + \frac{1}{\sqrt{3-2\sqrt{2}}}$

$f(3) = \frac{1}{\sqrt{\left(\sqrt{2}+1\right)^2}} + \frac{1}{\sqrt{\left(\sqrt{2}-1\right)^2}}$

$f(3) = \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{2}-1}$

$f(3) = \frac{\sqrt{2}-1+\sqrt{2}+1}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)} = 2\sqrt{2}$

92. (A) $f(x) = \begin{cases} \frac{5x-7\sin x}{x}, & x \ne 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$,

then $\lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{5x-7\sin x}{x} = k$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{5-7\cos x}{1} = k$

$\Rightarrow 5 - 7 \times 1 = k \Rightarrow k = -2$

93. (B) $f(x) = x^2 - |x|$

$f(x) = \begin{cases} x^2 - x & x \ge 0 \\ x^2 + x & x < 0 \end{cases}$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} (0-h)^2 + (0-h) = 0$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} (0-h)^2 - (h) = 0$

L.H.L. = R.H.L.

$f(x)$ is continuous at $x = 0$.

L.H.D. $= Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0} \frac{(-h)^2+(-h)-0}{-h}$

$= \lim_{h\to 0} \frac{h^2-h}{-h}$

$= \lim_{h\to 0} -h + 1 = 1$

R.H.D. $= Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} \frac{h^2-h-0}{h}$

$= \lim_{h\to 0} h - 1 = -1$

L.H.D. $\ne$ R.H.D.

$f(x)$ is not differentiable at $x = 0$.

94. (B) $\lim_{x\to\infty} x^{5/2}\left(\sqrt{x^5+1} - \sqrt{x^5-1}\right)$

$\Rightarrow \lim_{x\to\infty} x^{5/2}\left(\sqrt{x^5+1} - \sqrt{x^5-1}\right) \times \frac{\left(\sqrt{x^5+1}+\sqrt{x^5-1}\right)}{\left(\sqrt{x^5+1}+\sqrt{x^5-1}\right)}$

$\Rightarrow \lim_{x\to\infty} \frac{x^{5/2}(x^5+1-x^5+1)}{\sqrt{x^5+1}+\sqrt{x^5-1}}$

$\Rightarrow \lim_{x\to\infty} \frac{2.x^{5/2}}{x^{5/2}\sqrt{1+\frac{1}{x^5}}+\sqrt{1-\frac{1}{x^5}}}$

$\Rightarrow \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1$

95. (D) Centre is the intersection point of two diameters $2x + y = 6$ and $3x - y = 9$

So, centre = (3, 0)

circle passes through the point (–1,3),

then radius $(r) = \sqrt{(3+1)^2+(0-3)^2} = 5$

equation of circle

$(x-3)^2 + (y-0)^2 = 5^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = 25$

$\Rightarrow x^2 + y^2 - 6x - 16 = 0$

96. (B) equation of parabola

$x^2 + 4x - 16y + 24 = 0$

$\Rightarrow (x + 2)^2 - 4 - 16y + 24 = 0$

$\Rightarrow (x + 2)^2 = 16y - 20$

$\Rightarrow (x + 2)^2 = 16\left(y - \frac{5}{4}\right)$

$\Rightarrow X^2 = 16\,Y$ where $X = x + 2$, $Y = y - \frac{5}{4}$

$4a = 16 \Rightarrow a = 4$

equation of directrix

$Y = -a$

$\Rightarrow y - \frac{5}{4} = -4 \Rightarrow 4y + 11 = 0$

97. (D) **Statement I**

given that $a \times d = c \times b$ and $a \times c = d \times b$

$(d - c)\times(a - b) = d \times a - d \times b - c \times a + c \times b$

$= d \times a - a \times c + a \times c + a \times d$

$= -a \times d + a \times d = 0$

$(d - c)$ is parallel to $(a - b)$.

Statement I is correct.

Statement II

L.H.S. $= (a - d).[(d - c)\times(c - a)]$

$= (a - d).[d \times c - d \times a - c \times c + c \times a]$

$= (a - d).[d \times c - d \times a + c \times a]$

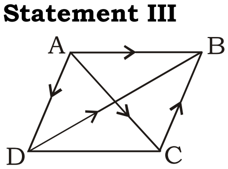
$= a.(d\times c) - a.(d\times a) + a.(c \times a) - d.(d \times c) + d.(d \times a) - d.(c \times a)$

$= [a\ \ d\ \ c] - 0 - 0 - [a\ \ d\ \ c]$

$= 0 =$ R.H.S.

Statement II is correct.

Statement III



AD + DB = AB ...(i)

AC + CB = AB ...(ii)

from (i) and eq (ii)

AD + DB = AC + CB

AD – CB = AC – DB

AD + BC = AC + BD

Statement III is correct.

98. (B) vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

$\cos\theta = \frac{1\times3 - 2\times1 + 3\times2}{\sqrt{1^2 + (-2)^2 + 3^2}\sqrt{3^2 + 1^2 + 2^2}}$

$\cos\theta = \frac{7}{\sqrt{14}\sqrt{14}}$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

Now, $\sin\theta = \sin 60 = \frac{\sqrt{3}}{4}$

99. (A) $\frac{\log x}{\log 3} = \frac{\log 64}{\log 4} = \frac{\log 343}{\log y}$

$\frac{\log x}{\log 3} = \frac{\log 64}{\log 4}$ and $\frac{\log 343}{\log y} = \frac{\log 64}{\log 4}$

$\frac{\log x}{\log 3} = \frac{3\log 4}{\log 4}$, $\frac{3\log}{\log y} = \frac{3\log 4}{\log 4}$

$\log x = 3\log 3$, $\log y = \log 7$

$x = 3^3 = 27$, $y = 7$

100. (B) $3^{\frac{4}{5}} . 3^{\frac{4}{5^2}} . 3^{\frac{4}{5^3}}$ .... ∞

$\Rightarrow 3^{\frac{4}{5}\left[1 + \frac{1}{5} + \frac{1}{5^2} \ldots\ldots\infty\right]}$

$\Rightarrow 3^{\frac{4}{5}\times\frac{1}{1-\frac{1}{5}}} \Rightarrow 3^{\frac{4}{5}\times\frac{5}{4}} = 3$

101. (A) $\begin{vmatrix} x & 4 & 3 \\ 4 & x & 4 \\ 3 & 3 & x \end{vmatrix} = 0$

$\Rightarrow x(x^2 - 12) - 4(4x - 12) + 3(12 - 3x) = 0$

$\Rightarrow x^3 - 12x - 16x + 48 + 36 - 9x = 0$

$\Rightarrow x^3 - 37x + 84 = 0$

$\Rightarrow (x - 3)(x - 4)(x + 7) = 0$

Hence third root = – 7

102. (C) $(1.03)^7 = (1 + 0.03)^7$

$= {}^7C_0 + {}^7C_1\ (0.03)^1 + {}^7C_2\ (0.03)^2 + \ldots.$

$= 1 + 0.21 + 0.0189 + \ldots\ldots\ldots$

$= 1.2289\ldots$

≈ 1.23

103. (A) There are 8 letters and 2 post-boxes

The number of ways $= 2^8 = 256$

104. (D) The required probability $= \frac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2}$

$= \frac{16}{\frac{52\times51}{2}} = \frac{8}{663}$

105. (B) $\frac{(110)_2^{(11)_2} - (101)_2^{(11)_2}}{(110)_2^{(10)_2} + (101)_2^{(10)_2} + (110)_2^{(01)_2}\,(101)_2^{(01)_2}}$

$\Rightarrow \frac{6^3 - 5^3}{6^2 + 5^2 + 6 + 5}$

$= (6 - 5) = 1 = (1)_2$

106. (B) $x^2 + y^2 + 4x - 2y = 0$

equation of tanget at point (– 4, 2)

$\Rightarrow x(-4) + y \times 2 + 2(x - 4) - 1(y + 2) = 0$

$\Rightarrow -4x + 2y + 2x - 8 - y - 2 = 0$

$\Rightarrow -2x + y - 10 = 0 \Rightarrow 2x - y + 10 = 0$

107. (B) $f(x) = \begin{cases} 2x^2+5, & x<4 \\ 6, & x=4 \\ 9-3x^2, & x>4 \end{cases}$

At x = 6

$f(x) = 9 - 3x^2$

$f'(x) = -3 \times 2x$

$f'(6) = -6 \times 6 = -36$

108. (C) $y = \cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$$\frac{d\theta}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$\Rightarrow y = \theta + \cos^{-1}\sqrt{1-\cos^2\theta}$

$\Rightarrow y = \theta + \cos^{-1}(\sin\theta)$

$\Rightarrow y = \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{2}-\theta\right)\right]$

$\Rightarrow y = \theta + \frac{\pi}{2} - \theta$

$\Rightarrow y = \frac{\pi}{2}$

$\Rightarrow \frac{dy}{dx} = 0$

109. (B) $y = \cos^2 t$ and $x = \sin t$

$\frac{dy}{dt} = 2\cos t(-\sin t)$ and $\frac{dx}{dt} = \cos t$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 2\cos t(-\sin t) \times \frac{1}{\cos t}$

$\Rightarrow \frac{dy}{dx} = -2\sin t$

$\Rightarrow \frac{d^2y}{dx^2} = -2\cos t\frac{dt}{dx}$

$\Rightarrow \frac{d^2y}{dx^2} = -2\cos t \times \frac{1}{\cos t} = -2$

110. (A) Parimeter of rectangle $2(l + b) = 48$

$l + b = 24$

$l = 24 - b$

Area A = lb

$\Rightarrow A = (24 - b)b$ …(i)

$\Rightarrow A = 24b - b^2$

$\Rightarrow \frac{dA}{db} = 24 - 2b$ …(ii)

$\Rightarrow \frac{d^2A}{db^2} = -2$ …(iii)

for maxima and minima

$\frac{dA}{db} = 0$

$\Rightarrow 24 - 2b = 0 \Rightarrow b = 12$

from eq (iii)

$\left(\frac{d^2A}{db^2}\right)_{(\text{at } b = 12)} = -2$(maxima)

from eq (i)

Maximum area A = (24 - 12) × 12

= 144 sq. cm

111. (D) $I = \int e^{5\log x}(x^6-1)^{-2}\,dx$

$I = \int \frac{e^{\log x^5}}{(x^6-1)^2}\,dx$

$I = \int \frac{x^5}{(x^6-1)^2}\,dx$

Let $x^6 - 1 = t$

$6x^5\,dx = dt \Rightarrow x^5\,dx = \frac{1}{6}\,dt$

$I = \int \frac{1}{6}\frac{1}{t^2}\,dt$

$I = \frac{1}{6} \times \frac{t^{-2+1}}{-2+1} + c$

$I = \frac{-1}{6t} + c$

$I = \frac{-1}{6(x^6-1)} + c$

112. (C) $I = \int_0^{2\pi}\cos^3 x\,dx$

$I = 2\int_0^{\pi}\cos^3 x\,dx$ $[\because f(2\pi - x) = f(x)]$

Now, $f(\pi - x) = \cos^3(\pi - x)$

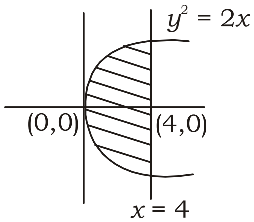
$f(\pi - x) = -\cos^3 x$

$f(\pi - x) = -f(x)$

$\because \int_0^{\pi}\cos^3 x\,dx = 0$

Hence $I = 2\int_0^{\pi}\cos^3 x = 0$

113. (B) $y^2 = 2x \Rightarrow y = \sqrt{2}\sqrt{x}$



$$\text{Area} = \int_0^4 y\ dx$$

$$= 2\int_0^4 \sqrt{2}\sqrt{x}\ dx$$

$$= 2\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

$$= 2\sqrt{2}\times\frac{2}{3}\left[4^{\frac{3}{2}} - 0\right]$$

$$= \frac{4\sqrt{2}}{3}\times 8 = \frac{32\sqrt{2}}{3} \text{ sq. unit}$$

114. (C) $(2y + 1)\ dx - (3x - 4)\ dy = 0$

$\Rightarrow (2y + 1)\ dx = (3x - 4)\ dy$

$$\Rightarrow \frac{dy}{2y+1} = \frac{dx}{3x-4}$$

On integrating

$$\Rightarrow \frac{\log(2y+1)}{2} = \frac{\log(3x-4)}{3} + \frac{\log\sqrt{c}}{3}$$

$$\Rightarrow \frac{\log(2y+1)}{2} = \frac{\log\sqrt{c}(3x-4)}{3}$$

$\Rightarrow$ 3 log $(2y + 1)$ = 2 log $\sqrt{c}\ (3x - 4)$

$\Rightarrow (2y + 1)^3 = c\ (3x - 4)^2$

$$\Rightarrow \frac{(2y+1)^3}{(3x-4)^2} = c$$

115. (B) Data 3, 3, 4, 6, 8, 10, 12, 15, 14

$$\sum_{i=1}^{n} x_i = 3 + 3 + 4 + 6 + 8 + 10 + 12 + 15 + 14$$

$= 75$

$$\sum_{i=1}^{n} x_i^2 = (3)^2 + (3)^2 + (4)^2 + (6)^2 + (8)^2 + (10)^2 + (12)^2 + (15)^2 + (14)^2 = 799$$

$$\text{Now, S.D.} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1} x_i\right)^2}$$

$$\Rightarrow \text{S.D.} = \sqrt{\frac{1}{9}\times 799 - \left(\frac{1}{9}\times 75\right)^2}$$

$$\Rightarrow \text{S.D.} = \sqrt{\frac{799}{9} - \frac{625}{9}}$$

$$\Rightarrow \text{S.D.} = \sqrt{\frac{174}{9}} = \sqrt{\frac{58}{3}}$$

116. (A) $$\text{Cov}(x, y) = \frac{\sum x_i y_i}{n} - \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n^2}$$

$$\text{Cov}(x, y) = \frac{183}{6} - \frac{30\times 36}{6\times 6}$$

$\text{Cov}(x, y) = 30.5 - 30 = 0.5$

117. (C) Mode = 18

118. (A) Parabola

$y^2 - 6y + 4x - 12 = 0$

$\Rightarrow (y - 3)^2 - 9 + 4x - 12 = 0$

$\Rightarrow (y - 3)^2 = -4x + 21$

$$\Rightarrow (y - 3)^2 = -4\left(x - \frac{21}{4}\right)$$

$\Rightarrow Y^2 = -4X$ where $Y = y - 3$, $X = x - \frac{21}{4}$

$4a = 4 \Rightarrow a = 1$

Length of latus rectum = $4a = 4$

119. (C) $y = a\sin x + b\cos x$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = a\cos x - b\sin x$$

Again, differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{d^2y}{dx^2} = -a\sin x - b\cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -(a\sin x + b\cos x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

120. (B) tan53. tan37. tan43. tan47

$\Rightarrow$ tan53. cot53. tan43. cot43 = 1

NDA (MATHS) MOCK TEST - 124 (Answer Key)

1.	(C)	21.	(A)	41.	(A)	61.	(A)	81.	(C)	101.	(A)
2.	(B)	22.	(B)	42.	(B)	62.	(C)	82.	(C)	102.	(C)
3.	(C)	23.	(C)	43.	(B)	63.	(D)	83.	(D)	103.	(C)
4.	(B)	24.	(D)	44.	(C)	64.	(A)	84.	(C)	104.	(D)
5.	(C)	25.	(B)	45.	(A)	65.	(C)	85.	(C)	105.	(B)
6.	(A)	26.	(B)	46.	(B)	66.	(B)	86.	(C)	106.	(B)
7.	(C)	27.	(C)	47.	(A)	67.	(C)	87.	(B)	107.	(B)
8.	(B)	28.	(B)	48.	(C)	68.	(D)	88.	(C)	108.	(C)
9.	(A)	29.	(D)	49.	(B)	69.	(A)	89.	(C)	109.	(B)
10.	(C)	30.	(A)	50.	(A)	70.	(C)	90.	(D)	110.	(A)
11.	(C)	31.	(C)	51.	(C)	71.	(B)	91.	(B)	111.	(A)
12.	(A)	32.	(B)	52.	(D)	72.	(B)	92.	(A)	112.	(C)
13.	(C)	33.	(C)	53.	(B)	73.	(A)	93.	(B)	113.	(B)
14.	(B)	34.	(B)	54.	(C)	74.	(B)	94.	(B)	114.	(C)
15.	(A)	35.	(D)	55.	(C)	75.	(C)	95.	(D)	115.	(B)
16.	(D)	36.	(B)	56.	(B)	76.	(B)	96.	(B)	116.	(A)
17.	(C)	37.	(D)	57.	(B)	77.	(B)	97.	(D)	117.	(C)
18.	(C)	38.	(A)	58.	(B)	78.	(C)	98.	(B)	118.	(A)
19.	(D)	39.	(C)	59.	(C)	79.	(C)	99.	(A)	119.	(C)
20.	(D)	40.	(D)	60.	(B)	80.	(A)	100.	(B)	120.	(B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777