KD Campus
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NDA MATHS MOCK TEST - 128 (SOLUTION)

1. (D) I = $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\ dx$ ...(i)

Prop. IV $\int_{-a}^{a} f(x)\ dx = \int_{-a}^{a} f(-x)\ dx$

I = $\int_{-\pi/2}^{\pi/2} \frac{\cos^2(-x)}{1+3^{-x}}\ dx$

I = $\int_{-\pi/2}^{\pi/2} \frac{3^x.\cos^2 x}{1+3^x}$ ...(ii)

from eq(i) and eq(ii)

2I = $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x(1+3^x)}{1+3^x}\ dx$

2I = $\int_{-\pi/2}^{\pi/2} \cos^2 x\ \ dx$

2I = $2\int_{0}^{\pi/2} \cos^2 x\ dx$

2I = $\int_{0}^{\pi/2} (1+\cos 2x)\ dx$

2I = $\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$

2I = $\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right)$ – (0 + 0)

2I = $\frac{\pi}{2}$ + 0 ⇒ I = $\frac{\pi}{4}$

2. (A) $8\cos x.\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)$

⇒ $4\cos x.2\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)$

⇒ $4\cos x\left[\cos\left(\frac{\pi}{6}+x+\frac{\pi}{6}-x\right)+\cos\left(\frac{\pi}{6}+x-\frac{\pi}{6}+x\right)\right]$

⇒ $4\cos x\left[\cos\frac{\pi}{3}+\cos 2x\right]$

⇒ $4\cos x\left[\frac{1}{2}+\cos 2x\right]$

⇒ $2\cos x[1 + 2\cos 2x]$

⇒ $2[\cos x + 2\cos x.\cos 2x]$

⇒ $2[\cos x + \cos 3x + \cos x]$

⇒ $2[2\cos x + \cos 3x]$

3. (C) Total students = 500

Fail students = 43

Total pass students $n(H \cup E)$ = 457

$n(H)$ = 226, $n(E)$ = 282

Now, $n(H \cap E) = n(H) + n(E) - n(H \cup E)$

⇒ $n(H \cap E)$ = 226 + 282 – 457

⇒ $n(H \cap E)$ = 508 – 457 = 51

4. (C) $y = \operatorname{cosec}^{-1} x$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$

5. (C) $(1 + x)^2(1 + x^2)^3$

⇒ $(1 + 2x + x^2)(1 + x^6 + 3x^2 + 3x^4)$

Hence coefficient of x^6 = 1 + 3 = 4

6. (B) Word "Chemistry"

The total no. of arrangement = ${}^9C_4 \times 4!$

$= \frac{9!}{4!5!} \times 4! = 3024$

7. (B) Maximum value of $\sin x$ = 1

8. (A) series $\frac{1^2}{2} + \frac{1^2+2^2}{2+4} + \frac{1^2+2^2+3^2}{2+4+6} + \ldots\ldots\ldots$

$T_n = \frac{1^2+2^2+3^2+\ldots\ldots+n^2}{2+4+\ldots\ldots+2n}$

$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{n(n+1)}$

$T_n = \frac{1}{6}(2n + 1)$

$S_n = \sum T_n$

$S_n = \frac{1}{3}\sum n + \frac{1}{6}\sum 1$

$S_n = \frac{1}{3} \times \frac{n(n+1)}{2} + \frac{1}{6} \times n$

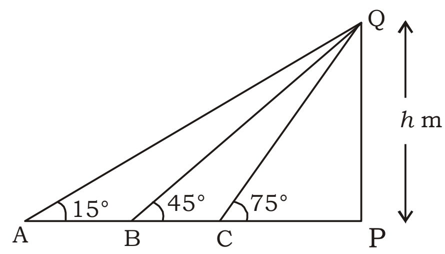
$S_n = \frac{1}{6}(n^2 + n + n)$

$S_n = \frac{1}{6}(n^2 + 2n)$

Now, $S_8 = \frac{1}{6}(8^2 + 2 \times 8)$

⇒ $S_8 = \frac{1}{6} \times 80 = \frac{40}{3}$

9. (C)



Let height of the tower PQ = h m

In ΔCPQ :-

$$\tan 75° = \frac{PQ}{CP}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{CP} \Rightarrow CP = h(2-\sqrt{3})$$

In ΔBPQ :-

$$\tan 45° = \frac{PQ}{BP}$$

$$\Rightarrow 1 = \frac{h}{BP}$$

$\Rightarrow BP = h$

$\Rightarrow BC + CP = h$

$\Rightarrow BC + h(2-\sqrt{3}) - h \Rightarrow BC = h(\sqrt{3}-1)$

In ΔAPQ:-

$$\tan 15° = \frac{PQ}{AP}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{AB + BP}$$

$\Rightarrow AB + BP = h(2+\sqrt{3})$

$\Rightarrow AB + h = h(2+\sqrt{3}) \Rightarrow AB = h(\sqrt{3}+1)$

Now, AB : BC = $h(\sqrt{3}+1) : h(\sqrt{3}-1)$

$= (\sqrt{3}+1) : (\sqrt{3}-1)$

10. (D) $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

Let $x = \tan\theta$

$$\Rightarrow \tan^{-1}y = \tan^{-1}(\tan\theta) + \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$\Rightarrow \tan^{-1}y = \theta + \tan^{-1}(\tan 2\theta)$

$\Rightarrow \tan^{-1}y = \theta + 2\theta$

$\Rightarrow \tan^{-1}y = 3\theta$

$\Rightarrow y = \tan 3\theta$

$\Rightarrow y = \tan 3\theta$

$$\Rightarrow y = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \frac{3x-x^3}{1-3x^2}$$

11. (D) Given that A = $\begin{bmatrix} 2a+b & 4 \\ a+b & -1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 2a+b & a+b \\ 4 & -1 \end{bmatrix}$$

Co-factors of A-

$$C = \begin{bmatrix} -1 & -(a+b) \\ -4 & 2a+b \end{bmatrix}$$

$$\text{Adj A} = C^T = \begin{bmatrix} -1 & -4 \\ -(a+b) & 2a+b \end{bmatrix}$$

Now, Adj A = A^T

$$\begin{bmatrix} -1 & -4 \\ -(a+b) & 2a+b \end{bmatrix} = \begin{bmatrix} 2a+b & a+b \\ 4 & -1 \end{bmatrix}$$

On comparing

$2a + b = -1$, $a + b = -4$

$-(a + b) = 4$, $2a + b = -1$

On solving

$a = 3$, $b = -7$

Then $4a - b = 4 \times 3 - (-7)$

$= 12 + 7 = 19$

12. (B) Let $z = \frac{1-2i\sin\theta}{1+2i\sin\theta}$

$$\Rightarrow z = \frac{(1-2i\sin\theta)(1-2i\sin\theta)}{(1+2i\sin\theta)(1-2i\sin\theta)}$$

$$\Rightarrow z = \frac{1+4i^2\sin^2\theta - 4i\sin\theta}{1-4i^2\sin^2\theta}$$

$$\Rightarrow z = \frac{1-4\sin^2\theta - 4i\sin\theta}{1+4\sin^2\theta}$$

$$\Rightarrow z = \frac{1-4\sin^2\theta}{1+4\sin^2\theta} - \frac{4i\sin\theta}{1+4\sin^2\theta}$$

z will be purely imaginary, when

$$\frac{1-4\sin^2\theta}{1+4\sin^2\theta} = 0$$

$\Rightarrow 1 - 4\sin^2\theta = 0$

$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

13. (C) $\lim_{x\to\infty} \frac{x^3-x-1}{4x^2-2x^3+6}$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3-x-1}{-2x^3+4x^2+6}$$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3\left(1-\frac{1}{x^2}-\frac{1}{x^3}\right)}{x^3\left(-2+\frac{4}{x}+\frac{6}{x^3}\right)}$$

$$\Rightarrow \lim_{x\to\infty} \frac{1-\frac{1}{x^2}-\frac{1}{x^3}}{-2+\frac{4}{x}+\frac{6}{x^3}}$$

$$\Rightarrow \frac{1-0}{-2+0} = \frac{-1}{2}$$

14. (B) $I = \int \frac{\cos x}{\sin(x-a)}\, dx$

Let $x - a = t \Rightarrow x = a + t$

$dx = dt$

$I = \int \frac{\cos(a+t)}{\sin t}\, dt$

$I = \int \frac{\cos a.\cos t - \sin a.\sin t}{\sin t}\, dt$

$I = \cos a \int \cot t\, dt - \sin a \int 1.\, dt$

$I = \cos a.\log \sin t - \sin a.\ (t) + C$

$I = \cos a.\log\sin(x-a) - (x-a)\sin a + C$

$I = \cos a.\log\sin(x-a) - x\sin a + a\sin a + C$

$I = \cos a.\ \log\sin(x-a) - x\sin a + c$

15. (C) Let $y = e^{\cos x}$ and $z = \sin x$

$\frac{dy}{dx} = e^{\cos x}(-\sin x)$, $\frac{dz}{dx} = \cos x$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = e^{\cos x}(-\sin x) \times \frac{1}{\cos x}$

$\Rightarrow \frac{dy}{dz} = -\tan x.e^{\cos x}$

16. (B) $I = \int_{ln2}^{ln3} \frac{\sin x}{\sin x + \sin(ln6 - x)}\, dx$ ...(i)

Prop.IV $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

$I = \int_{ln2}^{ln3} \frac{\sin(ln2 + ln3 - x)}{\sin(ln2 + ln3 - x) + \sin(ln6 - ln2 - ln3 + x)}$

$I = \int_{ln2}^{ln3} \frac{\sin(ln6 - x)}{\sin(ln6 - x) + \sin x}\, dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_{ln2}^{ln3} \frac{\sin x + \sin(ln6 - x)}{\sin x + \sin(ln6 - x)}\, dx$

$2I = \int_{ln2}^{ln3} 1.dx$

$2I = [x]_{ln2}^{ln3}$

$2I = ln3 - ln2$

$2I = ln\frac{3}{2} \Rightarrow I = \frac{1}{2} ln\frac{3}{2}$

17. (A) equations

$x^2 + ax - 1 = 0$

$x^2 + x + a = 0$

have one root in common,

Let one root = α

$\alpha^2 + a\alpha - 1 = 0$

$\alpha^2 + \alpha + a = 0$

Here, $\frac{\alpha^2}{a^2+1} = \frac{-\alpha}{a+1} = \frac{1}{1-a}$

$\alpha^2 = \frac{a^2+1}{1-a}$ and $\alpha = \frac{-a-1}{1-a}$

$\Rightarrow \left(\frac{-a-1}{1-a}\right)^2 = \frac{a^2+1}{1-a}$

$\Rightarrow \frac{(a^2+1+2a)}{1-a} = a^2 + 1$

On solving

$a^3 + 3a = 0$

$\Rightarrow a(a^2 + 3) = 0$

$a \neq 0,\ a^2 = -3$

Hence $a = \pm\sqrt{3}\, i$

18. (C) $\lim_{x\to\infty} x \sin\frac{1}{x}$

$\Rightarrow \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$ $\left[\frac{0}{0}\right]$ From

by L - Hospital's Rule

$\Rightarrow \lim_{x\to\infty} \frac{\cos\frac{1}{x}\left(\frac{-1}{x^2}\right)}{\left(\frac{-1}{x^2}\right)}$

$\Rightarrow \lim_{x\to\infty} \cos\frac{1}{x}$

$\Rightarrow \cos 0 = 1$

19. (C) An ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Point P(6, 4) satisfy the rquation

$\frac{36}{a^2} + \frac{16}{b^2} = 1$ ...(i)

equation of the tangent at point (6, 4) to the ellipse

$\frac{x\times 6}{a^2} + \frac{y\times 4}{b^2} = 1$

its passes through the point (12, 0)

$\Rightarrow \frac{12\times 6}{a^2} + 0 = 1$

$\Rightarrow a^2 = 72$

from eq (i)

$\Rightarrow \frac{32}{72}+\frac{16}{b^2} = 1$

$\Rightarrow \frac{16}{b^2} = 1 - \frac{1}{2}$

$\Rightarrow \frac{16}{b^2} = \frac{1}{2} \Rightarrow b^2 = 32$

Now, $e = \sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1-\frac{32}{72}}$

$\Rightarrow e = \sqrt{1-\frac{4}{9}} \Rightarrow e = \frac{\sqrt{5}}{3}$

20. (C)

21. (B) $\begin{vmatrix} 2! & 3! & 4! \\ 4! & 5! & 6! \\ 6! & 7! & 8! \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2! & 3\times 2! & 4\times 3\times 2! \\ 4! & 5\times 4! & 6\times 5\times 4! \\ 6! & 7\times 6! & 8\times 7\times 6! \end{vmatrix}$

$\Rightarrow 2! \times 4! \times 6! \begin{vmatrix} 1 & 3 & 12 \\ 1 & 5 & 30 \\ 1 & 7 & 56 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow 2! \times 4! \times 6! \begin{vmatrix} 1 & 3 & 12 \\ 0 & 2 & 18 \\ 0 & 4 & 44 \end{vmatrix}$

$2! \times 4! \times 6!\ [1(88 - 72) - 0]$

$2! \times 4! \times 6! \times 16 \Rightarrow 16 \times 2! \times 4! \times 6!$

22. (A) Determenant $\begin{vmatrix} 1 & 2 & 4 & -1 \\ 6 & 7 & -2 & 2 \\ 3 & 0 & 1 & 5 \\ 1 & 1 & 2 & 2 \end{vmatrix}$

Minor of the element 3 = $\begin{vmatrix} 2 & 4 & -1 \\ 7 & -2 & 2 \\ 1 & 2 & 2 \end{vmatrix}$

$= 2(-4 - 4) - 4(14 - 2) - 1(14 + 2)$

$= -16 - 48 - 16 = -80$

23. (D) $[A]_{(x-5)\times(y+3)}$ and $[B]_{(x-2)\times(6-y)}$

AB and BA and exist, then

$y + 3 = x - 2 \Rightarrow x - y = 5$ ...(i)

$6 - y = x - 5 \Rightarrow x + y = 11$ ...(ii)

from eq(i) and eq(ii)

$x = 8,\ y = 3$

24. (C) Plane $4x - 4y + 7z = 6$

and point (0, –3, 4)

Perpendicular distance

$= \frac{4\times 0 - 4(-3) + 7\times 4 - 6}{\sqrt{4^2 + (-4)^2 + 7^2}}$

$= \frac{0+12+28-6}{\sqrt{16+16+49}} = \frac{34}{9}$

25. (A) Vectors $\hat{i} + \hat{j} + \lambda\hat{k}$ and $(2\lambda - 3)\hat{i} + 3\hat{j} - 4\hat{k}$ are perpendicular,

then $1 \times (2\lambda - 3) + 1 \times 3 + \lambda(-4) = 0$

$\Rightarrow 2\lambda - 3 + 3 - 4\lambda = 0$

$\Rightarrow -2\lambda = 0 \Rightarrow \lambda = 0$

26. (B) $y = \sin x$ ...(i)

$x = \sin^{-1} y$

$\frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$

$\frac{d^2x}{dy^2} = \frac{-1}{2}(1-y^2)^{-3/2}(-2y)$

$\frac{d^2x}{dy^2} = \frac{y}{(1-y^2)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{(1-\sin^2 x)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{(\cos^2 x)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{\cos^3 x}$

$\frac{d^2x}{dy^2} = \tan x.\sec^2 x$

27. (A) Given that $S_{13} = 533$

$\Rightarrow \frac{13}{2}[2a + 12d] = 533$

$\Rightarrow 13[a + 6d] = 533$

$\Rightarrow a + 6d = 41$

Hence $T_7 = 41$

28. (B) A line makes the angles α, β and γ with the axes, then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2 - 3$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

29. (B) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\pi}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\pi)}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4\pi}}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\pi}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 2\pi}}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+2\cos 2\pi}}}$

$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 \pi}}}$

$\Rightarrow \sqrt{2+\sqrt{2+2\cos \pi}}$

$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2 \frac{\pi}{2}}}$

$\Rightarrow \sqrt{2+2\cos \frac{\pi}{2}}$

$\Rightarrow \sqrt{2\times 2\cos^2 \frac{\pi}{4}}$

$\Rightarrow 2\cos\frac{\pi}{4} = 2\times\frac{1}{\sqrt{2}} = \sqrt{2}$

30. (C) An angles of a triangle are in 3 : 2 : 1
Let Angles = $3x$, $2x$, x
$3x + 2x + x = 180$
$\Rightarrow 6x = 180 \Rightarrow x = 30$
Angles = 90, 60, 30
Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 90} = \frac{b}{\sin 60} = \frac{c}{\sin 30}$

$\Rightarrow \frac{a}{1} = \frac{b\times 2}{\sqrt{3}} = \frac{c\times 2}{1}$

$\frac{a}{2} = \frac{b}{\sqrt{3}} = \frac{c}{1}$

Hence $a : b : c = 2 : \sqrt{3} : 1$

31. (B) The required no. of triangles = ${}^{14}C_3 - {}^{8}C_3$
= 364 – 56
= 308

32. (C) Given that $\tan\theta = \frac{a}{b}$

Now, $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$\Rightarrow \frac{a\tan\theta - b}{a\tan\theta + b}$

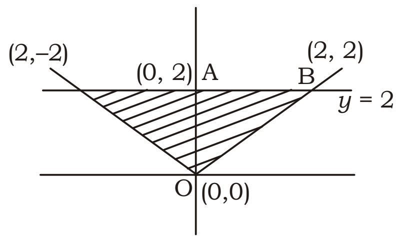
$\Rightarrow \frac{a\times\frac{a}{b} - b}{a\times\frac{a}{b} + b} = \frac{a^2 - b^2}{a^2 + b^2}$

33. (B) $\lim_{x\to 0}\frac{\sin 2x - \tan x}{x}$ $\left[\frac{0}{0}\right]$ Form
by L-Hospital Rule

$\Rightarrow \lim_{x\to 0}\frac{2\cos 2x - \sec^2 x}{1}$

$\Rightarrow$ 2 cos0 – $\sec^2 0$
$\Rightarrow$ 2 – 1 = 1

34. (A)



Lines $y = |x|$ and $y = 2$
Area = 2 × Area of ΔOAB

= $2\times\frac{1}{2}$ × OA × AB

= $2\times\frac{1}{2}$ × 2 × 2 = 4 sq. unit

35. (A) An ellipse

$\frac{x^2}{16} + \frac{y^2}{7} = 1$
$a^2 = 16$, $b^2 = 7$

Now, $e = \sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1-\frac{7}{16}} \Rightarrow e = \frac{3}{4}$

foci = $(ae, 0)$ = $\left(\pm 4\times\frac{3}{4}, 0\right)$

= (± 3, 0)
equation of circle whose centre (0, 3)
$(x-0)^2 + (y-3)^2 = r^2$ ...(i)
its passes through the point (3, 0)
$3^2 + 3^2 = r^2$
$9 + 9 = r^2 \Rightarrow r^2 = 18$
from eq(i)
$(x-0)^2 + (y-3)^2 = 18$
$\Rightarrow x^2 + y^2 + 9 - 6y = 18$
$\Rightarrow x^2 + y^2 - 6y - 9 = 0$

36. (C) $f(x) = \begin{cases} 3ax - 5b, & x < -2 \\ 2, & x = -2 \\ a - bx, & x > -2 \end{cases}$ is continous at

$x = -2$, then

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = f(-2)$

Now, $\lim_{x\to -2^-} f(x) = f(-2)$

$\Rightarrow \lim_{x\to -2} 3ax - 5b = 2$

$\Rightarrow 3a(-2) - 5b = 2$

$\Rightarrow -6a - 5b = 2$ ...(i)

and $\lim_{x\to -2^+} f(x) = f(-2)$

$\Rightarrow \lim_{x\to -2} a - bx = 2$

$\Rightarrow a - b(-2) = 2$

$\Rightarrow a + 2b = 2$ ...(ii)

from eq(i) and eq(ii)

$a = -2,\ b = 2$

37. (B) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$\Rightarrow \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$\Rightarrow \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\,.\cos\theta(\sin\theta-\cos\theta)}$

$\Rightarrow \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta.\cos\theta)}{\sin\theta\,.\cos\theta(\sin\theta-\cos\theta)}$

$\Rightarrow \frac{1+\sin\theta.\cos\theta}{\sin\theta\,.\cos\theta}$

$\Rightarrow \sec\theta.\mathrm{cosec}\theta + 1$

38. (A) $y = \mathrm{cosec}(\cot^{-1}x)$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = -\mathrm{cosec}(\cot^{-1}x).\cot(\cot^{-1}x)\left(\frac{-1}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \mathrm{cosec}(\cot^{-1}x)\ \frac{x}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{xy}{1+x^2}$ ...(i)

$\Rightarrow (1+x^2)\frac{dy}{dx} = xy$

Again, differentiating

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} \times 2x = \frac{xdy}{dx} + y.1$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} = y - \frac{xdy}{dx}$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} = y - x \times \frac{xy}{1+x^2}$ [from eq(i)]

$\Rightarrow \frac{d^2y}{dx^2} = y\left[\frac{1+x^2-x^2}{(1+x^2)^2}\right]$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{(1+x^2)^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{\mathrm{cosec}(\cot^{-1}x)}{(1+x^2)}$

$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x=1} = \frac{\mathrm{cosec}(\cot^{-1}1)}{(1+1)^2}$

$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x=1} = \frac{\mathrm{cosec}\frac{\pi}{4}}{4}$

$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x=1} = \frac{\sqrt{2}}{2^2} = \frac{1}{2\sqrt{2}}$

39. (C) $\tan^{-1}\frac{6}{7} + \cos^{-1}\frac{24}{25}$

$\Rightarrow \tan^{-1}\frac{6}{7} + \tan^{-1}\frac{7}{24}\left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow \tan^{-1}\left[\frac{\frac{6}{7}+\frac{7}{24}}{1-\frac{6}{7}\times\frac{7}{24}}\right]$

$\Rightarrow \tan^{-1}\left[\frac{193}{126}\right]$

40 (B) $z = 1 + \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$

$z = 2\cos^2\frac{\pi}{12} + i \times 2\sin\frac{\pi}{12}.\cos\frac{\pi}{12}$

$z = 2\cos\frac{\pi}{12}\left[\frac{\pi}{12} + i\sin\frac{\pi}{12}\right]$

Now, $|z| = 2\cos\frac{\pi}{12}$

41. (C) $\sqrt{3+2\sqrt{2}} = \sqrt{\left(\sqrt{2}\right)^2 + 1^2 + 2\times\sqrt{2}\times 1}$

$= \sqrt{\left(\sqrt{2}+1\right)^2} = \sqrt{2} + 1$

42. (D) 13-sided regular polygon

The no. of diagonals = $\frac{n(n-3)}{2}$

$= \frac{13\times 10}{2} = 65$

43. (D) equation whose roots are 4 and – 6

$(x-4)(x+6) = 0$

$\Rightarrow x^2 + 2x - 24 = 0$

Original equation

$x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - 6x + 4x - 24 = 0$

$\Rightarrow (x-6)(x+4) = 0$

Roots of originalar equation = 6, – 4

44. (C) Line

$(3x - 4y + 6) + \lambda(x + 2y + 1) = 0$

$(3+\lambda)x + (-4+2\lambda)y + 6 + \lambda = 0$

$y = -\left(\frac{3+\lambda}{-4+2\lambda}\right)x - \frac{6+\lambda}{-4+2\lambda}$

it is parallel to x-axis i.e.

m = 0

$\Rightarrow -\left(\frac{3+\lambda}{-4+2\lambda}\right) = 0$

$\Rightarrow \lambda + 3 = 0 \Rightarrow \lambda = -3$

45. (B) $A_1 = \int_0^{\pi/4} \cos x \ dx$

$A_1 = [\sin x]_0^{\pi/4}$

$A_1 = \sin\frac{\pi}{4} - \sin 0$

$A_1 = \frac{1}{\sqrt{2}}$

and $A_2 = \int_0^{\pi/4} \sin 2x \ dx$

$A_2 = -\left[\frac{\cos 2x}{2}\right]_0^{\pi/4}$

$A_2 = -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right]$

$A_2 = -\frac{1}{2}[0-1] = \frac{1}{2}$

Now, $\frac{A_1}{A_2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}$

$\Rightarrow \frac{A_1}{A_2} = \frac{\sqrt{2}}{1}$

Hence $A_1 : A_2 = \sqrt{2} : 1$

46. (C) Hyperbola

$\frac{x^2}{16} - \frac{y^2}{\lambda^2} = 1$

eccentricity $e = \sqrt{1+\frac{\lambda^2}{16}}$

$\Rightarrow e = \frac{\sqrt{16+\lambda^2}}{4}$

foci $(\pm ae, 0) = \left(\pm\sqrt{16+\lambda^2}, 0\right)$

Ellipse

$\frac{x^2}{36} + \frac{y^2}{9} = 1$

eccentricity $e = \sqrt{1-\frac{9}{36}}$

$\Rightarrow e = \sqrt{\frac{36-9}{25}} = \frac{\sqrt{27}}{5}$

foci$(\pm ae, 0) = \left(\pm\sqrt{27}, 0\right)$

Now, $\sqrt{16+\lambda^2} = \sqrt{27}$

$\Rightarrow 16 + \lambda^2 = 27 \Rightarrow \lambda^2 = 11$

47. (B) The required no. of ways = (9 – 1)!

= 8!

48. (A) The total Possible ways = 8 × 7 = 56

49. (C) $f(x) = \tan x - \tan^2 x + \tan^3 x - \ldots\ldots \infty$

$f(x) = \frac{\tan x}{1-(-\tan x)}$

$f(x) = \frac{\tan x}{1+\tan x}$

Let $I = \int_0^{\pi/2} \frac{f(x)}{\tan x} dx = \int_0^{\pi/2} \frac{\tan x}{(1+\tan x)\tan x} dx$

$I = \int_0^{\pi/2} \frac{1}{1+\tan x} \ dx$

$I = \int_0^{\pi/2} \frac{1}{1+\frac{\sin x}{\cos x}} dx$

$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ …(i)

$I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)} \ dx$

$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\, dx$

$2I = \int_0^{\pi/2} 1.dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

50. (C) Lines

$24x + 7y - 6 = 0$

$48x + 14y + 3 = 0$

$\Rightarrow 24x + 7y + \frac{3}{2} = 0$

The required distance $= \frac{\frac{3}{2} - (-6)}{\sqrt{(24)^2 + (7)^2}}$

$= \frac{\frac{3}{2} + 6}{\sqrt{(25)^2}}$

$= \frac{15}{2 \times 25} = \frac{3}{10}$

51. (A) Let $y = \sin^2\left[\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right]$

On putting $x = \cos\theta \Rightarrow \frac{dx}{d\theta}$

$y = \sin^2\left[\tan^{-1}\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right]$

$y = \sin^2\left[\tan^{-1}\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}}\right]$

$y = \sin^2\left[\tan^{-1}\left(\cot\frac{\theta}{2}\right)\right]$

$y = \sin^2\left[\tan^{-1}\left[\tan\left(\frac{\pi}{2} - \theta\right)\right]\right]$

$y = \sin^2\left(\frac{\pi}{2} - \theta\right)$

$y = \cos^2\theta$

$y = x^2$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = 2x$

52. (B) $\tan 7\frac{1}{2} = \frac{\sin 7\frac{1}{2}}{\cos 7\frac{1}{2}} \times \frac{2\sin 7\frac{1}{2}}{2\sin 7\frac{1}{2}}$

$\tan 7\frac{1}{2} = \frac{2\sin^2 7\frac{1}{2}}{2\sin 7\frac{1}{2}.\cos 7\frac{1}{2}}$

$\tan 7\frac{1}{2} = \frac{1-\cos 15}{\sin 15}$

$\tan 7\frac{1}{2} = \frac{1 - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

$\tan 7\frac{1}{2} = \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\tan 7\frac{1}{2} = \frac{2\sqrt{6} - 3 - \sqrt{3} + 2\sqrt{2} - \sqrt{3} - 1}{3 - 1}$

$\tan 7\frac{1}{2} = \frac{2\sqrt{6} - 2\sqrt{3} + 2\sqrt{2} - 4}{2}$

$\tan 7\frac{1}{2} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$

53. (C) $\text{cov}(x, y) = \frac{\sum x_i y_i}{n} - \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n^2}$

$= \frac{288}{8} - \frac{48 \times 36}{8 \times 8}$

$= 36 - 27 = 9$

54. (D) Given that $x + y = z$

Now, $\cos x + \cos y + \cos z + 1$

$\Rightarrow 2\cos\frac{x+y}{2}.\cos\frac{x-y}{2} + 2\cos^2\frac{z}{2}$

$\Rightarrow 2\cos\frac{z}{2}.\cos\frac{x-y}{2} + 2\cos^2\frac{z}{2}$ $[\because x + y = z]$

$\Rightarrow 2\cos\frac{z}{2}\left[\cos\frac{x-y}{2} + \cos\frac{z}{2}\right]$

$\Rightarrow 2\cos\frac{z}{2}\left[2\cos\frac{x-y+z}{4}.\cos\frac{x-y-z}{4}\right]$

$\Rightarrow 2\cos\frac{z}{2}\left[2\cos\frac{x+x}{4}.\cos\frac{-y-y}{2}\right]$

$[\because x + y = z]$

$\Rightarrow 2\cos\frac{z}{2}\left[2\cos\frac{x}{2}.\cos\frac{y}{2}\right]$

$\Rightarrow 4\cos\frac{x}{2}.\cos\frac{y}{2}.\cos\frac{z}{2}$

55. (B) $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A}$

$\Rightarrow \left(\frac{\sin 3A}{\sin A}\right)^2 - \left(\frac{\cos 3A}{\cos A}\right)^2$

$\Rightarrow \left(\frac{3\sin A - 4\sin^3 A}{\sin A}\right)^2 - \left(\frac{4\cos^2 A - 3\cos A}{\cos A}\right)^2$

$\Rightarrow (3 - 4\sin^2 A)^2 - (4\cos^2 A - 3)^2$

$\Rightarrow 9 + 16\sin^4 A - 24\sin^2 A - (16\cos^4 A + 9 - 24\cos^2 A)$

$\Rightarrow 9 + 16\sin^4 A - 24\sin^2 A - 16\cos^4 A - 9 + 24\cos^2 A$

$\Rightarrow 16(\sin^4 A - \cos^4 A) - 24(\sin^2 A - \cos^2 A)$

$\Rightarrow 16(\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A) - 24(\sin^2 A - \cos^2 A)$

$\Rightarrow 16(\sin^2 A - \cos^2 A) - 24(\sin^2 A - \cos^2 A)$

$\Rightarrow -8(\sin^2 A - \cos^2 A) = 8\cos 2A$

56. (C) $y = a^{x\log_a \sin x}$

$\Rightarrow y = a^{\log_a (\sin x)^x}$

$\Rightarrow y = (\sin x)^x$

taking log both side

$\Rightarrow \log y = x \log\sin x$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{\cos x}{\sin x} + \log\sin x \times 1$

$\Rightarrow \frac{dy}{dx} = y[x.\cot x + \log\sin x]$

$\Rightarrow \frac{dy}{dx} = (\sin x)^x [x.\cot x + \log\sin x]$

57. (C) 7

58. (B) The sum of focal radii of any point on an ellipse $= a + x + a - x$

$= 2a$ (length of major axis)

59. (A)

60. (C) 1 0 0 1 0

$0\times2^0 = 0$
$1\times2^1 = 2$
$0\times2^2 = 0$
$0\times2^3 = 0$
$1\times2^4 = 16$
18

0 . 1 1

$\frac{1}{2} = 1\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$

$(10010)_2 = (18)_{10}$ and $(0.11)_2 = (0.75)_{10}$

Hence $(10010.11)_2 = (18.75)_{10}$

61. (B) Word "SOURCE"

(O U E) S R C

The required arrangement = 4! × 3!

= 144

62. (C) Digits 3, 4, 5, 6, 8 and 9

4	5	4	3

= 4× 5 × 4× 3 = 240

(5, 6, 8, 9)

63. (A)

64. (C) sinθ, (3sinθ – 1) and (5 sinθ – 2) are in G.P.,

then $(3\sin\theta - 1)^2 = \sin\theta(-2 + 5\sin\theta)$

$\Rightarrow 9\sin^2\theta + 1 - 6\sin\theta = -2\sin\theta + 5\sin^2\theta$

$\Rightarrow 4\sin^2\theta - 4\sin\theta + 1 = 0$

$\Rightarrow (2\sin\theta - 1)^2 = 0$

$\Rightarrow 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Now, $\frac{1+\tan\theta}{2\cos\theta} = \frac{1+\tan\frac{\pi}{6}}{2\cos\frac{\pi}{6}}$

$\Rightarrow \frac{1+\tan\theta}{2\cos\theta} = \frac{1+\frac{1}{\sqrt{3}}}{2\times\frac{\sqrt{3}}{2}}$

$\Rightarrow \frac{1+\tan\theta}{2\cos\theta} = \frac{\sqrt{3}+1}{3}$

65. (D) Let $y = \log_x x$ and $z = 3^x$

$y = 1$, $\quad \frac{dz}{dx} = 3^x \log 3$

$\frac{dy}{dx} = 0$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = 0 \times \frac{1}{3^x \log 3} = 0$

66. (B)

67. (C) $I = \int \frac{1}{1+e^x}\,dx$

$I = \int \frac{1}{e^x(e^{-x}+1)}\,dx$

$I = \int \frac{e^{-x}}{e^{-x}+1}\,dx$

Let $e^{-x} + 1 = t$

$-e^{-x}dx = dt \Rightarrow e^{-x}dx = -dt$

$I = \int \frac{-dt}{t}$

$I = -\log t + c$

$I = -\log(e^{-x} + 1) + c$

$I = -\log\left(\frac{1+e^x}{e^x}\right) + c$

$I = \log\left(\frac{e^x}{1+e^x}\right) + c$

68. (C) Sphere $x^2 + y^2 + z^2 + 7x + 8y - 2z - 1 = 0$

$u = \frac{7}{2}$, $v = 4$, $w = -1$, $d = -1$

Now, radius $= \sqrt{u^2 + v^2 + w^2 - d}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + 4^2 + (-1)^2 - (-1)}$

$= \sqrt{\frac{49}{4} + 16 + 1 + 1}$

$= \sqrt{\frac{121}{4}} = \frac{11}{2}$ unit

69. (B) Rolle's Theorem-

(i) $f(x)$ is continuous on a closed interval $[a, b]$.

(ii) $f(x)$ is differentiable on an open interval (a, b).

(iii) $f(a) = f(b)$

(iv) $f'(c) = 0$

Given that $f(x) = 4x^3 + ax^2 - bx$

$f(x) = 12x^2 + 2ax - b$

(i) Function is continous on a interval $[-2,2]$.

(ii) Function is differentiable on a interval $(-2, 2)$.

(iii) $f(-2) = f(2)$

$\Rightarrow 4(-2)^3 + a(-2)^2 - b(-2) = 4(2)^3 + a(2)^2 - b \times 2$

$\Rightarrow -32 + 4a + 2b = 32 + 4a - 2b$

$\Rightarrow 4b = 64 \Rightarrow b = 16$

(iv) $f'(c) = 0$

$12c^2 + 2ac - b = 0$

on putting $c = 1$ and $b = 16$

$\Rightarrow 12 + 2a - 16 = 0 \Rightarrow a = 2$

Now, $a + 2b = 2 + 2 \times 16$

$= 34$

70. (C) Variance of 35 observations var(x) = 3

We know that

$\text{var}(\lambda x) = \lambda^2 \text{var}(x)$

If each observation multiplied by 3, then variance of new observations

$\text{var}(3x) = 3^2 \times \text{var}(x)$

$= 9 \times 3 = 27$

71. (A) $\tan^{-1}\left(\cot\frac{31\pi}{4}\right) = \tan^{-1}\left[\cot(4 \times 2\pi) - \frac{\pi}{4}\right]$

$= \tan^{-1}\left[-\cot\frac{\pi}{4}\right]$

$= \tan^{-1}\left[\tan^{-1}\left(-\frac{\pi}{4}\right)\right]$

$= -\frac{\pi}{4}$

72. (B) Given that P(A) = 0.6 and P(B) = 0.5

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

for minimum value of $P(A \cap B)$, $P(A \cup B) = 1$

$1 = 0.6 + 0.5 - P(A \cap B)$

$\Rightarrow P(A \cap B) = 1.1 - 1 = 0.1$

73. (C) line

$\frac{2x-1}{8} = \frac{y+6}{4} = \frac{z-1}{7}$

$\Rightarrow \frac{x - \frac{1}{2}}{4} = \frac{y+6}{4} = \frac{z-1}{7}$

and plane $2x + y + 2z + 4 = 0$

Let angle between line and plane = θ

Now, $\sin\theta = \frac{4 \times 2 + 4 \times 1 + 7 \times 2}{\sqrt{4^2 + 4^2 + 7^2}\sqrt{2^2 + 1^2 + 2^2}}$

$\Rightarrow \sin\theta = \frac{8 + 4 + 14}{9 \times 3}$

$\Rightarrow \sin\theta = \frac{26}{27} \Rightarrow \theta = \sin^{-1}\left(\frac{26}{27}\right)$

74. (A) p, q, r are in A.P.,

then $2q = p + r$ ...(i)

and a, b, c are in A.P,

then $2b = a + c$ ...(ii)

Now, $2(q + b) = (p + a) + (c + r)$

Hence $(b + a)$, $(q + b)$, $(c + r)$ also are in A.P.

75. (B) When $\theta = 180°$

$M = \frac{60}{11}(H \pm 6)$ when $- \rightarrow H > 6$

$+ \rightarrow H < 6$

H = 3 (between 3 and 4 O'clock)

$\Rightarrow M = \frac{60}{11}(3 + 6)$

$\Rightarrow M = \frac{60}{11} \times 9 = 49\frac{1}{11}$ minute

Time = $3 : 49\frac{1}{11}$

76. (C) $f(x) = \sqrt{8x^2 + 1} \Rightarrow f(1) = 3$

$f'(x) = \frac{16x}{2\sqrt{8x^2+1}} = \frac{8x}{\sqrt{8x^2+1}}$

Now, $\lim_{x \to 1} \frac{f(x) - 3}{x - 1}$ $\left[\frac{0}{0}\right]$ Form

By L- Hospital's Rule

$\Rightarrow \lim_{x \to 1} \frac{f'(x)}{1}$

$\Rightarrow \lim_{x \to 1} \frac{8x}{\sqrt{8x^2+1}} = \frac{8}{3}$

77. (D) $I = \int_0^1 \sum_{n=1}^{5} (x^{n-1} - x^n)\ dx$

$I = \int_0^1 \begin{bmatrix} (x^0 - x) + (x - x^2) + (x^2 - x^3) + \\ (x^3 - x^4) + (x^4 - x^5) \end{bmatrix} dx$

$I = \int_0^1 (x^0 - x^5)\ dx$

$I = \left(x - \frac{x^6}{6}\right)_0^1$

$I = 1 - \frac{1}{6} = \frac{5}{6}$

78. (C) Slope = $\frac{x}{y+2}$ and point = (–1, 2)

equation of the curve

$y - 2 = \frac{x}{y+2}(x + 1)$

$\Rightarrow y^2 - 4 = x^2 + x$
$\Rightarrow x^2 - y^2 + x + 4 = 0$

79. (B) $\left(-\sqrt{-1}\right)^{8n-1} + \left(-\sqrt{-1}\right)^{4n+3}$

$\Rightarrow (-i)^{8n-1} + (-i)^{4n+3}$
$\Rightarrow (-i)^{8n} (-i)^{-1} + (-i)^{4n} (-i)^3$

$\Rightarrow \frac{1}{-i} - i^3$

$\Rightarrow i + i = 2i$

80. (C) $x = 3 + 3^{1/3} + 3^{2/3}$
$\Rightarrow x - 3 = 3^{1/3} + 3^{2/3}$ ...(i)
$\Rightarrow (x - 3)^3 = (3^{1/3} + 3^{2/3})^3$
$\Rightarrow x^3 - 27 - 9x(x - 3) = 3 + 3^2 + 3 \times 3^{1/3} \times 3^{2/3} (3^{1/3} + 3^{2/2})$
$\Rightarrow x^3 - 27 - 9x^2 + 27x = 3 + 9 + 3 \times 3(x - 3)$ [from eq (i)]
$\Rightarrow x^3 - 9x^2 + 27x - 27 = 12 + 9x - 27$
$\Rightarrow x^3 - 9x^2 + 18x = 0$
$\Rightarrow x^3 - 9x^2 + 18x + 6 = 6$

81. (A) a = 4cm

$s = \frac{4+4+4}{2} = 6$ cm

$\Delta = \frac{\sqrt{3}}{4} a^2$

$\Delta = \frac{\sqrt{3}}{4} \times 4 \times 4 = 4\sqrt{3}$ sq. unit

Now, $r = \frac{\Delta}{s}$

$= \frac{4\sqrt{3}}{6}$

Area of in circle = πr^2

$= \pi \times \left(\frac{4\sqrt{3}}{6}\right)^2$

$= \frac{4}{3}\pi$ sq. unit

82. (A) In ΔABC, $c = 3$, B = 30°, $b = \sqrt{3}$
Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Now, $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin 30}{\sqrt{3}} = \frac{\sin C}{3}$

$\Rightarrow \frac{1}{2\sqrt{3}} = \frac{\sin C}{3}$

$\Rightarrow \sin C = \frac{\sqrt{3}}{2} \Rightarrow C = 60°$

83. (C) Curve
$3x^2 + 4y^2 + 24$

$\frac{3x^2}{24} + \frac{4y^2}{24} = 1$

$\frac{x^2}{8} + \frac{y^2}{6} = 1$

$a^2 = 8, b^2 = 6$

$a = 2\sqrt{2}$, $b = \sqrt{6}$

Area = πab

$= \pi \times 2\sqrt{2} \times \sqrt{6}$

$= 4\sqrt{3}\pi$ sq. unit

84. (B) Differetial equation

$\left(\frac{dy}{dx}\right)^3 = (xy)^2$

$\Rightarrow \frac{dy}{dx} = (xy)^{2/3}$

$\Rightarrow y^{-2/3}\ dy = x^{2/3}\ dx$
On integrating

$\Rightarrow \frac{y^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + 3c$

$\Rightarrow 3y^{1/3} = \frac{3}{5} x^{5/3} + 3c$

$\Rightarrow y^{1/3} = \frac{1}{5} x^{5/3} + c$

85. (C) $3 \times (n+1)!$, $2 \times n!$ and $(n-1)!$ are in G.P., then

$(2 \times n!)^2 = 3 \times (n+1)! \times (n-1)!$

$\Rightarrow 4 \times (n!)^2 = 3 \times (n+1)n! \times (n-1)!$

$\Rightarrow 4 \times n! \times n(n-1)! = 3 \times (n+1)n! \times (n-1)!$

$\Rightarrow 4n = 3(n+1)$

$\Rightarrow 4n = 3n + 3 \Rightarrow n = 3$

86. (C) $C(2n, 7) = C(2n, n-1)$

$\Rightarrow {}^{2n}C_7 = {}^{2n}C_{n-1}$

here $7 + n - 1 = 2n \Rightarrow n = 6$

Now, $C(12, n) = C(12, 6)$

$$= \frac{12!}{6! \times 6!} = 924$$

87. (B) In the expansion of $\left(3x - \frac{1}{x^2}\right)^7$

$T_{r+1} = {}^7C_r (3x)^{7-r} \left(\frac{-1}{x^2}\right)^r$

$= {}^7C_r\, 3^{7-r} (-1)^r x^{7-3r}$

Now, $7 - 3r = 1 \Rightarrow r = 2$

The coefficient of $x = {}^7C_2\, 3^5 (-1)^2$

$$= \frac{7!}{2!5!} \times 243 = 5103$$

88. (D) Equation $x^2 + 7|x| + 12 = 0$ has no root because sum of three positive numbers can not be zero.

89. (A) $\sec^{-1}\frac{x}{20} + \sec^{-1}\frac{x}{21} = \frac{\pi}{2}$

$\Rightarrow \sec^{-1}\frac{x}{20} = \frac{\pi}{2} - \sec^{-1}\frac{x}{21}$

$\Rightarrow \sec^{-1}\frac{x}{20} = \operatorname{cosec}^{-1}\frac{x}{21}$

$\Rightarrow \sec^{-1}\frac{x}{20} = \sec^{-1}\frac{x}{\sqrt{x^2 - 441}}$

$\Rightarrow \frac{x}{20} = \frac{x}{\sqrt{x^2 - 441}}$

$\Rightarrow 20 = \sqrt{x^2 - 441}$

$\Rightarrow 400 = x^2 - 441$

$\Rightarrow x^2 = 841 \Rightarrow x = 29$

90. (B) No. of element in set A = 35

No. of subsets = $2^5 = 32$

No. of element in set B = 3

No. of elements = $2^3 = 8$

91. (C) $\log_5\left(x - \frac{15}{2}\right) + \log_5 2x = 3$

$\Rightarrow \log_5 2x\left(x - \frac{15}{2}\right) = 3$

$\Rightarrow 2x\left(x - \frac{15}{2}\right) = 5^3$

$\Rightarrow 2x^2 - 15x = 125$

$\Rightarrow 2x^2 - 15x - 125 = 0$

$\Rightarrow (2x - 25)(x + 5) = 0$

Hence $x = \frac{25}{2}$, $x \neq -5$

92. (C) $3x = 2 + 4i$

$\Rightarrow 3x - 2 = 4i$

On squaring both side

$\Rightarrow 9x^2 + 4 - 12x = 16i^2$

$\Rightarrow 9x^2 - 12x = -16 - 4$

$\Rightarrow 9x^2 - 12x = -20$

93. (B) $A = \begin{bmatrix} 1+\omega^2 & \omega^2 & \omega^2 \\ \omega+\omega^2 & \omega & \omega^2 \\ 1+\omega & \omega & \omega \end{bmatrix}$

$|A| = \begin{vmatrix} 1+\omega^2 & \omega^2 & \omega^2 \\ \omega+\omega^2 & \omega & \omega^2 \\ 1+\omega & \omega & \omega \end{vmatrix}$

$= \begin{vmatrix} -\omega & \omega^2 & \omega^2 \\ -1 & \omega & \omega^2 \\ -\omega^2 & \omega & \omega \end{vmatrix}$ $[\because 1 + \omega + \omega^2 = 0]$

$= -\omega(\omega^2 - \omega^3) - \omega^2(-\omega + \omega^4) + \omega^2(-\omega + \omega^3)$

$= -\omega(\omega^2 - 1) - \omega^2(-\omega + \omega) + \omega^2(-\omega + 1)$

$= -\omega^3 + \omega + 0 - \omega^3 + \omega^2$

$= -1 - 1 - 1$ $[\because 1 + \omega + \omega^2 = 0]$

$= -3$

94. (C) Given that $x^2 + y^2 = 1$

Now, $\frac{1+x-iy}{1+x+iy} \times \frac{1+x-iy}{1+x-iy}$

$\Rightarrow \frac{1 + x^2 + (-iy)^2 + 2x - 2ixy - 2iy}{(1+x)^2 - (iy)^2}$

$\Rightarrow \frac{1 + x^2 - y^2 + 2x - 2iy(x+1)}{1 + x^2 + 2x + y^2}$

$\Rightarrow \frac{1 + x^2 + x^2 - 1 + 2x - 2iy(x+1)}{1 + 1 + 2x}$ $[\because x^2 + y^2 = 1]$

$\Rightarrow \frac{2x^2 + 2x - 2iy(x+1)}{2(1+x)}$

$\Rightarrow \frac{2x(x+1) - 2iy(x+1)}{2(x+1)}$

$\Rightarrow \frac{2(x+1)[x - iy]}{2(x+1)} = x - iy$

95. (B) Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$a = k \sin A$, $b = k \sin B$, $c = k \sin C$

Now, $\frac{a+b}{c} = \frac{k\sin A + k\sin B}{k\sin C}$

$$\Rightarrow \frac{a+b}{c} = \frac{\sin A + \sin B}{\sin C}$$

$$\Rightarrow \frac{a+b}{c} = \frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\sin\frac{C}{2}.\cos\frac{C}{2}}$$

$$\Rightarrow \frac{a+b}{c} = \frac{2\sin\frac{180-C}{2}.\cos\frac{A-B}{2}}{2\sin\frac{C}{2}.\cos\frac{C}{2}}$$

$$\Rightarrow \frac{a+b}{c} = \frac{2\cos\frac{C}{2}.\cos\frac{A-B}{2}}{2\sin\frac{C}{2}.\cos\frac{C}{2}}$$

$$\Rightarrow \frac{a+b}{c} = \frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}$$

96. (B) **Statement I**

In month of october = 31 days = 28 + 3

The Probability getting 5 fridays = $\frac{3}{7}$

In month of september = 30 days =28 +2

The Probability getting 5 Fridays = $\frac{2}{7}$

Statement I is incorrect.

Statement II

In leap year = 366 days = 52 weeks and 2 days

The Probability = $\frac{2}{7}$

In normal year = 365 days = 52 weeks and 1 days

The Probability = $\frac{1}{7}$

Statement II is correct.

97. (C) $\begin{vmatrix} a+b & a^2+b^2 & k \\ b+c & b^2+c^2 & k \\ c+a & c^2+a^2 & k \end{vmatrix} = (a-b)(b-c)(c-a)$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} a+b & a^2+b^2 & k \\ c-a & c^2-a^2 & 0 \\ c-b & c^2-b^2 & 0 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$\Rightarrow (c-a)(c-b)\begin{vmatrix} a+b & a^2+b^2 & k \\ 1 & c+a & 0 \\ 1 & c+b & 0 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$\Rightarrow \begin{vmatrix} a+b & a^2+b^2 & k \\ 1 & c+a & 0 \\ 1 & c+b & 0 \end{vmatrix} = -(a-b)$$

$R_3 \to R_3 - R_2$

$$\Rightarrow \begin{vmatrix} a+b & a^2+b^2 & k \\ 1 & c-a & 0 \\ 0 & b-a & 0 \end{vmatrix} = b-a$$

$$\Rightarrow (b-a)\begin{vmatrix} a+b & a^2+b^2 & k \\ 1 & c+a & 0 \\ 0 & 1 & 0 \end{vmatrix} = b-a$$

$\Rightarrow (a+b)\times 0 - (a^2+b^2)\times 0 + k(1) = 1$

$\Rightarrow k = 1$

98. (A) $S_{15} = \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \ldots\ldots\ldots\ldots + \frac{1}{31.33}$

$$S_{15} = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) \ldots. + \frac{1}{2}\left(\frac{1}{31} - \frac{1}{33}\right)$$

$$S_{15} = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{33}\right)$$

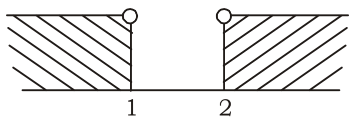
$$S_{15} = \frac{1}{2} \times \frac{11-1}{33}$$

$$S_{15} = \frac{10}{2\times 33} = \frac{5}{33}$$

99. (C) $\frac{\log_e(x^2-3x+2)}{x-1}$

here $x^2 - 3x + 2 > 0$ but $x - 1 \neq 0$

$(x-1)(x-2) > 0$ $x \neq 1$



Domain = $(-\infty, 1) \cup (2, \infty)$

100. (C)

2	23	1
2	11	1
2	5	1
2	2	0
2	1	1
	0	

↑

0 .25
× 2
[0].50
× 2
[1].00

$(23)_{10} = (10111)_2$ and $(0.25)_{10} = (0.01)_2$

Hence $(23.25)_{10} = (10111.01)_2$

101. (B) We know that

If $\sqrt{x}+\sqrt{y}=\sqrt{a}$, then

Area bounded by the curve = $\frac{a^2}{6}$

Now, curve $\sqrt{x}+\sqrt{y}$ = $\sqrt{5}$

Area bounded by the curve = $\frac{5^2}{6}$

$= \frac{25}{6}$ sq. unit

102. (A) Let $a + ib = \sqrt{-3+4i}$

On squaring both side

$(a^2 - b^2) + 2abi = -3 + 4i$

On comparing

$a^2 - b^2 = -3$ and $2ab = 4$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 9 + 16$

$\Rightarrow (a^2 + b^2)^2 = 25 \Rightarrow a^2 + b^2 = 5$ ...(ii)

from eq(i) and eq(ii)

$2a^2 = 2, \quad 2b^2 = 8$

$a = \pm 1, \quad b = \pm 2$

Hence $\sqrt{-3+4i} = \pm(1 + 2i)$

103. (C) cot10.cot20.......cot90.cot100 = 0

because cot90 = 0

104. (B) Let $f(x) = y = \frac{5^x - 5^x}{5^x + 5^x}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{y+1}{y-1} = \frac{5^x - 5^{-x} + 5^x + 5^{-x}}{5^x - 5^{-x} - 5^x - 5^{-x}}$

$\Rightarrow \frac{y+1}{y-1} = \frac{2.5^x}{-2.5^{-x}}$

$\Rightarrow \frac{y+1}{1-y} = 5^{2x}$

$\Rightarrow \log_5\left(\frac{1+y}{1-y}\right) = 2x$

$\Rightarrow x = \frac{1}{2}\log_5\left(\frac{1+y}{1-y}\right)$

$\Rightarrow f^{-1}(y) = \frac{1}{2}\log_5\left(\frac{1+y}{1-y}\right)$

$\Rightarrow f^{-1}(x) = \frac{1}{2}\log_5\left(\frac{1+x}{1-x}\right)$

105. (A) We know that

$(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n x^n$

Now, $(1.05)^5 = (1 + 0.05)^5$

$\Rightarrow (1.05)^5 = {}^5C_0 + {}^5C_1(0.05) + {}^5C_2(0.05)^2 + \ldots$

$\Rightarrow (1.05)^5 = 1 + 5 \times 0.05 + 10 \times 0.0025 + \ldots$

$\Rightarrow (1.05)^5 = 1 + 0.25 + 0.025 + \ldots\ldots$

$\Rightarrow (1.05)^5 \approx 1.275$

106. (B) The required Probability = $\frac{{}^4C_1 \times {}^4C_1}{{}^{52}C_2}$

$= \frac{4\times4}{\frac{52\times51}{2}} = \frac{8}{663}$

107. (A) $3f(x-3) + f\left(\frac{1}{x-3}\right) = 2x$ ...(i)

On putting $x = \frac{7}{2}$

$\Rightarrow 3f\left(\frac{1}{2}\right) + f(2) = 2 \times \frac{7}{2}$

$\Rightarrow 3f\left(\frac{1}{2}\right) + f(2) = 7$ ...(ii)

On putting $x = 5$ in eq(i)

$\Rightarrow 3f(2) + f\left(\frac{1}{2}\right) = 2 \times 5$

$\Rightarrow 3f(2) + f\left(\frac{1}{2}\right) = 10$ ...(iii)

from eq(ii) and eq(iii)

$f\left(\frac{1}{2}\right) = \frac{11}{8}$ and $f(2) = \frac{23}{8}$

108. (A) Given that $f(x) = 2[x]$ and $g(x) = x.f(x)$
$= 2x.[x]$

$I = \int_1^3 g(x)\, dx$

$I = \int_1^3 2x.[x]\, dx$

$I = \int_1^3 2x.1\, dx + \int_2^3 2x.2\, dx$

$I = 2\left[\frac{x^2}{2}\right]_1^2 + 4\left[\frac{x^2}{2}\right]_2^3$

$I = 2\left[2-\frac{1}{2}\right] + 4\left[\frac{9}{2}-2\right]$

$I = 3 + 10 = 13$

109. (D) $f(x) = 2[x]$

$$\text{L.H.L.} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$$

$$= \lim_{h\to 0} 2[2-h]$$

$$= 2 \times 1 = 2$$

$$\text{R.H.L.} = \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$$

$$= \lim_{h\to 0} 2[2+h]$$

$$= 2 \times 2 = 4$$

L.H.L. ≠ R.H.L.

Hence limit does not exist.

110. (C) $f(x) = x^3 + 2x^2 - 4x + 1$

$f'(x) = 3x^2 + 4x - 4$

$f''(x) = 6x + 4$

for maxima and minima

$f'(x) = 0$

$\Rightarrow 3x^2 + 4x - 4 = 0$

$\Rightarrow (3x-2)(x+2) = 0$

$\Rightarrow x = -2, \frac{2}{3}$

$f''(-2) = 6(-2) + 4 = -8$ (maxima)

$f''\left(\frac{2}{3}\right) = 6 \times \frac{2}{3} + 4 = 8$ (minima)

Maximum value of the function

$= (-2)^3 + 2(-2)^2 - 4(-2) + 1$

$= -8 + 8 + 8 + 1 = 9$

111. (B)

Class	0-10	10-20	20-30	30-40	40-50
Frequency	6	10	11	18	15

$f_1 = 18, f_0 = 11, f_2 = 15, l_1 = 30, l_2 = 40$

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times (l_2 - l_1)$$

$$= 30 + \frac{18-11}{2\times 18 - 11 - 15} \times (40-30)$$

$$= 30 + \frac{7}{10} \times 10 = 37$$

112. (A) sec315 + cosec315

$\Rightarrow$ sec(270 + 45) + cosec(270 + 45)

$\Rightarrow$ cosec45 – sec45

$\Rightarrow \sqrt{2} - \sqrt{2} = 0$

113. (C) $f(x) = \ln(1 + x^2)$

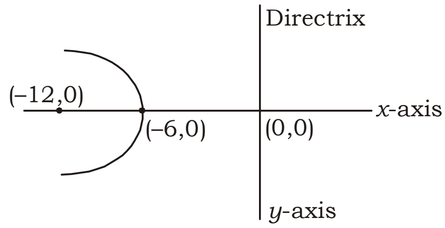
$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = \frac{(1+x^2).2 - 2x(2x)}{(1+x^2)^2}$$

$$f''(x) = \frac{2+2x^2-4x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{2-2x^2}{(1+x^2)^2}$$

114. (B)



$a = 6$

$4a = 24$

equation of parabola

$(y-0)^2 = -24(x+6)$

$\Rightarrow y^2 = -24x - 144$

$\Rightarrow y^2 + 24x + 144 = 0$

115. (A) S = (1× 2) + (2 × 4) + (3 × 6) + (4 × 8) +.................upto 15 terms

S = 2(1×1) +2(2 × 2) + 2(3 ×3) +2(4 × 4) +..........upto 15 terms

S = $2[1^2 + 2^2 + 3^2 + 4^2 + \ldots$..upto 15 terms]

$$S = 2 \times \frac{15}{6}(15+1)(2\times 15 + 1)$$

$S = 5 \times 16 \times 31 = 2480$

116. (C) $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$$\Rightarrow \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\times\frac{3}{4}}\right] = \tan^{-1}\left(\frac{56}{33}\right)$$

117. (B) $(2\vec{a} - \vec{b}) \times (5\vec{a} + 2\vec{b}) = k(\vec{a}\times\vec{b})$

$\Rightarrow 10(\vec{a}\times\vec{a}) - 5(\vec{b}\times\vec{a}) + 4(\vec{a}\times\vec{b}) - 2(\vec{b}\times\vec{b})$

$= k(\vec{a}\times\vec{b})$

$\Rightarrow 5(\vec{a}\times\vec{b}) + 4(\vec{a}\times\vec{b}) = k(\vec{a}\times\vec{b})$

$\Rightarrow 9(\vec{a}\times\vec{b}) = k(\vec{a}\times\vec{b}) \Rightarrow k = 9$

118. (B)

$$I = \int_0^{\pi/2} \frac{\phi(x)}{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)}\, dx \quad \ldots(i)$$

$$I = \int_0^{\pi/2} \frac{\phi\left(\frac{\pi}{2} - x\right)}{\phi\left(\frac{\pi}{2} - x\right) + \phi(x)}\, dx \quad \ldots(ii)$$

from eq(i) and eq(ii)

$$I + I = \int_0^{\pi/2} \frac{\phi(x) + \phi\left(\frac{\pi}{2} - x\right)}{\phi(x) + \left(\frac{\pi}{2} - x\right)}\, dx$$

$$2I = \int_0^{\pi/2} 1.dx$$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

119. (A) We know that

$\cos 2A = 1 - 2\sin^2 A$

$\Rightarrow 2\sin^2 A = 1 - \cos 2A$

$\Rightarrow \sin^2 A = \frac{1-\cos 2A}{2}$

$A = 22\frac{1}{2}$

$\Rightarrow \sin^2 22\frac{1}{2} = \frac{1-\cos 45}{2}$

$\Rightarrow \sin^2 22\frac{1}{2} = \frac{1-\frac{1}{\sqrt{2}}}{2}$

$\Rightarrow \sin^2 22\frac{1}{2} = \frac{\sqrt{2}-1}{2\sqrt{2}}$

120. (C) $I = \int_0^{2\pi} |\sin x|\, dx$

$I = 2\int_0^{\pi} \sin x\; dx$

$I = -2[\cos x]_0^{\pi}$

$I = -2[\cos\pi - \cos 0]$

$I = -2[-1 - 1]$

$I = -2(-2) = 4$

NDA (MATHS) MOCK TEST - 128 (Answer Key)

1. (D)	21. (B)	41. (C)	61. (B)	81. (A)	101. (B)
2. (A)	22. (A)	42. (D)	62. (C)	82. (A)	102. (A)
3. (C)	23. (D)	43. (D)	63. (A)	83. (C)	103. (C)
4. (C)	24. (C)	44. (C)	64. (C)	84. (B)	104. (B)
5. (C)	25. (A)	45. (B)	65. (D)	85. (C)	105. (A)
6. (B)	26. (B)	46. (C)	66. (B)	86. (C)	106. (B)
7. (B)	27. (A)	47. (B)	67. (C)	87. (B)	107. (A)
8. (A)	28. (B)	48. (A)	68. (C)	88. (D)	108. (A)
9. (C)	29. (B)	49. (C)	69. (B)	89. (A)	109. (D)
10. (D)	30. (C)	50. (C)	70. (C)	90. (B)	110. (C)
11. (D)	31. (B)	51. (A)	71. (A)	91. (C)	111. (B)
12. (B)	32. (C)	52. (B)	72. (B)	92. (C)	112. (A)
13. (C)	33. (B)	53. (C)	73. (C)	93. (B)	113. (C)
14. (B)	34. (A)	54. (D)	74. (A)	94. (C)	114. (B)
15. (C)	35. (A)	55. (B)	75. (B)	95. (B)	115. (A)
16. (B)	36. (C)	56. (C)	76. (C)	96. (B)	116. (C)
17. (A)	37. (B)	57. (C)	77. (D)	97. (C)	117. (B)
18. (C)	38. (A)	58. (B)	78. (C)	98. (A)	118. (B)
19. (C)	39. (C)	59. (A)	79. (B)	99. (C)	119. (A)
20. (C)	40. (B)	60. (C)	80. (C)	100. (C)	120. (C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777