NDA MATHS MOCK TEST - 130 (SOLUTION)

1. (A)

2. (B) Let $z = \dfrac{1}{(1-\cos\theta)+i\sin\theta}$

$$\Rightarrow z = \frac{1}{2\cos^2\frac{\theta}{2}+i\times 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$$

$$\Rightarrow z = \frac{1}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}$$

$$\Rightarrow z = \frac{1}{2}\sec\frac{\theta}{2}\times\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}-i^2\sin^2\frac{\theta}{2}}$$

$$\Rightarrow z = \frac{1}{2}\sec\frac{\theta}{2}\frac{\left[\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right]}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}}$$

$$\Rightarrow z = \frac{1}{2}\sec\frac{\theta}{2}\left[\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right]$$

$$\text{Im } z = \frac{1}{2}\sec\frac{\theta}{2}\left(-\sin\frac{\theta}{2}\right) = \frac{-1}{2}\tan\frac{\theta}{2}$$

3. (B)

4. (C) $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x+4}\right)^{x+3}$

$$\Rightarrow \lim_{x\to\infty}\left[\left(1+\frac{-2}{x+4}\right)^{\frac{x+4}{-2}}\right]^{\frac{-2}{x+4}\times(x+3)}$$

$$\Rightarrow e^{\left(\lim\limits_{x\to\infty}\frac{-2}{x+4}\times(x+3)\right)}\quad\left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$$

$$\Rightarrow e^{\lim\limits_{x\to\infty}\frac{-2x\left(1+\frac{3}{x}\right)}{x\left(1+\frac{4}{x}\right)}} = e^{-2}$$

5. (C) Determinant $\begin{vmatrix}1 & 0 & 2 & 4\\ 1 & -1 & 2 & 6\\ 8 & 9 & 10 & -3\\ 4 & 3 & 2 & 7\end{vmatrix}$

Co-factor of the element $3 = (1)^{4+2}\begin{vmatrix}1 & 2 & 4\\ 1 & 2 & 6\\ 8 & 10 & -3\end{vmatrix}$

$= 1(-6-60) - 2(-3-48) + 4(10-16)$
$= -66 + 102 - 24 = 12$

6. (D) Plane $-2x + 4y + 3z = 24$

$$\Rightarrow \frac{-2x}{24}+\frac{4y}{24}+\frac{3z}{24} = 1$$

$$\Rightarrow \frac{x}{-12}+\frac{y}{6}+\frac{z}{8} = 1$$

Intercepts of the Plane = (–12, 6, 8)

7. (A) Word "APPOINT"

Total arrangement = $\dfrac{7!}{2!}$ = 2520

(A O I) P P N T

arrangement when vowels always together = $\dfrac{5!}{2!}$ = 60

arrangement when vowels never come together = 2520 – 60 = 2460

8. (C) Given that $\Delta = \begin{vmatrix}2 & 3 & 4\\ -1 & 2 & 0\\ 6 & 4 & 2\end{vmatrix}$ and $\Delta' = \begin{vmatrix}2 & 3 & 4\\ 3 & 8 & 8\\ 3 & 2 & 1\end{vmatrix}$

Now, $\Delta = \begin{vmatrix}2 & 3 & 4\\ -1 & 2 & 0\\ 6 & 4 & 2\end{vmatrix}$

$R_2 \to R_2 + 2R_1$

$$\Rightarrow \Delta = \begin{vmatrix}2 & 3 & 4\\ 3 & 8 & 8\\ 6 & 4 & 2\end{vmatrix}$$

$$\Rightarrow \Delta = 2\begin{vmatrix}2 & 3 & 4\\ 3 & 8 & 8\\ 3 & 2 & 1\end{vmatrix}$$

$$\Rightarrow \Delta = 2\Delta'$$

9. (B)

2	67	1
2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(67)_{10} = (1000011)_2$

10. (A) $\frac{dy}{dx} = \sec(x-y)$

Let $x - y = t$

$$1 - \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$$

$\Rightarrow 1 - \frac{dt}{dx} = \sec t$

$\Rightarrow \frac{dt}{dx} = 1 - \sec t$

$\Rightarrow \frac{dt}{1-\sec t} = dx$

$\Rightarrow \frac{(1+\sec t)dt}{1-\sec^2 t} = dx$

$\Rightarrow \frac{(1+\sec t)dt}{-\tan^2 t} = dx$

$\Rightarrow \cot^2 t + \cot t.\text{cosec}\, t = -dx$

$\Rightarrow \text{cosec}^2 t - 1 + \cot t.\ \text{cosec}\, t = -\ dx$

On integrating

$\Rightarrow -\cot t - t - \text{cosec}\, t = -\ x - c$

$\Rightarrow \cot(x-y) + x - y + \text{cosec}(x-y) = x + c$

$\Rightarrow \cot(x-y) + \text{cosec}(x-y) = y + c$

11. (D) Given that no. of diagonals = 90

$\Rightarrow \frac{n(n-3)}{2} = 90$

$\Rightarrow n^2 - 3n = 180$

$\Rightarrow n^2 - 3n - 180 = 0$

$\Rightarrow (n-15)(n+12) = 0$

$\Rightarrow n = 15, -12$

Hence no. of sides = 15

12. (B) Given that $\cot^2\theta = 2\cot^2\phi + 1$

$\Rightarrow 1 + \cot^2\theta = 2\cot^2\phi + 2$

$\Rightarrow \text{cosec}^2\theta = 2\ \text{cosec}^2\phi$

$\Rightarrow \frac{1}{\sin^2\theta} = \frac{2}{\sin^2\phi}$

$\Rightarrow \sin^2\phi = 2\sin^2\theta$

$\Rightarrow \frac{1-\cos 2\phi}{2} = 1 - \cos 2\theta$

$\Rightarrow 1 - \cos 2\phi = 2 - 2\cos 2\theta$

$\Rightarrow 2\cos 2\theta = 2 - 1 + \cos 2\phi$

$\Rightarrow 2\cos 2\theta = 1 + \cos 2\phi$

13. (C) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}_{3\times1} \begin{bmatrix} m & n & l \end{bmatrix}_{1\times3} \begin{bmatrix} x & h & g \\ h & y & f \\ g & f & z \end{bmatrix}_{3\times3}$

Order = 3 × 3

14. (C) $y^2 = 4a(x-a)$ ...(i)

On differentiating both side w.r.t.'x'

$\Rightarrow 2yy_1 = 4a$

from eq(i) and eq(ii)

$\Rightarrow \frac{y^2}{2yy_1} = x - a$

$\Rightarrow a = x - \frac{y}{2y_1}$

On putting in eq(i)

$\Rightarrow y^2 = 4\left(x - \frac{y}{2y_1}\right)\left(\frac{y}{2y_1}\right)$

$\Rightarrow y^2 = 4 \times \frac{2xy_1 - y}{2y_1} \times \frac{y}{2y_1}$

$\Rightarrow yy_1^2 = 2xy_1 - y$

$\Rightarrow yy_1^2 + y = 2xy_1$

15. (B) Given that $f(x) = 6x - 5$

x	$f(x)$
2	7
3	13
⋮	⋮

so on

So function is injective but not surjective.

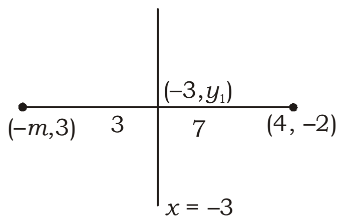
16. (C)

17. (D) A = {1, 3, 5, 7, 9,11,13} ; $n = 7$

Number of proper subsets = $2^n - 1$

$= 2^7 - 1 = 127$

18. (B)



Now, $\frac{3\times4 + 7\times(-m)}{3+7} = -3$

$\Rightarrow \frac{12-7m}{10} = -3$

$\Rightarrow 12 - 7m = -30 \Rightarrow m = 6$

19. (C) $I = \int \frac{\ln x}{x}\ dx$

$I = \ln x\int \frac{1}{x}\ dx - \int\left\{\frac{d}{dx}(\ln x).\int\frac{1}{x}\ dx\right\} dx$

$I = (\ln x)(\ln x) - \int \frac{1}{x}.\ln x\ dx + 2c$

$I = (\ln x)^2 - I + 2c$

$2I = (\ln x)^2 + 2c \Rightarrow I = \frac{(\ln x)^2}{2} + c$

20. (B) In the expansion $\left(3\sqrt{x}+\frac{1}{6x}\right)^7$

Middle terms = $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2}+1\right)^{th}$

= 4^{th} and 5^{th}

$T_4 = T_{3+1} = {}^7C_3\,(3\sqrt{x})^4\left(\frac{1}{6x}\right)^3$

$= 35 \times \frac{3^4}{6^3}x^{-1} = \frac{105}{8}\;x^{-1}$

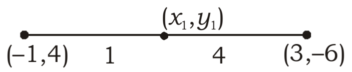
$T_5 = T_{4+1} = {}^7C_4\;(3\sqrt{x})^3\;\left(\frac{1}{6x}\right)^4$

$= 35 \times \frac{3^3}{6^4}x^{-5/2} = \frac{35}{48}x^{-5/2}$

The required sum = $\frac{105}{8} + \frac{35}{48}$

$= \frac{630+35}{48} = \frac{665}{48}$

21. (D) Given line $3x - 7y = 17$ ...(i)



Now, $x_1 = \frac{1\times3+4(-1)}{1+4}$ and $y_1 = \frac{1\times(-6)+4\times4}{1+4}$

$x_1 = \frac{-1}{5}$ and $y_1 = 2$

Point $(x_1, y_1) = \left(\frac{-1}{5}, 2\right)$

Equation of line which is parallel to line(i)

$3x - 7y = c$ ...(ii)

its passes through the point $\left(\frac{-1}{5}, 2\right)$

$\Rightarrow 3 \times \left(\frac{-1}{5}\right) - 7 \times 2 = c \Rightarrow c = \frac{-73}{5}$

from eq(i)

The required equation

$3x - 7y = \frac{-73}{5} \Rightarrow 15x - 35y + 73 = 0$

22. (C) $\frac{\cos 3x + 2\cos 2x + \cos x}{\sin 3x - \sin x}$

$\Rightarrow \frac{\cos 3x + \cos x + 2\cos 2x}{\sin 3x - \sin x}$

$\Rightarrow \frac{2\cos 2x.\cos x + 2\cos 2x}{2\cos 2x.\sin x}$

$\Rightarrow \frac{2\cos 2x(\cos x + 1)}{2\cos 2x.\sin x}$

$\Rightarrow \frac{2\cos^2 \frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}} = \cot\frac{x}{2}$

23. (D) $I = \int_0^\pi \frac{\sin\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}}\,dx$ ...(i)

$I = \int_0^\pi \frac{\sin\frac{\pi-x}{2}}{\sin\frac{\pi-x}{2}+\cos\frac{\pi-x}{2}}\,dx$ [Prop IV]

$I = \int_0^\pi \frac{\cos\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}\,dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^\pi \frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}}\,dx$

$2I = \int_0^\pi 1\,dx$

$2I = [x]_0^\pi$

$2I = \pi - 0 \Rightarrow I = \frac{\pi}{2}$

(24 - 26):-

$L_1 \Rightarrow \frac{1-2x}{8} = \frac{y-3}{4} = \frac{3+2z}{4}$

$L_1 \Rightarrow \frac{x-\frac{1}{2}}{-4} = \frac{y-3}{4} = \frac{z+\frac{3}{2}}{2}$

and $L_2 \Rightarrow \frac{x+1}{2} = \frac{y-3}{-4} = \frac{z+3}{4}$

24. (A) Let angle between the lines = θ

$\cos\theta = \left|\frac{-4\times2+4\times(-4)+2\times4}{\sqrt{(-4)^2+4^2+2^2}\sqrt{2^2+(-4)^2+4^2}}\right|$

$\cos\theta = \left|\frac{-16}{6\times6}\right|$

$\cos\theta = \frac{4}{9} \Rightarrow \theta = \cos^{-1}\frac{4}{9}$

25. (C) Direction ratio of $L_1 = \langle -4,\ 4,\ 2\rangle$

Direction cosine of $L_1 = \left\langle \frac{-4}{6}, \frac{4}{6}, \frac{2}{6}\right\rangle$

$= \left\langle \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$

26. (D) Direction ratio of $L_2 = \langle 2, -4, 4 \rangle$

27. (A) Given that $f(x) = ax + b$ and $g(x) = cx + d$
Now, $fog(x) = gof(x)$
$\Rightarrow f[g(x)] = g[f(x)]$
$\Rightarrow f[cx + d] = g[ax + b]$
$\Rightarrow a(cx + d) + b = c(ax + b) + d$
$\Rightarrow acx + ad + b = acx + bc + d$
$\Rightarrow ad + b = cb + d$
$\Rightarrow f(d) = g(b)$

28. (B) Given that $f(x) = \frac{1}{g(x)} = \frac{1}{x}$ and $g(x) = x$

From option B :-
L.H.S. $= f(g(g(g(g(f(x))))))$

$$= f\left(g\left(g\left(g\left(g\left(\frac{1}{x}\right)\right)\right)\right)\right)$$

$$= f\left(g\left(g\left(g\left(\frac{1}{x}\right)\right)\right)\right)$$

$$= f\left(g\left(g\left(\frac{1}{x}\right)\right)\right)$$

$$= f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{x}\right) = x$$

R.H.S. $= g(f(f(f(f(g(x))))))$
$= g(f(f(f(f(x)))))$

$$= g\left(f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)\right)$$

$= g(f(f(x)))$

$$= g\left(f\left(\frac{1}{x}\right)\right) = g(x) = x$$

L.H.S. = R.H.S.
Hence option B is correct.

29. (C) $x^x = e^{x+y}$
taking log both side
$\Rightarrow x\log x = x + y$
On differentiating both side w.r.t.'x'

$$\Rightarrow x \times \frac{1}{x} + \log x.\ 1 = 1 + \frac{dy}{dx}$$

$$\Rightarrow 1 + \log x = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \log x$$

30. (A) Let $y = 5^{x\sin x}$
On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = 5^{x\sin x}.\ln 5\ [x.\cos x + \sin x.1]$$

$$\Rightarrow \frac{dy}{dx} = 5^{x\sin x}.\ln 5\ [x.\cos x + \sin x]$$

31. (B) $2^{x+3} + 3.2^{y-2} = 16$ and $2^{x+1} + 2^{y-1} = 9$

$$2^x.8 + \frac{3}{4}.2^y = 16 \text{ and } 2.2^x + \frac{2^y}{2} = 9$$

Let $2^x = X$ and $2^y = Y$

$$8X + \frac{3}{4}Y = 16 \qquad \text{...(i)}$$

$$\text{and } 2X + \frac{Y}{2} = 9 \qquad \text{...(ii)}$$

from eq(i) and eq(ii)

$$X = \frac{1}{2},\ Y = 16$$

$\Rightarrow 2^x = 2^{-1},\ 2^y = 2^4$
$\Rightarrow x = -1,\ y = 4$

32. (C) $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{3}{8}$

We know that $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow \frac{3}{8} = \frac{P(A \cap B)}{\frac{1}{3}} \Rightarrow P(A \cap B) = \frac{1}{8}$$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{1/8}{1/2} = \frac{1}{4}$$

33. (B) $\begin{vmatrix} x-2 & x-3 & x-a \\ x-4 & x-5 & x-b \\ x-6 & x-7 & x-c \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} x-2 & -1 & -a+2 \\ x-4 & -1 & -b+4 \\ x-6 & -1 & -c+6 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} x+2 & -1 & -a+2 \\ -2 & 0 & -b+a+2 \\ -4 & 0 & -c+a+4 \end{vmatrix}$$

$\Rightarrow (x-2)\times 0 + 1(2c - 2a - 8 - 4b + 4a + 8) + (-a+2)\times 0$
$\Rightarrow 2c + 2a - 4b$
$\Rightarrow 2(c + a - 2b)$
a, b and c are in A.P. i.e $= 2b + c = a$
$\Rightarrow 2(c + a - c - a) = 0$

34. (B) word 'STATUS'

Total arrangement = $\frac{6!}{2!2!}$ = 180

Arrangement when T's appear together

= $\frac{5!}{2!}$ = 60

The required arrangement = 180 – 60 = 120

35. (C) Total students = 8

The table is round. One student is fixed.

Hence the no. of ways = (8 –1)! = 7! = 5040

36. (B) Differential equation

$$\frac{dy}{dx} + \frac{y}{\sqrt{x^2-1}} = \frac{x}{x+\sqrt{x^2-1}}$$

On comparing with general linear equation

$$P = \frac{1}{\sqrt{x^2-1}}, \quad Q = \frac{x}{x+\sqrt{x^2-1}}$$

$$\text{I.F.} = e^{\int P\,dx}$$

$$\text{I.F.} = e^{\int \frac{1}{\sqrt{x^2-1}}dx}$$

$$= e^{\ln\left(x+\sqrt{x^2-1}\right)} = x + \sqrt{x^2-1}$$

Solution of differential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\ dx$$

$$\Rightarrow y\times\left(x+\sqrt{x^2-1}\right) = \int \frac{x}{x+\sqrt{x^2-1}} \times \left(x+\sqrt{x^2-1}\right)dx$$

$$\Rightarrow y\left(x+\sqrt{x^2-1}\right) = \int x\ dx$$

$$\Rightarrow y\left(x+\sqrt{x^2-1}\right) = \frac{x^2}{2} + \frac{c}{2}$$

$$\Rightarrow 2y\left(x+\sqrt{x^2-1}\right) = x^2 + c$$

37. (C) $I = \int \frac{1+\ln x}{\cos(x\ln x)}\ dx$

Let $x\ln x = t$

$(1+\ln x)\ dx = dt$

$$I = \int \frac{dt}{\cos t}$$

$$I = \int \sec t\ dt$$

$I = \log|\sec t + \tan t| + c$

$I = \log|\sec(x\ln x) + \tan(x\ln x)| + c$

38. (A) $y = \left(1+\frac{1}{x}\right)\left(1+\frac{1}{x^{1/2}}\right)\left(1+\frac{1}{x^{1/4}}\right)\left(1-\frac{1}{x^{1/4}}\right)$

$$\Rightarrow y = \left(1+\frac{1}{x}\right)\left(1+\frac{1}{x^{1/2}}\right)\left(1-\frac{1}{x^{1/2}}\right)$$

$$\Rightarrow y = \left(1+\frac{1}{x}\right)\left(1-\frac{1}{x}\right)$$

$$\Rightarrow y = \frac{1}{x^2}$$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = 0 - \frac{(-2)}{x^3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{x^3}$$

39. (A) $\begin{bmatrix} x \\ y \\ y \end{bmatrix} + \begin{bmatrix} y \\ 0 \\ z \end{bmatrix} + \begin{bmatrix} z \\ x \\ 0 \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 4 \end{bmatrix}$

$x + y + z = 16$ ...(i)

$y + x = 7$ ...(ii)

$y + z = 4$ ...(iii)

On solving

$x = 12,\ y = -5,\ z = 9$

Now, $x + y - z = 12 - 5 - 9 = -2$

40. (C) $y = x.e^{-x} + \frac{\ln x}{x}$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = -x.e^{-x} + e^{-x}.1 + \frac{x\times\frac{1}{x} - \ln x.1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = -x.e^{-x} + e^{-x} + \frac{1-\ln x}{x^2}$$

$$\text{Now, } \left(\frac{dy}{dx}\right)_{\text{at } x=1} = -1.e^{-1} + e^{-1} + \frac{1-\ln 1}{1}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = \frac{-1}{e} + \frac{1}{e} + \frac{1-0}{1}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\text{at } x=1} = 1$$

41. (C) $\lim_{x\to 4} \frac{\sqrt{2x+1}-3}{x^2-16}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to 4} \frac{\frac{1\times 2}{2\sqrt{2x+1}} - 0}{2x - 0}$$

$$\Rightarrow \lim_{x\to 4} \frac{1}{2x\sqrt{2x+1}}$$

$$\Rightarrow \lim_{x\to 4} \frac{1}{2\times 4\sqrt{2\times 4+1}} = \frac{1}{8\times 3} = \frac{1}{24}$$

42. (C) We know that

$A(\text{Adj } A) = |A|\, I_n$ [where n is order]

Now, $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ here order = 2

$|A| = 5$

then $A(\text{Adj } A) = |A|\, I$

$\Rightarrow A(\text{Adj } A) = 5I$

43. (D) The required Probability

$$= \frac{6}{6\times6\times6\times6} = \frac{1}{216}$$

44. (A) $I = \int \cot^{-1}(\tan x + \sec x)\ dx$

$$I = \int \cot^{-1}\left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)\ dx$$

$$I = \int \cot^{-1}\left[\frac{\cos\left(\frac{\pi}{2} - x\right) + 1}{\sin\left(\frac{\pi}{2} - x\right)}\right]\ dx$$

$$I = \int \cot^{-1}\left[\frac{2\cos^2\left(\frac{\pi}{2} - \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right).\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]\ dx$$

$$I = \int \cot^{-1}\left[\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]\ dx$$

$$I = \int\left(\frac{\pi}{4} - \frac{x}{2}\right)\ dx$$

$$I = \frac{\pi}{4}x - \frac{x^2}{4} + c$$

45. (C) Given that A(–3, 4) and B(–1, 6)

Let $P(x, y)$

Now, PA = PB

$\Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x + 3)^2 + (y - 4)^2 = (x + 1)^2 + (y - 6)^2$

$\Rightarrow x^2 + 9 + 6x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow 4x + 4y = 12$

$\Rightarrow x + y = 3$ ...(i)

Area of $\Delta PAB = 24$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -3 & 4 & 1 \\ -1 & 6 & 1 \end{vmatrix} = \pm 24$$

$\Rightarrow x(4 - 6) - y(-3 + 1) + 1(-18 + 4) = \pm 48$

$\Rightarrow -2x + 2y - 14 = \pm 48$

$\Rightarrow -x + y - 7 = \pm 24$

$\Rightarrow -x + y - 7 = 24$ or $-x + y - 7 = -24$

$\Rightarrow y - x = 31$ ...(ii)

or

$y - x = -17$ ...(iii)

On solving eq(i) and eq(ii)

$x = -14,\ y = 17$

Similarly, on solving eq(i) and eq(ii)

$x = 10,\ y = -7$

Hence co-ordinates of P = (–14, 17) and (10, –7).

46. (B) Vectors $(2-2\lambda)\hat{i} + 5\hat{j} + 2\lambda\hat{k}$ and $4\hat{i} + (\lambda + 2)\hat{j} + 3\hat{k}$ are perpendicular,

then $(2 - 2\lambda)\times4 + 5\times(\lambda + 2) + 2\lambda\times(-3) = 0$

$\Rightarrow 8 - 8\lambda + 5\lambda + 10 - 6\lambda = 0$

$\Rightarrow 18 - 9\lambda = 0 \Rightarrow \lambda = 2$

47. (C) Equation $px^2 + qx + r = 0$

$\alpha + \beta = \frac{-q}{b}$

and equation $ax^2 + bx + c = 0$

$\alpha - h + \beta - h = \frac{-b}{a}$

$\Rightarrow \frac{-q}{p} - 2h = \frac{-b}{a}$

$\Rightarrow \frac{-q}{p} + \frac{b}{a} = 2h \Rightarrow h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

48. (A)

49. (C) Let $A = \begin{bmatrix} 2 & 4+3i \\ 4-3i & 6 \end{bmatrix}$

Now, $(\overline{A}) = \begin{bmatrix} 2 & 4-3i \\ 4+3i & 6 \end{bmatrix}$

and $(\overline{A})' = \begin{bmatrix} 2 & 4+3i \\ 4-3i & 6 \end{bmatrix}$

$(\overline{A})' = A$

Hence matrix A is a Hermitian matrix.

50. (A) Median is used for the measure of central tendency.

(51 - 52)

Given that $\lim_{x\to\infty}\left(\frac{3 + x^2}{1 + 2x} + Ax - B\right) = 4$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{3 + x^2 + Ax + 2Ax^2 - B - 2Bx}{1 + 2x}\right) = 4$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to\infty}\left(\frac{0 + 2x + A + 4Ax - 0 - 2B}{2}\right) = 4$$

$\Rightarrow \lim_{x\to\infty} \frac{(2+4A)x + A - 2B}{2} = 4$

On comparing both side

$\frac{2+4A}{2} = 0$ and $\frac{A-2B}{2} = 4$

$\Rightarrow A = \frac{-1}{2}$, $\frac{-1}{2} - 2B = 8 \Rightarrow B = -\frac{17}{4}$

51. (B) $A = \frac{-1}{2}$

52. (A) $B = -\frac{17}{4}$

53. (D) Given that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$

The required Probability

$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$= P(A).\ P(\overline{B}) + P(\overline{A}).\ P(B)$

$= \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{5}$

$= \frac{4}{15} + \frac{2}{15}$

$= \frac{6}{15} = \frac{2}{5}$

54. (A) Given that $\log_e\left(\frac{1-x}{1+x}\right)$, $g(x) = \frac{3+x^3}{1-2x^2}$

Now, $gof\left(\frac{e-1}{e+1}\right)$

$\Rightarrow g\left[f\left(\frac{e-1}{e+1}\right)\right]$

$\Rightarrow g\left[\log_e\left(\frac{1-\frac{e-1}{e+1}}{1+\frac{e-1}{e+1}}\right)\right]$

$\Rightarrow g\left[\log_e\left(\frac{e+1-e+1}{e+1+e-1}\right)\right]$

$\Rightarrow g\left[\log_e\left(\frac{2}{2e}\right)\right]$

$\Rightarrow g\left[\log_e\left(\frac{1}{e}\right)\right]$

$\Rightarrow g[-\log_e e]$

$\Rightarrow g(-1) = \frac{3+(-1)^3}{1-2} = \frac{2}{-1} = -2$

55. (A) If $B \subseteq A$, then $A \cup B = A$ and $A \cap B = B$

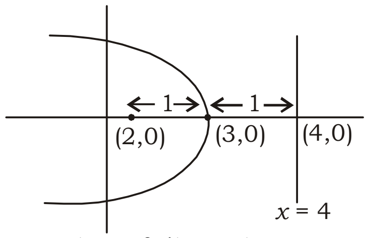
(*b*) $P(\overline{A} \cap B) = 0$

(*c*) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)}$

(*d*) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(A)} = 1$

(*a*) $P(A/A \cap B) = P(A/B) = 1$

56. (C)



equation of directrix

$x = 4$

57. (B) **Statement I**

If $\tan\theta = x$,

then $x + \frac{1}{x} = \tan\theta + \frac{1}{\tan\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{1}{\sin\theta.\cos\theta}$

$\Rightarrow x + \frac{1}{x} = \sec\theta.\text{cosec}\theta$

Statement I is correct.

Statement II

$x + \frac{1}{x} = 2\cos\theta$

On squaring both side

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (2\cos\theta)^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\cos^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = 4\cos^2\theta - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(2\cos^2\theta - 1)$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\cos 2\theta$

Statement II is incorrect.

Statement III

Minimum value of $(2\sin\theta + \sqrt{5}\cos\theta) =$

$= -\sqrt{2^2 + \left(\sqrt{5}\right)^2}$

$= -\sqrt{4+5} = -3$

Statement III is correct.

58. (A) Let $y = \cot^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $z = \cot^{-1}x$

On putting $x = \cot z$

$$\Rightarrow y = \cot^{-1}\left(\frac{\sqrt{1+\cot^2 z}-1}{\cot z}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\sqrt{\operatorname{cosec}^2 z}-1}{\cot z}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\operatorname{cosec} z-1}{\cot z}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\frac{1}{\sin z}-1}{\frac{\cos z}{\sin z}}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{1-\sin z}{\cos z}\right)$$

$$\Rightarrow y = \cot^{-1}\left[\frac{1-\cos\left(\frac{\pi}{2}-z\right)}{\sin\left(\frac{\pi}{2}-z\right)}\right]$$

$$\Rightarrow y = \cot^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4}-\frac{z}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{z}{2}\right).\cos\left(\frac{\pi}{4}-\frac{z}{2}\right)}\right]$$

$$\Rightarrow y = \cot^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{z}{2}\right)\right]$$

$$\Rightarrow y = \cot^{-1}\left[\cot\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}-\frac{z}{2}\right)\right\}\right]$$

$$\Rightarrow y = \cot^{-1}\left[\cot\left(\frac{\pi}{4}+\frac{z}{2}\right)\right]$$

$$\Rightarrow y = \frac{\pi}{4}+\frac{z}{2}$$

On differentiating both side w.r.t. 'z'

$$\Rightarrow \frac{dy}{dz} = 0 + \frac{1}{2} \Rightarrow \frac{dy}{dz} = \frac{1}{2}$$

59. (D) Angle between the regression lines will be

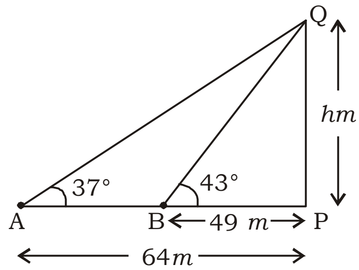
$$\tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow \tan 90 = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow \frac{1}{0}\left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow r\left(\sigma_x^2+\sigma_y^2\right) = 0 \Rightarrow r = 0$$

60. (B)



Let height of the tower = hm

In ΔAPQ :-

$$\tan 37° = \frac{PQ}{AP}$$

$$\Rightarrow \tan 37° = \frac{h}{64} \quad ...(i)$$

In ΔBPQ:-

$$\tan 43° = \frac{PQ}{BP}$$

$$\Rightarrow \tan 43° = \frac{h}{49} \quad ...(ii)$$

from eq(i) and eq(ii)

$$\tan 37° \times \tan 43° = \frac{h}{64}\times\frac{h}{49}$$

$$\Rightarrow 1 = \frac{h^2}{64\times 49} \Rightarrow h = 8 \times 7 = 56$$

Hence height of the tower = $56m$

61. (C) $x = \frac{a(1+t^2)}{1-t^2}$ and $y = \frac{2at}{1-t^2}$ ...(i)

$$\text{Now, } x^2 - y^2 = \frac{a^2(1+t^2)^2}{(1-t^2)^2} - \frac{(2at)^2}{(1-t^2)^2}$$

$$\Rightarrow x^2 - y^2 = a^2\left[\frac{1+t^4+2t^2-4t^2}{(1-t^2)^2}\right]$$

$$\Rightarrow x^2 - y^2 = a^2\left[\frac{(1-t^2)^2}{(1-t^2)^2}\right]$$

$$\Rightarrow x^2 - y^2 = a^2$$

It represents a rectangular hyperbola.

62. (A) $y = \frac{2at}{1-t^2}$

On differetiating both side w.r.t.'t'

$\Rightarrow \frac{dy}{dt} = 2a\left[\frac{(1-t^2).1 - t.(-2t)}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = 2a\left[\frac{1-t^2+2t^2}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = 2a\left[\frac{1+t^2}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = \frac{2x}{1-t^2}$ [from eq(i)]

$\Rightarrow (1-t^2)\frac{dy}{dt} = 2x$ ...(ii)

63. (C) $x = \frac{a(1+t^2)}{1-t^2}$

On differentiating both side w.r.t.'t'

$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1-t^2).2t-(1+t^2).(-2t)}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = a\left[\frac{2t-2t^3+2t+2t^3}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = a\left[\frac{4t}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = \frac{2y}{1-t^2}$ [from eq(i)]

$\Rightarrow (1-t^2)\frac{dx}{dt} = 2y$

64. (A) Differential equation

$\cos\left(\frac{dy}{dx}\right) - x = 0$

$\Rightarrow \cos\left(\frac{dy}{dx}\right) = x$

$\Rightarrow \frac{dy}{dx} = \cos^{-1}x$

$\Rightarrow dy = \cos^{-1}x\, dx$

On integrating

$\Rightarrow \int dy = \int \cos^{-1} x\ dx$

Let $\cos^{-1}x = t \Rightarrow x = \cos t$

$dx = -\sin t\, dt$

$\Rightarrow \int dy = \int t(-\sin t)\, dt$

$\Rightarrow y = -\left[t.\int \sin t\, dt - \int\left\{\frac{d}{dt}(t).\int \sin t\ dt\right\}dt\right]$

$\Rightarrow y = -\left[-t.\cos t - \int 1.(-\cos t)\,dt\right]$

$\Rightarrow y = -\left[-t.\cos t + \sin t\right] + c$

$\Rightarrow y = t\cos t - \sin t + c$

$\Rightarrow y = t\cos t - \sqrt{1-\cot^2 t} + c$

$\Rightarrow y = x.\cos^{-1}x - \sqrt{1-x^2} + c$

65. (C) Let Probability of success $(p) = \frac{1}{3}$

and Probability of unsuccess $(q) = 1-\frac{1}{3} = \frac{2}{3}$

x – binomial distribution $\left(5, \frac{1}{3}\right)$

$\therefore P(x) = {}^5C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$

(where $x = 0, 1, \ldots\ldots\ldots 5$)

$\therefore$ Required probability

$P(x \geq 3) = 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - \left[{}^5C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^5 + {}^5C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^4 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2\right]$

$= 1 - \left[\frac{32}{243} + \frac{5\times16}{243} + \frac{10\times8}{243}\right]$

$= 1 - \frac{192}{243}$

$= \frac{51}{243} = \frac{17}{81}$

66. (B) $(x-2)^3 = 27$

$(x-2) = 3(1)^{1/3}$

$x-2 = 3\times1$ or $x-2 = 3\omega$ or $x-2 = 3\omega^2$

$x = 5$ or $x = 2+3\omega$ or $x = 2+3\omega^2$

67. (C) Series $\frac{1^3}{1^2} + \frac{1^3+2^3}{1^2+2^2} + \frac{1^3+2^3+3^3}{1^2+2^2+3^2} + \ldots\ldots.$

$T_n = \frac{1^3+2^3+3^3+\ldots\ldots.+n^3}{1^2+2^2+3^2+\ldots\ldots.+n^3}$

$T_n = \frac{\frac{n^2}{4}(n+1)^2}{\frac{n}{6}(n+1)(2n+1)}$

$T_n = \frac{3}{2}\times\frac{n(n+1)}{2n+1}$

68. (A) Required probability = $^6C_3\left(\frac{1}{7}\right)^3\left(\frac{6}{7}\right)^3$

$= \frac{20\times 6^3}{7^6}$

69. (C) $\left|z-\frac{1}{z}\right| = 6$

$\Rightarrow z-\frac{1}{z} = \pm 6$

$\Rightarrow z^2 - 1 = \pm 6z$

$\Rightarrow z^2 - 6z - 1 = 0$ or $z^2 + 6z - 1 = 0$

$\Rightarrow z = \frac{6\pm\sqrt{36-4\times1(-1)}}{2\times1}$ or $z = \frac{-6\pm\sqrt{36-4\times1(-1)}}{2\times1}$

$\Rightarrow z = \frac{6\pm2\sqrt{10}}{2}$ or $z = \frac{-6\pm2\sqrt{10}}{2}$

$\Rightarrow z = 3 \pm\sqrt{10}$ or $z = -3 \pm\sqrt{10}$

$\Rightarrow z = 3-\sqrt{10}\ ,\ 3+\sqrt{10}$

or $z = -3-\sqrt{10}\ ,\ -3+\sqrt{10}$

Hence smallest value of $|z| = -3-\sqrt{10}$

70. (C) $v = 3s^2 + 5s + 9$

On differentiating both side w.r.t.'s'

$\Rightarrow \frac{dv}{ds} = 6s + 5$

Hence $\left(\frac{dy}{ds}\right)_{\text{at } s\,=\,6} = 6 \times 6 + 5 = 41$

71. (B) $\cos 2A = \lambda \cos 2B$

$\Rightarrow \frac{\cos 2A}{\cos 2B} = \frac{\lambda}{1}$

by componendo and Dividendo Rule

$\Rightarrow \frac{\cos 2A+\cos 2B}{\cos 2A-\cos 2B} = \frac{\lambda+1}{\lambda-1}$

$\Rightarrow \frac{2\cos(A+B).\cos(A-B)}{2\sin(A+B).\sin(B-A)} = \frac{\lambda+1}{\lambda-1}$

$\Rightarrow \frac{\cos(A+B).\cos(A-B)}{-\sin(A+B).\sin(A-B)} = -\frac{1+\lambda}{1-\lambda}$

$\Rightarrow \frac{\cot(A+B)}{\tan(A-B)} = \frac{1+\lambda}{1-\lambda}$

72. (D) Differential equation

$\sqrt{1-y^2}\,dx + \sqrt{1-x^2}\,dy = 0$

$\Rightarrow \sqrt{1-y^2}\,dx = -\sqrt{1-x^2}\,dy$

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = -\frac{dy}{\sqrt{1-y^2}}$

On intergrating

$\Rightarrow \sin^{-1}x = -\sin^{-1}y + \sin^{-1}c$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = \sin^{-1}c$

$\Rightarrow \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right) = \sin^{-1}c$

$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$

73. (B) $(1-\omega^2+\omega)^4 + (1-\omega+\omega^2)^4 + 32$

$\Rightarrow (-\omega^2-\omega^2)^4+(-\omega-\omega)^4+32\quad [\because 1+\omega+\omega^2=0]$

$\Rightarrow (-2\omega^2)^4 + (-2\omega)^4 + 32$

$\Rightarrow 16\omega^8 + 16\omega^4 + 32$

$\Rightarrow 16\omega^2 + 16\omega + 16 + 16$

$\Rightarrow 16(\omega^2+\omega+1) + 16 \Rightarrow 16\times 0 + 16 = 16$

74. (A)

75. (C) Given that $x + 2y = 11$

Now, $A = xy$

$\Rightarrow A = (11-2y)y$

$\Rightarrow A = 11y - 2y^2$

On differentiating both side w.r.t.'y'

$\Rightarrow \frac{dA}{dy} = 11 - 4y$

Again, differentiating

$\Rightarrow \frac{d^2A}{dy^2} = -4$

For maxima and minima

$\frac{dA}{dy} = 0$

$\Rightarrow 11-4y = 0 \Rightarrow y = \frac{11}{4}$ and $x = \frac{11}{2}$

Hence maximum value of $xy = \frac{11}{2}\times\frac{11}{4} = \frac{121}{8}$

76. (A) Given $x = y^{y^{y\ldots\infty}}$

$\Rightarrow x = y^x$

taking log both side

$\Rightarrow \log x = x\log y$ ...(i)

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{1}{x} = x\times\frac{1}{y}\frac{dy}{dx} + \log y.1$

$\Rightarrow \frac{1}{x} - \log y = \frac{x}{y}\frac{dy}{dx}$

$\Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{1-x\log y}{x}$

$\Rightarrow x^2\frac{dy}{dx} = y(1-x\log y)$

$\Rightarrow x^2\frac{dy}{dx} = y(1-\log x)$ [from eq(i)]

77. (B) $\lim_{x\to\infty}\left[8^x + 9^x\right]^{1/x}$

$\Rightarrow \lim_{x\to\infty} 9\left[\left(\frac{8}{9}\right)^x + 1\right]^{1/x}$

$\Rightarrow \lim_{x\to\infty} 9\left[1 + \frac{1}{\left(\frac{9}{8}\right)^x}\right]^{1/x}$

$\Rightarrow 9\left[1 + \frac{1}{\infty}\right]^{1/\infty} = 9$

78. (A) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} + 4\left(x + \frac{1}{x}\right)$

$\Rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$+ 4\left(x - \frac{1}{x}\right)$

Let $x + \frac{1}{x} = y$

$\Rightarrow f(y) = y^3 - 3y + 4y$

$\Rightarrow f(y) = y^3 + y$

Now, $f(-4) = (-4)^3 + (-4)$

$\Rightarrow f(-4) = -64 - 4 = -68$

79. (B)

80. (C) $I = \int_{-3}^{3} \frac{x^2}{1+3^x}\, dx$ ...(i)

Prop IV $\int_{-a}^{a} f(x)\, dx = \int_{-a}^{a} f(-x)\, dx$

$I = \int_{-3}^{3} \frac{(-x)^4}{1+3^{-x}}\, dx$

$I = \int_{-3}^{3} \frac{x^4 \cdot 3^x}{1+3^x}\, dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_{-3}^{3} \frac{x^4(1+3^x)}{1+3^x}\, dx$

$2I = \int_{-3}^{3} x^4\, dx$

$2I = 2\int_{0}^{3} x^4\, dx$

$I = \left[\frac{x^5}{5}\right]_0^3$

$I = \frac{243}{5} - 0 \Rightarrow I = 48\frac{3}{5}$

81. (B) $\left|\frac{z-2}{z+2}\right| = 3,\ z = x + iy$

$\Rightarrow \left|\frac{x+iy-2}{x+iy+2}\right| = 3$

$\Rightarrow \frac{\sqrt{(x-2)^2 + y^2}}{\sqrt{(x+2)^2 + y^2}} = 3$

On squaring both side

$\Rightarrow \frac{(x-2)^2 + y^2}{(x+2)^2 + y^2} = 9$

$\Rightarrow \frac{x^2 + 4 - 4x + y^2}{x^2 + 4 + 4x + y^2} = 9$

$\Rightarrow x^2 + 4 - 4x + y^2 = 9x^2 + 36 + 36x + 9y^2$

On solving

$\Rightarrow 8x^2 + 8y^2 + 40x + 32 = 0$

$\Rightarrow x^2 + y^2 + 5x + 4 = 0$

It is a circle.

82. (D) $n(S) = 6 \times 6 \times 6 = 216$

E = {(6, 6, 3), (6, 5, 4), (6 ,4, 5), (6, 3, 6), (5, 4, 6), (5, 5, 5),(5, 6, 4),(4, 6, 5), (4, 5, 6), (3,6,6)}

$n(E) = 10$

The required Probability = $\frac{10}{216} = \frac{5}{108}$

83. (D) $|z - 2i| > |z + 2i|$; $z = x + iy$

$\Rightarrow |x + iy - 2i| > |x + iy + 2i|$

$\Rightarrow \sqrt{x^2 + (y-2)^2} > \sqrt{x^2 + (y+2)^2}$

On squaring

$\Rightarrow x^2 + y^2 + 4 - 4y > x^2 + y^2 + 4 + 4y$

$\Rightarrow 0 > 8y \Rightarrow y < 0$

Hence Imz < 0

84. (A) $\cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{4}\right)\right]$

$\Rightarrow \cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \cos^{-1}\left[\cos\frac{\pi}{4}\right]$

$\Rightarrow \cos^{-1}\left[\cos\left(\frac{7\pi}{4}\right)\right] = \frac{\pi}{4}$

85. (D)

86. (B) The required no. of ways = ${}^{15-1}C_{11-1}$

$= {}^{14}C_{10} = 1001$

87. (D) 4 sin10.sin50.sin60.sin70

$\Rightarrow$ 4sin60[sin10.sin50.sin70]

$\Rightarrow 4\sin 60.\frac{1}{4}\sin(3 \times 10)$

$\left[\because \sin\theta.\sin(60-\theta).\sin(60+\theta) = \frac{1}{4}\sin 30\right]$

$\Rightarrow \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$

88. (C) $(A \cup C) - (A \cap B)$

89. (D) Equation of straight line which makes equal intercept on both axis

$x + y = a$ ...(i)

its passes through the point (–3, 4)

$-3 + 4 = a \Rightarrow a = 1$

from eq(i)

$x + y = 1$

90. (C) $\dfrac{\log_{\sqrt{3}} 9 \times \log_{25} \sqrt{5}}{\log_{36} 6}$

$\Rightarrow \dfrac{\log_{\sqrt{3}} \left(\sqrt{3}\right)^4 \times \log_{25} (25)^{1/4}}{\log_{36} (36)^{1/2}}$

$\Rightarrow \dfrac{4 \times \frac{1}{4}}{\frac{1}{2}} = 2$

91. (B) $f(x) = \begin{cases} 2x - \lambda, & x \le -1 \\ 3x^2 + 4, & x > -1 \end{cases}$ is continuous

at $x = -1$,

then $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x)$

$\Rightarrow \lim\limits_{x \to -1^-} 2x - \lambda = \lim\limits_{x \to -1^+} 3x^2 + 4$

$\Rightarrow 2(-1) - \lambda = 3(-1)^2 + 4$

$\Rightarrow -2 - \lambda = 3 + 4 \Rightarrow \lambda = -9$

92. (C) $x = \sqrt{3^{\sec^{-1} t}}$

On differentiating both side w.r.t.'t'

$\Rightarrow \dfrac{dx}{dt} = \dfrac{1}{2\sqrt{3^{\sec^{-1} t}}} \times 3^{\sec^{-1} t} \ln 3 \times \dfrac{1}{x\sqrt{x^2 - 1}}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{\ln 3 . 3^{\sec^{-1} t}}{2\sqrt{3^{\sec^{-1} t}}} \times \dfrac{1}{x\sqrt{x^2 - 1}}$

and $y = \sqrt{3^{\text{cosec}^{-1} t}}$

On differentiating both side w.r.t.'t'

$\Rightarrow \dfrac{dy}{dt} = \dfrac{1}{2\sqrt{3^{\text{cosec}^{-1} t}}} \times 3^{\text{cosec}^{-1} t} \ln 3 \times \dfrac{-1}{x\sqrt{x^2 - 1}}$

Now, $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{\dfrac{1}{2\sqrt{3^{\text{cosec}^{-1} t}}} \times 3^{\text{cosec}^{-1} t} \ln 3 \times \dfrac{-1}{x\sqrt{x^2 - 1}}}{\dfrac{\ln 3 . 3^{\sec^{-1} t}}{2\sqrt{3^{\sec^{-1} t}}} \times \dfrac{1}{x\sqrt{x^2 - 1}}}$$

$\Rightarrow \dfrac{dy}{dx} = -\dfrac{\sqrt{3^{\text{cosec}^{-1} x}}}{\sqrt{3^{\sec^{-1} x}}}$

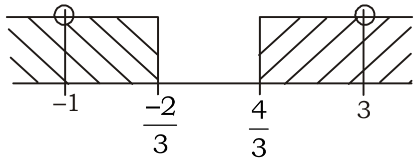
$\Rightarrow \dfrac{dy}{dx} = -\dfrac{y}{x}$

93. (B) $f(x) = \dfrac{\sqrt{9x^2 - 6x - 8}}{x^2 - 2x - 3}$

Now, $9x^2 - 6x - 8 \ge 0$ and $x^2 - 2x - 3 \ne 0$

$\Rightarrow (3x - 4)(3x + 2) \ge 0$ and $(x - 3)(x + 1) \ne 0$

$\Rightarrow x \le \dfrac{-2}{3}$, $x \ge \dfrac{4}{3}$ and $x \ne -1, 3$



Domain $= \left[\left(-\infty, \dfrac{-2}{3}\right] \cup \left[\dfrac{4}{3}, \infty\right)\right] - \{-1, 3\}$

94. (C)

2	51	1
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(51)_{10} = (110011)_2$

95. (D) Number can be formed y(0, 1, 2, 3) or (0, 2, 3, 4)

The required numbers = 3 × 3! + 2 × 3!

=18 ×12 = 30

96. (C)

97. (B) Data 21, 12, 31, 13, 41, 14, 51, 15, 61, 16

arranging in ascending order

12, 13, 14, 15, 16, 21, 31, 41, 51, 61

Middle terms = 16, 21

Median $= \dfrac{16 + 21}{2} = 18.5$

98. (A) Lines $x - 6y = 11$

and $12y - 2x = 7 \Rightarrow x - 6y = \dfrac{-7}{2}$

The required equation

$x - 6y = \dfrac{11 + \left(\frac{-7}{2}\right)}{2}$

$\Rightarrow x - 6y = \dfrac{15}{4} \Rightarrow 4x - 24y = 15$

99. (D)

100. (B) $\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}$

101. (A) $S = \frac{1}{1.5} + \frac{1}{5.9}$ +......upto 10 terms

$S = \frac{1}{1.5} + \frac{1}{5.9} + ... + \frac{1}{37.41}$

$S = \frac{1}{4}\left(1-\frac{1}{5}\right) + \frac{1}{4}\left(\frac{1}{5}-\frac{1}{9}\right) + ... + \frac{1}{4}\left(\frac{1}{37}-\frac{1}{41}\right)$

$S = \frac{1}{4}\left[1-\frac{1}{41}\right]$

$S = \frac{1}{4} \times \frac{40}{41} = \frac{10}{41}$

102. (C) Quadratic equation

$5x^2 + px + 6 = 0$ ...(i)

One root = 3

It is satisfy the equation

$5 \times 9 + p \times 3 + 6 = 0 \Rightarrow p = -17$

from eq(i)

$5x^2 - 17x + 6 = 0$

Let second root = α

$3 + \alpha = \frac{17}{5} \Rightarrow \alpha = \frac{2}{5}$

103. (D) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & -1 \\ 0 & 2 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 11 \\ -7 \end{bmatrix}$

Using elementary methed

Now, $[A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 3 & 2 & -1 & 11 \\ 0 & 2 & 5 & -7 \end{array}\right]$

$R_2 \to R_2 - 3R_1$

$\Rightarrow [A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -7 & 2 \\ 0 & 2 & 5 & -7 \end{array}\right]$

$R_3 \to R_3 - \frac{2}{5}R_2$

$\Rightarrow [A/B] = \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -7 & 2 \\ 0 & 0 & \frac{39}{5} & \frac{-39}{5} \end{array}\right]$

Now, $x - y + 2z = 3$ ...(i)

$5y - 7z = 2$ ...(ii)

$\frac{39}{5}z = \frac{-39}{5} \Rightarrow z = -1$

from eq(i)

$5y + 7 = 2 \Rightarrow y = -1$

from eq(ii)

$x + 1 - 2 = 3 \Rightarrow x = 4$

Hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$

104. (C) $\sec(\pi \sin\theta) = \text{cosec}(\pi \cos\theta)$

$\Rightarrow \sec(\pi \sin\theta) = \sec\left[\frac{\pi}{2} - \pi\cos\theta\right]$

$\Rightarrow \pi \sin\theta = \frac{\pi}{2} - \pi\cos\theta$

$\Rightarrow \pi \sin\theta + \pi\cos\theta = \frac{\pi}{2}$

$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2}$

$\Rightarrow \sin\theta.\frac{1}{\sqrt{2}} + \cos\theta.\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin\theta.\cos\frac{\pi}{4} + \cos\theta.\sin\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

105. (B) $\sqrt{2-\sqrt{2-\sqrt{2-2\cos 8A}}}$

$\Rightarrow \sqrt{2-\sqrt{2-\sqrt{2\times 2\sin^2 4A}}}$

$\Rightarrow \sqrt{2-\sqrt{2-2\sin 4A}}$

$\Rightarrow \sqrt{2-\sqrt{2-2\cos\left(\frac{\pi}{2}-4A\right)}}$

$\Rightarrow \sqrt{2-\sqrt{2\times 2\sin^2\left(\frac{\pi}{4}-2A\right)}}$

$\Rightarrow \sqrt{2-2\sin\left(\frac{\pi}{4}-2A\right)}$

$\Rightarrow \sqrt{2-2\cos\left(\frac{\pi}{4}+2A\right)}$

$= \sqrt{2\times 2\sin^2\left(\frac{\pi}{8}+A\right)} = 2\sin\left(\frac{\pi}{8}+A\right)$

106. (C) I = $\int_{\pi/4}^{\pi/2} \frac{\tan^3 x}{1+\tan^4 x}$

I = 0 [∵ Function is odd.]

107. (B) Data 3, 4, 6, 8, 10,12,14,16,18 ; n = 9

$\sum_{i=0}^{n} x_i$ = 2 + 4 +6 +8 +10+12+14+16+18=90

$\sum_{i=0}^{n} x_i^2$ =2^2+ 4^2+ 6^2+ 8^2+10^2+12^2 +14^2+ 16^2 +18^2 = 1140

S.D. (σ) = $\sqrt{\frac{\sum_{i=0}^{n} x_i^2}{n} - \left(\frac{\sum_{i=0}^{n} x_i}{n}\right)^2}$

S.D. (σ)= $\sqrt{\frac{1140}{9} - \left(\frac{90}{9}\right)^2}$

S.D. (σ)= $\sqrt{\frac{10260-8100}{9^2}}$

S.D. (σ)= $\sqrt{\frac{2160}{81}}$

Now, variance = $(\text{S.D.})^2$

$\Rightarrow$ variance $=\left(\sqrt{\frac{2160}{81}}\right)^2 = \frac{2160}{81} = \frac{80}{3} = 26\frac{2}{3}$

108. (D) Given that $\int_{-3}^{0} f(x)\, dx = 8$

Now, $\int_{-1}^{0} [6 + f(x)]\, dx = 16$

$\Rightarrow 6\int_{-1}^{0} dx + \int_{-1}^{0} f(x)\, dx = 16$

$\Rightarrow 6[x]_{-1}^{0} + \int_{-1}^{0} f(x)dx = 16$

$\Rightarrow 6[0-(-1)] + \int_{-1}^{0} f(x)dx = 16$

$\Rightarrow 6 + \int_{-1}^{0} f(x)dx = 16 \Rightarrow \int_{-1}^{0} f(x)dx = 10$

Now, $\int_{-3}^{0} f(x)\, dx = \int_{-3}^{-1} f(x)\, dx + \int_{-1}^{0} f(x)dx$

$\Rightarrow 8 = \int_{-3}^{-1} f(x)\, dx + 10$

$\Rightarrow \int_{-3}^{-1} f(x)\, dx = -2$

109. (A) Equation

$5x^2 + 6x + 1 = 0$

Roots are α and β, then

$\alpha + \beta = \frac{-6}{5}$ and $\alpha\beta = \frac{1}{5}$

For new quadratic equation

Sum of roots = $\alpha - 2\beta + 2\alpha - \beta$

$= 3(\alpha - \beta)$

$= 3\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= 3\sqrt{\left(\frac{-6}{5}\right)^2 - 4\times\frac{1}{5}}$

$= 3\sqrt{\frac{36}{25} - \frac{4}{5}}$

$= 3 \times \frac{4}{5} = \frac{12}{5}$

Product of equation = $(\alpha - 2\beta)(2\alpha - \beta)$

$= 2\alpha^2 - 4\alpha\beta - \alpha\beta + 2\beta^2$

$= 2(\alpha^2 + \beta^2) - 5\alpha\beta$

$= 2[(\alpha+\beta)^2 - 2\alpha\beta] - 5\alpha\beta$

$= 2\left[\frac{36}{25} - \frac{2}{5}\right] - 5 \times \frac{1}{5}$

$= 2\left[\frac{36-10}{25}\right] - 1$

$= \frac{52}{25} - 1 = \frac{27}{25}$

The required equation

$x^2 - \frac{12}{5}x + \frac{27}{25} = 0$

$\Rightarrow 25x^2 - 60x + 27 = 0$

110. (A) One leap year = 366 days

= 52 weeks and 2 days

The required Probability = $\frac{2}{7}$

111. (B) $(A \cap B \cap \overline{C})$

112. (C) $\lim_{x\to 0^+} \frac{e^{1/x} - 1}{e^{1/x} + 1} = \lim_{h\to 0} \frac{e^{1/h} - 1}{e^{1/h} + 1}$

$= \lim_{h\to 0} \frac{e^{1/h}(1 - e^{-1/h})}{e^{1/h}(1 + e^{-1/h})}$

$= \lim_{h\to 0} \frac{1 - e^{-1/h}}{1 + e^{-1/h}}$

$= \frac{1-0}{1+0} = 1$

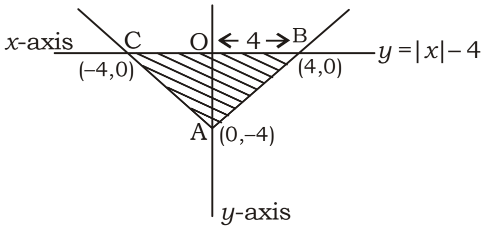
113. (C) Differential equation

$\left(\frac{d^2y}{dx^2} + y\right)^{3/4} = \frac{d^3y}{dx^3} + \frac{dy}{dx}$

$\Rightarrow \left(\frac{d^2y}{dx^2} + y\right)^3 = \left(\frac{d^3y}{dx^3} + \frac{dy}{dx}\right)^4$

Order = 3 and Degree = 4

114. (B)



The required Area = 2 × Area of ΔOAB

$= 2 \times \frac{1}{2} \times OA \times OB$

$= 2 \times \frac{1}{2} \times 4 \times 4 = 16$ sq. unit

115. (C) $x = a\cos\theta$

$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta$

and $y = a(\theta - \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a(1 + \sin\theta)$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = a(1 + \sin\theta) \times \frac{1}{-a\sin\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-(1+\sin\theta)}{\sin\theta}$

Slope at $\left(\theta = \frac{\pi}{2}\right) = \frac{-\left(1+\sin\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = -2$

Point$[a\cos\theta,\ a(\theta - \cos\theta)]$

$= \left[a\cos\frac{\pi}{2}, a\left(\frac{\pi}{2} - \cos\frac{\pi}{2}\right)\right] = \left[0, \frac{\pi a}{2}\right]$

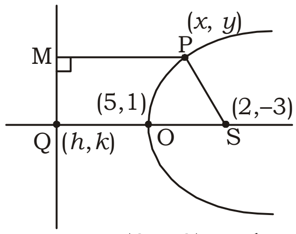
Equation of tangent at $\left(0, \frac{\pi a}{2}\right)$

$y - \frac{\pi a}{2} = -2(x - 0)$

$\Rightarrow y - \frac{\pi a}{2} = -2x$

$\Rightarrow 2x + y = \frac{\pi a}{2} \Rightarrow 4x + 2y = \pi a$

116. (A)



Focus at (2, –3) and vertex at (5,1)

Let co-ordinate of Q = (h, k)

Here O is mid-point of Q and S.

then, $\frac{h+2}{2} = 5 \Rightarrow h = 8$

and $\frac{k-3}{2} = 1 \Rightarrow k = 5$

Hence $Q(h, k) = (8, 5)$

slope of line QS $= \frac{-3-1}{2-5} = \frac{-4}{-3} = \frac{4}{3}$

slope of line QM $= \frac{-1}{4/3} = \frac{-3}{4}$

equation of directrix QM

$y - 5 = \frac{-3}{4}(x - 8)$

$\Rightarrow 4y - 20 = -3x + 24$

$\Rightarrow 3x + 4y - 44 = 0$

Condition of parabola

PS = PM

$\sqrt{(x-2)^2 + (y+3)^2} = \frac{3x + 4y - 44}{\sqrt{3^2 + 4^2}}$

On solving

$\Rightarrow 16x^2 + 9y^2 - 24xy + 164x + 502y = 1611$

117. (D) $\vec{a} = 4\hat{i} + 7\hat{j} - 9\hat{k}$ and $\vec{b} = 6\hat{i} - 6\hat{j} - 3\hat{k}$

Projection of $\vec{a}$ or $\vec{b}$ $= \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \frac{4 \times 6 + 7 \times (-6) - 9 \times (-3)}{\sqrt{6^2 + (-6)^2 + (-3)^2}}$

$= \frac{9}{\sqrt{81}} = 1$

118. (A)

119. (B) $I = \int e^x (2 + \tan x)\sqrt{\sec x}\ dx$

$I = 2\int e^x \left(\sqrt{\sec x} + \frac{\sqrt{\sec x}.\tan x}{2}\right) dx$

$I = 2\ e^x \sqrt{\sec x} + c$

$\left[\because \int e^x [f(x) + f'(x)dx = e^x f(x) + c\right]$

120. (C)

NDA (MATHS) MOCK TEST - 130 (Answer Key)

1.	(A)	21.	(D)	41.	(C)	61.	(C)	81.	(B)	101.	(A)
2.	(B)	22.	(C)	42.	(C)	62.	(A)	82.	(D)	102.	(C)
3.	(B)	23.	(D)	43.	(D)	63.	(C)	83.	(D)	103.	(D)
4.	(C)	24.	(A)	44.	(A)	64.	(A)	84.	(A)	104.	(C)
5.	(C)	25.	(C)	45.	(C)	65.	(C)	85.	(D)	105.	(B)
6.	(D)	26.	(D)	46.	(B)	66.	(B)	86.	(B)	106.	(C)
7.	(A)	27.	(A)	47.	(C)	67.	(C)	87.	(D)	107.	(B)
8.	(C)	28.	(B)	48.	(A)	68.	(A)	88.	(C)	108.	(D)
9.	(B)	29.	(C)	49.	(C)	69.	(C)	89.	(D)	109.	(A)
10.	(A)	30.	(A)	50.	(A)	70.	(C)	90.	(C)	110.	(A)
11.	(D)	31.	(B)	51.	(B)	71.	(B)	91.	(B)	111.	(B)
12.	(B)	32.	(C)	52.	(A)	72.	(D)	92.	(C)	112.	(C)
13.	(C)	33.	(B)	53.	(D)	73.	(B)	93.	(B)	113.	(C)
14.	(C)	34.	(B)	54.	(A)	74.	(A)	94.	(C)	114.	(B)
15.	(B)	35.	(C)	55.	(A)	75.	(C)	95.	(D)	115.	(C)
16.	(C)	36.	(B)	56.	(C)	76.	(A)	96.	(C)	116.	(A)
17.	(D)	37.	(C)	57.	(B)	77.	(B)	97.	(B)	117.	(D)
18.	(B)	38.	(A)	58.	(A)	78.	(A)	98.	(A)	118.	(A)
19.	(C)	39.	(A)	59.	(D)	79.	(B)	99.	(D)	119.	(B)
20.	(B)	40.	(C)	60.	(B)	80.	(C)	100.	(B)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777