NDA MATHS MOCK TEST - 132 (SOLUTION)

1. (B) Plane $3x - 2y + 6z = 48$

$$\Rightarrow \frac{3x}{48} - \frac{2y}{48} + \frac{6z}{48} = 1$$

$$\Rightarrow \frac{x}{16} + \frac{y}{-24} + \frac{z}{8} = 1$$

Intercept of the plane = $\langle 16, -24, 8 \rangle$

2. (C) Differential equation

$$\frac{dy}{dx} = \cos(x + y)$$

Let $x + y = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\Rightarrow \frac{dt}{dx} - 1 = \cos t$$

$$\Rightarrow \frac{dt}{dx} = 1 + \cos t$$

$$\Rightarrow \frac{dt}{1+\cos t} = dx$$

$$\Rightarrow \frac{dt}{2\cos^2 \frac{t}{2}} = dx$$

$$\Rightarrow \frac{1}{2}\sec^2\frac{t}{2}\, dt = dx$$

On intergrating

$$\Rightarrow \frac{1}{2} \times \frac{\tan\frac{t}{2}}{\frac{1}{2}} = x + c$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right) = x + c$$

3. (B) Total students = 11

The required no. of ways = (11– 1)!
= 10!

4. (D) Angle between the regression lines = $\frac{\pi}{2}$

$$\text{Now, } \tan\theta = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow \tan\frac{\pi}{2} = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow \frac{1}{0} = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\right)$$

$$\Rightarrow r\left(\sigma_x^2+\sigma_y^2\right) = 0 \Rightarrow r = 0$$

5. (C) Let $y = \sqrt{\cot x^4}$ and $z = x^2$

$$\Rightarrow y = \sqrt{\cot z^2}$$

On differentiating both side w.r.t. 'z'

$$\Rightarrow \frac{dy}{dz} = \frac{-\operatorname{cosec}^2 z^2}{2\sqrt{\cot z^2}}\,(2z)$$

$$\Rightarrow \frac{dy}{dz} = \frac{-z\operatorname{cosec}^2 z^2}{\sqrt{\cot z^2}}$$

$$\Rightarrow \frac{dy}{dz} = \frac{-x^2.\operatorname{cosec}^2 x^4}{\sqrt{\cot x^4}}$$

6. (A) Given that $f(x) = x^2 - 2x + 3$ ...(i)

$f'(x) = 2x - 2$

$a = 0,\ b = \frac{3}{4}$

$f(a) \Rightarrow f(0) = 3$

$$f(b) \Rightarrow f\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^2 - 2\times\frac{3}{4} + 3 = \frac{33}{16}$$

$$\text{Now, } f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow 2c - 2 = \frac{\frac{33}{16} - 3}{\frac{3}{4} - 0} \quad \text{[from eq(i)]}$$

$$\Rightarrow 2c - 2 = \frac{\frac{-15}{16}}{\frac{3}{4}}$$

$$\Rightarrow 2c - 2 = \frac{-5}{4} \Rightarrow c = \frac{3}{8}$$

7. (A) Differential equation

$$x\frac{dy}{dx} + y = \ln x$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{\ln x}{x}$$

On comparing with general equation

$$P = \frac{1}{x},\ Q = \frac{\ln x}{x}$$

$$\text{I.F.} = e^{\int P\, dx}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx}$$

$$\text{I.F.} = e^{\ln x} = x$$

Solution of diferential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.}\ dx$$

$$\Rightarrow y \times x = \int \frac{\ln x}{x} \times x\, dx$$

$$\Rightarrow xy = \int \ln x\ dx$$

$\Rightarrow xy = \ln x \int 1\,dx - \int \left\{ \frac{d}{dx}(\ln x) \int 1\,dx \right\} dx + c$

$\Rightarrow xy = (\ln x).\ x - \int \frac{1}{x} \times x\,dx + c$

$\Rightarrow xy = x\ln x - x + c$

8. (A) Given that $f(x)$ $|4x - 5|$ and $g(x) = x^2 + 1$

Now, $fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f[x^2 + 1]$

$\Rightarrow fog(x) = |4(x^2 + 1) - 5|$

$\Rightarrow fog(-1) = |4[(-1)^2 + 1] - 5|$

$\Rightarrow fog(-1) = |4 \times 2 - 5| = 3$

9. (B) $I = \int \left(x - \frac{1}{x^3} \right) e^{x + \frac{1}{x}}\,dx$

$I = \int \left(x + \frac{1}{x} \right) \left(1 - \frac{1}{x^2} \right) e^{x + \frac{1}{x}}\,dx$

Let $x + \frac{1}{x} = t$

$\left(1 - \frac{1}{x^2} \right) dx = dt$

$I = \int t.e^t dt$

$I = t.\int e^t dt - \int \left\{ \frac{d}{dt}(t).\int e^t dt \right\} + c$

$I = t.e^t - \int 1.e^t dt + c$

$I = t.e^t - e^t + c$

$I = e^t(t - 1) + c$

$I = e^{\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 1\right) + c}$

10. (C) $A = \begin{bmatrix} 2 & 0 & 8 \\ -1 & 2 & 3 \\ 2 & -1 & 5 \end{bmatrix}$

$[A/I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 8 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 \end{array}\right]$

$R_2 \to R_2 + \frac{1}{2} R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow [A/I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 8 & 1 & 0 & 0 \\ 0 & 2 & 7 & \frac{1}{2} & 1 & 0 \\ 0 & -1 & -3 & -1 & 0 & 1 \end{array}\right]$

$R_3 \to R_3 + \frac{1}{2} R_2$

$\Rightarrow [A/I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 8 & 1 & 0 & 0 \\ 0 & 2 & 7 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{-3}{4} & \frac{1}{2} & 1 \end{array}\right]$

$R_3 \to 2R_3$

$\Rightarrow [A/I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 8 & 1 & 0 & 0 \\ 0 & 2 & 7 & 1/2 & 1 & 0 \\ 0 & 0 & 1 & \frac{-3}{2} & 1 & 2 \end{array}\right]$

$R_2 \to R_2 - 7R_3$ and $R_1 \to R_1 - 8R_3$

$\Rightarrow [A/I] = \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 13 & -8 & -16 \\ 0 & 2 & 0 & 11 & -6 & -14 \\ 0 & 0 & 1 & \frac{-3}{2} & 1 & 2 \end{array}\right]$

$R_1 \to \frac{R_1}{2}$ and $R_2 \to \frac{R_2}{2}$

$\Rightarrow [A/I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{13}{2} & -4 & -8 \\ 0 & 1 & 0 & \frac{11}{2} & -3 & -7 \\ 0 & 0 & 1 & \frac{-3}{2} & 1 & 2 \end{array}\right]$

Hence $= A^{-1} = \begin{bmatrix} \frac{13}{2} & -4 & -8 \\ \frac{11}{2} & -3 & -7 \\ \frac{-3}{2} & 1 & 2 \end{bmatrix}$

11. (C) Plane $4x + 7y - 4z + 6 = 0$

Perpendicular distance from a point $(-1, 2, -4)$

$D = \frac{4 \times (-1) + 7 \times 2 - 4 \times (-4) + 6}{\sqrt{4^2 + 7^2 + (-4)^2}}$

$D = \frac{-4 + 14 + 16 + 6}{\sqrt{81}} = \frac{32}{9}$

12. (D) $f(x) = x^3 + 4x^2 + 4x - 1$

$f'(x) = 3x^2 + 8x + 4$

$f''(x) = 6x + 8$ ...(i)

for maxima and minima

$f'(x) = 0$

$\Rightarrow 3x^2 + 8x + 4 = 0$

$\Rightarrow (3x + 2)(x + 2) = 0$

$\Rightarrow x = \frac{-2}{3}, -2$

from eq(i)

$f''\left(-\frac{2}{3}\right) = 6\left(-\frac{2}{3}\right) + 8 = 4$ (minima)

$f''(-2) = 6(-2) + 8 = -4$ (maxima)

Hence function attains maximum value at $x = -2$.

13. (C)

14. (A) Given that $e = \frac{1}{4}$

and $\frac{a}{e} = 8$

$\Rightarrow \frac{a \times 4}{1} = 8 \Rightarrow a = 2$

Now, $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{16} = 1 - \frac{b^2}{4}$

$\Rightarrow \frac{15}{16} = \frac{b^2}{4} \Rightarrow b^2 = \frac{15}{4}$

equation of ellipse

$\frac{x^2}{4} + \frac{y^2}{15/4} = 1$

$\Rightarrow 15x^2 + 16y^2 = 60$

On differentiating

$\Rightarrow 15 \times 2x + 16 \times 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-15x}{16y}$

Slope at $\left(1, \frac{3\sqrt{5}}{4}\right) = \frac{-15 \times 1 \times 4}{16 \times 3\sqrt{5}} = \frac{-\sqrt{5}}{4}$

equation of tangent at point $\left(1, \frac{3\sqrt{5}}{4}\right)$

$y - \frac{3\sqrt{5}}{4} = \frac{-\sqrt{5}}{4}(x - 1)$

$\Rightarrow 4y - 3\sqrt{5} = -\sqrt{5}\,x + \sqrt{5}$

$\Rightarrow \sqrt{5}\,x + 4y = 4\sqrt{5}$

15. (A) Only I

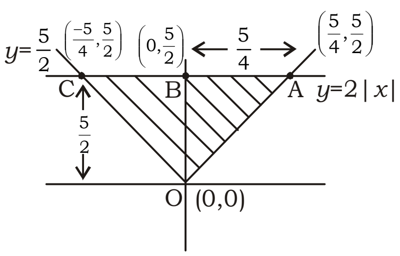
16. (C) $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\left(\frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}}\right)$

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{3}{4}$

$\Rightarrow \tan^{-1}\frac{4}{3} + \cot^{-1}\frac{4}{3} = \frac{\pi}{2}$

17. (C)



The required Area = 2 × Area of ΔOAB

$= 2 \times \frac{1}{2} \times \frac{5}{2} \times \frac{5}{4} = \frac{25}{8}$ sq.unit

18. (B) $f(x) = \begin{cases} x^2 - k, & x > 2 \\ 8x + 2, & x \le 2 \end{cases}$ is continuous at $x = 2$,

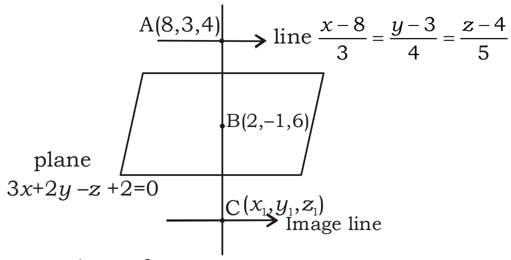
then $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow \lim_{x \to 2^-}(8x + 2) = \lim_{x \to 2^+}(x^2 - k)$

$\Rightarrow 8 + 2 + 2 = 2^2 - k$

$\Rightarrow 18 = 4 - k \Rightarrow k = -14$

19. (B)



equation of AB

$\frac{x-8}{3} = \frac{y-3}{2} = \frac{z-4}{-1} = \lambda$

Co-ordinate of point B

$x = 3\lambda + 8,\ y = 2\lambda + 3,\ z = -\lambda + 4$

Point satisfy the equation of plane

$3(3\lambda + 8) + 2(2\lambda + 3) - (-\lambda + 4) + 2 = 0$

$\Rightarrow 9\lambda + 24 + 4\lambda + 6 + \lambda - 4 + 2 = 0$

$\Rightarrow 14\lambda + 28 = 0 \Rightarrow \lambda = -2$

Co-ordinate of B = (2, –1, 6)

Let Co-ordinate of C = (x_1, y_1, z_1)

Now, $\frac{x_1 + 8}{2} = 2 \Rightarrow x_1 = -4$

$\frac{y_1 + 3}{2} = -1 \Rightarrow y_1 = -5$

$\frac{z_1 + 4}{2} = 6 \Rightarrow z_1 = 8$

Co-ordinate of C = (–4, –5, 8)

equation of line passing through the point C

$\frac{x+4}{3} = \frac{y+5}{4} = \frac{z-8}{5}$

20. (C) $\lim_{x \to 0} \frac{\sin 2x}{\tan x}$

$\Rightarrow \lim_{x \to 0} \frac{\sin 2x}{2x} \times \frac{x}{\tan x} \times \frac{2x}{x} = 2$

21. (C) Area of triangle $= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ a & -3 & 1 \\ 6 & 10 & 1 \end{vmatrix}$

$\Rightarrow 24 = \frac{1}{2}[3(-3-10) - 1(a-6) + 1(10a+18)]$

$\Rightarrow 48 = -39 - a + 6 + 10a + 18$

$\Rightarrow 48 = 9a - 15 \Rightarrow a = 7$

22. (C) $\dfrac{[1+(i^7)^{4n-1}]^{4n+1}}{[1+(i^3)^{4n+1}]^{4n-1}}$

$\Rightarrow \dfrac{[1+(-i)^{4n-1}]^{4n+1}}{[1+(-i)^{4n+1}]^{4n-1}}$

$\Rightarrow \dfrac{[1+(-i)^{4n}(-i)^{-1}]^{4n+1}}{[1+(-i)^{4n}(-i)^{-1}]^{4n-1}}$

$\Rightarrow \dfrac{\left[1-\frac{1}{i}\right]^{4n+1}}{[1-i]^{4n-1}}$

$\Rightarrow \dfrac{[1+i]^{4n+1}}{[1-i]^{4n-1}}$

$\Rightarrow \dfrac{(1+i)^{4n}(1+i)}{(1-i)^{4n}(1-i)^{-1}}$

$\Rightarrow \left[\dfrac{1+i}{1-i}\right]^{4n}$ $(1+i)(1-i)$

$\Rightarrow \left(\dfrac{(1+i)^2}{2}\right)^{4n}$ $(1 - i^2)$

$\Rightarrow (i)^{4n} \times 2 = 2$

23. (A) **Statement I**

$\sec\theta = x$

Now, $x - \dfrac{1}{x} = \sec\theta - \dfrac{1}{\sec\theta}$

$\Rightarrow x - \dfrac{1}{x} = \dfrac{1}{\cos\theta} - \cos\theta$

$\Rightarrow x - \dfrac{1}{x} = \dfrac{1-\cos^2\theta}{\cos\theta}$

$\Rightarrow x - \dfrac{1}{x} = \dfrac{\sin^2\theta}{\cos\theta}$

$\Rightarrow x - \dfrac{1}{x} = \sec\theta.\tan\theta$

Statement I is correct.

Statement II

$x + \dfrac{1}{x} = 2\sin\theta$

On squaring both side

$\Rightarrow x^2 + \dfrac{1}{x^2} + 2 = 4\sin^2\theta$

$\Rightarrow x^2 + \dfrac{1}{x^2} = 4\sin^2\theta - 2$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2(1 - 2\sin^2\theta)$

$\Rightarrow x^2 + \dfrac{1}{x^2} = -2\cos 2\theta$

Statement II is incorrect.

Statement III

$x = a\sin\theta$ and $y = b\cos\theta$

Now, $b^2x^2 + a^2y^2 = b^2a^2\sin^2\theta + a^2b^2\cos^2\theta$

$\Rightarrow b^2x^2 + a^2y^2 = b^2a^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow b^2x^2 + a^2y^2 = a^2b^2$

Statement III is correct.

24. (D) In the expansion of $\left(3x - \dfrac{1}{6x}\right)^{11}$

$T_{r+1} = {}^{11}C_r\,(3x)^{11-r}\left(\dfrac{-1}{6x}\right)^r$

$T_{r+1} = {}^{11}C_r\,3^{11-2r}\,x^{11-2r}\,(-2)^{-r}$

here $11 - 2r = 3 \Rightarrow r = 4$

Coefficient of $x^3 = {}^{11}C_4\,3^3\,(-2)^{-4}$

$= \dfrac{11!}{4!7!} \times \dfrac{27}{16}$

$= 110 \times 3 \times \dfrac{27}{16}$

$= \dfrac{55\times 81}{8} = \dfrac{4455}{8}$

25. (B) $\left[\dfrac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right]^3$

$\Rightarrow \left[\dfrac{2\cos^2\frac{\theta}{2}+i\times 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}-i\times 2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right]^3$

$\Rightarrow \dfrac{\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right]^3}{\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)\right]^3}$

$\Rightarrow \dfrac{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^3}{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^{-3}}$

$\Rightarrow \left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^6$

$\Rightarrow \cos\left(6\times\frac{\theta}{2}\right)+i\sin\left(6\times\frac{\theta}{2}\right)$

$\Rightarrow \cos 3\theta + i\sin 3\theta$

26. (C) Let $a - ib = \sqrt{5-12i}$

On squaring both side

$\Rightarrow (a^2 - b^2) - 2abi = 5 - 12i$

On comparing

$\Rightarrow a^2 - b^2 = 5$ and $2ab = 12$ …(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 5^2 + 12^2$

$\Rightarrow (a^2 + b^2)^2 = 169$

$\Rightarrow a^2 + b^2 = 13$ …(ii)

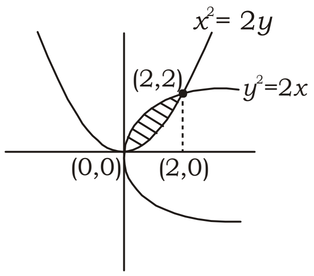
from eq(i) and eq(ii)

$2a^2 = 18,\ 2b^2 = 8$

$\Rightarrow a = \pm 3,\ b = \pm 2$

Hence $\sqrt{5-12i} = \pm(3-2i)$

27. (C)



Curves

$y_1 \Rightarrow y^2 = 2x$ and $y_2 \Rightarrow x^2 = 2y$

Area $= \int_0^2 (y_1 - y_2)\ dx$

Area $= \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right) dx$

Area $= \left[\sqrt{2} \times \frac{x^{3/2}}{3/2} - \frac{1}{2} \times \frac{x^3}{3}\right]_0^2$

Area $= \frac{2\sqrt{2}}{3} \times 2^{3/2} - \frac{1}{6} \times 8 - 0 + 0$

Area $= \frac{8}{3} - \frac{8}{6} = \frac{8}{6} = \frac{4}{3}$ sq.unit

28. (B) $\cos\left[\sin^{-1}\left(\sin\left(\frac{17\pi}{4}\right)\right)\right]$

$\Rightarrow \cos\left[\sin^{-1}\left(\sin\left(4\pi + \frac{\pi}{4}\right)\right)\right]$

$\Rightarrow \cos\left[\sin^{-1}\left(\sin\frac{\pi}{4}\right)\right]$

$\Rightarrow \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

29. (A) A.T.Q.

$2b = \frac{3}{4} \times 2a \Rightarrow b = \frac{3}{4}a$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 + \frac{9a^2}{16a^2}}$

$\Rightarrow e = \sqrt{\frac{25}{16}} \Rightarrow e = \frac{5}{4}$

30. (A) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

On putting $x = \tan\theta$

$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow y = \sin^{-1}(\sin 2\theta)$

$\Rightarrow y = 2\theta$

On differentiating both side w.r.t. 'θ'

$\Rightarrow \frac{dy}{d\theta} = 2$

and $z = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

On putting $x = \tan\theta$

$\Rightarrow z = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow z = \cos^{-1}(\cos 2\theta)$

$\Rightarrow z = 2\theta$

On differentiating both side w.r.t.'θ'

$\Rightarrow \frac{dz}{d\theta} = 2$

Now, $\frac{dy}{dz} = \frac{dy}{d\theta} \times \frac{d\theta}{dz}$

$\Rightarrow \frac{dy}{dz} = 2 \times \frac{1}{2} = 1$

31. (B) $\cos^2 31\frac{1}{2} + \cos^2 58\frac{1}{2}$

$\Rightarrow \cos^2 31\frac{1}{2} + \cos^2\left(90 - 31\frac{1}{2}\right)$

$\Rightarrow \cos^2 31\frac{1}{2} + \sin^2 31\frac{1}{2} = 1$

32. (D) Let $y = 3^{65}$

taking log both side

$\Rightarrow \log_{10}y = 65 \log_{10}3$

$\Rightarrow \log_{10}y = 65 \times 0.4771$

$\Rightarrow \log_{10}y = 31.0115$

The required no. of digits = 31 +1 = 32

33. (C) A' = co-factor A

|A'| = |co-factor of A|

$|A'| = (A)^5 - 1$ [∵ Order = 5]

$|A'| = A^4$

34. (B) Order = 3, Degree = 3

35. (C) Marks 54, 56, 71, 72, 84, 86, 91, 92, 63, 69, 70, 75, 84, 88, 85

Mean $= \frac{1140}{15} = 76$

The required number of students = 7

36. (D)

37. (C) $\begin{vmatrix} a^2 & 1-a & a^2-a \\ b^2 & 1-b & b^2-b \\ c^2 & 1-c & c^2-c \end{vmatrix}$

$C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} a^2 & 1-a & -a \\ b^2 & 1-b & -b \\ c^2 & 1-c & -c \end{vmatrix}$

$C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} a^2 & 1 & -a \\ b^2 & 1 & -b \\ c^2 & 1 & -c \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} a^2 & 1 & -a \\ b^2-a^2 & 0 & a-b \\ c^2-a^2 & 0 & a-c \end{vmatrix}$

$\Rightarrow (b-a)(c-a)\begin{vmatrix} a^2 & 1 & -a \\ b+a & 0 & -1 \\ c+a & 0 & -1 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow (b-a)(c-a)\begin{vmatrix} a^2 & 1 & -a \\ b+a & 0 & -1 \\ c-b & 0 & 0 \end{vmatrix}$

$\Rightarrow (b-a)(c-a)[a^2 \times 0 - 1(0 + c - b) - a \times 0]$

$\Rightarrow -(b-a)(c-a)(c-b)$

$\Rightarrow (a-b)(b-c)(a-c)$

38. (A) $I = \int_0^{\pi/2} \frac{\delta(x)}{\delta(x)+\delta\left(\frac{\pi}{2}-x\right)}\,dx$ ...(i)

Prop.IV $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$I = \int_0^{\pi/2} \frac{\delta\left(\frac{\pi}{2}-x\right)}{\delta\left(\frac{\pi}{2}-x\right)+\delta(x)}\,dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \frac{\delta(x)+\delta\left(\frac{\pi}{2}-x\right)}{\delta\left(\frac{\pi}{2}-x\right)+\delta(x)}\,dx$

$2I = \int_0^{\pi/2} 1\,dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

39. (A) A.T.Q.

$\frac{n(n-3)}{2} = 14$

$\Rightarrow n^2 - 3n = 28$

$\Rightarrow n^2 - 3n - 28 = 0$

$\Rightarrow (n-7)(n+4) = 0$

$\Rightarrow n = 7, -4$

The required number of sides = 7

40. (D) We know that

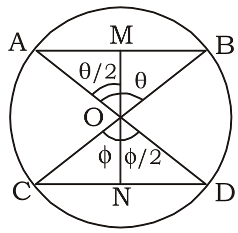
$T_p = q$, $T_q = p$, then $T_{p+q} = 0$

Similarly

$T_{56} = 321$, $T_{321} = 56$, then $T_{(56+321)} = 0$

The required term = 56 + 321 = 377

41. (A)



AD = 24 unit

AO = OD = 12 unit

Let $\angle AOB = \theta$ and $\angle COD = \phi$

Given that $\theta = \sin^{-1}\left(\frac{24}{25}\right)$ and $\phi = \cos^{-1}\left(\frac{7}{18}\right)$

$\Rightarrow \sin\theta = \frac{24}{25}$ and $\cos\phi = \frac{7}{18}$

$\Rightarrow \cos\theta = \frac{7}{25}$ and $2\cos^2\frac{\phi}{2} - 1 = \frac{7}{18}$

$\Rightarrow 2\cos^2\frac{\theta}{2} - 1 = \frac{7}{25}$ and $\cos\frac{\phi}{2} = \frac{5}{6}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{4}{5}$

In ΔOND :-

$\cos\frac{\phi}{2} = \frac{ON}{OD}$

$\Rightarrow \frac{5}{6} = \frac{ON}{12} \Rightarrow ON = 10$ unit

In ΔOMA:-

$\cos\frac{\theta}{2} = \frac{OM}{AO}$

$\Rightarrow \frac{4}{5} = \frac{OM}{12} \Rightarrow OM = \frac{48}{5}$ unit

The required distance = ON + OM

$= 10 + \frac{48}{5} = 19.6$ unit

42. (B)

4	3	2	1	4	5
T	R	E	A	T	Y
$\frac{3}{2!}$	$\frac{2}{1!}$	$\frac{1}{1!}$	$\frac{0}{1!}$	$\frac{0}{1!}$	$\frac{0}{1!}$
5!	4!	3!	2!	1!	0! +1

$5!\times\frac{3}{2!}+4!\times\frac{2}{1!}+3!\times\frac{1}{1!}+2!\times\frac{0}{1!}+1!\times\frac{0}{1!}+0!\times\frac{0}{1!}+1$

$\Rightarrow 180 + 48 + 6 + 0 + 0 + 0 + 1 \Rightarrow 235$

The required position = 235

43. (D) $\lim_{x\to4}\frac{\sqrt{6x+1}-5}{\sqrt{2x+1}-3}$ $\left[\frac{0}{0}\right]$ from

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to4}\frac{\frac{1\times6}{2\sqrt{6x+1}}-0}{\frac{1\times2}{2\sqrt{2x+1}}-0}$$

$$\Rightarrow \lim_{x\to4}\frac{6\sqrt{2x+1}}{2\sqrt{6x+1}}$$

$$\Rightarrow \frac{6\times3}{2\times5}=\frac{9}{5}$$

44. (A) A.T.Q.,

$$\frac{a+b}{2}=4\times\sqrt{ab}$$

$\Rightarrow a+b=8\sqrt{ab}$ ...(i)

Now, $(a-b)^2=(a+b)^2-4ab$

$\Rightarrow (a-b)^2=(8\sqrt{ab})^2-4ab$

$\Rightarrow (a-b)^2=64ab-4\ ab$

$\Rightarrow (a-b)^2=60ab$

$\Rightarrow a-b=2\ \sqrt{15}\ \sqrt{ab}$ ...(ii)

from eq(i) and eq(ii)

$$\frac{a+b}{a-b}=\frac{8\sqrt{ab}}{2\sqrt{15}\sqrt{ab}}$$

$$\Rightarrow \frac{a+b}{a-b}=\frac{4}{\sqrt{15}}$$

45. (C) $1-\frac{\cos^2 x}{1+\sin x}+\frac{1+\sin x}{\cos x}-\frac{\cos x}{1-\sin x}$

$$\Rightarrow \frac{1+\sin x-\cos^2 x}{1+\sin x}+\frac{(1+\sin x)(1-\sin x)-\cos^2 x}{(1-\sin x)\cos x}$$

$$\Rightarrow \frac{\sin^2 x+\sin x}{1+\sin x}+\frac{1-\sin^2 x-\cos^2 x}{(1-\sin x)\cos x}$$

$$\Rightarrow \frac{\sin x(1+\sin x)}{1+\sin x}+\frac{\cos^2 x-\cos^2 x}{(1-\sin x)\cos x}$$

$\Rightarrow \sin x + 0 = \sin x$

46. (D) $\frac{\cos\theta}{1+\sin\theta+\cos\theta}=x$

$$\Rightarrow \frac{\cos\theta}{1+\sin\theta+\cos\theta}\times\frac{1-\sin\theta-\cos\theta}{1-\sin\theta-\cos\theta}=x$$

$$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{1-(\sin\theta+\cos\theta)^2}=x$$

$$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{1-(1+2\sin\theta.\cos\theta)}=x$$

$$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{-2\sin\theta.\cos\theta}=x$$

$$\Rightarrow \frac{1-\sin\theta-\cos\theta}{-2\sin\theta}=x$$

$$\Rightarrow \frac{1-\sin\theta-\cos\theta}{\sin\theta}=-2x$$

47. (A) $\begin{vmatrix}5! & 4! & 3!\\4! & 3! & 2!\\6! & 5! & 4!\end{vmatrix}$

$$\Rightarrow \begin{vmatrix}5\times4\times3! & 4\times3! & 3!\\4\times3\times2! & 3\times2! & 2!\\6\times5\times4! & 5\times4! & 4!\end{vmatrix}$$

$$\Rightarrow 2!\times3!\times4!\begin{vmatrix}20 & 4 & 1\\12 & 3 & 1\\30 & 5 & 1\end{vmatrix}$$

$R_2\to R_2-R_1$ and $R_3\to R_3-R_1$

$$\Rightarrow 2!\times3!\times4!\begin{vmatrix}20 & 4 & 1\\-8 & -1 & 0\\10 & 1 & 0\end{vmatrix}$$

$R_3\to R_3+R_2$

$$\Rightarrow 2!\times3!\times4!\begin{vmatrix}20 & 4 & 1\\-8 & -1 & 0\\2 & 0 & 0\end{vmatrix}$$

$\Rightarrow 2!\times3!\times4!\ [20\times0-4\times0+1\ (0+2)]$

$\Rightarrow 2\times2!\times3!\times4! = 576$

48. (B) Angle describe in 12 hr by hour-hand = 360°

Angle describe in 1 hr(60 min) by hour-hand = $\frac{360}{12}$

Angle decribe in 1 min by hour-hand = $\frac{360}{12\times60}$

Angle describe in 24 min by hour-hand = $\frac{360}{12\times60}\times24=12°$

49. (A) **In ΔABC:-**

$a = 2,\ b = \sqrt{2}$ and C = 45°

Now, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos 45° = \frac{4+2-c^2}{2\times 2\times\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{6-c^2}{4\sqrt{2}}$

$\Rightarrow 4 = 6 - c^2 \Rightarrow c = \sqrt{2}$

50. (A)

51. (B) Plane $2x + y - 2z + 6 = 0$ and

line $\frac{x-1}{4} = \frac{y+1}{7} = \frac{z-6}{-4}$

Let angle between plane and line = θ

Now, $\sin\theta = \frac{2\times4\times1\times7-2\times(-4)}{\sqrt{2^2+1^2+(-2)^2}\sqrt{4^2+7^2+(-4)^2}}$

$\Rightarrow \sin\theta = \frac{23}{3\times 9}$

$\Rightarrow \sin\theta = \frac{23}{27} \Rightarrow \theta = \sin^{-1}\left(\frac{23}{27}\right)$

52. (C) $9! \times C(17,9) = k \times P(18, 9)$

$\Rightarrow 9! \times \frac{17!}{9!8!} = k \times \frac{18!}{9!}$

$\Rightarrow \frac{17!}{8!} = k \times \frac{18\times17!}{9\times8!}$

$\Rightarrow 1 = k \times \frac{18}{9} \Rightarrow k = \frac{1}{2}$

53. (B) $\frac{\sqrt{3}}{\sin 225°} + \frac{1}{\cos 315°}$

$\Rightarrow \frac{\sqrt{3}}{\sin(180+45)} + \frac{1}{\cos(360-45)}$

$\Rightarrow \frac{\sqrt{3}}{-\sin 45} + \frac{1}{\cos 45}$

$\Rightarrow \frac{\sqrt{3}\times\sqrt{2}}{-1} + \frac{1\times\sqrt{2}}{1} \Rightarrow \sqrt{2}\,(1-\sqrt{3})$

54. (C) $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 5\hat{k}$

Now, $\vec{b} + 2\vec{a} = (2\hat{i} - \hat{j} + 5\hat{k}) + 2(2\hat{i} + \hat{j} - \hat{k})$

$\Rightarrow \vec{b} + 2\vec{a} = 6\hat{i} + \hat{j} + 3\hat{k}$

and $\vec{a} - 3\vec{b} = (2\hat{i} + \hat{j} - \hat{k}) - 3(2\hat{i} - \hat{j} + 5\hat{k})$

$\Rightarrow \vec{a} - 3\vec{b} = -4\hat{i} + 4\hat{j} - 16\hat{k}$

Now, $(\vec{b} + \vec{a}).(\vec{a} - 3\vec{b})$

$\Rightarrow (6\hat{i} + \hat{j} + 3\hat{k}).(4-4\hat{i} + \hat{j} - 16\hat{k})$

$\Rightarrow 6\times(-4)+1\times4 + 3\times(-16)$

$\Rightarrow -24 + 4 - 48 = -68$

55. (B) $y = \cot^{-1}\left[\frac{1+x}{1-x}\right]$

On putting $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow y = \cot^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$

$\Rightarrow y = \cot^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$

$\Rightarrow y = \cot^{-1}\left[\cot\left(\frac{\pi}{2}-\frac{\pi}{4}-\theta\right)\right]$

$\Rightarrow y = \frac{\pi}{4} - \theta$

$\Rightarrow y = \frac{\pi}{4} - \tan^{-1}x$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = \frac{-1}{1+x^2}$

56. (A) $I = \int \frac{dx}{x(x^7+1)}$

$I = \int \frac{x^6}{x^7(x^7+1)}\,dx$

Let $x^7 = t$

$7x^6dx = dt \Rightarrow x^6\,dx = \frac{1}{7}dt$

$I = \frac{1}{7}\int \frac{1}{t(t+1)}\,dt$

$I = \frac{1}{7}\int\left[\frac{1}{t} - \frac{1}{t+1}\right]dt$

$I = \frac{1}{7}[\log t - \log(t+1)] + c$

$I = \frac{1}{7}\log\left(\frac{t}{t+1}\right) + c$

$I = \frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right) + c$

57. (B) $\sec^{-1}(-2) = \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$

$\Rightarrow \sec^{-1}(-2) = \sec^{-1}\left[\sec\left(\pi - \frac{\pi}{3}\right)\right]$

$\Rightarrow \sec^{-1}(-2) = \sec^{-1}\left(\sec\frac{2\pi}{3}\right) = \frac{2\pi}{3}$

58. (A) Probability of Kapil's selection $P(K) = \frac{3}{4}$

and $P(\overline{K}) = 1 - \frac{3}{4} = \frac{1}{4}$

Probabilty of Tani's selection $P(T) = \frac{1}{5}$

and $P(\overline{T}) = 1 - \frac{1}{5} = \frac{4}{5}$

The required Probability $= \frac{3}{4} \times \frac{4}{5} \times \frac{1}{4} \times \frac{1}{5}$

$= \frac{3}{5} + \frac{1}{20} = \frac{13}{20}$

59. (C) We know that

Curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Area $= \frac{a^2}{b}$

Now, Curve $\sqrt{x} + \sqrt{y} = \sqrt{3}$

The required Area $= \frac{(3)^2}{6}$

$= \frac{9}{6} = \frac{3}{2}$ sq.unit

60. (B) $z = \frac{1+2i}{2-i} - \frac{1-3i}{3+i}$

$z = \frac{1+2i}{2-i} \times \frac{2+i}{2+i} - \frac{1-3i}{3+i} \times \frac{3-i}{3-i}$

$z = \frac{5i}{5} - \frac{-10i}{10}$

$z = i + i = 2i$

Now, $z^2 + z\overline{z} = (2i)^2 + 2i(-2i)$

$\Rightarrow z^2 + z\overline{z} = 4i^2 - 4i^2 = 0$

61. (C) $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots\ldots\ldots\infty}}}$

$\Rightarrow y = \sqrt{\tan x + y}$

On squaring both side

$\Rightarrow y^2 = \tan x + y$

On differentiating both side w.r.t. 'x'

$\Rightarrow 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}(2y - 1) = \sec^2 x$

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

62. (A) The required number of elementary events $= {}^9C_2 \times 2! = 72$

63. (C) $I = \int_0^3 x(3-x)^6\, dx$

Prop.IV $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$I = \int_0^3 (3-x)x^6\, dx$

$I = \int_0^3 3.x^6\, dx - \int_0^3 x^7\, dx$

$I = 3\left[\frac{x^7}{7}\right]_0^3 - \left[\frac{x^8}{8}\right]_0^3$

$I = 3\left[\frac{3^7}{7} - 0\right] - \left[\frac{3^8}{8} - 0\right]$

$I = \frac{3^7}{7} - \frac{3^8}{8}$

$I = 3^8 \times \frac{1}{56} = \frac{6561}{56}$

64. (B) $\dfrac{\sec(\pi+\theta).\tan\left(\frac{9\pi}{2}-\theta\right).\sec^2(2\pi-\theta)}{\tan(2\pi-\theta).\mathrm{cosec}^2(\pi-\theta).\sec\left(\frac{3\pi}{2}+\theta\right)}$

$\Rightarrow \dfrac{(-\sec\theta).\tan\left[4\pi+\left(\frac{\pi}{2}-\theta\right)\right].\sec^2\theta}{(-\tan\theta).\mathrm{cosec}^2\theta.\sec\left[2\pi-\left(\frac{\pi}{2}-\theta\right)\right]}$

$\Rightarrow \dfrac{\sec\theta.\tan\left(\frac{\pi}{2}-\theta\right).\sec^2\theta}{\tan\theta.\mathrm{cosec}^2\theta.\sec\left(\frac{\pi}{2}-\theta\right)}$

$\Rightarrow \dfrac{\sec\theta.\cot\theta.\sec^2\theta}{\tan\theta.\mathrm{cosec}^2\theta.\mathrm{cosec}\,\theta}$

$\Rightarrow \dfrac{\frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} \times \frac{1}{\cos^2\theta}}{\frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin^2\theta} \times \frac{1}{\sin\theta}} = \tan\theta$

65. (D) I = $\int \cot^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right) dx$

Let $x = \cos 2\theta$

$dx = -2\sin 2\theta \, d\theta$

I = $\int \cot^{-1}\left(\sqrt{\frac{1+\cos 2\theta}{1-\cos 2\theta}}\right).(-2\sin 2\theta).\, d\theta$

I = $-2\int \cot^{-1}\left(\sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}\right).\sin 2\theta \; d\theta$

I = $-2\int \cot^{-1}(\cot\theta).\sin 2\theta \, d\theta$

I = $-2\int \theta.\sin 2\theta . d\theta$

I = $-2\left[\theta.\int \sin 2\theta \, d\theta - \int\left\{\frac{d}{d\theta}(\theta).\int \sin 2\theta \, d\theta\right\} d\theta\right]$

I = $-2\left[-2\theta.\cos 2\theta - \int 1.(-2\cos 2\theta) d\theta\right] + c$

I = $-2\left[-2\theta.\cos 2\theta + 2\int \cos 2\theta . d\theta\right] + c$

I = $-2\left[-2\theta.\cos 2\theta + 2\times 2\sin 2\theta\right] + c$

I = $4\theta.\cos 2\theta - 8\sin 2\theta + c$

I = $2x.\cos^{-1}x - 8\sqrt{1-x^2} + c$

66. (D)

67. (B) A bag contains 7 black and 5 while

The requiredProbability

$= \frac{{}^5C_1 \times {}^7C_1 + {}^5C_2 \times {}^7C_0}{{}^{12}C_2}$

$= \frac{5\times 7 + 10\times 1}{66}$

$= \frac{45}{66} = \frac{15}{22}$

68. (D) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Now, $(\vec{a}.\hat{i})\hat{i} + (\vec{a}.\hat{j})\hat{j} + (\vec{a}.\hat{k})\hat{k}$

$\Rightarrow [(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).\hat{i}]\hat{i} + [(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).\hat{j}]\hat{j}$

$+ [(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).\hat{k}]\hat{k}$

$\Rightarrow a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$

69. (A)

70. (D) The required number of ways = 12^9

71. (B) Circle $x^2 + y^2 + 3x - 5y + 17 = 0$ ...(i)

equtión of circle which is concentric with circle (i)

$x^2 + y^2 + 3x - 5y + c = 0$ ...(ii)

its passes through the point (–1, 2)

$(-1)^2 + 2^2 + 3(-1) - 5\times 2 + c = 0$

$\Rightarrow 1 + 4 - 3 - 10 + c = 0 \Rightarrow c = 8$

from eq(ii)

$x^2 + y^2 + 3x - 5y + 8 = 0$

72. (C) $\cos^2\frac{\pi}{10} + \cos^2\frac{3\pi}{10} + \cos^2\frac{\pi}{5} + \cos^2\frac{2\pi}{5}$

$\Rightarrow \cos^2\left(\frac{\pi}{2}-\frac{2\pi}{5}\right) + \cos^2\left(\frac{\pi}{2}-\frac{\pi}{5}\right) + \cos^2\frac{\pi}{5} + \cos^2\frac{2\pi}{5}$

$\Rightarrow \sin^2\frac{2\pi}{5} + \sin^2\frac{\pi}{5} + \cos^2\frac{\pi}{5} + \cos^2\frac{2\pi}{5}$

$\Rightarrow \sin^2\frac{2\pi}{5} + \cos^2\frac{2\pi}{5} + \sin^2\frac{\pi}{5} + \cos^2\frac{\pi}{5}$

$\Rightarrow 1 + 1 = 2$

73. (A)

74. (B) $\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\times 2 + 4\times 0 & 2\times(-3) + 4\times 6 \\ -1\times 2 + 2\times 0 & -1\times(-3) + 2\times 6 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 4 & 18 \\ -2 & 15 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$

On comparing

$k = 18$

75. (B) Given that $b_{yx} = \frac{-16}{3}$ and $b_{xy} = \frac{-1}{2}$

Now, $r = \sqrt{b_{yx}\times b_{xy}}$

$\Rightarrow r = \sqrt{\left(\frac{-16}{3}\right)\times\left(\frac{-1}{12}\right)}$

$\Rightarrow r = \sqrt{\frac{16}{36}}$

$\Rightarrow r = \sqrt{\frac{4}{9}} \Rightarrow r = -\frac{2}{3}$

76. (C) $x = \sin\theta + \cos\theta$ and $y = \sin\theta.\cos\theta$

Now, $x^2 = (\sin\theta + \cos\theta)^2$

$\Rightarrow x^2 = 1 + 2\sin\theta.\cos\theta$

$\Rightarrow x^2 = 1 + 2\sin\theta.\cos\theta$

$\Rightarrow x^2 = 1 + 2y \Rightarrow x^2 - 2y - 1 = 0$

77. (C) Let point = (h, k, l)

A.T.Q.,

$\sqrt{(h-1)^2 + (k+2)^2 + (l-3)^2} = \sqrt{(h+3)^2 + k^2 + (l-5)^2}$

On solving

$\Rightarrow 8h - 4k - 4l + 20 = 0$

$\Rightarrow 2h - k - l + 5 = 0$

locus of a point

$\Rightarrow 2x - y - z + 5 = 0$

78. (D) Let $f(x) = \frac{x}{[x]}$

L.H.L. $= \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} \frac{2-h}{[2-h]}$

$= \lim_{h\to 0} \frac{2-h}{1} = 2$

R.H.L. $= \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} \frac{2+h}{[2+h]}$

$= \lim_{h\to 0} \frac{2+h}{2} = 1$

L.H.L. $\neq$ R.H.L.
Hence limit does not exist.

79. (A) Given that A $= \frac{19\pi}{3}$

$\tan A = \tan\frac{19\pi}{3}$

$\tan A = \tan\left(3\times 2\pi + \frac{\pi}{3}\right)$

$\tan A = \tan\frac{\pi}{3} = \sqrt{3}$

Now, $\frac{3-3\tan^2 A}{3\tan A + \tan^3 A} = \frac{3-3\times 3}{3\times\sqrt{3}+3\sqrt{3}}$

$\Rightarrow \frac{3-3\tan^2 A}{3\tan A+\tan^3 A} = \frac{-6}{6\sqrt{3}} = \frac{-1}{\sqrt{3}}$

80. (A) $10^{-x\sin x}\left[\frac{d}{dx}10^{x\sin x}\right]$

$\Rightarrow 10^{-x\sin x}\left[10^{x\sin x}\ln 10\{x\cos x + \sin x.1\}\right]$

$\Rightarrow (x\cos x + \sin x)\ln 10$

81. (C) **Statement I**
Now, $(\omega^{43}+1)^7 + \omega^2$
$\Rightarrow (\omega+1)^7 + \omega^2$ $[\because \omega^3 = 1]$
$\Rightarrow (-\omega^2)^7 + \omega^2$ $[\because 1+\omega+\omega^2 = 0]$
$\Rightarrow -\omega^{14} + \omega^2$
$\Rightarrow -\omega^2 + \omega^2 = 0$
Statement I is correct.
Statement II
Now, $(\omega^{142}+\omega^2)^3 + 1$
$\Rightarrow (\omega+\omega^2)^3 + 1$ $[\because \omega^3 = 1]$
$\Rightarrow (-1)^3 + 1 = -1 + 1 = 0$ $[\because 1+\omega+\omega^2 = 0]$
Statement II is correct.

82. (B) Points $(a, 0)$, $(at_1^2, 2at_1)$ and $(at_1^2, 2at_2)$ are collinear,

then $\begin{vmatrix} a & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = 0$

$\Rightarrow a\times 2a\begin{vmatrix} 1 & 0 & 1 \\ t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ t_1^2-1 & t_1 & 0 \\ t_2^2-1 & t_2 & 0 \end{vmatrix} = 0$

$\Rightarrow 1\times 0 + 0 + 1[\,t_1^2 t_2 - t_2 - t_1 t_2^2 + t_1] = 0$
$\Rightarrow t_1 t_2(t_1 - t_2) + 1(t_1 - t_2) = 0$
$\Rightarrow (t_1 - t_2)(t_1 t_2 + 1) = 0$
$\Rightarrow t_1 t_2 + 1 = 0,\ t_1 - t_2 \neq 0$
$\Rightarrow t_1 t_2 = -1$

83. (B) $\begin{vmatrix} \log_7 7 & \log_2 4 & 3 \\ 1 & \log_3 9 & 10 \\ \log_e e & 2 & 12 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & \log_2 2^2 & 3 \\ 1 & \log_3 3^2 & 10 \\ 1 & 2 & 12 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 10 \\ 1 & 2 & 12 \end{vmatrix}$

$\Rightarrow 2\begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & 1 & 12 \end{vmatrix} = 0$

[$\because$ Two columns are identical.]

84. (D) Let $y = \log_{10}(5x^3 - 2x)$ and $z = x^2$

$y = \log_{10}e \times \log_{10}(5x^3 - 2x)$, $\frac{dz}{dx} = 2x$

$\Rightarrow \frac{dy}{dx} = \log_{10}e \times \frac{1}{5x^3-2x} \times (15x^2 - 2)$,

$\Rightarrow \frac{dy}{dx} = (15x^2 - 2)\log_{10}e \times \frac{1}{5x^3-2x}$

Now, $\frac{dy}{dz} = \frac{dy}{dx}\times\frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = (15x^2-2)\log_{10}e \times \frac{1}{5x^3-2x}\times\frac{1}{2x}$

$\Rightarrow \frac{dy}{dz} = \frac{1}{2}\times\frac{(15x^2-2)\log_{10}e}{2x^2(5x^2-2)}$

85. (C) Let $y = \sqrt{6+5\sqrt{6+5\sqrt{6+........\infty}}}$

$\Rightarrow y = \sqrt{6+5y}$

On squaring

$\Rightarrow y^2 = 6 + 5y$

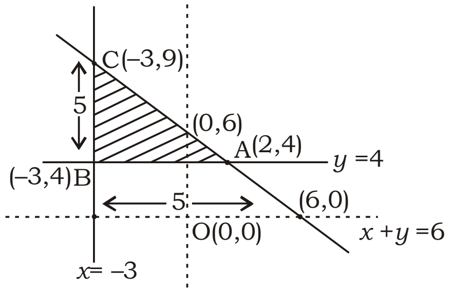
$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow (y - 6)(y + 1) = 0$

$\Rightarrow y = 6, -1$

Hence $\sqrt{6+5\sqrt{6+5\sqrt{6+........\infty}}} = 6$

86. (C)



The required Area $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ sq.unit

87. (A) Function is one-one but onto.

88. (C) We know that

$(1+x)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n x^n$

On putting $x = 1$

$\Rightarrow (1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n$

$\Rightarrow 2^n$ = sum of the coefficients

$\Rightarrow$ sum of the coeffients $= 2^n$

89. (B) A = {1, 3, 6}, B = {1, 6, 8} and C = {3, 7}

$(A \cap B) = \{1, 6\}$

Now, $(A \cap B) \times C = \{1, 6\} \times \{3, 7\}$

$\Rightarrow (A \cap B) \times C = \{(1, 3), (1,7), (6, 3), (6,7)\}$

Hence the number of elements = 4

90. (C) Curve

$2x^2 + 6y^2 = 36$

$\frac{x^2}{18} + \frac{y^2}{6} = 1$

$a^2 = 18 \Rightarrow a = 3\sqrt{2}$ and $b^2 = 6 \Rightarrow b = \sqrt{6}$

Area of an ellipse $= \pi ab$

$= \pi \times 3\sqrt{2} \times \sqrt{6}$

$= 6\sqrt{3}\,\pi$ sq. unit

91. (D) Conic

$4x^2 + 6y^2 + 8x + 12y - 26 = 0$

$\Rightarrow 4(x^2 + 2x) + 6(y^2 + 2y) - 26 = 0$

$\Rightarrow 4(x + 1)^2 - 4 + 6(y + 1)^2 - 6 - 26 = 0$

$\Rightarrow 4(x + 1)^2 + 6(y + 1)^2 = 36$

$\Rightarrow \frac{(x+1)^2}{9} + \frac{(y+1)^2}{6} = 1$

$a^2 = 9 \Rightarrow a = 3,\ b^2 = 6 \Rightarrow b = \sqrt{6}$

Let X = $x + 2$, Y = $y + 1$

$\Rightarrow \frac{X^2}{9} + \frac{Y^2}{6} = 1$

Vertices (X,Y) = $(\pm a, 0)$

X = $\pm a$, Y = 0

$\Rightarrow x + 1 = \pm 3,\ y + 1 = 0$

$\Rightarrow x + 1 = 3$ or $x + 1 = -3,\ y = -1$

$\Rightarrow x = 2$ or $x = -4,\ y = -1$

Hence vertices = (2, –1) and (–4, –1)

92. (B) $y = \omega^2 - \omega + 4$

$\Rightarrow y - 4 = \omega^2 - \omega$

On squaring

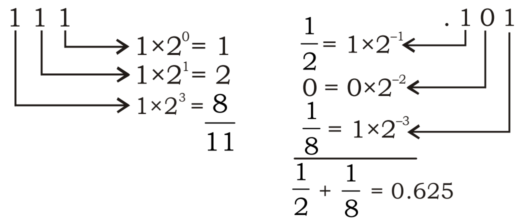
$\Rightarrow y^2 + 16 - 8y = \omega^4 + \omega^2 - 2\omega^3$

$\Rightarrow y^2 + 16 - 8y = \omega + \omega^2 - 2$

$\Rightarrow y^2 + 16 - 8y = -1 - 2 \quad [\because 1+\omega+\omega^2 = 0]$

$\Rightarrow y^2 - 8y = -19$

93. (C)



Hence $(111.101)_2 = (11.625)_{10}$

94. (B) $\frac{\log_3 \sqrt{3} \times \log_{64} 4}{\log_2 16 \times \log_{36} \sqrt{6}}$

$\Rightarrow \frac{\log_3 3^{1/2} \times \log_{64}(64)^{1/4}}{\log_2 2^4 \times \log_3 (36)^{1/4}}$

$\Rightarrow \frac{\frac{1}{2}\log_3 3 \times \frac{1}{4}\log_{64} 64}{4\log_2 2 \times \frac{1}{4}\log_{36} 36} \Rightarrow \frac{\frac{1}{2} \times \frac{1}{4}}{4 \times \frac{1}{4}} = \frac{1}{8}$

95. (C) Let Ratio = $m : 1$

A(–1,–4) —— m —— C(0,–3) —— 1 —— B(2,–1)

Now, $\frac{m \times 2 + 1(-1)}{m+1} = 0$ and $\frac{m \times (-1) + 1 \times (-4)}{m+1} = -3$

$\Rightarrow 2m - 1 = 0$ and $-m - 4 = -3m - 3$

$\Rightarrow m = \frac{1}{2}$ and $2m = 1 \Rightarrow m = \frac{1}{2}$

Hence Ratio = 1 : 2

96. (C) $I = \int_{-1}^{2} |x-1|\, e^x dx$

$I = \int_{-1}^{1} (1-x)e^x dx + \int_{1}^{2} (x-1)e^x dx$

$$I = \left[(1-x).\int e^x dx - \int\left\{\frac{d}{dx}(1-x).\int e^x dx\right\}dx\right]_{-1}^{1}$$

$$+\left[(x-1)\int e^x dx - \int\left\{\frac{d}{dx}(x-1).\int e^x dx\right\}dx\right]_{1}^{2}$$

$$I = \left[(1-x)e^x - \int -1.e^x dx\right]_{-1}^{1} + \left[(x-1)e^x - \int 1.e^x dx\right]_{1}^{2}$$

$$I = \left[(1-x)e^x + e^x\right]_{-1}^{1} + \left[(x-1)e^x - e^x\right]_{1}^{2}$$

$I = [(0.e^1+e^1)-(2.e^{-1}+e^{-1})]+[(1.e^2-e^2)-(0.e^1-e^1)]$

$I = e - 3e^{-1} + 0 + e$

$$I = 2e - \frac{3}{e}$$

97. (B) $I_n = \int_{\pi/3}^{\pi/3} \tan^n x\, dx$

$$I_n = \int_{\pi/6}^{\pi/3} \tan^{n-2} x.\tan^2 x\, dx$$

$$I_n = \int_{\pi/6}^{\pi/3} \tan^{n-2} x.(\sec^2 x - 1)\, dx$$

$$I_n = \int_{\pi/6}^{\pi/3} \tan^{n-2} x.\sec^2 x\, dx - \int_{\pi/6}^{\pi/3} \tan^{n-2} x\, dx$$

$$I_n = \left[\frac{[\tan x]^{n-2+1}}{n-2+1}\right]_{\pi/6}^{\pi/3} - I_{n-2}$$

$$I_n + I_{n-2} = \frac{1}{n-1}\left[\left(\tan\frac{\pi}{3}\right)^{n-1} - \left(\tan\frac{\pi}{6}\right)^{n-1}\right]$$

$n = 3$

$$I_3 + I_1 = \frac{1}{3-1}\left[\left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right]$$

$$I_3 + I_1 = \frac{1}{2}\left[3 - \frac{1}{3}\right] = \frac{4}{3}$$

98. (B) cos(525) = cos(360 + 165)

⇒ cos(525) = cos165

⇒ cos(525) = cos(180 – 15)

⇒ cos(525) = – cos15

$$\Rightarrow \cos(525) = -\frac{\sqrt{3}+1}{2\sqrt{2}}$$

99. (C) $n(S) = 6 \times 6 = 36$

$$E = \begin{Bmatrix} (6,3),(5,4),(4,5),(3,6) \text{ for sum} = 9 \\ (6,4),(5,5),(4,6) \text{ for sum} = 10 \\ (6,5),(5,6) \text{ for sum} = 11 \\ (6,6) \text{ for sum} = 12 \end{Bmatrix}$$

$n(E) = 10$

The required Probability $= \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

100. (B) Given that A = B ∩ C

Now, (U – (U – (U – (U – A))))

⇒ (U – (U – (U – A')))

⇒ (U – (U – A))

⇒ (U – A') = A = B ∩ C

101. (C) tanθ + 2tan2θ + 4cot4θ

⇒ cotθ – (cotθ – tanθ) + 2tan2θ + 4cot4θ

We know that

cotA – tanA = 2 cot2A

⇒ cotθ – 2cot2θ + 2tan2θ + 4cot4θ

⇒ cotθ – 2(cot2θ – tan2θ) + 4cot4θ

⇒ cot – 2 × 2 cot4θ + 4 cot4θ = cotθ

102. (A) Differential equation

$xdy - xdx = ydx$

$\Rightarrow xdy - ydx = xdx$

$$\Rightarrow \frac{xdy - ydx}{x^2} = \frac{1}{x}dx$$

$$\Rightarrow d\left(\frac{y}{x}\right) = \frac{1}{x}dx$$

On integrating

$$\Rightarrow \frac{y}{x} = \log x + c$$

$\Rightarrow y = x\log x + cx$

103. (B) Tangents of a circle

$5x - 12y + 25 = 0$

and $24y - 10y + 2 = 0 \Rightarrow -2(5y - 12y - 1) = 0$

$\Rightarrow 5y - 12y - 1 = 0$

Both lines are parallel.

Now, radius of circle $= \frac{1}{2}$ (Distance between tangents)

$$\Rightarrow \text{radius of circle} = \frac{1}{2}\left|\frac{25+1}{\sqrt{5^2+(-12)^2}}\right|$$

$$\Rightarrow \text{radius of circle} = \frac{1}{2} \times \frac{26}{13} = 1$$

Hence Area of circle $= \pi r^2 = \pi \times 1^2 = \pi$ sq.unit

104. (C) $\sum_{r=0}^{3} C(43+r,4) + C(43,5)$

$\Rightarrow {}^{43}C_4 + {}^{44}C_4 + {}^{45}C_4 + {}^{46}C_4 + {}^{43}C_5$

$\Rightarrow {}^{43}C_4 + {}^{43}C_5 + {}^{44}C_4 + {}^{45}C_4 + {}^{46}C_4$

We know that

${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$\Rightarrow {}^{44}C_5 + {}^{44}C_4 + {}^{45}C_4 + {}^{46}C_4$

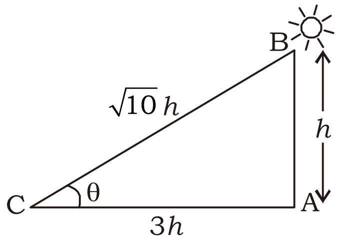
$\Rightarrow {}^{45}C_5 + {}^{44}C_4 + {}^{45}C_4 + {}^{46}C_4$

$\Rightarrow {}^{45}C_5 + {}^{45}C_4 + {}^{46}C_4$

$\Rightarrow {}^{46}C_5 + {}^{46}C_4 = {}^{47}C_5$

105. (D) The required Probability = $\frac{1}{7}$

106. (B)



Let height = h m and AC = $3h$

$\angle BCA = \theta$

In ΔABC :-

$\sin\theta = \frac{AB}{BC}$

$\Rightarrow \sin\theta = \frac{h}{\sqrt{10}h} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$

107. (A) $\frac{1+\tan 122.\tan 182}{\cot 148-\cot 88}$

$\Rightarrow \frac{1+\tan(90+32).\tan(90+92)}{\cot(180-32)-\cot(180-92)}$

$\Rightarrow \frac{1+(-\cot 32).(-\cot 92)}{-\cot 32+\cot 92}$

$\Rightarrow -\frac{1+\cot 32.\cot 92}{\cot 32-\cot 92}$

$\Rightarrow -\cot(92-32)$

$\Rightarrow -\cot 60 = -\frac{1}{\sqrt{3}}$

108. (C) Given that $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ ...(i)

Now, $x^n - y^n = 4$

On differentiating

$\Rightarrow nx^{n-1} - ny^{n-1}\frac{dy}{dx} = 0$

$\Rightarrow ny^{n-1}\frac{dy}{dx} = nx^{n-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{x}{y}\right)^{n-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right)^{1-n}$

On comparing with eq(i)

$1-n = \frac{1}{3} \Rightarrow n = \frac{2}{3}$

109. (B) Word "SUGGESTION"

The required number of permutations

$= \frac{10!}{2!2!} = 907200$

110. (C) $y = a^{x+a^{x+a^{x+\ldots\ldots\infty}}}$

$\Rightarrow y = a^{x+y}$

taking log both side

$\Rightarrow \log y = (x+y)\log a$

On differetiating both side w.r.t.'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(1+\frac{dy}{dx}\right)\log a$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log a + \log a.\frac{dy}{dx}$

$\Rightarrow \left(\frac{1}{y}-\log a\right)\frac{dy}{dx} = \log a$

$\Rightarrow \left(\frac{1-y\log a}{y}\right)\frac{dy}{dx} = \log a$

$\Rightarrow \frac{dy}{dx} = \frac{y\log a}{1-y\log a}$

111. (D) When θ = 180

M = $\frac{60}{11}$(H ± 6) where + → H < 6

– H > 6

H = 2 (between 2 and 3 o'clock)

Now, M = $\frac{60}{11}(2+6)$

$\Rightarrow M = \frac{60}{11}\times 8$

$\Rightarrow M = \frac{480}{11} = 43\frac{7}{11}$

The required time = 2 : $43\frac{7}{11}$

112. (C) Plane $3x-12y+4z+16=0$ and point (2,–1,3)

Perpendicular Distance

$= \frac{3\times 2-12\times(-1)+4\times 3+16}{\sqrt{3^2+(-12)^2+4^2}}$

$= \frac{6+12+12+16}{13} = \frac{46}{13}$

113. (C) A = $\begin{bmatrix} 1 & 0 & -3 \\ 2 & -1 & 5 \\ 6 & 2 & -1 \end{bmatrix}$

Co-factors of A -

$C_{11}=(-1)^{1+1}\begin{vmatrix}-1 & 5\\ 2 & -1\end{vmatrix}$, $C_{12}=(-1)^{1+2}\begin{vmatrix}2 & 5\\ 6 & -1\end{vmatrix}$, $C_{13}=(-1)^{1+3}\begin{vmatrix}2 & -1\\ 6 & 2\end{vmatrix}$

$= -9$ $\quad = 32$ $\quad = 10$

$C_{12}=(-1)^{2+1}\begin{vmatrix}0 & -3\\ 2 & -1\end{vmatrix}$, $C_{22}=(-1)^{2+2}\begin{vmatrix}1 & -3\\ 6 & -1\end{vmatrix}$, $C_{23}=(-1)^{2+3}\begin{vmatrix}1 & 0\\ 6 & 2\end{vmatrix}$

$= -6$ $\quad = 17$ $\quad = -2$

$C_{31}=(-1)^{3+1}\begin{vmatrix}0 & -3\\ -1 & 5\end{vmatrix}$, $C_{32}=(-1)^{3+2}\begin{vmatrix}1 & -3\\ 2 & 5\end{vmatrix}$, $C_{33}=(-1)^{3+3}\begin{vmatrix}1 & 0\\ 2 & -1\end{vmatrix}$

$= -3$ $\quad = -11$ $\quad = -1$

$$C = \begin{bmatrix}-9 & 32 & 10\\ -6 & 17 & -2\\ -3 & -11 & -1\end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix}-9 & -6 & -3\\ 32 & 17 & -11\\ 10 & -2 & -1\end{bmatrix}$$

114. (C) Line = $\frac{x}{2} + 2y = 1$

$\Rightarrow x + 4y = 2$

Slope of line = $\frac{-1}{4}$

The required slope = $\frac{-1}{4}$

115. (B) $n = 121!$

Now, $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots + \frac{1}{\log_{121} n}$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 + \ldots\ldots\ldots + \log_n 121$

$\Rightarrow \log_n(2.3.4\ldots\ldots\ldots 121)$

$\Rightarrow \log_n 121! = \log_n n = 1$

116. (D)

$$\begin{array}{r} 1\ x\ 1\ 1\ 0\\ 1\ 0\ y\ 0\ 1\\ \hline 1\ 1\ 0\ z\ 1\ 1\\ \hline \end{array}$$

$x = 1,\ y = 1,\ z = 0$

117. (B) $\tan^2 x = 3$

$\Rightarrow \tan^2 x = \tan^2 \frac{\pi}{3}$

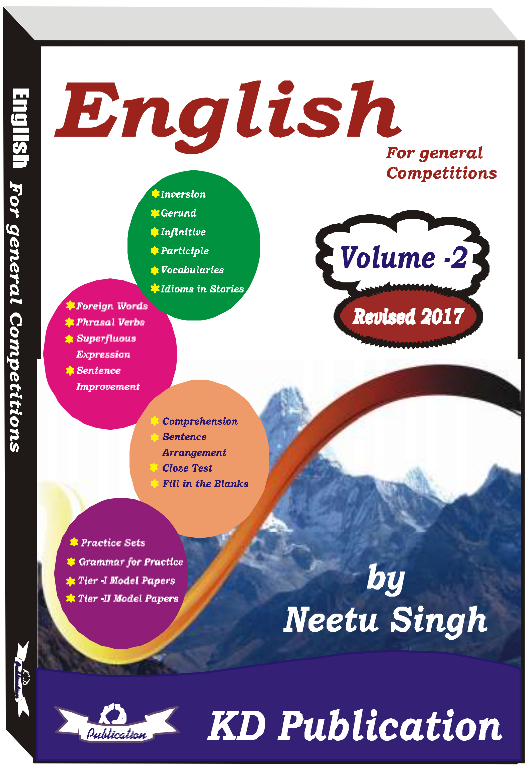
$\Rightarrow x = n\pi \pm \frac{\pi}{3}$

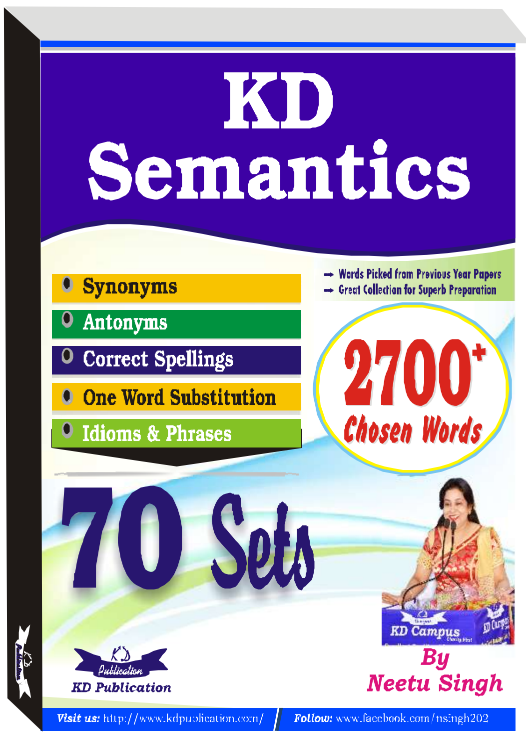
118. (C)

119. (D) The required no. of hand shakes in party

$= {}^{15}C_2 = \frac{15 \times 14}{2} = 105$

120. (A) Mode = 3 Median – 2 Mean





NDA (MATHS) MOCK TEST - 132 (Answer Key)

1. (B)	21. (C)	41. (A)	61. (C)	81. (C)	101. (C)
2. (C)	22. (C)	42. (B)	62. (A)	82. (B)	102. (A)
3. (B)	23. (A)	43. (D)	63. (C)	83. (B)	103. (B)
4. (D)	24. (D)	44. (A)	64. (B)	84. (D)	104. (C)
5. (C)	25. (B)	45. (C)	65. (D)	85. (C)	105. (D)
6. (A)	26. (C)	46. (D)	66. (D)	86. (C)	106. (B)
7. (A)	27. (C)	47. (A)	67. (B)	87. (A)	107. (A)
8. (A)	28. (B)	48. (B)	68. (D)	88. (C)	108. (C)
9. (B)	29. (A)	49. (A)	69. (A)	89. (B)	109. (B)
10. (C)	30. (A)	50. (A)	70. (D)	90. (C)	110. (C)
11. (C)	31. (B)	51. (B)	71. (B)	91. (D)	111. (D)
12. (D)	32. (D)	52. (C)	72. (C)	92. (B)	112. (C)
13. (C)	33. (C)	53. (B)	73. (A)	93. (C)	113. (C)
14. (A)	34. (B)	54. (C)	74. (B)	94. (B)	114. (C)
15. (A)	35. (C)	55. (B)	75. (B)	95. (C)	115. (B)
16. (C)	36. (D)	56. (A)	76. (C)	96. (C)	116. (D)
17. (C)	37. (C)	57. (B)	77. (C)	97. (B)	117. (B)
18. (B)	38. (A)	58. (A)	78. (D)	98. (B)	118. (C)
19. (B)	39. (A)	59. (C)	79. (A)	99. (C)	119. (D)
20. (C)	40. (D)	60. (B)	80. (A)	100. (B)	120. (A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777