

NDA MATHS MOCK TEST - 136 (SOLUTION)

1. (C) $I = \int \sqrt{x} \cdot e^{\sqrt{x}} dx$

$$I = 2 \int x \cdot \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx$$

Let $\sqrt{x} = t \Rightarrow x = t^2$

$$\frac{1}{2\sqrt{x}} dx = dt$$

$$I = 2 \int t^2 \cdot e^t dt$$

$$I = 2 \left[t^2 \int e^t dt - \int \left\{ \frac{d}{dt}(t^2) \cdot \int e^t dt \right\} dt \right]$$

$$I = 2 \left[t^2 \cdot e^t - \int 2t \cdot e^t dt \right] + c$$

$$I = 2t^2 \cdot e^t - 4 \int t \cdot e^t dt + c$$

$$I = -2t^2 \cdot e^t - 4 \left[t \cdot e^t - \int 1 \cdot e^t dt \right] + c$$

$$I = -2t^2 \cdot e^t - 4t \cdot e^t + 4 \int e^t dt + c$$

$$I = 2t^2 \cdot e^t - 4t \cdot e^t + 4e^t + c$$

$$I = 2x e^{\sqrt{x}} - 4\sqrt{x} \cdot e^{\sqrt{x}} + 4e^{\sqrt{x}} + c$$

2. (B) $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \frac{c+a\omega+b\omega^2}{a+b\omega+c\omega^2}$

$$\Rightarrow \frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)} + \frac{\omega(c+a\omega+b\omega^2)}{\omega(a+b\omega^2+c\omega^2)}$$

$$\Rightarrow \frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a\omega^3+b\omega^4} + \frac{\omega(c+a\omega+b\omega^2)}{a\omega+b\omega^2+c\omega^3}$$

$$\Rightarrow \frac{\omega^2(a+b\omega+c\omega^2)}{a+b\omega+c\omega^2} + \frac{\omega(c+a\omega+b\omega^2)}{c+a\omega+b\omega^2}$$

$$\Rightarrow \omega^2 + \omega = -1 \quad [\because 1 + \omega + \omega^2 = 0]$$

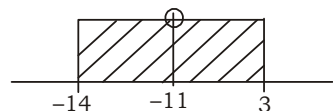
3. (C) $f(x) = \frac{\sqrt{\log_e(43-11x-x^2)}}{x+11}$

Now, $\log_e(43-11x-x^2) \geq 0$ and $x+11 \neq 0$

$$\Rightarrow 43-11x-x^2 \geq 1, \quad x \neq -11$$

$$\Rightarrow x^2+11x-42 \leq 0$$

$$\Rightarrow (x+14)(x-3) \leq 0$$



Domain = $[-14, 3] - \{-11\}$

4. (D) $f(x) = |2x^2 - 11|$ and $g(x) = 2x - 1$

Now, $f \circ g(x) = f[g(x)]$

$$\Rightarrow f \circ g(x) = f[2x-1]$$

$$\Rightarrow f \circ g(x) = |2(2x-1)^2 - 11|$$

$$\Rightarrow f \circ g(x) = |2(4x^2 + 1 - 4x) - 11|$$

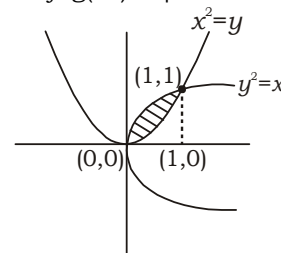
$$\Rightarrow f \circ g(x) = |8x^2 + 2 - 8x - 11|$$

$$\Rightarrow f \circ g(x) = |8x^2 - 8x - 9|$$

Now, $f \circ g(-1) = |8(-1)^2 - 8(-1) - 9|$

$$\Rightarrow f \circ g(-1) = |8 + 8 - 9| = 7$$

5. (C)



$$y_1 \Rightarrow y^2 = x \text{ and } y_2 \Rightarrow x^2 = y$$

$$\text{Area} = \int_0^1 (y_1 - y_2) dx$$

$$\text{Area} = \int_0^1 (\sqrt{x} - x^2) dx$$

$$\text{Area} = \left[2 \times \frac{x^{3/2}}{3} - \frac{x^3}{3} \right]_0^1$$

$$\text{Area} = \left[\frac{2}{3} - \frac{1}{3} - 0 \right]$$

$$\text{Area} = \frac{1}{3} \text{ sq. unit}$$

6. (D) The required no. of ways = ${}^5C_2 \times {}^{11}C_9$

7. (C) Given that $X = \{9(n-1) : n \in \mathbb{N}\}$

$$n = 1, 2, 3, 4, \dots$$

$$X = \{0, 9, 18, 27, \dots\}$$

$$Y = \{4^n - 3n - 1 : n \in \mathbb{N}\}$$

$$n = 1, 2, 3, 4, \dots$$

$$Y = \{0, 9, 54, 243, \dots\}$$

$$(X \cap Y) = \{0, 9, 54, 243\} = Y$$

8. (C) Differential equation

$$x dy - y dx = x^2 y dx$$

$$\Rightarrow \frac{xdy - ydx}{xy} = x dx$$

$$\Rightarrow \frac{dy}{y} - \frac{dx}{x} = x dx$$

On integrating

$$\Rightarrow \log y - \log x = \frac{x^2}{2} + c$$

$$\Rightarrow \log \frac{y}{x} = \frac{x^2}{2} + c$$

9. (B) Let $y = 3\sqrt{\tan x^2}$
On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{dy}{dx} = 3 \times \frac{1}{2} (\tan x^2)^{-1/2} (\sec^2 x^2)(2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x \sec^2 x^2}{\sqrt{\tan x^2}}$$

10. (C) $n(S) = 2^5 = 32$
 $n(E) = {}^5C_3 + {}^5C_4 + {}^5C_5$
 $n(E) = 10 + 5 + 1 = 16$

The required probability $P(E) = \frac{n(E)}{n(S)}$

$$\Rightarrow P(E) = \frac{16}{32} = \frac{1}{2}$$

11. (D) $S = 0.4 + 0.44 + 0.444 + \dots n$ terms

$$S = \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \dots n \text{ terms}$$

$$S = \frac{4}{9} \left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots n \text{ terms} \right]$$

$$S = \frac{4}{9} \left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \dots n \text{ terms} \right]$$

$$S = \frac{4}{9} (1 + 1 + 1 + \dots n \text{ terms})$$

$$- \frac{4}{9} \left[\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots n \text{ terms} \right]$$

$$S = \frac{4}{9} n - \frac{4}{9} \times \frac{10 \left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}$$

$$S = \frac{4}{9} n - \frac{4}{9} \times \frac{1}{9} \left[1 - \frac{1}{10^n}\right]$$

$$S = \frac{4}{9} \left[n - \frac{1}{9} \left(1 - \frac{1}{10^n}\right) \right]$$

12. (B) Curve $y = 3x^2 + 8x + 9$

$$\Rightarrow \frac{dy}{dx} = 6x + 8$$

Slope at $(-1, 4)$ (m_1) = $6(-1) + 8 = 2$

Slope at $(-2, 5)$ = $6(-2) + 8 = -4$

Now, $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$$\Rightarrow \tan \theta = \left| \frac{2 + 4}{1 + 2(-4)} \right|$$

$$\Rightarrow \tan \theta = \frac{6}{7} \Rightarrow \theta = \tan^{-1} \left(\frac{6}{7} \right)$$

The required angle = $\theta = \tan^{-1} \left(\frac{6}{7} \right)$

13. (C) $\lim_{x \rightarrow \infty} \left[\frac{x^2 + 3x - 5}{x^2 + x - 5} \right]^x \Rightarrow \lim_{x \rightarrow \infty} \left[\frac{1 + \frac{3}{x} - \frac{5}{x^2}}{1 + \frac{1}{x} - \frac{5}{x^2}} \right]^x$
[1^∞ form]

$$\Rightarrow e^{\lim_{x \rightarrow \infty} x \left[\frac{x^2 + 3x - 5}{x^2 + x - 5} - 1 \right]} \Rightarrow e^{\lim_{x \rightarrow \infty} x \left[\frac{2x}{x^2 + x - 5} \right]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} \left[\frac{2}{1 + \frac{1}{x} - \frac{5}{x^2}} \right]} \Rightarrow e^{\frac{1}{1+0-0}} = e^2$$

14. (B) Vectors $3\hat{i} + (\lambda + 1)\hat{j} + \lambda\hat{k}$ and $(\lambda - 2)\hat{i} + 2\hat{j} - 4\hat{k}$ are perpendicular,

then $3 \times (\lambda - 2) + (\lambda + 1) \times 2 + \lambda \times (-4) = 0$
 $\Rightarrow 3\lambda - 6 + 2\lambda + 2 - 4\lambda = 0 \Rightarrow \lambda = 4$

15. (D)

16. (A) $(1 + x)^3(1 + x^2)^2$
 $\Rightarrow (1 + x^3 + 3x^2 + 3x)(1 + x^2 + 2x^2)$
Coefficient of $x^5 = 2 + 3 = 5$

17. (C) A line makes the angles α, β and γ with the axes,
then $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
 $\Rightarrow 1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1$
 $\Rightarrow 3 - (\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma) = 1$
 $\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

18. (B) Given that $\tan \theta = \frac{b}{a}$

Now, $\frac{a \cos \theta + b \sin \theta}{a \cos \theta - b \sin \theta}$

$$\Rightarrow \frac{a + b \tan \theta}{a - b \tan \theta}$$

$$\Rightarrow \frac{a + b \times \frac{b}{a}}{a - b \times \frac{b}{a}} = \frac{a^2 + b^2}{a^2 - b^2}$$

19. (A) $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos \frac{16\pi}{3}}}}}}$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 \frac{8\pi}{3}}}}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos \frac{8\pi}{3}}}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 \frac{4\pi}{3}}}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos \frac{4\pi}{3}}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 \frac{2\pi}{3}}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + 2 \cos \frac{2\pi}{3}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 \times 2 \cos^2 \frac{\pi}{3}}}$$

$$\Rightarrow \sqrt{2 + 2 \cos \frac{\pi}{3}}$$

$$\Rightarrow \sqrt{2 \times 2 \cos^2 \frac{\pi}{6}}$$

$$\Rightarrow 2 \times \cos \frac{\pi}{6} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

20. (B) A.T.Q,

$$\frac{17}{2} [2a + (17-1)d] = 867$$

$$\Rightarrow \frac{1}{2} [2a + 16d] = 51$$

$$\Rightarrow a + 8d = 51 \Rightarrow T_9 = 51$$

21. (A) The required no. of triangles = ${}^{11}C_3 - {}^6C_3$
 $= 165 - 20$
 $= 145$

22. (B) $\sin^{-1}(\log_3 2x)$

$$\text{Here } -1 \leq \log_3 2x \leq 1$$

$$\Rightarrow 3^{-1} \leq 2x \leq 3^1$$

$$\Rightarrow \frac{1}{3} \leq 2x \leq 3$$

$$\Rightarrow \frac{1}{6} \leq x \leq \frac{3}{2}$$

$$\text{Domain} = \left[\frac{1}{6}, \frac{3}{2} \right]$$

23. (C) Series $\frac{1^2}{2} + \frac{1^2 + 2^2}{2+4} + \frac{1^2 + 2^2 + 3^2}{2+4+6} + \dots$

$$T_n = \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{2 + 4 + 6 + \dots + n}$$

$$T_n = \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{2(1 + 2 + 3 + \dots + n)}$$

$$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{2 \times \frac{n(n+1)}{2}} = \frac{2n+1}{6}$$

24. (A) $f(x) = x^2 + 5x - 6$

$$f'(x) = 2x + 5 \Rightarrow f'(c) = 2c + 5$$

$$a = -1, b = \frac{1}{2}$$

$$f(a) \Rightarrow f(-1) = (-1)^2 + 5(-1) - 6 = -10$$

$$f(b) \Rightarrow f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 5 \times \frac{1}{2} - 6 = \frac{-13}{4}$$

$$\text{Now, } f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 2c + 5 = \frac{\frac{-13}{4} + 10}{\frac{1}{2} + 1}$$

$$\Rightarrow 2c + 5 = \frac{9}{2}$$

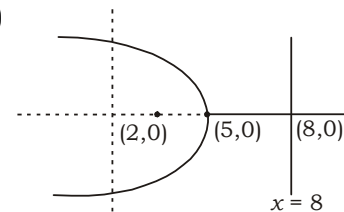
$$\Rightarrow 4c + 10 = 9 \Rightarrow c = \frac{-1}{4}$$

25. (A) $I = \int_{-\pi/2}^{\pi/2} \frac{\sin x}{1 + \cos x} dx = 0$

We know that

$$\int_{-a}^a f(x) dx = \begin{cases} 0, \text{ function is odd} \\ 2 \int_0^a f(x) dx, \text{ function is even} \end{cases}$$

26. (B)



equation of directrix

$$x = 8$$

27. (C) Direction ratios of lines are $(-1, 2, -4)$ and $(-2, x, -3)$.

A.T.Q.,

$$\cos \frac{\pi}{2} = \frac{-1 \times (-2) + 2 \times x + (-4) \times (-3)}{\sqrt{(-1)^2 + 2^2 + (-4)^2} \sqrt{(-2)^2 + x^2 + (-3)^2}}$$

$$\Rightarrow 0 = \frac{2 + 2x + 12}{\sqrt{21} \sqrt{x^2 + 13}}$$

$$\Rightarrow 0 = 2x + 14 \Rightarrow x = -7$$

28. (B) $y = \tan^{-1} \left[\frac{x^{1/2}(x^{1/2} - 1)}{1 + x^{3/2}} \right]$

$$y = \tan^{-1} \left[\frac{x - x^{1/2}}{1 + x \cdot x^{1/2}} \right]$$

Let $x = \tan A$ and $x^{1/2} = \tan B$

$$y = \tan^{-1} \left[\frac{\tan A - \tan B}{1 + \tan A \tan B} \right]$$

$y = \tan^{-1}[\tan(A - B)]$
 $y = A - B$
 $y = \tan^{-1}x - \tan^{-1}(x^{1/2})$
 On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{1+x} \times \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2} - \frac{1}{2\sqrt{x}(1+x)}$$

29. (A) We know that
 A.M. \geq G.M. \geq H.M.
 $\Rightarrow \frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$
 $\Rightarrow \frac{2ab}{a+b} \leq \sqrt{ab} \leq \frac{a+b}{2}$

30. (C) A.T.Q.,
 Mean = $\frac{a^{n-9} + b^{n-9}}{a^{n-10} + b^{n-10}}$
 $\Rightarrow \frac{a+b}{2} = \frac{a^{n-9} + b^{n-9}}{a^{n-10} + b^{n-10}}$

On comparing
 $n - 9 = 1 \Rightarrow n = 10$

31. (B) The required remainder = 4
 32. (D) $S = 3 + 6 + 9 + \dots + 99$
 $S = 3(1 + 2 + 3 + \dots + 33)$

$$S = 3 \times \frac{33 \times 34}{2}$$

$$S = 33 \times 51 = 1683$$

33. (C) Let $a + ib = \sqrt{1 + 2\sqrt{2}i}$
 On squaring both side
 $\Rightarrow (a^2 - b^2)^2 + 2abi = 1 + 2\sqrt{2}i$
 On comparing
 $\Rightarrow a^2 - b^2 = 1$ and $2ab = 2\sqrt{2}$... (i)
 Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
 $\Rightarrow (a^2 + b^2)^2 = 1 + 8$
 $\Rightarrow a^2 + b^2 = 3$... (ii)
 from eq(i) and eq(ii)
 $\Rightarrow 2a^2 = 4$ and $2b^2 = 2$
 $\Rightarrow a = \pm \sqrt{2}$, $b = \pm 1$

Hence $\sqrt{1 + 2\sqrt{2}i} = \pm (\sqrt{2} + i)$

34. (B) Let $y = 7^{22}$
 taking log both sides
 $\Rightarrow \log_{10}y = 22\log_{10}7$
 $\Rightarrow \log_{10}y = 22 \times 0.8451$
 $\Rightarrow \log_{10}y = 18.5922$
 The required number of digits = $18 + 1 = 19$

35. (C) $\vec{a} = 3\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} + 4\hat{k}$
 Now, $\vec{b} - \vec{a} = (-2\hat{i} + 3\hat{j} + 4\hat{k}) - (3\hat{i} - \hat{j} + \hat{k})$
 $\Rightarrow \vec{b} - \vec{a} = -5\hat{i} + 4\hat{j} + 3\hat{k}$
 and $\vec{a} + 3\vec{b} = (3\hat{i} - \hat{j} + \hat{k}) + 3(-2\hat{i} + 3\hat{j} + 4\hat{k})$
 $\Rightarrow \vec{a} + 3\vec{b} = -3\hat{i} + 8\hat{j} + 13\hat{k}$
 Now, $(\vec{b} - \vec{a}) \cdot (\vec{a} + 3\vec{b})$
 $\Rightarrow (-5\hat{i} + 4\hat{j} + 3\hat{k}) \cdot (-3\hat{i} + 8\hat{j} + 13\hat{k})$
 $\Rightarrow -5 \times (-3) + 4 \times 8 + 3 \times 13 = 86$

36. (C) Total students = 13
 The required no. of ways = $(13 - 1)! = 12!$

37. (B) Differential equation
 $x dy = (x^2 + y^2 + y) dx$
 $\Rightarrow xdy - ydx = (x^2 + y^2) dx$
 $\Rightarrow \frac{xdy - ydx}{x^2 + y^2} = dx$

On integrating

$$\int d\left[\tan^{-1}\frac{y}{x}\right] = \int dx$$

$$\Rightarrow \tan^{-1}\frac{y}{x} = x + c$$

$$\Rightarrow \frac{y}{x} = \tan(x + c)$$

$$\Rightarrow y = x \tan(x + c)$$

38. (B) Total students = 500
 Passed students $n(E \cup H) = 500 - 29 = 471$
 $n(E) = 247$ and $n(H) = 307$
 Now, $n(E \cap H) = n(E) + n(H) - n(E \cup H)$
 $\Rightarrow n(E \cap H) = 247 + 307 - 471$
 $\Rightarrow n(E \cap H) = 83$
 The required no. of students = 83

39. (C) $\lim_{x \rightarrow 0} \frac{4x^4 + 2x^2 + 1}{3x^3 - 2x^4 + 6}$
 $\Rightarrow \frac{4 \times 0 + 2 \times 0 + 1}{3 \times 0 - 2 \times 0 + 6} = \frac{1}{6}$

40. (D) $[A]_{(x+2) \times (y-3)}$ and $[B]_{(x+1) \times (8-y)}$
 Both AB and BA exist
 $y - 3 = x + 1 \Rightarrow x - y = -4$... (i)
 and $x + 2 = 8 - y \Rightarrow x + y = 6$... (ii)
 On solving
 $2x = 2 \Rightarrow x = 1$ and $2y = 10 \Rightarrow y = 5$

41. (C) The required possible ways = $9 \times 8 = 72$

42. (D) $f(x) = \cot x - \cot^2 x + \cot^3 x \dots \infty$

$$\Rightarrow f(x) = \frac{\cot x}{1 + \cot x}$$

Now, $I = \int_0^{\pi/2} \tan x \cdot f(x) dx$

$$\Rightarrow I = \int_0^{\pi/2} \tan x \times \frac{\cot x}{1 + \cot x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1}{1 + \cot x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \dots(i)$$

Prop. IV $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \dots(ii)$$

from eq(i) and eq(ii)

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2}$$

$$\Rightarrow 2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

43. (A) Hyperbola $\frac{x^2}{25} - \frac{y^2}{\lambda} = 1$

$$a^2 = 25, b^2 = \lambda$$

Now, $e = \sqrt{1 + \frac{a^2}{b^2}}$

$$\Rightarrow e = \sqrt{1 + \frac{25}{\lambda}} \Rightarrow e = \sqrt{\frac{\lambda + 25}{\lambda}}$$

$$\text{foci} = (0, \pm be) = \left(0, \pm \sqrt{\lambda} \times \sqrt{\frac{\lambda + 25}{\lambda}}\right)$$

$$= (0, \pm \sqrt{\lambda + 25})$$

and ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$

$$a^2 = 9, b^2 = 36$$

Now, $e = \sqrt{1 - \frac{a^2}{b^2}}$

$$\Rightarrow e = \sqrt{1 - \frac{9}{36}} \Rightarrow e = \frac{\sqrt{3}}{2}$$

$$\text{foci} = (0, \pm be) = \left(0, \pm 6 \times \frac{\sqrt{3}}{2}\right)$$

$$= (0, \pm 3\sqrt{3})$$

A.T.Q.,

$$\sqrt{\lambda + 25} = 3\sqrt{3}$$

$$\Rightarrow \lambda + 25 = 27 \Rightarrow \lambda = 2$$

44. (C) Lines $10x - 24y + 6 = 0 \Rightarrow 5x - 12y + 3 = 0$
and $5x - 12y + 16 = 0$

The required distance = $\frac{16 - 3}{\sqrt{5^2 + (-12)^2}}$

$$= \frac{13}{13} = 1$$

45. (B) Given $x + z = y$

Now, $\sin x + \sin y + \sin z$

$$\Rightarrow \sin x + \sin z + \sin y$$

$$\Rightarrow 2 \sin \frac{x+z}{2} \cdot \cos \frac{x-z}{2} + 2 \sin \frac{y}{2} \cdot \cos \frac{y}{2}$$

$$\Rightarrow 2 \sin \frac{y}{2} \cdot \cos \frac{x-z}{2} + 2 \sin \frac{y}{2} \cdot \cos \frac{y}{2}$$

$$\Rightarrow 2 \sin \frac{y}{2} \left[\cos \frac{x-z}{2} + \cos \frac{y}{2} \right]$$

$$\Rightarrow 2 \sin \frac{y}{2} \times 2 \cos \frac{\frac{x-z}{2} + \frac{y}{2}}{2} \cdot \cos \frac{\frac{x-z}{2} - \frac{y}{2}}{2}$$

$$\Rightarrow 4 \sin \frac{y}{2} \times \cos \frac{x+y-z}{4} \times \cos \frac{x-z-y}{4}$$

$$\Rightarrow 4 \sin \frac{y}{2} \times \cos \frac{x}{2} \times \cos \frac{z}{2} \quad [\because x+z=y]$$

$$\Rightarrow 4 \cos \frac{x}{2} \times \sin \frac{y}{2} \times \cos \frac{z}{2}$$

46. (C) $\cot\left(7\frac{1}{2}\right)^\circ = \frac{\cos 7\frac{1}{2}}{\sin 7\frac{1}{2}} = \frac{2 \cos 7\frac{1}{2}}{2 \cos 7\frac{1}{2}}$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{2 \cos^2 7\frac{1}{2}}{2 \sin 7\frac{1}{2} \cdot \cos 7\frac{1}{2}}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{1 + \cos 15}{\sin 15}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{2\sqrt{6} + 3 + \sqrt{3} + 2\sqrt{2} + \sqrt{3} + 1}{3-1}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \frac{2\sqrt{6} + 2\sqrt{3} + 2\sqrt{2} + 4}{2}$$

$$\cot\left(7\frac{1}{2}\right)^\circ = \sqrt{6} + \sqrt{3} + \sqrt{2} + 2$$

47. (C) Word "DISEASE"

$$\text{Total no. of words} = \frac{7!}{2!2!} = 1260$$

When vowels come together

$$\text{no. of words} = \frac{4!}{2!} \times \frac{4!}{2!} = 144$$

$$\text{The required no. of words} = 1260 - 144 = 1116$$

48. (A) A.T.Q.,

$$\frac{\sqrt{3}}{4} a^2 = 16\sqrt{3}$$

$$\Rightarrow a^2 = 64 \Rightarrow a = 8$$

$$\text{Now, } R = \frac{a^3}{4\Delta}$$

$$\Rightarrow R = \frac{8 \times 8 \times 8}{4 \times 16\sqrt{3}} \Rightarrow R = \frac{8}{\sqrt{3}}$$

Area of circumcircle = πR^2

$$= \pi \times \left(\frac{8}{\sqrt{3}}\right)^2 = \frac{64\pi}{3} \text{ cm}^2$$

49. (B) $(-\sqrt{-1})^{4n+1} + (-\sqrt{-1})^{4n-3}$

$$\Rightarrow (-i)^{4n+1} + (-i)^{4n-3}$$

$$\Rightarrow (-i)^{4n} (-i)^1 + (-i)^{4n} (-i)^{-3}$$

$$\Rightarrow -i + \frac{-1}{i^3}$$

$$\Rightarrow -i + \frac{-1}{-i}$$

$$\Rightarrow -i - i = -2i$$

50. (C) Equation $x^2 + 5|x| + 6 = 0$ has no root because sum of three positive number can not zero.

51. (B) We know that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\text{Now, } \frac{b+c}{a} = \frac{k \sin B + k \sin C}{\sin A}$$

$$\Rightarrow \frac{b+c}{a} = \frac{\sin B + \sin C}{\sin A}$$

$$\Rightarrow \frac{b+c}{a} = \frac{2 \sin \frac{B+C}{2} \cdot \cos \frac{B-C}{2}}{2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}}$$

$$[\because A+B+C = 180]$$

$$\Rightarrow \frac{b+c}{a} = \frac{2 \cos \frac{A}{2} \cdot \cos \frac{B-C}{2}}{2 \sin \frac{A}{2} \cdot \cos \frac{A}{2}}$$

$$\Rightarrow \frac{b+c}{a} = \frac{\cos \frac{B-C}{2}}{\sin \frac{A}{2}}$$

52. (C)

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
0		

$$\begin{array}{r} 0.25 \\ \times 2 \\ \hline 0.50 \\ \times 2 \\ \hline 1.00 \end{array}$$

$$(0.25)_{10} = (0.01)_2$$

$$(37)_{10} = (100101)_2$$

$$\text{Hence } (37.25)_{10} = (100101.01)_2$$

53. (B)
$$\begin{vmatrix} x^2 + y^2 & x + y & k \\ y^2 + z^2 & y + z & k \\ z^2 + x^2 & z + x & k \end{vmatrix} = (x-y)(y-z)(z-x)$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} x^2 + y^2 & x + y & k \\ z^2 - x^2 & z - x & 0 \\ z^2 - y^2 & z - y & 0 \end{vmatrix} = (x-y)(y-z)(z-x)$$

$$\Rightarrow (z-x)(z-y) \begin{vmatrix} x^2 + y^2 & x + y & k \\ z + x & 1 & 0 \\ z + y & 1 & 0 \end{vmatrix} = -(x-y)(z-y)(z-x)$$

$$R_3 \rightarrow R_3 - R_2$$

$$\Rightarrow \begin{vmatrix} x^2 + y^2 & x + y & k \\ z + x & 1 & 0 \\ y - x & 0 & 0 \end{vmatrix} = -(x - y)$$

$$\Rightarrow (y - x) \begin{vmatrix} x^2 + y^2 & x + y & k \\ z + x & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} = y - x$$

$$\Rightarrow (x^2 + y^2) \times 0 - (x + y) \times 0 + k(0 - 1) = 1$$

$$\Rightarrow -k = 1 \Rightarrow k = -1$$

54. (C) Let $y = x^5 + 5^x$
On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{dy}{dx} = 5x^4 + 5^x \cdot \log 5$$

55. (A) Let $f(x) = y = \frac{7^x + 7^{-x}}{7^x - 7^{-x}}$
by Componendo & Dividendo Rule

$$\Rightarrow \frac{y+1}{y-1} = \frac{7^x + 7^{-x} + 7^x - 7^{-x}}{7^x + 7^{-x} - 7^x + 7^{-x}}$$

$$\Rightarrow \frac{y+1}{y-1} = \frac{2 \times 7^x}{2 \times 7^{-x}}$$

$$\Rightarrow \frac{y+1}{y-1} = 7^{2x}$$

taking log both side

$$\Rightarrow \log_7 \left(\frac{y+1}{y-1} \right) = 2x \Rightarrow x = \frac{1}{2} \left[\log_7 \left(\frac{y+1}{y-1} \right) \right]$$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} \left[\log_7 \left(\frac{y+1}{y-1} \right) \right]$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \left[\log_7 \left(\frac{x+1}{x-1} \right) \right]$$

56. (D) $4f(x+1) + f\left(\frac{1}{x+1}\right) = 3x$... (i)

On putting $x = 1$

$$4f(2) + f\left(\frac{1}{2}\right) = 3$$
 ... (ii)

On putting $x = \frac{-1}{2}$ in eq(i)

$$4f\left(\frac{1}{2}\right) + f(2) = \frac{-3}{2}$$
 ... (iii)

On solving eq(ii) and eq(iii)

$$15f(2) = 12 + \frac{3}{2}$$

$$\Rightarrow 15f(2) = \frac{27}{2} \Rightarrow f(2) = \frac{9}{10}$$

57. (C) $I = \int_0^\pi |\cos x| dx$

$$I = 2 \int_0^{\pi/2} \cos x$$

$$I = 2 [\sin x]_0^{\pi/2}$$

$$I = 2 \left[\sin \frac{\pi}{2} - \sin 0 \right]$$

$$I = 2[1 - 0] = 2$$

58. (B) $S = \frac{1}{9.12} + \frac{1}{12.15} + \frac{1}{15.18} + \dots$ upto 10 terms

$$S = \frac{1}{9.12} + \frac{1}{12.15} + \frac{1}{15.18} + \dots + \frac{1}{36.39}$$

$$S = \frac{1}{3} \left[\left(\frac{1}{9} - \frac{1}{12} \right) + \left(\frac{1}{12} - \frac{1}{15} \right) + \dots + \left(\frac{1}{36} - \frac{1}{39} \right) \right]$$

$$S = \frac{1}{3} \left[\frac{1}{9} - \frac{1}{39} \right]$$

$$S = \frac{1}{3} \times \frac{39-9}{9 \times 39}$$

$$S = \frac{1}{3} \times \frac{30}{9 \times 39} = \frac{10}{351}$$

59. (D) $\lim_{x \rightarrow 0} \frac{\sin x + \cos x - 1}{\tan x}$ $\left[\frac{0}{0} \right]$ from

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\cos x - \sin x}{\sec^2 x}$$

$$\Rightarrow \frac{\cos 0 - \sin 0}{\sec^2 0}$$

$$\Rightarrow \frac{1-0}{1} = 1$$

60. (C) Ratio of angles = 8 : 5 : 2

Let Angles = 8x, 5x, 2x

Now, $8x + 5x + 2x = 180$

$$\Rightarrow 15x = 180 \Rightarrow x = 12$$

Angles = 96, 60, 24

Now, $\cos 96 + \cos 60 + \cos 24$

$$\Rightarrow \cos 96 + \cos 24 + \cos 60$$

$$\Rightarrow 2 \cos \frac{96+24}{2} \cdot \cos \frac{96-24}{2} + \frac{1}{2}$$

$$\Rightarrow 2 \cos 60 \cdot \cos 36 + \frac{1}{2}$$

$$\Rightarrow 2 \times \frac{1}{2} \cos 36 + \frac{1}{2}$$

$$\Rightarrow \frac{\sqrt{5}+1}{4} + \frac{1}{2} = \frac{\sqrt{5}+3}{4}$$

61. (C) $\frac{\cot \theta}{1 + \sin \theta} - \frac{\tan \theta}{1 + \cos \theta}$

$$\Rightarrow \frac{\cot \theta(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} - \frac{\tan \theta(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$\Rightarrow \frac{\cos \theta(1 - \sin \theta)}{\sin \theta \times \cos^2 \theta} - \frac{\sin \theta(1 - \cos \theta)}{\cos \theta \times \sin^2 \theta}$$

$$\Rightarrow \frac{1 - \sin \theta}{\sin \theta \cdot \cos \theta} - \frac{1 - \cos \theta}{\sin \theta \cdot \cos^2 \theta}$$

$$\Rightarrow \frac{1 - \sin \theta - 1 + \cos \theta}{\sin \theta \cdot \cos \theta}$$

$$\Rightarrow \frac{\cos \theta - \sin \theta}{\sin \theta \cdot \cos \theta} = \operatorname{cosec} \theta - \sec \theta$$

62. (C) Equations $2x + y + 2z = 4$, $4x + y + 2z = 6$ and $5x - 3y - z = 11$
Using elementary method

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 4 & 1 & 2 & 6 \\ 5 & -3 & -1 & 11 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 - \frac{5}{2}R_1$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & -\frac{11}{2} & -6 & 1 \end{array} \right]$$

$$R_3 \rightarrow 2R_3$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & -11 & -12 & 2 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 11R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 10 & 24 \end{array} \right]$$

Rank A = 3 and Rank [A/B] = 3
Hence solution is consistent with an unique solution.

63. (B) Number of elements in set B = 4
Number of subsets of a set B = $2^4 = 16$
Number of subsets of set A = $16 + 48 = 64 = 2^6$
Hence no. of elements in set A = 6

64. (A) **Statement I**

In a leap year = 366 days
= 52 weeks and 2 days

The probability = $\frac{2}{7}$

In a normal year = 365 days = 52 weeks and 1 days

The probability = $\frac{1}{7}$

Statement I is correct.

Statement II

In month of July = 31 days = 28 + 3

The probability = $\frac{3}{7}$

In month of June = 30 days = 28 + 2

The probability = $\frac{2}{7}$

Statement II is incorrect.

65. (C) $4 \sin x \cdot \sin\left(\frac{\pi}{3} + x\right) \cdot \sin\left(\frac{\pi}{3} - x\right)$

$$\Rightarrow 2 \sin x \cdot \left[2 \sin\left(\frac{\pi}{3} + x\right) \cdot \sin\left(\frac{\pi}{3} - x\right) \right]$$

$$\Rightarrow 2 \sin x \left[\cos\left(\frac{\pi}{3} + x - \frac{\pi}{3} + x\right) \cdot \cos\left(\frac{\pi}{3} + x + \frac{\pi}{3} - x\right) \right]$$

$$\Rightarrow 2 \sin x \left[\cos 2x - \cos \frac{2\pi}{3} \right]$$

$$\Rightarrow 2 \sin x \cdot \cos 2x - 2 \sin x \cdot \cos \frac{2\pi}{3}$$

$$\Rightarrow \sin(x + 2x) + \sin(x - 2x) - 2 \sin x \left(\frac{-1}{2}\right)$$

$$\Rightarrow \sin 3x - \sin x + \sin x = \sin 3x$$

66. (D) $I = \int \frac{\sin x}{\cos(x+a)} dx$

Let $x + a = t \Rightarrow x = t - a \Rightarrow dx = dt$

$$I = \int \frac{\sin(t-a)}{\cos t} dt$$

$$I = \int \frac{\sin t \cdot \cos a - \cos t \cdot \sin a}{\cos t} dt$$

$$I = \cos a \int \tan t dt - \sin a \int 1 dt$$

$$I = \cos a \cdot \log \sec(x+a) - \sin a \cdot (x+a) + c$$

$$I = \cos a \cdot \log \sec(x+a) - x \sin a - a \cdot \sin a + c$$

$$I = \cos a \cdot \log \sec(x+a) - x \sin a + c$$

67. (C) $\frac{1+2i\cos\theta}{1-2i\cos\theta}$

$$\Rightarrow \frac{1+2i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta}$$

$$\Rightarrow \frac{1+4i^2\cos^2\theta+4i\cos\theta}{1-4i^2\cos^2\theta}$$

$$\Rightarrow \frac{1-4\cos^2\theta+4i\cos\theta}{1+4\cos^2\theta}$$

it is purely imaginary,
then $1-4\cos^2\theta=0$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

68. (A) $y = \cos(\ln x)$... (i)
On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = -\sin(\ln x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -\sin(\ln x)$$

Again, differentiating

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 1 = -\cos(\ln x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y \quad [\text{from eq(i)}]$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

69. (C) $\frac{\log_{81} 27 + \log_{64} 16}{\log_3 9 + \log_{16} 2}$

$$\Rightarrow \frac{\log_{3^4} 3^3 + \log_{4^3} 4^2}{\log_3 3^2 + \log_{2^4} 2}$$

$$\Rightarrow \frac{\frac{3}{4} + \frac{2}{3}}{2 + \frac{1}{4}} \quad \left[\because \log_{a^b} a^c = \frac{c}{b} \right]$$

$$\Rightarrow \frac{\frac{17}{12}}{\frac{9}{4}} = \frac{17}{27}$$

70. (C) Given roots are -14 and -4
equation $(x+14)(x+4)=0$
 $x^2+18x+56=0$
New equation
 $x^2-15x+56=0$
 $(x-7)(x-8)=0$
 $x=7, 8$
Hence roots of new equation are 7, 8.

71. (C) $\frac{\sin 3x + \sin 5x}{\sin 3x - \sin 5x}$

$$\Rightarrow \frac{2\sin \frac{3x+5x}{2} \cdot \cos \frac{3x-5x}{2}}{2\cos \frac{3x+5x}{2} \cdot \sin \frac{3x-5x}{2}}$$

$$\Rightarrow \frac{\sin 4x \cdot \cos x}{\cos 4x \cdot (-\sin x)} \Rightarrow \frac{-\tan 4x}{\tan x}$$

72. (A) Rolle's Theorem-
- (i) $f(x)$ is continuous on a closed interval $[a, b]$.
 - (ii) $f(x)$ is differentiable on an open interval (a, b) .
 - (iii) $f(a) = f(b)$
 - (iv) $f'(c) = 0$

Given that $f(x) = 3x^3 + ax^2 + 2bx$
 $f'(x) = 9x^2 + 2ax + 2b$

- (i) Function is continuous on a interval $[-1, 1]$.
- (ii) Function is differentiable on a interval $(-1, 1)$.
- (iii) $f(-1) = f(1)$

$$\Rightarrow 3(-1)^3 + a(-1)^2 + 2b(-1) = 3(1)^3 + a(1)^2 + 2b \times 1$$

$$\Rightarrow -3 + a - 2b = 3 + a + 2b$$

$$\Rightarrow -6 = 4b \Rightarrow b = \frac{-3}{2}$$

- (iv) $f'(c) = 0$
 $\Rightarrow 9c^2 + 2ac + 2b = 0$

$$c = \frac{-1}{2} \text{ and } b = \frac{-3}{2}$$

$$\Rightarrow 9\left(\frac{-1}{2}\right)^2 + 2a\left(\frac{-1}{2}\right) + 2\left(\frac{-3}{2}\right) = 0$$

$$\Rightarrow \frac{9}{4} - a - 3 = 0 \Rightarrow a = \frac{-3}{4}$$

$$\text{Now, } 2a - 3b = 2\left(\frac{-3}{4}\right) - 3 \times \left(\frac{-3}{2}\right)$$

$$\Rightarrow 2a - 3b = \frac{-3}{2} + \frac{9}{2} = 3$$

73. (C) $\frac{1+x+iy}{1-x-iy}$

$$\Rightarrow \frac{1+x+iy}{1-x-iy} \times \frac{1-x+iy}{1-x+iy}$$

$$\Rightarrow \frac{(1+iy)^2 - x^2}{(1-x)^2 - (iy)^2}$$

$$\Rightarrow \frac{1+i^2y^2+2iy-x^2}{1+x^2-2x-i^2y^2}$$

$$\Rightarrow \frac{1-y^2+2iy-x^2}{1+x^2-2x+y^2}$$

$$\Rightarrow \frac{x^2+2iy-x^2}{1+1-2x} \quad [\because x^2+y^2=1]$$

$$\Rightarrow \frac{2iy}{2(1-x)} = \frac{iy}{1-x}$$

74. (B) $f(x) = \begin{cases} 2ax+7b, & x < -3 \\ 4, & x = -3 \text{ is continuous} \\ a+bx, & x > -3 \end{cases}$

at $x = -3$,

then $\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^+} f(x) = f(-3)$

Now, $\lim_{x \rightarrow -3^-} f(x) = f(-3)$

$$\Rightarrow \lim_{x \rightarrow -3^-} 2ax + 4b = 4$$

$$\Rightarrow 2a(-3) + 4b = 4$$

$$\Rightarrow -6a + 4b = 4 \quad \dots(i)$$

and $\lim_{x \rightarrow -3^+} f(x) = f(-3)$

$$\Rightarrow \lim_{x \rightarrow -3^+} a + bx = 4$$

$$\Rightarrow a + b(-3) = 4$$

$$\Rightarrow a - 3b = 4 \quad \dots(ii)$$

On solving eq(i) and eq(ii)

$$a = -2, b = -2$$

75. (B) $\cot^{-1} \frac{24}{7} + \sin^{-1} \frac{3}{5}$

$$\Rightarrow \tan^{-1} \frac{7}{24} + \tan^{-1} \frac{3}{4} \quad \left[\because \sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} \right]$$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{7}{24} + \frac{3}{4}}{1 - \frac{7}{24} \times \frac{3}{4}} \right]$$

$$\Rightarrow \tan^{-1} \left[\frac{100}{75} \right] = \tan^{-1} \left(\frac{4}{3} \right)$$

76. (C) $\sqrt{5+2\sqrt{6}} = (\sqrt{3} + \sqrt{2})$

77. (D) Total terms in the expansion = 4

78. (C) sphere $x^2 + y^2 + z^2 + 6x + 5y + 3z - 5 = 0$
On comparing with general equation

$$u = 3, v = \frac{5}{2}, w = \frac{3}{2}, d = -5$$

$$\text{radius} = \sqrt{u^2 + v^2 + w^2 - d}$$

$$= \sqrt{(3)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 5}$$

$$= \sqrt{9 + \frac{25}{4} + \frac{9}{4} + 5}$$

$$= \sqrt{\frac{90}{4}} = \sqrt{\frac{45}{2}} = \frac{3\sqrt{5}}{\sqrt{2}}$$

79. (B) $I = \int x \cos x \, dx$

$$I = x \int \cos x \, dx - \int \left\{ \frac{d}{dx}(x) \cdot \int \cos x \, dx \right\} dx$$

$$I = x \sin x - \int 1 \cdot \sin x \, dx$$

$$I = x \sin x + \cos x + c$$

80. (C) 'PROBABILITY'

$$\text{No. of permutation} = \frac{11!}{2!2!} = 9979200$$

81. (A) Equation $3x^2 - 5x + 1 = 0$

roots are α and β , then

$$\alpha + \beta = \frac{5}{3} \text{ and } \alpha\beta = \frac{1}{3}$$

$$\text{Now, } (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\Rightarrow (\alpha - \beta)^2 = \frac{25}{9} - \frac{4}{3}$$

$$\Rightarrow (\alpha - \beta)^2 = \frac{13}{9} \Rightarrow \alpha - \beta = \frac{\sqrt{13}}{3}$$

$$\text{then } \frac{\alpha - \beta}{\beta - \alpha} = \frac{\alpha^2 - \beta^2}{\alpha\beta}$$

$$\Rightarrow \frac{\alpha - \beta}{\beta - \alpha} = \frac{(\alpha + \beta)(\alpha - \beta)}{\alpha\beta}$$

$$\Rightarrow \frac{\alpha - \beta}{\beta - \alpha} = \frac{\frac{5}{3} \times \frac{\sqrt{13}}{3}}{\frac{1}{3}} = \frac{5\sqrt{13}}{3}$$

82. (D) Let $y = x^2 \ln \sin x$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = x^2 \frac{d}{dx} (\ln \sin x) + (\ln \sin x) \times 2x$$

$$\frac{dy}{dx} = x^2 \cdot \frac{\cos x}{\sin x} + (\ln \sin x) \times 2x$$

$$\frac{dy}{dx} = x^2 \cdot \cot x + 2x \ln \sin x$$

83. (A) $\tan^{-1} \frac{1}{7} + \sin^{-1} \frac{7}{25}$

$$\Rightarrow \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{7}{24} \left[\because \sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} \right]$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{7} + \frac{7}{24}}{1 - \frac{1}{7} \times \frac{7}{24}} \right) = \tan^{-1} \left(\frac{73}{161} \right)$$

84. (A) $A = \{x \in \mathbb{R} : x^2 + 4x + 3 < 0\}$
 $A = \{x \in \mathbb{R} : -3 < x < -1\}$
 $B = \{x \in \mathbb{R} : x^2 - 7x + 12 > 0\}$
 $B = \{x \in \mathbb{R} : -\infty < x < 3 \text{ and } 4 < x < \infty\}$

Statement 1

$$A \cap B = \{x \in \mathbb{R} : -3 < x < -1\}$$

Statement 1 is correct.

Statement 2

$$A - B = \{\emptyset\}$$

Statement 2 is incorrect.

85. (B) $\sqrt{\frac{\omega}{1+\omega^2}}$

$$\Rightarrow \sqrt{\frac{\omega}{-\omega}} \quad [\because 1 + \omega + \omega^2 = 0]$$

$$\Rightarrow \sqrt{\frac{1}{-1}} = \sqrt{-1} = i$$

86. (D) We know that

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$A = 22 \frac{1}{2}$$

$$\Rightarrow \tan 45 = \frac{2 \tan 22 \frac{1}{2}}{1 - \tan^2 22 \frac{1}{2}}$$

$$\Rightarrow 1 = \frac{2 \tan 22 \frac{1}{2}}{1 - \tan^2 22 \frac{1}{2}}$$

$$\Rightarrow \tan^2 22 \frac{1}{2} + 2 \tan 22 \frac{1}{2} - 1 = 0$$

$$\Rightarrow \tan 22 \frac{1}{2} = \frac{-2 \pm \sqrt{(-2)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$\Rightarrow \tan 22 \frac{1}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow \tan 22 \frac{1}{2} = -1 \pm \sqrt{2}$$

$$\text{Hence } \tan 22 \frac{1}{2} = \sqrt{2} - 1$$

87. (B) $\frac{dy}{dx} + y \cdot \tan x = \sec x$

On comparing with general equation
 $P = \tan x$ and $Q = \sec x$

$$\text{I.F.} = e^{\int P dx}$$

$$\text{I.F.} = e^{\int \tan x dx}$$

$$\text{I.F.} = e^{\log \sec x} = \sec x$$

Solution of the differential equation

$$y \times \text{I.F.} = Q \times \text{I.F.} dx$$

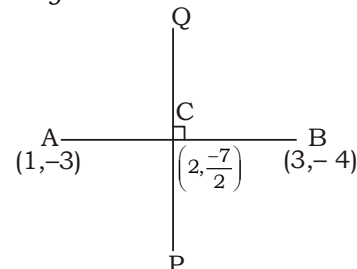
$$\Rightarrow y \times \sec x = \int \sec x \cdot \sec x dx$$

$$\Rightarrow y \sec x = \tan x + c$$

$$\Rightarrow \frac{y}{\cos x} = \frac{\sin x}{\cos x} + c$$

$$\Rightarrow y = \sin x + c \cdot \cos x$$

88. (C)



Mid-point of line joining the points

$$= \left(\frac{1+3}{2}, \frac{-3-4}{2} \right) = \left(2, \frac{-7}{2} \right)$$

$$\text{Slope of line AB } (m_1) = \frac{-4+3}{3-1} = \frac{-1}{2}$$

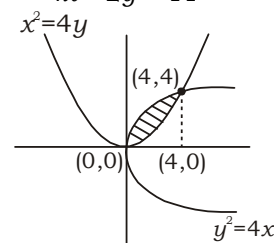
$$\text{Slope of line PQ } (m_2) = \frac{-1}{\frac{-1}{2}} = 2$$

equation of line PQ

$$y + \frac{7}{2} = 2(x - 2)$$

$$\Rightarrow 4x - 2y = 11$$

89. (C)



Curve

$$y_1 \Rightarrow y = 2\sqrt{x}$$

$$y_2 \Rightarrow y = \frac{x^2}{4}$$

The required Area (A) = $\int_0^4 (y_1 - y_2) dx$

$$A = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$$

$$A = \left[2 \times \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{4 \times 3} \right]_0^4$$

$$A = \left[\frac{4}{3} \times (4)^{3/2} - \frac{1}{12} (4)^3 - 0 - 0 \right]$$

$$A = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq. unit}$$

90. (A) $I = \int \frac{1}{\sqrt{1 - \sin x}} dx$

$$I = \frac{1}{\sqrt{1 - \cos\left(\frac{\pi}{2} - x\right)}} dx$$

$$I = \frac{1}{\sqrt{2 \sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}} dx$$

$$I = \frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) dx$$

$$I = \frac{1}{\sqrt{2}} \frac{\log \left| \operatorname{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) - \cot\left(\frac{\pi}{4} - \frac{x}{2}\right) \right|}{\frac{-1}{2}} + c$$

$$I = \sqrt{2} \log \left| \operatorname{cosec}\left(\frac{\pi}{4} - \frac{x}{2}\right) + \cot\left(\frac{\pi}{4} - \frac{x}{2}\right) \right| + c$$

91. (D) Equation of rectangular hyperbola

$$x^2 - y^2 = 1$$

$$a = 1, b = 1$$

$$\text{Now, } e^2 = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow e^2 = 1 + 1 \Rightarrow e = \sqrt{2}$$

92. (A) eccentricity $e = \frac{1}{\sqrt{2}}$ and distance

$$\text{between foci } 2ae = \sqrt{3}$$

$$\Rightarrow 2a \times \frac{1}{\sqrt{2}} = \sqrt{3} \Rightarrow a = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\text{then } e^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{1}{2} = 1 - \frac{2b^2}{3}$$

$$\Rightarrow \frac{2b^2}{3} = \frac{1}{2} \Rightarrow b^2 = \frac{3}{4}$$

equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{2x^2}{3} + \frac{4y^2}{3} = 1$$

$$\Rightarrow 2x^2 + 4y^2 = 3$$

93. (C) In the expansion of $\left(x^3 - \frac{1}{2x^2}\right)^{13}$

$$T_{r+1} = {}^{13}C_r (x^3)^{13-r} \left(\frac{-1}{2x^2}\right)^r$$

$$T_{r+1} = {}^{13}C_r (-1)^r x^{39-5r} \left(\frac{1}{2}\right)^r$$

$$\text{Now, } 39 - 5r = 9$$

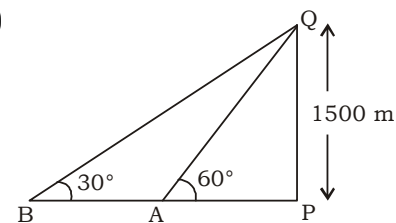
$$\Rightarrow 5r = 30 \Rightarrow r = 6$$

$$\text{Coefficient of } x^9 = {}^{13}C_6 (-1)^6 \left(\frac{1}{2}\right)^6$$

$$= \frac{13!}{6!7!} \times \frac{1}{64}$$

$$= \frac{13 \times 11 \times 3 \times 4}{64} = \frac{429}{16}$$

94. (B)



Let AP = x m

In ΔAPQ :-

$$\tan 60^\circ = \frac{PQ}{AP}$$

$$\Rightarrow \sqrt{3} = \frac{1500}{x} \Rightarrow x = \frac{1500}{\sqrt{3}} \quad \dots(i)$$

In ΔBPQ :-

$$\tan 30^\circ = \frac{PQ}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500}{x + AB}$$

$$\Rightarrow x + AB = 1500\sqrt{3}$$

$$\Rightarrow AB = 1500\sqrt{3} - x$$

$$\Rightarrow AB = 1500\sqrt{3} - \frac{1500}{\sqrt{3}}$$

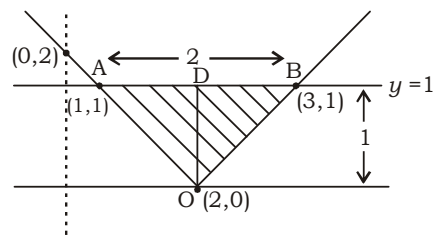
$$\Rightarrow AB = 1500 \times \frac{2}{\sqrt{3}}$$

time taken by aeroplane A to B = 20 sec

$$\text{Now, speed of aeroplane} = \frac{1500 \times \frac{2}{\sqrt{3}}}{20} \text{ m/sec}$$

$$= 50\sqrt{3} \text{ m/sec}$$

95. (C)

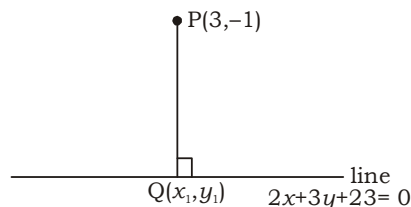


$$y = |x - 2|$$

$$\text{Area} = \frac{1}{2} \times OD \times AB$$

$$= \frac{1}{2} \times 1 \times 2 = 1 \text{ sq. unit}$$

96. (C)



Let $Q = (x_1, y_1)$

Line $2x + 3y + 23 = 0$

$$\text{slope of line } (m_1) = \frac{-2}{3}$$

$$\text{slope of line } PQ(m_2) = \frac{y_1 + 1}{x_1 - 3}$$

$$\text{Now, } m_1 \times m_2 = -1$$

$$\Rightarrow \frac{-2}{3} \times \frac{y_1 + 1}{x_1 - 3} = -1$$

$$\Rightarrow -2y_1 - 2 = -3x_1 + 9$$

$$\Rightarrow 3x_1 - 2y_1 = 11 \quad \dots(i)$$

Point $Q(x_1, y_1)$ passes through the line $2x + 3y + 23 = 0$

$$\text{then } 2x_1 + 3y_1 + 23 = 0 \quad \dots(ii)$$

from eq(i) and eq(ii)

$$x = -1 \text{ and } y = -7$$

Hence co-ordinate of $Q = (-1, -7)$

97. (C) Equations $2x + y + z = 4$, $6x + 7y + 11z = 2$ and $2x - 3y + z = 5$

$$\text{Let } A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 7 & 11 \\ 2 & -3 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

Using elementary method

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 6 & 7 & 11 & 2 \\ 2 & -3 & 1 & 5 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 3R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 0 & 4 & 8 & -10 \\ 0 & -4 & 0 & 1 \end{array} \right]$$

$$R_3 \rightarrow R_3 + R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 2 & 1 & 1 & 4 \\ 0 & 4 & 8 & -10 \\ 0 & 0 & 8 & -9 \end{array} \right]$$

$$\text{Rank}(A) = \text{Rank}(A/B)$$

Hence equation have unique solution.

98. (D) $4 \times n!$, $3 \times (n+1)!$ and $2 \times (n+2)!$ are in G.P,

$$\text{then } [3 \times (n+1)!]^2 = (4 \times n!) \times (2 \times (n+2)!)$$

$$\Rightarrow 9 \times (n+1)! \times (n+1)! = 4 \times n! \times 2 \times (n+2)!$$

$$\Rightarrow 9 \times (n+1)n! \times (n+1)! = 4 \times n! \times 2 \times (n+2)(n+1)!$$

$$\Rightarrow 9(n+1) = 8(n+2)$$

$$\Rightarrow 9n + 9 = 8n + 16 \Rightarrow n = 7$$

99. (D) $m = \tan \theta = \tan 30 = \frac{1}{\sqrt{3}}$ and $c = 52$

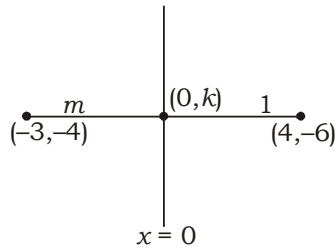
The equation of line

$$y = mx + c$$

$$\Rightarrow y = \frac{1}{\sqrt{3}}x + 52$$

$$\Rightarrow \sqrt{3}y - x = 52\sqrt{3}$$

100. (B)



Let the $x = 0$ divides the line joining the points $(-3, -4)$ and $(4, -6)$ in the ratio $m : 1$,

$$\text{then } \frac{4m - 3}{m + 1} = 0$$

$$\Rightarrow m = \frac{3}{4}$$

The required ratio = $3 : 4$

101. (C) $(3x + 4y - 5) + \lambda(5x - y + 11) = 0$
 $(3 + 5\lambda)x + (4 - \lambda)y - 5 + 11\lambda = 0$

$$y = \frac{-(3 + 5\lambda)}{(4 - \lambda)}x + \frac{5 - 11\lambda}{4 - \lambda}$$

$$\text{Slope } m = \frac{-(3 + 5\lambda)}{4 - \lambda}$$

Given straight line parallel to x -axis i.e $\theta = 0 \Rightarrow m = 0$

$$\text{then } \frac{-(3 + 5\lambda)}{4 - \lambda} = 0$$

$$\Rightarrow \lambda = \frac{-3}{5}$$

102. (A) $A = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 20 & 24 \\ 24 & 32 \end{bmatrix}$

From option A

$$A^2 - 6A - 8I = \begin{bmatrix} 20 & 24 \\ 24 & 32 \end{bmatrix} - 6 \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 - 6A - 8I = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$A^2 - 6A - 8I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 6A - 8I = 0$$

103. (D) $\begin{vmatrix} 1 & \omega & 3\omega^2 \\ 3 & 3\omega^2 & 9\omega^3 \\ 2 & 2\omega^3 & 6\omega^4 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 1 & \omega & 3\omega^2 \\ 3 & 3\omega^2 & 9 \\ 2 & 2 & 6\omega \end{vmatrix}$$

$$\begin{aligned} &\Rightarrow 1(18\omega^3 - 18) - \omega(18\omega - 18) + 3\omega^2(6 - 6\omega^2) \\ &\Rightarrow 1(18 - 18) - 18\omega^2 + 18\omega + 18\omega^2 - 18\omega^4 \\ &\Rightarrow 0 + 18\omega - 18\omega = 0 \end{aligned}$$

104. (B) $z = 1 + \cos \frac{\pi}{12} + i \sin \frac{\pi}{12}$

$$z = 2 \cos^2 \frac{\pi}{24} + i \times 2 \sin \frac{\pi}{24} \times \cos \frac{\pi}{24}$$

$$z = 2 \cos \frac{\pi}{24} \left[\cos \frac{\pi}{24} + i \sin \frac{\pi}{24} \right]$$

$$\text{Hence } |z| = 2 \cos \frac{\pi}{24}$$

105. (C) $z = \left[\frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \right]^3$

$$z = \left[\frac{(1 + \sqrt{3}i)(1 + \sqrt{3}i)}{(1 - \sqrt{3}i)(1 + \sqrt{3}i)} \right]^3$$

$$z = \left[\frac{1 + 3i^2 + 2\sqrt{3}i}{1 - 3i^2} \right]^3$$

$$z = \left[\frac{-2 + 2\sqrt{3}i}{4} \right]^3$$

$$z = \left[\frac{-1 + \sqrt{3}i}{2} \right]^3$$

$$z = \omega^3 = 1$$

106. (B) Sum of n terms

$$S_n = n^2 + 3n \quad \dots(i)$$

$$S_{n-1} = (n-1)^2 + 3(n-1)$$

$$S_{n-1} = n^2 + n - 2 \quad \dots(ii)$$

n^{th} term of the series

$$T_n = S_n - S_{n-1}$$

$$T_n = (n^2 + 3n) - (n^2 + n - 2)$$

$$T_n = 2n + 2$$

107. (B) $\sqrt{(4 - \sqrt{5})} = \sqrt{\frac{8 - 2\sqrt{15}}{2}}$

$$= \sqrt{\frac{(\sqrt{5} - \sqrt{3})^2}{2}}$$

$$= \frac{\sqrt{5} - \sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{10} - \sqrt{6}}{2}$$

108. (C) Equation whose roots 3 and -15

$$(x - 3)(x + 15) = 0$$

$$x^2 + 12x - 45 = 0$$

Now, original equation

$$x^2 + 4x - 45 = 0$$

Roots of original equation are -9 and 5.

109. (D) Series $0.8 + 0.08 + 0.008 + \dots \infty$

$$\text{Sum of the series} = \frac{0.8}{1 - 0.1}$$

$$= \frac{0.8}{0.9} = \frac{8}{9}$$

110. (B) Vectors $-\hat{i} + m\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$

Angle between the vectors

$$\cos\theta = \frac{-1 \times 1 + m \times 1 + 1 \times 1}{\sqrt{(-1)^2 + m^2 + 1^2} \sqrt{1^2 + (-1)^2 + 1^2}}$$

$$\Rightarrow \cos \frac{\pi}{3} = \frac{m}{\sqrt{m^2 + 2} \times \sqrt{3}}$$

$$\Rightarrow \frac{1}{2} = \frac{m}{\sqrt{3}\sqrt{m^2 + 2}}$$

$$\Rightarrow \frac{1}{4} = \frac{m^2}{3(m^2 + 2)}$$

$$\Rightarrow m^2 = 6 \Rightarrow m = \sqrt{6}$$

111. (C) $n(S) = 6 \times 6 = 36$

$$E = \{(6, 3), (3, 6), (5, 4), (4, 5)\}$$

$$n(E) = 4$$

$$\text{The required Probability} = \frac{4}{36} = \frac{1}{9}$$

112. (A) Days (in February 2017) = 28

The required Probability = 0

113. (D) No. of diagonals = $\frac{n(n-3)}{2}$

$$n = 11$$

$$\text{No. of diagonals} = \frac{11 \times 8}{2} = 44$$

114. (D) $\frac{\sin^2 \frac{3A}{2} - \cos^2 \frac{3A}{2}}{\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2}}$

$$\Rightarrow \left(\frac{\sin \frac{3A}{2}}{\sin \frac{A}{2}} \right)^2 - \left(\frac{\cos \frac{3A}{2}}{\cos \frac{A}{2}} \right)^2$$

$$\Rightarrow \left(\frac{3 \sin \frac{A}{2} - 4 \sin^3 \frac{A}{2}}{\sin \frac{A}{2}} \right)^2 - \left(\frac{4 \cos^3 \frac{A}{2} - 3 \cos \frac{A}{2}}{\cos \frac{A}{2}} \right)^2$$

$$\Rightarrow \left(3 - 4 \sin^2 \frac{A}{2} \right)^2 - \left(4 \cos^2 \frac{A}{2} - 3 \right)^2$$

$$\Rightarrow 9 + 16 \sin^4 \frac{A}{2} - 24 \sin^2 \frac{A}{2} - 16 \cos^4 \frac{A}{2} - 9$$

$$+ 24 \cos^2 \frac{A}{2}$$

$$\Rightarrow 16 \sin^4 \frac{A}{2} - 16 \cos^4 \frac{A}{2} - 24 \left(\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2} \right)$$

$$\Rightarrow 16 \left(\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2} \right) \left(\sin^2 \frac{A}{2} + \cos^2 \frac{A}{2} \right)$$

$$- 24 \left(\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2} \right)$$

$$\Rightarrow \left(\sin^2 \frac{A}{2} - \cos^2 \frac{A}{2} \right) (16 - 24)$$

$$\Rightarrow 8 \cos A$$

115. (C) We know that

$$\cos^2 A = 1 - 2 \sin^2 A$$

$$\Rightarrow 2 \sin^2 A = 1 - \cos^2 A$$

$$A = 22 \frac{1}{2}$$

$$\Rightarrow 2 \sin^2 22 \frac{1}{2} = 1 - \cos 45$$

$$\Rightarrow 2 \sin^2 22 \frac{1}{2} = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin^2 22 \frac{1}{2} = \frac{\sqrt{2} - 1}{2\sqrt{2}}$$

$$\Rightarrow \sin 22 \frac{1}{2} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

$$\text{then } \cos \left(247 \frac{1}{2} \right) = \cos \left(270 - 22 \frac{1}{2} \right)$$

$$= - \sin 22 \frac{1}{2}$$

$$= - \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

116. (A) $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \dots(i)$

Prop.IV $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \dots(ii)$$

from eq(i) and eq(ii)

$$I + I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

117. (C) $y = x^2 - e^x$
On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = 2x - e^x$$

$$\frac{dx}{dy} = \frac{1}{2x - e^x} \quad \dots(i)$$

On differentiating both side w.r.t. 'y'

$$\frac{d^2x}{dy^2} = (-1)(2x - e^x)^{-1-1} (2 - e^x) \frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = \frac{-1}{(2x - e^x)^2} (2 - e^x) \times \frac{1}{(2x - e^x)}$$

$$\frac{d^2x}{dy^2} = \frac{e^x - 2}{(2x - e^x)^3}$$

118. (B) $x = g(t)$ and $y = f(t)$

$$\Rightarrow \frac{dx}{dt} = g'(t), \frac{dy}{dt} = f'(t)$$

$$\Rightarrow \frac{d^2x}{dt^2} = g''(t), \frac{d^2y}{dt^2} = f''(t)$$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{f'(t)}{g'(t)}$$

On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{[g'(t)f''(t) - f'(t) \cdot g''(t)] \frac{dt}{dx}}{\{g'(t)\}^2}$$

Given that $\frac{d^2y}{dx^2} = 0$

$$\Rightarrow g'(t) f''(t) - f'(t) \cdot g''(t) = 0$$

$$\Rightarrow \frac{dx}{dt} \cdot \frac{d^2y}{dx^2} - \frac{dy}{dt} \cdot \frac{d^2x}{dt^2} = 0$$

$$\Rightarrow \frac{dx}{dt} \cdot \frac{d^2y}{dx^2} = \frac{dy}{dt} \cdot \frac{d^2x}{dt^2}$$

119. (A) Given that $y = a^{x \log_a \tan x}$

$$\Rightarrow y = a^{\log_a (\tan x)^x}$$

$$\Rightarrow y = (\tan x)^x$$

On taking log both side

$$\Rightarrow \log y = x \log \tan x$$

On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = x \cdot \frac{\sec^2 x}{\tan x} + \log \tan x \times 1$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{2x}{\sin 2x} + \log \tan x$$

$$\Rightarrow \frac{dy}{dx} = y(2x \cdot \operatorname{cosec} 2x + \log \tan x)$$

120. (D) Let $y = \log_3 x$ and $z = \log_x 3$

$$\Rightarrow y = \frac{\log x}{\log 3}, \quad z = \frac{\log 3}{\log x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x \log 3}, \quad \frac{dz}{dx} = -(\log 3)(\log x)^{-2} \times \frac{1}{x}$$

$$\frac{dz}{dx} = \frac{-\log 3}{x(\log x)^2}$$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$$\Rightarrow \frac{dy}{dz} = \frac{1}{x \log 3} \times \frac{-x(\log x)^2}{\log 3}$$

$$\Rightarrow \frac{dy}{dz} = -\left(\frac{\log x}{\log 3}\right)^2 = -(\log_3 x)^2$$

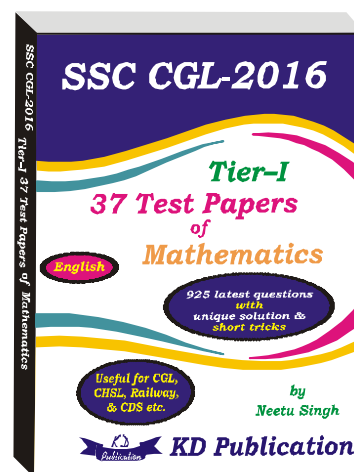
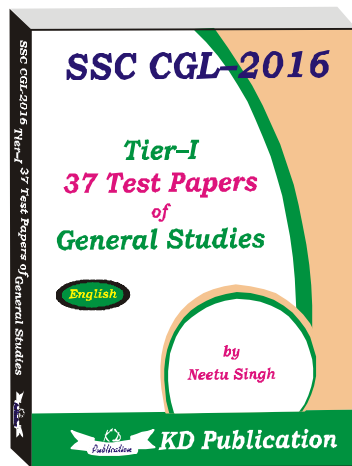


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NDA (MATHS) MOCK TEST - 136 (Answer Key)

1. (C)	21. (A)	41. (C)	61. (C)	81. (A)	101. (C)
2. (B)	22. (B)	42. (D)	62. (C)	82. (D)	102. (A)
3. (C)	23. (C)	43. (A)	63. (B)	83. (A)	103. (D)
4. (D)	24. (A)	44. (C)	64. (A)	84. (A)	104. (B)
5. (C)	25. (A)	45. (B)	65. (C)	85. (B)	105. (C)
6. (D)	26. (B)	46. (C)	66. (D)	86. (D)	106. (B)
7. (C)	27. (C)	47. (C)	67. (C)	87. (B)	107. (B)
8. (C)	28. (B)	48. (A)	68. (A)	88. (C)	108. (C)
9. (B)	29. (A)	49. (B)	69. (C)	89. (C)	109. (D)
10. (C)	30. (C)	50. (C)	70. (C)	90. (A)	110. (B)
11. (D)	31. (B)	51. (B)	71. (C)	91. (D)	111. (C)
12. (B)	32. (D)	52. (C)	72. (A)	92. (A)	112. (A)
13. (C)	33. (C)	53. (B)	73. (C)	93. (C)	113. (D)
14. (B)	34. (B)	54. (C)	74. (B)	94. (B)	114. (D)
15. (D)	35. (C)	55. (A)	75. (B)	95. (C)	115. (C)
16. (A)	36. (C)	56. (D)	76. (C)	96. (C)	116. (A)
17. (C)	37. (B)	57. (C)	77. (D)	97. (C)	117. (C)
18. (B)	38. (B)	58. (B)	78. (C)	98. (D)	118. (B)
19. (A)	39. (C)	59. (D)	79. (B)	99. (D)	119. (A)
20. (B)	40. (D)	60. (C)	80. (C)	100. (B)	120. (D)



Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777