NDA MATHS MOCK TEST - 146 (SOLUTION)

1. (B) Equation whose roots are –7 and –6, then $(x + 7)(x + 6) = 0$
$\Rightarrow x^2 + 13x + 42 = 0$
Original equation
$x^2 + 17x + 42 = 0$
$\Rightarrow (x + 14)(x + 3) = 0$
Hence roots are –14 and –3.

2. (D) $A = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & -1 \\ -2 & 18 \end{bmatrix}$

Now, $A^2 + A - 14I = \begin{bmatrix} 11 & -1 \\ -2 & 18 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 2 & -4 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^2 + A - 14I = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

$\Rightarrow A^2 + A - 14I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 + A - 14I = 0$

3. (C) $\int_0^2 \{k^2 + (2+k)x + 3x^2\} dx \le 36$

$\Rightarrow \left[k^2 x + (2+k)\frac{x^2}{2} + 3 \times \frac{x^3}{3}\right]_0^2 \le 36$

$\Rightarrow 2k^2 + (2 + k)\times 2 + 8 \le 36$
$\Rightarrow 2k^2 + 2k - 24 \le 0$
$\Rightarrow (2k - 6)(k + 4) \le 0$
$\Rightarrow (k - 3)(k + 4) \le 0$
Hence $-4 \le k \le 3$

4. (C) $f(x) = \frac{1}{\sqrt{29 - x^2}} \Rightarrow f'(x) = \frac{x}{(29 - x^2)^{3/2}}$

Now, $\lim_{x\to 2} \frac{f(2) - f(x)}{x^3 - 8}$ $\quad \left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 2} \frac{-f'(x)}{3x^2}$

$\Rightarrow \lim_{x\to 2} \frac{\frac{-x}{(29 - x^2)^{3/2}}}{3x^2}$

$\Rightarrow \frac{\frac{-2}{(29 - 4)^{3/2}}}{3 \times 4}$

$\Rightarrow \frac{-2}{12 \times 125} = \frac{-1}{750}$

5. (B) Series $\frac{1}{1\times 4} + \frac{1}{4\times 7} + \frac{1}{7\times 10} + ... + \frac{1}{34\times 37}$

$\Rightarrow \frac{1}{3}\left[\left(1 - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + ... + \left(\frac{1}{34} - \frac{1}{37}\right)\right]$

$\Rightarrow \frac{1}{3}\left[1 - \frac{1}{37}\right]$

$\Rightarrow \frac{1}{3} \times \frac{36}{37} = \frac{12}{37}$

6. (A) $\overline{A} \cap B \cap C$

7. (C) $n(S) = {}^{14}C_4 = 1001$
$n(E) = {}^6C_3 \times {}^3C_1 \times {}^5C_0 + {}^6C_3 \times {}^3C_0 \times {}^5C_1 + {}^6C_4 \times {}^3C_0 \times {}^5C_0$
$n(E) = 20\times 3\times 1 + 20\times 1\times 5 + 15\times 1\times 1$
$n(E) = 60 + 100 + 15 = 175$

The required Probability $= \frac{175}{1001} = \frac{25}{143}$

8. (B)

9. (C) The required numbers $= 9\times 9\times 8\times 7$
$= 4536$

10. (A) $I = \int (x+1)e^x . \ln x$

$I = (x+1).\ln x \int e^x dx - \int\left\{\frac{d}{dx}((x+1)\ln x).\int e^x dx\right\} dx$

$I = (x+1).\ln x.(e^x) - \int\left((x+1)\times\frac{1}{x} + (\ln x).1\right)e^x\, dx$

$I = (x+1)e^x.\ln x - \int e^x dx - \int \frac{e^x}{x} dx - \int e^x.\ln x\, dx$

$I = (x+1)e^x.\ln x - e^x - \left[e^x \int \frac{1}{x}.dx - \int\left\{\frac{d}{dx}(e^x).\int \frac{1}{x} dx\right\} dx\right.$

$- \int e^x.\ln x\, dx + c$

$I = (x+1)e^x.\ln x - e^x - \left[e^x.\ln x - \int e^x.\ln x\, dx\right]$

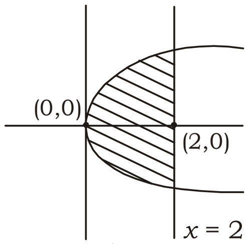
$- \int e^x.\ln x\, dx + c$

$I = (x+1)e^x.\ln x - e^x - e^x.\ln x + \int e^x.\ln x\, dx -$

$\int e^x \ln x\, dx + c$

$I = x.e^x.\ln x + e^x.\ln x - e^x - e^x.\ln x + c$
$I = x.e^x.\ln x - e^x + c$

11. (C)



curve $2y^2 = x$ and line $x = 2$

Area $= 2\int_0^2 y\,dx$

Area $= 2\int_0^2 \sqrt{\frac{x}{2}}\,dx$

$= \sqrt{2}\,.\left[\frac{x^{3/2}}{3/2}\right]_0^2$

$= \sqrt{2} \times \frac{2}{3}\left[2^{3/2} - 0\right]$

$= \sqrt{2} \times \frac{2}{3} \times 2\sqrt{2} = \frac{8}{3}$ sq.unit

12. (C) A be an 4 × 4 matrix.

$\det(\lambda A) = \lambda^r(\det A)$

then $r = 4$

13. (A) 110010 . 101

$0\times2^0 = 0$, $1\times2^1 = 2$, $0\times2^2 = 0$, $0\times2^3 = 0$, $1\times2^4 = 16$, $1\times2^5 = 32$; total $= 50$

$\frac{1}{2} = 1\times2^{-1}$, $0 = 0\times2^{-2}$, $\frac{1}{8} = 1\times2^{-3}$

$\frac{1}{2} + \frac{1}{8} = \frac{5}{8} = 0.625$

Hence $(110010.101)_2 = (50.625)_{10}$

14. (C) If 35° < 45°,

then sin35° < sin45° and cos35° > cos45°

⇒ sin35° < cos45° < cos35°

⇒ sin35° – cos35 < 0

Hence the value of sin 35° – cos45° is negative but greater than –1.

15. (A) $f(x) = \tan^{-1}\left[\frac{\sqrt{x}(1-x)}{1+x^2}\right]$

$\Rightarrow f(x) = \tan^{-1}\left[\frac{x^{1/2} - x^{3/2}}{1 + x^{1/2}.x^{3/2}}\right]$

$\Rightarrow f(x) = \tan^{-1}(x^{1/2}) - \tan^{-1}(x^{3/2})$

On differentiating both side w.r.t 'x'

$\Rightarrow f'(x) = \frac{1}{1+(x^{1/2})^2} \times \frac{1}{2\sqrt{x}} - \frac{1}{1+(x^{3/2})^2} \times \frac{3\sqrt{x}}{2}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}(1+x)} - \frac{3\sqrt{x}}{2(1+x^3)}$

$\Rightarrow f'(4) = \frac{1}{2\sqrt{4}(1+4)} - \frac{3\sqrt{4}}{2(1+4^3)}$

$\Rightarrow f'(4) = \frac{1}{2\times2\times5} - \frac{3\times2}{2\times65}$

$\Rightarrow f'(4) = \frac{1}{20} - \frac{3}{65} = \frac{1}{260}$

16. (A) $f(x) = \log\left[\frac{1+x}{1-x}\right]$ and $g(x) = \frac{3x+x^3}{1+3x^2}$

Now, $fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f\left[\frac{3x+x^3}{1+3x^2}\right]$

$\Rightarrow fog(x) = \log\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right]$

$\Rightarrow fog(x) = \log\left[\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right]$

$\Rightarrow fog(x) = \log\left[\frac{(1+x)^3}{(1-x)^3}\right]$

$\Rightarrow fog(x) = 3\log\left[\frac{1+x}{1-x}\right]$

$\Rightarrow fog(x) = 3\,f(x)$

17. (C) $I = \int_0^{\frac{\sqrt{3}}{4}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

let $\sin^{-1}x = t$ when $x = 0$, $t = 0$

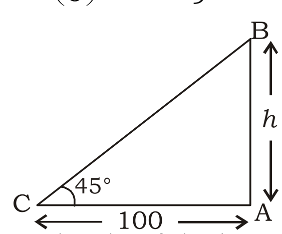
$\frac{1}{\sqrt{1-x^2}}\,dx = dt$ $x = \frac{\sqrt{3}}{4}$, $t = \frac{\pi}{3}$

$I = \int_0^{\pi/3} t\,dt$

$I = \left[t^2\right]_0^{\pi/3}$

$I = \left(\frac{\pi}{3}\right)^2 - 0 = \frac{\pi^2}{9}$

18. (A)



Let height of the lamp-post = h m

In ΔABC

$\tan 45° = \frac{AB}{AC}$

$\Rightarrow 1 = \frac{h}{100} \Rightarrow h = 100$

Height of the lamp-post = 100 m

19. (A) $\frac{\cos(x+y)}{\cos(x-y)} = \frac{a-b}{a+b}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\cos(x+y)+\cos(x-y)}{\cos(x+y)-\cos(x-y)} = \frac{a-b+a+b}{a-b-a-b}$

$\Rightarrow \frac{2\cos\left(\frac{x+y+x-y}{2}\right).\cos\left(\frac{x+y-x+y}{2}\right)}{2\sin\left(\frac{x+y+x-y}{2}\right).\sin\left(\frac{x-y-x-y}{2}\right)} = \frac{2a}{-2b}$

$\Rightarrow \frac{2\cos x.\cos y}{-2\sin x.\sin y} = \frac{-a}{b}$

$\Rightarrow \cot x.\cot y = \frac{a}{b}$

$\Rightarrow \frac{\cot y}{\tan x} = \frac{a}{b}$

20. (D) $\sin x.\frac{dy}{dx} + x = y$

$\Rightarrow \frac{dy}{dx} + \frac{x}{\sin x} = \frac{y}{\sin x}$

$\Rightarrow \frac{dy}{dx} - \frac{y}{\sin x} = \frac{-x}{\sin x}$

$\Rightarrow \frac{dy}{dx} - y.\text{cosec}x = -x.\text{cosec}x$

It is linear equation.

21. (C) Given that $\int_{-2}^{0} f(x)dx = 4$

and $\int_{-2}^{-1}[4 - f(x)]\, dx = 11$

$\Rightarrow \int_{-2}^{-1} 4\, dx - \int_{-2}^{-1} f(x)dx = 11$

$\Rightarrow 4[x]_{-2}^{-1} - \int_{-2}^{-1} f(x)dx = 11$

$\Rightarrow 4[-1-(-2)] - 11 = \int_{-2}^{-1} f(x)\, dx$

$\Rightarrow \int_{-2}^{-1} f(x)\, dx = -7$

Now, $\int_{-2}^{-1} f(x)\, dx + \int_{-1}^{0} f(x)dx = \int_{-2}^{0} f(x)$

$\Rightarrow -7 + \int_{-1}^{0} f(x)\, dx = 4$

$\Rightarrow \int_{-1}^{0} f(x)\, dx = 11$

$\Rightarrow \int_{-1}^{0} 2f(x)\, dx = 2\times 11 = 22$

22. (B) Let one number = 10

other = $10 - x$

Now, $A = x^3(10 - x)^2$

$\Rightarrow A = x^3(100 + x^2 - 20x)$

$\Rightarrow A = 100x^3 + x^5 - 20x^4$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dA}{dx} = 300x^2 + 5x^4 - 80x^3$

Again, differentiating

$\Rightarrow \frac{d^2A}{dx^2} = 600x + 20x^3 - 240x^2$

$\Rightarrow \frac{d^2A}{dx^2} = 20x(x^2 - 12x + 30)$

For maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 300x^2 + 5x^4 - 80x^3 = 0$

$\Rightarrow 5x^2(x^2 - 16x + 60) = 0$

$\Rightarrow x^2(x-10)(x-6) = 0$

$x = 0,\ x = 6,\ x = 10$

$\left(\frac{d^2A}{dx^2}\right)_{(\text{at } x=0)} = 0$

$\left(\frac{d^2A}{dx^2}\right)_{(\text{at } x=6)} = 20\times6(6^2 - 12\times6 + 30)$

$= 120(-6) = -720$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{(\text{at } x=10)} = 20\times10(10^2 - 12\times10 + 30)$

$= 120(10) = 1200$ (minima)

The required numbers = 6 and 4

23. (C) Quadratic equation

$9x^2 - 12x - c = 0$

Roots are real and equal, then

$B^2 - 4AC = 0$

$\Rightarrow (-12)^2 - 4\times9(-c) = 0$

$\Rightarrow 144 + 36c = 0 \Rightarrow c = -4$

24. (C) Area $= \int_0^2 (x.e^x - x.e^{-x})dx$

$= \left[x.\int e^x - \int 1.e^x dx\right]_0^2 - \left[x.\int e^{-x}dx - \int 1.(-e^{-x})dx\right]_0^2$

$= \left[x.e^x - e^x\right]_0^2 - \left[-x.e^{-x} - e^{-x}\right]_0^2$

$= \left[(2.e^2 - e^2) - (0 - e^0)\right] - \left[(-2.e^{-2} - e^{-2}) - (0 - e^0)\right]$

$= \left[e^2 + 1\right] - \left[-3.e^{-2} + 1\right]$

$= e^2 + 1 + 3.e^{-2} - 1$

$= \left(e^2 + \frac{3}{e^2}\right)$ sq.unit

25. (B) Series = $1.3^2 + 3.5^2 + 5.7^2 + \ldots\ldots$

$T_n = (2n-1)(2n+1)^2$

$T_n = 8n^3 + 4n^2 - 2n - 1$

$S_n = \sum T_n$

$S_n = 8\sum n^3 + 4\sum n^2 - 2\sum n - \sum 1$

$S_n = 8\times\frac{n^2(n+1)^2}{4} + 4\times\frac{n}{6}(n+1)(2n+1) - 2\times\frac{n(n+1)}{2} - n$

$S_n = 2n^2(n+1)^2 + \frac{2}{3}n(n+1)(2n+1) - n(n+1) - n$

$S_n = \frac{n}{3}[6n(n+1)^2 + 2(n+1)(2n+1) - 3(n+1) - 3]$

$S_n = \frac{n}{3}[6n(n^2+1+2n) + 2(2n^2+3n+1) - 3n - 3 - 3]$

$S_n = \frac{n}{3}[6n^3 + 16n^2 + 9n - 4]$

26. (A) Differential equation

$\frac{dy}{dx} = \sin(x+y)$

Let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\Rightarrow \frac{dt}{dx} - 1 = \sin t$

$\Rightarrow \frac{dt}{dx} = 1 + \sin t$

$\Rightarrow \frac{dt}{1+\sin t} = dx$

$\Rightarrow \frac{dt}{1+\cos\left(\frac{\pi}{2} - t\right)} = dx$

$\Rightarrow \frac{dt}{2\cos^2\left(\frac{\pi}{4} - \frac{t}{2}\right)} = dx$

$\Rightarrow \frac{1}{2}\sec^2\left(\frac{\pi}{4} - \frac{t}{2}\right) = dx$

On integrating

$\Rightarrow \frac{\frac{1}{2}\tan\left(\frac{\pi}{4} - \frac{t}{2}\right)}{\frac{-1}{2}} = x - c$

$\Rightarrow -\tan\left(\frac{\pi}{4} - \frac{t}{2}\right) = x - c$

$\Rightarrow x + \tan\left(\frac{\pi}{4} - \frac{t}{2}\right) = c$

$\Rightarrow x + \tan\left(\frac{\pi}{4} - \frac{x+y}{2}\right) = c$

$\Rightarrow x + \tan\left(\frac{\pi - 2x - 2y}{4}\right) = c$

27. (C) $y = 2^{\frac{1}{\log_x 8}}$

$\Rightarrow y = 2^{\log_8 x}$ $\left[\because \log_a b = \frac{1}{\log_b a}\right]$

$\Rightarrow y = 2^{\frac{1}{3}\log_2 x}$

$\Rightarrow y = 2^{\log_2 (x)^{1/3}}$

$\Rightarrow y = x^{1/3} \Rightarrow x = y^3$

28. (D)

29. (C) Given that $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

and $\vec{c} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

Now, $(\vec{b}+\vec{c})\times\vec{a} + (\vec{c}+\vec{a})\times\vec{b} + (\vec{a}+\vec{b})\times\vec{c}$

$\Rightarrow \vec{b}\times\vec{a} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b} + \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c}$

$\Rightarrow -\vec{a}\times\vec{b} + \vec{c}\times\vec{a} - \vec{b}\times\vec{c} + \vec{a}\times\vec{b} - \vec{c}\times\vec{a} + \vec{b}\times\vec{c}$

$\Rightarrow 0$

30. (C) $\lim_{x\to\infty}\left(\frac{x+2}{x-5}\right)^{x+1}$

$\Rightarrow \lim_{x\to\infty}\left(1 + \frac{7}{x-5}\right)^{\frac{x-5}{7}\times\frac{(x+1)\times 7}{x-5}}$

$\Rightarrow e^{\lim_{x\to\infty}\frac{7(x+1)}{x-5}}$ $\left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$

$\Rightarrow e^{\lim_{x\to\infty}\frac{7x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{5}{x}\right)}}$

$\Rightarrow e^{\frac{7(1+0)}{(1-0)}} = e^7$

31. (A) In the expansion of $\left(x^3 - \frac{2}{x}\right)^{11}$

$T_{r+1} = {}^{11}C_r (x^3)^{11-r}\left(\frac{-2}{x}\right)^r$

$= {}^{11}C_r\, x^{33-4r}(-2)^r$

Hence $33 - 4r = -3 \Rightarrow r = 9$

The coefficient of $x^{-3} = {}^{11}C_9(-2)^9$

$= \frac{11!}{9!2!}\times 2^9$

Again, $33 - 4r = 5 \Rightarrow r = 7$

The coefficient of $x^5 = {}^{11}C_7(-2)^7$

$= -\frac{11!}{7!4!}\times 2^7$

The required ratio $= \frac{-\frac{11!}{9!2!}\times 2^9}{-\frac{11!}{7!4!}\times 2^7}$

$= \frac{2}{3} = 2 : 3$

32. (C) $z = 1 + \cos\frac{\pi}{6} 4i\sin\frac{\pi}{6}$

$z = 2\cos^2\frac{\pi}{12} + i\times 2\sin\frac{\pi}{12}\times\cos\frac{\pi}{12}$

$z = 2\cos\frac{\pi}{12}\left[\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right]$

Now, $\arg(z) = \tan^{-1}\left(\frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}}\right)$

$\Rightarrow \arg(z) \Rightarrow \tan^{-1}\left(\tan\frac{\pi}{12}\right) = \frac{\pi}{12}$

33. (C) Given that $f(x) = ax + c$ and $g(x) = bx + d$

Now, $fog(x) = gof(x)$

$\Rightarrow f[g(x)] = g[f(x)]$

$\Rightarrow f[bx + d] = g[ax + c]$

$\Rightarrow a(bx + d) + c = b(ac + c) + d$

$\Rightarrow abx + ad + c + abx + bc + d$

$\Rightarrow ad + c = bc + d$

$\Rightarrow f(d) = g(c)$

34. (C) $I = \int \frac{1}{\sqrt{x^2 - 6x + 25}}\,dx$

$I = \int \frac{1}{\sqrt{(x-3)^2 + 5^2}}\,dx$

$I = \cosh^{-1}\left(\frac{x-3}{5}\right) + c$

35. (C) $x = \omega^2 - \omega + 3$

$\Rightarrow x - 3 = \omega^2 - \omega$

On squaring

$\Rightarrow (x-3)^2 = (\omega^2 - \omega)^2$

$\Rightarrow x^2 + 9 - 6x = \omega^4 + \omega^2 - 2\omega^3$

$\Rightarrow x^2 - 6x + 9 = \omega + \omega^2 - 2$

$\Rightarrow x^2 - 6x + 9 = -1 - 2 \quad [\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow x^2 - 6x = -12$

$\Rightarrow x^2 - 6x + 5 = -12 + 5 = -7$

36. (A) Line $(2x - 3y + 5) + \lambda(3x - 2y + 7) = 0$

$\Rightarrow (2 + 3\lambda)x + (-3 - 2\lambda)y + 5 + 7\lambda = 0$

$\Rightarrow (2 + 3\lambda)x + 5 + 7\lambda = (3 + 2\lambda)y$

$\Rightarrow y = \frac{2+3\lambda}{3+2\lambda}x + \frac{5+7\lambda}{3+2\lambda}$

line is parallel to y-axis

$\frac{2+3\lambda}{3+2\lambda} = \frac{1}{0}$

$\Rightarrow 3 + 2\lambda = 0 \Rightarrow \lambda = \frac{-3}{2}$

37. (C) $\cos\left(\cos^{-1}\frac{4}{5} + \cos^{-1}x\right) = 0$

$\Rightarrow \cos^{-1}\frac{4}{5} + \cos^{-1}x = \cos^{-1}0$

$\Rightarrow \cos^{-1}\frac{4}{5} + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \cos^{-1}\frac{4}{5}$

$\Rightarrow \cos^{-1}x = \sin^{-1}\frac{4}{5}$

$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{3}{5} \Rightarrow x = \frac{3}{5}$

38. (B) $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

On multiply by x

$\Rightarrow x(1 + x)^n = C_0x + C_1x^2 + C_2x^3 + \ldots + C_nx^{n+1}$

On differentiating both side w.r.t 'x'

$\Rightarrow x \times n(1+x)^{n-1} + (1+x)^n.1 = C_0 + 2C_1x + 3C_2x^2 + \ldots + (n+1)C_nx^n$

$\Rightarrow nx(1+x)^{n-1} + (1+x)^n = C_0 + 2C_1x + 3C_2x^2 + \ldots + (n+1)C_nx^n$ ...(ii)

$x \to \frac{1}{x}$ in eq(i)

$\left(1 + \frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots + \frac{C_n}{x^{n+1}}$ ...(iii) from eq(ii) and eq(iii)

coefficient of x^0 in $\left(1+\frac{1}{x}\right)^n$ $[nx(1+x)^{n+1}$ $+(1+x)^n]$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow$ coefficient of x^0 in

$$\left[\frac{(1+x)^n}{x^n}\times nx(1+x)^{n-1}+\frac{(1+x)^n}{x^n}\times(1+x)^n\right]$$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow$ coefficient of x^{n-1} in $[n(1+x)^{2n-1}]$ + coefficient of x^n in $(1+x)^{2n}$ = $C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow n\times{}^{2n-1}C_{n-1}+{}^{2n}C_n = C_0^2 + 2C_1^2 + \ldots. + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!}+\frac{2n!}{n!n!} = C_0^2 + 2C_1^2 \ldots + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!}+\frac{2n(2n-1)!}{n(n-1)!n!} = C_0^2 + 2C_1^2 + \ldots.$ $\ldots. + (n+1)C_n^2$

$\Rightarrow \frac{(2n-1)!}{(n-1)!n!}[n+2] = C_0^2 + 2C_1^2 + \ldots. + (n+1)C_n^2$

Hence $C_0^2 + 2C_1^2 + \ldots\ldots.. + (n+1)C_n^2$ $= (n+2)\ {}^{2n-1}C_{n-1}$

39. (C) $I = \int_0^{\pi/2} \sin 2x.\log(\tan x)\,dx$ ...(i)

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \sin 2\left(\frac{\pi}{2}-x\right).\log\left[\tan\left(\frac{\pi}{2}-x\right)\right]dx$

$I = \int_0^{\pi/2} \sin 2x.\log[\cot x]\,dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \sin 2x.[\log(\tan x)+\log(\cot x)]\,dx$

$2I = \int_0^{\pi/2} \sin 2x.[\log(\tan x.\cot x]\,dx$

$2I = 0 \Rightarrow I = 0$

40. (A) Total number of arrangements $=\frac{9!}{2!3!}=\frac{9!}{12}$

The total number of arrangements when N's come together $= \frac{8!}{3!} = \frac{8!}{6}$

The total number of arrangements when N's do not come together $=\frac{9!}{12}-\frac{8!}{6}$

$=\frac{9\times8!}{12}-\frac{8!}{6}$

$=\frac{3\times8!}{4}-\frac{8!}{6}=\frac{7\times8!}{12}$

41. (A) $[(A\cap B)\cup(B\cap C)]'$

$\Rightarrow (A\cap B)'\cap(B\cap C)'$

$\Rightarrow (A'\cup B')\cap(B'\cup C')$

42. (B) Three points $(x, -3)$, $(-1, 5)$ and $(-2, 1)$ are collinear, then

$$\begin{vmatrix} x & -3 & 1 \\ -1 & 5 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow x(5-1)+3(-1+2)+1(-1+10) = 0$

$\Rightarrow 4x + 3 + 9 = 0 \Rightarrow x = -3$

43. (C) $\tan(\sin^{-1}x)$

$\Rightarrow \tan\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right) = \frac{x}{\sqrt{1-x^2}}$

44. (D) $I = \int_0^{\pi/2}\frac{\sin x-\cos x}{1+\sin x.\cos x}dx$ ...(i)

$I = \int_0^{\pi/2}\frac{\sin\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-x\right)}{1+\sin\left(\frac{\pi}{2}-x\right).\cos\left(\frac{\pi}{2}-x\right)}dx$

$I = \int_0^{\pi/2}\frac{\cos x-\sin x}{1+\cos x.\sin x}dx$

$I = -\int_0^{\pi/2}\frac{\sin x-\cos x}{1+\sin x.\cos x}dx$

$I = -I$ [from eq(i)]

$2I = 0 \Rightarrow I = 0$

45. (B) $\log_{128}1024$

$\Rightarrow \log_{2^7}2^{10}$

$\Rightarrow \frac{10}{7}\log_2 2 = \frac{10}{7}$

46. (B) Let $y = \frac{\ln\sin x}{\tan x}$

$y = \cot x.\ln\sin x$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \cot x \times \frac{\cos x}{\sin x} + \ln\sin x(-\operatorname{cosec}^2 x)$

$\frac{dy}{dx} = \frac{\cos^2 x}{\sin^2 x} - \frac{\ln\sin x}{\sin^2 x}$

$\frac{dy}{dx} = \frac{\cos^2 x - \ln\sin x}{\sin^2 x}$

47. (B) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & c-b & a-b \\ c & a-c & b-c \end{vmatrix}$

$\Rightarrow (a+b+c)[1\{[c-b)(b-c) - (a-c)[a-b)\} - 0]$
$\Rightarrow (a+b+c)[bc - b^2 - c^2 + bc) - (a^2 - ac - ab + bc)]$
$\Rightarrow (a+b+c)[ab + bc + ca - a^2 - b^2 - c^2)$
$\Rightarrow -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$\Rightarrow -(a^3 + b^3 + c^3 - 3abc)$
$\Rightarrow 3abc - a^3 - b^3 - c^3$

48. (A)

49. (B) $2^{x+2} + 3.2^{y-1} = 4$

$\Rightarrow 4.2^x + \frac{3}{2} \times 2^y = 4$

Let $2^x = X$ and $2^y = Y$

$\Rightarrow 4X + \frac{3}{2}Y = 4$ ...(i)

and $3.2^{x-1} + 2^{y+1} = \frac{35}{8}$

$\Rightarrow \frac{3}{2} \times 2^x + 2.2^y = \frac{35}{8}$

$\Rightarrow \frac{3}{2}X + 2Y = \frac{35}{8}$ ...(ii)

On solving eq(i) and eq(ii)

$X = \frac{1}{4}$ and $Y = 2$

$\Rightarrow 2^x = 2^{-2}$ and $2^y = 2$
$\Rightarrow x = -2$ and $y = 1$

50. (C) $\sum_{n=1}^{7} (i^{n+1} - i^n)$

$\Rightarrow (i^2 - i) + (i^3 - i^2) + \ldots\ldots + (i^7 - i^6) + (i^8 - i^7)$
$\Rightarrow -i + i^8$
$\Rightarrow -i + 1 = 1 - i$

51. (A) $\begin{vmatrix} -1 & -\omega^2 & 2\omega^4 \\ -2 & -2\omega & 4\omega^5 \\ 4 & 4\omega^4 & -8\omega^8 \end{vmatrix}$

$\Rightarrow -1 \times (-2) \times 4 \begin{vmatrix} 1 & \omega^2 & -2\omega^4 \\ 1 & \omega & -2\omega^5 \\ 1 & \omega^4 & -2\omega^8 \end{vmatrix}$

$\Rightarrow 8 \times (-2) \begin{vmatrix} 1 & \omega^2 & \omega^4 \\ 1 & \omega & \omega^5 \\ 1 & \omega^4 & \omega^8 \end{vmatrix}$

$\Rightarrow -16 \begin{vmatrix} 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \\ 1 & \omega & \omega^2 \end{vmatrix}$

= 0 [$\because$ Two rows are identical]

52. (B) **Statement I**

Prime numbers
2, 3, 5, 7, 11, 13, 17, 19, 23
The required sum = $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 + 23^2$
= 4 + 9 + 25 + 49 + 121 + 169 + 289 + 361 + 529
= 1556
Statement I is incorrect.

Statement II

Odd natural numbers
1, 3, 5, 7, 9, 11, 13
The required sum = $1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3$
= 1 + 27 + 125 + 343 + 729 + 1331 + 2197
= 4753
Statements II is correct.

53. (C) We Know that

$\sin 36° = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$ and $\cos 18 = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

Now, sin36°.cos18°

$\Rightarrow \frac{\sqrt{10 - 2\sqrt{5}}}{4} \times \frac{\sqrt{10 + 2\sqrt{5}}}{4}$

$\Rightarrow \frac{\sqrt{100 - 20}}{16}$

$\Rightarrow \frac{\sqrt{80}}{16}$

$\Rightarrow \frac{4\sqrt{5}}{16} = \frac{\sqrt{5}}{4}$

54. (C) $\operatorname{cosec}\alpha = \dfrac{25}{24}$

$\sin\alpha = \dfrac{24}{25}$ and $\cot\alpha = \dfrac{7}{24}$

Now, $\sin\alpha.\cot\alpha = \dfrac{24}{25} \times \dfrac{7}{24}$

$\Rightarrow \sin\alpha.\cot\alpha = \dfrac{7}{25}$

55. (C) Ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

$a^2 = 9 \Rightarrow a = 3$, $b^2 = 4 \Rightarrow b = 2$

The sum of the focal distance

= length of major- axis

$= 2a = 2 \times 3 = 6$

56. (A) $x^n - y^n = 1$

On differentiating both side w.r.t. 'x'

$\Rightarrow nx^{n-1} - ny^{n-1}\dfrac{dy}{dx} = 0$

$\Rightarrow x^{n-1} = y^{n-1}\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \left(\dfrac{x}{y}\right)^{n-1}$

$\Rightarrow \dfrac{dy}{dx} = \left(\dfrac{y}{x}\right)^{1-n}$

On comparing with $\dfrac{dy}{dx} = \sqrt{\dfrac{y}{x}}$

$1 - n = \dfrac{1}{2} \Rightarrow n = \dfrac{1}{2}$

57. (C) $(\lambda\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - \hat{j} + 4\hat{k}) = 11\hat{i} + 13\hat{j} - 5\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & 3 \\ 3 & -1 & 4 \end{vmatrix} = 11\hat{i} + 13\hat{j} - 5\hat{k}$$

$\Rightarrow \hat{i}(8 + 3) - \hat{j}(4\lambda - 9) + \hat{k}(-\lambda - 6) = 11\hat{i} + 13\hat{j} - 5\hat{k}$

$\Rightarrow 11\hat{i}\,(9 - 4\lambda)\,\hat{j} + \hat{k}(-\lambda - 6) = 11\hat{i} + 13\hat{j} - 5\hat{k}$

On comparing

$9 - 4\lambda = 13 \Rightarrow \lambda = -1$

58. (C) Even numbers

2, 4, 6......upto n

Mean $= \dfrac{2 + 4 + 6 + ... + \text{upto } n}{n}$

$= \dfrac{2(1 + 2 + 3.......\text{upto } n)}{n}$

$= \dfrac{2 \times n(n+1)}{2 \times n} = n + 1$

59. (C) In a leap year, Days = 366 days

= 52 weeks and 2days

The required Probability $= \dfrac{2}{7}$

60. (C) $y = \ln(e^{mx} + e^{-mx})$

On differentiating both side w.r.t. 'x'

$\dfrac{dy}{dx} = \dfrac{1}{e^{mx} + e^{-mx}} \times (me^{mx} - me^{-mx})$

$\dfrac{dy}{dx} = m \times \dfrac{e^{mx} - e^{-mx}}{e^{mx} + e^{-mx}}$

$\dfrac{dy}{dx}_{(\text{at } x = 0)} = m \times \dfrac{e^0 - e^0}{e^0 + e^0} = 0$

61. (A) $f(x) = \sqrt{\log_e(1 - x^2 + 3x)}$

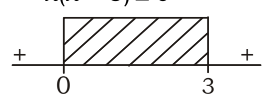
Now, $\log_e(1 - x^2 + 3x) \ge 0$

$\Rightarrow 1 - x^2 + 3x \ge 1$

$\Rightarrow -x^2 + 3x \ge 0$

$\Rightarrow x^2 - 3x \le 0$

$\Rightarrow x(x - 3) \le 0$



Domain = [0, 3]

62. (B)

63. (D)

64. (B) $z = \dfrac{\sqrt{2} - i}{\sqrt{2} + i}$

$z = \dfrac{\sqrt{2} - i}{\sqrt{2} + i} \times \dfrac{\sqrt{2} - i}{\sqrt{2} - i}$

$z = \dfrac{(\sqrt{2} - i)^2}{2 - i^2}$

$z = \dfrac{2 + i^2 - 2\sqrt{2}i}{2 + 1}$

$z = \dfrac{2 - 1 - 2\sqrt{2}i}{3}$

$z = \dfrac{1 - 2\sqrt{2}i}{3}$

$|z| = \dfrac{\sqrt{1^2 + (2\sqrt{2})^2}}{3}$

$|z| = \dfrac{\sqrt{1 + 8}}{3}$

$|z| = \dfrac{9}{3} = 3$

65. (B) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$\Rightarrow \cos^{-1}\left[\cos\left(\pi+\frac{\pi}{4}\right)\right]$

$\Rightarrow \cos^{-1}\left(-\cos\frac{\pi}{4}\right)$

$\Rightarrow \cos^{-1}\left(\cos\left(\pi-\frac{\pi}{4}\right)\right)$

$\Rightarrow \cos^{-1}\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

66. (A) $\sin^2 56\frac{1}{2} - \sin^2 33\frac{1}{2}$

$\Rightarrow \sin^2\left(90-33\frac{1}{2}\right) - \sin^2 33\frac{1}{2}$

$\Rightarrow \cos^2 33\frac{1}{2} - \sin^2 33\frac{1}{2}$

$\Rightarrow \cos\left(2\times 33\frac{1}{2}\right) = \cos 67$

67. (C) $\frac{\cos 9x - \cos 5x}{\sin 9x - 2\sin 7x + \sin 5x}$

$\Rightarrow \frac{-2\sin 2x.\sin 7x}{\sin 9x + \sin 5x - 2\sin 7x}$

$\Rightarrow \frac{-2\sin 2x.\sin 7x}{2\sin 7x.\cos 2x - 2\sin 7x}$

$\Rightarrow \frac{-2\sin 2x.\sin 7x}{2\sin 7x(\cos 2x - 1)}$

$\Rightarrow \frac{-\sin 2x}{-(1-\cos 2x)}$

$= \frac{2\sin x.\cos x}{2\sin^2 x} = \cot x$

68. (D) $\lim_{x\to 0}\frac{7^x - 1}{x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{7^x\log 7 - 0}{1}$

$\Rightarrow 7^0 \log 7 = \log 7$

69. (B) Given that $f(x) = \log x$, $g(x) = e^x$

Now, $y = fog(x)$

$\Rightarrow y = f[g(x)]$

$\Rightarrow y = f[e^x]$

$\Rightarrow y = \log e^x$

$\Rightarrow y = x$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = 1$

$\frac{dy}{dx}\left(\text{at } x = \frac{\pi}{4}\right) = 1$

70. (C) Given that $e = \frac{1}{\sqrt{2}}$

and $ae = 3$

$\Rightarrow a \times \frac{1}{\sqrt{2}} = 3 \Rightarrow a = 3\sqrt{2}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow b^2 = 18\left(1-\frac{1}{2}\right)$

$\Rightarrow b^2 = 18\times\frac{1}{2} \Rightarrow b^2 = 9$

Equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{18} + \frac{y^2}{9} = 1 \Rightarrow x^2 + 2y^2 = 18$

71. (C) $y = x.\ln x - e^x$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x.1 - e^x$

$\frac{dy}{dx} = 1 + \ln x - e^x$

Again, differentiating

$\frac{d^2y}{dx^2} = 0 + \frac{1}{x} - e^x$

$\frac{d^2y}{dx^2} = \frac{1 - x.e^x}{x}$

72. (A) The required Probability

$= {}^6C_3\left(\frac{1}{7}\right)^3\left(\frac{6}{7}\right)^3 = \frac{20\times 6^3}{7^6}$

73. (D) $T_n = \frac{n^2+3n}{2}$

Now, $S_n = \sum T_n$

$\Rightarrow S_n = \sum\frac{n^2+3n}{2}$

$\Rightarrow S_n = \frac{1}{2}\sum n^2 + \frac{3}{2}\sum n$

$\Rightarrow S_n = \frac{1}{2}\times\frac{n}{6}(n+1)(2n+1) + \frac{3}{2}\times\frac{n(n+1)}{2}$

$\Rightarrow S_n = \frac{n(n+1)}{12}[2n+1+9]$

$\Rightarrow S_n = \frac{n(n+1)}{12}(2n+10)$

$\Rightarrow S_n = \frac{n(n+1)(n+5)}{6}$

74. (C) $T_7 = 39$

$\Rightarrow a + 6d = 39$

$\Rightarrow 2a + 12d = 78$

$\Rightarrow \frac{13}{2}[2a + 12d] = \frac{13}{2} \times 78$

$\Rightarrow S_{13} = 507$

75. (B) $y = \frac{2at}{1+t^2}$

On differentiating both side w.r.t.'t'

$\frac{dy}{dt} = 2a \times \frac{(1+t^2).1 - t(2t)}{(1+t^2)^2}$

$\frac{dy}{dt} = 2a \times \frac{1+t^2-2t^2}{(1+t^2)^2}$

$\frac{dy}{dt} = \frac{2a(1-t^2)}{(1+t^2)^2}$

and $x = \frac{a(1-t^2)}{1+t^2}$

On differentiating both side w.r.t.'t'

$\frac{dx}{dt} = a \times \frac{(1+t^2)(-2t) - (1-t^2) \times 2t}{(1+t^2)^2}$

$\frac{dx}{dt} = a \times \left[\frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2}\right]$

$\frac{dx}{dt} = \frac{-4at}{(1+t^2)^2}$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2a(1-t^2)}{(1+t^2)^2} \times \frac{(1+t^2)^2}{-4at}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{2t}$

76. (C) $\frac{[1+(i^3)^{4n+3}]^{4n-3}}{[1+(i^3)^{4n-3}]^{4n+3}} \Rightarrow \frac{[1+(-i)^{4n+3}]^{4n-3}}{[1+(-i)^{4n-3}]^{4n+3}}$

$\Rightarrow \frac{[1+(-i)^{4n}.(-i)^3]^{4n-3}}{[1+(-i)^{4n}(-i)^{-3}]^{4n+3}} \Rightarrow \frac{[1-i^3]^{4n-3}}{\left[1-\frac{1}{i^3}\right]^{4n+3}}$

$\Rightarrow \frac{[1+i]^{4n-3}}{\left[1-\frac{1}{-i}\right]^{4n+3}} \Rightarrow \frac{[1+i]^{4n-3}}{[1-i]^{4n+3}}$

$\Rightarrow [1 + i]^{4n-3-4n-3} \Rightarrow [1 + i]^{-6}$

$\Rightarrow \frac{1}{(1+i)^6} \Rightarrow \frac{1}{[(1+i)^2]^3}$

$\Rightarrow \frac{1}{[1+i^2+2i]^3} \Rightarrow \frac{1}{(2i)^3}$

$\Rightarrow \frac{1}{8i^3} \Rightarrow \frac{1}{-8i} = \frac{i}{8}$

77. (A) $\begin{vmatrix} 0 & a & b \\ b & 0 & a \\ a & b & 0 \end{vmatrix} = 0$

$\Rightarrow -a(0 - a^2) + b(b^2 - 0) = 0$

$\Rightarrow a^3 + b^3 = 0$

78. (C) The required number of ways $= {}^{15-1}C_{11-1}$

$= {}^{14}C_{10} = 1001$

79. (D) Given that $f(x) = \frac{x-1}{x+1}$

Now, $\frac{f(x)+1}{f(x)-1} + x$

$\Rightarrow \frac{\frac{x-1}{x+1}+1}{\frac{x-1}{x+1}-1} + x$

$\Rightarrow \frac{x-1+x+1}{x-1-x-1} + x$

$\Rightarrow \frac{2x}{-2} + x = 0$

80. (C) $f(f(x)) = f[f(x)]$

$\Rightarrow f(f(x)) = f\left[\frac{x-1}{x+1}\right]$

$\Rightarrow f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}$

$\Rightarrow f(f(x)) = \frac{x-1-x-1}{x-1+x+1}$

$\Rightarrow f(f(x)) = \frac{-2}{2x} = \frac{-1}{x}$

81. (A) A.T.Q,

$\frac{\text{AM}}{\text{G.M}} = \frac{5}{4}$

$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

by Componedo & Dividendo Rule

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$

$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{9}{1}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{1}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{3+1}{3-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{4}{2}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{2}{1}$

On squaring

$\Rightarrow \frac{a}{b} = \frac{4}{1}$

Hence = $a : b = 4 : 1$

82. (C) Given that $g(x) = x$, $f(x) = \frac{1}{g(x)} = \frac{1}{x}$

L.H.S. = $f(g(g(f(x))))$

$= f\left(g\left(g\left(\frac{1}{x}\right)\right)\right)$

$= f\left(g\left(\frac{1}{x}\right)\right)$

$= f\left(\frac{1}{x}\right) = x$

R.H.S = $g(f(f(g(x))))$

$= g(f(f(x)))$

$= g\left(f\left(\frac{1}{x}\right)\right)$

$= g(x) = x$

L.H.S = R.H.S

Hence option (C) is correct.

83. (A)

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

Hence $(37)_{10} = (100101)_2$

84. (B) B is a 2×3 matrix.

85. (C) The required no. of triangles = ${}^{14}C_3 - {}^{8}C_3$
= 364 – 56
= 308

86. (D) Centre is (–1, 2, –3) and radius(r) = 4
Equation of sphere
$(x+1)^2 + (y-2)^2 + (z+3)^2 = 4^2$
$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4y + z^2 + 9 + 6z = 16$
$\Rightarrow x^2 + y^2 + z^2 + 2x - 4y + 6z = 2$

87. (B) Given that $|\vec{a}| = 3$ and $|\vec{b}| = 2$

and $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

Now, $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$

$\Rightarrow \sqrt{2^2 + (-2)^2 + 1^2} = 3\times 2 \times \sin\theta$

$\Rightarrow 3 = 3\times 2\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

88. (D) $\cos^{-1}\left(\cos\frac{7\pi}{4}\right)$

$\Rightarrow \cos^{-1}\left[\cos\left(2\pi - \frac{\pi}{4}\right)\right]$

$\Rightarrow \cos^{-1}\left(\cos\frac{\pi}{4}\right) = \frac{\pi}{4}$

89. (C) A.T.Q,
$ar^2 = 4$
Now, $a.ar.ar^2.ar^3.ar^4 = a^5r^{10}$
$= (ar^2)^5$
$= 4^5 = 1024$

90. (C) $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{k}\times\hat{i})$

$\Rightarrow \hat{i}.(\hat{i}) + \hat{j}.(\hat{j})$

$\Rightarrow 1 + 1 = 2$

91. (A) $\begin{vmatrix} 1000 & 1001 & 1002 \\ 1003 & 1004 & 1005 \\ 1006 & 1007 & 1008 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1000 & 1 & 2 \\ 1003 & 1 & 2 \\ 1006 & 1 & 2 \end{vmatrix}$

$\Rightarrow 2\begin{vmatrix} 1000 & 1 & 1 \\ 1003 & 1 & 1 \\ 1006 & 1 & 1 \end{vmatrix}$

$\Rightarrow 0$ [∵ Two columns are identical.]

92. (C) $I = \int \frac{dx}{1+\cos x}$

$I = \int \frac{dx}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}\, dx$

$I = \int \frac{1-\cos x}{1-\cos^2 x}\, dx$

$I = \int \frac{1-\cos x}{\sin^2 x}\, dx$

$I = \int (\text{cosec}^2 x - \text{cosec}\, x.\cot x)\, dx$

$I = -\cot x + \text{cosec}\, x + c$

$I = \text{cosec}\, x - \cot x + c$

93. (B) $\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} + \cos^{-1}\frac{63}{65}$

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{16}{63}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\times\frac{5}{12}}\right] + \tan^{-1}\frac{16}{63}$

$\Rightarrow \tan^{-1}\left[\frac{48+15}{36-20}\right] + \tan^{-1}\frac{16}{63}$

$\Rightarrow \tan^{-1}\frac{63}{16} + \cot^{-1}\frac{63}{16} = \frac{\pi}{2}$

94. (B) $2X + 5Y = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ ...(i)

and $3X - 2Y = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ ...(ii)

eq(i)×3 – eq(ii)×2

$19Y = \begin{bmatrix} -1 & 11 \\ 4 & 6 \end{bmatrix}$ ...(iii)

eq(i)×2 + eq(ii)×5

$19X = \begin{bmatrix} 12 & 1 \\ 9 & -15 \end{bmatrix}$ ...(iv)

from eq(iii) and eq(iv)

$19X - 19Y = \begin{bmatrix} 12 & 1 \\ 9 & -15 \end{bmatrix} - \begin{bmatrix} -1 & 11 \\ 4 & 6 \end{bmatrix}$

$19X - 19Y = \begin{bmatrix} 13 & -10 \\ 5 & -21 \end{bmatrix}$

95. (D) $I = \int_0^\pi \frac{x \sec x.\tan x}{1+\sec^2 x}\, dx$ ...(i)

$I = \int_0^\pi \frac{(\pi - x)\sec(\pi - x).\tan(\pi - x)}{1+\sec^2(\pi - x)}\, dx$

$I = \int_0^\pi \frac{(\pi - x)\sec x.\tan x}{1+\sec^2 x}\, dx$

$I = \int_0^\pi \frac{\pi.\sec x.\tan x}{1+\sec^2 x}\, dx - \int_0^\pi \frac{x \sec x.\tan x}{1+\sec^2 x}\, dx$

$I = \pi\int_0^\pi \frac{\frac{1}{\cos x}.\frac{\sin x}{\cos x}}{1+\frac{1}{\cos^2 x}}\, dx - I$

$2I = \pi\int_0^\pi \frac{\sin x}{\cos^2 x + 1}\, dx$

let $\cos x = t$ when $x = 0,\ t = 1$

$-\sin dx = dt$ $\quad x = \pi,\ t = -1$

$\sin x.dx = -dt$

$2I = -\pi\int_1^{-1} \frac{dt}{1+t^2}$

$2I = \pi\int_{-1}^{1} \frac{dt}{1+t^2}$

$2I = \pi \times 2\int_0^1 \frac{dt}{1+t^2}$

$I = \pi\left[\tan^{-1} t\right]_0^1$

$I = \pi\left[\tan^{-1}1 - \tan^{-1}0\right]$

$I = \pi\left[\frac{\pi}{4} - 0\right] = \frac{\pi^2}{4}$

96. (C) $f(x) = \begin{cases} ax-1, & x \le 2 \\ bx-5, & x > 2 \end{cases}$ is continuous at $x = 2$, then

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\Rightarrow \lim_{x\to 2}(ax-1) = \lim_{x\to 2}(bx-5)$

$\Rightarrow a \times 2 - 1 = b \times 2 - 5$

$\Rightarrow 2a - 2b = 1 - 5$

$\Rightarrow 2a - 2b = -4 \Rightarrow a - b = -2$

97. (B) $f(x) = (1+x)(1+x^2)(1+x^4)$

On differentiating both side w.r.t.'x'

$f'(x) = (1+x)(1+x^2)\frac{d}{dx}(1+x^4) + (1+x)(1+x^4)\frac{d}{dx}(1+x^2) + (1+x^2)(1+x^4)\frac{d}{dx}(1+x)$

$f'(x) = (1+x)(1+x^2)\times 4x^3 + (1+x)(1+x^4)\times 2x + (1+x^2)(1+x^4)\times 1$

$f'(x) = 4x^3(1+x)(1+x^2) + 2x(1+x)(1+x^4) + (1+x^2)(1+x^4)$

$f'(1) = 4 \times 1(1+1)(1+1) + 2 \times 1(1+1)(1+1) + (1+1)(1+1)$

$f'(1) = 4\times2\times2 + 2\times2\times2 + 2\times2$

$f'(1) = 16 + 8 + 4 = 28$

98. (C) $A = \begin{bmatrix} 0 & -3 & b \\ a & 0 & 1 \\ 4 & -1 & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} 0 & a & 4 \\ -3 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

A is skew symmetrix matrix, then
$A = -A^T$

$\Rightarrow \begin{bmatrix} 0 & -3 & b \\ a & 0 & 1 \\ 4 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 4 \\ -3 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & -3 & b \\ a & 0 & 1 \\ 4 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -4 \\ 3 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$

On camparing
$a = 3$ and $b = -4$

99. (C) Let $y = \cot^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$

$y = \cot^{-1}\left(\dfrac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$y = \cot^{-1}\left(\tan\dfrac{x}{2}\right)$

$y = \cot^{-1}\left[\cot\left(\dfrac{\pi}{2}-\dfrac{x}{2}\right)\right]$

$y = \dfrac{\pi}{2}-\dfrac{x}{2}$

On differentiating both side w.r.t.'x'

$\dfrac{dy}{dx} = -\dfrac{1}{2}$

100. (B) $y = \sin(\sin x)$ ...(i)

On differentiating both side w.r.t. 'x'

$\dfrac{dy}{dx} = \cos(\sin x).\cos x$ ...(ii)

Again, differentiating

$\dfrac{d^2y}{dx^2} = -\sin(\sin x).\cos x.\cos x + \cos(\sin x).(-\sin x)$

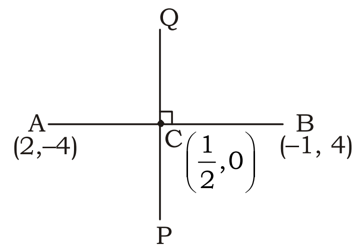
$\dfrac{d^2y}{dx^2} = -\cos^2 x.\sin(\sin x) - \sin x.\cos(\sin x)$

$\dfrac{d^2y}{dx^2} = -y\cos^2 x - \sin x.\dfrac{\frac{dy}{dx}}{\cos x}$

$\dfrac{d^2y}{dx^2} = -y\cos^2 x - \tan x\dfrac{dy}{dx}$

$\dfrac{d^2y}{dx^2} + \tan x.\dfrac{d^2y}{dx^2} = -y\cos^2 x$

101. (B)



Mid point of line joining the points = $\left(\dfrac{2-1}{2},\dfrac{-4+4}{2}\right) = \left(\dfrac{1}{2},0\right)$

Slope of line AB(m_1) $= \dfrac{4+4}{-1-2} = \dfrac{8}{-3} = \dfrac{-8}{3}$

Slope of line PQ(m_2) $= \dfrac{-1}{m_1} = \dfrac{-1}{-8/3} = \dfrac{3}{8}$

Equation of line PQ

$y - 0 = \dfrac{3}{8}\left(x-\dfrac{1}{2}\right)$

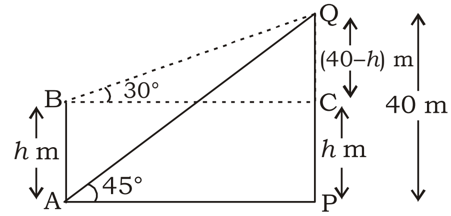
$\Rightarrow y = \dfrac{3}{8}\times\dfrac{2x-1}{2}$

$\Rightarrow 16y = 6x - 3 \Rightarrow 6x - 16y = 3$

102. (C) Given that $S_p = q$ and $S_q = p$
then $S_{(p+q)} = -(p+q)$

103. (D) $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$
$\Rightarrow (1+\omega)(1+\omega^2)(1+1)\times 0$ $[\because 1+\omega+\omega^2 = 0]$
$\Rightarrow 0$

104. (B)



Let height of the pole (AB) = h m

In ΔQBC

$\tan 30° = \frac{QC}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40-h}{BC} \Rightarrow BC = (40-h)\sqrt{3} = AP$ ...(i)

In ΔAPQ

$\tan 45° = \frac{PQ}{AP}$

$\Rightarrow 1 = \frac{40}{(40-h)\sqrt{3}}$ [from eq(i)]

$\Rightarrow 40\sqrt{3} - h\sqrt{3} = 40$

$\Rightarrow h\sqrt{3} = 40(\sqrt{3}-1) \Rightarrow h = \frac{40\left(\sqrt{3}-1\right)}{\sqrt{3}}$

Hence height of the pole $= \frac{40\left(\sqrt{3}-1\right)}{\sqrt{3}}$ m

105. (C) $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots\ldots \infty$

$\Rightarrow \frac{1}{\log_3 e} + \frac{1}{2\log_3 e} + \frac{1}{4\log_3 e} + \ldots\ldots. \infty$

$\Rightarrow \log_e 3 + \frac{1}{2}\log_e 3 + \frac{1}{4}\log_e 3 + \ldots\ldots.. \infty$

$\Rightarrow \log_e 3\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\infty\right]$

$\Rightarrow \log_e 3 \times \frac{1}{1-\frac{1}{2}}$

$\Rightarrow \log_e 3 \times \frac{1}{1/2} \Rightarrow 2\log_e 3$

106. (B) $\tan^{-1}\frac{1}{3} + 2\tan^{-1}\frac{4}{9}$

$\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\left[\frac{2\times\frac{4}{9}}{1-\left(\frac{4}{9}\right)^2}\right]$

$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\left(\frac{8/9}{65/81}\right)$

$\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{72}{65}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{3}+\frac{72}{65}}{1-\frac{1}{3}\times\frac{72}{65}}\right]$

$\Rightarrow \tan^{-1}\left[\frac{65+216}{195-72}\right] = \tan^{-1}\left[\frac{281}{123}\right]$

107. (A) $y = \sec x^{\sec x^{\sec x \ldots\ldots\infty}}$

$\Rightarrow y = (\sec x)^y$

On taking log

$\Rightarrow \log y = y \log \sec x$ ...(i)

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{1}{y}\times\frac{dy}{dx} = \frac{y}{\sec x}\times \sec x.\tan x + \log\sec x \times 1$

$\Rightarrow \frac{dy}{dx} = y^2.\tan x + y.\log\sec x$

$\Rightarrow \frac{dy}{dx} = y^2.\tan x + \log y$ [from eq(i)]

108. (C) $n(S) = {}^9C_4 = 126$

$n(E) = {}^3C_2 \times {}^6C_2 + {}^3C_3 \times {}^6C_1$

$n(E) = 3\times15 + 1\times6 = 51$

The required Probability $= \frac{n(E)}{n(S)}$

$= \frac{51}{126} = \frac{17}{42}$

109. (B) Coefficient of correlation $= \sqrt{r_1 \times r_2}$

$= \sqrt{0.4\times0.9}$

$= 0.6$

110. (D) Given that $B = A\cap C$

Now, $U-(U-(U-(U-(A\cap C))))$

$\Rightarrow U-(U-(U-(U-B)))$

$\Rightarrow U-(U-(U-B'))$

$\Rightarrow U-(U-B)$

$\Rightarrow U-B'$

$= B = A\cap C$

111. (D) $\frac{1+\tan 158.\tan 8}{\tan 22 - \tan 172}$

$\Rightarrow \frac{1+\tan(90+68).\tan(90-82)}{\tan(90-68)-\tan(90+82)}$

$\Rightarrow \frac{1+(-\cot 68).\cot 82}{\cot 68+\cot 82}$

$\Rightarrow -\left[\frac{\cot 68.\cot 82-1}{\cot 82+\cot 68}\right]$

$\Rightarrow -\cot(82+68)$

$\Rightarrow -\cot(150)$

$\Rightarrow -\cot(90+60)$

$\Rightarrow \tan 60° = \sqrt{3}$

112. (D) Lines $x - 3y = 6$ and $2x - y = 7$
Intersecting points of both lines = (3, 1)
Equation of line which is parallel to the line $2x - 5y = 9$
$2x - 5y = c$
its passes through the point (3,–1)
$2\times3 - 5\times(-1) = c$
$\Rightarrow 6 + 5 = c \Rightarrow c = 11$
The required equation
$2x - 5y = 11$

113. (C) Planes $2x - 3y + 6z = 5$
and $4x - 6y + 12z = 9$

$\Rightarrow 2x - 3y + 6z = \frac{9}{2}$

Distance between Planes

$$D = \frac{\left|\frac{9}{2} - 5\right|}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

$$D = \frac{\frac{1}{2}}{\sqrt{49}} \Rightarrow D = \frac{1}{14}$$

114. (B) Conic
$4x^2 + 9y^2 + 8x - 18y + 12 = 0$
$\Rightarrow (4x^2 + 8x) + (9y^2 - 18y) + 12 = 0$
$\Rightarrow 4(x^2 + 2x) + 9(y^2 - 2y) + 12 = 0$
$\Rightarrow 4(x+1)^2 - 4 + 9(y-1)^2 - 9 + 12 = 0$
$\Rightarrow 4(x+1)^2 + 9(y-1)^2 = 1$

$$\Rightarrow \frac{(x+1)^2}{1/4} + \frac{(y+1)^2}{1/9} = 1$$

$\Rightarrow a^2 = \frac{1}{4}, b^2 = \frac{1}{9}$

Now, $b^2 = a^2(1 - e^2)$

$\Rightarrow \frac{1}{9} = \frac{1}{4}(1 - e^2)$

$\Rightarrow \frac{4}{9} = 1 - e^2$

$\Rightarrow e^2 = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

115. (C) We know that
curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Area $= \frac{a^2}{b}$

Now, curve $\sqrt{x} + \sqrt{y} = 3 = \sqrt{9}$

The required Area $= \frac{9^2}{6} = \frac{27}{2}$ sq.unit

116. (B) A.T.Q,

$\frac{n(n-3)}{2} = 35$

$\Rightarrow n^2 - 3n = 70$
$\Rightarrow n^2 - 3n - 70 = 0$
$\Rightarrow (n + 7)(n - 10) = 0$
$n = -7, 10$
Hence number of sides = 10

117. (A)

118. (C) x, $2y$ and $3z$ are in A.P., then
$2 \times 2y = x + 3z$
$4y = x + 3z$ …(i)
x, y and z are in G.P., then

$\frac{y}{x} = \frac{z}{y}$ …(ii)

from eq(i)

$\frac{4y}{x} = \frac{x}{x} + \frac{3z}{x}$

$\Rightarrow \frac{4y}{x} = 1 + \frac{3}{x} \times \frac{y^2}{x}$ [from eq(ii)]

Let $\frac{y}{x} = a$

$\Rightarrow 4a = 1 + 3a^2$
$\Rightarrow 3a^2 - 4a + 1 = 0$
$\Rightarrow (3a - 1)(a - 1) = 0$
$\Rightarrow a = 1, \frac{1}{3}$

Hence common ratio $= \frac{y}{x} = \frac{1}{3}$

119. (D)

120. (C) Time = 8 : 35

$$\text{Angle} = \left|\frac{11M - 60H}{2}\right|$$

$$= \left|\frac{11 \times 35 - 60 \times 8}{2}\right|$$

$$= \left|\frac{385 - 480}{2}\right|$$

$$= \frac{95}{2} = 47.5°$$

NDA (MATHS) MOCK TEST - 146 (Answer Key)

1.	(B)	21.	(C)	41.	(A)	61.	(A)	81.	(A)	101.	(B)
2.	(D)	22.	(B)	42.	(B)	62.	(B)	82.	(C)	102.	(C)
3.	(C)	23.	(C)	43.	(C)	63.	(D)	83.	(A)	103.	(D)
4.	(C)	24.	(C)	44.	(D)	64.	(B)	84.	(B)	104.	(B)
5.	(B)	25.	(B)	45.	(B)	65.	(B)	85.	(C)	105.	(C)
6.	(A)	26.	(A)	46.	(B)	66.	(A)	86.	(D)	106.	(B)
7.	(C)	27.	(C)	47.	(B)	67.	(C)	87.	(B)	107.	(A)
8.	(B)	28.	(D)	48.	(A)	68.	(D)	88.	(D)	108.	(C)
9.	(C)	29.	(C)	49.	(B)	69.	(B)	89.	(C)	109.	(B)
10.	(A)	30.	(C)	50.	(C)	70.	(C)	90.	(C)	110.	(D)
11.	(C)	31.	(A)	51.	(A)	71.	(C)	91.	(A)	111.	(D)
12.	(C)	32.	(C)	52.	(B)	72.	(A)	92.	(C)	112.	(D)
13.	(A)	33.	(C)	53.	(C)	73.	(D)	93.	(B)	113.	(C)
14.	(C)	34.	(C)	54.	(C)	74.	(C)	94.	(B)	114.	(B)
15.	(A)	35.	(C)	55.	(C)	75.	(B)	95.	(D)	115.	(C)
16.	(A)	36.	(A)	56.	(A)	76.	(C)	96.	(C)	116.	(B)
17.	(C)	37.	(C)	57.	(C)	77.	(A)	97.	(B)	117.	(A)
18.	(A)	38.	(B)	58.	(C)	78.	(C)	98.	(C)	118.	(C)
19.	(A)	39.	(C)	59.	(C)	79.	(D)	99.	(C)	119.	(D)
20.	(D)	40.	(A)	60.	(C)	80.	(C)	100.	(B)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777