NDA MATHS MOCK TEST - 156 (SOLUTION)

1. (A) In the expansion of $\left(x^3 - \frac{2}{x}\right)^{11}$

$T_{r+1} = {}^{11}C_r (x^3)^{11-r} \left(\frac{-2}{x}\right)^r$

$T_{r+1} = {}^{11}C_r\, x^{33-4r}(-2)^r$

Here $33 - 4r = -3 \Rightarrow r = 9$

The coefficient of $x^{-3} = {}^{11}C_9(-2)^9$

$= \frac{11!}{9!2!} \times 2^9$

Again, $33 - 4r = 5 \Rightarrow r = 7$

The coefficient of $x^5 = {}^{11}C_7(-2)^7$

$= -\frac{11!}{7!4!} \times 2^7$

The required ratio $= \dfrac{-\frac{11!}{9!2!}\times 2^9}{-\frac{11!}{7!4!}\times 2^7}$

$= \frac{2}{3} = 2 : 3$

2. (B) $z = 1 + \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$z = 2\cos^2\frac{\pi}{6} + i \times 2\sin\frac{\pi}{6} \times \cos\frac{\pi}{6}$

$z = 2\cos\frac{\pi}{6}\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$

Now, $\arg(z) = \tan^{-1}\left(\dfrac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$

$\Rightarrow \arg(z) \Rightarrow \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$

3. (C) Given that $f(x) = ax + c$ and $g(x) = bx + d$

Now, $fog(x) = gof(x)$

$\Rightarrow f[g(x)] = g[f(x)]$

$\Rightarrow f[bx + d] = g[ax + c]$

$\Rightarrow a(bx + d) + c = b(ac + c) + d$

$\Rightarrow abx + ad + c + abx + bc + d$

$\Rightarrow ad + c = bc + d$

$\Rightarrow f(d) = g(c)$

4. (C) $I = \int \frac{1}{\sqrt{x^2 - 4x + 29}}\, dx$

$I = \int \frac{1}{\sqrt{(x-2)^2 + 5^2}}\, dx$

$I = \cosh^{-1}\left(\frac{x-2}{5}\right) + c$

5. (B) $x = \omega^2 - \omega + 3$

$\Rightarrow x - 3 = \omega^2 - \omega$

On squaring

$\Rightarrow (x-3)^2 = (\omega^2 - \omega)^2$

$\Rightarrow x^2 + 9 - 6x = \omega^4 + \omega^2 - 2\omega^3$

$\Rightarrow x^2 - 6x + 9 = \omega + \omega^2 - 2$

$\Rightarrow x^2 - 6x + 9 = -1 - 2 \quad [\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow x^2 - 6x = -12$

$\Rightarrow x^2 - 6x + 7 = -12 + 7 = -5$

6. (A) $f(x) = \sqrt{\log_e(1 - x^2 + 3x)}$

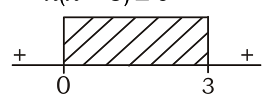
Now, $\log_e(1 - x^2 + 3x) \geq 0$

$\Rightarrow 1 - x^2 + 3x \geq 1$

$\Rightarrow -x^2 + 3x \geq 0$

$\Rightarrow x^2 - 3x \leq 0$

$\Rightarrow x(x-3) \leq 0$



Domain = [0, 3]

7. (B)

8. (D)

9. (B) $z = \frac{\sqrt{2} - i}{\sqrt{2} + i}$

$z = \frac{\sqrt{2} - i}{\sqrt{2} + i} \times \frac{\sqrt{2} - i}{\sqrt{2} - i}$

$z = \frac{\left(\sqrt{2} - i\right)^2}{2 - i^2}$

$z = \frac{2 + i^2 - 2\sqrt{2}i}{2 + 1}$

$z = \frac{2 - 1 - 2\sqrt{2}i}{3}$

$z = \frac{1 - 2\sqrt{2}i}{3}$

$|z| = \frac{\sqrt{1^2 + \left(2\sqrt{2}\right)^2}}{3}$

$|z| = \frac{\sqrt{1+8}}{3}$

$|z| = \frac{9}{3} = 3$

10. (B) $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$

$\Rightarrow \cos^{-1}\left[\cos\left(\pi+\frac{\pi}{4}\right)\right]$

$\Rightarrow \cos^{-1}\left(-\cos\frac{\pi}{4}\right)$

$\Rightarrow \cos^{-1}\left(\cos\left(\pi-\frac{\pi}{4}\right)\right)$

$\Rightarrow \cos^{-1}\left(\cos\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

11. (D) $\lim\limits_{x\to\infty}\frac{2x^2+3x-5}{1+2x-x^2}$

$\Rightarrow \lim\limits_{x\to\infty}\frac{x^2\left(2+\frac{3}{x}-\frac{5}{x^2}\right)}{x^2\left(\frac{1}{x^2}+\frac{2}{x}-1\right)}$

$\Rightarrow \frac{3+0-0}{0+0-1} = \frac{2}{-1} = -2$

12. (A) Let $a + ib = \sqrt{-3+4\sqrt{7}i}$

On squaring both side

$\Rightarrow (a^2 - b^2) + (2ab)\, i = -3 + 4\sqrt{7}\, i$

On comparing

$a^2 - b^2 = -3$ and $2ab = 4\sqrt{7}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 9 + 112$

$\Rightarrow (a^2 + b^2)^2 = 9 + 121 \Rightarrow a^2+b^2 = 11$ ..(ii)

From eq (i) and eq (ii)

$2a^2 = 8,$ $2b^2 = 14$

$a^2 = 4 \Rightarrow a = \pm 2,$ $b^2 = 7 \Rightarrow b = \pm\sqrt{7}$

Hence $\sqrt{-3+4\sqrt{7}i} = \pm\,(2+\sqrt{7}\, i)$

13. (C) $\vec{a} = -4\hat{i} + \lambda\hat{j} - 2\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} + 9\hat{k}$

Now, $\vec{a}.\vec{b} = |\vec{a}|\,|\vec{b}|\,\cos\theta$

$\Rightarrow -4 \times 3 + \lambda \times (-6) + (-2) \times 9$

$= \sqrt{(-4)^2+\lambda^2+(-2)^2}\ \sqrt{3^2+(-6)^2+9^2}\ .\cos\frac{\pi}{2}$

$\Rightarrow -6\lambda - 30 = 0 \Rightarrow \lambda = -5$

14. (C) A.M. $= \frac{1^2+2^2+3^2.........+n^2}{n}$

$= \frac{n}{6}(n+1)(2n+1) \times \frac{1}{n} = \frac{1}{6}(n+1)(2n+1)$

15. (D) Determinant $\begin{vmatrix} -1 & 0 & 4 & 6 \\ 2 & -3 & 1 & 0 \\ -5 & 6 & 2 & -2 \\ -6 & -1 & 7 & 2 \end{vmatrix}$

Co-factor of 4 = $(-1)^{1+3}\begin{vmatrix} 2 & -3 & 0 \\ -5 & 6 & -2 \\ -6 & -1 & 2 \end{vmatrix}$

$= 2\,(12 - 2) + 3\,(-10 - 12)$

$= 20 - 66 = -46$

16. (B) We know that

$(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

On putting $x = 1$

$(1 + 1)^n = C_0 + C_1 + C_2 + ... + C_n$

$\Rightarrow C_0 + C_1 + C_2 + ... + C_n = 2^n$

17. (C) 7, 8, 18, 21, 22, 23, 25, 27, 29

Mean $(\bar{x}) = \frac{7+8+18+21+22+23+25+27+29}{9}$

$\bar{x} = \frac{180}{9} = 20$

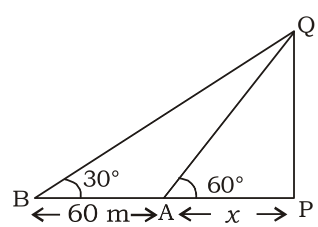
$\Sigma(x-\bar{x})^2 = (7-20)^2 + (8-20)^2 + (18-20)^2 + (21-20)^2 + (22-20)^2 + (23-20)^2 + (25-20)^2 + (27-20)^2 + (29-20)^2$

$\Sigma(x-\bar{x})^2 = 169 + 144 + 4 + 1 + 4 + 9 + 25 + 49 + 81 = 486$

Standard Deviation $= \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{486}{9}} = \sqrt{54} = 3\sqrt{6}$

18. (A)



Let breadth of the river = x m

In ΔAPQ

$\tan 60° = \frac{PQ}{AP}$

$\Rightarrow \sqrt{3} = \frac{PQ}{x} \Rightarrow PQ = \sqrt{3}\,x$

In ΔBPQ

$\tan 30° = \frac{PQ}{BP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+40}$

$\Rightarrow x + 60 = 3x$

$\Rightarrow 2x = 60 \Rightarrow x = 30$

Hence breadth of the river = 30 m

19. (C) $\sum_{n=1}^{10}(i^{n-1}-i^{n})$

$\Rightarrow (i^0 - i^1) + (i^1 - i^2) + (i^2 - i^3) + ... + (i^9 - i^{10})$

$\Rightarrow 1 - i^{10}$

$\Rightarrow 1 + 1 = 2$

20. (B) $I = \int \frac{x+2}{(x+5)(x-3)} dx$

$I = \int \left(\frac{-3}{2(x+5)} + \frac{5}{8(x-3)}\right) dx$

$I = \frac{-3}{2}\log(x + 5) + \frac{5}{8}\log(x - 3) + c$

21. (C) $L_1 \Rightarrow \frac{1-\sqrt{2}x}{4} = \frac{y+4}{7} = \frac{z-1}{8}$

$L_1 \Rightarrow \frac{x-\frac{1}{\sqrt{2}}}{-2\sqrt{2}} = \frac{y+4}{7} = \frac{z-1}{8}$

and $L_2 \Rightarrow \frac{x-0}{\sqrt{2}} = \frac{y-0}{1} = \frac{z-0}{1}$

Angle between lines

$\cos\theta = \frac{-2\sqrt{2}\times\sqrt{2}+7\times1+8\times1}{\sqrt{(-2\sqrt{2})^2+(7)^2+(8)^2}\sqrt{(\sqrt{2})^2+(1)^2+(1)^2}}$

$\cos\theta = \frac{11}{11\times2} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

22. (B) Direction ratio of L_1 = $(-2\sqrt{2}, 7, 8)$

23. (D) Direction cosine of L_2 = $< \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2} >$

24. (C) $n(S) = 8$

E = (HTT), (THT), (HTT) ; $n(E) = 3$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$

25. (B) We know that

sin35 < sin45 and cos 35 > cos 45

sin 35 < cos45

then sin35 < cos45 < cos35

⇒ sin35 < cos35

⇒ sin35 – cos35 < 0

Negative but greater than – 1.

26. (C) Differential equation

$\frac{dy}{dx} - \frac{y}{x^2} = 2.e^{\frac{-1}{x}}$

On comparing with general equation

$P = -\frac{1}{x^2}$ and $Q = 2.e^{\frac{-1}{x}}$

I.F. = $e^{\int P\,dx}$

I.F. = $e^{\int -\frac{1}{x^2}dx} = e^{\frac{1}{x}}$

Solution of differential equation

$y \times$ I.F. $= \int Q \times$ I.F. dx

$\Rightarrow y \times e^{\frac{1}{x}} = \int 2.e^{\frac{-1}{x}}.e^{\frac{1}{x}} dx$

$\Rightarrow y \times e^{\frac{1}{x}} = 2x + c$

27. (B) Let angles of a triangle = $3x, 4x, 3x$

$3x + 4x + 3x = 180$

$\Rightarrow 10x = 180 \quad \Rightarrow x = 18$

Angles of a triangle = 54°, 72°, 54°

Sine Rule

$\frac{a}{\sin 54} = \frac{b}{\sin 72} = \frac{c}{\sin 54}$

$\Rightarrow \frac{a}{\frac{\sqrt{5}+1}{4}} = \frac{b}{\frac{\sqrt{10+2\sqrt{5}}}{4}} = \frac{c}{\frac{\sqrt{5}+1}{4}}$

$\Rightarrow \frac{a}{\sqrt{5}+1} = \frac{b}{\sqrt{10+2\sqrt{5}}} = \frac{c}{\sqrt{5}+1}$

$\Rightarrow a : b : c = (\sqrt{5}+1) : \left(\sqrt{10+2\sqrt{5}}\right) : (\sqrt{5}+1)$

28. (B)

29. (B) $\begin{bmatrix} 2x+y+z & y & z \\ x & x+2y+z & z \\ x & y & x+y+2z \end{bmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{bmatrix} 2(x+y+z) & y & z \\ 2(x+y+z) & x+2y+z & z \\ 2(x+y+z) & y & x+y+2z \end{bmatrix}$

$\Rightarrow 2(x+y+z)\begin{bmatrix} 1 & y & z \\ 1 & x+2y+z & z \\ 1 & y & x+y+2z \end{bmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow 2(x+y+z)\begin{bmatrix} 1 & y & z \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{bmatrix}$

$\Rightarrow 2(x+y+z)\,[1(x+y+z)^2 - 0 - 0]$

$\Rightarrow 2(x+y+z)^3$

(30 - 31)

$$I = \int_0^{\pi} \ln(\cos x)dx$$

$$I = 2\int_0^{\pi/2} \ln(\cos x)dx \quad ...(i)$$

$$I = 2\int_0^{\pi/2} \ln\left[\cos\left(\frac{\pi}{2} - x\right)\right]dx$$

$$I = 2\int_0^{\pi/2} \ln(\sin x)\, dx \quad ...(ii)$$

30. (D) From eq(ii)

$$I = 2\int_0^{\pi/2} \ln(\sin x)dx$$

$$\Rightarrow \int_0^{\pi/2} \ln(\sin x)dx = I/2$$

31. (A) From eq(i)

$$I = 2\int_0^{\pi/2} \ln(\cos x)\, dx$$

$$\Rightarrow \int_0^{\pi/2} \ln(\cos x)\, dx = I/2$$

32. (C) $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$\Rightarrow (a+b+c)[1(bc-a^2)-b(b-a)+c(a-c)]$

$\Rightarrow (a+b+c)[bc-a^2-b^2+ab+ac-c^2]$

$\Rightarrow -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$\Rightarrow -\frac{1}{2}(a+b+c)[(a-b^2)+(b-c)^2+(c-a)^2]$

$\Rightarrow$ Negative value

33. (D) 1. We have, AB = A

$\therefore A^2 = (AB).(AB) = A.(BA)B$

$\Rightarrow A^2 = ABB \quad (\because BA = B)$

$\Rightarrow A^2 = ABB = AB = A \quad (\because AB = B)$

2. $B^2 = (BA).(BA)$

$\Rightarrow B^2 = B.(AB).A$

$\Rightarrow B^2 = B.A.A \quad (\because AB = A)$

$\Rightarrow B^2 = B.A$

$\Rightarrow B^2 = B \quad (\because BA = B)$

3. $(AB)^2 = (A)B.(AB)$

$\Rightarrow (AB)^2 = A.(BA)B$

$\Rightarrow (AB)^2 = A.B.B \quad (\because BA = B)$

$\Rightarrow (AB)^2 = AB$

$\Rightarrow (AB)^2 = A \quad (\because AB = A)$

34. (C) $\int_0^{2\pi} |\sin x|\, dx = 2\int_0^{\pi} \sin x\, dx$

$= 2[-\cos x]_0^{\pi}$

$= 2[-\cos\pi + \cos 0]$

$= 2[+1+1] = 4$

35. (A) Let $y = x.\ln\cot x$

On differential both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = x \times \frac{1}{\cot x} \times (-\text{cosec}^2 x) + \ln\cot x$$

$$\Rightarrow \frac{dy}{dx} = -x.\sec x.\text{cosec}\, x + \ln\cot x$$

36. (C) We know that

$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + ... + {}^nC_n x^n$

On putting $x = 1$

$\Rightarrow (1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n$

$$\Rightarrow 2^n = 1 + \sum_{r=1}^{n} C(n,r)$$

$$\Rightarrow \sum_{r=1}^{n} C(n,r) = 2^n - 1$$

37. (C) Deteminant $\begin{vmatrix} 1 & 2 & 0 & 4 \\ -1 & 3 & -2 & 4 \\ 0 & -5 & -4 & 2 \\ 6 & -1 & 0 & -3 \end{vmatrix}$

Cofactor of 3 $= (-1)^{2+2}\begin{vmatrix} 1 & 0 & 4 \\ 0 & -4 & 2 \\ 6 & 0 & -3 \end{vmatrix}$

$= 1(12-0) - 0 + 4(0+24)$

$= 12 + 96 = 108$

38. (D) Hyperbola $3x^2 - 4y^2 = 1$

$$\Rightarrow \frac{x^2}{1/3} - \frac{y^2}{1/4} = 1$$

$$a^2 = \frac{1}{3},\ b^2 = \frac{1}{4}$$

Now, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow e = \sqrt{1 + \frac{1/4}{1/3}} \Rightarrow e = \frac{\sqrt{7}}{2}$$

foci = $(\pm ae, 0)$

$$= \left(\pm\frac{1}{\sqrt{3}} \times \frac{\sqrt{7}}{2}, 0\right) = \left(\pm\frac{1}{2}\sqrt{\frac{7}{3}}, 0\right)$$

39. (A) $f(x) = 3x^3 + 5x^2 - 6x + 7$
$f'(x) = 9x^2 + 10x - 6$
$f''(x) = 18x + 10$
Now, $3f'(0) - 4f''(-1)$
$\Rightarrow 3[9\times0+10\times0-6] - 4[18\times(-1)+10]$
$\Rightarrow -18 - 4\times(-8) = 14$

40. (B) $I = \int \sin^2\theta\, d\theta + \int \cot^2\theta . \sin^2\theta\, d\theta$
$I = \int\left(\sin^2\theta + \frac{\cos^2\theta}{\sin^2\theta}.\sin^2\theta\right) d\theta$
$I = \int\left(\sin^2\theta + \cos^2\theta\right) d\theta$
$I = \int 1\, d\theta$
$I = \theta + c$

41. (A) The required Probability $= \frac{1}{2}\left[\frac{2}{5}\times\frac{2}{6}+\frac{3}{5}\times\frac{4}{6}\right]$
$= \frac{1}{2}\times\frac{16}{30} = \frac{4}{15}$

42. (B) The required Probability
$= \frac{1}{2}\left[\frac{3}{5}\times\frac{2}{6}+\frac{2}{5}\times\frac{4}{6}+\frac{3}{5}\times\frac{4}{6}\right]$
$= \frac{1}{2}\left[\frac{6}{30}+\frac{8}{30}+\frac{12}{30}\right]$
$= \frac{1}{2}\times\frac{26}{30} = \frac{13}{30}$

43. (A) $y = a^{\frac{1}{1-\log_a z}} \Rightarrow a = y^{1-\log_a z}$
$\Rightarrow \log_a a = (1-\log_a z)\log_a y$
$\Rightarrow \log_a y = \frac{1}{1-\log_a z}$
and $x = a^{\frac{1}{1-\log_a y}} \Rightarrow a = x^{1-\log_a y}$
$\Rightarrow \log_a a = (1-\log_a y)\log_a x$
$\Rightarrow \log_a x = \frac{1}{1-\log_a y}$
$\Rightarrow \log_a x = \frac{1}{1-\frac{1}{\log_a z}}$
$\Rightarrow \log_a x = \frac{1-\log_a z}{-\log_a z}$
Now, $\frac{1}{1-\log_a x} = \frac{1}{1+\frac{1-\log_a z}{\log_a z}}$
$\Rightarrow \frac{1}{1-\log_a x} = \frac{\log_a z}{1}$
$\Rightarrow \log_a z = \frac{1}{1-\log_a x} \Rightarrow z = a^{\frac{1}{1-\log_a x}}$

44. (C) Let $y = 3^{82}$
taking log both side
$\Rightarrow \log y = 82 \log_{10} 3$
$\Rightarrow \log_{10} y = 82\times0.4771$
$\Rightarrow \log_{10} y = 39.1222$
The number of digits = 39 + 1 = 40

45. (D) $\cos\theta = \frac{1}{2}\left(x+\frac{1}{x}\right)$
$\Rightarrow x + \frac{1}{x} = 2\cos\theta$
$\Rightarrow x^3 + \frac{1}{x^3} = (2\cos\theta)^3 - 3\times(2\cos\theta)$
$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$
$\Rightarrow x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta)$
$\Rightarrow x^3 + \frac{1}{x^3} = 2\cos3\theta$
$\Rightarrow \frac{1}{2}\left(x^3+\frac{1}{x^3}\right) = \cos3\theta$

46. (A) $\sin\frac{\pi}{3} + \sin\frac{5\pi}{9} - \sin\frac{7\pi}{9} - \sin\frac{8\pi}{9}$
$\Rightarrow \sin60 + \sin100 - \sin140 - \sin160$
$\Rightarrow \sin60 + \sin(90+10) - 2\sin\frac{140+160}{2}.$
$\cos\frac{160-140}{2}$
$\Rightarrow \frac{\sqrt{3}}{2} + \cos10 - 2\sin150.\cos10$
$\Rightarrow \frac{\sqrt{3}}{2} + \cos10 - 2\times\frac{1}{2}\cos10 = \frac{\sqrt{3}}{2}$

47. (B) **Statement I**
$\tan^{-1}1 + \tan^{-1}(2+\sqrt{3}) = \tan^{-1}\left(\frac{1+2+\sqrt{3}}{1-1(2+\sqrt{3})}\right)$
$\Rightarrow \tan^{-1}1 + \tan^{-1}(2+\sqrt{3}) = \tan^{-1}\left(\frac{3+\sqrt{3}}{-1-\sqrt{3}}\right)$
$\Rightarrow \tan^{-1}1 + \tan^{-1}(2+\sqrt{3}) = \tan^{-1}\left(\frac{\sqrt{3}(\sqrt{3}+1)}{-1(\sqrt{3}+1)}\right)$
$\Rightarrow \tan^{-1}1 + \tan^{-1}(2+\sqrt{3}) = \tan^{-1}(-\sqrt{3})$
Statement I is incorrect.
Statement II
$\sin^{-1}\frac{7}{25} + \sin^{-1}\frac{24}{25} = \sin^{-1}\frac{7}{25} + \cos^{-1}\frac{7}{25}$
$\Rightarrow \sin^{-1}\frac{7}{25} + \sin^{-1}\frac{24}{25} = \frac{\pi}{2}$
Statement II is correct.

48. (C) $\begin{vmatrix} x-y-z & 1-x & y+z \\ y-z-x & 1-y & z+x \\ z-x-y & 1-z & x+y \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1 & 1-x & y+z \\ 1 & 1-y & z+x \\ 1 & 1-z & x+y \end{vmatrix}$

$C_3 \to C_3 - C_2 + C_1$

$\Rightarrow \begin{vmatrix} 1 & 1-x & x+y+z \\ 1 & 1-y & x+y+z \\ 1 & 1-z & x+y+z \end{vmatrix}$

$\Rightarrow (x+y+z) \begin{vmatrix} 1 & 1-x & 1 \\ 1 & 1-y & 1 \\ 1 & 1-z & 1 \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two columns are indentical.]

49. (A) a, b, c are in G.P., then
$b^2 = ac$ ...(i)
p, q, r are in G.P., then
$q^2 = pr$ ...(ii)
from eq(i) and eq(ii)
$b^2q^2 = ac \times pr$
$(bq)^2 = ap \times cr$
Hence ap, bq, cr are in G.P.

50. (C) $I = \int \frac{1}{e^x - 1}\, dx$

$I = \int \frac{1}{e^x(1-e^{-x})}\, dx$

$I = \int \frac{e^{-x}}{1-e^{-x}}\, dx$

Let $1 - e^{-x} = t \Rightarrow e^{-x}\, dx = dt$

$I = \int \frac{dt}{t}$

$I = \log t + c$

$I = \log(1 - e^{-x}) + c$

$I = \log\left(\frac{e^x - 1}{e^x}\right) + c$

51. (D) $\frac{\sin 330°.\tan 150°.\cot 135°}{\sec 240°.\text{cosec}\,120°.\cos 225°}$

$\Rightarrow \frac{\sin(360° - 30°).\tan(180° - 30°).\cot(180° - 45°)}{\sec(180° + 60°).\text{cosec}(180° - 60°).\cos(180° + 45°)}$

$\Rightarrow \frac{(-\sin 30)(-\tan 30)(-\cot 45)}{(-\sec 60)(\text{cosec}\,60)(-\cos 45)}$

$\Rightarrow \frac{-\frac{1}{2} \times \frac{1}{\sqrt{3}} \times 1}{2 \times \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}}} = -\frac{1}{4\sqrt{2}}$

52. (D) Given line $3x + 7y = 22$

$\Rightarrow y = -\frac{3}{7}x + \frac{22}{7}$

Slope of line $m_1 = -\frac{3}{7}$

Slope of required line $m_2 = \frac{-1}{m_1} = \frac{-1 \times 7}{-3} = \frac{7}{3}$

Equation of line which passes through the point (–2, 5)

$y - 5 = \frac{7}{3}(x + 2)$

$\Rightarrow 7x - 3y + 29 = 0$

53. (C) $I = \int_{-3}^{3} (ax^3 + bx^2 + c)\, dx$

$I = \int_{-3}^{3} ax^3\, dx + \int_{-3}^{3} bx^2\, dx + \int_{-3}^{3} c\, dx$

$I = 0 + 2\int_0^3 bx^2\, dx + 2\int_0^3 c\, dx$

$I = 2b\left[\frac{x^3}{3}\right]_0^3 + 2c[x]_0^3$

$I = 2b\left[\frac{27}{3} - 0\right] + 2c[3 - 0]$

$I = 18b + 6c$

Hence it depends on values of b and c.

54. (B) $\begin{vmatrix} 1+i & \omega^2 & -\omega \\ \omega^2 + i & \omega & i \\ 1+2i+\omega^2 & \omega^2+\omega & i-\omega \end{vmatrix}$

$R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} 1+i & \omega^2 & -\omega \\ -1-i & -\omega^2 & \omega \\ 1+2i+\omega^2 & \omega^2+\omega & i-\omega \end{vmatrix}$

$\Rightarrow - \begin{vmatrix} 1+i & \omega^2 & -\omega \\ 1+i & \omega^2 & -\omega \\ 1+2i+\omega^2 & \omega^2+\omega & i-\omega \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two rows are identical.]

55. (D) $\begin{vmatrix} 2a & p & -x \\ 6b & 3q & -3y \\ 2c & r & -z \end{vmatrix} = \lambda \begin{vmatrix} x & -a & 2p \\ y & -b & 2q \\ z & -c & 2r \end{vmatrix}$

$\Rightarrow 2\times3\times(-1)\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \lambda\times(-1)\times2\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

$\Rightarrow -6\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = -2\lambda\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

$\Rightarrow -6 = -2\lambda \Rightarrow \lambda = 3$

56. (B) $\dfrac{\sin 80 - \cos 80}{\sin 80 + \cos 80}$

$\Rightarrow \dfrac{\sin 80 - \sin 10}{\sin 80 + \sin 10}$

$\Rightarrow \dfrac{2\cos\frac{80+10}{2}.\sin\frac{80-10}{2}}{2\sin\frac{80+10}{2}.\cos\frac{80-10}{2}}$

$\Rightarrow \dfrac{2\cos 45.\sin 35}{2\sin 45.\cos 35}$

$\Rightarrow \dfrac{2\times\frac{1}{\sqrt{2}}.\sin 35}{2\times\frac{1}{\sqrt{2}}.\cos 35} = \tan 35$

57. (C) $\lim\limits_{x\to 0} \dfrac{\sqrt{a+x}-\sqrt{a-x}}{2}$ $\left[\dfrac{0}{0}\right]$ from

$\Rightarrow \lim\limits_{x\to 0} \dfrac{\frac{1}{2\sqrt{a+x}}+\frac{1}{2\sqrt{a-x}}}{2}$

$\Rightarrow \dfrac{\frac{1}{2\sqrt{a}}+\frac{1}{2\sqrt{a}}}{2} = \dfrac{1}{2\sqrt{a}}$

58. (D) $f(x) = |3x^2 - 10|$ and $g(x) = x - 1$

Now, $fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f[x-1]$

$\Rightarrow fog(x) = |3(x-1)^2 - 10|$

$\Rightarrow fog(x) = |3(x^2 + 1 - 2x) - 10|$

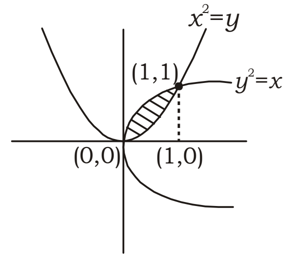
$\Rightarrow fog(x) = |3x^2 + 3 - 6x - 10|$

$\Rightarrow fog(x) = |3x^2 - 6x - 7|$

Now, $fog(-2) = |3(-2)^2 - 6(-2) - 7|$

$\Rightarrow fog(-2) = |12 + 12 - 7| = 17$

59. (C)



$y_1 \Rightarrow y^2 = x$ and $y_2 \Rightarrow x^2 = y$

Area = $\int_0^1 (y_1 - y_2)\,dx$

Area = $\int_0^1 \left(\sqrt{x} - x^2\right)dx$

Area = $\left[2\times\dfrac{x^{3/2}}{3} - \dfrac{x^3}{3}\right]_0^1$

Area = $\left[\dfrac{2}{3} - \dfrac{1}{3} - 0\right]$

Area = $\dfrac{1}{3}$ sq.unit

60. (D) The required no. of ways = ${}^6C_3 \times {}^{10}C_8$

61. (C) Given that $X = \{9(n-1) : n \in N\}$

$n = 1, 2, 3, 4, ...$

$X = \{0, 9, 18, 27, ...\}$

$Y = \{4^n - 3n - 1 : n \in N\}$

$n = 1, 2, 3, 4, ...$

$Y = \{0, 9, 54, 243, ...\}$

$(X \cap Y) = \{0, 9, 54, 243\} = Y$

62. (C) Differential equation

$x\,dy - y\,dx = x^2 y\,dx$

$\Rightarrow \dfrac{xdy - ydx}{xy} = x\,dx$

$\Rightarrow \dfrac{dy}{y} - \dfrac{dx}{y} = x\,dx$

On integrating

$\Rightarrow \log y - \log x = \dfrac{x^2}{2} + c$

$\Rightarrow \log\dfrac{y}{x} = \dfrac{x^2}{2} + c$

63. (B) The required no. of triangles = ${}^{11}C_3 - {}^4C_3$

$= 165 - 4$

$= 161$

64. (B) $\sin^{-1}(\log_3 2x)$

Here $-1 \le \log_3 2x \le 1$

$\Rightarrow 3^{-1} \le 2x \le 3^1$

$\Rightarrow \dfrac{1}{3} \le 2x \le 3$

$\Rightarrow \dfrac{1}{6} \le x \le \dfrac{3}{2}$

Domain = $\left[\dfrac{1}{6}, \dfrac{3}{2}\right]$

65. (C) Series $\frac{1^2}{2}+\frac{1^2+2^2}{2+4}+\frac{1^2+2^2+3^2}{2+4+6}+.......$

$T_n = \frac{1^2+2^2+3^2+.....+n^2}{2+4+6+.....+n}$

$T_n = \frac{1^2+2^2+3^2+.....+n^2}{2(1+2+3+.....+n)}$

$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{2\times\frac{n(n+1)}{2}} = \frac{2n+1}{6}$

66. (A) $f(x) = x^2 + 5x - 6$

$f'(x) = 2x + 5 \Rightarrow f'(c) = 2c + 5$

$a = -1,\ b = \frac{1}{2}$

$f(a) \Rightarrow f(-1) = (-1)^2 + 5(-1) - 6 = -10$

$f(b) \Rightarrow f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 5\times\frac{1}{2} - 6 = \frac{-13}{4}$

Now, $f'(c) = \frac{f(b)-f(a)}{b-a}$

$\Rightarrow 2c + 5 = \frac{\frac{-13}{4}+10}{\frac{1}{2}+1}$

$\Rightarrow 2c + 5 = \frac{9}{2}$

$\Rightarrow 4c + 10 = 9 \Rightarrow c = \frac{-1}{4}$

67. (B) $0.1\overline{37} = \frac{137-1}{990} = \frac{136}{990} = \frac{68}{495}$

68. (A) R is symmetric only.

69. (B) $S \subset R$

70. (A) $f(x) = x^2 - 3x + 2$

Now, $f\{f(x)\} = f(x^2 - 3x + 2)$

$\Rightarrow f\{f(x)\} = (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$\Rightarrow f\{f(x)\} = x^4 - 6x^3 + 10x^2 - 3x$

71. (C) $(f + 3g)(x) = f(x) + 3g(x)$

$(f + 3g)(x) = [x] + 3[x]$

$(f + 3g)(-2.5) = [-2.5] + 3[-2.5]$

$(f + 3g)(-2.5) = -3 + 3 \times (-3)$

$(f + 3g)(-2.5) = -3 - 9 = -12$

72. (C) If one regression coefficient be unity, them the other will be less than or equal to unity.

73. (A) $\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}\{3\times0.4 - 4\times(0.4)^3\}]$

$\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}\{1.2 - 0.256\}]$

$\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}(0.944)] = 0.944$

74. (D)

75. (B) $\left(\frac{1-i}{1+i}\right)$ is purely imaginary, so

$\frac{1-i}{1+i}\times\frac{1-i}{1-i} = \frac{(1-2)^2}{1-2^2} = \frac{1+i^2-2i}{2}$

$\Rightarrow \frac{1-i}{1+i}\times\frac{1-i}{1-i} = -\frac{2i}{2} = -i$

So, $(-i)^n \Rightarrow$ Purely imaginary with positive part.

So, n must be equal to 3.

76. (D) The set of all prime numbers

77. (C) The required Probability $= \frac{4}{52}\times\frac{4}{51}$

$= \frac{4}{13\times51} = \frac{4}{663}$

78. (C) Let the natural number be x.

Then, sum of 11 consecutive natural numbers

$\Rightarrow x + (x + 1) + (x + 2) + (x + 3) + (x + 4) + (x + 5) + (x + 6) + (x + 7) + (x + 8) + (x + 9) + (x + 10) = 2761$

$\Rightarrow 11x + 55 = 2761$

$\Rightarrow 11x = 2761 - 55$

$\Rightarrow 11x = 2706 \Rightarrow x = \frac{2706}{11} = 246$

Middle term = $(x + 5) = 246 + 5 = 251$

79. (A) SD = $\sqrt{\text{Variance}} = \sqrt{V}$

80. (C) Equation of Y-axis is $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$

81. (D) $I = \int\frac{\sec^2\left(2\tan^{-1}x\right)}{1+x^2}\,dx$

Let $2\tan^{-1}x = t \Rightarrow 2\times\frac{1}{1+x^2}\,dx = dt$

$\Rightarrow \frac{1}{1+x^2}\,dx = \frac{1}{2}dt$

$I = \int\sec^2 t\times\frac{dt}{2} = \frac{1}{2}\cdot\int\sec^2 t\,dt$

$I = \frac{1}{2}\tan t + c = \frac{1}{2}\tan(2\tan^{-1}x) + c$

82. (C) $1 - \frac{\cos^2 x}{1+\sin x} + \frac{1+\sin x}{\cos x} - \frac{\cos x}{1-\sin x}$

$\Rightarrow \frac{1+\sin x-\cos^2 x}{1+\sin x} + \frac{(1+\sin x)(1-\sin x)-\cos^2 x}{(1-\sin x)\cos x}$

$\Rightarrow \frac{\sin^2 x+\sin x}{1+\sin x} + \frac{1-\sin^2 x-\cos^2 x}{(1-\sin x)\cos x}$

$\Rightarrow \frac{\sin x(1+\sin x)}{1+\sin x} + \frac{\cos^2 x-\cos^2 x}{(1-\sin x)\cos x}$

$\Rightarrow \sin x + 0 = \sin x$

83. (D) $\frac{\cos\theta}{1+\sin\theta+\cos\theta} = x$

$\Rightarrow \frac{\cos\theta}{1+\sin\theta+\cos\theta} \times \frac{1-\sin\theta-\cos\theta}{1-\sin\theta-\cos\theta} = x$

$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{1-(\sin\theta+\cos\theta)^2} = x$

$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{1-(1+2\sin\theta.\cos\theta)} = x$

$\Rightarrow \frac{\cos\theta(1-\sin\theta-\cos\theta)}{-2\sin\theta.\cos\theta} = x$

$\Rightarrow \frac{1-\sin\theta-\cos\theta}{-2\sin\theta} = x$

$\Rightarrow \frac{1-\sin\theta-\cos\theta}{\sin\theta} = -2x$

84. (A) $\begin{vmatrix} 5! & 4! & 3! \\ 4! & 3! & 2! \\ 6! & 5! & 4! \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 5\times4\times3! & 4\times3! & 3! \\ 4\times3\times2! & 3\times2! & 2! \\ 6\times5\times4! & 5\times4! & 4! \end{vmatrix}$

$\Rightarrow 2! \times 3! \times 4! \begin{vmatrix} 20 & 4 & 1 \\ 12 & 3 & 1 \\ 30 & 5 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow 2! \times 3! \times 4! \begin{vmatrix} 20 & 4 & 1 \\ -8 & -1 & 0 \\ 10 & 1 & 0 \end{vmatrix}$

$R_3 \to R_3 + R_2$

$\Rightarrow 2! \times 3! \times 4! \begin{vmatrix} 20 & 4 & 1 \\ -8 & -1 & 0 \\ 2 & 0 & 0 \end{vmatrix}$

$\Rightarrow 2! \times 3! \times 4!\ [20 \times 0 - 4 \times 0 + 1\ (0 + 2)]$

$\Rightarrow 2 \times 2! \times 3! \times 4! = 576$

85. (B) Angle describe in 12 hr by hour-hand = 360°

Angle describe in 1 hr(60 min) by hour-hand = $\frac{360}{12}$

Angle decribe in 1 min by hour-hand = $\frac{360}{12\times60}$

Angle describe in 36 min by hour-hand

$= \frac{360}{12\times60} \times 36 = 18°$

86. (A) **In ΔABC:-**

$a = 2$, $b = \sqrt{2}$ and C = 45°

Now, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos45° = \frac{4+2-c^2}{2\times2\times\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{6-c^2}{4\sqrt{2}}$

$\Rightarrow 4 = 6 - c^2 \Rightarrow c = \sqrt{2}$

87. (A)

88. (B) Plane $2x + y - 2z + 6 = 0$ and

line $\frac{x-1}{4} = \frac{y+1}{7} = \frac{z-6}{-4}$

Let angle between plane and line = θ

Now, $\sin\theta = \frac{2\times4\times1\times7-2\times(-4)}{\sqrt{2^2+1^2+(-2)^2}\sqrt{4^2+7^2+(-4)^2}}$

$\Rightarrow \sin\theta = \frac{23}{3\times9}$

$\Rightarrow \sin\theta = \frac{23}{27} \Rightarrow \theta = \sin^{-1}\left(\frac{23}{27}\right)$

89. (B) Points $(a, 0)$, $(at_1^2, 2at_1)$ and $(at_1^2, 2at_2)$ are collinear,

then $\begin{vmatrix} a & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = 0$

$\Rightarrow a \times 2a \begin{vmatrix} 1 & 0 & 1 \\ t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ t_1^2-1 & t_1 & 0 \\ t_2^2-1 & t_2 & 0 \end{vmatrix} = 0$

$\Rightarrow 1\times 0 + 0 + 1[\ t_1^2 t_2 - t_2 - t_1 t_2^2 + t_1] = 0$

$\Rightarrow t_1t_2(t_1 - t_2) + 1(t_1 - t_2) = 0$

$\Rightarrow (t_1 - t_2)(t_1t_2 + 1) = 0$

$\Rightarrow t_1t_2 + 1 = 0,\ t_1 - t_2 \neq 0$

$\Rightarrow t_1t_2 = -1$

90. (B) $\begin{vmatrix} \log_7 7 & \log_2 4 & 3 \\ 1 & \log_3 9 & 10 \\ \log_e e & 2 & 12 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & \log_2 2^2 & 3 \\ 1 & \log_3 3^2 & 10 \\ 1 & 2 & 12 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 10 \\ 1 & 2 & 12 \end{vmatrix}$

$\Rightarrow 2\begin{vmatrix} 1 & 1 & 3 \\ 1 & 1 & 10 \\ 1 & 1 & 12 \end{vmatrix} = 0$

[$\because$ Two columns are identical.]

91. (D) Let $y = \log_{10}(5x^3 - 2x)$ and $z = x^2$

$y = \log_{10}e \times \log_e(5x^3 - 2x)$, $\frac{dz}{dx} = 2x$

$\Rightarrow \frac{dy}{dx} = \log_{10}e \times \frac{1}{5x^3 - 2x} \times (15x^2 - 2)$,

$\Rightarrow \frac{dy}{dx} = (15x^2 - 2)\log_{10}e \times \frac{1}{5x^3 - 2x}$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = (15x^2 - 2)\log_{10}e \times \frac{1}{5x^3 - 2x} \times \frac{1}{2x}$

$\Rightarrow \frac{dy}{dz} = \frac{(15x^2 - 2)\log_{10}e}{2x^2(5x^2 - 2)}$

92. (C) Let $y = \sqrt{6 + 5\sqrt{6 + 5\sqrt{6 + \ldots\ldots\infty}}}$

$\Rightarrow y = \sqrt{6 + 5y}$

On squaring

$\Rightarrow y^2 = 6 + 5y$

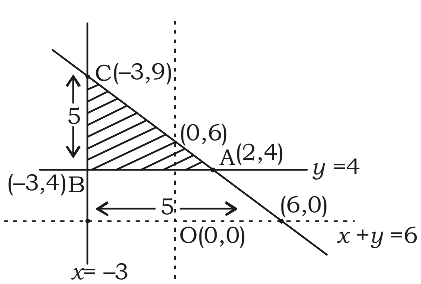
$\Rightarrow y^2 - 5y - 6 = 0$

$\Rightarrow (y - 6)(y + 1) = 0$

$\Rightarrow y = 6, -1$

Hence $\sqrt{6 + 5\sqrt{6 + 5\sqrt{6 + \ldots\ldots\infty}}} = 6$

93. (C)



The required Area $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ sq.unit

94. (A) Function is one-one but onto.

95. (C)

2	51	1
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(51)_{10} = (110011)_2$

96. (D) Number can be formed $y(0, 1, 2, 3)$ or $(0, 2, 3, 4)$

The required numbers = 3 × 3! + 2 × 3!

=18 ×12 = 30

97. (C)

98. (B) Data 21, 22, 32, 13, 41, 42, 51, 52, 61, 62

Middle terms = 41, 42

Median = $\frac{41 + 42}{2}$ = 41.5

99. (A) Lines $x - 6y = 11$

and $12y - 2x = 7 \Rightarrow x - 6y = \frac{-7}{2}$

The required equation

$x - 6y = \frac{11 + \left(\frac{-7}{2}\right)}{2}$

$\Rightarrow x - 6y = \frac{15}{4} \Rightarrow 4x - 24y = 15$

100. (D)

101. (D) $\sin 18° = \frac{\sqrt{5} - 1}{4}$

102. (A) $S = \frac{1}{1.5} + \frac{1}{5.9} + \ldots\ldots$upto 10 terms

$S = \frac{1}{1.5} + \frac{1}{5.9} + \ldots\ldots + \frac{1}{37.41}$

$S = \frac{1}{4}\left(1 - \frac{1}{5}\right) + \frac{1}{4}\left(\frac{1}{5} - \frac{1}{9}\right) + \ldots. + \frac{1}{4}\left(\frac{1}{37} - \frac{1}{41}\right)$

$S = \frac{1}{4}\left[1 - \frac{1}{41}\right]$

$S = \frac{1}{4} \times \frac{40}{41} = \frac{10}{41}$

103. (D) Quadratic equation

$5x^2 + px - 8 = 0$ ...(i)

One root = 2

It is satisfy the equation

$5 \times 4 + p \times 2 - 8 = 0 \Rightarrow p = -6$

from eq(i)

$5x^2 - 6x - 8 = 0$

Let second root = α

$2 + \alpha = \frac{6}{5} \Rightarrow \alpha = -\frac{4}{5}$

104. (D) Let A = $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & -1 \\ 0 & 2 & 5 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 3 \\ 11 \\ -7 \end{bmatrix}$

Using elementary methed

Now, [A/B] = $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 3 & 2 & -1 & 11 \\ 0 & 2 & 5 & -7 \end{array}\right]$

$R_2 \to R_2 - 3R_1$

$\Rightarrow$ [A/B] = $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -7 & 2 \\ 0 & 2 & 5 & -7 \end{array}\right]$

$R_3 \to R_3 - \frac{2}{5} R_2$

$\Rightarrow$ [A/B] = $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 0 & 5 & -7 & 2 \\ 0 & 0 & \frac{39}{5} & \frac{-39}{5} \end{array}\right]$

Now, $x - y + 2z = 3$ ...(i)

$5y - 7z = 2$ ...(ii)

$\frac{39}{5} z = \frac{-39}{5} \Rightarrow z = -1$

from eq(i)

$5y + 7 = 2 \Rightarrow y = -1$

from eq(ii)

$x + 1 - 2 = 3 \Rightarrow x = 4$

Hence $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix}$

105. (B) Given that $f(x) = 3x + 7$

x	$f(x)$
1	10
2	13
⋮	⋮

so on

So function is injective but not surjective.

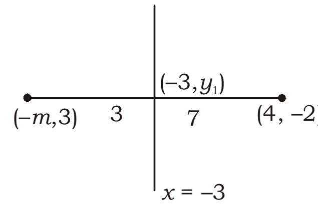
106. (C)

107. (A) A = {1, 2, 4, 5, 7, 8, 11,13} ; $n = 8$

Number of proper subsets = $2^n - 1$

$= 2^8 - 1 = 255$

108. (B)



Now, $\frac{3 \times 4 + 7 \times (-m)}{3 + 7} = -3$

$\Rightarrow \frac{12 - 7m}{10} = -3$

$\Rightarrow 12 - 7m = -30 \Rightarrow m = 6$

109. (C) $I = \int \frac{\ln x}{x} \, dx$

$I = \ln x \int \frac{1}{x} \, dx - \int \left\{ \frac{d}{dx}(\ln x) . \int \frac{1}{x} \, dx \right\} dx$

$I = (\ln x)(\ln x) - \int \frac{1}{x} . \ln x \, dx + 2c$

$I = (\ln x)^2 - I + 2c$

$2I = (\ln x)^2 + 2c \Rightarrow I = \frac{(\ln x)^2}{2} + c$

110. (B) In the expansion $\left(3\sqrt{x} + \frac{1}{6x}\right)^7$

Middle terms = $\left(\frac{7+1}{2}\right)^{th}$ and $\left(\frac{7+1}{2} + 1\right)^{th}$

$= 4^{th}$ and 5^{th}

$T_4 = T_{3+1} = {}^7C_3 (3\sqrt{x})^4 \left(\frac{1}{6x}\right)^3$

$= 35 \times \frac{3^4}{6^3} x^{-1} = \frac{105}{8} x^{-1}$

$T_5 = T_{4+1} = {}^7C_4 (3\sqrt{x})^3 \left(\frac{1}{6x}\right)^4$

$= 35 \times \frac{3^3}{6^4} x^{-5/2} = \frac{35}{48} x^{-5/2}$

The required sum = $\frac{105}{8} + \frac{35}{48}$

$= \frac{630 + 35}{48} = \frac{665}{48}$

111. (D) equation whose roots are 4 and – 6

$(x - 4)(x + 6) = 0$

$\Rightarrow x^2 + 2x - 24 = 0$

Original equation

$x^2 - 2x - 24 = 0$

$\Rightarrow x^2 - 6x + 4x - 24 = 0$

$\Rightarrow (x - 6)(x + 4) = 0$

Roots of originalar equation = 6, – 4

112. (C) Line

$(3x - 4y + 6) + \lambda(x + 2y + 1) = 0$

$(3 + \lambda)x + (-4 + 2\lambda)y + 6 + \lambda = 0$

$y = -\left(\frac{3+\lambda}{-4+2\lambda}\right)x - \frac{6+\lambda}{-4+2\lambda}$

it is parallel to x-axis i.e.

m = 0

$\Rightarrow -\left(\frac{3+\lambda}{-4+2\lambda}\right) = 0$

$\Rightarrow \lambda + 3 = 0 \Rightarrow \lambda = -3$

113. (B) $A_1 = \int_0^{\pi/4} \cos x \; dx$

$A_1 = [\sin x]_0^{\pi/4}$

$A_1 = \sin\frac{\pi}{4} - \sin 0$

$A_1 = \frac{1}{\sqrt{2}}$

and $A_2 = \int_0^{\pi/4} \sin 2x \; dx$

$A_2 = -\left[\frac{\cos 2x}{2}\right]_0^{\pi/4}$

$A_2 = -\frac{1}{2}\left[\cos\frac{\pi}{2} - \cos 0\right]$

$A_2 = -\frac{1}{2}[0 - 1] = \frac{1}{2}$

Now, $\frac{A_1}{A_2} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}$

$\Rightarrow \frac{A_1}{A_2} = \frac{\sqrt{2}}{1}$

Hence $A_1 : A_2 = \sqrt{2} : 1$

114. (C) Hyperbola

$\frac{x^2}{16} - \frac{y^2}{\lambda^2} = 1$

eccentricity $e = \sqrt{1 + \frac{\lambda^2}{16}}$

$\Rightarrow e = \frac{\sqrt{16+\lambda^2}}{4}$

foci $(\pm ae, 0) = \left(\pm\sqrt{16+\lambda^2}, 0\right)$

Ellipse

$\frac{x^2}{36} + \frac{y^2}{9} = 1$

eccentricity $e = \sqrt{1 - \frac{9}{36}}$

$\Rightarrow e = \sqrt{\frac{36-9}{25}} = \frac{\sqrt{27}}{5}$

foci$(\pm ae, 0) = \left(\pm\sqrt{27}, 0\right)$

Now, $\sqrt{16+\lambda^2} = \sqrt{27}$

$\Rightarrow 16 + \lambda^2 = 27 \Rightarrow \lambda^2 = 11$

115. (B) The required no. of ways = (8 – 1)!
= 7!

116. (B) The total possible ways = 7 × 6 = 42

117. (C) $f(x) = \sqrt{8x^2+1} \Rightarrow f(1) = 3$

$f'(x) = \frac{16x}{2\sqrt{8x^2+1}} = \frac{8x}{\sqrt{8x^2+1}}$

Now, $\lim_{x\to 1} \frac{f(x)-3}{x-1}$ $\left[\frac{0}{0}\right]$ Form

By L- Hospital's Rule

$\Rightarrow \lim_{x\to 1} \frac{f'(x)}{1}$

$\Rightarrow \lim_{x\to 1} \frac{8x}{\sqrt{8x^2+1}} = \frac{8}{3}$

118. (D) $I = \int_0^1 \sum_{n=1}^{5} (x^{n-1} - x^n) \; dx$

$I = \int_0^1 \begin{bmatrix} (x^0 - x) + (x - x^2) + (x^2 - x^3) + \\ (x^3 - x^4) + (x^4 - x^5) \end{bmatrix} dx$

$I = \int_0^1 (x^0 - x^5) \; dx$

$I = \left(x - \frac{x^6}{6}\right)_0^1$

$I = 1 - \frac{1}{6} = \frac{5}{6}$

119. (C) Slope = $\frac{x}{y+2}$ and point = (–1, 2)

equation of the curve

$y - 2 = \frac{x}{y+2}(x + 1)$

$\Rightarrow y^2 - 4 = x^2 + x$

$\Rightarrow x^2 - y^2 + x + 4 = 0$

120. (B) $\left(-\sqrt{-1}\right)^{8n-1} + \left(-\sqrt{-1}\right)^{4n+3}$

$\Rightarrow (-i)^{8n-1} + (-i)^{4n+3}$

$\Rightarrow (-i)^{8n}(-i)^{-1} + (-i)^{4n}(-i)^3$

$\Rightarrow \frac{1}{-i} - i^3$

$\Rightarrow i + i = 2i$

NDA (MATHS) MOCK TEST - 156 (Answer Key)

1.	(A)	21.	(C)	41.	(A)	61.	(C)	81.	(D)	101.	(D)
2.	(B)	22.	(B)	42.	(B)	62.	(C)	82.	(C)	102.	(A)
3.	(C)	23.	(D)	43.	(A)	63.	(B)	83.	(D)	103.	(D)
4.	(C)	24.	(C)	44.	(C)	64.	(B)	84.	(A)	104.	(D)
5.	(B)	25.	(B)	45.	(D)	65.	(C)	85.	(B)	105.	(B)
6.	(A)	26.	(C)	46.	(A)	66.	(A)	86.	(A)	106.	(C)
7.	(B)	27.	(B)	47.	(B)	67.	(B)	87.	(A)	107.	(A)
8.	(D)	28.	(B)	48.	(C)	68.	(A)	88.	(B)	108.	(B)
9.	(B)	29.	(B)	49.	(A)	69.	(B)	89.	(B)	109.	(C)
10.	(B)	30.	(D)	50.	(C)	70.	(A)	90.	(B)	110.	(B)
11.	(D)	31.	(A)	51.	(D)	71.	(C)	91.	(D)	111.	(D)
12.	(A)	32.	(C)	52.	(D)	72.	(C)	92.	(C)	112.	(C)
13.	(A)	33.	(D)	53.	(C)	73.	(A)	93.	(C)	113.	(B)
14.	(C)	34.	(C)	54.	(B)	74.	(D)	94.	(A)	114.	(C)
15.	(D)	35.	(A)	55.	(D)	75.	(B)	95.	(C)	115.	(B)
16.	(B)	36.	(C)	56.	(B)	76.	(D)	96.	(D)	116.	(B)
17.	(C)	37.	(C)	57.	(C)	77.	(C)	97.	(C)	117.	(C)
18.	(A)	38.	(D)	58.	(D)	78.	(C)	98.	(B)	118.	(D)
19.	(B)	39.	(A)	59.	(C)	79.	(A)	99.	(A)	119.	(C)
20.	(B)	40.	(B)	60.	(D)	80.	(C)	100.	(D)	120.	(B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777