NDA MATHS MOCK TEST - 160 (SOLUTION)

1. (B) Word "ELECTION"

LCTN (EEIO) — as one word

The required no. of words = $5! \times \frac{4!}{2!}$

$= 120 \times 2 = 1440$

2. (D) $A = \begin{bmatrix} 1 & -1 & \lambda \\ 8 & -5 & -1 \\ 3 & -2 & 0 \end{bmatrix}$

A is not inverible,

then $|A| = 0$

$\Rightarrow \begin{vmatrix} 1 & -1 & \lambda \\ 8 & -5 & -1 \\ 3 & -2 & 0 \end{vmatrix} = 0$

$\Rightarrow 1(0-2) + 1(0+3) + \lambda(-16+15) = 0$

$\Rightarrow -2 + 3 - \lambda = 0 \Rightarrow \lambda = 1$

3. (B) $\tan\theta$, $2\tan\theta + 2$, $3\tan\theta + 3$ are in G.P.,

then $(2\tan\theta + 2)^2 = \tan\theta.(3\tan\theta + 3)$

$\Rightarrow 4\tan^2\theta + 4 + 8\tan\theta = 3\tan^2\theta + 3\tan\theta$

$\Rightarrow \tan^2\theta + 5\tan\theta + 4 = 0$

$\Rightarrow (\tan\theta + 4)(\tan + 1) = 0$

$\Rightarrow \tan\theta + 4 = 0 \qquad [\tan\theta \neq -1]$

$\Rightarrow \tan\theta = -4$

Now, $\frac{7 - 4\tan\theta}{8 - 2\sqrt{\sec^2\theta - 1}}$

$\Rightarrow \frac{7 - 4\tan\theta}{8 - 2\tan\theta}$

$\Rightarrow \frac{7 - 4(-4)}{8 - 2(-4)}$

$\Rightarrow \frac{7+16}{8+8} = \frac{23}{16}$

4. (C) Let $a + ib = \sqrt{-1 + 3\sqrt{11}i}$

On squaring

$\Rightarrow (a^2 - b^2) + 2abi = -1 + 3\sqrt{11}\,i$

On comparing

$\Rightarrow a^2 - b^2 = -1$ and $2ab = 3\sqrt{11}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = (-1)^2 + (3\sqrt{11})^2$

$\Rightarrow (a^2 + b^2)^2 = 1 + 99$

$\Rightarrow (a^2 + b^2)^2 = 100 = 10$ ...(ii)

from eq(i) and eq(ii)

$2a^2 = 9 \Rightarrow a = \pm\frac{3}{\sqrt{2}}$, $2b^2 = 11 \Rightarrow b = \pm\frac{\sqrt{11}}{\sqrt{2}}$

Hence $\sqrt{-1 + 3\sqrt{11}i} = \pm\left(\frac{3}{\sqrt{2}} + \frac{\sqrt{11}}{\sqrt{2}}i\right)$

$\Rightarrow \sqrt{-1 + 3\sqrt{11}i} = \pm\frac{1}{\sqrt{2}}\left(3 + \sqrt{11}i\right)$

5. (A) Digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

9	9	8	7

= 4536

6. (D) $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$ ...(i)

Prop. IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2} \frac{\sin^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx$

$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x} dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x} dx$

$2I = \int_0^{\pi/2} 1.dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

7. (A) Let $y = 3^{93}$

taking log

$\Rightarrow \log y = 93 \log 3$

$\Rightarrow \log y = 93 \times 0.4771$

$\Rightarrow \log y = 44.3703$

The number of digits = 44 +1 = 45

8. (B) $\phi = \{\ \}$

9. (C) $\begin{vmatrix} 3a & -3b & 6c \\ 5p & -5q & 10r \\ -4l & 4m & -8n \end{vmatrix} = \lambda \begin{vmatrix} l & m & n \\ p & q & r \\ a & b & c \end{vmatrix}$

$\Rightarrow 3\times5(-4)\begin{vmatrix} a & -b & 2c \\ p & -q & 2r \\ l & -m & 2n \end{vmatrix} = -\lambda \begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix}$

$\Rightarrow -60\times(-1)\times2\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix} = -\lambda \begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix}$

$\Rightarrow 120 = -\lambda \Rightarrow \lambda = -120$

10. (C) $\cos36°.\cos72°.\cos108°.\cos144°$

$\Rightarrow \cos36°.\cos(90-18)°.\cos(90+18)°.\cos(180-36)°$

$\Rightarrow \cos36°.\sin18°(-\sin18°).(-\cos36°)$

$\Rightarrow \cos^2 36°.\sin^2 18°$

$\Rightarrow \left(\frac{\sqrt{5}+1}{4}\right)^2 \left(\frac{\sqrt{5}-1}{4}\right)^2$

$\Rightarrow \left[\frac{\sqrt{5}+1}{4}\times\frac{\sqrt{5}-1}{4}\right]^2$

$\Rightarrow \left(\frac{5-1}{16}\right)^2$

$\Rightarrow \left(\frac{4}{16}\right)^2 = \frac{1}{16}$

11. (A) $I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ ...(i)

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi} \frac{(\pi-x)dx}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}\,dx$

$I = \int_0^{\pi} \frac{(\pi-x)dx}{a^2\cos^2 x + b^2\sin^2 x}$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi} \frac{x+\pi-x}{a^2\cos^2 x + b^2\sin^2 x}\,dx$

$2I = \int_0^{\pi} \frac{\pi}{a^2\cos^2 x + b^2\sin^2 x}\,dx$

$I = 2\int_0^{\pi} \frac{\pi}{a^2\cos^2 x + b^2\sin^2 x}\,dx$

$I = \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$

$I = \pi\int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$

Let $b\tan x = t$ when $x\to 0,\ t\to 0$

$\Rightarrow b\sec^2 x\,dx = dt$ $x\to\pi/2,\ t\to\infty$

$\Rightarrow \sec^2 x\,dx = \frac{1}{b}dt$

$I = \frac{\pi}{b}\int_0^{\infty}\frac{dt}{a^2+t^2}$

$I = \frac{\pi}{ab}\left[\tan^{-1}\frac{t}{a}\right]_0^{\infty}$

$I = \frac{\pi}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$

$I = \frac{\pi}{ab}\left[\frac{\pi}{ab}-0\right] = \frac{\pi^2}{2ab}$

12. (B) $y = \tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]$

Let $x = \cos2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$y = \tan^{-1}\left[\frac{\sqrt{1+\cos2\theta}-\sqrt{1-\cos2\theta}}{\sqrt{1+\cos2\theta}+\sqrt{1-\cos2\theta}}\right]$

$y = \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$

$y = \tan^{-1}\left[\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right]$

$y = \tan^{-1}\left[\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}\right]$

$y = \tan^{-1}\left[\frac{\cos\theta-\sin\theta)}{\cos\theta+\sin\theta)}\right]$

$y = \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$

$y = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$

$y = \frac{\pi}{4} - \theta$

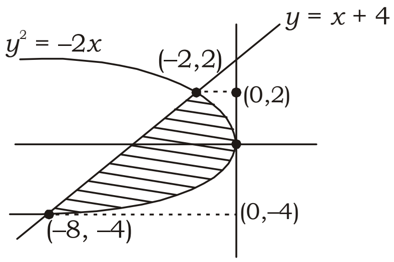
$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

On differentiating both sides w.r.t.'x'

$\frac{dy}{dx} = \frac{-1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)$

$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

13. (D)



curve $y^2 = -2x \Rightarrow x_1 \Rightarrow x = \frac{-y^2}{2}$

and line $y = x + 4 \Rightarrow x_2 \Rightarrow x = y - 4$

Area $= \int_{-4}^{2}(x_2 - x_1)\,dy$

Area $= \int_{-4}^{2}\left(y - 4 + \frac{y^2}{2}\right)dy$

Area $= \left[\frac{y^2}{2} - 4y + \frac{1}{2}\times\frac{y^3}{3}\right]_{-4}^{2}$

Area $= \left(\frac{2^2}{2} - 4\times 2 + \frac{1}{6}\times 2^3\right) - \left(\frac{(-4)^2}{2} - 4\times(-4) + \frac{1}{6}\times(-4)^3\right)$

Area $= \left(2 - 8 + \frac{4}{3}\right) - \left(8 + 16 - \frac{32}{3}\right)$

Area $= \left|-\frac{14}{3} - \frac{40}{3}\right| = 18$ sq. unit

14. (C) Equation $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a real sphere, then $u^2 + v^2 + w^2 - d > 0$

15. (B) $z = \frac{(1-3i)(2+i)}{3+2i}$

$z = \frac{2 - 6i + i - 3i^2}{3+2i}$

$z = \frac{5-5i}{3+2i}\times\frac{3-2i}{3-2i}$

$z = \frac{15 - 15i - 10i + 10i^2}{9 - 4i^2}$

$z = \frac{5 - 25i}{13}$

Now, argument $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{\frac{-25}{13}}{\frac{5}{13}}\right)$

$\Rightarrow \theta = \tan^{-1}(-5)$

16. (C) In the expansion of $\left(x^{1/3} + \frac{1}{2x^{2/3}}\right)^9$

$T_{r+1} = {}^9C_r (x^{1/3})^{9-r}\left(\frac{1}{2x^{2/3}}\right)^r$

$T_{r+1} = {}^9C_r\left(\frac{1}{2}\right)^r x^{\frac{9-3r}{3}}$

Here $\frac{9-3r}{3} = -5$

$\Rightarrow 9 - 3r = -15$

$\Rightarrow 24 = 3r \Rightarrow r = 8$

Coefficient of $x^{-5} = {}^9C_8\left(\frac{1}{2}\right)^8$

$= \frac{9}{2^8} = \frac{9}{256}$

17. (D) $\sin\left(\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{1}{x}\right) = 1$

$\Rightarrow \sin\left(\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{1}{x}\right) = \sin\frac{\pi}{2}$

$\Rightarrow \cos^{-1}\frac{3}{5} + \cos^{-1}\frac{1}{x} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\frac{1}{x} = \frac{\pi}{2} - \cos^{-1}\frac{3}{5}$

$\Rightarrow \cos^{-1}\frac{1}{x} = \sin^{-1}\frac{3}{5}$

$\Rightarrow \cos^{-1}\frac{1}{x} = \cos^{-1}\frac{4}{5}$

$\Rightarrow \frac{1}{x} = \frac{4}{5} \Rightarrow x = \frac{5}{4}$

18. (C) Equation $x^2 - 3x + 21 = 0$

Now, $B^2 - 4AC = (-3)^2 - 4\times1\times21$

$\Rightarrow B^2 - 4AC = 9 - 84 = -75 < 0$

Hence Root are imaginary

19. (B) A.T.Q,

$\Rightarrow \frac{n(n-3)}{2} = 77$

$\Rightarrow n^2 - 3n = 154$

$\Rightarrow n^2 - 3n - 154 = 0$

$\Rightarrow (n-14)(n+11) = 0$

$\Rightarrow n = 14, -11$

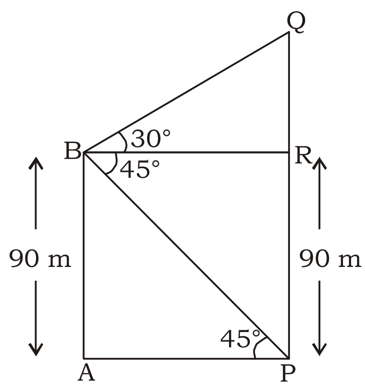
Hence no. of sides = 14

20. (B) $\sin(-2850) = -\sin(2850)$

$\sin(-2850) = -\sin(360\times8 - 30)$

$\sin(-2850) = -(-\sin30) = \sin30 = \frac{1}{2}$

21. (C)



In ΔABP

$\tan45° = \frac{AB}{AP}$

$\Rightarrow 1 = \frac{AB}{AP}$

$\Rightarrow 1 = \frac{90}{AP} \Rightarrow AP = 90 = BR$

In ΔBRQ

$\tan30° = \frac{RQ}{BR}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{RQ}{90}$

$\Rightarrow RQ = \frac{90}{\sqrt{3}} = 30\sqrt{3}$

Height of the second building = PR + RQ

$= 90 + 30\sqrt{3}$

$= 30(3+\sqrt{3})$m

22. (B) Equation

$ax^2 + bx + c = 0$

Let roots are α and β

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...(i)

New equation

$x^2 + 45x + 30 = 0$

New roots are 3α and 3β

$3\alpha + 3\beta = -45$ and $3\alpha.3\beta = 30$

$\Rightarrow \alpha + \beta = -15$ and $\alpha\beta = \frac{10}{3}$ ...(ii)

from eq(i) and eq(ii)

$\frac{-b}{a} = -15 \Rightarrow \frac{b}{a} = 15$

and $\frac{c}{a} = \frac{10}{3}$

Now, $\frac{b/a}{c/a} = \frac{15}{10/3}$

$\Rightarrow \frac{b}{c} = \frac{9}{2}$

$= b : c = 9 : 2$

23. (A) $\frac{b}{a} \times \frac{c}{a} = 15\times\frac{10}{3}$

$\Rightarrow \frac{bc}{a^2} = 50 \Rightarrow bc = 50a^2$

24. (C)

25. (B) $I = \int (x^2-1)^{3/2} x\, dx$

Let $x^2 - 1 = t$

$\Rightarrow 2x\, dx = dt \Rightarrow x\, dx = \frac{1}{2} dt$

$I = \frac{1}{2}\int t^{3/2}\, dt$

$I = \frac{1}{2} \times \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$

$I = \frac{1}{2} \times \frac{t^{5/2}}{5/2} + c$

$I = \frac{1}{5} t^{5/2} + c$

$I = \frac{1}{5} (x^2-1)^{5/2} + c$

26. (D) $S = 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots\ldots$

$\Rightarrow S = \frac{1}{1-\left(\frac{-1}{3}\right)} \Rightarrow S = \frac{1}{1+\frac{1}{3}}$

$\Rightarrow S = \frac{1}{4/3} = \frac{3}{4}$

27. (C)

28. (B) $I = \int \frac{\tan^{-1} x}{1+x^2} dx$

Let $\tan^{-1}x = t$

$\Rightarrow \frac{1}{1+x^2} dx = dt$

$I = \int t\, dt$

$I = \frac{t^2}{2} + c$

$I = \frac{1}{2}(\tan^{-1}x)^2 + c$

29. (A) **Statement I**

$\int \ln 10\, dx = \ln 10 \int 1\, dx$

$\int \ln 10\, dx = (\ln 10)x + c$

$\int \ln 10\, dx = x.\ln 10 + c$

Statement I is correct.

Statement II

$\int 5^x dx = \frac{5^x}{\ln 5} + c$

Statement II is incorrect.

30. (C) $A^2 = A.A$

$A^2 = AB.AB$ [$\because$ AB = A]

$A^2 = A.BA.B$ [$\because$ BA = B]

$A^2 = AI = A$

31. (B) $\text{cosec}\theta - \cot\theta = \sqrt{3}$ ...(i)

$\text{cosec}\theta + \cot\theta = \frac{1}{\sqrt{3}}$ ...(ii)

from eq(i) and eq(ii)

$\Rightarrow 2\text{cosec}\theta = \sqrt{3} + \frac{1}{\sqrt{3}}$

$\Rightarrow 2\text{cosec}\theta = \frac{4}{\sqrt{3}} \Rightarrow \text{cosec}\theta = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

32. (C) $n(S) = 6\times6\times6 = 216$

E = {(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5) (6,6,6)}

$n(E) = (6)$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{6}{216} = \frac{1}{36}$

33. (B)

34. (D) $\frac{\text{cosec}\,\theta}{\sin\theta} - \frac{\sec\theta}{\cos\theta} - \frac{\cot\theta}{\tan\theta}$

$\Rightarrow \text{cosec}^2\theta - \sec^2\theta - \cot^2\theta$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta - \sec^2\theta$

$\Rightarrow 1 - \sec^2\theta$

$\Rightarrow -(\sec^2\theta - 1) = -\tan^2\theta$

35. (C) $\cos\left\{\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right\}$

$\Rightarrow \cos\left\{\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12}\right\}$ $\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$

$\Rightarrow \cos\left\{\tan^{-1}\left(\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right)\right\}$

$\Rightarrow \cos\left\{\tan^{-1}\left(\frac{\frac{36+20}{48}}{\frac{48-15}{48}}\right)\right\}$

$\Rightarrow \cos\left\{\tan^{-1}\left(\frac{56}{33}\right)\right\}$

$\Rightarrow \cos\left\{\tan^{-1}\left(\frac{33}{65}\right)\right\} = \frac{33}{65}$

36. (B) Equation $x^2 + bx + c = 0$

Roots are tan17° and tan28°.

A.T.Q,

$\tan 17° + \tan 28 = -b$

and $\tan 17.\tan 28° = c$

Now, $\tan(17 + 28) = \frac{\tan 17 + \tan 28}{1 - \tan 17.\tan 28}$

$\Rightarrow \tan 45° = \frac{-b}{1-c}$

$\Rightarrow 1 = \frac{-b}{1-c}$

$\Rightarrow 1 - c = -b \Rightarrow c = b + 1$

37. (B) A and B are symmetric matrices

$\therefore$ A' = A and B' = B ...(i)

Now, (AB – BA)' = (AB)' – (BA)'

$\Rightarrow$ (AB – BA)' = B'A' – A'B'

$\Rightarrow$ (AB – BA)' = BA – AB [From eq(i)]

$\Rightarrow$ (AB – BA) = – (AB – BA)

Hence (AB – BA) is a skew–symmetric matrix.

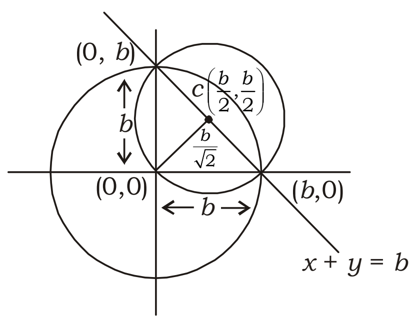
38. (B) tan15° – cot15°

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{3+1-2\sqrt{3}-3-1-2\sqrt{3}}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$\Rightarrow \frac{-4\sqrt{3}}{3-1} = -2\sqrt{3}$$

39. (C)



Radius of new circle = $r \Rightarrow \frac{b}{\sqrt{2}}$

and Centre = $\left(\frac{b}{2}, \frac{b}{2}\right)$

Equation of circle

$$\left(x-\frac{b}{2}\right)^2 + \left(y-\frac{b}{2}\right)^2 = \left(\frac{b}{\sqrt{2}}\right)^2$$

$$\Rightarrow x^2 + \frac{b^2}{2} - bx + y^2 + \frac{b^2}{4} - by = \frac{b^2}{2}$$

$\Rightarrow x^2 + y^2 - bx - by = 0$

$\Rightarrow x^2 + y^2 = bx + by$

40. (B) Series 4, –16, 64, –256,.......

$\Rightarrow 2^2, -2^4, 2^6, -2^8$,........

$T_n = (-1)^{n+1}\, 2^{2n}$

41. (C) C(26, n –1) = C(26, 4n + 2)

$\Rightarrow {}^{26}C_{n-1} = {}^{26}C_{4n+2}$

here $n - 1 + 4n + 2 = 26$

$\Rightarrow 5n + 1 = 26$

$\Rightarrow 5n = 25 \Rightarrow n = 5$

42. (B) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & -4 \\ 2 & 6 & -1 \end{bmatrix}$

Co-factors of A-

$C_{11}=(-1)^{1+1}\begin{vmatrix} 0 & -4 \\ 6 & -1 \end{vmatrix}$, $C_{12}=(-1)^{1+2}\begin{vmatrix} 4 & -4 \\ 2 & -1 \end{vmatrix}$, $C_{13}=(-1)^{1+3}\begin{vmatrix} 4 & 0 \\ 2 & 6 \end{vmatrix}$

= 24 = –4 = 24

$C_{21}=(-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 6 & -1 \end{vmatrix}$, $C_{22}=(-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$, $C_{23}=(-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix}$

= 20 = –7 = –2

$C_{31}=(-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 0 & -4 \end{vmatrix}$, $C_{32}=(-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 4 & -4 \end{vmatrix}$, $C_{33}=(-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix}$

= –8 =16 = –8

$$C = \begin{bmatrix} 24 & -4 & 24 \\ 20 & -7 & -2 \\ -8 & 16 & -8 \end{bmatrix}$$

Adj A = C^T

$$\text{Adj } A = \begin{bmatrix} 24 & 20 & -8 \\ -4 & -7 & 16 \\ 24 & -2 & -8 \end{bmatrix}$$

43. (B) Differential equation

$$\frac{dy}{dx} = \frac{x}{y}$$

$\Rightarrow y\, dy = x\, dx$

On integrating

$\Rightarrow \int y\, dy = \int x\, dx$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + \frac{c}{2}$$

$\Rightarrow y^2 - x^2 = c$

44. (C) $\sin\theta = \frac{-\sqrt{3}}{2}$ and $\sec\theta = -2$

then θ lies in the third quadrant.

45. (A) 1/4, 1/x, 1/10 are in H.P.

$\Rightarrow$ 4, x, 10 are in A.P.

then, $2x = 4 + 10$

$\Rightarrow 2x = 14 \Rightarrow x = 7$

46. (D) $A = \begin{bmatrix} 11 & 3 \\ -4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -4 & -6 \end{bmatrix}$

$$AB = \begin{bmatrix} 11 & 3 \\ -4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -4 & -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11\times1+3\times(-4) & 11\times2+3\times(-6) \\ -4\times1+(-1)\times(-4) & -4\times2+(-1)\times(-6) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 4 \\ 0 & -2 \end{bmatrix} \Rightarrow |AB| = 2 - 0 = 2$$

Co-factors of AB-

$C_{11} = (-1)^{1+1}(-2) = 2$, $C_{12} = (-1)^{1+2}(0) = 0$

$C_{21} = (-1)^{2+1}(4) = -4$, $C_{22} = (-1)^{2+2}(-1) = -1$

$$C = \begin{bmatrix} 2 & 0 \\ -4 & -1 \end{bmatrix}$$

$\text{Adj}(AB) = C^T$

$$\text{Adj}(AB) = \begin{bmatrix} 2 & -4 \\ 0 & -1 \end{bmatrix}$$

Now, $(AB)^{-1} = \dfrac{\text{Adj}(AB)}{|AB|}$

$$\Rightarrow B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & -1 \end{bmatrix}$$

47. (A) $(\tan\alpha - \tan\beta)^2 - \sec^2\alpha.\sec^2\beta + (1+\tan\alpha.\tan\beta)^2$

$\Rightarrow \tan^2\alpha + \tan^2\beta - 2\tan\alpha.\tan\beta - \sec^2\alpha.\sec^2\beta + 1 + \tan^2\alpha.\tan^2\beta + 2\tan\alpha.\tan\beta$

$\Rightarrow \tan^2\alpha + \tan^2\beta - 2\tan\alpha.\tan\beta - (1+\tan^2\alpha)(1+\tan^2\beta) + 1 + \tan^2\alpha.\tan^2\beta + 2\tan\alpha.\tan\beta$

$\Rightarrow \tan^2\alpha + \tan^2\beta - 2\tan\alpha.\tan\beta - 1 - \tan^2\alpha - \tan^2\beta - \tan^2\alpha.\tan^2\beta + 1 + \tan^2\alpha.\tan^2\beta + 2\tan\alpha.\tan\beta$

$\Rightarrow 0$

48. (C) Equation $3x^2+6y^2 = 15$

$$\Rightarrow \frac{3x^2}{15} + \frac{6y^2}{15} = 1$$

$$\Rightarrow \frac{x^2}{5} + \frac{y^2}{5/2} = 1$$

It is an ellipse.

49. (B) $\sin\alpha - \cos\alpha = A$

On squaring

$\Rightarrow \sin^2\alpha + \cos^2\alpha - 2\sin\alpha.\cos\alpha = A^2$

$\Rightarrow 1 - 2\sin\alpha.\cos\alpha = A^2$

$\Rightarrow 1 - A^2 = 2\sin\alpha.\cos\alpha$

$\Rightarrow 1 - A^2 = \sin2\alpha$

$\Rightarrow \sin^2 2\alpha = (1 - A^2)^2$

50. (D) $\begin{vmatrix} x & 3i & i \\ y & -2 & 3i \\ 0 & -i & i \end{vmatrix} = 9 + 12i$

$\Rightarrow x(-2i + 3i^2) - 3i(yi) + 1(-yi) = 9+12i$

$\Rightarrow -2xi - 3x + 3y - yi = 9+12i$

$\Rightarrow (-3x + 3y) + (-2x - y)i = 9 + 12i$

On comparing

$-3x + 3y = 9$ ...(i)

and $-2x - y = 12$ ...(ii)

from eq(i) and eq(ii)

$x = -5$ and $y = -2$

51. (B) $I = \int e^{a\ln x} dx$

$I = \int e^{\ln x^a} dx$

$I = \int x^a dx$

$I = \dfrac{x^{a+1}}{a+1} + c$

52. (C) $\vec{a} = 3\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \lambda\hat{j} + 10\hat{k}$ are perpendicular, then

$\vec{a}.\vec{b} = 0$

$\Rightarrow (3\hat{i} + 4\hat{j} - \hat{k}).(-2\hat{i} + \lambda\hat{j} + 10\hat{k}) = 0$

$\Rightarrow -6 + 4\lambda - 10 = 0$

$\Rightarrow 4\lambda - 16 = 0 \Rightarrow \lambda = 4$

53. (C) Let $y = \text{cosec}^2(\cot^{-1}x)$

$y = 1 + [\cot(\cot^{-1}x)]^2$

$y = 1 + x^2$

On differentiating both sides w.r.t. 'x'

$$\frac{dy}{dx} = 2x$$

54. (B) $S_n = n(n+3)$

$S_n = n^2 + 3n$

$S_{n-1} = (n-1)^2 + 3(n-1)$

$S_{n-1} = n^2 + 1 - 2n + 3n - 3$

$S_{n-1} = n^2 + n - 2$

Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = S_n - S_{n-1}$

$\Rightarrow T_n = n^2 + 3n - (n^2 + n - 2)$

$\Rightarrow T_n = 2n + 2$

Now, $T_6 = 2 \times 6 + 2 = 14$

55. (B) curve $y = x^2 - 5x + 4$

it cuts x–axis i.e $y = 0$

$x^2 - 5x + 4 = 0$

$\Rightarrow (x-4)(x-1) = 0$

$\Rightarrow x = 1, 4$

Hence two tangents are parallel to x-axis for the curve.

56. (A) $\lim\limits_{x\to\infty} \dfrac{x^3 + 5x^2 - 6}{1 + 2x - 4x^3}$

$$\Rightarrow \lim_{x\to\infty} \frac{x^3\left(1 + \frac{5}{x} - \frac{6}{x^3}\right)}{x^3\left(\frac{1}{x^3} + \frac{2}{x^2} - 4\right)}$$

$$\Rightarrow \lim_{x\to\infty} \frac{1+\frac{5}{x}-\frac{6}{x^3}}{-4+\frac{1}{x^3}+\frac{2}{x^2}}$$

$$\Rightarrow \frac{1+0-0}{-4+0+0} = \frac{-1}{4}$$

57. (A)

58. (B) $[(3x-4y)^3(3x+4y)^3]^4$
$\Rightarrow [(3x-4y)(3x+4y)]^{12}$
$\Rightarrow (9x^2-16y^2)^{12}$
Total terms = 12 +1 = 13

59. (B) Differential equation

$$\frac{dy}{dx} = \sin|y-x|+1$$

$$\Rightarrow \frac{dy-dx}{dx} = \sin|y-x|$$

$$\Rightarrow \frac{dy-dx}{\sin|y-x|} = dx$$

$$\Rightarrow \int \frac{d(y-x)}{\sin(y-x)} = \int dx$$

$$\Rightarrow \int \operatorname{cosec}|y-x|\, d(y-x) = \int dx$$

$$\Rightarrow \log|\operatorname{cosec}(y-x)-\cot(y-x)| = x + C$$

$$\Rightarrow e^x|\operatorname{cosec}(y-x)-\cot(y-x)| = c$$

60. (C) $y = a\sin 3x + b\cos 3x$ ...(i)
On differentiating both sides w.r.t 'x'

$$\frac{dy}{dx} = 3a\cos 3x - 3b\sin 3x$$

Again, differentiating

$$\frac{d^2y}{dx^2} = 3a\times(-3)\sin 3x - 3b \times 3\cos 3x$$

$$\frac{d^2y}{dx^2} = -9a\sin 3x - 9b\cos 3x$$

$$\frac{d^2y}{dx^2} = -9[a\sin 3x + b\cos 3x]$$

$$\frac{d^2y}{dx^2} = -9y \quad \text{[from eq(i)]}$$

$$\frac{d^2y}{dx^2} + 9y = 0$$

61. (C) Equation $x^2 + 8|x| + 12 = 0$ has no root because sum of three positive numbers can not be zero.

62. (B) $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Now, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

$$\Rightarrow \cos\theta = \frac{2\times 3 - 3\times(-2) + 6\times 6}{\sqrt{2^2+(-3)^2+6^2}\sqrt{3^2+(-2)^2+6^2}}$$

$$\Rightarrow \cos\theta = \frac{6+6+36}{7\times 7}$$

$$\Rightarrow \cos\theta = \frac{48}{49}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{48}{49}\right)$$

63. (B) The differential equation of the system of circles touching the x-axis at the origin is
$(x-0)^2+(y-\alpha)^2 = \alpha^2$
$\Rightarrow x^2 + y^2 + \alpha^2 - 2y\alpha = \alpha^2$
$\Rightarrow x^2 + y^2 - 2y\alpha = 0$

$$\Rightarrow \frac{x^2}{y} + y - 2\alpha = 0$$

$\Rightarrow$ On differenting both sides w.r.t 'x'

$$\Rightarrow \frac{2xy - x^2\frac{dy}{dx}}{y^2} + 1 = 0$$

$$\Rightarrow 2xy - x^2\frac{dy}{dx} + y^2 = 0$$

64. (B) Vertices A(–1, 2, – 3), B(– 2, 3, 4) and C(–4, –1, 6)

$$\text{Centroid} = \left(\frac{-1-2-4}{3}, \frac{2+3-1}{3}, \frac{-3+4+6}{3}\right)$$

$$\text{Centroid} = \left(\frac{-7}{3}, \frac{4}{3}, \frac{7}{3}\right)$$

65. (C) Plane $3x - y + 2z = 8$ and $4x - y + z = 2$
The required equation of plane
$(3x - y + 2z - 8) + \lambda(4x - y + z - 2) = 0$ ...(i)
it passes through the point (–1, 2, 3)
$(3\times(-1) - 2 + 2\times 3 - 8) + \lambda(4\times(-1) - 2 + 3 - 2) = 0$
$\Rightarrow (-3 - 2 + 6 - 8) + \lambda(-4 - 2 + 3 - 2) = 0$

$$\Rightarrow -7 + \lambda(-5) = 0 \Rightarrow \lambda = \frac{-7}{5}$$

From eq(i)

$$(3x - y + 2z - 8) - \frac{7}{5}(4x - y + z - 2) = 0$$

$\Rightarrow 5(3x - y + 2z - 8) - 7(4x - y + z - 2) = 0$
$\Rightarrow 15x - 5y + 10z - 40 - 28x + 7y - 7z + 14 = 0$
$\Rightarrow -13x + 2y + 3z - 26 = 0$
$\Rightarrow 13x - 2y - 3z + 26 = 0$

66. (B) Direction cosine of y-axis = $\langle 0, \; 1, \; 0 \rangle$

67. (A) $f(x) = \ln\left(\sqrt{x^2+1}+x\right)$

$f(-x) = \ln\sqrt{(-x)^2+1} - x$

$f(-x) = \ln\left(\sqrt{x^2+1}-x\right)$

Now, $f(x) + f(-x) = \ln\left(\sqrt{x^2+1}+x\right) + \ln\left(\sqrt{x^2+1}-x\right)$

$\Rightarrow f(x) + f(-x) = \ln\left[\left(\sqrt{x^2+1}+x\right)\left(\sqrt{x^2+1}-x\right)\right]$

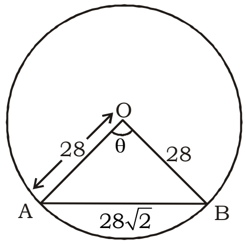
$\Rightarrow f(x) + f(-x) = \ln\left[x^2+1-x^2\right]$

$\Rightarrow f(x) + f(-x) = \ln 1$

$\Rightarrow f(x) + f(-x) = 0$

Hence function is an odd.

68. (B)



Let $\angle AOB = \theta$

$$\cos\theta = \frac{(28)^2 + (28)^2 - \left(28\sqrt{2}\right)^2}{2\times 28\times 28}$$

$$\cos\theta = \frac{2(28)^2 - 2(28)^2}{2\times 28\times 28}$$

$\cos\theta = 0 \Rightarrow \theta = 90°$

Area of major arc = $\frac{360-90}{360}\times 2\pi r$

$= \frac{270}{360}\times 2\times\frac{22}{9}\times 28$

= 132 cm

69. (B) Given that P(A) = 0.4, P(B) = 0.3 and $P(A\cap B) = 0.2$

Now, $P\left(\overline{A\cup B}\right) = 1 - P(A\cup B)$

$\Rightarrow P\left(\overline{A\cup B}\right) = 1 - P(A) - P(B) + P(A\cap B)$

$\Rightarrow P\left(\overline{A\cup B}\right) = 1 - 0.4 - 0.3 + 0.2$

$\Rightarrow P\left(\overline{A\cup B}\right) = 1.2 - 0.7 = 0.5$

70. (A) Given that the no. of white balls in the bag = 8
Let the no. of black balls in the bag = x
$n(S) = 8 + x$
Probability of drawing white ball from the bag = $\frac{8}{8+x}$
Probability of drawing black ball from the bag = $\frac{x}{8+x}$
A.T.Q.,

$$\frac{x}{8+x} = 3\times\frac{8}{8+x}$$

$x = 24$
Hence the no. of black balls in the bag = 24

71. (B) S = {1, 2, 3, 4}
R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}
Reflexive:-
1R1, 2R2, 3R3
R is reflexive.
Symmetric:-
1R2, 2 R̸ 1
R is not symmetric.
Transitive:-
1R2, 2R3 and 1R3
R is transitive.
Hence R is reflexive and transitive, but not symmetric.

72. (C) $I = \int e^{\tan x}(1+\tan^2 x)\; dx$

$I = \int e^{\tan x}\sec^2 x\; dx$

Let $\tan x = t$
$\Rightarrow \sec^2 x\, dx = dt$

$I = \int e^t\, dt$

$I = e^t + c$
$I = e^{\tan x} + c$

73. (C) We know that
Minimum value of $a\sec^2\theta + b\operatorname{cosec}^2\theta = \left(\sqrt{a}+\sqrt{b}\right)^2$
Now,
Minimum value of $18\sec^2\theta + 72\operatorname{cosec}^2\theta$

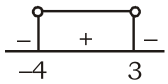
$= \left(\sqrt{18}+\sqrt{72}\right)^2 = \left(3\sqrt{2}+6\sqrt{2}\right)^2$

$= \left(9\sqrt{2}\right)^2 = 162$

74. (B) $f(x) = \dfrac{1}{\sqrt{12 - x - x^2}}$

Now, $12 - x - x^2 > 0$

$\Rightarrow (3 - x)(4 + x) > 0$



Domain of the function = (–4, 3)

75. (C) $\sin A = \dfrac{3}{5} \Rightarrow \cos A = \dfrac{4}{5}$

$\cos A = 2\cos^2\dfrac{A}{2} - 1$

$\Rightarrow \dfrac{4}{5} = 2\cos^2\dfrac{A}{2} - 1$

$\Rightarrow 2\cos^2\dfrac{A}{2} = \dfrac{4}{5} + 1$

$\Rightarrow 2\cos^2\dfrac{A}{2} = \dfrac{9}{5} \Rightarrow \cos\dfrac{A}{2} = \sqrt{\dfrac{9}{10}}$

Now, $\cos\left(\dfrac{A}{2}\right) . \cos\left(\dfrac{3A}{2}\right)$

$\Rightarrow \cos\left(\dfrac{A}{2}\right) . \left[4\cos^3\dfrac{A}{2} - 3\cos\dfrac{A}{2}\right]$

$\Rightarrow 4\cos^4\dfrac{A}{2} - 3\cos^2\dfrac{A}{2}$

$\Rightarrow 4\left(\sqrt{\dfrac{9}{10}}\right)^4 - 3\left(\sqrt{\dfrac{9}{10}}\right)^2$

$\Rightarrow 4 \times \dfrac{81}{100} - 3 \times \dfrac{9}{10}$

$\Rightarrow \dfrac{81}{25} - \dfrac{27}{10}$

$\Rightarrow \dfrac{162 - 135}{50} = \dfrac{27}{50}$

76. (C) Given line $5x + 4y = 7$

Equation of line which is parallel to given line

$5x + 4y = c$ ...(i)

Mid point of the points (–1, 3) and (2, 6)

$= \left(\dfrac{-1+2}{2}, \dfrac{3+6}{2}\right) = \left(\dfrac{1}{2}, \dfrac{9}{2}\right)$

point $\left(\dfrac{1}{2}, \dfrac{9}{2}\right)$ lies on the eq(i)

$5 \times \dfrac{1}{2} + 4 \times \dfrac{9}{2} = c$

$\Rightarrow \dfrac{5+36}{2} = c \Rightarrow c = \dfrac{41}{2}$

from eq (i)

$5x + 4y = \dfrac{41}{2}$

$\Rightarrow 10x + 8y = 41$

77. (A) $\sin^{-1}\dfrac{12}{13} + \sec^{-1}\dfrac{13}{12}$

$\Rightarrow \sin^{-1}\dfrac{12}{13} + \cos^{-1}\dfrac{12}{13}$

$\Rightarrow \dfrac{\pi}{2}$ $\left[\because \sin^{-1}A + \cos^{-1}A = \dfrac{\pi}{2}\right]$

78. (C) Ellipse $8x^2 + 50y^2 = 400$

$\Rightarrow \dfrac{8x^2}{400} + \dfrac{50y^2}{400} = 1$

$\Rightarrow \dfrac{x^2}{50} + \dfrac{y^2}{8} = 1$

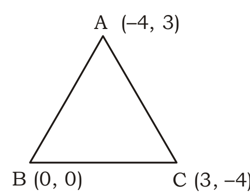
$a^2 = 50 \Rightarrow a = 5\sqrt{2}$, $b^2 = 8 \Rightarrow b = 2\sqrt{2}$

Area of an ellipse = πab

$= \pi \times 5\sqrt{2} \times 2\sqrt{2}$

$= 20\pi$ sq. unit

79. (A)



$AB = \sqrt{(-4-0)^2 + (3-0)^2}$

$AB = \sqrt{16+9}$

AB = 5

$BC = \sqrt{(3-0)^2 + (-4-0)^2}$

$BC = \sqrt{9+16} = 5$

$AC = \sqrt{(-4-3)^2 + (3+4)^2}$

$AC = \sqrt{49+49} = 7\sqrt{2}$

Perimeter of a triangle = AB + BC + AC

$= 5 + 5 + 7\sqrt{2}$

$= 10 + 7\sqrt{2}$

80. (B) Equation $(a - b)x^2 + (b - c)x + (c - a) = 0$

One root is 1.

Let other root = α

Product of root

$\alpha.1 = \dfrac{c-a}{a-b} \Rightarrow \alpha = \dfrac{c-a}{a-b}$

81. (C) A = {x, y, z}, B = {1, 2, 3}, C = {2, 3, 4}

($B \cap C$) = {2, 3}

Now, A × ($B \cap C$) = {x, y, z} × {2, 3)

$\Rightarrow$ A × ($B \cap C$) = {(x, 2), (x, 3) (y, 2), (y, 3), (z, 2), (z, 3)}

Hence no. of element in A × ($B \cap C$) = 6

82. (B) 1101

$1 \times 2^0 = 1$

$0 \times 2^1 = 0$

$1 \times 2^2 = 4$

$1 \times 2^3 = 8$

13

0.11

$2^{-1} = 1 \times 2^{-1}$

$2^{-2} = 1 \times 2^{-2}$

$2^{-1} + 2^{-2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$

Hence $(1101.11)_2 = (13.75)_{10}$

83. (C) A × (B – C) = (A × B) – (A × C)

84. (A) $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$A^2 = A.A$

$A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^2 = I$

85. (B) Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = -1$

Cofacters of A

$C_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$

= 0 = –1 = 0

$C_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

= –1 = 0 = 0

$C_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

= 0 = 0 = –1

$C = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$\text{Adj } A = C^T = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj}A}{|A|}$

$A^{-1} = \dfrac{1}{-1}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

86. (B) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and $C = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Now, $A\sin\alpha + B\cos\alpha$

$\Rightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\sin\alpha + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\cos\alpha$

$\Rightarrow \begin{bmatrix} 0 & \sin\alpha \\ -\sin\alpha & 0 \end{bmatrix} + \begin{bmatrix} \cos\alpha & 0 \\ 0 & \cos\alpha \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = C$

Hence $A\sin\alpha + B\cos\alpha = C$

87. (B) $|x^2 - x - 6| = x + 2$

$\Rightarrow x^2 - x - 6 = x + 2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = -2, 4$

and $-x^2 + x + 6 = x + 2$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Hence roots are 2, –2, 4.

88. (C) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}}\right] + \tan^{-1}\left[\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{5+3}{15-1}\right] + \tan^{-1}\left[\frac{8+7}{56-1}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{8}{14}\right] + \tan^{-1}\left[\frac{15}{55}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{4}{7}\right] + \tan^{-1}\left[\frac{3}{11}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{44+21}{77-12}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{65}{65}\right] = \tan^{-1}(1) = \frac{\pi}{4}$$

89. (B) A.T.Q.

$$180 - \frac{360}{n} = 140$$

$$\Rightarrow 40 = \frac{360}{n} \Rightarrow n = 9$$

No. of diagonals $= \frac{n(n-3)}{2}$

$$= \frac{9\times(9-3)}{2}$$

$$= 9 \times 3 = 27$$

90. (B) $2\cos\frac{12\pi}{13} . \cos\frac{4\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$$\Rightarrow 2\cos\frac{12\pi}{13} .\cos\frac{4\pi}{13} + 2\cos\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2} . \cos\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}$$

$$\Rightarrow 2\cos\frac{12\pi}{13} .\cos\frac{4\pi}{13} + 2\cos\frac{4\pi}{13} . \cos\frac{\pi}{13}$$

$$\Rightarrow -2\cos\frac{\pi}{13} . \cos\frac{4\pi}{13} + 2\cos\frac{4\pi}{13} . \cos\frac{\pi}{13} = 0$$

91. (B) $2\sin^{-1}x = \sin^{-1}\left[2x\sqrt{1-x^2}\right]$

We know that

$$-\frac{\pi}{2} \le \sin^{-1}\theta \le \frac{\pi}{2}$$

Now, $-\frac{\pi}{2} \le \sin^{-1}\left[2x\sqrt{1-x^2}\right] \le \frac{\pi}{2}$

$$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$$

$$x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

92. (C) $I = \int_3^4 \frac{dx}{x^2-2x}$

$$I = \int_3^4 \frac{dx}{x(x-2)}$$

$$I = \int_3^4 \frac{1}{2}\left(\frac{1}{x-2} - \frac{1}{x}\right) dx$$

$$I = \frac{1}{2}\left[\ln(x-2)\right]_3^4 - \frac{1}{2}\left[\ln x\right]_3^4$$

$$I = \frac{1}{2}\left[\ln(4-2) - \ln(3-2)\right] - \frac{1}{2}\left[\ln 4 - \ln 3\right]$$

$$I = \frac{1}{2}\ln 2 - \ln 2 + \frac{1}{2}\ln 3$$

$$I = \frac{1}{2}\ln 3 - \frac{1}{2}\ln 2 = \frac{1}{2}\ln\frac{3}{2}$$

93. (C) $\frac{1}{bc}$, $\frac{1}{ca}$ and $\frac{1}{ab}$ are in A.P.,

then $\frac{2}{ca} = \frac{1}{bc} + \frac{1}{ab}$

$$\Rightarrow \frac{2}{ca} = \frac{a+c}{abc}$$

$\Rightarrow 2b = a + c$

Hence a, b and c are in A.P.

94. (B) $I = \int \cot^2 x.\operatorname{cosec}^4 x \; dx$

$I = \int \cot^2 x.\operatorname{cosec}^2 x.\operatorname{cosec}^2 x \; dx$

$I = \int \cot^2 x.(1+\cot^2 x).\operatorname{cosec}^2 x \; dx$

$I = \int (\cot^2 x + \cot^4 x).\operatorname{cosec}^2 x \; dx$

Let $\cot x = t$

$\Rightarrow -\operatorname{cosec}^2 x \, dx = dt \Rightarrow \operatorname{cosec}^2 x \, dx = -dt$

$I = -\int (t^2 + t^4) \; dt$

$I = -\left[\frac{t^3}{3} + \frac{t^5}{5}\right] + c$

$I = -\left[\frac{\cot^3 x}{3} + \frac{\cot^5 x}{5}\right] + c$

95. (B) Data 21, 21 , 20, 21, 22, 23, 20, 20, 21, 21, 24, 25

Mode = 21

96. (A) Let $z = x + iy$ and $\bar{z} = x - iy$

Now, $z = \bar{z}$

$\Rightarrow x + iy = x - iy$

$\Rightarrow 2iy = 0 \Rightarrow y = 0$

Hence the imaginary part of z is zero.

97. (C) A leap year = 366 days

= 52 weeks and 2 days

The required Probability = $\frac{2}{7}$

98. (A) $\sin\frac{\pi}{12} = \sin 15°$

$= \sin(45–30)$

$= \sin45.\cos30–\cos45.\sin30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

99. (C) $n(S) = 6\times6 = 36$

$$E = \begin{cases} (6,3),(3,6),(5,4),(4,5) \text{ for sum}=9 \\ (6,4),(4,6),(5,5) \text{ for sum}=10 \\ (6,5),(5,6) \text{ for sum}=11 \\ (6,6) \text{ for sum}=12 \end{cases}$$

$n(E) = 10$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{10}{36} = \frac{5}{18}$

100. (B) $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_n x^n$

$x = 1$

$\Rightarrow (1 + 1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n x^n$

$\Rightarrow {}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

Sum of even binomial coefficients = $\frac{2^n}{2}$

$= 2^{n-1}$

101. (B) $9^8 + 8.9^7 + 28.9^6 + ... + 1 = k \times 2^5 \times 5^7$

$\Rightarrow (1+9)^8 = k \times 2^5 \times 5^7$

$\Rightarrow 10^8 = k \times 2^5 \times 5^7$

$\Rightarrow 2^8 \times 5^8 = k \times 2^5 \times 5^7$

$\Rightarrow k = \frac{2^8 \times 5^8}{2^5 \times 5^7}$

$\Rightarrow k = 2^3 \times 5 \Rightarrow k = 40$

102. (D) a, b, c are in A.P, then

$b = \frac{a+c}{2}$ ...(i)

and a, b, c are in G.P, then

$b^2 = ac$

$\Rightarrow \left(\frac{a+c}{2}\right)^2 = ac$ [from eq(i)]

$\Rightarrow \frac{a^2 + c^2 + 2ac}{4} = ac$

$\Rightarrow a^2 + c^2 + 2ac = 4ac$

$\Rightarrow (a - c)^2 = 0 \Rightarrow a = c$

from eq(i)

$b = \frac{c+c}{2} \Rightarrow b = c$

Hence $a = b = c$

103. (A)

104. (A)

105. (C) Sphere $x^2 + y^2 + z^2 - 2x + 3y - 5z + 5 = 0$

On comparing with general equation

$u = -1, \; v = \frac{3}{2}, \; w = \frac{-5}{2}, \; d = 5$

radius $(r) = \sqrt{u^2 + v^2 + w^2 - d}$

$\Rightarrow r = \sqrt{(-1)^2 + \left(\frac{3}{2}\right)^2 + \left(-\frac{5}{2}\right) - 5}$

$\Rightarrow r = \sqrt{1 + \frac{9}{4} + \frac{25}{4} - 5} \Rightarrow r = \sqrt{\frac{34}{4} - 4}$

$\Rightarrow r = \sqrt{\frac{34-16}{4}} \Rightarrow r = \sqrt{\frac{18}{4}}$

$\Rightarrow r = \frac{3\sqrt{2}}{2}$

Diameter = $2r = 2\times\frac{3\sqrt{2}}{2} = 3\sqrt{2}$

106. (B) I = $\int \frac{e^x(x+1)}{\cos^2(x.e^x)}dx$

Let $x.e^x = t$

$\Rightarrow (x.e^x + e^x.1)\ dx = dt$

$\Rightarrow e^x(x+1)\ dx = dt$

I = $\int \frac{dt}{\cos^2 t}$

I = $\int \sec^2 t\, dt$

I = $\tan t + c$

I = $\tan(x.e^x) + c$

107. (C) conic $\frac{x^2}{a^2} - \frac{y^2}{31} = 1$

$e = \sqrt{1-\frac{b^2}{a^2}}$

$e = \sqrt{1-\frac{31}{a^2}}$

$e = \sqrt{\frac{a^2-31}{a^2}} \Rightarrow ae = \sqrt{a^2-31}$

foci $(\pm ae, 0) = \left(\pm\sqrt{a^2-31}, 0\right)$

and conic $\frac{x^2}{144} - \frac{y^2}{81} = 1$

$e = \sqrt{1+\frac{b^2}{a^2}}$

$e = \sqrt{1+\frac{81}{144}}$

$e = \sqrt{\frac{225}{144}} \Rightarrow e = \frac{15}{12} = \frac{5}{4}$

foci $(\pm ae, 0) = \left(\pm 12\times\frac{5}{4}, 0\right) = (\pm 15, 0)$

A.T.Q.,

$\sqrt{a^2-31} = 15$

$\Rightarrow a^2 - 31 = 225$

$\Rightarrow a^2 = 225 + 31$

$\Rightarrow a^2 = 256 \Rightarrow a = 16$

108. (A) Equation

$x^2 - 7x + 12 = 0$

$\Rightarrow (x-3)(x-4) = 0$

$\Rightarrow x = 3, 4$

$\alpha = 3,\ \beta = 4$

Now, $\alpha^3 + \beta^2 = (3)^3 + (4)^3$

$\Rightarrow \alpha^3 + \beta^2 = 27 + 64 = 91$

109. (D)

110. (C) $f(x) = 2x - 7$ and $g(x) = 3x^2 + 1,\ x \in R$

Now, $fog(x) = f[g(x)]$

$\Rightarrow fog(x) = f[3x^2 + 1]$

$\Rightarrow fog(x) = 2(3x^2 + 1) - 7$

$\Rightarrow fog(x) = 6x^2 + x - 7$

$\Rightarrow fog(x) = 6x^2 - 5$

$\Rightarrow fog(2) = 6\times(2)^2 - 5$

$\Rightarrow fog(2) = 24 - 5 = 19$

111. (C) I = $\int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right]$

I = $e^x.\frac{1}{x} + c$

$\left[\because \int e^x[f(x) + f'(x)]dx = e^x f(x) + c\right]$

I = $\frac{e^x}{x} + c$

112. (B) $\sqrt{x} + \sqrt{y} = 3$

On differentiating both sides w.r.t.'x'

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$

$\Rightarrow \frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = -\frac{3-\sqrt{x}}{\sqrt{x}}$ [from eq(i)]

$\Rightarrow \left(\frac{dy}{dx}\right)_{at\, x=1} = -\frac{3-1}{1} = -2$

113. (C) $f(x) = \frac{x(x-3)}{x^2-9}$

$f(x) = \frac{x(x-3)}{(x-3)(x+3)}$

$f(x) = \frac{x}{x+3}$

$f(3) = \frac{3}{x+3} = \frac{3}{6} = \frac{1}{2}$

114. (C) $y = \sin^{-1}\left(\frac{6x}{1+9x^2}\right)$

$y = \sin^{-1}\left(\frac{2\times 3x}{1+(3x)^2}\right)$

Let $3x = \tan\theta \Rightarrow \theta = \tan^{-1}(3x)$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta)$
$y = \sin^{-1}(\sin 2\theta)$
$y = 2\theta$
$y = 2\tan^{-1}3x$

On differentiating both sides w.r.t. 'x'

$\frac{dy}{dx} = 2\times\frac{1}{1+(3x)^2}\times 3$

$\frac{dy}{dx} = \frac{6}{1+9x^2}$

115. (C)

```
  1011        11001
+ 1110       - 1101
 11001         1100
```

Hence $(1011)_2 + (1110)_2 - (1101)_2 = (1100)_2$

116. (B) The required no. of ways = ${}^{15-3}C_{11-3}$
$= {}^{12}C_8 = 495$

117. (D) Equation of line

$\frac{x}{-3} + \frac{y}{5} = 1$

$\Rightarrow -\frac{x}{3} + \frac{y}{5} = 1$

$\Rightarrow \frac{-5x+3y}{15} = 1$

$\Rightarrow -5x + 3y = 15$
$\Rightarrow 5x - 3y + 15 = 0$

118. (B)

119. (C) Conic $9x^2 + 4y^2 + 18x - 8y - 5 = 0$
$\Rightarrow 9x^2 - 18x + 4y^2 - 8y - 5 = 0$
$\Rightarrow 9(x^2 - 2x) + 4(y^2 - 2y) - 5 = 0$
$\Rightarrow 9(x-1)^2 - 9 + 4(y-1)^2 - 4 - 5 = 0$
$\Rightarrow 9(x-1)^2 + 4(y-1)^2 = 18$

$\Rightarrow \frac{9(x-1)^2}{18} + \frac{4(y-1)^2}{18} = 1$

$\Rightarrow \frac{(x-1)^2}{2} + \frac{(y-1)^2}{9/2} = 1$

Now, $e = \sqrt{1-\frac{a^2}{b^2}} \Rightarrow e = \sqrt{1-\frac{2}{9/2}}$

$\Rightarrow e = \sqrt{1-\frac{4}{9}} \Rightarrow e = \sqrt{\frac{5}{9}} \Rightarrow e = \frac{\sqrt{5}}{3}$

120. (D) S = 0.1 + 0.11 + 0.111 + ... 10 terms

$S = \frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + ...10 \text{ terms}$

$S = \frac{1}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...10\text{ terms}\right]$

$S = \frac{1}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + ...10\text{ terms}\right]$

$S = \frac{1}{9}(1+1+.....10\text{ terms})$

$-\frac{1}{9}\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ...10\text{ terms}\right)$

$S = \frac{1}{9}\left[10 - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{10}\right)}{1-\frac{1}{10}}\right]$

$S = \frac{1}{9}\left[10 - \frac{1/10}{9/10}\left(1-\frac{1}{10^{10}}\right)\right]$

$S = \frac{1}{9}\left[10 - \frac{1}{9}\left(1-\frac{1}{10^{10}}\right)\right]$

NDA (MATHS) MOCK TEST - 160 (Answer Key)

1.	(B)	21.	(C)	41.	(C)	61.	(C)	81.	(C)	101.	(B)
2.	(D)	22.	(B)	42.	(B)	62.	(B)	82.	(B)	102.	(D)
3.	(B)	23.	(A)	43.	(B)	63.	(B)	83.	(C)	103.	(A)
4.	(C)	24.	(C)	44.	(C)	64.	(B)	84.	(A)	104.	(A)
5.	(A)	25.	(B)	45.	(A)	65.	(C)	85.	(B)	105.	(C)
6.	(D)	26.	(D)	46.	(D)	66.	(B)	86.	(B)	106.	(B)
7.	(A)	27.	(C)	47.	(A)	67.	(A)	87.	(B)	107.	(C)
8.	(B)	28.	(B)	48.	(C)	68.	(B)	88.	(C)	108.	(A)
9.	(C)	29.	(A)	49.	(B)	69.	(B)	89.	(B)	109.	(D)
10.	(C)	30.	(C)	50.	(D)	70.	(A)	90.	(B)	110.	(C)
11.	(A)	31.	(B)	51.	(B)	71.	(B)	91.	(B)	111.	(C)
12.	(B)	32.	(C)	52.	(C)	72.	(C)	92.	(C)	112.	(B)
13.	(D)	33.	(B)	53.	(C)	73.	(C)	93.	(C)	113.	(C)
14.	(C)	34.	(D)	54.	(B)	74.	(B)	94.	(B)	114.	(C)
15.	(B)	35.	(C)	55.	(B)	75.	(C)	95.	(B)	115.	(C)
16.	(C)	36.	(B)	56.	(A)	76.	(C)	96.	(A)	116.	(B)
17.	(D)	37.	(B)	57.	(A)	77.	(A)	97.	(C)	117.	(D)
18.	(C)	38.	(B)	58.	(B)	78.	(C)	98.	(A)	118.	(B)
19.	(B)	39.	(C)	59.	(B)	79.	(A)	99.	(C)	119.	(C)
20.	(B)	40.	(B)	60.	(C)	80.	(B)	100.	(B)	120.	(D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777