

NDA MATHS MOCK TEST - 162 (SOLUTION)

1. (C) Let $f(x) = \log x$ and $f(y) = \log y$
Now, $f(xy) = \log(xy)$

$$\Rightarrow f(xy) = \log x + \log y$$

$$\Rightarrow f(xy) = f(x) + f(y)$$

2. (C) $A = \{x \in \mathbb{R}, x^2 + 5x - 6 < 0\}$
 $x^2 + 5x - 6 < 0$

$$\Rightarrow (x+6)(x-1) < 0$$

$$\Rightarrow A = (-6, 1)$$

and $B = \{x \in \mathbb{R} : x^2 + 8x + 12 > 0\}$

$$\Rightarrow x^2 + 8x + 12 > 0$$

$$\Rightarrow (x+6)(x+2) > 0$$

$$x \in (-\infty, -6) \cup (-2, \infty)$$

Statement I

$$(A \cap B) = (-2, 1)$$

Statement I is correct.

Statement II

$$A - B = (-6, -2)$$

Statement II is correct.

Hence both statements are correct.

3. (D)

2	14	0	↑	0.125
2	7	1		<u>0.250</u>
2	3	1		<u>0.500</u>
2	1	1		<u>1.000</u>
0				

$$(14)_{10} = (1110)_2 \quad (0.125)_{10} = (0.001)_2$$

$$\text{Hence } (14.125)_{10} = (1110.001)_2$$

4. (C) Planes $x + y + z + 1 = 0$ and $2x - 2y + 2z + 1 = 0$

Let angle between planes = θ

$$\text{Now, } \cos \theta = \frac{1 \times 2 + 1 \times (-2) + 1 \times 2}{\sqrt{1^2 + 1^2 + 1^2} \sqrt{2^2 + (-2)^2 + 2^2}}$$

$$\Rightarrow \cos \theta = \frac{2}{\sqrt{3} \sqrt{12}}$$

$$\Rightarrow \cos \theta = \frac{2}{6} \Rightarrow \cos \theta = \frac{1}{3}$$

5. (B) $I = \int_{-1}^1 x^2 |x| dx$

$$I = \int_{-1}^0 x^2 (-x) dx + \int_0^1 x^2 \cdot x dx$$

$$I = -\int_{-1}^0 x^3 dx + \int_0^1 x^3 dx$$

$$I = -\left[\frac{x^4}{4}\right]_{-1}^0 + \left[\frac{x^4}{4}\right]_0^1$$

$$I = -\left[0 - \frac{(-1)^4}{4}\right] + \left[\frac{1}{4} - 0\right]$$

$$I = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

6. (C) Let $y = \tan^{-1} \frac{2x}{1-x^2}$

$$\text{let } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$y = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

$$y = \tan^{-1}(\tan 2\theta)$$

$$y = 2\theta$$

$$y = 2 \tan^{-1} x$$

On differentiating both sides w.r.t. 'x'

$$\frac{dy}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

and Let $z = \sin^{-1} \frac{2x}{1+x^2}$

$$\Rightarrow z = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow z = \sin^{-1}(\sin 2\theta)$$

$$z = 2\theta$$

$$z = 2 \tan^{-1} x$$

On differentiating both sides w.r.t. 'x'

$$\frac{dz}{dx} = 2 \times \frac{1}{1+x^2}$$

$$\frac{dz}{dx} = \frac{2}{1+x^2}$$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$$\Rightarrow \frac{dy}{dz} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\Rightarrow \frac{dy}{dz} = 1$$

7. (C) $I = \int_a^b \frac{\phi(x)}{\phi(x) + \phi(a+b-x)} dx \quad \dots(i)$

Prop.IV $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$$I = \int_a^b \frac{\phi(a+b-x)}{\phi(a+b-x) + \phi(x)} dx \quad \dots(ii)$$

from eq(i) and eq(ii)

$$2I = \int_a^b \frac{\phi(x) + \phi(a+b-x)}{\phi(x) + \phi(a+b-x)} dx$$

$$2I = \int_a^b 1 \cdot dx$$

$$2I = [x]_a^b$$

$$2I = b - a \Rightarrow I = \frac{b-a}{2}$$

8. (C) $n(S) = {}^9C_2 = 36$
 $E = \{(1, 2), (1, 4), (1, 6), (2, 3), (2, 5), (2, 9), (3, 4), (3, 8), (4, 7), (4, 9), (5, 6), (5, 8), (6, 7), (8, 9)\}$
 $n(E) = 14$

The required Probability = $\frac{14}{36} = \frac{7}{18}$

9. (B) Differential equation

$$\left(\frac{dy}{dx} - y\right)^{1/2} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

$$\Rightarrow \left(\frac{dy}{dx} - y\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Order = 2 and Degree = 2

10. (C) $\lim_{x \rightarrow \infty} \left(\frac{x+5}{x+3}\right)^{x+2}$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x+3}\right)^{x+2}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x+3}\right)^{\frac{x+3}{2} \times \frac{2(x+2)}{x+3}}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} \frac{2(x+2)}{x+3}} \left[\because \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e \right]$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} \frac{2x(1+\frac{2}{x})}{x(1+\frac{3}{x})}}$$

$$\Rightarrow e^{2\left(\frac{1+0}{1+0}\right)} = e^2$$

11. (A) $\frac{1}{\log_3 30} + \frac{1}{\log_2 30} + \frac{1}{\log_5 30}$

$$\Rightarrow \log_{30} 3 + \log_{30} 2 + \log_{30} 5 \left[\because \log_a b = \frac{1}{\log_b a} \right]$$

$$\Rightarrow \log_{30} (3 \times 2 \times 5)$$

$$\Rightarrow \log_{30} 30 = 1$$

12. (C) $I = \int e^{\ln x} \cdot \cos x \, dx$

$$I = \int x \cdot \cos x \, dx$$

$$I = x \cdot \int \cos x \, dx - \int \left\{ \frac{d}{dx}(x) \int \cos x \, dx \right\} dx$$

$$I = x \cdot \sin x - \int 1 \cdot \sin x \, dx$$

$$I = x \cdot \sin x + \cos x + c$$

13. (B) In ΔABC ,

$$a = 6, b = 8, c = 10$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{8^2 + 10^2 - 6^2}{2 \times 8 \times 10}$$

$$\cos A = \frac{64 + 100 - 36}{160}$$

$$\cos A = \frac{4}{5}$$

Now, $\cos 2A = 2\cos^2 A - 1$

$$\Rightarrow \cos 2A = 2 \times \left(\frac{4}{5}\right)^2 - 1$$

$$\Rightarrow \cos 2A = \frac{32}{25} - 1 \Rightarrow \cos 2A = \frac{7}{25}$$

14. (A) Sphere $x^2 + y^2 + z^2 + x + y + z = 0$
 On comparing with general equation

$$u = \frac{1}{2}, v = \frac{1}{2}, w = \frac{1}{2}, d = 0$$

Now, $r = \sqrt{u^2 + v^2 + w^2 - d}$

$$\Rightarrow r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - 0}$$

$$\Rightarrow r = 1 \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}}$$

$$\Rightarrow r = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

15. (D) $\vec{a} = \hat{i} - 3x\hat{j} - 4y\hat{k}$ and $\vec{b} = \hat{i} + 4x\hat{j} + 3y\hat{k}$
 are orthogonal to each other,

then $\vec{a} \cdot \vec{b} = 0$

$$\Rightarrow 1 \times 1 - 3x \times 4x - 4y \times 3y = 0$$

$$\Rightarrow 1 - 12x^2 - 12y^2 = 0$$

$$\Rightarrow 12x^2 + 12y^2 = 1 \Rightarrow x^2 + y^2 = \frac{1}{12}$$

Hence it is circle.

16. (C) $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} a+x & a-x & a-x \\ -2x & 2x & 0 \\ -2x & 0 & 2x \end{vmatrix} = 0$$

$$R_2 \rightarrow R_2 - R_3$$

$$\Rightarrow \begin{vmatrix} a+x & a-x & a-x \\ 0 & 2x & -2x \\ -2x & 0 & 2x \end{vmatrix} = 0$$

$$\Rightarrow (a+x)(4x^2 - 0) - (a-x)(0 - 4x^2) + (a-x)(0 + 4x^2) = 0$$

$$\Rightarrow 4ax^2 + 4x^3 + 4ax^2 - 4x^3 + 4ax^2 - 4x^3 = 0$$

$$\Rightarrow 12ax^2 - 4x^3 = 0$$

$$\Rightarrow 4x^2(3a - x) = 0$$

$$\Rightarrow x = 0, x = 3a$$

17. (B) Let $\vec{a} = p(6\hat{i} + 3\hat{j} - 2\hat{k})$

Given that $|\vec{a}| = 7$

$$\Rightarrow \sqrt{p^2[6^2 + 3^2 + (-2)^2]} = 7$$

$$\Rightarrow p^2 \times 49 = 49$$

$$\Rightarrow p = 1$$

Hence $p = 1$

18. (C) Word "SORRY"

$$\text{Total arrangement} = \frac{5!}{2!} = 60$$

$$\text{Total arrangement when two 'R's come together} = \frac{4!}{2!} = 12$$

$$\text{Total arrangement when two 'R's should not be together} = 60 - 12 = 48$$

19. (C) Given that $A = \{1, 2, 3, 5, 7\}$

$$n(A) = 5$$

$$P(A) = 2^5 = 32$$

20. (D) $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{1^2 + 2^2 + 3^2 + \dots + n^2}$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{\frac{n^2(n+1)^2}{4}}{\frac{n}{6}(n+1)(2n+1)}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{n^2 \times n^2 \left(1 + \frac{1}{n}\right)^2}{\frac{n^3}{6} \left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{3n}{2} \frac{\left(1 + \frac{1}{n}\right)^2}{\left(1 + \frac{1}{n}\right) \left(2 + \frac{1}{n}\right)} = \infty$$

21. (D)

22. (B) $n(S) = {}^8C_2 = 28$

$$n(E) = {}^5C_2 \times {}^4C_0 = 10 \times 1 = 10$$

$$\text{The required Probability} = \frac{n(E)}{n(S)} = \frac{10}{28} = \frac{5}{14}$$

23. (B) $I = \int x^2 \cdot \cos^2 x^3 \cdot \sin x^3 dx$

Let $\cos x^3 = t$

$$\Rightarrow -3x^2 \cdot \sin x^3 dx = dt$$

$$\Rightarrow x^2 \sin x^3 dx = \frac{-1}{3} dt$$

$$I = \frac{-1}{3} \int t^2 dt$$

$$I = \frac{-1}{3} \times \frac{t^3}{3} + c$$

$$I = -\frac{1}{9} \cos^3 x^3 + c$$

24. (B) $f(x) = 2^{\cos x}$

On differentiating both sides w.r.t. 'x'

$$f'(x) = 2^{\cos x} \cdot \log 2 \cdot (-\sin x)$$

$$f'(x) = -\sin x \cdot \log 2 \cdot 2^{\cos x}$$

25. (C) $\tan 195^\circ + \cot 255^\circ$

$$\Rightarrow \tan(180 + 15) + \cot(180 + 75)$$

$$\Rightarrow \tan 15 + \cot 75$$

$$\Rightarrow \tan 15 + \tan 15$$

$$\Rightarrow 2 \tan 15$$

$$\Rightarrow 2 \left(\frac{\sqrt{3}-1}{\sqrt{3}+1} \right) \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow 2 \times \frac{(\sqrt{3}-1)^2}{3-1}$$

$$\Rightarrow 2 \times \frac{(3+1-2\sqrt{3})}{2} = 4 - 2\sqrt{3}$$

26. (B) $I = \int \frac{dx}{\sqrt{9+x^2}}$

$$I = \log |x + \sqrt{x^2 + 9}| + c$$

27. (D) $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$

$$\Rightarrow (\vec{a} \times \vec{a}) + (\vec{b} \times \vec{a}) - (\vec{a} \times \vec{b}) - (\vec{b} \times \vec{b})$$

$$\Rightarrow 0 - (\vec{a} \times \vec{b}) - (\vec{a} \times \vec{b}) = -2(\vec{a} \times \vec{b})$$

28. (A) $y = \tan^{-1} \left(\frac{3 - 2 \tan \sqrt{x}}{2 + 3 \tan \sqrt{x}} \right)$

$$y = \tan^{-1} \left(\frac{\frac{3}{2} - \tan \sqrt{x}}{1 + \frac{3}{2} \tan \sqrt{x}} \right)$$

$$\text{Let } \frac{3}{2} = \tan \phi \Rightarrow \phi = \tan^{-1} \left(\frac{3}{2} \right)$$

$$y = \tan^{-1} \left(\frac{\tan \phi - \tan \sqrt{x}}{1 + \tan \phi \cdot \tan \sqrt{x}} \right)$$

$$y = \tan^{-1} [\tan(\phi - \sqrt{x})]$$

$$y = \phi - \sqrt{x}$$

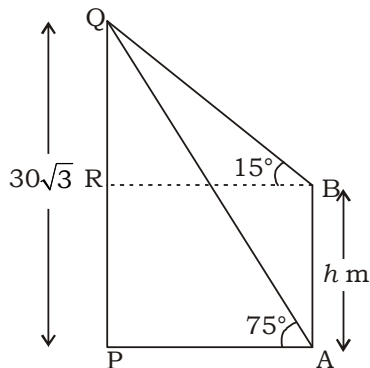
$$y = \tan^{-1} \left(\frac{3}{2} \right) - \sqrt{x}$$

On differentiating both sides w.r.t. 'x'

$$\frac{dy}{dx} = 0 - \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x}}$$

29. (B)



Let $AB = h \text{ m} = PR$
and $QR = 30\sqrt{3} - h$

In ΔQRB

$$\tan 15^\circ = \frac{QR}{RB}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{30\sqrt{3}-h}{RB} \quad \dots(i)$$

In ΔAPQ

$$\tan 75^\circ = \frac{PQ}{PA}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{30\sqrt{3}}{RB} \quad \dots(ii)$$

from eq(i) and eq(ii)

$$\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2} = \frac{30\sqrt{3}-h}{30\sqrt{3}}$$

$$\Rightarrow \frac{4-2\sqrt{3}}{4+2\sqrt{3}} = \frac{30\sqrt{3}-h}{30\sqrt{3}}$$

$$\Rightarrow \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{30\sqrt{3}-h}{30\sqrt{3}}$$

$$\Rightarrow 60\sqrt{3}-90 = 60\sqrt{3}-2h+90-\sqrt{3}h$$

$$\Rightarrow (2-\sqrt{3})h = 180$$

$$\Rightarrow h = \frac{180}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$h = 180(2+\sqrt{3})$$

Hence height of the pole = $180(2+\sqrt{3}) \text{ m}$

30. (C) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$\vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\vec{a} \times \hat{i} = \hat{i}(0) - \hat{j}(-a_3) + \hat{k}(-a_2)$$

$$\vec{a} \times \hat{i} \Rightarrow a_3\hat{j} - a_2\hat{k}$$

$$|\vec{a} \times \hat{i}|^2 = |a_3\hat{j} - a_2\hat{k}|^2$$

$$|\vec{a} \times \hat{i}|^2 = a_3^2 + a_2^2$$

Similarly

$$|\vec{a} \times \hat{j}|^2 = a_1^2 + a_3^2 \text{ and } |\vec{a} \times \hat{k}|^2 = a_1^2 + a_2^2$$

$$\text{Now, } |\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$$

$$\Rightarrow a_3^2 + a_2^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$$

$$\Rightarrow 2(a_1^2 + a_2^2 + a_3^2) = 2|\vec{a}|^2$$

31. (C) Let the equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$... (i)

it passes through the point (1, 2)

$$1^2 + 2^2 + 2g \times 1 + 2f \times 2 + c = 0$$

$$\Rightarrow 4g + 4f + c = -5 \quad \dots(ii)$$

eq(i) passes through the point (-1, 0)

$$(-1)^2 + 0^2 + 2g \times (-1) + 2f \times 0 + c = 0$$

$$\Rightarrow -2g + c = -1 \quad \dots(iii)$$

eq(i) passes through the point (3, -4)

$$3^2 + (-4)^2 + 2g \times 3 + 2f \times (-4) + c = 0$$

$$\Rightarrow 6g - 8f + c = -25 \quad \dots(iv)$$

On solving eq(ii), (iii) and (iv)

$$g = \frac{-8}{5}, f = \frac{7}{5}, c = \frac{-21}{5}$$

from eq(i)

$$x^2 + y^2 + 2\left(\frac{-8}{5}\right)x + 2\left(\frac{7}{5}\right)y - \frac{21}{5} = 0$$

$$\Rightarrow 5x^2 + 5y^2 - 16x + 14y - 21 = 0$$

32. (B) $y = (\sec x)^{\sec x} \dots \dots \dots$

$$\Rightarrow y = (\sec x)^y$$

taking log both sides

$$\Rightarrow \log y = y \log \sec x \quad \dots(i)$$

On differentiating both sides w.r.t.'x'

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y \times \frac{\sec x \cdot \tan x}{\sec x} + \frac{dy}{dx} \cdot \log \sec x$$

$$\Rightarrow \frac{dy}{dx} = y^2 \cdot \tan x + \frac{dy}{dx} (y \log \sec x)$$

$$\Rightarrow \frac{dy}{dx} = y^2 \cdot \tan x + \log y \cdot \frac{dy}{dx} \quad [\text{from eq(i)}]$$

$$\Rightarrow (1 - \log y) \frac{dy}{dx} = y^2 \cdot \tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 \cdot \tan x}{1 - \log y}$$

33. (D) Equation

$$x^2 + \alpha x + \beta = 0$$

Roots are α and β ,

$$\text{then } \alpha + \beta = -\alpha$$

$$\Rightarrow 2\alpha + \beta = 0 \quad \dots(i)$$

$$\text{and } \alpha\beta = \beta \Rightarrow \alpha = 1$$

from eq(i)

$$2 \times 1 + \beta = 0 \Rightarrow \beta = -2$$

New quadratic equation

$$\Rightarrow -x^2 - \alpha x - \beta$$

$$\Rightarrow -x^2 - x + 2$$

$$\Rightarrow -x^2 - x - \frac{1}{4} + \frac{1}{4} + 2$$

$$\Rightarrow -\left(x + \frac{1}{2}\right)^2 + \frac{9}{4}$$

Hence greatest value of the equation = $\frac{9}{4}$

34. (B) Equation

$$5x^2 + 4 = 0$$

Roots are $\sin\alpha$ and $\sin\beta$,

$$\text{then } \sin\alpha + \sin\beta = 0$$

$$\text{and } \sin\alpha \cdot \sin\beta = \frac{4}{5}$$

$$\Rightarrow \operatorname{cosec}\alpha \cdot \operatorname{cosec}\beta = \frac{5}{4}$$

35. (B) Equation

$$|x-4|^2 + |x-4| - 2 = 0$$

$$\text{Let } x-4 = y$$

$$y^2 + |y| - 2 = 0$$

(i) When $y \geq 0$

$$y^2 + y - 2 = 0$$

$$\Rightarrow (y+2)(y-1) = 0$$

$$\Rightarrow y = -2, 1$$

$$\Rightarrow y = 1$$

$$\Rightarrow x-4 = 1 \Rightarrow x = 5$$

(ii) when $y < 0$

$$y^2 - y - 2 = 0$$

$$\Rightarrow (y-2)(y+1) = 0$$

$$\Rightarrow y = 2, -1$$

$$\Rightarrow y = -1$$

$$\Rightarrow x-4 = -1 \Rightarrow x = 3$$

Hence sum of all real roots = $5 + 3 = 8$

36. (B) No. of triangle = ${}^9C_3 - {}^4C_3$

$$= 84 - 4 = 80$$

37. (C) Equation $4x^2 - 8x - \log_2 A = 0$ has real roots,

$$\text{then } B^2 - 4AC \geq 0$$

$$\Rightarrow (-8)^2 - 4 \times 4(-\log_2 A) \geq 0$$

$$\Rightarrow 16\log_2 A \geq -64$$

$$\Rightarrow \log_2 A \geq -4$$

$$\Rightarrow A \geq 2^{-4} \Rightarrow A \geq \frac{1}{16}$$

Hence minimum value of $A = \frac{1}{16}$

38. (A) Equation $ax^2 + x + b = 0$

Let roots are α and β .

$$\alpha + \beta = \frac{-1}{a} \quad \text{and} \quad \alpha\beta = \frac{b}{a}$$

A.T.Q

$$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \frac{-1}{a} = \frac{1 - \frac{2b}{a}}{\frac{b^2}{a^2}}$$

$$\Rightarrow \frac{-1}{a} = \frac{1 - 2ab}{b^2}$$

$$\Rightarrow \frac{-1}{a} = \frac{1}{b^2} - \frac{2a}{b}$$

$$\Rightarrow \frac{2a}{b} = \frac{1}{a} + \frac{1}{b^2}$$

Hence $\frac{1}{a}, \frac{a}{b}, \frac{1}{b^2}$ are in A.P.

$$\Rightarrow \frac{ab}{a}, \frac{a \cdot ab}{b}, \frac{ab}{b^2} \text{ are in A.P.}$$

$$\Rightarrow b, a^2, \frac{a}{b} \text{ are in A.P.}$$

39. (D) Let $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{2 \times 2 - 1 \times (-1) - 2 \times 2}{\sqrt{2^2 + (-1)^2 + 2^2}}$$

$$= \frac{4 + 1 - 4}{3} = \frac{1}{3}$$

40. (C) Equation $x^2 + kx + c = 0$

Let roots are α and β ,

$$\alpha + \beta = -k \quad \text{and} \quad \alpha\beta = c$$

A.T.Q

$$\alpha^2 + \beta^2 = 4c$$

$$\Rightarrow (\alpha + \beta)^2 = 2\alpha\beta = 4c$$

$$\Rightarrow k^2 - 2c = 4c$$

$$\Rightarrow k^2 = 6c$$

41. (B) $(3\omega^2 + 2 + 2\omega)^{23}$

$$\Rightarrow [3\omega^2 + 2(1 + \omega)]^{23}$$

$$\Rightarrow [3\omega^2 - 2\omega^2]^{23} \quad [\because 1 + \omega + \omega^2 = 0]$$

$$\Rightarrow (\omega^2)^{23}$$

$$\Rightarrow \omega^{46} = \omega^{3 \times 45 + 1} = \omega \quad [\because \omega^3 = 1]$$

42. (A) In ΔABC , $a + b = 2c$

$$s = \frac{a+b+c}{2}$$

$$s = \frac{2c+c}{2} = \frac{3c}{2}$$

Now, $\tan \frac{A}{2} \cdot \tan \frac{B}{2}$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)(s-a)(s-c)}{s^2(s-a)(s-b)}}$$

$$\Rightarrow \frac{s-c}{s} = \frac{\frac{3c}{2} - c}{\frac{3c}{2}} = \frac{\frac{c}{2}}{\frac{3c}{2}} = \frac{1}{3}$$

43. (B) Differential equation

$$\tan\left(\frac{dy}{dx}\right) = x$$

$$\Rightarrow \frac{dy}{dx} = \tan^{-1}x$$

$$\Rightarrow dy = \tan^{-1}x \, dx$$

On integrating

$$\Rightarrow \int dy = \int \tan^{-1}x \, dx$$

$$\Rightarrow y = \tan^{-1}x \int 1 \cdot dx - \int \left\{ \frac{d}{dx}(\tan^{-1}x) \cdot \int 1 \cdot dx \right\} dx$$

$$\Rightarrow y = (\tan^{-1}x) \cdot x - \int \frac{1}{1+x^2} \cdot x \, dx$$

Let $1 + x^2 = t$

$$2x \, dx = dt \Rightarrow x \, dx = \frac{1}{2} dt$$

$$\Rightarrow y = x \cdot \tan^{-1}x - \frac{1}{2} \int \frac{1}{t} dx$$

$$\Rightarrow y = x \cdot \tan^{-1}x - \frac{1}{2} \log t + c$$

$$\Rightarrow y = x \cdot \tan^{-1}x - \frac{1}{2} \log(1+x^2) + c$$

44. (C) $[3 \ -5 \ x] \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 0 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = [0]$

$$\Rightarrow [3 \ -5 \ x] \begin{bmatrix} 1 \times 1 + 2 \times 0 + 3 \times (-2) \\ 4 \times 1 - 1 \times 0 + 0 \times (-2) \\ 3 \times 1 + 2 \times 0 + 5 \times (-2) \end{bmatrix} = [0]$$

$$\Rightarrow [3 \ -5 \ x] \begin{bmatrix} -5 \\ 4 \\ -7 \end{bmatrix} = [0]$$

$$\Rightarrow [3 \times (-5) - 5 \times 4 + x \times (-7)] = [0]$$

$$\Rightarrow [-15 - 20 - 7x] = [0]$$

$$\Rightarrow -35 = 7x \Rightarrow x = -5$$

45. (D) Let $y = \tan^{-1}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$

and $z = \cos^{-1}x \Rightarrow x = \cos z$

$$y = \tan^{-1}\left(\frac{1-\sqrt{1-\cos^2 z}}{\cos z}\right)$$

$$y = \tan^{-1}\left(\frac{1-\sin z}{\cos z}\right)$$

$$y = \tan^{-1}\left[\frac{1-\cos\left(\frac{\pi}{2}-z\right)}{\sin\left(\frac{\pi}{2}-z\right)}\right]$$

$$y = \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4}-\frac{z}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{z}{2}\right) \cdot \cos\left(\frac{\pi}{4}-\frac{z}{2}\right)}\right]$$

$$y = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{4}-\frac{z}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{z}{2}\right)}\right]$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{z}{2}\right)\right)$$

$$y = \frac{\pi}{4} - \frac{z}{2}$$

On differentiating both sides w.r.t. 'z'

$$\frac{dy}{dz} = \frac{-1}{2}$$

46. (C) In the expansion of $\left(x^3 - \frac{1}{x^5}\right)^9$

$$T_{r+1} = {}^9C_r (x^3)^{9-r} \left(\frac{-1}{x^5}\right)^r$$

$$T_{r+1} = {}^9C_r x^{27-8r} (-1)^r$$

Here $27 - 8r = 3$

$$\Rightarrow 8r = 24 \Rightarrow r = 3$$

47. (D) Equation of line which passes through the point $(-1, 3)$ and $(3, 4)$

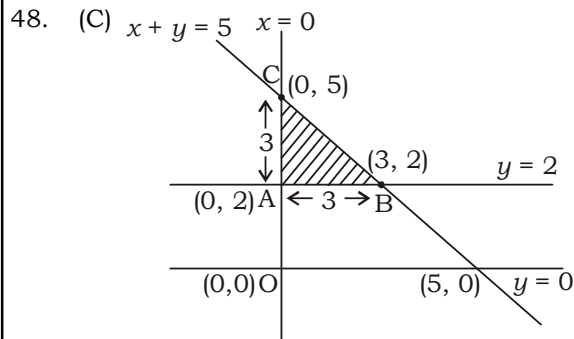
$$y - 3 = \frac{4-3}{3+1}(x+1)$$

$$\Rightarrow y - 3 = \frac{1}{4}(x+1)$$

$$\Rightarrow x - 4y + 13 = 0 \quad \dots(i)$$

Now, length of the perpendicular from the point $(4, -2)$ on the line (i)

$$p = \frac{4 - 4(-2) + 13}{\sqrt{1^2 + (-4)^2}} \Rightarrow p = \frac{25}{\sqrt{27}}$$



$$\text{Area of } \triangle ABC = \frac{1}{2} \times AB \times AC$$

$$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2} \text{ sq.unit}$$

49. (B) $y = \frac{x^2}{2!} + \frac{x^2}{3!} + \dots \infty$

$$y = \left(1 - x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty \right) - 1 - x$$

$$y = e^x - 1 - x$$

On differentiating both sides w.r.t 'x'

$$\frac{dy}{dx} = e^x - 0 - 1$$

$$\frac{dy}{dx} = e^x - 1$$

50. (B) $\begin{vmatrix} a^3+1 & a & a^2 \\ b^3+1 & b & b^2 \\ c^3+1 & c & c^2 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a^3 & a & a^2 \\ b^3 & b & b^2 \\ c^3 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

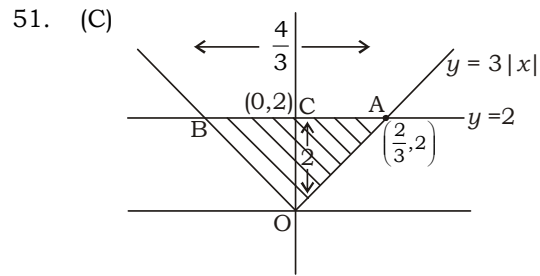
$$\Rightarrow abc \begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (abc + 1) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow abc + 1 = 0, \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\Rightarrow abc = -1$$



$$\begin{aligned} \text{Area of } \triangle AOB &= \frac{1}{2} \times OC \times AB \\ &= \frac{1}{2} \times 2 \times \frac{4}{3} = \frac{4}{3} \text{ sq.unit} \end{aligned}$$

52. (B) Let $a + ib = \sqrt{i}$

On squaring both sides
 $(a^2 - b^2) + 2abi = i$

On comparing
 $a^2 - b^2 = 0$ and $2ab = 1$... (i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
 $\Rightarrow (a^2 + b^2)^2 = 0 + 1$
 $\Rightarrow a^2 + b^2 = 1$... (ii)

$$2a^2 = 1 \Rightarrow a^2 = \frac{1}{2} \Rightarrow a = \pm \frac{1}{\sqrt{2}}$$

$$\text{and } 2b^2 = 1 \Rightarrow b^2 = \frac{1}{2} \Rightarrow b = \pm \frac{1}{\sqrt{2}}$$

$$\text{Hence } \sqrt{i} = \pm \left(\frac{1+i}{\sqrt{2}} \right)$$

53. (C) $\log_4[\log_4[\log_4 x]] = \log_4 4$
 $\Rightarrow \log_4[\log_4 x] = 4$
 $\Rightarrow \log_4 x = 4^4$
 $\Rightarrow \log_4 x = 256$
 $\Rightarrow x = 4^{256}$

54. (B) $\sin 10^\circ + \sin 50^\circ - \sin 70^\circ$
 $\Rightarrow \sin 10 + 2 \cos \frac{50+70}{2} \cdot \sin \frac{50-70}{2}$
 $\Rightarrow \sin 10 + 2 \cos 60 \cdot \sin(-10)$
 $\Rightarrow \sin 10 - 2 \times \frac{1}{2} \sin 10 = 0$

55. (D) $f(x) = \frac{\tan(e^{x-3}) - 1}{\ln(x-2)}$

Now, $\lim_{x \rightarrow 3} f(x)$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{\tan(e^{x-3}) - 1}{\ln(x-2)} \quad \left[\frac{0}{0} \right]$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 3} \frac{\sec^2(e^{x-3}) \cdot e^{x-3}}{\frac{1}{x-2}}$$

$$\Rightarrow \lim_{x \rightarrow 3} (x-2) \cdot e^{x-3} \cdot \sec^2(e^{x-3} - 1)$$

$$\Rightarrow (3-2) \cdot e^{3-3} \cdot \sec^2(e^{3-3} - 1)$$

$$\Rightarrow 1 \cdot e^0 \cdot \sec^2 0 = 1 \times 1 \times 1 = 1$$

56. (A) Given that $A = 12$, $H = 27$
We know that
 $G^2 = AH$
 $\Rightarrow G^2 = 12 \times 27 \Rightarrow G = 18$

57. (D) $\cot\left(7\frac{1}{2}\right) = \frac{\cos 7\frac{1}{2}}{\sin 7\frac{1}{2}}$
$$\Rightarrow \frac{2\cos^2 7\frac{1}{2}}{2\sin 7\frac{1}{2} \cdot \cos 7\frac{1}{2}} \Rightarrow \frac{1 + \cos 15}{\sin 15}$$

$$\Rightarrow \frac{1 + \frac{\sqrt{3} + 1}{2\sqrt{2}}}{\frac{\sqrt{3} - 1}{2\sqrt{2}}} \Rightarrow \frac{2\sqrt{2} + \sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow \frac{2\sqrt{6} + 3 + \sqrt{3} + 2\sqrt{2} + \sqrt{3} + 1}{3 - 1}$$

$$\Rightarrow \frac{2\sqrt{6} + 2\sqrt{3} + 2\sqrt{2} + 4}{2}$$

$$\Rightarrow \sqrt{6} + \sqrt{3} + \sqrt{2} + 2$$

58. (C) $\lim_{x \rightarrow \infty} \frac{(2x - 5)(x + 3)}{(x + 1)(3x - 4)}$
$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2 \left(2 - \frac{5}{x}\right) \left(1 + \frac{3}{x}\right)}{x^2 \left(1 + \frac{1}{x}\right) \left(3 - \frac{4}{x}\right)}$$

$$\Rightarrow \frac{(2 - 0)(1 + 0)}{(1 + 0)(3 - 0)} = \frac{2}{3}$$

59. (B) $I = \int \frac{dx}{\sin^2 x \cdot \cos^2 x}$
$$I = \int \frac{(\sin^2 x + \cos^2 x) dx}{\sin^2 x \cdot \cos^2 x} [\because \sin^2 x + \cos^2 x = 1]$$

$$I = \int (\sec^2 x + \operatorname{cosec}^2 x) dx$$

$$I = \tan x - \cot x + c$$

60. (C) $\operatorname{cosec}^{-1}\left(\frac{1 + x^2}{2x}\right)$
Let $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$
$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1 + \tan^2 \theta}{2 \tan \theta}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{\sec^2 \theta}{2 \tan \theta}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\frac{\cos^2 \theta}{2 \sin \theta}}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{2 \sin \theta \cdot \cos \theta}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}\left(\frac{1}{\sin 2\theta}\right)$$

$$\Rightarrow \operatorname{cosec}^{-1}(\operatorname{cosec} 2\theta) = 2\theta = 2 \tan^{-1} x$$

61. (D) $\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} + 2X = \begin{bmatrix} 4 & 3 \\ 2 & -6 \end{bmatrix}$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 3 \\ 2 & -6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 1 \\ 5 & -10 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3/2 & 1/2 \\ 5/2 & -5 \end{bmatrix}$$

62. (B) $y = x^x$
taking log both sides
 $\Rightarrow \log y = x \cdot \log x$
On differentiating both sides w.r.t. 'x'

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = x \times \frac{1}{x} + (\log x) \cdot 1$$

$$\Rightarrow \frac{dy}{dx} = y(1 + \log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$$

63. (D) $6\cos^2 \theta - 2\cos 2\theta - 3 = 0$
$$\Rightarrow 6\cos^2 \theta - 2(2\cos^2 \theta - 1) - 3 = 0$$

$$\Rightarrow 6\cos^2 \theta - 4\cos^2 \theta - 1 = 0$$

$$\Rightarrow 2\cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow \theta = 45^\circ$$

Now, $\sec^2 3\theta = \sec^2 135^\circ$
$$\Rightarrow \sec^2 3\theta = \sec^2(90 + 45)$$

$$\Rightarrow \sec^2 3\theta = (-\operatorname{cosec} 45^\circ)^2$$

$$\Rightarrow \sec^2 3\theta = (\sqrt{2})^2 = 2$$

64. (D) $f(x) = \begin{cases} \frac{a \sin 2x - b \cos x}{\frac{\pi}{2} - x}, & x > \frac{\pi}{2} \\ 4, & x = \frac{\pi}{2} \\ \frac{2b \cos x}{\frac{\pi}{2} - x}, & x < \frac{\pi}{2} \end{cases}$ is

continuous function, then

$$\lim_{x \rightarrow \left(\frac{\pi}{2}\right)} f(x) = \lim_{x \rightarrow \left(\frac{\pi}{2}\right)} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\text{Now, } = \lim_{x \rightarrow \left(\frac{\pi}{2}\right)} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{2b \cos x}{\frac{\pi}{2} - x} = 4$$

by L-Hospital Rule

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{-2b \sin x}{-1} = 4$$

$$\Rightarrow 2b \sin \frac{\pi}{2} = 4$$

$$\Rightarrow 2b = 4 \Rightarrow b = 2$$

$$\text{and } \lim_{x \rightarrow \left(\frac{\pi}{2}\right)} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{a \sin 2x - b \cos x}{\frac{\pi}{2} - x} = f\left(\frac{\pi}{2}\right)$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow \frac{\pi}{2}} \frac{2a \cos x + b \sin x}{-1} = 4$$

$$\Rightarrow \frac{2a \cos \pi + b \sin \pi}{-1} = 4$$

$$\Rightarrow \frac{-2a + 0}{-1} = 4 \Rightarrow a = 2$$

$$\text{Hence } = a - b = 2 - 2 = 0$$

65. (A) Linear equations

$$x + 4y - 3z = 2$$

$$2x + 7y - 4z = \alpha$$

$$-x - 5y + 5z = \beta$$

$$\text{Let } A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & 7 & -4 \\ -1 & -5 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 \\ \alpha \\ \beta \end{bmatrix}$$

Using Elementary method

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 2 & 7 & -4 & \alpha \\ -1 & -5 & 5 & \beta \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 + R_1$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 0 & -1 & 2 & \alpha - 4 \\ 0 & -1 & 2 & \beta + 2 \end{array} \right]$$

$$R_3 \rightarrow R_3 - R_2$$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 4 & -3 & 2 \\ 0 & -1 & 2 & \alpha - 4 \\ 0 & 0 & 0 & \beta - \alpha + 6 \end{array} \right]$$

has infinitely many solutions, then

$$\beta - \alpha + 6 = 0 \Rightarrow \alpha - \beta = 6$$

Hence pair (α, β) cannot take the value $(-3, 3)$.

66. (C) $T_n = (n+2)(n+3)$

$$T_n = n^2 + 5n + 6$$

$$\text{Now, } S_n = \Sigma(T_n)$$

$$\Rightarrow S_n = \Sigma(n^2 + 5n + 6)$$

$$\Rightarrow S_n = \Sigma n^2 + 5\Sigma n + 6\Sigma 1$$

$$\Rightarrow S_n = \frac{n}{6}(n+1)(2n+1) + 5 \times \frac{n(n+1)}{2} + 6n$$

$$\Rightarrow S_n = \frac{n}{6} [2n^2 + 3n + 1 + 15n^2 + 15n + 36n]$$

$$\Rightarrow S_n = \frac{n}{6} [17n^2 + 54n + 1]$$

$$S_5 = \frac{5}{6} [17 \times 25 + 54 \times 25 + 1]$$

$$S_5 = \frac{5}{6} \times 1776 = 1480$$

67. (A) Differential equation

$$x \frac{dy}{dx} + 2y = x^3$$

$$\Rightarrow \frac{dy}{dx} + \frac{2}{x} y = x^2$$

On comparing with general equation

$$P = \frac{2}{x}, Q = x^2$$

$$\text{I.F.} = e^{\int P dx}$$

$$\text{I.F.} = e^{\int \frac{2}{x} dx}$$

$$\text{I.F.} = e^{2 \log x} = e^{\log x^2} = x^2$$

Solution of the differential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx$$

$$y \times x^2 = \int x^2 \times x^2 dx$$

$$\Rightarrow yx^2 = \int x^4 dx$$

$$\Rightarrow yx^2 = \frac{x^5}{5} + c$$

$$\Rightarrow y = \frac{x^3}{5} + \frac{c}{x^2}$$

68. (A) Quadratic equation

$$x^2 + px + q = 0$$

One root is $3 + \sqrt{8}$ and other roots is $3 - \sqrt{8}$.

$$\text{Sum of roots} = 3 + \sqrt{8} + 3 - \sqrt{8}$$

$$\Rightarrow -p = 6 \Rightarrow p = -6$$

$$\text{and products of roots} = (3 + \sqrt{8})(3 - \sqrt{8})$$

$$\Rightarrow q = 9 - 8 \Rightarrow q = 1$$

Quadratic equation

$$x^2 - 6x + 1 = 0$$

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69. (A) Given that $e = \frac{3}{5}$
and $2be = 36$
 $\Rightarrow 2b \times \frac{3}{5} = 36 \Rightarrow b = 30$

Now, $e^2 = 1 - \frac{a^2}{b^2}$
 $\Rightarrow \frac{9}{25} = 1 - \frac{a^2}{900}$
 $\Rightarrow \frac{a^2}{900} = 1 - \frac{9}{25}$
 $\Rightarrow \frac{a^2}{900} = \frac{16}{25} \Rightarrow a^2 = 576$
Equation of ellipse
 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
 $\Rightarrow \frac{x^2}{576} + \frac{y^2}{900} = 1$

70. (B) $I = \int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$
Let $\sin^{-1} x = t$ when $x \rightarrow 0, t \rightarrow 0$
 $\frac{1}{\sqrt{1-x^2}} dx = dt$ $x \rightarrow 1, t \rightarrow \frac{\pi}{2}$
 $I = \int_0^{\pi/2} t dx$
 $I = \left[\frac{t^2}{2} \right]_0^{\pi/2}$
 $I = \frac{1}{2} \times \frac{\pi^2}{4} = \frac{\pi^2}{8}$

71. (D) $\lim_{x \rightarrow 3} \frac{4^{x/2} - 8}{2^{2x} - 64}$ $\left[\frac{0}{0} \right]$ form
by L-Hospital's Rule
 $\Rightarrow \lim_{x \rightarrow 3} \frac{4^{x/2}(\log 4) \times \left(\frac{1}{2}\right) - 0}{2^{2x}(\log 2) \times (2) - 0}$
 $\Rightarrow \lim_{x \rightarrow 3} \frac{2^x \times \frac{1}{2} \times 2 \log 2}{2^{2x} \times 2 \log 2}$
 $\Rightarrow \frac{1}{2} \times \frac{2^3}{2^6} = \frac{1}{16}$

72. (C) Straight line
 $\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-1}{-2}$ and $\frac{x+1}{-2} = \frac{y-4}{4} = \frac{z+5}{5}$
Let angle between the straight lines = θ
Now, $\cos \theta = \frac{3 \times (-2) + 4 \times 4 + (-2) \times 5}{\sqrt{3^2 + 4^2 + (-2)^2} \sqrt{(-2)^2 + 4^2 + 5^2}}$
 $\Rightarrow \cos \theta = 0$
 $\Rightarrow \theta = 90^\circ = \frac{\pi}{2}$

73. (B) Determinant $\begin{vmatrix} 2 & 5 & 1 \\ 6 & 4 & 3 \\ 2 & -1 & 0 \end{vmatrix}$

Cofactor of 0 = $(-1)^{3+3} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$
 $= 8 - 30 = -22$

74. (B) $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} -2 & -4 \\ 3 & -\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} -4+3 & -8-\lambda \\ -8+3 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} -1 & -8-\lambda \\ -5 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$
On comparing
 $-8 - \lambda = -2 \Rightarrow \lambda = -6$

75. (D) $I = \int \frac{dx}{x(1+\log x)^3}$
Let $1 + \log x = t \Rightarrow \frac{1}{x} dx = dt$
 $I = \int \frac{dt}{t^3}$
 $I = \frac{t^{-3+1}}{-3+1} + c$
 $I = \frac{-1}{2} \times \frac{1}{t^2} + c$
 $I = \frac{-1}{2(1+\log x)^2} + c$

76. (A)

II	I
(sin θ , cosec θ) \rightarrow '+' other \rightarrow '-'	All positive
(tan θ , cot θ) \rightarrow '+' other \rightarrow '-'	(cos θ , sec θ) \rightarrow '+' other \rightarrow '-'
III	IV

77. (C) The number of ways = $10 \times 9 = 90$

78. (D) $I = \int_0^{\pi/2} \frac{f(x)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx \quad \dots(i)$

Prop. IV $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$I = \int_0^{\pi/2} \frac{f\left(\frac{\pi}{2} - x\right)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx \quad \dots(ii)$

From eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \frac{f(x) + f\left(\frac{\pi}{2} - x\right)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx$

$2I = \int_0^{\pi/2} 1 dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

79. (A) Equation $8x - 6y = 7$
and $6x + 8y = 8$

$\frac{8}{6} \neq \frac{-6}{8} \neq \frac{7}{8}$

Equations have a unique solutions.

80. (B) Let $a + ib = \sqrt{4 + 6\sqrt{5}i}$

On squaring both sides

$(a^2 - b^2) + 2abi = 4 + 6\sqrt{5}i$

On comparing

$a^2 - b^2 = 4$ and $2ab = 6\sqrt{5} \quad \dots(i)$

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$\Rightarrow (a^2 + b^2)^2 = 16 + 180$

$\Rightarrow a^2 + b^2 = 14 \quad \dots(ii)$

from eq(i) and eq(ii)

$2a^2 = 18, 2b^2 = 10$

$a = \pm 3, b = \pm \sqrt{5}$

Square root of $(4 + 6\sqrt{5}i)$ is $\pm(3 + \sqrt{5}i)$.

81. (C) $z = \frac{1}{\sin \theta + i(1 + \cos \theta)}$

$z = \frac{1}{2\sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} + i \times 2\cos^2 \frac{\theta}{2}}$

$z = \frac{1}{2\cos \frac{\theta}{2}} \left[\frac{1}{\sin \frac{\theta}{2} + i \cos \frac{\theta}{2}} \right]$

$z = \frac{1}{2} \sec \frac{\theta}{2} \left[\frac{\sin \frac{\theta}{2} - i \cos \frac{\theta}{2}}{\sin^2 \frac{\theta}{2} - i^2 \cos^2 \frac{\theta}{2}} \right]$

$z = \frac{1}{2} \sec \frac{\theta}{2} \left[\sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \right]$

$z = \frac{1}{2} \left[\tan \frac{\theta}{2} - i \right]$

Imaginary part of $z = \frac{-1}{2}$

82. (A) Let $z = \begin{bmatrix} \omega & \omega^2 & 1 + \omega^2 \\ 1 & \omega & \omega + \omega^2 \\ \omega^2 & 1 & 1 + \omega \end{bmatrix}$

$|z| = \begin{vmatrix} \omega & \omega^2 & 1 + \omega^2 \\ 1 & \omega & \omega + \omega^2 \\ \omega^2 & 1 & 1 + \omega \end{vmatrix}$

$C_1 \rightarrow C_1 + C_3$

$|z| = \begin{vmatrix} 1 + \omega + \omega^2 & \omega^2 & 1 + \omega^2 \\ 1 + \omega + \omega^2 & \omega & \omega + \omega^2 \\ 1 + \omega + \omega^2 & 1 & 1 + \omega \end{vmatrix}$

$|z| = \begin{vmatrix} 0 & \omega^2 & 1 + \omega^2 \\ 0 & \omega & \omega + \omega^2 \\ 0 & 1 & 1 + \omega \end{vmatrix}$

$|z| = 0 = 1 + \omega + \omega^2$

83. (A) $\frac{d}{dx} \left(\frac{\sin^{-1} x}{\sqrt{1-x^2}} \right)$

$\Rightarrow \frac{\sqrt{1-x^2} \times \frac{1}{\sqrt{1-x^2}} - (\sin^{-1} x) \times \frac{-2x}{2\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2}$

$\Rightarrow \frac{1 + \frac{x}{\sqrt{1-x^2}} \sin^{-1} x}{1-x^2}$

$\Rightarrow \frac{\sqrt{1-x^2} + x \cdot \sin^{-1} x}{(1-x^2)^{3/2}}$

84. (C) $AA^T = 1$

$\Rightarrow |AA^T| = 1$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = \pm 1$

85. (C) $\lim_{x \rightarrow \infty} \frac{2x^4 + 5x^3 + 6x}{5x^5 + 3x + 4x^2}$

$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^4 \left(2 + \frac{5}{x} + \frac{6}{x^2} \right)}{x^5 \left(5 + \frac{3}{x^4} + \frac{4}{x^3} \right)}$

$\Rightarrow \lim_{x \rightarrow \infty} \frac{\left(2 + \frac{5}{x} + \frac{6}{x^2} \right)}{x \left(5 + \frac{3}{x^4} + \frac{4}{x^3} \right)} = \frac{1}{\infty} = 0$

86. (B) $\int_1^2 \{k^2 + (1-k)x + 2x^3\} dx \leq 10$

$$\Rightarrow \left[k^2x + (1-k)\frac{x^2}{2} + \frac{2x^4}{4} \right]_1^2 \leq 10$$

$$\Rightarrow (2k^2 + (1-k) \times 2 + 8) - \left(k^2 + (1-k) \times \frac{1}{2} + \frac{1}{2} \right) \leq 10$$

$$\Rightarrow k^2 - \frac{3k}{2} + 9 \leq 10$$

$$\Rightarrow 2k^2 - 3k + 18 \leq 20$$

$$\Rightarrow 2k^2 - 3k - 2 \leq 0$$

$$\Rightarrow (2k+1)(k-2) \leq 0$$

$$\frac{+}{-\frac{1}{2}} \frac{+}{2}$$

Hence $\frac{-1}{2} \leq k \leq 2$

87. (A) $\sin\left(\sin^{-1}\frac{12}{13} + \cos^{-1}x\right) = 1$

$$\Rightarrow \sin^{-1}\frac{12}{13} + \cos^{-1}x = \sin^{-1}(1)$$

$$\Rightarrow \sin^{-1}\frac{12}{13} + \cos^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{12}{13}$$

$$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{12}{13} \Rightarrow x = \frac{12}{13}$$

88. (A) $f(x) = \frac{1}{\sqrt{55+x^2}}$

On differentiating both sides w.r.t. 'x'

$$f'(x) = \frac{-(2x)}{2(55+x^2)^{\frac{3}{2}}}$$

$$f'(x) = \frac{-x}{(55+x^2)^{\frac{3}{2}}}$$

Now, $\lim_{x \rightarrow 3} \frac{f(3) - f(x)}{x^3 - 27} \quad \left[\frac{0}{0} \right] \text{Form}$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 3} \frac{-f'(x)}{3x^2}$$

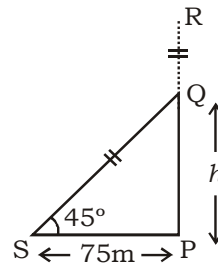
$$\Rightarrow \lim_{x \rightarrow 3} \frac{x}{3x^2(55+x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{1}{3x(55+x^2)^{\frac{3}{2}}}$$

$$\Rightarrow \frac{1}{3 \times 3(55+3^2)^{\frac{3}{2}}} = \frac{1}{4608}$$

89. (C) The required Probability $P(A) = \frac{n(E)}{n(S)} = \frac{1}{52}$

90. (B)



We know that $QR = QS$

Let $PQ = h$ m

In ΔPSQ :

$$\tan 45^\circ = \frac{PQ}{PS}$$

$$\Rightarrow 1 = \frac{h}{75} \Rightarrow h = 75 \quad \dots(i)$$

Now, $\sin 45^\circ = \frac{PQ}{QS}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{h}{QS}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{75}{QS} \Rightarrow QS = 75\sqrt{2} = QR$$

Length of a tree = $PQ + QR$

$$= 75 + 75\sqrt{2} = 75(\sqrt{2} + 1) \text{ m}$$

91. (B) Given that $f(x) = bx + c$, $g(x) = ax + d$

Now, $f \circ g(x) = g \circ f(x)$

$$\Rightarrow f[g(x)] = g[f(x)]$$

$$\Rightarrow f[ax + d] = g[bx + c]$$

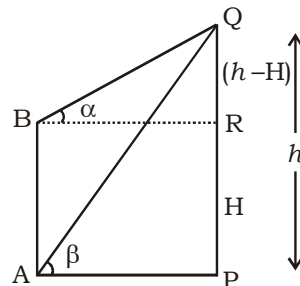
$$\Rightarrow b(ax + d) + c = a(bx + c) + d$$

$$\Rightarrow abx + bd + c = abx + ac + d$$

$$\Rightarrow bd + c = ac + d$$

$$\Rightarrow f(d) = g(c)$$

92. (A)



Let height of a building (AB) = H m

In ΔRBQ :

$$\tan \alpha = \frac{RQ}{BR} = \frac{h-H}{BR}$$

$$\Rightarrow AP = BR = \frac{h-H}{\tan \alpha} \quad \dots(i)$$

In ΔAPQ :-

$$\tan \beta = \frac{PQ}{AP}$$

$$\Rightarrow \tan \beta = \frac{h \times \tan \alpha}{h-H} \quad [\text{from eq. (i)}]$$

$$\Rightarrow h \tan \beta - H \tan \beta = h \tan \alpha$$

$$\Rightarrow H \tan \beta = h(\tan \beta - \tan \alpha)$$

$$\Rightarrow H = \frac{h(\tan \beta - \tan \alpha)}{\tan \beta}$$

93. (D) $f(x) = \tan^{-1} \left[\frac{\sqrt{x}(1+x)}{1-x^2} \right]$

$$\Rightarrow f(x) = \tan^{-1} \left[\frac{\sqrt{x} + x^{\frac{3}{2}}}{1 - \sqrt{x} \cdot x^{\frac{3}{2}}} \right]$$

$$\Rightarrow f(x) = \tan^{-1}(x^{\frac{1}{2}}) + \tan^{-1}(x^{\frac{3}{2}})$$

On differentiating both side w.r.t. 'x'

$$\Rightarrow f'(x) = \frac{1}{1+x} \times \frac{1}{2} \times \frac{1}{\sqrt{x}} + \frac{1}{1+x^3} \times \frac{3}{2} x^{\frac{1}{2}}$$

$$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}} \times \frac{1}{1+x} + \frac{3\sqrt{x}}{2(1+x^3)}$$

$$\Rightarrow f'(4) = \frac{1}{2\sqrt{4}} \times \frac{1}{1+4} + \frac{3\sqrt{4}}{2(1+64)}$$

$$\Rightarrow f'(4) = \frac{1}{4} \times \frac{1}{5} + \frac{3 \times 2}{2 \times 65} = \frac{29}{260}$$

94. (C) $I = \int \frac{dx}{\sqrt{4x^2 - 20x + 29}}$

$$I = \frac{1}{2} \int \frac{dx}{\sqrt{x^2 - 5x + \frac{29}{4}}}$$

$$I = \frac{1}{2} \int \frac{dx}{\sqrt{\left(x - \frac{5}{2}\right)^2 + 1^2}}$$

$$I = \frac{1}{2} \log \left| x - \frac{5}{2} + \sqrt{\left(x - \frac{5}{2}\right)^2 + 1^2} \right| + c$$

$$I = \frac{1}{2} \log | 2x - 5 + \sqrt{4x^2 - 20x + 29} | + c$$

95. (D) $y = a^{x^2 \log_a \cos x}$

$$\Rightarrow y = a^{\log_a (\cos x)^{x^2}}$$

$$\Rightarrow y = (\cos x)^{x^2}$$

taking log both sides

$$\Rightarrow \log y = x^2 \cdot \log \cos x$$

On differentiating both sides w.r.t. 'x'

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = x^2 \times \frac{(-\sin x)}{\cos x} + (\log \cos x) \times 2x$$

$$\Rightarrow \frac{dy}{dx} = y(-x^2 \cdot \tan x + 2x \cdot \log \cos x)$$

96. (A) 11011.011_2

$$\text{Hence } (11011.011)_2 = (27.375)_{10}$$

97. (C) $\left(\frac{d^2y}{dx^2} + y \frac{dy}{dx} \right)^{\frac{1}{2}} = \left(\frac{d^3y}{dx^3} \right)^{\frac{1}{4}}$

$$\Rightarrow \left(\frac{d^2y}{dx^2} + y \frac{dy}{dx} \right)^2 = \frac{d^3y}{dx^3}$$

Order = 3, Degree = 1

98. (B) 5

99. (D) $A = \{0, 1, 2, 3\}$ and $B = \{0, 1, 4, 5\}$

$$A \times B = \{(0, 0), (0, 1), (0, 4), (0, 5), (1, 0), (1, 1), (1, 4), (1, 5), (2, 0), (2, 1), (2, 4), (2, 5), (3, 0), (3, 1), (3, 4), (3, 5)\}$$

$$B \times A = \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (4, 0), (4, 1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2), (5, 3)\}$$

$$\text{Now, } (B \times A) \cap (A \times B) = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$$

100. (D) $I = \int \frac{\log(x - \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$

$$\text{Let } \log(x - \sqrt{1+x^2}) = t$$

$$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \left[1 - \frac{1 \times 2x}{2\sqrt{1+x^2}} \right] dx = dt$$

$$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} - x}{2\sqrt{1+x^2}} dx = dt$$

$$\Rightarrow \frac{-1}{2\sqrt{1+x^2}} dx = dt \Rightarrow \frac{1}{\sqrt{1+x^2}} dx = -2dt$$

$$I = \int t(-2dt)$$

$$I = -2 \times \frac{t^2}{2} + C$$

$$I = -\left[\log(x - \sqrt{1+x^2}) \right]^2 + c$$

101. (B) Let $y = 4^{67}$

taking log both sides

$$\Rightarrow \log_{10} y = 67 \log_{10} 4$$

$$\Rightarrow \log_{10} y = 67 \times 2 \log_{10} 2$$

$$\Rightarrow \log_{10} y = 134 \times 0.3010$$

$$\Rightarrow \log_{10} y = 40.334$$

The required no. of digits = 40 + 1 = 41

102. (C) Given that $s = 5$ cm and $r = 3$ cm

$$\text{Now } \theta = \frac{s}{r}$$

$$\Rightarrow \theta = \left(\frac{5}{3}\right)^c$$

$$\Rightarrow \theta = \left(\frac{5}{3} \times \frac{180}{\pi}\right)^0$$

$$\Rightarrow \theta = \left(\frac{300}{\pi}\right)^0$$

103. (D) $A = \begin{bmatrix} -2 & 1 \\ -6 & 5 \end{bmatrix}$

$$|A| = -10 + 6 = -4$$

We know that

$$A(\text{Adj}A) = |A|^{n-1}I \quad [\text{where } n \text{ is order}]$$

$$\text{Now, } A(\text{Adj}A) = |A|^{2-1}I$$

$$\Rightarrow A(\text{Adj}A) = |A| I$$

$$\Rightarrow A(\text{Adj}A) = -4I$$

104. (C) Lines $\frac{2x-1}{12} = \frac{1-y}{3} = \frac{z-1}{-2}$

$$\Rightarrow \frac{2\left(x-\frac{1}{2}\right)}{12} = \frac{-(y-1)}{3} = \frac{z-1}{-2}$$

$$\Rightarrow \frac{x-\frac{1}{2}}{6} = \frac{y-1}{-3} = \frac{z-1}{-2}$$

$$\text{Distance ratios} = \langle 6, -3, -2 \rangle$$

105. (C) Given that $e = \frac{1}{\sqrt{6}}$

$$\text{and } \frac{2a}{e} = \sqrt{12}$$

$$\Rightarrow \frac{2 \times a}{1/\sqrt{6}} = \sqrt{12} \Rightarrow a = \frac{1}{\sqrt{2}}$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = \frac{1}{2} \times \left(1 - \frac{1}{6}\right)$$

$$\Rightarrow b^2 = \frac{1}{2} \times \frac{5}{6} \Rightarrow b^2 = \frac{5}{12}$$

Equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{1/2} + \frac{y^2}{5/12} = 1$$

$$\Rightarrow 10x^2 + 12y^2 = 5$$

106. (B) Word "OFFICER"

$$\begin{aligned} \text{The number of Permutations} &= \frac{7!}{2!} \\ &= 2520 \end{aligned}$$

107. (B) $I = \int e^x \left(\frac{3x-1}{x^{4/3}}\right) dx$

$$I = 3 \int e^x \left(\frac{3x-1}{3x^{4/3}}\right) dx$$

$$I = 3 \int e^x \left(x^{-1/3} - \frac{1}{3}x^{-4/3}\right) dx$$

We know that $\int e^x [f(x) + f'(x)] = e^x \cdot f(x) + c$

$$I = 3e^x \cdot x^{-1/3} + c$$

$$I = \frac{3e^x}{x^{1/3}} + c$$

108. (B) In the expansion of $\left(\frac{2x}{y} - \frac{y}{6x}\right)^6$

$$\text{Total term} = 6 + 1 = 7$$

$$\text{Middle term} = \left(\frac{6}{2} + 1\right)^{\text{th}} = 4^{\text{th}}$$

$$T_4 = T_{3+1} = {}^6C_3 \left(\frac{2x}{y}\right)^3 \left(\frac{-y}{6x}\right)^3$$

$$T_4 = 20 \times \frac{8x^3}{y^3} \left(\frac{-y^3}{216x^3}\right) = \frac{-20}{27}$$

109. (A)

110. (C) Digits 0, 1, 2, 4, 5, 7, 9
when last digit is '0'

$$\begin{array}{|c|c|} \hline 4 & 1 \\ \hline \end{array} = 4$$

↓
0

when last digit is '2'

$$\begin{array}{|c|c|} \hline 4 & 1 \\ \hline \end{array} = 4$$

↓
2

when last digit is '4'

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} = 3$$

↓
4

The required numbers = 4 + 4 + 3 = 11

111. (B) $n(S) = 16$

$$E = \{(HHHT), (HTHH), (HHTH), (THHH)\}$$

$$n(E) = 4$$

$$\text{The required Probability} = \frac{n(E)}{n(S)} = \frac{4}{16} = \frac{1}{4}$$

112. (C) $\tan 75^\circ - \cot 33^\circ$

$$\Rightarrow \tan(72^\circ + 3^\circ) - \cot(36^\circ - 3^\circ)$$

$$\Rightarrow \tan 30^\circ + \cot 30^\circ$$

$$\Rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}}$$

113. (D)
$$\frac{\sin 330 \cdot \cos 75 \cdot \tan 135}{\cos 435 \cdot \sin 750 \cdot \cot 225}$$

$$\Rightarrow \frac{\sin(360 - 30) \cdot \cos 75 \cdot \tan(90 + 45)}{\cos(360 + 75) \cdot \sin(720 + 30) \cdot \cot(180 + 45)}$$

$$\Rightarrow \frac{-\sin 30 \cdot \cos 75 \cdot (-\tan 45)}{\cos 75 \cdot \sin 30 \cdot \cot 45}$$

$$\Rightarrow \frac{-\frac{1}{2} \times \cos 75 \times (-1)}{\cos 75 \times \frac{1}{2} \times 1} = 1$$

114. (B) Curve $2x^2 + 3y^2 = 10$

$$\Rightarrow \frac{2x^2}{10} + \frac{3y^2}{10} = 1$$

$$\Rightarrow \frac{x^2}{5} + \frac{y^2}{10/3} = 1$$

here, $a^2 = 5 \Rightarrow a = \sqrt{5}$

$$b^2 = 10/3 \Rightarrow b = \sqrt{\frac{10}{3}}$$

Area = πab

$$= \pi \times \sqrt{5} \times \sqrt{\frac{10}{3}} = 5\sqrt{\frac{2}{3}} \pi \text{ sq. unit}$$

115. (C) Differential equation

$$\frac{dy}{dx} + y \cot x = \sin x$$

On comparing with general equation
P = cotx, Q = sinx

I.F. = $e^{\int P dx}$

$$= e^{\int \cot x dx}$$

$$= e^{\ln \sin x} = \sin x$$

Solution of differential equation

$$\Rightarrow y \times \text{I.F.} = \int Q \times \text{I.F.} dx$$

$$\Rightarrow y \cdot \sin x = \int \sin x \cdot \sin x dx$$

$$\Rightarrow y \cdot \sin x = \int \frac{1 - \cos 2x}{2} dx$$

$$\Rightarrow y \cdot \sin x = \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] + c$$

$$\Rightarrow y \cdot \sin x = \frac{1}{4} (2x - \sin 2x) + c$$

$$\Rightarrow 4y \cdot \sin x = 2x - \sin 2x + c$$

116. (D) $f(x) = \frac{\sqrt{\log_e(36 - 2x - x^2)}}{x - 1}$

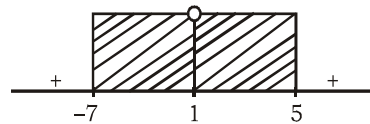
Now, $\log_e(36 - 2x - x^2) \geq 0$

$$\Rightarrow 36 - 2x - x^2 \geq 1$$

$$\Rightarrow x^2 + 2x + 1 - 36 \leq 0$$

$$\Rightarrow x^2 + 2x - 35 \leq 0$$

$$\Rightarrow (x + 7)(x - 5) \leq 0$$



Domain = $[-7, 5] - \{1\}$

117. (B) $\sin(-3180) = -\sin 3180$

$$= -\sin(360 \times 9 - 60)$$

$$= -(-\sin 60) = \frac{\sqrt{3}}{2}$$

118. (D) $A = \begin{bmatrix} -7 & 12 \\ -2 & 3 \end{bmatrix}$

$|A| = -21 + 24 = 3$

Co-factors of A -

$$C_{11} = (-1)^{1+1}(3) = 3, C_{12} = (-1)^{1+2}(-2) = 2$$

$$C_{21} = (-1)^{2+1}(12) = -12, C_{22} = (-1)^{2+2}(-7) = -7$$

$$C = \begin{bmatrix} 3 & 2 \\ -12 & -7 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} 3 & -12 \\ 2 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & -12 \\ 2 & -7 \end{bmatrix} \Rightarrow 3A^{-1} = \begin{bmatrix} 3 & -12 \\ 2 & -7 \end{bmatrix}$$

Now, $A + 2A^{-1}$

$$\Rightarrow \begin{bmatrix} -7 & 12 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -12 \\ 2 & -7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 0 \\ 0 & -4 \end{bmatrix}$$

119. (C)
$$\frac{\cos 9x - \cos 5x}{\sin 9x - 2\sin 7x + \sin 5x}$$

$$\Rightarrow \frac{2\sin 7x \cdot \sin(-2x)}{\sin 9x + \sin 5x - 2\sin 7x}$$

$$\Rightarrow \frac{-2\sin 2x \cdot \sin 7x}{2\sin 7x \cdot \cos 2x - 2\sin 7x}$$

$$\Rightarrow \frac{-2\sin 2x \cdot \sin 7x}{2\sin 7x(\cos 2x - 1)}$$

$$\Rightarrow \frac{-\sin 2x}{-(1 - \cos 2x)}$$

$$\Rightarrow \frac{2\sin x \cdot \cos x}{2\sin^2 x} = \cot x$$

120. (D) $\lim_{x \rightarrow 0} \frac{7^x - 1}{x}$ $\left[\frac{0}{0} \right]$ form

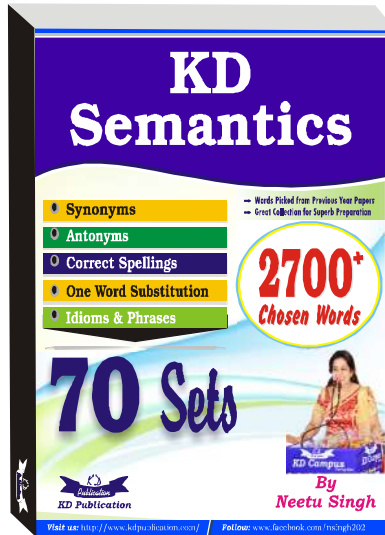
by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{7^x \log 7 - 0}{1}$$

$$\Rightarrow 7^0 \log 7 = \log 7$$

NDA (MATHS) MOCK TEST - 162 (Answer Key)

1. (C)	21. (D)	41. (B)	61. (D)	81. (C)	101. (B)
2. (C)	22. (B)	42. (A)	62. (B)	82. (A)	102. (C)
3. (D)	23. (B)	43. (B)	63. (D)	83. (A)	103. (D)
4. (C)	24. (B)	44. (C)	64. (B)	84. (C)	104. (C)
5. (B)	25. (C)	45. (D)	65. (A)	85. (C)	105. (C)
6. (C)	26. (B)	46. (C)	66. (C)	86. (B)	106. (B)
7. (C)	27. (D)	47. (D)	67. (A)	87. (A)	107. (B)
8. (C)	28. (A)	48. (C)	68. (A)	88. (A)	108. (B)
9. (B)	29. (B)	49. (B)	69. (A)	89. (C)	109. (A)
10. (C)	30. (C)	50. (B)	70. (B)	90. (B)	110. (C)
11. (A)	31. (C)	51. (C)	71. (D)	91. (B)	111. (B)
12. (C)	32. (B)	52. (B)	72. (C)	92. (A)	112. (C)
13. (B)	33. (D)	53. (C)	73. (B)	93. (D)	113. (D)
14. (A)	34. (B)	54. (B)	74. (B)	94. (C)	114. (B)
15. (D)	35. (B)	55. (D)	75. (D)	95. (D)	115. (C)
16. (C)	36. (B)	56. (A)	76. (A)	96. (A)	116. (D)
17. (B)	37. (C)	57. (D)	77. (C)	97. (C)	117. (B)
18. (C)	38. (A)	58. (C)	78. (D)	98. (B)	118. (D)
19. (C)	39. (D)	59. (B)	79. (A)	99. (D)	119. (C)
20. (D)	40. (C)	60. (C)	80. (B)	100. (D)	120. (D)



Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777