NDA MATHS MOCK TEST - 166 (SOLUTION)

1. (C) $\dfrac{\sin 14°.\cos 196° - \sin 16°.\sin 104°}{\cos 88°.\cos 88° + \cos 178°.\sin 268°}$

$\Rightarrow \dfrac{\sin 14°.\cos(180°+16°) - \sin 16°.\sin(90°+14°)}{\cos 88°.\cos(90°-2°) + \cos(180°-2°).\sin(180°+88°)}$

$\Rightarrow \dfrac{\sin 14°.(-\cos 16°) - \sin 16°.\cos 14°}{\cos 88°.\sin 2° + (-\cos 2°).(-\sin 88°)}$

$\Rightarrow \dfrac{-\sin 14°.\cos 16° - \cos 14°.\sin 16°}{\cos 88°.\sin 2° + \cos 2°.\sin 88°}$

$\Rightarrow \dfrac{-\sin(14°+16°)}{\sin(2°+88°)}$

[$\because$ sin(A + B) = sinA.cosB + cosA.sinB]

$\Rightarrow -\dfrac{\sin 30°}{\sin 90°}$

$\Rightarrow -\dfrac{1/2}{1} = -\dfrac{1}{2}$

2. (A) tan36° = tan(45° – 9°)

$\tan 36° = \dfrac{\tan 45° - \tan 9°}{1+\tan 45°.\tan 9°}$

$\left[\because \tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A.\tan B}\right]$

$\tan 36° = \dfrac{1-\tan 9°}{1+\tan 9°}$

$\tan 36° = \dfrac{1-\dfrac{\sin 9°}{\cos 9°}}{1+\dfrac{\sin 9°}{\cos 9°}}$

$\tan 36° = \dfrac{\cos 9° - \sin 9°}{\cos 9° + \sin 9°}$

3. (A) $\cos A = \dfrac{8}{15}$

$\Rightarrow 1 - 2\sin^2\dfrac{A}{2} = \dfrac{8}{15}$

$\Rightarrow 2\sin^2\dfrac{A}{2} = \dfrac{7}{15}$

$\Rightarrow \sin^2\dfrac{A}{2} = \dfrac{7}{16}$

Now, $\sin\dfrac{A}{2}.\sin\dfrac{3A}{2}$

$\Rightarrow \sin\dfrac{A}{2}\left(3\sin\dfrac{A}{2} - 4\sin^3\dfrac{A}{2}\right)$

[$\because$ sin3θ = 3sinθ – 4sin³θ]

$\Rightarrow 3\sin^2\dfrac{A}{2} - 4\sin^4\dfrac{A}{2}$

$\Rightarrow 3 \times \dfrac{7}{16} - 4 \times \left(\dfrac{7}{16}\right)^2$

$\Rightarrow \dfrac{21}{16} - \dfrac{49}{64} = \dfrac{35}{64}$

4. (D) $(1+\tan\alpha.\tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha.\sec^2\beta$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + 2\tan\alpha.\tan\beta + \tan^2\alpha + \tan^2\beta - \tan\alpha.\tan\beta - \sec^2\alpha.\sec^2\beta$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + \tan^2\alpha + \tan^2\beta - (1+\tan^2\alpha)(1+\tan^2\beta)$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + \tan^2\alpha + \tan^2\beta - 1 - \tan^2\alpha - \tan^2\beta - \tan^2\alpha.\tan^2\beta$

$\Rightarrow 0$

5. (D) sin3θ = cos2θ

$\Rightarrow$ sin3θ = sin(90 – 2θ)

$\Rightarrow$ 3θ = 90 – 2θ

$\Rightarrow$ 5θ = 90 $\Rightarrow$ θ = 18°

Hence cos4θ $\Rightarrow$ cos(4×18)

$\Rightarrow \cos 72 = \dfrac{\sqrt{5}-1}{4}$

6. (C) $\dfrac{2\cot\theta}{1+\cot^2\theta} \Rightarrow \dfrac{\dfrac{2\cos\theta}{\sin\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}}$

$\Rightarrow \dfrac{2\sin\theta.\cos\theta}{\cos^2\theta+\sin^2\theta} = \dfrac{\sin 2\theta}{1} = \sin 2\theta$

7. (D) sin2A + sin2B – sin2C

$\Rightarrow 2\sin\dfrac{2A+2B}{2}.\cos\dfrac{2A-2B}{2} - \sin 2C$

$\left[\because \sin C + \sin D = 2\sin\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}\right]$

$\Rightarrow$ 2 sin(A + B).cos(A – B) – sin2C

$\Rightarrow$ 2 sin(180 – C).cos(A – B) – 2sinC.cosC

$\Rightarrow$ 2sinC.cos(A – B) + 2sinC.cos(A+B)

[$\because$ A + B + C = 180]

$\Rightarrow$ 2sinC[cos(A – B) + cos(A + B)]

$\Rightarrow 2\sin C.2\cos\dfrac{A-B+A+B}{2}.\cos\dfrac{A-B-A-B}{2}$

$\Rightarrow$ 4sinC.cosA.cosB

$\Rightarrow$ 4cosA.cosB.sinC

8. (C) Given that $\sin x = \frac{3}{\sqrt{10}}$, $\sin y = \frac{1}{\sqrt{5}}$

$\cos x = \frac{1}{\sqrt{10}}$ and $\cos y = \frac{2}{\sqrt{5}}$

Now, $\sin(x - y) = \sin x.\cos y - \cos x.\sin y$

$\Rightarrow \sin(x - y) = \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}}$

$\Rightarrow \sin(x - y) = \frac{5}{\sqrt{50}}$

$\Rightarrow \sin(x - y) = \frac{5}{5\sqrt{2}}$

$\Rightarrow \sin(x - y) = \frac{1}{\sqrt{2}} \Rightarrow x + y = \frac{\pi}{4}$

9. (B) $\frac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$

$\Rightarrow \frac{2\cos\frac{4x+2x}{2}.\sin\frac{4x-2x}{2}}{2\cos\frac{4x+2x}{2}.\cos\frac{4x-2x}{2}}$

$\Rightarrow \frac{\sin x}{\cos x} = \tan x$

10. (B) $\sin 75° + \cos 75°$

$\Rightarrow \sin(30° + 45°) + \cos(30° + 45°)$

$\Rightarrow \sin 30°.\cos 45° + \cos 30°.\sin 45° + \cos 30°.\cos 45° - \sin 30°.\sin 45°$

$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{2\sqrt{3}}{2\sqrt{2}} = \sqrt{\frac{3}{2}}$

11. (A) Let angles are x, x, $2x$.

$x + x + 2x = 180$

$\Rightarrow 4x = 180 \Rightarrow x = 45$

Angles = 45°, 45°, 90°

Sine Rule

$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 45} = \frac{b}{\sin 45} = \frac{c}{\sin 90}$

$\Rightarrow \frac{a}{1/\sqrt{2}} = \frac{b}{1/\sqrt{2}} = \frac{c}{1}$

Hence $a : b : c = 1 : 1 : \sqrt{2}$

12. (D) Given that $a = 3$, $b = 2$ and $\sin A = \frac{3}{4}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Now, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{3/4}{3} = \frac{\sin B}{2}$

$\Rightarrow \frac{1}{4} = \frac{\sin B}{2}$

$\Rightarrow \sin B = \frac{1}{2} \Rightarrow B = \frac{\pi}{6}$

13. (A) Given that $a + b - 2c = 0 \Rightarrow a + b = 2c$

$s = \frac{a+b+c}{2}$

$s = \frac{2c+c}{2} = \frac{3c}{2}$

Now, $\cot\frac{A}{2}.\cot\frac{B}{2}$

$\Rightarrow \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}.\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$\Rightarrow \sqrt{\frac{s^2(s-a)(s-b)}{(s-b)(s-c)(s-a)(s-c)}}$

$\Rightarrow \frac{s}{s-c} \Rightarrow \frac{\frac{3c}{2}}{\frac{3c}{2} - c} \Rightarrow \frac{\frac{3}{2}c}{\frac{c}{2}} = 3$

14. (B) **Statement 1**

$\sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{180 - C}{2}\right)$

$\Rightarrow \sin\left(\frac{A+B}{2}\right) = \sin\left(90 - \frac{C}{2}\right) = \cos\frac{C}{2}$

Statement 1 is correct.

Statement 2

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{180 - C}{2}\right)$

$\Rightarrow \tan\left(\frac{A+B}{2}\right) = \tan\left(90 - \frac{C}{2}\right) = \cot\frac{C}{2}$

Statement 2 is correct.

Statement 3

sin(A + B) = sin(180 – C)

sin(A + B) = sinC

Statement 3 is incorrect.

Statement 4

tan(A + B) = tan(180 – C)

tan(A + B) = – tanC

Statement 4 is incorrect.

Hence statement 1 and 2 are correct.

15. (B) $\sin^{-1}\frac{3}{5} + \sec^{-1}\frac{5}{3} - \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5} - \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{2} - \frac{\pi}{2} = 0 \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

16. (A) $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-p^2}{1+p^2} = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = \tan^{-1}\frac{2x}{1-x^2}$

$\left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$

$\Rightarrow 2[\tan^{-1}p - \tan^{-1}q] = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}\frac{p-q}{1+pq} = 2\tan^{-1}x$

On comparing

$x = \frac{p-q}{1+pq}$

17. (A) $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$

$\Rightarrow \sin^{-1}\left[\sin\left(2\pi - \frac{\pi}{3}\right)\right]$

$\Rightarrow \sin^{-1}\left[-\sin\frac{\pi}{3}\right]$

$\Rightarrow \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$

18. (C) $\cos^{-1}\frac{4}{5} + \cot^{-1}7$

$\Rightarrow \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} \quad \left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow \tan^{-1}\left[\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right] \quad \Rightarrow \tan^{-1}\left[\frac{21+4}{28-3}\right]$

$\Rightarrow \tan^{-1}\left(\frac{25}{25}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

19. (D) $\cos(2\sin^{-1}0.6) \Rightarrow \cos\left(2\sin^{-1}\frac{3}{5}\right)$

$\Rightarrow \cos\left(2\tan^{-1}\frac{3}{4}\right)\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)\right]$

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{3/2}{7/16}\right)\right]$

$\Rightarrow \cos\left[\tan^{-1}\left(\frac{24}{7}\right)\right]$

$\Rightarrow \cos\left[\cos^{-1}\left(\frac{7}{25}\right)\right] = \frac{7}{25} = 0.28$

20. (B) **Statement 1**

$\tan^{-1}(\tan\theta) = \theta\ ;\ \theta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Statement 1 is incorrect.

Statement 2

$\Rightarrow \sin^{-1}\frac{1}{3} - \sin^{-1}\frac{1}{5}$

$\Rightarrow \sin^{-1}\left[\frac{1}{3}\sqrt{1-\left(\frac{1}{5}\right)^2} - \frac{1}{5}\sqrt{1-\left(\frac{1}{3}\right)^2}\right]$

$\Rightarrow \left[\because \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]\right]$

$\Rightarrow \sin^{-1}\left[\frac{1}{3}\times\frac{\sqrt{24}}{5} - \frac{1}{5}\times\frac{\sqrt{8}}{3}\right]$

$\Rightarrow \sin^{-1}\left[\frac{2\sqrt{6}}{15} - \frac{2\sqrt{2}}{15}\right] \Rightarrow \sin^{-1}\left[\frac{2\sqrt{6}-2\sqrt{2}}{15}\right]$

$\Rightarrow \sin\left[\frac{2\sqrt{2}(\sqrt{3}-1)}{15}\right]$

Statement 2 is correct.

21. (A) **Statement 1**

$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

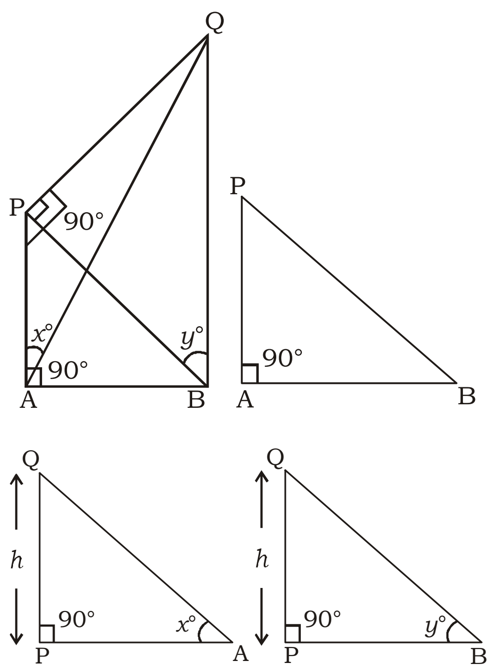
Statement 1 is correct.

Statement 2

$\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, when $x = y$

Statement 2 is incorrect.

22. (B) Given that AB = z and height of tower (PQ) = h m



$AP^2 + AB^2 = BP^2$

$AB^2 = BP^2 - AP^2$

$z^2 = BP^2 - AP^2$ ...(i)

In ΔAPQ

$\tan A = \frac{PQ}{AP}$

$\Rightarrow \tan x° = \frac{h}{AP} \Rightarrow AP = h.\cot x°$

In ΔBPQ

$\tan B = \frac{PQ}{BP}$

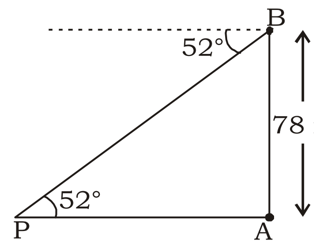
$\Rightarrow \tan y = \frac{h}{BP} \Rightarrow BP = h.\cot y°$

from eq(i)

$z^2 = (h \cot y)^2 - (h \cot x)^2$

$z^2 = h^2(\cot^2 y - \cot^2 x)$

23. (D)



Let length of the bridge (AP) = x m

In ΔABP

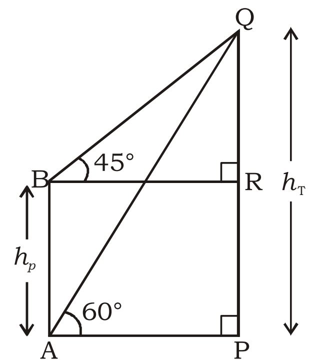
$\Rightarrow \tan 52° = \frac{78}{x}$

$\Rightarrow x = 78 \cot 52°$

$\Rightarrow x = 78 \tan 38°$

Hence length of the bridge = 78 tan38° m

24. (C)



In ΔAPQ

$\tan 60° = \frac{PQ}{AP}$

$\Rightarrow \sqrt{3} = \frac{h_T}{AP} \Rightarrow AP = \frac{h_T}{\sqrt{3}} = BR$

In ΔBRQ

$\tan 45° = \frac{QR}{BR}$

$\Rightarrow 1 = \frac{\sqrt{3}\left(h_T - h_p\right)}{h_T}$

$\Rightarrow h_T = \frac{\sqrt{3}h_p}{\sqrt{3} - 1}$ ...(i)

Statement 1

L.H.S. $= \frac{2h_p h_T}{3 + \sqrt{3}}$

$= \frac{2h_p}{\sqrt{3}(\sqrt{3} + 1)} \times \frac{\sqrt{3}h_p}{(\sqrt{3} - 1)}$

$= \frac{2h_p^2}{3 - 1} = h_p^2 =$ R.H.S.

Statement 1 is correct.

Statement 2

L.H.S. = $\frac{h_T - h_p}{\sqrt{3}+1}$

$= \frac{\frac{\sqrt{3}h_p}{\sqrt{3}-1} - h_p}{\sqrt{3}+1}$

$= \frac{h_p(\sqrt{3}-\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{h_p}{2}$ = R.H.S.

Statement 2 is correct.

Statement 3

L.H.S. = $\frac{2(h_p + h_T)}{h_p}$

$= \frac{2\left(h_p + \frac{\sqrt{3}h_p}{\sqrt{3}-1}\right)}{h_p}$

$= \frac{2h_p\left(\frac{\sqrt{3}-1+\sqrt{3}}{\sqrt{3}-1}\right)}{h_p}$

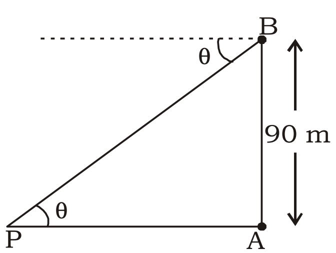
$= \frac{2(2\sqrt{3}-1)}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{2(6-\sqrt{3}+2\sqrt{3}-1)}{3-1}$

$= 5 + \sqrt{3} \neq$ R.H.S.

Statement 3 is incorrect.

25. (B)



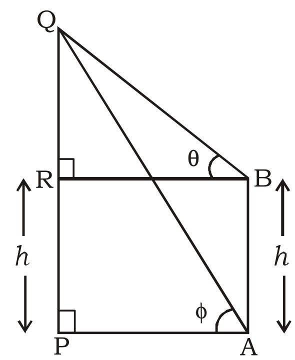
Let $\theta = \tan^{-1}\frac{3}{4} \Rightarrow \tan\theta = \frac{3}{4}$

In ΔABP

$\tan\theta = \frac{AB}{AP} \Rightarrow \frac{3}{4} = \frac{90}{AP} \Rightarrow AP = 120$ m

The distance between the boat and the lighthouse = 120 m

26. (C)



In ΔQRB

$\tan\theta = \frac{QR}{RB}$ …(i)

In ΔAPQ

$\tan\phi = \frac{PQ}{AP}$

$\Rightarrow \tan\phi = \frac{h + QR}{RB}$ …(ii)

from eq(i) and eq(ii)

$\frac{\tan\theta}{\tan\phi} = \frac{QR}{h+QR}$

$\Rightarrow \frac{\cot\phi}{\cot\theta} = \frac{QR}{h+QR}$

$\Rightarrow h.\cot\phi + QR.\cot\phi = QR.\cot\theta$

$\Rightarrow h.\cot\phi = QR(\cot\theta - \cot\phi)$

$\Rightarrow QR = \frac{h.\cot\phi}{\cot\theta - \cot\phi}$

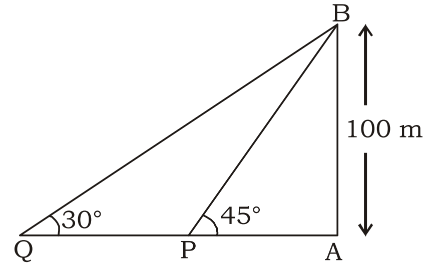
Height of the hill = h + QR

$= h + \frac{h.\cot\phi}{\cot\theta - \cot\phi}$

$= h\left[\frac{\cot\theta - \cot\phi + \cot\phi}{\cot\theta - \cot\phi}\right]$

$= \frac{h.\cot\theta}{\cot\theta - \cot\phi}$

27. (C)



Let PQ = x m

In ΔABP

$\tan 45° = \frac{AB}{AP}$

$\Rightarrow 1 = \frac{100}{AP} \Rightarrow AP = 100$

In ΔABQ

$\tan 30° = \frac{AB}{AQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{100+x} \Rightarrow x = 100(\sqrt{3}-1)$

Time taken by boat from P to Q = 4 minutes = $\frac{4}{60}$ hrs

Speed of the boat $= \frac{100(\sqrt{3}-1)\times 60}{4}$

$= 1500(\sqrt{3}-1)$

28. (D) Equation $x^2 + \alpha x - 2\beta = 0$
Roots are α and β,
then $\alpha + \beta = -\alpha$
$\Rightarrow 2\alpha + \beta = 0$ ...(i)
$\alpha.\beta = -2\beta \Rightarrow \alpha = -2$
from eq(i)
$2(-2) + \beta = 0 \Rightarrow \beta = 4$
Another equation $= -x^2 + \alpha x + \beta$
$= -x^2 - 2x + 4$
$= -(x+1)^2 + 3$
Greatest value of the equation = 3

29. (B) Equation $6x^2 - 5 = 0$
Roots are cosα and cosβ,
then $\cos\alpha + \cos\beta = 0$
and $\cos\alpha.\cos\beta = \frac{-5}{6}$
$\Rightarrow \sec\alpha.\sec\beta = \frac{-6}{5}$

30. (B) Let α_1, β_1 are roots of $ax^2 + bx + c = 0$ and α_2, β_2 are roots of $px^2 + qx + r = 0$.
$D_1 = b^2 - 4ac$, $D_2 = q^2 - 4pr$
and $\alpha_1 + \beta_1 = \frac{-b}{a}$, $\alpha_1.\beta_1 = \frac{c}{a}$
$\alpha_2 + \beta_2 = \frac{-q}{p}$, $\alpha_2.\beta_2 = \frac{r}{p}$
Given that $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$
By Componendo and Dividendo Rule
$\frac{\alpha_1+\beta_1}{\alpha_1-\beta_1} = \frac{\alpha_2+\beta_2}{\alpha_2-\beta_2}$
$\Rightarrow \frac{(\alpha_1+\beta_1)^2}{(\alpha_1+\beta_1)^2 - 4\alpha_1.\beta_1} = \frac{(\alpha_2+\beta_2)^2}{(\alpha_2+\beta_2)^2 - 4\alpha_2.\beta_2}$
$\Rightarrow \frac{\frac{b^2}{a^2}}{\frac{b^2}{a^2} - 4\frac{c}{a}} = \frac{\frac{q^2}{p^2}}{\frac{q^2}{p^2} - 4\frac{r}{p}}$
$\Rightarrow \frac{b^2}{b^2 - 4ac} = \frac{q^2}{q^2 - 4pr}$
$\Rightarrow \frac{b^2}{D_1} = \frac{q^2}{D_2}$
Hence $\frac{D_1}{D_2} = \frac{b^2}{q^2}$

31. (D) Equation $|2 - x| + x^2 = 6$
Now, $2 - x + x^2 = 6$
$\Rightarrow x^2 - x - 4 = 0$
$b^2 - 4ac = \sqrt{(-1)^2 - 4\times(-4)} = \sqrt{17}$
Roots are irrational.
and $-(2 - x) + x^2 = 6$
$\Rightarrow x^2 + x - 8 = 0$
$b^2 - 4ac = \sqrt{(-1)^2 - 4\times(-8)} = \sqrt{33}$
Roots are irrational.
Hence equation has two irrational roots.

32. (A) Equation $|x-4|^2 + 2|x-4| - 3 = 0$
Let $x - 4 = y$
$y^2 + 2|y| - 3 = 0$
(i) when $y \geq 0$
$y^2 + 2y - 3 = 0$
$\Rightarrow (y+3)(y-1) = 0$
$\Rightarrow y = -3, 1$
Hence $y = 1$
$x - 4 = 1 \Rightarrow x = \mathbf{5}$
(ii) When $y < 0$
$y^2 - 2y - 3 = 0$
$\Rightarrow (y-3)(y+1) = 0$
$\Rightarrow y = 3, -1$
Hence $y = -1$
$x - 4 = -1 \Rightarrow x = \mathbf{3}$
Hence sum of all real roots = 5 + 3 = 8

33. (A) Given that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Now, $A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
$\Rightarrow A^2 = \begin{bmatrix} 0\times0+1\times1 & 0\times1+1\times0 \\ 1\times0+0\times1 & 1\times1+0\times0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = I$

Hence A is an Involuntory matrix.

34. (A)

35. (B) A is an orthogonal matrix, then $A' = A^{-1}$

36. (A)

37. (B) Let $A = \begin{bmatrix} 3 & 6 \\ -8 & x \end{bmatrix}$

A is non-invertible, then $|A| = 0$

$\Rightarrow \begin{vmatrix} 3 & 6 \\ -8 & x \end{vmatrix} = 0$

$\Rightarrow 3x + 48 = 0 \Rightarrow x = -16$

38. (A) $u = ab^{p-1}$

$\ln u = \ln a + (p-1)\ln b$ ...(i)

Similarly

$\ln v = \ln a + (q-1)\ln b$ ...(ii)

and $\ln w = \ln a + (r-1)\ln b$ ...(iii)

from eq(i) and eq(ii)

$\ln v - \ln u = (q-p)\ln b$ ...(iv)

from eq(i) and eq(iii)

$\ln w - \ln u = (r-p)\ln b$ ...(v)

Now, $\begin{vmatrix} \ln u & p & 1 \\ \ln v & q & 1 \\ \ln w & r & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} \ln u & p & 1 \\ \ln v - \ln u & q-p & 0 \\ \ln w - \ln u & r-p & 0 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} \ln u & p & 1 \\ (q-p)\ln b & q-p & 0 \\ (r-p)\ln b & r-p & 0 \end{vmatrix}$

[from eq(iv) and eq(v)]

$\Rightarrow (q-p)(r-p)\begin{vmatrix} \ln u & p & 1 \\ \ln b & 1 & 0 \\ \ln b & 1 & 0 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$= (q-p)(r-p)\begin{vmatrix} \ln u & p & 1 \\ \ln b & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$

39. (D) Let $f(x) = \dfrac{x}{[x]}$

L.H.L. $= \lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2-h)$

$= \lim\limits_{h \to 0} \dfrac{2-h}{[2-h]}$

$= \lim\limits_{h \to 0} \dfrac{2-h}{1} = 2$

R.H.L. $= \lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} f(2+h)$

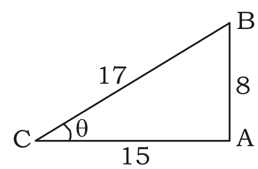
$= \lim\limits_{h \to 0} \dfrac{2+h}{[2+h]}$

$= \lim\limits_{h \to 0} \dfrac{2+h}{2} = 1$

L.H.L. $\neq$ R.H.L.

Hence limit does not exist.

40. (D)



$\sec\theta = \dfrac{-17}{15}$ and $\cos\theta = \dfrac{-15}{17}$

Now, $5\sin\theta - 3\cos\theta$

$\Rightarrow 5 \times \left(\dfrac{-8}{17}\right) - 3\left(\dfrac{-15}{17}\right)$

$\Rightarrow \dfrac{-40}{17} + \dfrac{45}{17} = \dfrac{5}{17}$

41. (D) Let Locus of a point = (h, k, l)

A.T.Q.,

$\sqrt{(h+1)^2 + (k-2)^2 + (l+3)^2}$

$= \sqrt{(h+2)^2 + (k-4)^2 + (l+5)^2}$

$\Rightarrow h^2 + 1 + 2h + k^2 + 4 - 4k + l^2 + 9 + 6l$

$= h^2 + 4 + 4h + k^2 + 16 - 8k + l^2 + 25 + 10l$

On solving

$2h - 4k + 4l + 31 = 0$

Locus of a point

$2x - 4y + 4z + 31 = 0$

42. (B) Digits are 2, 3, 5, 7, 8, 9.

$n(S) = {}^6C_3 = 20$

E = {(2, 3, 8), (2, 7, 8), (2, 8, 9), (3, 5, 9), (3, 7, 9)}

$n(E) = 5$

The required Probability $= \dfrac{n(E)}{n(S)} = \dfrac{5}{20} = \dfrac{1}{4}$

43. (A) In a year = 365 days

= 52 weeks and 1 days

The required Probability $= \dfrac{1}{7}$

44. (C) $\begin{bmatrix} x & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 6 & -2 \\ 0 & 2 & 4 \\ -2 & 1 & 2 \end{bmatrix}\begin{bmatrix} -9 \\ 3 \\ 4 \end{bmatrix} = [0]$

$\Rightarrow \begin{bmatrix} x & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 22 \\ 29 \end{bmatrix} = [0]$

$\Rightarrow [x - 22 + 58] = [0]$

$\Rightarrow x + 36 = 0 \Rightarrow x = -36$

45. (C) $I = \int e^x[x^2.\ln x + 2x\ln x + x]dx$

$I = e^x.x^2 \ln x + c\left[\because \int e^x[f(x) + f'(x)]dx = e^x.f(x) + c\right]$

$I = x^2.e^x.\ln x + c$

46. (A) We know that

$(1+x)^n = C_1 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$ ...(i)

$x \to \frac{1}{x}$

$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots\ldots + \frac{C_n}{x^n}$ ...(ii)

from eq(i) and eq(ii)

Coefficient of x^0 in $(1+x)^n\left(1+\frac{1}{x}\right)^n$ =

$C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$ = Coefficient of x^n in $\frac{(1+x)^{2n}}{x^n}$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$ = Coefficient of x^0 in $(1+x)^{2n}$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = \frac{(2n)!}{n!n!}$

47. (C) We know that

$\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Now, $(-1-i\sqrt{3})^{72} = 2^{72}\left(\frac{-1-i\sqrt{3}}{2}\right)^{72}$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72}(\omega^2)^{72}$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72} \times 1$ $[\because \omega^3 = 1]$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72}$

48. (B) $\frac{\sec 34}{\sec 112} + \frac{\text{cosec}\, 34}{\text{cosec}\, 112}$

$\Rightarrow \frac{\cos 112}{\cos 34} + \frac{\sin 112}{\sin 34}$

$\Rightarrow \frac{\sin 34.\cos 112 + \cos 34.\sin 112}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin(34+112)}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin 146}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin(180-34)}{\sin 34.\cos 34} = \frac{\sin 34}{\sin 34.\cos 34} = \sec 34$

49. (C) Given that

$a = 2,\ b = \frac{7}{2}$

$f(x) = x^2 + x - 2$

$f'(x) = 2x + 1$

$f'(c) = 2c + 1$

$f(a) \Rightarrow f(2) = 4,\ f(b) \Rightarrow f\left(\frac{7}{2}\right) = \frac{55}{4}$

Now, $f'(c) = \frac{f(b) - f(a)}{b - a}$

$\Rightarrow 2c + 1 = \frac{\frac{55}{4} - 4}{\frac{7}{2} - 2}$

$\Rightarrow 2c + 1 = \frac{39/4}{3/2}$

$\Rightarrow 2c + 1 = \frac{13}{2} \Rightarrow c = \frac{11}{4}$

50. (C)

51. (D) Sphere $x^2 + y^2 + z^2 - 4x + 6y + 16z - 4 = 0$

$u = -2,\ v = 3,\ w = 8,\ d = -4$

$r = \sqrt{u^2 + v^2 + w^2 - d}$

$r = \sqrt{(-2)^2 + 3^2 + 8^2 + 4} = 9$

Diameter = $2r$ = 18 unit

52. (C) Two circles

$x^2 + y^2 + 4x - 8y + 16 = 0$

and $x^2 + y^2 - 3x + 4y + \lambda = 0$

Condition of orthogonality

$2gg' + 2ff' = c + c'$

$\Rightarrow 2\times2\times\left(\frac{-3}{2}\right) + 2\times(-4)\times2 = 16 + \lambda$

$\Rightarrow -6 - 16 = 16 + \lambda \Rightarrow \lambda = -38$

53. (C) Given that $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

Now, $[(\vec{a} + 2\vec{b}) \times (\vec{b} - 3\vec{a})].\vec{a}$

$\Rightarrow [(\vec{a} \times \vec{b}) + 2(\vec{b} \times \vec{b}) - 3(\vec{a} \times \vec{a}) - 6(\vec{b} \times \vec{a})].\vec{a}$

$\Rightarrow [(\vec{a} \times \vec{b}) + 6(\vec{a} \times \vec{b})].\vec{a}$

$\Rightarrow 7(\vec{a} \times \vec{b}).\vec{a} = 0$

54. (B) Series 1.3 + 2.4 + 3.5 +......+ $n(n + 2)$

$T_n = n(n + 2)$

$T_n = n^2 + 2n$

Now, $S_n = \sum T_n$

$\Rightarrow S_n = \sum (n^2 + 2n)$

$\Rightarrow S_n = \sum n^2 + 2\sum n$

$\Rightarrow S_n = \frac{n}{6}(n+1)(2n+1) + 2\times\frac{n(n+1)}{2}$

$\Rightarrow S_n = \frac{n(n+1)}{6}[2n + 1 + 6]$

$\Rightarrow S_n = \frac{n(n+1)}{6} \times (2n + 7)$

$\Rightarrow S_n = \frac{n(n+1)(2n+7)}{6}$

55. (A) $\begin{vmatrix} x+2 & x+3 & x+5 \\ x+7 & x+9 & x+12 \\ x+14 & x+17 & x+21 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} x+2 & x+3 & x+5 \\ 5 & 6 & 7 \\ 12 & 14 & 16 \end{vmatrix}$

$R_3 \to R_3 - 2R_2$

$\Rightarrow \begin{vmatrix} x+2 & x+3 & x+5 \\ 5 & 6 & 7 \\ 2 & 2 & 2 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} x+2 & 1 & 3 \\ 5 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix}$

$\Rightarrow (x + 2)\times 0 - 1(0 - 4) + 3(-2)$

$= 4 - 6 = -2$

56. (B) The required no. of hand shakes in party

$= {}^{14}C_2 = 91$

57. (D) $\theta = \left|\frac{11M - 60H}{2}\right|$

Time = 5 : 20

$\theta = \left|\frac{11\times 20 - 60\times 5}{2}\right|$

$\theta = 40°$

$\theta = 40\times\frac{\pi}{180} = \frac{2\pi}{9}$

58. (B) We know that

Mode = 3 Median – 2 Mean

Mode = 3×27 – 2 × 37

Mode = 81 – 74 = 7

59. (C) $z = \frac{3+2i}{2-3i} - \frac{2-3i}{3+2i}$

$z = \frac{(3+2i)(2+3i)}{(2-3i)(2+3i)} - \frac{(2-3i)(3-2i)}{(3+2i)(3-2i)}$

$z = \frac{13i}{4-9i^2} - \frac{-13i}{9-4i^2}$

$z = \frac{13i}{13} + \frac{13i}{13}$

$z = i + i = 2i$ and $\bar{z} = -2i$

Now, $z^2 + z\bar{z} = z(z + \bar{z})$

$\Rightarrow z^2 + z\bar{z} = 2i(2i - 2i) = 0$

60. (C) Digits 0, 1, 3, 5, 7, 8

| 4 | 6 | 6 | 6 | = 4×6×6×6 = 864

(3,5,7,8)

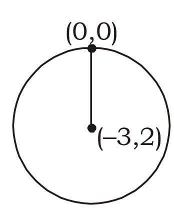
61. (C) Given that $b_{xy} = \frac{-13}{8}$ and $b_{yx} = \frac{-2}{13}$

Now, $r = \sqrt{b_{xy} \times b_{yx}}$

$\Rightarrow r = \sqrt{\left(\frac{-13}{8}\right)\times\left(\frac{-2}{13}\right)}$

$\Rightarrow r = \sqrt{\frac{1}{4}} = \frac{-1}{2}$

62. (C)



Equation of circle

$x^2 + y^2 + 6x - 4y = 0$

$\Rightarrow (x+3)^2 - 9 + (y-2)^2 - 4 = 0$

$\Rightarrow (x+3)^2 + (y-2)^2 = 13$

Equation of diameter

$y - 0 = \frac{2-0}{-3-0}(x-0)$

$\Rightarrow 2x + 3y = 0$

63. (D) Given that $\vec{a} = 6\hat{i} - 2\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 2\hat{j} - 5\hat{k}$

Now, $(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})$

$\Rightarrow 2(\vec{a}\times\vec{a}) + 4(\vec{b}\times\vec{a}) - (\vec{a}\times\vec{b}) - 2(\vec{b}\times\vec{b})$

$\Rightarrow 0 - 4(\vec{a}\times\vec{b}) - (\vec{a}\times\vec{b}) - 0$

$\Rightarrow -5(\vec{a}\times\vec{b}) = -5(16\hat{i} + 18\hat{j} + 20\hat{k})$

64. (D) In ΔABC, $\frac{1}{b+c} + \frac{1}{a+b} = \frac{3}{a+b+c}$

$\Rightarrow \frac{a+b+b+c}{(b+c)(a+b)} = \frac{3}{a+b+c}$

$\Rightarrow \frac{a+2b+c}{(b+c)(a+b)} = \frac{3}{a+b+c}$

$\Rightarrow (a+2b+c)(a+b+c) = 3(b+c)(a+b)$

$\Rightarrow a^2 + ab + ac + 2ab + 2b^2 + 2bc + ac + bc + c^2 = 3(ab + b^2 + ca + bc)$

$\Rightarrow a^2 + 2b^2 + c^2 + 3ab + 2ac + 3bc = 3ab + 3b^2 + 3ca + 3bc$

$\Rightarrow a^2 + c^2 - b^2 = ac$

$\Rightarrow \frac{a^2+c^2-b^2}{2ac} = \frac{1}{2}$

$\Rightarrow \cos B = \cos\frac{\pi}{3} \Rightarrow B = \frac{\pi}{3}$

65. (B) $\cos(\cot^{-1}x) = \cos\left[\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right]$

$\Rightarrow \cos(\cot^{-1}x) = \frac{x}{\sqrt{1+x^2}}$

66. (C) $\frac{1}{ab}, \frac{1}{bc}$ and $\frac{1}{ca}$ are in A.P.,

then, $\frac{2}{bc} = \frac{1}{ab} + \frac{1}{ca}$

$\Rightarrow \frac{2}{bc} = \frac{c+b}{abc} \Rightarrow 2a = b + c$

Hence b, a and c are in A.P.

67. (B) $\sin^{-1}\left(\cos\left(\cos^{-1}\left(\sin\frac{5\pi}{4}\right)\right)\right)$

$\Rightarrow \sin^{-1}\left(\cos\left(\cos^{-1}\left(\sin\left(\pi + \frac{\pi}{4}\right)\right)\right)\right)$

$\Rightarrow \sin^{-1}\left(\cos\left(\cos^{-1}\left(-\sin\frac{\pi}{4}\right)\right)\right)$

$\Rightarrow \sin^{-1}\left(\cos\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)\right) \Rightarrow \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) \Rightarrow -\frac{\pi}{4}$

68. (C) Given that $X = \{9(n-1) : n \in N\}$

$n = 1, 2, 3, 4, \ldots\ldots$

$X = \{0, 9, 18, 27, \ldots\ldots\}$

$Y = \{4^n - 3n - 1 : n \in N\}$

$n = 1, 2, 3, 4, \ldots\ldots$

$Y = \{0, 9, 54, 243, \ldots\ldots\}$

$(X \cap Y) = \{0, 9, 54, 243\} = Y$

69. (C) Differential equation

$x\,dy - y\,dx = x^3 y\,dx$

$\Rightarrow \frac{x\,dy - y\,dx}{xy} = x^2\,dx$

$\Rightarrow \frac{dy}{y} - \frac{dx}{y} = x^2\,dx$

On integrating

$\Rightarrow \log y - \log x = \frac{x^3}{3} + \frac{c}{3}$

$\Rightarrow \log\frac{y}{x} = \frac{x^3}{3} + \frac{c}{3}$

$\Rightarrow 3\log\frac{y}{x} = x^3 + c$

70. (A) The required no. of triangles $= {}^{10}C_3 - {}^{3}C_3$
$= 120 - 1$
$= 119$

71. (B) $\sin^{-1}(\log_3 2x)$

Here $-1 \le \log_3 2x \le 1$

$\Rightarrow 3^{-1} \le 2x \le 3^1$

$\Rightarrow \frac{1}{3} \le 2x \le 3$

$\Rightarrow \frac{1}{6} \le x \le \frac{3}{2}$

Domain $= \left[\frac{1}{6}, \frac{3}{2}\right]$

72. (C) Series $\frac{1^2}{2} + \frac{1^2+2^2}{2+4} + \frac{1^2+2^2+3^2}{2+4+6} + \ldots\ldots$

$$T_n = \frac{1^2+2^2+3^2+\ldots..+n^2}{2+4+6+\ldots..+n}$$

$$T_n = \frac{1^2+2^2+3^2+\ldots..+n^2}{2(1+2+3+\ldots..+n)}$$

$$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{2\times\frac{n(n+1)}{2}} = \frac{2n+1}{6}$$

73. (D) Let $y = \log_{10}(5x^3 - 2x)$ and $z = x^2$

$\Rightarrow y = \log_{10}e \times \log_e(5x^3 - 2x)$, $\frac{dz}{dx} = 2x$

$\Rightarrow \frac{dy}{dx} = \log_{10}e \times \frac{1}{5x^3-2x} \times (15x^2 - 2)$

$\Rightarrow \frac{dy}{dx} = (15x^2 - 2)\log_{10}e \times \frac{1}{5x^3-2x}$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = (15x^2-2)\log_{10}e \times \frac{1}{5x^3-2x} \times \frac{1}{2x}$

$\Rightarrow \frac{dy}{dz} = \frac{(15x^2-2)\log_{10}e}{2x^2(5x^2-2)}$

74. (C) Let $y = \sqrt{4+3\sqrt{4+3\sqrt{4+\ldots\ldots..\infty}}}$

$\Rightarrow y = \sqrt{4+3y}$

On squaring

$\Rightarrow y^2 = 4 + 3y$

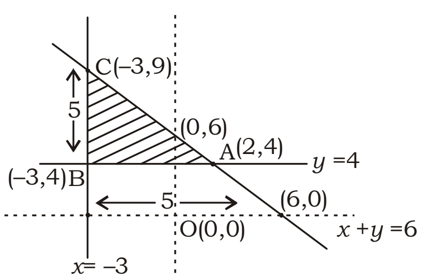
$\Rightarrow y^2 - 3y - 4 = 0$

$\Rightarrow (y-4)(y+1) = 0$

$\Rightarrow y = 4, -1$

Hence $\sqrt{4+3\sqrt{4+3\sqrt{4+\ldots\ldots..\infty}}} = 4$

75. (C)



The required Area $= \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$ sq.unit

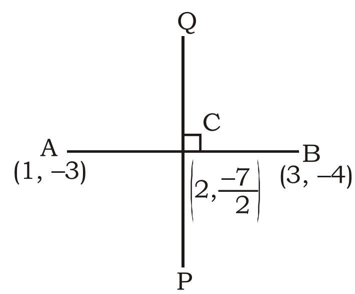
76. (A) Function is one-one but onto.

77. (C)

2	51	1
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(51)_{10} = (110011)_2$

78. (C)



mid-point of line joining

the points $= \left(\frac{1+3}{2}, \frac{-3-4}{2}\right) = \left(2, \frac{-7}{2}\right)$

Slope of line AB $(m_1) = \frac{-4+3}{3-1} = \frac{-1}{2}$

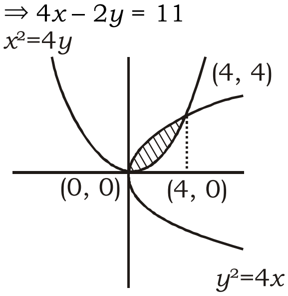
Slope of line PQ $(m_2) = \frac{-1}{\frac{-1}{2}} = 2$

Equation of line PQ

$y + \frac{7}{2} = 2(x-2)$

$\Rightarrow 4x - 2y = 11$

79. (C)



$y_1 \Rightarrow y = 2\sqrt{x}$ and $y_2 \Rightarrow y = \frac{x^2}{4}$

The required Area $= \int_0^4 (y_1 - y_2)\,dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \left[2\times\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{4\times3}\right]_0^4$

$= \left[\frac{4}{3}\times(4)^{\frac{3}{2}} - \frac{1}{12}(4)^3\right] = \frac{37}{3} - \frac{16}{3}$

$= \frac{16}{3}$ sq. unit

80. (A) $I = \int \frac{1}{\sqrt{1-\sin x}}\,dx$

$I = \int \frac{1}{\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

$I = \frac{1}{\sqrt{2}}\int \text{cosec}\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$

$I = \frac{\frac{1}{\sqrt{2}}\log\left|\text{cosec}\left(\frac{\pi}{4}-\frac{x}{2}\right)-\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right|}{-\frac{1}{2}} + c$

$I = \sqrt{2}\log\left|\text{cosec}\left(\frac{\pi}{4}-\frac{x}{2}\right)+\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + c$

81. (C) In the expansion of $\left(x^3 - \frac{1}{2x^2}\right)^{13}$

$T_{r+1} = {}^{13}C_r\,(x^3)^{13-r}\left(\frac{-1}{2x^2}\right)^r$

$T_{r+1} = {}^{13}C_r\,(-1)^r\,x^{39-5r}\left(\frac{1}{2}\right)^r$

Here, $39 - 5r = 9$

$\Rightarrow 5r = 30 \Rightarrow r = 6$

Coefficient of $x^9 = {}^{13}C_6(-1)^6\left(\frac{1}{2}\right)^6$

$= \frac{429\times 4}{64} = \frac{429}{16}$

82. (B) $m = \tan\theta = \tan 60 = \sqrt{3}$ and $c = 24$

The equation of line

$y = mx + c$

$\Rightarrow y = \sqrt{3}\,x + 24 \Rightarrow \sqrt{3}\,x - y + 24 = 0$

83. (B) Let the $x = 0$ divides the line joining the points $(-3, -4)$ and $(4, -6)$ in the ratio $m : 1$.

Now, $\frac{4m-3}{m+1} = 0 \Rightarrow m = \frac{3}{4}$

The required ratio = 3 : 4

84. (C) $(3x + 4y - 5) + \lambda(5x - y + 11) = 0$

$\Rightarrow (3 + 5\lambda)x + (4 - \lambda)y - 5 + 11\lambda = 0$

$\Rightarrow y = \frac{-(3+5\lambda)}{(4-\lambda)}x + \frac{5-11}{4-\lambda}$

Slope $m = \frac{-(3+5\lambda)}{(4-\lambda)}x$

given straight line parallel to x-axis

i.e $\theta = 0 \Rightarrow m = 0$

then $\frac{-(3+5\lambda)}{4-\lambda} = 0 \Rightarrow \lambda = \frac{-3}{5}$

85. (B) Variance of 25 observations var $(x) = 6$

We know that

Var $(\lambda x) = \lambda^2$ var(x)

If each observation multiplied by 3 then variance of new observations

var $(3x) = 3^2 \times$ var(x)

var $(3x) = 3^2 \times 6 = 54$

86. (A) $n(S) = {}^{12}C_4 = 495$

$n(E) = {}^3C_2\times{}^5C_1\times{}^4C_1 + {}^3C_2\times{}^5C_2\times{}^4C_0 + {}^3C_2\times{}^5C_0\times{}^4C_2 + {}^3C_3\times{}^5C_1\times{}^4C_0 + {}^3C_3\times{}^5C_0\times{}^4C_1$

$n(E) = 3\times5\times4 + 3\times10\times1 + 3\times1\times6 + 1\times5\times1 + 1\times1\times4$

$n(E) = 117$

The required Probability $= \frac{n(E)}{n(S)}$

$= \frac{117}{495} = \frac{13}{55}$

87. (A) a, b, c are in A.P.

$2b = a + c$ ...(i)

l, m, n in A.P

$2m = l + n$ ...(ii)

from eq. (i) and eq. (ii)

$2(b + m) = (a + l) + (c + n)$

then $(a + l), (b + m), (c + n)$ also are in A.P.

88. (B) Digits 0, 1, 2, 3, 4, 7, 9

| 5 | 5 | 4 | $= 5 \times 5 \times 4 = 100$

'0' can not put here; only (1,3,7,9) for odd number

89. (B) We know that

$C_0 + C_1x + C_2x^2 + \ldots C_{n-1}x^{n-1} + C_nx^n = (1+x)^n$ ...(i)

Multiply by x

$\Rightarrow C_0x + C_1x^2 + \ldots + C_{n-1}x^n + C_nx^{n+1} = x(1+x)^n$

On differentiate both side w.r.t. 'x'

$\Rightarrow C_0 + 2C_1x + 3C_2x^2 + \ldots + nC_{n-1}x^{n-1} + (n+1)C_nx^n = nx.(1+x)^{n-1} + (1+x)^n$ ...(ii)

$x \to \frac{1}{x}$ in eq(i)

$\Rightarrow C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots + \frac{C_{n-1}}{x^{n-1}} + \frac{C_n}{x^n} = \left(1+\frac{1}{x}\right)^n$ ...(iii) from eq(ii) and eq(iii)

$\Rightarrow$ coeff. of x^0 in $\left(1+\frac{1}{x}\right)^n [nx(x+1)^{n-1}+(1+x)^n]$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow$ coeff. of x^{n-1} in $n(1+x)^{2n-1}$ + coeff. of x^n in $(1+x)^{2n} = C_0^2 + 2C_1^2 + \ldots + nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow n\,{}^{2n-1}C_{n-1} + {}^{2n}C_n = C_0^2 + 2C_1^2 + \ldots + nC_{n-1}^2 + (n+1)C_2^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{(2n)!}{n!n!}$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots + nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{2n(2n-1)!}{n(n-1)!n!}$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots + nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow \frac{(2n-1)!}{(n-1)!n!}(n+2) = C_0^2 + \ldots + (n+1)C_n^2$

$\Rightarrow (n+2)\,{}^{2n-1}C_{n-1} = C_0^2 + 2C_1^2 \ldots + (n+1)C_n^2$

90. (C) Total students = 8
the table is round. One students is fixed.
No. of ways = (8 – 1)! = 7! = 5040

91. (A) Let $y = 5^{79}$
taking log both sides
$\log_{10}y = 79 \log_{10}5$
$\log_{10}y = 79 \times 0.699$
$\log_{10}y = 55.221$
No. of digits = 55 + 1 = 56

92. (B) $C(3n, 6) = C(3n, n)$
$\Rightarrow 3n = 6 + n \Rightarrow 2n = 6$
then $C(9, 2n) = C(9, 6) = \frac{9!}{6!3!} = 84$

93. (C) When $\theta = 180^\circ$
$M = \frac{60}{11}(H \pm 6)$ when $- \rightarrow H > 6$
$+ \rightarrow H < 6$
H = 7 (between 7 and 8 O'clock)
$M = \frac{60}{11}(7-6)$
$M = \frac{60}{11} = 5\frac{5}{11}$ minute
Time = $7 : 5\frac{5}{11}$

94. (B) $(\log_2 x)(\log_x 4x)(\log_{4x} y) = \log_y y^3$
$\Rightarrow \frac{\log x}{\log 2}\times\frac{\log 4x}{\log x}\times\frac{\log y}{\log 4x} = 3\log_y y$
$\Rightarrow \frac{\log y}{\log 2} = 3 \Rightarrow \log_2 y = 3 \Rightarrow y = 2^3 = 8$

95. (B) $\begin{vmatrix} 1 & \omega & \omega^5 \\ \omega^2 & \omega & \omega^6 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & \omega & 1 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$\Rightarrow C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega & 1 \\ 1+\omega+\omega^2 & \omega^2 & \omega \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 0 & \omega & \omega^5 \\ 0 & \omega & 1 \\ 0 & \omega^2 & \omega \end{vmatrix} = 0 \; [\because \omega^2 + \omega + 1 = 0]$

96. (D)
```
 1101
 +110
10011
```

97. (C) $\log_3[\log_3(\sqrt{3\sqrt{3}})] \Rightarrow \log_3[\log_3 3^{\frac{3}{4}}]$
$\Rightarrow \log_3\left[\frac{3}{4}\log_3 3\right] \Rightarrow \log_3\left(\frac{3}{4}\right)$
$\Rightarrow \log_3 3 - \log_3 4 = 1 - 2\log_3 2$

98. (B) Equation of line which makes equal intercept on coordinate axes
$x + y = c$ …(i)
Its passes through the point (– 2, 5)
$-2 + 5 = c \Rightarrow c = 3$
from eq. (i)
$x + y = 3$

99. (A) $I = \int \frac{1+\ln x}{\sin^2(x\ln x)}dx$
Let $x\,lnx = t \Rightarrow (1 + lnx)dx = dt$
$I = \int \frac{dt}{\sin^2 t}$
$I = \int \text{cosec}^2\, t\, dt$
$I = -\cot t + c$
$I = -\cot(x.\ln x) + c$

100. (C) $\int_0^1 \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx = \frac{1}{2}\left[e^{\pi}-1\right]$

Let $m\sin^{-1}x = t$ when $x \to 0,\ t \to 0$

$\frac{m}{\sqrt{1-x^2}}dx = dt$ $x \to 1,\ t \to \frac{m\pi}{2}$

$\frac{1}{\sqrt{1-x^2}}dx = \frac{1}{m}dt$

$\Rightarrow \frac{1}{m}\int_0^{\frac{m\pi}{2}} e^t\, dt = \frac{1}{2}[e^{\pi}-1]$

$\Rightarrow \frac{1}{m}\left[e^t\right]_0^{\frac{m\pi}{2}} = \frac{1}{2}[e^{\pi}-1]$

$\Rightarrow \frac{1}{m}\left[e^{\frac{m\pi}{2}}-1\right] = \frac{1}{2}[e^{\pi}-1]$

On comparing

$\Rightarrow m = 2$

101. (C) $I = \int\sqrt{2-2\cos 2x}\ dx$

$I = \int\sqrt{2[1-\cos 2x]}\ dx$

$I = \int\sqrt{2\times 2\sin^2 x}\ dx$

$I = \int 2\sin x\ dx$

$I = -2\cos x + c$

102. (B) Given that

$\int x^2.e^{3x}dx = ax^2.e^{3x} + bx.e^{3x} + c.e^{3x} + k$ ...eq.(i)

Let $I = \int x^2.e^{3x}dx$

$I = x^2.\int e^{3x}dx - \int\left\{\frac{d}{dx}(x^2).\int e^{3x}dx\right\}dx + k$

$I = x^2.\frac{e^{3x}}{3} - \int 2x.\frac{e^{3x}}{3}dx + k$

$I = \frac{1}{3}x^2.e^{3x} - \frac{2}{3}\left[x.\int e^{3x}dx - \int\left\{\frac{d}{dx}(x).\int e^{3x}dx\right\}dx\right] + k$

$I = \frac{1}{3}x^2.e^{3x} - \frac{2}{3}\left[x.\frac{e^{3x}}{3} - \int 1.\frac{e^{3x}}{3}dx\right] + k$

$I = \frac{1}{3}x^2.e^{3x} - \frac{2}{3}\left[\frac{x}{3}.e^{3x} - \frac{1}{3}\frac{e^{3x}}{3}\right] + k$

$I = \frac{1}{3}x^2.e^{3x} - \frac{2}{9}x.e^{3x} + \frac{2}{27}e^{3x} + k$

On comparing eq(i)

$a = \frac{1}{3},\ b = \frac{-2}{9},\ c = \frac{2}{27}$

103. (B) $[(A\cap B)\cup(B\cap C)\cup(C\cap A)] - (A\cap B\cap C)$

104. (B) Points (1, 2) and (2, –1)

Slope of line joining the points

$m_1 = \frac{-1-2}{2-1} = -3$

Slope of perpendicular line $m_2 = \frac{-1}{-3} = \frac{1}{3}$

Mid-point $= \left(\frac{1+2}{2},\frac{2-1}{2}\right) = \left(\frac{3}{2},\frac{1}{2}\right)$

The required equation of line

$y - \frac{3}{2} = \frac{1}{3}\left(x - \frac{1}{2}\right)$

$\Rightarrow 2y - 3 = \frac{1}{3}(2x - 1)$

$\Rightarrow 6y - 9 = 2x - 1$

$\Rightarrow 2x - 6y + 8 = 0 \Rightarrow x - 3y + 4 = 0$

105. (A) Rectanglar hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

Now, $e = \sqrt{1+\frac{a^2}{a^2}}$

$\Rightarrow e = \sqrt{1+1} \Rightarrow e = \sqrt{2}$

106. (C) Points (1, 0, 1) and (1, –3, 5)

Direction ratio = $\langle 1-1, -3-0, 5-1\rangle$

$= \langle 0, -3, 4\rangle$

Direction Cosine

$\left\langle \frac{0}{\sqrt{0^2+(-3)^2+4^2}}, \frac{-3}{\sqrt{0^2+(-3)^2+4^2}}, \frac{4}{\sqrt{0^2+(-3)^2+4^2}}\right\rangle$

$= \left\langle 0, \frac{-3}{5}, \frac{4}{5}\right\rangle$

107. (C) a, A_1, A_2, A_3, b

here $d = \frac{b-a}{4}$

$A_1 = a + d \Rightarrow A_1 = a + \frac{b-a}{4} = \frac{3a+b}{4}$

$A_2 = a + 2d \Rightarrow A_2 = \frac{2b-2a}{4} = \frac{a+b}{2}$

$A_3 = a + 3d \Rightarrow A_3 = a + \frac{3b-3a}{4} = \frac{a+3b}{4}$

a, G_1, G_2, G_3, b

here $r = \left(\frac{b}{a}\right)^{1/4}$

$G_1 = ar \Rightarrow G_1 = a\left(\frac{b}{a}\right)^{1/4} = a^{3/4}.b^{1/4}$

$G_2 = ar^2 \Rightarrow G_2 = a\left(\frac{b}{a}\right)^{1/2} = a^{1/2}.b^{1/2}$

$G_3 = ar^3 \Rightarrow G_3 = a\left(\frac{b}{a}\right)^{3/4} = a^{1/4}.b^{3/4}$

Now, $\frac{A_1+A_2+A_3}{G_1G_2G_3} = \frac{\frac{3a+b}{4}+\frac{a+b}{2}+\frac{a+3b}{4}}{a^{3/4}.b^{1/4}.a^{1/2}.b^{1/2}.a^{1/4}.b^{3/4}}$

$= \frac{\frac{6a+6b}{4}}{a^{3/2}b^{3/2}}$

$= \frac{\frac{6}{4}(a+b)}{(ab)^{3/2}} = \frac{3}{2}\frac{a+b}{(ab)^{3/2}}$

108. (A) Data 21,22,11,24, 26, 15, 22, 27, 18, 35
On arranging in ascending order
11, 15, 18, 21, 22, 22, 24, 26, 27, 35

Middle terms = $\left(\frac{10}{2}\right)^{th}$ and $\left(\frac{10}{2}+1\right)^{th}$

= 5^{th} and 6^{th}

Median = $\frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2}$

Median = $\frac{22+22}{2}$ = 22

109. (A) Given $x = y^{y^{y\ldots\infty}}$

$\Rightarrow x = y^x$

taking log both sides

$\Rightarrow \log x = x.\log y$ ...(i)

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{1}{x} = x \times \frac{1}{y}\frac{dy}{dx} + \log y.1$

$\Rightarrow \frac{1}{x} - \log y = \frac{x}{y}\frac{dy}{dx}$

$\Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{1 - x\log y}{x}$

$\Rightarrow x^2\frac{dy}{dx} = y(1 - x\log y)$

$\Rightarrow x^2\frac{dy}{dx} = y(1 - \log x)$ [from eq(i)]

110. (B) $\lim_{x\to\infty}\left[8^x + 9^x\right]^{1/x} \Rightarrow \lim_{x\to\infty} 9\left[\left(\frac{8}{9}\right)^x + 1\right]^{1/x}$

$\Rightarrow \lim_{x\to\infty} 9\left[1 + \frac{1}{\left(\frac{9}{8}\right)^x}\right]^{1/x} \Rightarrow 9\left[1+\frac{1}{\infty}\right]^{1/\infty} = 9$

111. (A) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3} + 4\left(x+\frac{1}{x}\right)$

$\Rightarrow f\left(x+\frac{1}{x}\right) = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$+ 4\left(x-\frac{1}{x}\right)$

Let $x + \frac{1}{x} = y$

$\Rightarrow f(y) = y^3 - 3y + 4y$

$\Rightarrow f(y) = y^3 + y$

Now, $f(-4) = (-4)^3 + (-4)$

$\Rightarrow f(-4) = -64 - 4 = -68$

112. (B)

113. (C) $I = \int_{-3}^{3}\frac{x^4}{1+3^x}\,dx$ ...(i)

Prop IV $\int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} f(-x)\,dx$

$I = \int_{-3}^{3}\frac{(-x)^4}{1+3^{-x}}\,dx$

$I = \int_{-3}^{3}\frac{x^4 \cdot 3^x}{1+3^x}\,dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_{-3}^{3}\frac{x^4(1+3^x)}{1+3^x}\,dx$

$2I = \int_{-3}^{3} x^4\,dx$

$2I = 2\int_{0}^{3} x^4\,dx$

$I = \left[\frac{x^5}{5}\right]_0^3$

$I = \frac{243}{5} - 0 \Rightarrow I = 48\frac{3}{5}$

114. (B) $\left|\frac{z-2}{z+2}\right| = 3,\ z = x + iy$

$\Rightarrow \left|\frac{x+iy-2}{x+iy+2}\right| = 3$

$\Rightarrow \frac{\sqrt{(x-2)^2+y^2}}{\sqrt{(x+2)^2+y^2}} = 3$

On squaring both side

$\Rightarrow \frac{(x-2)^2+y^2}{(x+2)^2+y^2} = 9$

$\Rightarrow \frac{x^2+4-4x+y^2}{x^2+4+4x+y^2} = 9$

$\Rightarrow x^2 + 4 - 4x + y^2 = 9x^2 + 36 + 36x + 9y^2$

On solving

$\Rightarrow 8x^2 + 8y^2 + 40x + 32 = 0$

$\Rightarrow x^2 + y^2 + 5x + 4 = 0$

It is a circle.

115. (A) We know that

$\sinh A + \sinh B = 2\sinh\frac{A+B}{2} . \cosh\frac{A-B}{2}$

and $\cosh A - \cosh B = 2\sinh\frac{A+B}{2} . \sinh\frac{A-B}{2}$

Now, $\frac{\sinh x + \sinh y}{\cosh x - \cosh y}$

$\Rightarrow \frac{2\sinh\frac{x+y}{2} . \cosh\frac{x-y}{2}}{2\sinh\frac{x+y}{2} . \sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$

116. (C) $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$

We know that

$\lim_{x\to a} [f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$, if $[1^\infty]$ form

$\Rightarrow e^{\lim_{x\to\pi/2} \tan x(\sin x - 1)}$

$\Rightarrow e^{\lim_{x\to\pi/2} \frac{\sin x(\sin x-1)}{\cos x}}$ $\left[\frac{0}{0}\right]$ form

by L - Hospital's Rule

$\Rightarrow e^{\lim_{x\to\pi/2} \frac{\sin x.\cos x+(\sin x-1)\cos x}{-\sin x}}$

$\Rightarrow e^{\frac{\sin\frac{\pi}{2}.\cos\frac{\pi}{2}+\left(\sin\frac{\pi}{2}-1\right)\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}}$

$\Rightarrow e^0 = 1$

117. (D) Three-digit odd numbers

101, 103, 105, 107............999

Now, $l = a + (n-1)d$

$\Rightarrow 999 = 101 + (n-1)\times 2$

$\Rightarrow 898 = (n-1)\times 2 \Rightarrow n = 450$

Sum = $\frac{n}{2}[2a+(n-1)d]$

$= \frac{450}{2}[2\times 101 + (450-1)\times 2]$

$= 450\,[101 + 449]$

$= 450 \times 550 = 247500$

118. (B) $S_n = n^2 - 3n + 5$

$S_{n-1} = (n-1)^2 - 3(n-1) + 5$

$S_{n-1} = n^2 - 5n + 9$

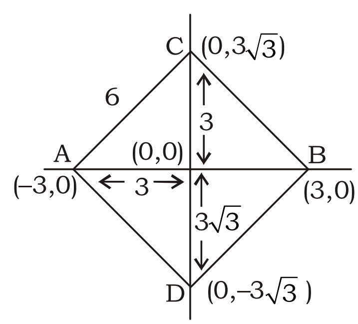
Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = n^2 - 3n + 5 - n^2 + 5n - 9$

$\Rightarrow T_n = 2n - 4$

$\Rightarrow T_{11} = 2 \times 11 - 4 = 18$

119. (C)



Hence third vertex of an equilateral triangle = $(0, \pm 3\sqrt{3})$

120. (D)

NDA (MATHS) MOCK TEST - 166 (Answer Key)

1.	(C)	21.	(A)	41.	(D)	61.	(C)	81.	(C)	101.	(C)
2.	(A)	22.	(B)	42.	(B)	62.	(C)	82.	(B)	102.	(B)
3.	(A)	23.	(D)	43.	(A)	63.	(D)	83.	(B)	103.	(B)
4.	(D)	24.	(C)	44.	(C)	64.	(D)	84.	(C)	104.	(B)
5.	(D)	25.	(B)	45.	(C)	65.	(B)	85.	(B)	105.	(A)
6.	(C)	26.	(C)	46.	(A)	66.	(C)	86.	(A)	106.	(C)
7.	(D)	27.	(C)	47.	(C)	67.	(B)	87.	(A)	107.	(C)
8.	(C)	28.	(D)	48.	(B)	68.	(C)	88.	(B)	108.	(A)
9.	(B)	29.	(B)	49.	(C)	69.	(C)	89.	(B)	109.	(A)
10.	(B)	30.	(B)	50.	(C)	70.	(A)	90.	(C)	110.	(B)
11.	(A)	31.	(D)	51.	(D)	71.	(B)	91.	(A)	111.	(A)
12.	(D)	32.	(A)	52.	(C)	72.	(C)	92.	(B)	112.	(B)
13.	(A)	33.	(A)	53.	(C)	73.	(D)	93.	(C)	113.	(C)
14.	(B)	34.	(A)	54.	(B)	74.	(C)	94.	(B)	114.	(B)
15.	(B)	35.	(B)	55.	(A)	75.	(C)	95.	(B)	115.	(A)
16.	(A)	36.	(A)	56.	(B)	76.	(A)	96.	(D)	116.	(C)
17.	(A)	37.	(B)	57.	(D)	77.	(C)	97.	(C)	117.	(D)
18.	(C)	38.	(A)	58.	(B)	78.	(C)	98.	(B)	118.	(B)
19.	(D)	39.	(D)	59.	(C)	79.	(C)	99.	(A)	119.	(C)
20.	(B)	40.	(D)	60.	(C)	80.	(A)	100.	(C)	120.	(D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777