NDA MATHS MOCK TEST - 168 (SOLUTION)

1. (D) Let A = $\begin{bmatrix} \sin(-\theta) & \cos(-\theta) \\ -\cos(-\theta) & \sin(-\theta) \end{bmatrix}$

A = $\begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix}$

Co-factors of A-

$C_{11} = (-1)^{1+1}(-\sin\theta)$, $C_{12} = (-1)^{1+2}(-\cos\theta)$

$= -\sin\theta$ $\quad$ $= \cos\theta$

$C_{21} = (-1)^{2+1}(\cos\theta)$, $C_{22} = (-1)^{2+2}(-\sin\theta)$

$= -\cos\theta$ $\quad$ $= -\sin\theta$

C = $\begin{bmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{bmatrix}$

Adj A = C^T = $\begin{bmatrix} -\sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$

2. (B) Let A = $\begin{bmatrix} 3 & x \\ 4 & -8 \end{bmatrix}$

A is non-invertible,
then |A| = 0

$\Rightarrow \begin{vmatrix} 3 & x \\ 4 & -8 \end{vmatrix} = 0$

$\Rightarrow -24 - 4x = 0 \Rightarrow x = -6$

3. (C) $\lim_{x\to 0} \frac{1-\sqrt{1-x}}{x}$ $\quad \left[\frac{0}{0}\right]$

by L - Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{0 - \frac{1\times(-1)}{2\sqrt{1-x}}}{1}$

$\Rightarrow \lim_{x\to 0} \frac{1}{2\sqrt{1-x}} = \frac{1}{2\sqrt{1-0}} = \frac{1}{2}$

4. (D) $T_{m+n} = a + (m+n-1)d$

$T_{m-n} = a + (m-n-1)d$

Now, $T_{m+n} + T_{m-n} = 2a + 2(2m-2)d$

$\Rightarrow T_{m+n} + T_{m-n} = 2(a + (m-1)d)$

$\Rightarrow T_{m+n} + T_{m-n} = 2T_m$

Hence the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A.P. will be equal to twice the m^{th} term.

5. (C) cot65° = cot(45 + 20)

cot65° = $\frac{\cot 45.\cot 20 - 1}{\cot 45 + \cot 20}$

cot65° = $\frac{1.\cot 20 - 1}{1 + \cot 20} = \frac{\cot 20 - 1}{\cot 20 + 1}$

6. (B) Equation $ax^2 + bx + c = 0$

A.T.Q,

tan31 + tan14 = $\frac{-b}{a}$

and tan31.tan14 = $\frac{c}{a}$

Now, tan(31 + 14) = $\frac{\tan 31 + \tan 14}{1 - \tan 31.\tan 14}$

$\Rightarrow \tan 45 = \frac{\frac{-b}{a}}{1 - \frac{c}{a}}$

$\Rightarrow 1 = \frac{-b}{a-c}$

$\Rightarrow a - c = -b \Rightarrow a + b = c$

7. (B) Ratio of angles = 1 : 3 : 2

Let angles = x, $3x$, $2x$

$x + 3x + 2x = 180$

$6x = 180 \Rightarrow x = 30°$

Hence angles = 30, 90, 60

Now, Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 30} = \frac{b}{\sin 90} = \frac{c}{\sin 60}$

$\Rightarrow \frac{a}{1/2} = \frac{b}{1} = \frac{c}{\sqrt{3}/2}$

$\Rightarrow \frac{a}{1} = \frac{b}{2} = \frac{c}{\sqrt{3}}$

Hence $a : b : c = 1 : 2 : \sqrt{3}$

8. (C) Circle $x^2 + y^2 - 8x + 5y + 12 = 0$

it cuts the x-axis i.e. $y = 0$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 2, 6$

Intercept = 6 – 2 = 4

9. (C) Line $3y - x = 5$

Slope of line $m_1 = \frac{1}{3}$

Slope of perpendicular line $(m_2) = \frac{-1}{1/3} = -3$

Equation of new line passes through the point (–1, 6)

$\Rightarrow y - 6 = -3(x + 1)$

$\Rightarrow y - 6 = -3x - 3$

$\Rightarrow 3x + y = 3$

10. (C) $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} + \tan^2\frac{\pi}{4}$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + (1)^2$

$\Rightarrow \frac{1}{4} + \frac{1}{4} + 1 = 1\frac{1}{2}$

11. (B) $\frac{\sin 10 - \sin 30}{\sin^2 10 - \cos^2 10}$

$\Rightarrow \frac{\sin 30 - \sin 10}{\cos^2 10 - \sin^2 10}$

$\Rightarrow \frac{2\cos\frac{30+10}{2}.\sin\frac{30-10}{2}}{2\cos(2\times 10)}$

$\Rightarrow \frac{2\cos 20.\sin 10}{2\cos 20} = \sin 10$

12. (B) $I = \int_0^{\pi/2} (\sqrt{\cos\theta}.\sin\theta)^3$

$I = \int_0^{\pi/2} (\cos\theta)^{3/2}.\sin^3\theta\, d\theta$

$I = \int_0^{\pi/2} (\cos\theta)^{3/2}(1-\cos^2\theta).\sin\theta d\theta$

$I = \int_0^{\pi/2} [(\cos\theta)^{3/2} - (\cos\theta)^{7/2}].\sin\theta.d\theta$

Let $\cos\theta = t$ when $\theta \to 0,\ t \to 1$

$\Rightarrow -\sin\theta.d\theta = dt$ $\quad \theta \to \pi/2,\ t \to 0$

$\Rightarrow \sin\theta.d\theta - dt$

$I = \int_1^0 -(t^{3/2} - t^{7/2})dt$

$I = \int_0^1 (t^{3/2} - t^{7/2})dt$

$I = \left[\frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{t^{\frac{7}{2}+1}}{\frac{7}{2}+1}\right]_0^1$

$I = \left[\frac{2}{5}t^{5/2} - \frac{2}{9}t^{9/2}\right]_0^1$

$I = \frac{2}{5}\times 1 - \frac{2}{9}\times 1 - 0$

$I = \frac{18-10}{45} = \frac{8}{45}$

13. (A) $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}y = \frac{\pi}{2} - \cos^{-1}x$

$\Rightarrow \cos^{-1}y = \sin^{-1}x$

$\Rightarrow \cos^{-1}y = \cos^{-1}\sqrt{1-x^2}$

$\Rightarrow y = \sqrt{1-x^2}$ ...(i)

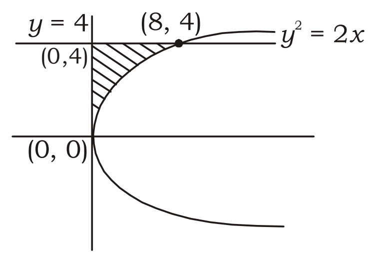
On differentiating w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}} \times (-2x)$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$ [from eq(i)]

14. (C)



curve $y^2 = 2x$

$x_1 \Rightarrow x = \frac{y^2}{2}$

Area $= \int_0^4 x_1\, dy$

$= \int_0^4 \frac{y^2}{2}\, dy$

$= \left[\frac{y^3}{6}\right]_0^4 = \frac{4^3}{6} - 0 = \frac{32}{3}$ sq.unit

15. (A) $C(27, 2r) = C(27, 2r-1)$

$\Rightarrow 2r + 2r - 1 = 27$

$\Rightarrow 4r = 28 \Rightarrow r = 7$

16. (B) $\frac{\operatorname{cosec}\theta}{\sin\theta} + \frac{\sec\theta}{\cos\theta}$

$\Rightarrow \frac{1}{\sin\theta.\sin\theta} + \frac{1}{\cos\theta.\cos\theta}$

$\Rightarrow \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta.\cos^2\theta}$

$\Rightarrow \frac{1}{\sin^2\theta.\cos^2\theta} = \operatorname{cosec}^2\theta.\sec^2\theta$

17. (C) Conic $6x^2 + 8y^2 = 48$

$\Rightarrow \frac{x^2}{8} + \frac{y^2}{6} = 1$

$a^2 = 8,\ b^2 = 6$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{6}{8}}$

$e = \sqrt{\frac{2}{8}} = \frac{1}{2}$

18. (C) $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 12 = 3 \times 8 \cos\theta$

$\Rightarrow \frac{1}{2} = \cos\theta \Rightarrow \theta = 60°$

Now, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$\Rightarrow |\vec{a} \times \vec{b}| = 3 \times 8 \times \sin 60$

$\Rightarrow |\vec{a} \times \vec{b}| = 24 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

19. (A)

20. (C) $I = \int_0^{\pi/2} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$

$I = \int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 \tan^2 x + b^2}$

Let $a \tan x = t$ when $x \to 0,\ t = 0$

$\Rightarrow a \sec^2 x\, dx = dt$ $x \to \pi/2,\ t = \infty$

$\Rightarrow \sec^2 x\, dx = \frac{1}{a} dt$

$I = \frac{1}{a} \int_0^{\infty} \frac{dt}{t^2 + b^2}$

$I = \frac{1}{a} \times \frac{1}{b} \left[\tan^{-1} \frac{t}{b}\right]_0^{\infty}$

$I = \frac{1}{ab} \left[\tan^{-1}\infty - \tan^{-1} 0\right]$

$I = \frac{1}{ab} \left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$

21. (D)

2	11	1
2	5	1
2	2	0
2	1	1
	0	

0 .125
× 2
[0].250
× 2
[0].500
× 2
[1].000

$(11)_{10} = (1011)_2$ $(0.125)_{10} = (0.001)_2$

Hence $(11.125)_{10} = (1011.001)_2$

22. (B) $(2 - 3\omega^2 + 2\omega)^{23}$

$\Rightarrow [2(1+\omega) - 3\omega^2]^{23}$

$\Rightarrow [-2\omega^2 - 3\omega^2]^{23}$ $[\because 1 + \omega + \omega^2 = 0]$

$\Rightarrow (-5\omega^2)^{23}$

$\Rightarrow -5^{23}\omega^{46} = -5^{23}\omega$ $[\because \omega^3 = 1]$

23. (B) Series $S = 2 - 1 + \frac{1}{2} - \frac{1}{4} + \ldots\ldots$

$\Rightarrow S = \frac{2}{1 - \left(\frac{-1}{2}\right)}$ $\Rightarrow S = \frac{2}{1 + \frac{1}{2}}$

$\Rightarrow S = \frac{2}{3/2} = \frac{4}{3}$

24. (C) A.T.Q

$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{17}{15}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{17}{15}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{a + b + 2\sqrt{ab}}{a + b - 2\sqrt{ab}} = \frac{17+15}{17-15}$

$\Rightarrow \frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{32}{2}$

$\Rightarrow \left(\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}}\right)^2 = \frac{16}{1}$

$\Rightarrow \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{4}{1}$

Again, Componendo & Dividendo Rule

$\Rightarrow \frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b}} = \frac{4+1}{4-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{5}{3} \Rightarrow \frac{a}{b} = \frac{25}{9}$

Hence $a : b = 25 : 9$

25. (C) $\frac{1}{\log_2 e} + \frac{1}{\log_2 e^2} + \frac{1}{\log_2 e^4} + \ldots\ldots$

$\Rightarrow \frac{1}{\log_2 e} + \frac{1}{2\log_2 e} + \frac{1}{4\log_2 e}$

$\Rightarrow \frac{1}{\log_2 e}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots..\right)$

$\Rightarrow \left(\frac{1}{1 - \frac{1}{2}}\right) \log_e 2 \Rightarrow \frac{1}{1/2} \log_e 2$

$\Rightarrow 2\log_e 2 \Rightarrow \log_e 4$

26. (C) Curves $y = a\cos(bx + c)$ ...(i)

On differentiating both sides w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = -ab\sin(bx + c)$

Again, differentiating

$\Rightarrow \frac{d^2y}{dx^2} = -ab \times b\cos(bx + c)$

$\Rightarrow \frac{d^2y}{dx^2} = -b^2 \times a\cos(bx + c)$

$\Rightarrow \frac{d^2y}{dx^2} = -b^2y$

$\Rightarrow \frac{d^2y}{dx^2} + b^2y = 0$

27. (D) Differential equation

$\left(\frac{d^3y}{dx^3}\right)^2 + 3\left(\frac{d^2y}{dx^2}\right)^3 + 4y = 0$

Order = 3, Degree = 2

28. (B) $A \subseteq (R \times R)$

29. (C) Word "FOOTBALL"

No. of permutations $= \frac{8!}{2!2!} = 10080$

30. (B) Circle $x^2 + y^2 + 3x + 2y + c = 0$ ...(i)

it passes through the point (0, 0)

$0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$

from eq(i)

$x^2 + y^2 + 3x + 2y = 0$

$\Rightarrow \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + (y + 1)^2 - 1 = 0$

$\Rightarrow \left(x + \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{13}{4}$

$\Rightarrow \left(x + \frac{3}{2}\right)^2 + (y + 1)^2 = \left(\frac{\sqrt{13}}{2}\right)^2$

Hence radius $= \frac{\sqrt{13}}{2}$

31. (B) Differential equation

$x\frac{dy}{dx} + y = 0$

$\Rightarrow xdy + ydx = 0$

$\Rightarrow d(xy) = 0$

On intgrating

$\Rightarrow \int d(xy) = \int 0$

$\Rightarrow xy = c$

32. (A) The foot of perpendicular drawn from (2, 5, –3) on the $y = 0$ is (2, 0, –3).

33. (C) $\lim_{x\to 0} \frac{6^x - 1}{x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{6^x.\log_e 6}{1}$

$\Rightarrow 6^0.\log_e 6 = \log_e 6$

34. (B) $\log_{10}\left(\frac{3}{4}\right) - \log_{10}\left(\frac{8}{9}\right) + \log_{10}\left(\frac{32}{27}\right)$

$\Rightarrow \log_{10}\left(\frac{3}{4} \times \frac{9}{8} \times \frac{32}{27}\right)$

$\Rightarrow \log_{10} 1 = 0$

35. (D) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix}$

$\Rightarrow (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 0 & c^2+ca-b^2-ab \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow (b-a)(c-a)\begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+a^2+ab \\ 0 & 0 & c^2+ca-b^2-ab \end{vmatrix}$

$\Rightarrow (b-a)(c-a)[1(c^2 + ca - b^2 - ab) - 0]$

$\Rightarrow (b-a)(c-a)(a+b+c)(c-b)$

$\Rightarrow (a-b)(b-c)(c-a)(a+b+c)$

36. (A) Lines $5x - 12y = 10$

and $-10x + 24y = 13$

$\Rightarrow 5x - 12y = \frac{-13}{2}$

Perpendicular distance $= \frac{10 + \frac{13}{2}}{\sqrt{5^2 + (-12)^2}}$

$= \frac{33}{2 \times 13} = \frac{33}{26}$

37. (A) Let $f(x) = \int_{-\pi/2}^{\pi/2} \frac{dx}{(\sin^3 x + \sin x)}$

$f(-x) = \int_{-\pi/2}^{\pi/2} \frac{dx}{\{\sin^3(-x) + \sin(-x)\}}$

$f(-x) = \int_{-\pi/2}^{\pi/2} \frac{dx}{-\sin^3 x - \sin x}$

$f(-x) = -\int_{-\pi/2}^{\pi/2} \frac{dx}{\sin^3 x + \sin x}$

$f(-x) = -f(x)$

function is odd function, then

$\int_{-\pi/2}^{\pi/2} \frac{dx}{\sin^3 x + \sin x} = 0$

38. (D) $\sin y = x.\cos(x + y)$

On differentiating both sides w.r.t.'x'

$\Rightarrow \cos y \frac{dy}{dx} = -x.\sin(x+y)\left(1 + \frac{dy}{dx}\right) + \cos(x+y).1$

$\Rightarrow \cos y.\frac{dy}{dx} = -x.\sin(x+y) - x.\sin(x+y)$

$\frac{dy}{dx} + \cos(x+y)$

$\Rightarrow [\cos y + x.\sin(x+y)]\frac{dy}{dx} = \cos(x+y) - x.\sin(x+y)$

$\Rightarrow \frac{dy}{dx} = \frac{\cos(x+y) - x.\sin(x+y)}{\cos y + x.\sin(x+y)}$

39. (C) In the expansion of $\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)^8$

$T_{r+1} = {}^8C_r (\sqrt{x})^{8-r}\left(\frac{1}{2\sqrt{x}}\right)^r$

$T_{r+1} = {}^8C_r\, x^{\frac{8-2r}{2}}.\left(\frac{1}{2}\right)^r$

Here $\frac{8-2r}{2} = 3$

$\Rightarrow 8 - 2r = 6 \Rightarrow r = 1$

Now, Coefficient of $x^3 = {}^8C_1 \times \left(\frac{1}{2}\right)^1$

Coefficient of $x^3 = 8 \times \frac{1}{2} = 4$

40. (D) $\frac{a+b+c+d+e}{5} = M$

$\Rightarrow a + b + c + d + e = 5M$

Now, $(a - M) + (b - M) + (c - M) + (d - M) + (e - M)$

$\Rightarrow a + b + c + d + e - 5M$

$\Rightarrow 5M - 5M = 0$

41. (A)

42. (C) In ΔABC, A = 60°, B = 75° and C = 45°

Now, $a\sqrt{2} + c$

$\Rightarrow k \sin A(\sqrt{2}) + k \sin C$

$\Rightarrow k[\sqrt{2}.\sin 60° + \sin 45]$

$\Rightarrow k\left[\sqrt{2} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\right]$

$\Rightarrow k\left[\frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

$\Rightarrow 2k\left[\frac{\sqrt{3}+1}{2\sqrt{2}}\right]$

$\Rightarrow 2k \sin 75$

$\Rightarrow 2k \sin B = 2b$

43. (B) $I = \int_{-2}^{2}[x]dx$

$I = \int_{-2}^{1}[x]dx + \int_{-1}^{0}[x]dx + \int_{0}^{1}[x]dx + \int_{1}^{2}[x]dx$

$I = \int_{-2}^{1}(-2)dx + \int_{-1}^{0}(-1)dx + \int_{0}^{1}(0)dx + \int_{1}^{2}1.dx$

$I = -2[x]_{-2}^{-1} + (-1)[x]_{-1}^{0} + 0 + [x]_1^2$

$I = -2[-1+2] - 1[0+1] + [2-1]$

$I = -2 \times 1 - 1 + 1 = -2$

44. (C) Vertices of the triangle are A(6, 3, –1), B(4, 2, –1) and C(3, –2, 5),

AB = (4 – 6, 2 – 3, –1 + 1) = (–2, –1, 0)

AC = (3 – 6, –2 – 3, 5 + 1) = (–3, –5, 6)

Area of triangle = $\frac{1}{2}|AB \times AC|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -1 & 0 \\ -3 & -5 & 6 \end{vmatrix}$

$= \frac{1}{2}\left\|\hat{i}(-6-0) - \hat{j}(-12-0) + \hat{k}(10-3)\right\|$

$= \frac{1}{2}\left\|-6\hat{i} + 12\hat{j} - 3\hat{k}\right\|$

$= \frac{1}{2}\sqrt{(-6)^2 + (12)^2 + (-3)^2}$

$= \frac{1}{2}\sqrt{36+144+9} = \frac{1}{2}\sqrt{189} = \frac{3}{2}\sqrt{21}$

45. (C) $\log_4 m + \log_4 \frac{1}{6} = \frac{3}{2}$

$\Rightarrow \log_4\left(m \times \frac{1}{6}\right) = \frac{3}{2}$

$\Rightarrow \frac{m}{6} = 4^{3/2}$

$\Rightarrow \frac{m}{6} = 8 \Rightarrow m = 48$

46. (B) A = (–2, 1), B = (4, –2) and C = (0, 0)

$AB = \sqrt{(4+2)^2 + (-2-1)^2} = \sqrt{45} = 3\sqrt{5}$

$BC = \sqrt{(0-4)^2 + (0+2)^2} = \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(0+2)^2 + (0-1)^2} = \sqrt{5}$

Perimeter of the triangle = $3\sqrt{5} + 2\sqrt{5} + \sqrt{5} = 6\sqrt{5}$

47. (C) $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}}\right) + \tan^{-1}\frac{1}{7}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{8}{15}}{\frac{14}{15}}\right) + \tan^{-1}\frac{1}{7}$

$\Rightarrow \tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\frac{1}{7}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{4}{7}+\frac{1}{7}}{1-\frac{4}{7}\times\frac{1}{7}}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{5}{7}}{\frac{45}{49}}\right) = \tan^{-1}\left(\frac{7}{9}\right)$

48. (A)

49. (B) A.T.Q,

$180 - \frac{360}{n} = 135$

$\Rightarrow 45 = \frac{360}{n} \Rightarrow n = 8$

50. (C) $x^2 - 4x + 6 = 0$

$D = b^2 - 4ac$

$D = (-4)^2 - 4 \times 1 \times 6$

$D = 16 - 24 = -8 < 0$

Hence roots are imaginary.

51. (A) **Statement 1**

$\int \ln 5\, dx = \ln 5 \int 1\, dx$

$\int \ln 5\, dx = (\ln 5).x + c$

$\int \ln 5\, dx = x.\ln 5 + c$

Statement I is correct.

Statement II

$\int 10^x\, dx = \frac{10^x}{\ln 10} + c$

Statement II is incorrect.

52. (B) $I = \int e^{2\ln x} dx$

$I = \int e^{\ln x^2} dx$

$I = \int x^2 dx$

$I = \frac{x^3}{3} + c$

53. (A) The differential equation of the system of circles touching the y-axis at the origin is

$(x - \alpha)^2 + (y - 0)^2 = \alpha^2$

$\Rightarrow x^2 + \alpha^2 - 2x\alpha + y^2 = \alpha^2$

$\Rightarrow x^2 + y^2 - 2x\alpha = 0$

$\Rightarrow x + \frac{y^2}{x} - 2\alpha = 0$

On differentiating both sides w.r.t 'x'.

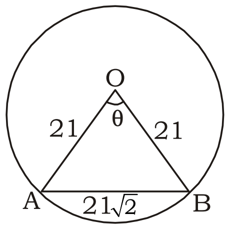
$\Rightarrow 1 + \frac{x.2y\frac{dy}{dx} - y^2.1}{x^2} = 0$

$\Rightarrow x^2 + 2xy\frac{dy}{dx} - y^2 = 0$

$\Rightarrow 2xy\frac{dy}{dx} + x^2 - y^2 = 0$

54. (C) Direction cosine of z-axis = <0, 0, 1>

55. (D)



Let $\angle AOB = \theta$

$$\cos\theta = \frac{(21)^2 + (21)^2 - (21\sqrt{2})^2}{2\times 21\times 21}$$

$\cos\theta = 0 \Rightarrow \theta = 90°$

Length of minor arc = $\frac{90}{360} \times 2\pi r$

$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 21$

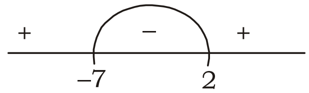
= 33 cm

56. (C) $f(x) = \frac{1}{\sqrt{14 - 5x - x^2}}$

Now, $14 - 5x - x^2 > 0$

$\Rightarrow x^2 + 5x - 14 < 0$

$\Rightarrow (x + 7)(x - 2) < 0$



Domain of the function = (–7, 2)

57. (B) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\Rightarrow \cos\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{3}.\cos\frac{\pi}{4} + \sin\frac{\pi}{3}.\sin\frac{\pi}{4}$

$\Rightarrow \cos\left(\frac{\pi}{12}\right) = \frac{1}{2}\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

58. (C) The required Probability = $\frac{7+3}{16} = \frac{5}{8}$

59. (A) The required Probability = $\frac{6!\times 2!}{7!}$

$= \frac{6!\times 2}{7\times 6!} = \frac{2}{7}$

60. (C) A = {1, 2, 3}, B = {1, 4, 5} and C = {2, 5, 6}

A – B = {1, 2, 3} – {1, 4, 5} = {2, 3}

$(A \cap C)$ = {1, 2, 3} $\cap$ {2, 5, 6} = {2}

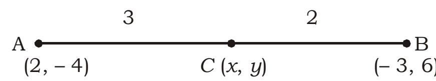
Now, (A – B) × $(A \cap C)$ = {2, 3} × {2}

$= \{(2,2),(3,2)\}$

61. (B) The required no. of hand shakes in party

$= {}^8C_2 = \frac{8\times 7}{2} = 28$

62. (C)



$x = \frac{3\times(-3)+2\times 2}{3+2}$ and $y = \frac{3\times 6 + 2\times(-4)}{3+2}$

$x = \frac{-9+4}{5} = -1,\qquad y = \frac{18-8}{5} = 2$

Co-ordinate of C = (–1, 2)

63. (A) $\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -4 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 1\times 2 + (-1)\times(-4) \\ 0\times 2 + 3\times(-4) \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 6 \\ -12 \end{bmatrix} = 0$

$\Rightarrow 6x - 24 = 0 \Rightarrow x = 4$

64. (C) Let

$I = \int \frac{\sin 2\theta}{\sin^3\theta - \cos^3\theta}\, d\theta$

$I = \int \frac{8\sin 2\theta}{(\sin 2\theta)^3}\, d\theta$

$I = 8\int \text{cosec}^2 2\theta\, d\theta$

$I = -8\times\frac{\cot 2\theta}{2} + c$

$I = -4\cot 2\theta + c$

65. (D) Given that

Mean = 42 and Mode = 57

We know that

Mode = 3Median – 2Mean

$\Rightarrow$ 57 = 3Median – 2 × 42

$\Rightarrow$ 3Median = 57 + 84

$\Rightarrow$ 3Median = 141 $\Rightarrow$ Median = 47

66. (C) Given that ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

then

$${}^nC_r + {}^nC_{r+1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$

$$= \frac{n!(r+1)}{(r+1)r!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)(n-r-1)!}$$

$$= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)n!}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)!(n-r)!} = {}^{n+1}C_{r+1}$$

67. (B) $A = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2\times 2 - 2\times 1 & 2\times(-2)+(-2)\times(-1) \\ 1\times 2+(-1)\times 1 & 1\times(-2)+(-1)\times(-1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$A^2 = A$

Hence Matrix A is an Idempotent matrix.

68. (A) $y = e^{2x}(a\sin x - b\cos x)$ ....(i)

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = e^{2x}(a\cos x + b\sin x) + 2(a\sin x - b\cos x)e^{2x}$$

$$\frac{dy}{dx} = e^{2x}(a\cos x + b\sin x) + 2y \quad ..(ii)$$

Again, differentiating

$$\frac{d^2y}{dx^2} = e^{2x}(-a\sin x + b\cos x)$$

$$+2(a\cos x + b\sin x)e^{2x} + 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -e^{2x}(a\sin x - b\cos x) + 2e^{2x}(a\cos x + b\sin x) + \frac{2dy}{dx}$$

$$\frac{d^2y}{dx^2} = -y + 2\left(\frac{dy}{dx} - 2y\right) + \frac{2dy}{dx}$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$$

69. (A)

70. (A) Equation of straight line which makes equal intercept on the co-ordinate areas, then $x + y = a$ .....(i)

it passes through the point (5, –2)

$5 - 2 = a \Rightarrow a = 3$

from eq. (i)

$x + y = 3$

71. (D) $\tan\left(2\tan^{-1}\frac{3}{4} - \frac{\pi}{4}\right)$

$$\Rightarrow \tan\left[\tan^{-1}\frac{24}{7} - \tan^{-1}1\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{24}{7}-1}{1+\frac{24}{7}\times 1}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{17}{7}}{\frac{31}{7}}\right)\right] = \frac{17}{31}$$

72. (B) $\dfrac{\cos 3A + 3\cos A}{\cos A} - \dfrac{\sin 3A + 3\sin A}{\sin A}$

$$\Rightarrow \frac{4\cos^3 A - 3\cos A + 3\cos A}{\cos A} - \frac{3\sin A - 4\sin^3 A - 3\sin A}{\sin A}$$

$\Rightarrow 4\cos^2 A + 4\sin^2 A = 4$

73. (A) Length of diagonal = $\sqrt{(24)^2 + (7)^2}$

$$\Rightarrow a\sqrt{2} = \sqrt{576+49}$$

$$\Rightarrow a\sqrt{2} = 25 \Rightarrow a = \frac{25}{\sqrt{2}}$$

Now, Area of square $= a^2$

$$= \frac{25}{\sqrt{2}} \times \frac{25}{\sqrt{2}}$$

= 312.5 sq. unit

74. (A) $y = \ln(x - \cos x)$, $z = x + \sin x$

$$\frac{dy}{dx} = \frac{1+\sin x}{x - \cos x}, \quad \frac{dz}{dx} = 1 + \cos x$$

Now, $\dfrac{dy}{dz} = \dfrac{dy}{dx} \times \dfrac{dx}{dz}$

$$\Rightarrow \frac{dy}{dz} = \frac{1+\sin x}{x-\cos x} \times \frac{1}{1+\cos x}$$

$$\Rightarrow \frac{dy}{dz} = \frac{1+\sin x}{(x-\cos x)(1+\cos x)}$$

75. (A) Differential equation

$\frac{dy}{dx} + y.\tan x = \sec x$

here P = $\tan x$ and Q = $\sec x$

I.F. = $e^{\int P.dx}$

I.F. = $e^{\int \tan x\, dx} = e^{\int \log \sec x} = \sec x$

Solution of differential equation

$y \times$ I.F. = $\int Q \times \text{I.F.} dx$

$\Rightarrow y \times \sec x = \int \sec x.\sec x\, dx$

$\Rightarrow y \times \sec x = \tan x + c$

$\Rightarrow y = \sin x + c.\cos x$

76. (B) The required Probability = $\frac{1+1}{7} = \frac{2}{7}$

77. (B) Given that U= {1, 2, 3, 4, 5, 6, 7, 8 , 9},

$A = \{7,8,3\}$, $B = \{3,8,9\}$ and $C = \{9,3,4\}$

Now, $(A \cup B) = \{3,7,8,9\}$, $(B \cap C) = \{3\}$

and $(A \cap C) = \{3\}$

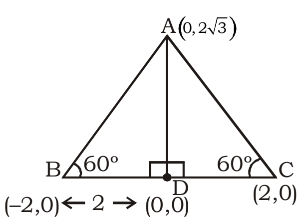
$\{(A \cup B) - (B \cap C)\} \times (A \cap C)$

$= [\{3,7,8,9\} - \{3\}] \times \{3\}$

$= \{7,8,9\} \times \{3\}$

$= \{(7,3),(8,3),(9,3)\}$

78. (C)

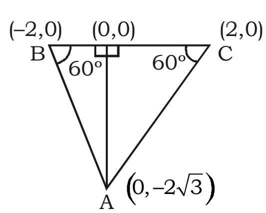


In ΔABD

$\tan 60° = \frac{AD}{BD}$

$\sqrt{3} = \frac{AD}{2} \Rightarrow AD = 2\sqrt{3}$

Similarly



Hence A = $(0, 2\sqrt{3})$ and $(0, -2\sqrt{3})$

79. (C) Given that $S_n = n^2 + 3n - 2$

$S_{n-1} = (n-1)^2 + 3(n-1) - 2$

$S_{n-1} = n^2 + n - 4$

Now, $T_n = S_n - S_{n-1}$

$T_n = n^2 + 3n - 2 - n^2 - n + 4$

$T_n = 2n + 2$

$T_7 = 2 \times 7 + 2 = 16$

80. (A) $= \begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}\begin{bmatrix} 3 & \alpha \\ \alpha & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9+\alpha^2 & 3\alpha+3\alpha \\ 3\alpha+3\alpha & \alpha^2+9 \end{bmatrix}$

$A^2 = \begin{bmatrix} 9+\alpha^2 & 6\alpha \\ 6\alpha & \alpha^2+9 \end{bmatrix}$

Given that det (A^2) = 0

$\Rightarrow \begin{bmatrix} 9+\alpha^2 & 6\alpha \\ 6\alpha & \alpha^2+9 \end{bmatrix} = 0$

$\Rightarrow (9+\alpha^2)^2 - 36\alpha^2 = 0$

$\Rightarrow 81 + \alpha^4 + 18\alpha^2 - 36\alpha^2 = 0$

$\Rightarrow \alpha^4 - 18\alpha^2 + 81 = 0$

$\Rightarrow (\alpha^2 - 9)^2 = 0$

$\Rightarrow \alpha^2 = 9 \Rightarrow \alpha = \pm 3$

81. (B) Let point (h, k)

According to question

$\frac{4h-3k-7}{\sqrt{4^2+(-3)^2}} = \frac{8h-15k-9}{\sqrt{8^2+(-15)^2}}$

$\Rightarrow \frac{4h-3k-7}{5} = \frac{8h-15k-9}{17}$

On solving

$\Rightarrow 14h + 12k = 37$

locus of point

$14x + 12y = 37$

82. (B) $\tan(\sin^{-1}x) + \tan(\cos^{-1}x)$

$\Rightarrow \tan\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)\right)$

$\Rightarrow \frac{x}{\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{x}$

$\Rightarrow \frac{x^2+1-x^2}{x\sqrt{1-x^2}} = \frac{1}{x\sqrt{1-x^2}}$

83. (C) $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right) dx$

$I = e^x.\sin^{-1}x + c$

$\left[\because \int e^x \left[f(x) + f'(x)\right]dx = e^x f(x) + c\right]$

84. (D) $\left(1-\cos\frac{\pi}{3}\right)\left(1-\cos\frac{2\pi}{3}\right)\left(1-\cos\frac{4\pi}{3}\right)$

$\left(1-\cos\frac{5\pi}{3}\right)$

$\Rightarrow \left(1-\cos\frac{\pi}{3}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{\pi}{3}\right)$

$\left(1-\cos\frac{\pi}{3}\right)$

$\Rightarrow \left(1-\cos^2\frac{\pi}{3}\right)\left(1-\cos^2\frac{\pi}{3}\right)$

$\Rightarrow \left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right) = \frac{9}{16}$

85. (B) $s = t\sqrt{t^2-1}$

On differentiating both sides w.r.t. 't'

$\frac{ds}{dt} = t \times \frac{1\times 2t}{2\sqrt{t^2-1}} + \sqrt{t^2-1}.1$

$\frac{ds}{dt} = \frac{t^2}{\sqrt{t^2-1}} + \sqrt{t^2-1} = \frac{2t^2-1}{\sqrt{t^2-1}}$

86. (C) Digits 0, 1, 3, 5, 8, 9, 6

6	6	5

= 6 × 6 × 5 = 180

87. (A) Given that $x^2 + y^2 = 8$

Let $A = x^2y^2$

$\Rightarrow A = x^2(8 - x^2)$

$\Rightarrow A = 8x^2 - x^4$

$\Rightarrow \frac{dA}{dx} = 16x - 4x^3$

$\Rightarrow \frac{d^2A}{dx^2} = 16 - 12x^2$

for maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 16x - 4x^3 = 0$

$\Rightarrow 4x(4 - x^2) = 0$

$\Rightarrow x = 0, 2, -2$

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=0} = 16 - 2 \times 0 = 16$ (minima)

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=2} = 16 - 12\times 2^2 = -32$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=-2} = 16 - 12(-2)^2 = -32$ (maxima)

Function minimum at $x = 0$, $y = 2\sqrt{2}$

Minimum value of $x^2y^2 = 0$

88. (C) We know that

$\sin ix = \frac{e^x - e^{-x}}{-2i}$ and $\cos ix = \frac{e^x + e^{-x}}{2}$

Now, $\cos ix - i\sin ix = \frac{e^x + e^{-x}}{2} - i \times \frac{e^x - e^{-x}}{-2i}$

$\Rightarrow \cos ix - i\sin ix = \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = e^x$

89. (B) Word "STATEMENT"

The total no. of arrangement $= \frac{9!}{3!2!} = \frac{9!}{12}$

No. of arrangement when T's come together $= \frac{7!}{2!} = \frac{7!}{2}$

No. of arrangement when T's don't come together $= \frac{9!}{12} - \frac{7!}{2}$

$= 6 \times 7! - \frac{7!}{2} = \frac{11 \times 7!}{2}$

90. (C) $y = \text{cosec}(\cot^{-1}x)$ ...(i)

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = -\text{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\frac{-1}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \text{cosec}(\cot^{-1}x).\frac{x}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{yx}{1+x^2}$ [from eq (i)]

$\Rightarrow (1 + x^2)dy = yx\,dx$

91. (B) $f(x) = \begin{cases} 3x^2 - 4, & 2 \le x < 4 \\ \lambda x + x^2, & 4 \le x < 6 \end{cases}$ is continuous at $x = 4$,

then $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

$\Rightarrow \lim_{x \to 4} 3x^2 - 4 = \lim_{x \to 4} \lambda x + x^2$

$\Rightarrow 3 \times 16 - 4 = \lambda \times 4 + 16$

$\Rightarrow 44 = 4\lambda + 16 \Rightarrow \lambda = 7$

92. (C) $\begin{bmatrix} x+7 & 13 \\ 5 & 2x \end{bmatrix} = \begin{bmatrix} y+8 & y+9 \\ y+1 & 10 \end{bmatrix}$

On comparing

$x + 7 = y + 8 \Rightarrow x - y = 1$, $13 = y + 9 \Rightarrow y = 4$

$5 = y + 1 \Rightarrow y = 4$, $2x = 10 \Rightarrow x = 5$

93. (A) $f(x) = \begin{cases} 2x^2 - 5, & -1 < x \le 3 \\ x - \lambda, & 3 < x \le 7 \end{cases}$ is

continuous at $x = 3$, then

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\Rightarrow 2 \times 3^2 - 5 = 3 - \lambda \Rightarrow \lambda = -10$

94. (B) $y = e^{\tan x}.\cos^2 x$

On differentiating both sides w.r.t 'x'

$\Rightarrow \frac{dy}{dx} = e^{\tan x}.\sec^2 x.\cos^2 x + e^{\tan x}.2\cos x.(-\sin x)$

$\Rightarrow \frac{dy}{dx} = e^{\tan x} - e^{\tan x}.\sin 2x$

$\Rightarrow \frac{dy}{dx} = e^{\tan x}(1 - \sin 2x)$

95. (A) A = $\{x \in R, x^2 + 3x - 28 \le 0\}$

$x^2 + 3x - 28 \le 0$

$(x + 7)(x - 4) \le 0$

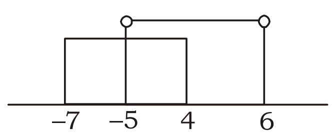
$-7 \le x \le 4$

and B = $\{x \in R, x^2 - x - 30 < 0\}$

$\Rightarrow x^2 - x - 30 < 0$

$\Rightarrow (x - 6)(x + 5) < 0$

$\Rightarrow -5 < x < 6$



Statement I

$(A \cup B) = \{x \in R, -7 \le x < 6\}$

Statement I is correct.

Statement II

$(A \cup B) = \{x \in R, -5 < x \le 4\}$

Statement II is incorrect.

96. (B) $\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} p & f & g \\ f & q & h \\ g & h & r \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ap + bf + cg & af + bq + ch & ag + bh + cr \end{bmatrix}$

97. (C) $\cos(2\sin^{-1}0.6)$

$\Rightarrow \cos\left(2\sin^{-1}\frac{3}{5}\right)$

$\Rightarrow \cos\left(2\tan^{-1}\frac{3}{4}\right)$ $\left[\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$

$\Rightarrow \cos\left(\tan^{-1}\frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}}\right)$ $\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right)$

$\Rightarrow \cos\left(\cos^{-1}\frac{7}{25}\right)$ $\left[\because \tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}\right]$

$\Rightarrow \frac{7}{25}$

98. (B) Series $1.2 + 2.3 + 3.4 + \ldots\ldots\ldots + n(n + 1)$

$T_n = n(n+1)$

$S_n = \sum T_n$

$S_n = \sum n(n+1)$

$S_n = \sum n^2 + \sum n$

$S_n = \frac{n}{6}(n+1)(2n+1) + \frac{n(n+1)}{2}$

$S_n = \frac{n(n+1)(n+2)}{3}$

99. (D) Differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y = \frac{1}{\left(\frac{d^2y}{dx^2}\right)^2}$

$\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{dy}{dx}\right)^3\left(\frac{d^2y}{dx^2}\right)^2 + y\left(\frac{d^2y}{dx^2}\right)^2 = 1$

Degree = 4

100. (C) $\sin(60 - x) + \sin(60 + x)$

We know that

$\sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2}$

$\Rightarrow 2\sin\frac{60-x+60+x}{2}.\cos\frac{60-x-60-x}{2}$

$\Rightarrow 2\sin 60.\cos(-x)$

$\Rightarrow 2 \times \frac{\sqrt{3}}{2}\cos x = \sqrt{3}\cos x$

101. (A) $\tan 2475 + \sin 2475$

$\Rightarrow \tan(360 \times 7 - 45) + \sin(360 \times 7 - 45)$

$\Rightarrow -\tan 45 - \sin 45$

$\Rightarrow -1 - \frac{1}{\sqrt{2}} = -\frac{\sqrt{2}+1}{\sqrt{2}}$

102. (C) $(\log_2 x)(\log_3 9) = \log_5 y$

$(\log_2 x)(\log_3 3^2) = \log_5 y$

$2(\log_2 x)(\log_3 3) = \log_5 y$

$(\log_2 x^2) = \log_5 y$ or $(\log_2 x)(\log_3 3) = \frac{1}{2}\log_5 y$

$x^2 = 2$ and $y = 5$ or $(\log_2 x) = \log_5 \sqrt{y}$

$x = \sqrt{2}$ and $y = 5$ or $x = 2$ and $\sqrt{y} = 5$

$x = \sqrt{2}$ and $y = 5$ or $x = 2$ and $y = 25$

103. (C) $y = a^{x+a^{x+a^{x+\ldots\ldots\infty}}}$

$\Rightarrow y = a^{x+y}$

On differentiating both sides w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = a^{x+y}\log_e a\left(1+\frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(1-a^{x+y}\log_e a\right) = a^{x+y}\log_e a$

$\Rightarrow \frac{dy}{dx} = \frac{a^{x+y}\log_e a}{1-a^{x+y}\log_e a}$

104. (A) $\sin^2 5 + \sin^2 10 + \sin^2 15 + \ldots\ldots + \sin^2 90$

$\Rightarrow \sin^2 5 + \sin^2 10 + \ldots. + \sin^2 40 + \sin^2 45 + \sin^2 50 + \ldots\ldots + \sin^2 80 + \sin^2 85 + 1$

$\Rightarrow (\sin^2 5 + \sin^2 85)(\sin^2 10 + \sin^2 80) + \ldots. \ldots. + (\sin^2 40 + \cos^2 50) + \sin^2 45 + 1$

$\Rightarrow (\sin^2 5 + \cos^2 5) + (\sin^2 10 + \cos^2 10) + \ldots. + (\sin^2 40 + \cos^2 40) + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$\Rightarrow 1 + 1 + \ldots\ldots$ 8 times $+ \frac{1}{2} + 1$

$\Rightarrow 8 + \frac{1}{2} + 1 = 9 + \frac{1}{2} = 9\frac{1}{2}$

105. (B) $x = a\sec\alpha.\cos\beta$, $y = b\tan\alpha$, $z = c\sec\alpha.\sin\beta$

Now, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}$

$\Rightarrow \frac{a^2\sec^2\alpha - \cos^2\beta}{a^2} - \frac{b^2\tan^2\alpha}{b^2} + \frac{c^2\sec^2\alpha.\sin^2\beta}{c^2}$

$\Rightarrow \sec^2\alpha.\cos^2\beta - \tan^2\alpha + \sec^2\alpha.\sin^2\beta$

$\Rightarrow \sec^2\alpha.\cos^2\beta + \sec^2\alpha.\sin^2\beta - \tan^2\alpha$

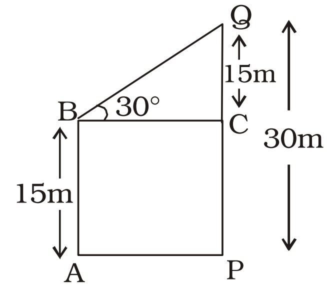
$\Rightarrow \sec^2\alpha(\cos^2\beta + \sin^2\beta) - \tan^2\alpha$

$\left[\because \sin^2 A + \cos^2 A = 1\right]$

$\Rightarrow \sec^2\alpha - \tan^2\alpha$ $\left[\because \sec^2 A - \tan^2 A = 1\right]$

$\Rightarrow 1$

106. (B)



In Δ BCQ :-

$\tan 30° = \frac{QC}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{BC} \Rightarrow BC = 15\sqrt{3}$

Distance between the poles = $15\sqrt{3}$ m

107. (B) Let $a + ib = \sqrt{-2+2\sqrt{35}i}$

On squaring both side

$(a^2 - b^2) + 2abi = -2 + 2\sqrt{35}\,i$

On comparing

$a^2 - b^2 = -2$ and $2ab = 2\sqrt{35}$ …(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 4 + 4 \times 35$

$\Rightarrow (a^2 + b^2)^2 = 144$

$\Rightarrow a^2 + b^2 = 12$ …(ii)

from eq(i) and eq(ii)

$2a^2 = 10$, $2b^2 = 14$

$a = \pm\sqrt{5}$, $b = \pm\sqrt{7}$

Hence $\sqrt{-2+2\sqrt{35}i} = \pm\left(\sqrt{5}+\sqrt{7}i\right)$

108. (C) Given that

$\begin{bmatrix} x+y & 3x+w \\ 2w+z & x-y \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ 6 & -4 \end{bmatrix}$

On comparing

$x + y = 12$, $3x + w = -7$

$2w + z = 6$, $x - y = -4$

On solving

$x = 4$, $y = 8$, $z = 44$, $w = -19$

109. (A) $I = \int \frac{5^x}{5^x - 1}\,dx$

Let $5^x - 1 = t$

$\Rightarrow 5^x \log 5\, dx = dt \Rightarrow 5^x\, dx = \frac{1}{\log 5}\,dt$

$I = \int \frac{1}{\log 5}\frac{1}{t}\,dt$

$I = \frac{1}{\log 5}\log t + c$

$I = \frac{\log(5^x - 1)}{\log 5} + c$

$I = \log_5(5^x - 1) + c$

110. (D) $i^{501} + i^{502} + i^{503} + i^{504} + i^{505}$

$\Rightarrow i^{501}(1 + i + i^2 + i^3 + i^4)$

$\Rightarrow i^{3\times167}(1 + i - 1 - i + 1)$

$\Rightarrow 1 \times 1 = 1$

111. (B) $^{23}C_4 + \sum_{r=1}^{4} {}^{23+r}C_3$

$\Rightarrow {}^{23}C_4 + {}^{23}C_3 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$

We know that

$^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{24}C_4 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$

$\Rightarrow {}^{25}C_4 + {}^{25}C_3 + {}^{26}C_3$

$\Rightarrow {}^{26}C_4 + {}^{26}C_3 = {}^{27}C_4$

112. (B) $\cos(x - iy) = A + iB$

$\Rightarrow \cos x.\cos iy + \sin x.\sin iy = A + iB$

We know that

$\cos iA = \cosh A$ and $\sin iA = i\sinh A$

$\Rightarrow \cos x.\cosh y + i\sin x.\sinh y = A + iB$

On comparing

$A = \cos x.\cosh y$, $B = \sin x.\sinh y$

113. (C) $x = a\cos\theta - b\sin\theta$ and $y = b\cos\theta + a\sin\theta$

$x^2 + y^2 = (a\cos\theta - \sin\theta)^2 + (b\cos\theta + a\sin\theta)^2$

$\Rightarrow x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta.\cos\theta + b^2\cos^2\theta + a^2\sin^2\theta + 2ab\sin\theta.\cos\theta$

$\Rightarrow x^2 + y^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

114. (A) We know that

$\sin ix = i\sinh y$

Now, $\sinh\left(\frac{i\pi}{3}\right) = -i\sin\left[i\left(\frac{i\pi}{3}\right)\right]$

$\Rightarrow \sinh\left(\frac{i\pi}{3}\right) = -i\sin\left(\frac{-\pi}{3}\right)$

$\Rightarrow \sinh\left(\frac{i\pi}{3}\right) = i\sin\frac{\pi}{3} = \frac{\sqrt{3}i}{2}$

115. (B) $\cosh x + \sinh x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = e^x$

116. (B) lines $x - 3y = -4$ ...(i)

$2x - y = 7$ ...(ii)

$4x - 5y = 11$ ...(iii)

Intersecting point of line (i) and (ii) is (5, 3).

Let the equation of line which is perpendicular to the line (iii)

$5x + 4y = c$ ...(iv)

its passes through the point (5, 3)

$5 \times 5 + 4 \times 3 = c \Rightarrow c = 37$

from eq (iv)

$5x + 4y = 37$

117. (B) $b(c\cos A - a\cos C)$

$\Rightarrow b\left[c.\frac{b^2 + c^2 - a^2}{2bc} - a.\frac{a^2 + b^2 - c^2}{2ab}\right]$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2} - \frac{a^2 + b^2 - c^2}{2} \Rightarrow c^2 - a^2$

118. (A) $I = \int \cos(\log x)dx$

Let $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$I = \int \cos t\, e^t\,dt$ ...(i)

$I = \cos t\int e^t dt - \int\left\{\frac{d}{dt}(\cos t).\int e^t dt\right\}$

$I = \cos t.\, e^t - \int -\sin t.e^t dt$

$I = e^t.\cos t + \int \sin t.e^t dt$

$I = e^t.\cos t + \sin t.\int e^t dt - \int\left\{\frac{d}{dt}(\sin t).\int e^t dt\right\} dt$

$I = e^t.\cos t + \sin t.e^t - \int \cos t.e^t dt + c$

$I = e^t(\sin t + \cos t) - I + c$ [from eq(i)]

$2I = e^t(\sin t + \cos t) + c$

$I = \frac{1}{2}e^t(\sin t + \cos t) + c$

$I = \frac{1}{2}x[\sin(\log x) + \cos(\log x)] + c$

119. (C) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} x+a & b & c \\ -x & x & 0 \\ -x & 0 & x \end{vmatrix} = 0$

$\Rightarrow (x + a)x^2 - b(-x^2) + cx^2 = 0$

$\Rightarrow x^3 + ax^2 + bx^2 + cx^2 = 0$

$\Rightarrow x^2(x + a + b + c) = 0$

$x + a + b + c = 0$, $x = 0$

Hence $x = -(a + b + c)$

120. (B) $7x - 6y + 20 = 0$

and $7x - 6y - 12 = 0$

The required line

$7x - 6y + 4 = 0$

NDA (MATHS) MOCK TEST - 168 (Answer Key)

1.	(D)	21.	(D)	41.	(A)	61.	(B)	81.	(B)	101.	(A)
2.	(B)	22.	(B)	42.	(C)	62.	(C)	82.	(B)	102.	(C)
3.	(C)	23.	(B)	43.	(B)	63.	(A)	83.	(C)	103.	(C)
4.	(D)	24.	(C)	44.	(C)	64.	(C)	84.	(D)	104.	(A)
5.	(C)	25.	(C)	45.	(C)	65.	(D)	85.	(B)	105.	(B)
6.	(B)	26.	(C)	46.	(B)	66.	(C)	86.	(C)	106.	(B)
7.	(B)	27.	(D)	47.	(C)	67.	(B)	87.	(A)	107.	(B)
8.	(C)	28.	(B)	48.	(A)	68.	(A)	88.	(C)	108.	(C)
9.	(C)	29.	(C)	49.	(B)	69.	(A)	89.	(B)	109.	(A)
10.	(C)	30.	(B)	50.	(C)	70.	(A)	90.	(C)	110.	(D)
11.	(B)	31.	(B)	51.	(A)	71.	(D)	91.	(B)	111.	(B)
12.	(B)	32.	(A)	52.	(B)	72.	(B)	92.	(C)	112.	(B)
13.	(A)	33.	(C)	53.	(A)	73.	(A)	93.	(A)	113.	(C)
14.	(C)	34.	(B)	54.	(C)	74.	(A)	94.	(B)	114.	(A)
15.	(A)	35.	(D)	55.	(D)	75.	(A)	95.	(A)	115.	(B)
16.	(B)	36.	(A)	56.	(C)	76.	(B)	96.	(B)	116.	(B)
17.	(C)	37.	(A)	57.	(B)	77.	(B)	97.	(C)	117.	(B)
18.	(C)	38.	(D)	58.	(C)	78.	(C)	98.	(B)	118.	(A)
19.	(A)	39.	(C)	59.	(A)	79.	(C)	99.	(D)	119.	(C)
20.	(C)	40.	(D)	60.	(C)	80.	(A)	100.	(C)	120.	(B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777