NDA MATHS MOCK TEST - 174 (SOLUTION)

1. (A) $fof\,(x) = f\,(f\,(x))$
$= f\{(3 - x^3)^{1/3}\}$
$= [3 - \{(3 - x^3)^{1/3}\}^3]^{1/3}$
$= (x^3)^{1/3} = x$

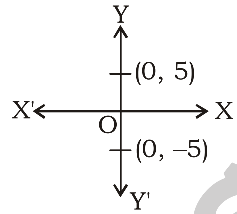
2. (C) It is given,
$f(\theta) = \sin\theta(\sin\theta + \sin3\theta)$
$= \sin\theta(\sin\theta + 3\sin\theta - 4\sin^3\theta)$
$= \sin\theta(4\sin\theta - 4\sin^3\theta)$
$= 4\sin^2\theta(1 - \sin^2\theta)$
$= 4\sin^2\theta\cos^2\theta$
$= (2\sin\theta\cos\theta)^2 = (\sin2\theta)^2 \geq 0$
which is true $\forall\theta$ (Real)

3. (D) Given, $f(x) = \cos(\log x)$
$\therefore f(x).f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$
$= \cos(\log x).\cos(\log y) -$
$\frac{1}{2}\left[\cos\left[\log\left(\frac{x}{y}\right)\right] + \cos[\log x(xy)]\right]$
$= \cos(\log x).\cos(\log y) - \frac{1}{2}[\cos[\log x - \log y] + \cos[\log x + \log y]]$
$= \cos(\log x).\cos(\log y) - \frac{1}{2}[2\cos(\log x).\cos(\log y)]$
$= \cos(\log x).\cos(\log y) - \cos(\log x).\cos(\log y)$
$= 0$

4. (C) We know that
$|A^n| = |A|^n$
$|A^3| = 125 \Rightarrow |A|^3 = 125$
$\Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$
$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha = \pm 3$

5. (B) Since, $|w| = 1 \Rightarrow \left|\frac{1 - iz}{z - i}\right| = 1$
$\Rightarrow |z - i| = |1 - iz|$
$\Rightarrow |z - i| = |z + i|$
$[\because |1 - iz| = |i|\,|z + i| = |z + i|]$
$\therefore$ It is a perpendicular bisector of (0, 1) and (0, – 1)
i.e. X-axis. Thus, z lies on the real axis.

6. (D) Given, $\left|\frac{z - 5i}{z + 5i}\right| = 1 \Rightarrow |z - 5i| = |z + 5i|$
[$\because$ if $|z - z_1| = |z - z_2|$, then it is a perpendicular bisector of z_1 and z_2]



$\therefore$ Perpendicular bisector of (0, 5) and (0, – 5) is X-axis.

7. (B) $\sum_{n=1}^{13}(i^n + i^{n+1}) = \sum_{n=1}^{13} i^n(1 + i) = (1 + i)\sum_{n=1}^{13} i^n$
$= (1 + i)(i + i^2 + i^3 + \ldots.. + i^{13}) = (1 + i)\left[\frac{i(1 - i)}{1 - i}\right]$
$= (1 + i)\,i = i - 1$

8. (D) Given that, $z = \cos\theta + i\sin\theta = e^{i\theta}$
$\therefore \sum_{m=1}^{15}\text{Im}(z^{2m-1}) = \sum_{m=1}^{15}\text{Im}(e^{i\theta})^{2m-1}$
$= \sum_{m=1}^{15}\text{Im}\, e^{i(2m-1)\theta}$
$= \sin\theta = \sin3\theta + \sin5\theta + \ldots. + \sin29\theta$
$= \frac{\sin\left(\frac{\theta + 29\theta}{2}\right)\sin\left(\frac{15 \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$
$= \frac{\sin(15\theta)\sin(15\theta)}{\sin\theta} = \frac{1}{4\sin 2^\circ}$

9. (B) Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 - 1 \\ 1 & \omega^2 & \omega \end{vmatrix}$
Applying $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$
$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2-\omega^2 & \omega^2 - 1 \\ 0 & \omega^2 - 1 & \omega - 1 \end{vmatrix}$
$= (-2 - \omega^2)(\omega - 1) - (\omega^2 - 1)^2$
$= -2\omega + 2 - \omega^2 + (\omega^4 - 2\omega^2 + 1)$
$= 3\omega^2 - 3\omega = 3\omega(\omega - 1)$ $[\omega^4 = \omega]$

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10. (B) Sum of roots = $\frac{\alpha^2+\beta^2}{\alpha\beta}$ and product = 1

Given $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$

$\Rightarrow (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = q$

$\therefore\ \alpha^2 + \beta^2 - \alpha\beta = \frac{-q}{p}$ ...(i)

and $(\alpha + \beta)^2 = p^2$ ...(ii)

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = p^2$

From eq(i) and eq(ii), we get

$\alpha^2 + \beta^2 = \frac{p^3-2q}{3p}$ and $\alpha\beta = \frac{p^3+q}{3p}$

$\therefore$ Required equation is, $x^2 - \frac{(p^3-2q)x}{(p^3+q)} + 1 = 0$

$\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

11. (B) Given, $x^2 - |x + 2| + x > 0$ ...(i)

Case I When $x + 2 > 0$

$\therefore\ x^2 - x - 2 + x > \Rightarrow 0\ x^2 - 2 > 0$

$\Rightarrow x < -\sqrt{2}$ or $x > \sqrt{2}$

$\Rightarrow x \in [-2, -\sqrt{2}) \cup (\sqrt{2}, \infty)$ ...(ii)

Case II When $x + 2 < 0$

$\therefore\ x^2 + x + 2 + x > 0$

$\Rightarrow x^2 + 2x + 2 > 0$

$\Rightarrow (x + 1)^2 + 1 > 0$

which is true for all x.

$\therefore\ x \le -2$ or $x \in (-\infty, -2)$ ...(ii)

From eq(ii) and eq(iii), we get

$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

12. (A) Given, $x - \frac{2}{x-1} = 1 - \frac{2}{x-1} \Rightarrow x = 1$

But at $x = 1$, the given equation is not defined.

Hence, no solution exist.

13. (C) We have,

$225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 5bc = 0$

$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - (15a)(5c) - (15a)(3b) - (3b)(5c) = 0$

$\Rightarrow \frac{1}{2}(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2 = 0$

$\Rightarrow 15a = 3b,\ 3b = 5c$

and $5c = 15a$

$\therefore\ 15a = 3b = 5c$

$\Rightarrow \frac{a}{1} = \frac{b}{5} = \frac{c}{3} = \lambda$(say)

$\Rightarrow a$, c and b are in AP.

14. (C) Let $T_m = a + (m - 1)d = \frac{1}{n}$ ...(i)

and $T_n = a + (n - 1)d = \frac{1}{m}$ ...(ii)

On subtracting eq(ii) from eq(i), we get

$(m - n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$

$\Rightarrow d = \frac{1}{mn}$

Again, $T_{mn} = a + (mn - 1)d = a + (mn - n + n - 1)d$

$= a + (n - 1)d + (mn - n)d$

$= T_n + n(m - 1)\frac{1}{mn}$

$= \frac{1}{m} + \frac{(m-1)}{m} = 1$

15. (C) Since, $(\alpha + \beta)$, $(\alpha^2 + \beta^2)$, $(\alpha^3 + \beta^3)$ are in GP.

$\Rightarrow (\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = \alpha^4 + \beta^2 + \alpha\beta^3 + \beta\alpha^3$

$\Rightarrow \alpha\beta(\alpha^2 + \beta^2 + 2\alpha\beta^2) = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha = \beta$

$\Rightarrow \frac{c}{a} = 0$ or $\Delta = 0$

$\Rightarrow c\Delta = 0$

16. (B) Here, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \le 0$

$\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \le 0$

$\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \le 0$

[since, sum of squares is never less than zero]

Since, each of the squares is zero.

$\therefore\ (ap - b)^2 = (bp - c)^2 = (cp - d)^2 = 0$

$\Rightarrow p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

$\therefore$ a, b, c, d are in GP.

17. (D) Let a, ar, ar^2 are in GP, where $(r > 1)$.

On multiplying middle term by , we have a, $2ar$, ar^2 are in an AP.

$\Rightarrow 4ac = a + ar^2$

$\Rightarrow r^2 - 4ar = a + ar^2$

$\Rightarrow r^2 - 4r + 1 = 0$

$\Rightarrow r = \frac{4 \pm \sqrt{16-4}}{2}$

$= 2 \pm \sqrt{3}$

$\Rightarrow r = 2 + \sqrt{3}$ [since, AP is increasing]

18. (A) Since, a, b, c, d are in AP.

$\Rightarrow \frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd}$ are in AP.

$\Rightarrow \frac{1}{bcd}, \frac{1}{cda}, \frac{1}{abd}, \frac{1}{abc}$ are in AP.

$\Rightarrow bcd, cda, abd, abc$ are in HP.

$\Rightarrow abc\ abd, cda, bcd$ are in HP.

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19. (B) Let the common ratio of the GP be r. Then, $y = xr$ and $z = xr^2$

$\Rightarrow \ln y = \ln x + \ln r$ and $\ln z = \ln x + 2\ln r$

Let $A = 1 + \ln x$, $D = \ln r$

Then, $\frac{1}{1+\ln z} = \frac{1}{A}$, $\frac{1}{1+\ln y} = \frac{1}{1+\ln x+\ln r}$

$= \frac{1}{A+D}$

and $\frac{1}{1+\ln z} = \frac{1}{1+\ln x+2\ln r} = \frac{1}{A+2D}$

Therefore, $\frac{1}{1+\ln z}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in HP.

20. (A) Since, AM $\geq$ GM, then

$\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)}$ $M \leq 1$

Also, $(a + b) + (c + d) > 0$ [$\because a,b,c,d > 0$]

$\therefore 0 < M \leq 1$

21. (B) Let a, b be the roots of given quadratic equation. Then, $\alpha + \beta = \frac{4+\sqrt{5}}{5+\sqrt{2}}$ and $\alpha\beta = \frac{8+2\sqrt{5}}{5+\sqrt{2}}$

Let H be the harmonic mean between a and b, then

$H = \frac{2\alpha\beta}{\alpha+\beta} = \frac{16+4\sqrt{5}}{4+\sqrt{5}} = 4$

22. (C) Let n be the number of newspapers which are read by the students.

Then, $60n = (300)\times 5$

$\Rightarrow n = 25$

23. (B) Total number of five letters words formed from ten different letters $= 10\times10\times10\times10\times10 = 10^5$

Number of five letters words having no repetition

$= 10\times9\times8\times7\times6 = 30240$

24. (B) Given, $T_n = {}^nC_3 \Rightarrow T_{n+1} = {}^{n+1}C_3$

$\therefore T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$ [Given]

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 10$

[$\therefore {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$]

$\Rightarrow {}^nC_2 = 10$

$\Rightarrow n = 5$

25. (D) We have, $X = \{4^n - 3n - 1 : n \in N\}$

$X = \{0, 9, 54, 243, ...\}$ [put $n = 1,2,3,...$]

$Y = \{9, n - 1: n \in N\}$

$Y = \{0, 9, 18, 27....\}$ [put $n = 1,2,3,...$]

It is clear that, $X \subset Y$

$\therefore X \cup Y = Y$

26. (C) $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{(x-1)}{x-x^{1/2}}\right]^{10}$

$= \left[\frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1} - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$= \left[\frac{(x^{1/3}+1)(x^{2/3}+1-x^{1/3})}{x^{2/3}-x^{1/3}+1} - \frac{\{(\sqrt{x})^2-1\}}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$= \left[(x^{1/3}+1) - \frac{(\sqrt{x}+1)}{\sqrt{x}}\right]^{10} = (x^{1/3} - x^{-1/2})^{10}$

$\therefore$ The general term is

$T^{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r = {}^{10}C_r (-1)^r \, x^{\frac{10-r}{3}-\frac{r}{2}}$

For independent of x, put

$\frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow 20 = 5r \Rightarrow r = 4$

$\therefore T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$

27. (D) Since, three distinct numbers are to be selected from first 100 natural numbers.

$\Rightarrow n(S) = 100C3$

$E_{(\text{favouable events})}$ = All three of them are divisible by both 2 and 3.

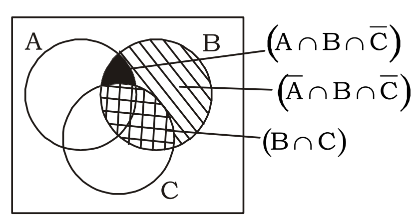
$\Rightarrow$ Divisible by i.e. {6, 12, 18,....., 96}

Thus, out of 16 we have to select 3.

$\therefore n(E) = {}^{16}C_3$

$\therefore$ Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3} = \frac{4}{1155}$

28. (A) Given $= P(B) = \frac{3}{4}, \left(A \cap B \cap \overline{C}\right) = \frac{1}{3}$



and $P\left(\overline{A} \cap B \cap \overline{C}\right) = \frac{1}{3}$

Which can be shown in venn diagram.

$\therefore P(B \cap C) = P(B) - \{P(A \cap B \cap \overline{C} + \overline{A} \cap B \cap \overline{C}))\}$

$= \frac{3}{4} - \left(\frac{1}{3} + \frac{1}{3}\right) = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

= Probability that the first machine tested is faulty × Probability that the second machine tested is faulty = $\frac{2}{4}\times\frac{1}{3}=\frac{1}{6}$

30. (A) Given, $P(\overline{A\cup B})=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$, $P(\overline{A})=\frac{1}{4}$

$\therefore\ P(A\cup B) = 1 - P(\overline{A\cup B}) = 1-\frac{1}{6}=\frac{5}{6}$

and $P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4}=\frac{3}{4}$

$\therefore\ P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{5}{6}=\frac{3}{4}+P(B)-\frac{1}{4}$

$\Rightarrow P(B)=\frac{1}{3}\Rightarrow$ A and B are not equally likely

$P(A\cap B) = P(A).P(B) = \frac{1}{4}$

So, events are independent.

31. (B) Given , $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0\end{vmatrix}=0$

Applying $C_3 \to C_3 - (\alpha C_1 + C_2)$

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -(a\alpha^2+2b\alpha+c)\end{vmatrix}=0$$

$\Rightarrow -(a\alpha^2 + 2b\alpha + c)\ (ac - b^2) = 0$

$\Rightarrow a\alpha^2 + 2b\alpha + c = 0$ or $b^2 = ac$

$\Rightarrow x - \alpha$ is a factor of $ax^2 + 2bx + c$ or a, b c, are in GP.

32. (C) Clearly, for f to be continuous at $x = \pi$,

$f(x) = \lim\limits_{x\to\pi}\frac{1-\sin x+\cos x}{1+\sin x+\cos x}$ $\left[\frac{0}{0}\text{ from}\right]$

$f(x) = \lim\limits_{x\to\pi}\frac{-\cos x-\sin x}{\cos x-\sin x}$

[by L' Hospital rule]

$= \left[\frac{-\cos\pi-\sin\pi}{\cos\pi-\sin\pi}\right]=\frac{-(-1)-0}{-1-0}=\frac{1}{-1}=-1$

33. (D) Given, $\lim\limits_{x\to0}\frac{2(1-\cos x)}{x^2}=\lim\limits_{x\to0}\frac{2\left(1-1+2\sin^2\frac{x}{2}\right)}{x^2}$

$\left[\because \cos x = 1-2\sin^2\frac{x}{2}\right]$

$\lim\limits_{x\to0}\frac{4\sin^2\frac{x}{2}}{x^2}=\lim\limits_{x\to0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = (1)^2 = 1$

$\left[\because \lim\limits_{\theta\to0}\frac{\sin\theta}{\theta}=1\right]$

34. (A) Consider,

$\lim\limits_{x\to\infty}\left(\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}\right)$

$=\lim\limits_{x\to\infty}\left(\frac{a^2x^2+ax+1-a^2x^2-1}{\sqrt{a^2x^2+ax+1}+\sqrt{a^2x^2+1}}\right)$

[by rationalising]

$=\lim\limits_{x\to\infty}\frac{a}{\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}}$

$=\frac{a}{\sqrt{a^2}+\sqrt{a^2}}=\frac{a}{2a}=\frac{1}{2}$

35. (D) Consider, $\lim\limits_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$

$=\lim\limits_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+4}{5}\cdot\frac{5}{x+1}(x+1)}$

$=\lim\limits_{x\to\infty}\left[\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{5\cdot\frac{x+4}{x+1}}$

$= e^{5\lim\limits_{x\to\infty}\frac{1+\frac{4}{x}}{1+\frac{1}{x}}} = e^5$ $\left[\because \lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e\right]$

36. (A) Let $f(x) = \frac{(x-1)^2}{|x-1|}$

$\Rightarrow f(x) = \begin{cases}(x-1), & x\ge1\\ -(x-1), & x<1\end{cases}$

Now, LHL = $\lim\limits_{h\to0}f(1-h)$

$\lim\limits_{h\to0}[-(1-h-1)]=\lim\limits_{h\to0}h=0$

and RHL = $\lim\limits_{h\to0}f(1+h)$

$=\lim\limits_{h\to0}(1+h-1) = \lim\limits_{h\to0}h=0$

$\because$ LHL = RHL

$\Rightarrow \lim\limits_{x\to1}\frac{(x-1)^2}{|x-1|}=0$

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37. (A) AM : GM = $m : n$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \; \frac{(a+b)^2}{4ab} = \frac{m^2}{n^2} \quad \ldots(i)$$

and $\frac{(a+b)^2 - 4ab}{4ab} = \frac{m^2 - n^2}{n^2}$

[by componendo]

$$\Rightarrow \frac{(a-b)^2}{4ab} = \frac{m^2 - n^2}{n^2} \quad \ldots(ii)$$

From eq(i) and eq(ii),

$$\frac{(a+b)^2}{(a-b)^2} = \frac{m^2}{m^2 - n^2} \Rightarrow \frac{(a+b)}{(a-b)} = \frac{m}{\sqrt{m^2 - n^2}}$$

[using componendo and dividendo rule]

$$\Rightarrow \frac{a}{b} = \frac{m + \sqrt{m^2 - n^2}}{m - \sqrt{m^2 - n^2}}$$

38. (C) We have, the series

$\sqrt{2} + 2\sqrt{2} + 3\sqrt{2} + 4\sqrt{2} + \ldots.$ which is an AP.

$\because a = \sqrt{2}$ and $d = \sqrt{2}$

$$\because S_n = \frac{n}{2}[2a + (n+1)d]$$

$$\because S_n = \frac{n}{2}\left[2\sqrt{2} + (n-1)\sqrt{2}\right]$$

$$= \frac{n}{2}\left[\sqrt{2}n + \sqrt{2}\right] = \frac{n}{2}\sqrt{2}(n+1)$$

$$= \frac{n}{\sqrt{2}}(n+1)$$

39. (A) series = 0.9 + 0.09 + 0.009 + ...

= 9{0.1 + 0.01 + 0.001 + ...}

$$= \left\{\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots..\right\}$$

$= 9\{10^{-1} + 10^{-2} + 10^{-3} + \ldots.\}$

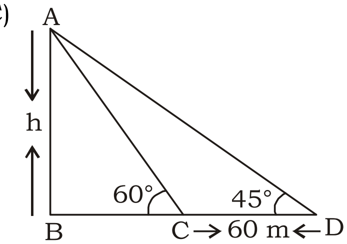
$$= \frac{9}{10}\left\{1 + \left(\frac{1}{10}\right)^1 + \left(\frac{1}{10}\right)^1 + \ldots..\right\}$$

$$= \left[\begin{array}{l}\text{which form an infinite GP} \\ \text{with common ratio } \left(\frac{1}{10}\right)\end{array}\right]$$

$$= \frac{9}{10} \times \frac{1}{\left(1 - \frac{1}{10}\right)} = \frac{9}{10} \times \frac{10}{9} = 1$$

$$\left[\because S_\infty = \frac{a}{1-r}\right]$$

40. (C)



AB height of tower

In ΔABC

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC} \Rightarrow AB : BD = \sqrt{3} : 1 \quad \ldots(i)$$

In ΔABD

$$\tan 45° = \frac{AB}{BC}$$

$$1 = \frac{AB}{BD} \Rightarrow AB : BD = 1 : 1 \quad \ldots(ii)$$

Now,

BD	: AB :		BC	
1	: 1			
	$\sqrt{3}$	:	1	
$\sqrt{3}$	: $\sqrt{3}$	:	1	

CD = BD – BC

$= \left(\sqrt{3} - 1\right)$ unit

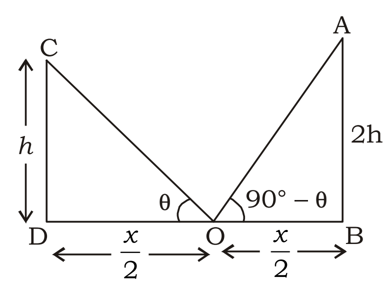
$\Rightarrow \left(\sqrt{3} - 1\right)$ unit = 60 metre

$$1 \text{ unit} = \frac{60}{\sqrt{3} - 1}$$

$$AB = \sqrt{3} \text{ units} = \frac{60}{\sqrt{3} - 1} \times \sqrt{3}$$

$= 30\left(3 + \sqrt{3}\right)$ m

41. (A)



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From figure,

OB = OD = $\frac{x}{2}$

In ΔOCD,

$\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$ ...(i)

In ΔAOB

$\tan(90° - \theta) = \frac{AB}{OB}$

$\cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x}$ ...(ii)

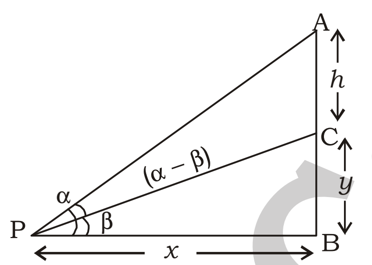
multiplying both equations,

$\tan\theta.\cot\theta = \frac{2h}{x} \times \frac{4h}{x}$

$\Rightarrow x^2 = 8h^2$

$\Rightarrow h^2 = \frac{x^2}{8} \Rightarrow h = \frac{x}{2\sqrt{2}}$ metre

42. (A) Let the height of the tower = y = BC



Now, In ΔCPB,

$\Rightarrow \tan\beta = \frac{y}{x}$

and in ΔAPB,

$\Rightarrow \tan\alpha = \frac{AB}{BP} = \frac{AC+BC}{BP}$

$\Rightarrow \tan\alpha = \frac{h+y}{x}$

$\Rightarrow y + h = x.\tan\alpha$

$\Rightarrow y + h = \frac{y}{\tan\beta}.\tan\alpha$

[from eq(i)]

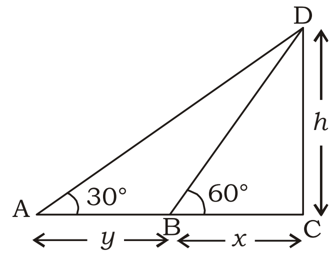
$\Rightarrow y\left(1 - \frac{\tan\alpha}{\tan\beta}\right) = -h$

$y(\tan\alpha - \tan\beta) = h\tan\beta$

∴ Height of the power,

$y = \frac{h\tan\beta}{(\tan\alpha - \tan\beta)}$

43. (C) Let DC be the height of the tree and BC = x m = h m



In ΔACD, $\tan 30° = \frac{CD}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$

$\Rightarrow x + y = h\sqrt{3}$ ...(i)

and in ΔCBD, $\tan 60° = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(ii)

From eq(i) and eq(ii),

$\frac{h}{\sqrt{3}} + y = h\sqrt{3} \Rightarrow y = h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}$

$\therefore h = \frac{\sqrt{3}y}{2}$ m

44. (D) $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{2x+3x}{1-6x^2}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow (6x - 1)(x + 1) = 0$

$\Rightarrow x = \frac{1}{6}$ or $x = -1$

45. (A) $\sin(-600°) = -\sin 600°$

$= -\sin(2 \times 360° - 120°)$

$= \sin 120° = \sin(180° - 60°)$

$= \sin 60° = \sin\frac{\pi}{3}$

$\sin^{-1}[\sin(-600°)] = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$

$= \left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{6}$

Given Exp. = $\left(\frac{\pi}{3} + \frac{5\pi}{6}\right) = \frac{7\pi}{6}$

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46. (A) Given Exp.

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\frac{1-\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)}\right]$$

$$= \tan^{-1}\frac{x}{y} - \left[\tan^{-1}1 - \tan^{-1}\frac{y}{x}\right]$$

$$= \left(\tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y}\right) - \frac{\pi}{4}$$

$$= \left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \frac{\pi}{4}$$

47. (B) Putting $x^2 = \cos 2\theta$, we get :

$$\text{L.H.S. } \tan^{-1}\left\{\frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1-\tan\theta}{1+\tan\theta}\right\}$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \left(\frac{\pi}{4} - \theta\right)$$

$$\alpha = \frac{\pi}{4} - \theta$$

$$\Rightarrow 2\alpha = \left(\frac{\pi}{2} - 2\theta\right)$$

$$\Rightarrow \sin 2\alpha = \sin\left(\frac{\pi}{2} - 2\theta\right) = \cos 2\theta = x^2.$$

48. (A) $2\tan^{-1}x = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$

$$\Rightarrow \tan^{-1}x = \frac{1}{2}\cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$$

Putting $\frac{1-x^2}{1+x^2} = \frac{\sqrt{2}}{3}$, we get

$$x^2 = \frac{(3-\sqrt{2})}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})} = \frac{(3-\sqrt{2})^2}{7}$$

$$\Rightarrow x = \frac{3-\sqrt{2}}{\sqrt{7}}$$

$$\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{2}}{3}\right) = \tan(\tan^{-1}x)$$

$$= \frac{(3-\sqrt{2})}{\sqrt{7}}$$

49. (A) $\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$ ...(i)

$$\because 2\tan^{-1}\frac{1}{5} = \tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right)$$

$$\left[\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} \; for\, |x| < 1\right]$$

$$= \tan^{-1}\left(\frac{5}{12}\right)$$

Now, from eq.(i),

$$\tan\left(\tan^{-1}\frac{5}{12} - \frac{\pi}{4}\right) = \frac{\tan\left(\tan^{-1}\frac{5}{12}\right) - \tan\frac{\pi}{4}}{1+\tan\left(\tan^{-1}\frac{5}{12}\right).\tan\frac{\pi}{4}}$$

$$\left[\because \tan(A-B) = \frac{\tan A - \tan B}{1+\tan.\tan B}\right]$$

$$= \frac{\frac{5}{12} - 1}{1+\frac{5}{12}(1)} = \frac{-7}{17}$$

50. (A) $bc\cos^2\frac{A}{2} + ca\cot\frac{B}{2} + \text{ab co}2\frac{C}{2}$

$$= \text{bc}.\frac{s(s-a)}{bc} + ca.\frac{s(s-b)}{ca} + ab.\left(s\frac{(s-c)}{ab}\right)$$

= s(s – a) + s (s – b) + s (s – c)

= s[(s – a) + (s – b) + (s – c)]

= s (3s –(a + b + c)] = s (3s –2s) =s^2

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51. (C) $\tan\frac{A}{2}.\tan\frac{C}{2} = \left(\frac{5}{6}\times\frac{2}{5}\right) = \frac{1}{3}$

$\Rightarrow \tan^2\frac{A}{2}.\tan^2\frac{C}{2} = \frac{1}{9}$

$\Rightarrow \frac{(s-b)(s-c)}{s(s-a)}.\frac{(s-a)(s-b)}{s(s-c)} = \frac{1}{9}$

$\Rightarrow \frac{s-b}{s} = \frac{1}{3}$

$\Rightarrow 3s - 3b = s$

$\Rightarrow 2s - 3b = 0$

$\Rightarrow (a + b + c) - 3b = 0$

$\Rightarrow 2b = a + c$

$\Rightarrow a, b, c$ and in A.P.

52. (A) $\frac{R}{r} = \frac{abc}{4\Delta}.\frac{s}{\Delta} = \frac{abc}{4(s-a)(s-b)(s-c)}$

Let $a = 3K$, $b = 7K$, $c = 8K$

Then, $s = 9K$, $(s - a) = 6K$

$(s - b) = 2K$ and $(s - c) = K$

$\therefore \frac{R}{r} = \frac{3K\times7K\times8K}{4\times6K\times2K\times K} = \frac{7}{2}$

53. (C) Given in ΔABC, $a = 6$ cm

$b = 10$ cm,

$c = 14$ cm

Since, c is the long side among three sides and the obtuse angle of ΔABC is the corresponding angle of side c.

By cosine law,

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(6)^2+(10)^2-(14)^2}{2.6.10}$

$\Rightarrow \cos C = \frac{36+100-196}{2.6.10} = \frac{136-196}{2.6.10}$

$\Rightarrow \cos C = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\therefore C = \frac{2\pi}{3} = 120°$

54. (A) Let $\angle A = 30°$, $\angle B = 45°$ and

AB = $\angle 3 + 1$

Then $\angle C = 180° - (\angle A + \angle B)$

[since, then sum of internal angles of a triangle is 180°]

$\Rightarrow \angle C = 180° - (30° + 45°)$

$= 180° - 75° = 105°$

Now, by sine rule, $\frac{\sin 30°}{BC} = \frac{\sin 105°}{\sqrt{3}+1}$

$\therefore BC = (\sqrt{3}+1)\times\left(\frac{2\sqrt{2}}{\sqrt{3}+1}\right)\times\frac{1}{2} = \sqrt{2}$

$$\left[\begin{aligned}&\because \sin 105° = \sin(60+45°)\\&= \sin 60°.\cos 45° + \cos 60°.\sin 45°\\&= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}\end{aligned}\right]$$

Again, now by sine rule,

$\frac{\sin 45°}{AC} = \frac{\sin 105°}{\sqrt{3}+1}$

$\Rightarrow AC = \frac{(\sqrt{3}+1)}{\sqrt{2}}\times\frac{2\sqrt{2}}{(\sqrt{3}+1)} = 2$

$\therefore$ After of ΔABC, $\frac{1}{2}\times BC\times AC\times \sin 105°$

$= \frac{1}{2}\times 2\times\sqrt{2}\times\frac{(\sqrt{3}+1)}{2\sqrt{2}} = \frac{(\sqrt{3}+1)}{2}$ cm^2

55. (B) $\frac{\cos 7x - \cos 3x}{\sin 7x - 2\sin 5x + \sin 3x}$

$= \frac{-2\sin\frac{7x+3x}{2}.\sin\frac{7x-3x}{2}}{2\sin\frac{7x+3x}{7}.\cos\frac{7x-3x}{2} - 2\sin 5x}$

$$\left[\begin{aligned}&\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)\\&\textit{and}\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)\end{aligned}\right]$$

$= \frac{-2\sin 5x.\sin 2x}{2\sin 5x.\cos 2x - 2\sin 5x}$

$= \frac{-2\sin 5x.\sin 2x}{2\sin 5x[1-\cos 2x]}$

$= \frac{\sin 2x}{1-1+2\sin^2 x}$

[$\because \cos 2A = 1 - 2\sin^2 A$]

$= \frac{2\sin x\cos x}{2\sin^2 x} = \cot x$

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56. (B) $(1 - \sin A + \cos A)^2$
$= 1 + \sin^2 A + \cos^2 A - 2\sin A - 2\sin A\cos A + 2\cos A$
$[\because (a + b + c)^2 = a^2 + b^2 + c^3 + 2ab + 2bc + 2ca]$
$= 2 - 2\sin A - \sin 2A + 2\cos A$
$= 2(1 + \cos A) - 2\sin A(1 + \cos A)$
$= 2(1 - \sin A)(1 + \cos A)$

57. (D) I. Let $f_1(x) = \sin|x| + \cos|x|$ depend on its angles.
$\because [0, \pi/2] \rightarrow \sin x > 0$ and $\cos x > 0$
$[\pi/2, \pi] \rightarrow \sin x > 0$ and $\cos x < 0$
$\left[\pi, \frac{3\pi}{2}\right] \rightarrow \sin x < 0$ and $\cos x < 0$
$[3\pi/2, 2\pi] \rightarrow \sin x < 0$ and $\cos x > 0$
We see that, in interval $x \in [\pi, 3\pi/2]$, then value of $\sin|x| + \cos|x|$ is always negative.
So, it is not necessary that $\sin|x| + \cos|x|$ is not always positive.
II. Given that, let
$f_2(x) = \sin(x^2) + \cos(x^2)$
If we take the values of x^2 between any value which lies in the interval, $\left[\pi, \frac{3\pi}{2}\right]$,
then value of, f_2, $(x) = \sin(x^2) + \cos(x^2)$ is always negative.
If $x^2 = 225° \Rightarrow x = 15°$, then $f_2(x)$
$= (\sin x^2 + \cos x^2) < 0$
So, it's also not necessary that $\sin x^2 + \cos x^2$ is not always positive.
Note If $x \in [-\pi/2, 0] \rightarrow \sin x < 0$ and $\cos x < 0$ but $\sin|x| + \cos|x|$ is always positive.
If we take $x = -15° \Rightarrow x^2 = 225°$, then $\sin x^2 + \cos x^2$ is nagative.

58. (A) I. We know that, $\sin\theta \in [-1, 1]$; $\theta \in R$
i.e. the value of $\sin\theta$ lies between –1 to 1.
II. We know that, $\cos\theta \in [-1, 1]$; $\theta \in R$
i.e., the value of $\cos\theta$ also lies between –1 to 1.

59. (B) Given that, A + B = 90°
Now, $\sqrt{\sin A \sec B - \sin A \cos B}$
$= \sqrt{\sin A.\sec(90° - A) - \sin A\cos(90° - A)}$
$= \sqrt{\sin A.\operatorname{cosec} A - \sin A.\sin A}$
$= \sqrt{\sin A.\frac{1}{\sin A} - \sin^2 A}$
$= \sqrt{1 - \sin^2 A} = \sqrt{\cos^2 A} = \cos A$

60. (C) Given that, $\operatorname{cosec}\theta + \cot\theta = c$
$\Rightarrow \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = c$
$\Rightarrow \frac{1 + \cos\theta}{\sin\theta} = c$
$\Rightarrow \frac{1 + 2\cos^2\frac{\theta}{2} - 1}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}} = c$
$\left[\because \cos A = 2\cos^2\frac{A}{2} - 1, \sin 2A = 2\sin\frac{A}{2}.\cos\frac{A}{2}\right]$
$\Rightarrow \frac{\cos\theta/2}{\sin\theta/2} = \cot\theta/2 = c$
$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{c}$ ...(i)
$\therefore \cos\theta = \frac{1 - \tan^2\theta/2}{1 + \tan^2\theta/2}$
$\left[\because \cos A = \frac{1 - \tan^2 A/2}{1 + \tan^2 A/2}\right]$
$= \frac{1 - (1/c)^2}{1 + (1/c)^2}$ [from eq. (ii)]
$= \frac{c^2 - 1}{c^2 + 1}$

61. (C) Let $I = \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$
$= \int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$
[divide numerator and denominator $\cos^2 x$]
Let $\tan x = t \Rightarrow \sec^2 x\,dx = dt$
When $x = 0$, then $t = 0$
and when $x = \frac{\pi}{2}$ then $t = \infty$
$I = \int_0^{\infty} \frac{dt}{a^2 + b^2 t^2} = \frac{1}{b^2}\int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$
$= \frac{1}{b^2}\left(\frac{1}{\frac{a}{b}}\right)\left[\tan^{-1}\left(\frac{bt}{a}\right)\right]_0^{\infty}$
$\left[\because \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C\right]$
$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$

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62. (A) Let $I = \int_0^{\pi/2} \sin 2x \ln(\cot x)\, dx$ ...(i)

By property of definite integral,

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$I = \int_0^{\pi} \sin 2\left(\frac{\pi}{2} - x\right) \ln\left\{\cot\left(\frac{\pi}{2} - x\right)\right\} dx$$

$$I = \int_0^{\pi/2} \sin(\pi - 2x)\log(\tan x)\, dx$$

$$I = \int_0^{\pi/2} \sin 2x \log(\cot x)^{-1}\, dx$$

$$I = -\int_0^{\pi/2} \sin 2x . \log \cot x\, dx$$

$I = -I$ [form eq. (i)]

$2I = 0$

$\Rightarrow I = 0$

63. (B) $I = \int_3^5 x\left(1 + \frac{4}{x^2-4}\right)dx$

$$= \int_3^5 dx + 4\int_3^5 \frac{1}{(x^2-4)}dx$$

$$= [x]_3^5 + \left[4.\frac{1}{4}\log\left|\frac{x-2}{x+2}\right|\right]_3^5$$

$$= 2 + \log\frac{15}{7} = 2 + \log 15 - \log 7$$

64. (B) $\because \int \frac{\cos^n x\,dx}{(\cos^n x + \sin^n x)} = \frac{\pi}{4}$

$$\therefore \int_0^{\pi/2} \frac{\cos^{\frac{1}{2}} x}{\cos^{\frac{1}{2}} x + \sin^{\frac{1}{2}} x}\, dx = \frac{\pi}{4}$$

65. (B) $\int_0^a f(a-x)\, dx + \int_0^a f(a+x)\, dx$

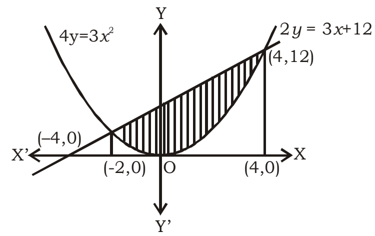
[Putting a – x = t in I_1 & a + x = z in I_2]

$$= -\int_a^0 f(t)\, dt + \int_a^{2a} f(z)\, dz$$

$$= \int_0^a f(x)dx + \int_a^{2a} f(x)dx$$

$$= \int_0^{2a} f(x)dx.$$

66. (A) Area of enclosed by the parabola and the line



$$= \int_{-2}^4 \left[\frac{(3x+12)}{2} - \frac{3x^2}{4}\right] dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^4 - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^4$$

$$= \frac{1}{2}\left[\left\{\frac{3(4)^2}{2} + 12(4)\right\} - \left\{\frac{3(-2)^2}{2} + 12(-2)\right\}\right]$$

$$-\frac{3}{4}\left(\frac{4^3}{3} - \frac{(-2)^3}{3}\right)$$

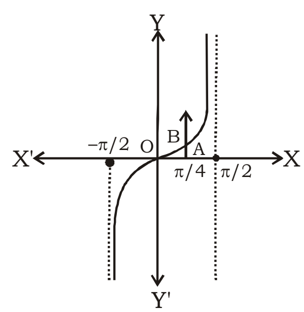
$$= \frac{1}{2}(24 + 48 - 6 + 24) - \frac{3}{4}\left(\frac{64+8}{3}\right)$$

$$= \frac{1}{2}(90) - 18 = 45 - 18 = 27 \text{ sq units.}$$

67. (B) Given equation of curves

y = tanx ....(i)

and $y = 0$ and $x = \frac{\pi}{4}$ ....(ii)



$\therefore$ Required area

$$= \int_0^{\pi/4} y\,dx = \int_0^{\pi/4} \tan x\, dx$$

$$= [\log|\sec x|]_0^{\pi/4} = \log\left|\sec\frac{\pi}{4}\right| - \log|\sec 0|$$

$$= \log|\sqrt{2}| - \log|1| = \log\sqrt{2} - 0$$

$$= \frac{1}{2}\log 2 \text{ sq units}$$

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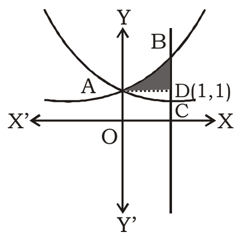
68. (C) The equation of curves are $y = e^x$, $y = e^{-x}$

$\therefore e^x = \frac{1}{e^x} \Rightarrow e^{2x} = e^0$

$\Rightarrow x = 0$



$\therefore$ Required area $= \int_0^1 \left(e^x - e^{-x}\right) dx$

$= \left[e^x + e^{-x}\right]_0^1$

$= e + e^{-1} - e^0 - e^0$

$= \left(e + \frac{1}{e} - 2\right)$ sq unit

69. (B) Consider the given differential equation,

$\frac{ydx - xdy}{y^2} = 0$

$\Rightarrow d\left(\frac{y}{y}\right) = 0$

$\left[\because d\left(\frac{u}{v}\right) - \frac{v.du - u.dv}{v^2}\right]$

On integrating both sides, we get

$\int d\left(\frac{x}{y}\right) = C_1$

$\Rightarrow \frac{x}{y} = C_1$

$\Rightarrow x = C_1 y$

$\Rightarrow y = \frac{1}{C_1} x$

$\Rightarrow y = Cx$, where $C = \frac{1}{C_1}$

70. (A) Consider the given differential equation,

$\sin\left(\frac{dy}{dx}\right) - a = 0$

$\Rightarrow \sin\left(\frac{dy}{dx}\right) = a$

$\Rightarrow \frac{dy}{dx} = \sin^{-1} a$

$\Rightarrow dy = (\sin^{-1}a)\, dx$

On integrating both sides w.r.t.x., we get

$\int dy = \int (\sin^{-1} a)\, dx$

$\Rightarrow y = (\sin^{-1}a) \int 1\, dx$

$\Rightarrow y = (\sin^{-1}a)\, x + c$

71. (A) Given differential equation is

$\frac{dy}{dx} = |x|$

$dy = |x|\, dx$ ....(i)

Case I. if $x > 0$

$\int dy = \int xdx$

[on integrating both sides]

$y = \frac{x^2}{2} + C$

$y = \frac{x.(x)}{2} + C$ ....(ii)

Cas II if $x < 0$

$\int dy = -\int xdx$

[on integrating both sides]

$y = \frac{-x^2}{2} + C$

$y = \frac{x.(-x)}{2} + C$ ...(iii)

When we combined both cases, we get the required solution

$y = \frac{x|x|}{2} + C$

72. (B) Given curve is

$y = \sin x$ ...(i)

On differentiating w.r.t.x, we get

$\frac{dy}{dx} = \cos x$ ...(ii)

[Again, differentiating w.r.t.x, we get]

$\frac{d^2y}{dx^2} = -\sin x = -y$ [from eq. (i)]

$\Rightarrow \frac{d^2y}{dx^2} + y = 0$

73. (C) A (2, – 4) —3— C (x, y) —2— B (– 3, 6)

$x = \frac{3\times(-3)+2\times 2}{3+2}$ and $y = \frac{3\times 6+2\times(-4)}{3+2}$

$x = \frac{-9+4}{5} = -1,\qquad y = \frac{18-8}{5} = 2$

Co-ordinate of C = (–1, 2)

74. (B) $y = \frac{e^{-\theta}-e^{\theta}}{e^{-\theta}+e^{\theta}}$

$\Rightarrow y = -\tanh\theta$

On differentiating both side w.r.t. 'θ'

$\Rightarrow \frac{dy}{d\theta} = -\operatorname{sech}^2\theta$

$\Rightarrow \frac{dy}{d\theta} = -\left[\frac{2}{e^{\theta}+e^{-\theta}}\right]^2$

$\Rightarrow \frac{dy}{d\theta} = \frac{-4}{e^{2\theta}+e^{-2\theta}+2}$

$\Rightarrow \frac{dy}{d\theta} = \frac{-4e^{2\theta}}{e^{4\theta}+1+2e^{2\theta}}$

75. (A) $\sin^{-1}\frac{x}{2} + \cos^{-1}\frac{x}{2} = \frac{\pi}{2}$

76. (B) $z = \frac{1-i}{1-\sqrt{3}i}$

$z = \frac{1-i}{1-\sqrt{3}i} \times \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$

$z = \frac{(\sqrt{3}+1)+(\sqrt{3}-1)i}{4}$

Amplitude of $z = \tan^{-1}\left(\frac{\frac{\sqrt{3}-1}{4}}{\frac{\sqrt{3}+1}{4}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{12}\right) = \frac{\pi}{12}$

77. (B) differential equation

$y^2 = x\left(\frac{dy}{dx}\right)^2 - \frac{3}{\frac{dy}{dx}}$

$y^2\,\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^3 - 3$

Hence order = 1 and degree = 3

78. (B) Equation

$(4\lambda+1)\,x^2 + 3\lambda x + 1 = 0$

roots are equal,

then $b^2 - 4ac = 0$

$\Rightarrow (3\lambda)^2 - 4\,(4\lambda+1)\times 1 = 0$

$\Rightarrow 9\lambda^2 - 16\lambda - 4 = 0$

$\Rightarrow (\lambda-2)(9\lambda+2) = 0$

$\lambda = 2, \frac{-2}{9}$

79. (C) Given that ${}^nC_r = \frac{n!}{r!(n-r)!}$

then

${}^nC_r + {}^nC_{r+1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$

$= \frac{n!(r+1)}{(r+1)r!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)(n-r-1)!}$

$= \frac{n!(r+1)}{(r+1)!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)!}$

$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$

$= \frac{(n+1)n!}{(r+1)!(n-r)!}$

$= \frac{(n+1)!}{(r+1)!(n-r)!} = {}^{n+1}C_{r+1}$

80. (B) $A = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2\times 2-2\times 1 & 2\times(-2)+(-2)\times(-1) \\ 1\times 2+(-1)\times 1 & 1\times(-2)+(-1)\times(-1) \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$A^2 = A$

Hence Matrix A is an Idempotent matrix.

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81. (A) $y = e^{2x}(a \sin x - b \cos x)$ ....(i)

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = e^{2x}(a\cos x + b\sin x) + 2(a\sin x - b\cos x)\, e^{2x}$$

$$\frac{dy}{dx} = e^{2x}\,(a \cos x + b \sin x) + 2y \quad ..(ii)$$

Again, differentiating

$$\frac{d^2y}{dx^2} = e^{2x}(-\,a \sin x + b \cos x)$$

$$+2(a \cos x + b \sin x)\, e^{2x} + 2\frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -\,e^{2x}(a \sin x - b\cos x) + 2e^{2x}(a \cos x +$$

$$b \sin x) + \frac{2dy}{dx}$$

$$\frac{d^2y}{dx^2} = -y + 2\left(\frac{dy}{dx} - 2y\right) + \frac{2dy}{dx}$$

$$\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} + 5y = 0$$

82. (A) Curve $y^2 = 3x$

$$\Rightarrow 2y\frac{dy}{dx} = 3$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2y} \quad ....(i)$$

Slope of Tangent at point (12, 6)= $\frac{3}{2\times 6}$

$= \frac{1}{4}$

Slope of Normal = $\frac{-1\times 4}{1} = -\,4$

Equation of Normal at (12, 6)

$y - 6 = -\,4\,(x - 12)$

$4x + y = 54$ ......(ii)

From eq (i)

$$\frac{dy}{dx} = \frac{3}{2y}$$

Slope of Tangent at point (12,–6) = $\frac{3}{2(-6)}$

$= \frac{-1}{4}$

Slope of Normal = $\frac{-1\times 4}{-1} = 4$

Equation of Normal at point (12, –6)

$(y + 6) = 4(x - 12)$

$4x - y = 54$ .....(ii)

from eq (ii) and eq (iii)

$$x = \frac{27}{2} \text{ and } y = 0$$

Hence intersection point = $\left(\frac{27}{2}, 0\right)$

83. (C) Adjacent sides of the prallelogram

$\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Area of parallelogram = $|\vec{a}\times\vec{b}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= \left|i(-3+8) - \hat{j}(-2+4) + \hat{k}(4-3)\right|$$

$$= \left|5\hat{i} - 2\hat{j} + \hat{k}\right|$$

$$= \sqrt{(5)^2 + (-2)^2 + (1)^2} = \sqrt{30}$$

84. (B) $\lim_{x\to 0} \frac{\log(1+x) - x.e^x}{x^2}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to 0} \frac{\frac{1}{1+x} - x.e^x - e^x.1}{2x}$$

$$\Rightarrow \lim_{x\to 0} \frac{1 - x(1+x).e^x - (1+x)e^x}{2x(1+x)} \left[\frac{0}{0}\right]\text{form}$$

by L-Hospital's Rule

$$\Rightarrow \frac{0 - 2(1+0).e^0 - 0 - e^0}{0+2} \Rightarrow \frac{-3}{2}$$

85. (B) $\vec{a} = a\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - b\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + c\hat{k}$ are mutually orthogonal, then

$\vec{a}\,.\,\vec{b} = 0$

$2a + b - 2 = 0$

$2a + b = 2$ ......(i)

and $\vec{b}\,.\,\vec{c} = 0$

$2 + b + c = 0$

$b + c = -2$ ......(ii)

and $\vec{a}\,.\,\vec{c} = 0$

$a + 1 - 2c = 0$

$a - 2c = -1$ ......(iii)

On Solving eq(i), (ii) and (iii)

$a = 3,\ b = -\,4$ and $c = 2$

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86. (A) Equations $x + 2y + 3z = 3$, $x - y - 2z = 2$, and $-x + y - 3z = 0$

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & -2 \\ -1 & 1 & -1 \end{vmatrix}$$

$= 1(1 + 2) - 2(-1 - 2) + 3(1 - 1)$

$= 3 + 6 = 9 \neq 0$

Hence equation has a unique solution.

87. (D) Conic $2x^2 + 6y^2 = 18$

$$\frac{x^2}{9} + \frac{y^2}{3} = 1$$

Now, eccentricity $e^2 = 1 - \frac{b^2}{a^2}$

$$e^2 = 1 - \frac{3}{9}$$

$$e^2 = \frac{2}{3} \Rightarrow e = \sqrt{\frac{2}{3}}$$

88. (C) $a^{-\tan x}\left[\frac{d}{dx}a^{\tan x}\right]$

$\Rightarrow a^{-\tan x}.\ a^{\tan x}.\ \log_e a.\ \sec^2 x$

$\Rightarrow \sec^2 x.\log_e a$

89. (B) $y = a^{x+a^{x+a^{x+a^{x+\ldots\ldots\infty}}}}$

$y = a^{x+y}$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = a^{x+y}.\ \log_e a\left(1 + \frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = y.\log_e a\left(1 + \frac{dy}{dx}\right) \quad \text{from eq.}(i)$$

$$(1 - y\log_e a)\frac{dy}{dx} = y.\log_e a$$

$$\frac{dy}{dx} = \frac{y\log_e a}{1 - y\log_e a}$$

90. (A) $y = \operatorname{cosec} 2\theta$

$$\Rightarrow y = \frac{1}{\sin 2\theta} = \frac{1}{2\sin\theta.\cos\theta}$$

$$\Rightarrow 2\sin\theta.\cos\theta = \frac{1}{y}$$

and $x = \sin\theta + \cos\theta$

$\Rightarrow x^2 = (\sin\theta + \cos\theta)^2$

$\Rightarrow x^2 = 1 + 2\sin\theta.\cos\theta$

$$\Rightarrow x^2 = 1 + \frac{1}{y} \Rightarrow x^2 y = y + 1$$

91. (D) Let $f(x) = \frac{x}{[x]}$

$$\text{L.H.L.} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2 - h)$$

$$= \lim_{h \to 0} \frac{2 - h}{[2 - h]}$$

$$= \frac{2 - 0}{1} = 2$$

$$\text{R.H.L.} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2 + h)$$

$$= \lim_{h \to 0} \frac{2 + h}{[2 + h]}$$

$$= \frac{2 + 0}{2} = 1$$

L.H.L. ≠ R.H.L.

Hence limit does not exist.

92. (D)

93. (A) Given that $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$

and $\vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$

Now, $\vec{a}\cdot(\vec{b} - \vec{c}) + \vec{b}\cdot(\vec{c} - \vec{a}) + \vec{c}\ (\vec{a} - \vec{b})$

$\Rightarrow \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} - \vec{b}\cdot\vec{a} + \vec{c}\cdot\vec{a} - \vec{c}\cdot\vec{b} = 0$

94. (B) Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 4 \\ 3 & -1 & 2 \end{bmatrix}$

Co-factors of A —

$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 4 \\ -1 & 2 \end{vmatrix}$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix}$, $C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -2 \\ 3 & -1 \end{vmatrix}$

$= 0$, $= 8$, $= 4$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 3 \\ -1 & 2 \end{vmatrix}$, $C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}$, $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix}$

$= -3$, $= -7$, $= 1$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 3 \\ -2 & 4 \end{vmatrix}$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$, $C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix}$

$= 6$, $= 2$, $= -2$

$$C = \begin{bmatrix} 0 & 8 & 4 \\ -3 & -7 & 1 \\ 6 & 2 & -2 \end{bmatrix}, \text{AdjA} = \begin{bmatrix} 0 & -3 & 6 \\ 8 & -7 & 2 \\ 4 & 1 & -2 \end{bmatrix}$$

95. (B) 1 0 1

$1\times2^0 = 1$
$0\times2^1 = 0$
$1\times2^2 = 4$
$\quad\quad\quad\;\; 5$

.11

$\frac{1}{2} = 1\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}=0.75$

$(101)_2 = (5)_{10}$, $\quad (0.11)_2 = (0.75)_{10}$
Hence $(101.11)_2 = (5.75)_{10}$

96. (C) line $\frac{x}{3}-\frac{y}{7} = 2$

$7x - 3y = 42$

Slope of line $=\frac{7}{3}$

Slope of line which is parallel to given line $=\frac{7}{3}$

97. (B) $\vec{a} = 3\hat{i} - 6\hat{j} + 7\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Projection of $\vec{a}$ on $\vec{b} = \frac{3\times2-6\times(-3)+7\times6}{\sqrt{(2)^2+(-3)^2+(6)^2}}$

$= \frac{6+18+42}{\sqrt{49}} = \frac{66}{7}$

98. (A) Given equations $x - 2y + 3z = 0$, $2x - y + 2z = 4$ and $3x + y - z = 5$

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$$

by elementary Row method

$$[A/B] = \left[\begin{array}{ccc|c} \mathbf{1} & -2 & 3 & 0 \\ 2 & -1 & 2 & 4 \\ 3 & 1 & -1 & 5 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & \mathbf{3} & -4 & 4 \\ 0 & 7 & -10 & 5 \end{array}\right]$$

$R_3 \to R_3 - \frac{7}{3}R_2$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 3 & -4 & 4 \\ 0 & 0 & -2/3 & \frac{-13}{3} \end{array}\right]$$

$x - 2y + 3z = 0$ ...(i)

$3y - 4z = 4$ ...(ii)

$\frac{-2}{3}z = \frac{-13}{3}$ ...(iii)

On solving eq (i), (ii) and (iii)

$x = \frac{1}{2}$, $y = 10$, $z = \frac{13}{2}$

99. (C) Let $I = \int_1^2 [x^2]^2\, dx$

$I = \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 4.dx + \int_{\sqrt{3}}^{2} 9.dx$

$I = [x]_1^{\sqrt{2}} + 4[x]_{\sqrt{2}}^{\sqrt{3}} + 9[x]_{\sqrt{3}}^{2}$

$I = \sqrt{2} - 1 + 4\left(\sqrt{3}-\sqrt{2}\right) + 9\left(2-\sqrt{3}\right)$

$I = 17 - 3\sqrt{2} - 5\sqrt{3}$

100. (B) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \frac{3\pi}{2}$

$\cos^{-1}x = \frac{\pi}{2}$, $\quad \cos^{-1}y = \frac{\pi}{2}$, $\quad \cos^{-1}z = \frac{\pi}{2}$

$\Rightarrow x = \cos\frac{\pi}{2} = 0$, $y = \cos\frac{\pi}{2} = 0$, $z = \cos\frac{\pi}{2} = 0$

Now, $x^{2002} + y^{2002} + z^{2002} = 0 + 0 + 0 = 0$

101. (C) $s = \sqrt{t^2 - 1}$ ...(i)

On differentiating both side w.r.t. 't'

$\Rightarrow \frac{ds}{dt} = \frac{1\times2t}{2\sqrt{t^2-1}}$

$\Rightarrow \frac{ds}{dt} = \frac{t}{\sqrt{t^2-1}}$

$\Rightarrow \frac{d^2s}{dt^2} = \frac{\sqrt{t^2-1}.1 - t.\frac{1\times2t}{2\sqrt{t^2-1}}}{\left(\sqrt{t^2-1}\right)^2}$

$\Rightarrow \frac{d^2s}{dt^2} = \frac{t^2-1-t^2}{\sqrt{t^2-1}.(t^2-1)}$

$\Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{(t^2-1)^{3/2}}$

$\Rightarrow \frac{d^2s}{dt^2} = \frac{-1}{s^3}$

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102. (D) $I = \int e^x \left[(x+1)^2 \tan^{-1} x + 1\right] dx$

$I = \int e^x \left[(x^2+1+2x)\tan^{-1} x + 1\right] dx$

$I = \int e^x \left[\{(1+x^2)\tan^{-1} x\} + \{1 + 2x\tan^{-1} x\}\right] dx$

$I = e^x (1 + x^2) \tan^{-1}x + c$

$\left[\because \int e^x \left[f(x) + f'(x)\right] dx = e^x . f(x) + c\right]$

103. (B) Given that $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{7,8,3\}$, $B = \{3,8,9\}$ and $C = \{9,3,4\}$

Now, $(A \cup B) = \{3,7,8,9\}$, $(B \cap C) = \{3\}$

and $(A \cap C) = \{3\}$

$\{(A \cup B) - (B \cap C)\} \times (A \cap C)$

$= [\{3,7,8,9\} - \{3\}] \times \{3\}$

$= \{7,8,9\} \times \{3\}$

$= \{(7,3),(8,3),(9,3)\}$

104. (A) $S = 0.2 + 0.22 + 0.222 + \ldots\ldots\ldots\ldots n$ terms

$S = \frac{2}{9}[0.9 + 0.99 + 0.999 + \ldots\ldots n \text{ terms}]$

$S = \frac{2}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots. n \text{ terms}\right]$

$S = \frac{2}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \ldots. n \text{ terms}\right]$

$S = \frac{2}{9}\left[(1 + 1 + \ldots n \text{ terms}) - \left(\frac{1}{10} + \frac{1}{100} + \ldots. n \text{ terms}\right)\right]$

$S = \frac{2}{9}\left[n - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^n\right)}{1 - \frac{1}{10}}\right]$

$S = \frac{2}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

$S = \frac{2}{81}\left[9n - 1 + \frac{1}{10^n}\right]$

105. (D) $\sin\theta = \frac{8}{17}$, $\sin\phi = \frac{15}{17}$

$\cos\theta = \frac{15}{17}$, $\cos\phi = \frac{8}{17}$

$\cos(\theta - \phi) = \cos\theta . \cos\phi + \sin\theta . \sin\phi$

$= \frac{15}{17} \times \frac{8}{17} + \frac{8}{17} \times \frac{15}{17} = \frac{240}{289}$

Now, $\sin\left(\frac{\theta - \phi}{2}\right) = \sqrt{\frac{1 - \cos(\theta - \phi)}{2}}$

$= \sqrt{\frac{1 - \frac{240}{289}}{2}}$

$= \sqrt{\frac{49}{2 \times 289}} = \frac{7}{17\sqrt{2}}$

106. (A) In ΔABC, A(3, –2), B(–3, 4) and C(–1, 0)

Co-ordinate of centroid

$\bar{x} = \frac{3 - 3 - 1}{3} = \frac{-1}{3}$, $\bar{y} = \frac{-2 + 4 + 0}{3} = \frac{2}{3}$

Co-ordinate of centroid $= \left(\frac{-1}{3}, \frac{2}{3}\right)$

107. (C) Given that $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{5}$

$P(\bar{A}) = \frac{1}{2}$, $P(\bar{B}) = \frac{3}{5}$

The Probability $= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A).P(\bar{B}) + P(\bar{A}).P(B)$

$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{5} = \frac{5}{10} = \frac{1}{2}$

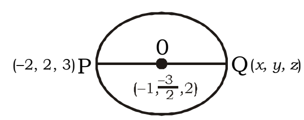
108. (B) Equation of sphere

$x^2 + y^2 + z^2 + 2x + 3y - 4z = 15$

$u = 1$, $v = \frac{3}{2}$, $w = -2$

centre $\left(-1, \frac{-3}{2}, 2\right)$

Let co-ordinate of Q = (x, y, z)



$\frac{x - 2}{2} = -1$, $\frac{y + 2}{2} = \frac{-3}{2}$, $\frac{z + 3}{2} = 2$

$x = 0$, $y = -5$, $z = 1$

Hence end point of a diameter = (0, – 5, 1)

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109. (B) $f(x) = \begin{cases} 3 - x^2, & 0 \le x < 1 \\ 2\lambda + x, & 1 \le x < 2 \end{cases}$ is continuous

at $x = 1$, then

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$3 - 1 = 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$$

110. (C) The total no. of arrangement $= \frac{9!}{2!2!} = \frac{9!}{4}$

No. of arrangement when I's come together $= \frac{8!}{2!} = \frac{8!}{2}$

No. of arrangement when I's don't come together $= \frac{9!}{4} - \frac{8!}{2} = \frac{7 \times 8!}{4}$

The required Probability $= \dfrac{\frac{7 \times 8!}{4}}{\frac{9!}{4}} = \frac{7}{9}$

111. (B) $y = \sec(\tan^{-1}x)$ ....(i)

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = \sec(\tan^{-1}x).\ \tan(\tan^{-1}x).\frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{y.x}{1+x^2} \quad [\text{from eq (i)}]$$

$$(1 + x^2)\frac{dy}{dx} = xy$$

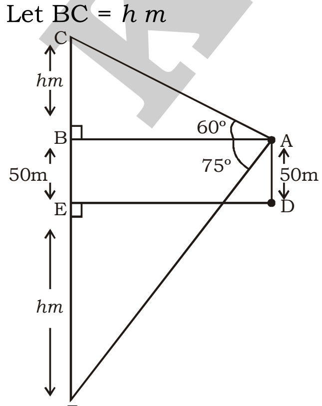
112. (D) $(A \cap B) \cup (B \cap C)$

113. (C) Equation $\lambda x^2 + 3x + (\lambda - 1) = 0$

product of roots = – 2

$$\frac{\lambda - 1}{\lambda} = -2 \Rightarrow \lambda = \frac{1}{3}$$

114. (C) Let BC = h m



In ΔABC

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{h}{AB} \quad \text{..............(i)}$$

In ΔABF

$$\tan 75° = \frac{BF}{AB}$$

$$2 + \sqrt{3} = \frac{h + 50}{h/\sqrt{3}} \quad \text{.............(ii)}$$

$$2h + h\sqrt{3} = h\sqrt{3} + 50\sqrt{3} \Rightarrow h = 25\sqrt{3}$$

height of the aeroplane above the lake level = h + 50

$= 25\sqrt{3} + 50 = 25(2 + \sqrt{3})$m

115. (B) Equation

$x^3 + 4x^2 - 9x - 36 = 0$

Let roots are $\alpha, -\alpha, \beta$

$\alpha - \alpha + \beta = -4 \Rightarrow \beta = -4$

and $\alpha(-\alpha)\beta = -(-36)$

$-\alpha^2(-4) = 36$

$\alpha = -3, 3$

Hence roots are – 3, 3, – 4.

116. (C) ${}^nC_{r-1} + 2\,{}^nC_r + {}^nC_{r+1}$

$\Rightarrow {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$

$\Rightarrow {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

117. (B)

118. (B)

119. (D) Given data

12, 8, 14, 6, 17, 8, 19, 15

arrange in ascending both

6, 8, 8, 12, 14, 15, 17, 19

middle terms = 12 and 14

Median $= \frac{12 + 14}{2} = 13$

120. (C) $\cot\theta + \cos\theta = x$

$\cot^2\theta + \cos^2\theta + 2\cot\theta.\cos\theta = x^2$

and $\cot\theta.\cos\theta = y$

$\cot^2\theta + \cos^2\theta - 2\cot\theta.\cos\theta = y^2$

$x^2 - y^2 = 4\cot\theta.\cos\theta$ and $xy = \cot^2\theta - \cos^2\theta$

$$x^2 - y^2 = 4\frac{\cos\theta}{\sin\theta}.\cos\theta, \quad xy = \frac{\cos^4\theta}{\sin^2\theta}$$

$$x^2 - y^2 = 4\frac{\cos^2\theta}{\sin\theta}, \quad \sqrt{xy} = \frac{\cos^2\theta}{\sin\theta}$$

$$x^2 - y^2 = 4\sqrt{xy}$$

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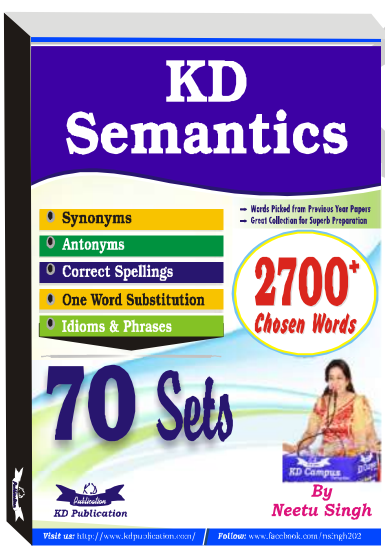
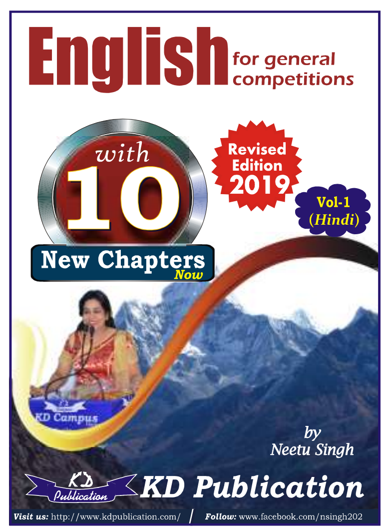
NDA (MATHS) MOCK TEST - 174 (Answer Key)

1.	(A)	21.	(B)	41.	(A)	61.	(C)	81.	(A)	101.	(C)
2.	(C)	22.	(C)	42.	(A)	62.	(A)	82.	(A)	102.	(D)
3.	(D)	23.	(B)	43.	(C)	63.	(B)	83.	(C)	103.	(B)
4.	(C)	24.	(B)	44.	(D)	64.	(B)	84.	(B)	104.	(A)
5.	(B)	25.	(D)	45.	(A)	65.	(B)	85.	(B)	105.	(D)
6.	(D)	26.	(C)	46.	(A)	66.	(A)	86.	(A)	106.	(A)
7.	(B)	27.	(D)	47.	(B)	67.	(B)	87.	(D)	107.	(C)
8.	(D)	28.	(A)	48.	(A)	68.	(C)	88.	(C)	108.	(B)
9.	(B)	29.	(B)	49.	(A)	69.	(B)	89.	(B)	109.	(B)
10.	(B)	30.	(A)	50.	(A)	70.	(A)	90.	(A)	110.	(C)
11.	(B)	31.	(B)	51.	(C)	71.	(A)	91.	(D)	111.	(B)
12.	(A)	32.	(C)	52.	(A)	72.	(B)	92.	(D)	112.	(D)
13.	(C)	33.	(D)	53.	(C)	73.	(C)	93.	(A)	113.	(C)
14.	(C)	34.	(A)	54.	(A)	74.	(B)	94.	(B)	114.	(C)
15.	(C)	35.	(D)	55.	(B)	75.	(A)	95.	(B)	115.	(B)
16.	(B)	36.	(A)	56.	(B)	76.	(B)	96.	(C)	116.	(C)
17.	(D)	37.	(A)	57.	(D)	77.	(B)	97.	(B)	117.	(B)
18.	(A)	38.	(C)	58.	(A)	78.	(B)	98.	(A)	118.	(B)
19.	(B)	39.	(A)	59.	(B)	79.	(C)	99.	(C)	119.	(D)
20.	(A)	40.	(C)	60.	(C)	80.	(B)	100.	(B)	120.	(C)

Campus





Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777