NDA MATHS MOCK TEST - 176 (SOLUTION)

1. (A) $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} = \frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2}$

$= \frac{\sqrt{a_1}-\sqrt{a_2}}{-d}$

Now,

$-\frac{1}{d}\left[\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+\sqrt{a_3}-\sqrt{a_4}+....\sqrt{a_{n-1}}-\sqrt{a_n}\right]$

$\Rightarrow -\frac{1}{d}\left[\sqrt{a_1}-\sqrt{a_n}\right]$

$\Rightarrow -\frac{1}{d}\left[\frac{\left(\sqrt{a_1}-\sqrt{a_n}\right)}{\sqrt{a_1}+\sqrt{a_n}}\left(\sqrt{a_1}+\sqrt{a_n}\right)\right]$

$\Rightarrow -\frac{1}{d}\left[\frac{a_1-a_n}{\sqrt{a_1}+\sqrt{a_n}}\right] = \frac{1}{d}\left[\frac{a_n-a_1}{\sqrt{a_1}+\sqrt{a_n}}\right]$

from equestion a_1, a_2, a_3..... an in A.P.

$a_n = a_1 + (n-1)d$

$a_n - a_1 = (n-1)d$

Now,

$\frac{1}{d}\left[\frac{(n-1)d}{\sqrt{a_1}+\sqrt{a_n}}\right] = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

2. (B) A.T.Q,

100 < 13k + 2 < 8000

98 < 13k < 7998

7.8 < k < 615.2

Now,

k = 8, 9, 10, 11,......., 615

Now, Number are = 13k + 2

= 106, 119, 132.....7997

$a = 106$, $d = 13$

$T_n = a + (n-1)d$

$\Rightarrow 7997 = 106 + (n-1)13$

$\Rightarrow n = 608$

$S_n = \frac{n}{2}[2a + (n+1)d]$

$= \frac{608}{2}[212 + 607 \times 13]$

= 304[212 + 7891]

= 2463312

3. (D) Let α be the common root of the two equations.

Then $a\alpha^2 + b\alpha + c = 0$ ...(i)

and , $b\alpha^2 + c\alpha + a = 0$ ...(ii)

after solving eq^n (i) and (ii), we get

$\frac{\alpha^2}{ab-c^2} = \frac{\alpha}{bc-a^2} = \frac{1}{ac-b^2}$

$\Rightarrow \alpha^2 = \frac{ab-c^2}{ac-b^2}$ and $\alpha = \frac{bc-a^2}{ac-b^2}$

$\Rightarrow \left(\frac{bc-a^2}{ac-b^2}\right)^2 = \left(\frac{ab-c^2}{ac-b^2}\right)$ $[\because \alpha^2 = (\alpha)^2]$

$\Rightarrow (ab-c^2)(ac-b^2) = (bc-a^2)^2$

$\Rightarrow a(a^3+b^3+c^3-3abc) = 0$

$\Rightarrow a^3+b^3+c^3-3abc = 0$ $[\because a \neq 0$

$\Rightarrow a^3+b^3+c^3 = 3abc$

4. (B) Let $f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3 = 0$

Now, differentiate w.r.t x

$f(x) = 3[(x-a)^2 + (x-b)^2 + (x-c)^2]$

clearly,

$f'(x) > 0 \; \forall \; x$.

So, $f'(x) = 0$ has no real roots. so, $f(x) = 0$ has two imaginary roots and one real root.

5. (B) $|x|^2 + |x| - 6 = 0$

$\Rightarrow (|x|+3)(|x|-2) = 0$

$\Rightarrow (|x|+3) \neq 0 \Rightarrow (|x|-2) = 0$

$\Rightarrow |x| = 2$

$\Rightarrow x = \pm 2$

6. (C) Let first term of AP is 'a' and common difference is 'd'

$a + (p-1)d = q$ ...(i)

$a + (q-1)d = p$ ...(ii)

after solving eq^n(i) and (ii), we get

$d = -1$ $a = q + p - 1$

Now,

$T_r = a + (r-1)d$

$T_r = q + p - 1 + (r-1)(-1)$

$T_r = p + q - r$

7. (D) $\frac{1}{5}, \frac{1}{9}, \frac{1}{13}$..... 7th term (H.P.)

5, 9, 13,........(A.P.)

$a = 5$, $d = 9 - 5 = 4$

$T_n = a + (n-1)d$

$T_7 = a + 6d$

(A.P.) → $T_7 = 5 + 6 \times 4 = 29$

(H.P.) → $T_7 = \frac{1}{29}$

8. (C) Let $x = \sqrt{-i}$

$x^2 = -i$

taking $x^2 + i = 0$ ...(i)

Now, by taking option (C), Put $x = \pm \frac{1}{\sqrt{2}}(1-i)$

$\Rightarrow$ eq^n (i) is setisfy

$\Rightarrow$ square root of $(-i) = \pm \frac{1}{\sqrt{2}}(1-i)$

9. (B) $\left(3x^2 - \frac{1}{3x}\right)^9$

find coefficient of x^6

$r = \frac{9\times 2 - 6}{2+1}$

$r = 4$

We know that

$\left(ax^p + \frac{b}{x^q}\right)^n$

Find coefficient of x^M

$r = \frac{np - M}{p+q}$

(coefficient of x^M) $T_{r+1} = {}^nC_r\, x^{n-r} a^r$

Coefficient of x^6 $(T_{r+1}) = {}^9C_4\,(3)^5\left(\frac{-1}{3}\right)^4$

$= \frac{9\times 8\times 7\times 6}{1\times 2\times 3\times 4}\times 3$

$= 378$

10. (B) We know that $a\sin\theta + b\cos\theta$

max. value $\sqrt{a^2+b^2}$

Now, $7\sin\theta + 3\cos\theta$

max. value $= \sqrt{7^2+3^2} = \sqrt{49+9} = \sqrt{58}$

11. (C) We know that

$\tan\frac{c}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Now, $s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9$

$\Rightarrow \tan\frac{c}{2} = \sqrt{\frac{(9-5)(9-6)}{9(9-7)}}$

$= \sqrt{\frac{4\times 3}{9\times 2}} = \sqrt{\frac{2}{3}}$

12. (B) We know

$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= 1(1-4) - 2(2-4) + 2(4-2)$

$= 5 \neq 0$

Thus, A is non-singular matrix of order 3.

Therefore r(A) = 3

13. (B)

14. (D) We have

$(ABC)^{-1} = [A(BC)]^{-1}$

$= (BC)^{-1}A^{-1}$

$= C^{-1}B^{-1}A^{-1}$

15. (A) Let A be a symmetric matrix.

Since, $AA^{-1} = I \Rightarrow (AA^{-1})^T = I$

$\Rightarrow (A^{-1})^T A^T = I \Rightarrow (A^{-1})^T = (A^T)^{-1}$

$\Rightarrow (A^{-1})^T = A^{-1}$ $[\because A^T = A]$

$\Rightarrow A^{-1}$ is a symmetric martrix.

16. (B)

17. (A) We have

$(A+B)(A-B) = A^2 - AB + BA - B^2$

So, (A) is not true.

18. (C) Since α, β, γ are the roots of the given eq^n,

therefore $\alpha + \beta + \gamma = -a$

$\alpha\beta + \beta\gamma + \gamma\alpha = 0$

and $\alpha\beta\gamma = -b$

Now,

$\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = -(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta+\beta\gamma+\gamma\alpha)$

$= -(\alpha+\beta+\gamma)\{(\alpha+\beta+\gamma)^2 - 3(\alpha\beta+\beta\gamma+\gamma\alpha)\}$

$= -(-a)\{(a^2-0)\} = a^3$

19. (A) Let $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$

Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ and $R_3 \to R_3(c)$,

We get

$\Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ba \\ ca^2b^2 & abc & ac+bc \end{vmatrix}$

$[\because R_1, R_2, R_3$ are multiplied by a, b and c respectively therefore we divide by $abc]$

$= \frac{1}{abc}(abc)^2\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ba \\ ab & 1 & ac+bc \end{vmatrix}$

$$= abc\begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix} \begin{bmatrix}\text{Appling} \\ C_3 \to C_3 + C_1\end{bmatrix}$$

$$= abc(ab + bc + ca)\begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$$

$$= abc(ab + bc + ca).\ 0 \quad \begin{bmatrix}\because C_2 \text{ and } C_3 \\ \text{are identitical}\end{bmatrix}$$

$= 0$

20. (B) We have

$$\sum_{n=1}^{\infty}\frac{2n}{(2n+1)!} = \sum_{n=1}^{\infty}\frac{2n+1-1}{(2n+1)!}$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{(2n)!} - \frac{1}{(2n+1)!}\right)$$

$$= \sum_{n=1}^{\infty}\frac{1}{(2n)!} - \sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$$

$$= \left(\frac{e+e^{-1}}{2} - 1\right) - \left(\frac{e-e^{-1}}{2} - 1\right)$$

$= e^{-1}$

21. (D) We have $T_3 = 1000$

$\Rightarrow T_{2+1} = 1000 \Rightarrow {}^5C_2\left(\frac{1}{x}\right)^{5-2}(x^{\log_{10} x})^2 = 1000$

$\Rightarrow 10x^2\log_{10}x \Rightarrow x^{-3} = 1000$

$\Rightarrow x^2\log_{10}x - 3$

$\Rightarrow 2\log_{10}x - 3 = 10^2$

$\Rightarrow 2\log_{10}x - 3 = \log_x 10^2$

$\Rightarrow 2\log_{10}x - 3 = \frac{2}{\log_{10}x}$ let $y = \log_{10}x$

$\Rightarrow 2y - 3 = \frac{2}{y}$

$\Rightarrow 2y^2 - 3y - 2 = 0$

$\Rightarrow (2y + 1)(y - 2) = 0$

$\Rightarrow y = 2 \left(\because y \neq -\frac{1}{2}\right) \Rightarrow \log_{10}x = 2$

$\Rightarrow x = 10^2 = 100$

22. (B) Let $(\sqrt{2}+1)^6 = I + F$,

where I is an integer and $0 \le F \le 1$.

Let $G = (\sqrt{2}-1)^6$

then $I + F + G = (\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

$= 2[{}^6C_0(\sqrt{2})^6 + \ldots] =$ an integer ...(i)

$\therefore F + G = 1$

Substituting F + G = 1 in eq^n(i), we get

$I + 1 = 2[{}^6C_0(\sqrt{2})^6 + {}^6C_2(\sqrt{2})^4 + {}^6C_4(\sqrt{2})^2 + {}^6C_6(\sqrt{2})]$

$I + 1 = 2[8 + 60 + 30 + 1]$

$I + 1 = 198$

$I = 197$

23. (A) ${}^nC_{15} = {}^nC_8 \Rightarrow 15 + 8 = 23$

$\left[{}^nC_x = {}^nC_y \Rightarrow x + y = n\right]$

$\therefore {}^nC_{21} = {}^{23}C_{21}$

$$= \frac{(23)!}{(21)!(23-21)!} = \frac{(23)!}{(21)!(2)!}$$

$$= \frac{23 \times 22 \times 21!}{21! \times 2} = 23 \times 11$$

$= 253$

24. (C) We have

${}^nP_r = {}^nP_{r+1}$

$$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\Rightarrow \frac{1}{(n-r)(n-r-1)!} = \frac{1}{(n-r-1)!}$$

$\Rightarrow n - r = 1$ ...(i)

And,

${}^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(r-1)!(n-r)!} = \frac{1}{(r-1)!(n-r+1)(n-r)!}$$

$\Rightarrow n - r + 1 = r$

$n - 2r = -1$ ...(ii)

After solving eq^n (i) and (ii) we get $n = 3$ and $r = 2$

25. (D) Since roots are imaginary, therefore

$b^2 - 4ac < 0$

the roots α and β are given by

$$\alpha = \frac{-b + i\sqrt{4ac - b^2}}{2a}$$

$$\beta = \frac{-b - i\sqrt{4ac - b^2}}{2a}$$

Clearly, $\alpha = \bar{\beta}$, therefore, $|\alpha| = |\beta|$

Furthermore,

$$|\alpha| = \sqrt{\frac{b^2}{4a^2} + \frac{4ac - b^2}{4a^2}}$$

$$|\alpha| = \sqrt{\frac{c}{a}} \quad [\because c > a]$$

$\Rightarrow |\alpha| > 1$

26. (A) Let $f(x) = x^2 - 2ax + a^2 + a - 3$.
Since $f(x)$ has real roots both less than 3. Therefore
Disc ≥ 0 and and $f(3) > 0$
$4a^2 - 4(a^2 + a - 3) \geq 0$ and $a^2 - 5a + 6 > 0$
$a^2 - a^2 - a + 3 \geq 0$ and $(a-2)(a-3) > 0$
$a \leq 3$ and $a < 2$ or $a > 3$
$\Rightarrow a < 2$

27. (A) $\int x^2 \cot^{-1} x\, dx$
Integration by part

$$= \cot^{-1}x\left[\frac{x^3}{3}\right] - \int \frac{-1}{1+x^2}.\frac{x^3}{3}dx$$

$$= \cot^{-1}x\left[\frac{x^3}{3}\right] - \int \frac{-1}{1+x^2}.\frac{x^3}{3}dx$$

$$= \frac{x^3}{3}\cot^{-1}x + \frac{1}{3}\int \frac{x^3}{1+x^2}dx \quad \ldots(i)$$

Consider

$$\int \frac{x^3}{1+x^2}dx = \frac{1}{2}\int \frac{(t-1)}{t}\,dt$$

$$= \frac{1}{2}\int \left(1 - \frac{1}{t}\right)dt$$

[Let $1 + x^2 = t \Rightarrow 2x\,dx = dt$]

$$= \frac{1}{2}[t - \log|t|]$$

$$= \frac{1}{2}[(1+x^2) - \log|1+x^2|] \quad \ldots(ii)$$

Substituting in (i) we get

$$\int x^2 \cot^{-1} x = \frac{x^3}{3}\cot^{-1}x + \frac{1}{6}(1+x^2) - \frac{1}{6}\log|1+x^2| + c$$

28. (A) We have

$$\int \frac{(x-4)}{(x-2)^3}e^x dx$$

$$= \int \frac{(x-2)-2}{(x-2)^3}e^x dx$$

$$= \int e^x\left\{\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3}\right\}dx$$

Let $f(x) = \frac{1}{(x-2)^2} \Rightarrow f'(x) = \frac{-2}{(x-2)^3}$

Now using formula

$$\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + c$$

$$\Rightarrow \int \frac{(x-4)}{(x-2)^3}e^x dx = e^x\frac{1}{(x-2)^2} + c$$

29. (D) Consider, $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

Let $\frac{(x^2+1)}{(x-1)^2(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+3)}$ ...(i)

After solving eq[4] (i) we get

$A = \frac{3}{8}$, $B = \frac{1}{2}$, $C = \frac{5}{8}$

$$\therefore \int \frac{x^2+1}{(x+3)(x-1)^2}dx$$

$$= \frac{3}{8}\int \frac{1}{(x-1)}dx + \frac{1}{2}\int \frac{1}{(x-1)^2}dx + \frac{5}{8}\int \frac{dx}{(x+3)}$$

$$= \frac{3}{8}\log|x-1| - \frac{1}{2(x+1)} + \frac{5}{8}\log|x+3| + c$$

30. (C) We have

$$\int 5^{5^{5^x}}.5^{5^x}.5^x dx$$

Put $5^{5^{5^x}} = t$

$5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x (\log_e 5)^3\, dx = dt$

Now,

$$\int 5^{5^{5^x}}.5^{5^x}.5^x dx = \int \frac{dt}{(\log_3 5)^3}$$

$$= \frac{t}{(\log_e 5)^3} + c$$

$$= \frac{5^{5^{5^x}}}{(\log_e 5)^3} + c$$

31. (A) Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ ...(i)

using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$ ...(i)

Adding eq[n] (i) and (ii)

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$
$x = 0 \Rightarrow t = 1$
$x = \pi \Rightarrow t = -1$

$\Rightarrow 2I = -\pi\int_{1}^{-1}\frac{dt}{(1+t^2)}$

$\Rightarrow 2I = -\pi\left[\tan^{-1}t\right]_{1}^{-1}$

$\Rightarrow 2I = -\pi\left[\tan^{-1}(-1)-\tan^{-1}(1)\right]$

$\Rightarrow 2I = -\pi\left[\frac{-\pi}{4}-\frac{\pi}{4}\right] = \frac{\pi^2}{2}$

$\Rightarrow I = \frac{\pi^2}{4}$

32. (B) Let $I = \int_0^{2\pi}\frac{1}{1+e^{\sin x}}dx$ ...(i)

using property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\Rightarrow I = \int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx$

$\Rightarrow I = \int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx$

$\Rightarrow I = \int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx$ ...(ii)

Adding eq^n (i) and (ii)

$\Rightarrow 2I = \int_0^{2\pi}\left(\frac{1}{1+e^{\sin x}}+\frac{e^{\sin x}}{1+e^{\sin x}}\right)dx$

$\Rightarrow 2I = \int_0^{2\pi}\frac{1+e^{\sin x}}{1+e^{\sin x}}dx$

$\Rightarrow 2I = [x]_0^{2\pi} = [2\pi - 0]$

$\Rightarrow I = \pi$

33. (A) $I = \int_1^4 f(x)dx$

given $f(x) = \begin{cases} 2x+8, & 1 \le x \le 2 \\ 6x, & 2 \le x \le 4 \end{cases}$

$= \int_1^2 (2x+8)dx + \int_2^4 6x\ dx$

$= \left[\frac{2x^2}{2}+8x\right]_1^2 + \left[\frac{6x^2}{2}\right]_2^4$

$= \left[x^2+8x\right]_1^2 + \left[3x^2\right]_2^4$

= [(4 +16) – (1 + 8)] + [48 – 12]
= 47

34. (D) $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|\,|\vec{a}|}$

$\cos\theta = \frac{1-1-1}{\sqrt{3}\sqrt{3}} = -\frac{1}{3}$

$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

Angle bet^n two vectors is $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

35. (C) By example :-

Rational No. $\Rightarrow \frac{1}{2}$ = 0.5 (Terminating)

Rational No. $\Rightarrow \frac{1}{3}$ = 0.3333....(Non terminating, recurring)

Irrational No. $\Rightarrow \sqrt{2}$ = 1.414...(Non terminating, non recurring)

36. (D) $x = \sqrt{3}+\sqrt{2}$

$x^{-2} = \frac{1}{x^2} = \frac{1}{(\sqrt{3}+\sqrt{2})^2} = \frac{1}{5+2\sqrt{6}} = 5-2\sqrt{6}$

37. (B) $x^2 - 5|x| + 6 = 0$

$|x|^2 - 5|x| + 6 = 0$

$x^2 = |x|^2$
$\Rightarrow |x| = a$
$x = \pm a$

$(|x|-3)(|x|-2) = 0$

$|x| = 3 \Rightarrow x = \pm 3$

$|x| = 2 \Rightarrow x = \pm 2$

x = 3, –3, 2, –2

four real roots

38. (C) $x = y\sqrt{1-y^2}$

diff. w.r. to x

$\Rightarrow 1 = \frac{dy}{dx}\sqrt{1-y^2} + y\frac{1}{2\sqrt{1-y^2}}(-2y)\frac{dy}{dx}$

$\Rightarrow 1 = \frac{dy}{dx}\left[\sqrt{1-y^2}-\frac{y^2}{\sqrt{1-y^2}}\right]$

$\Rightarrow 1 = \frac{dy}{dx}\left[\frac{1-2y^2}{\sqrt{1-y^2}}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{(1-2y^2)}$

39. (C) $x^m y^n = 2(x+y)^{m+n}$

taking log both sides

$\Rightarrow \log(x^m y^n) = \log 2(x+y)^{m+n}$

$\Rightarrow m\log x + n\log y = \log 2 + (m+n)\log(x+y)$

Now, diff. w.r . to x

$$\Rightarrow \frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0 + \frac{(m+n)}{(x+y)}\left[1+\frac{dy}{dx}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{(m+n)}{(x+y)} - \frac{n}{y}\right] = \left[\frac{m}{x} - \frac{m+n}{x+y}\right]$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{my+ny-nx-ny}{y(x+y)}\right] = \left[\frac{mx+my-mx+nx}{x(x+y)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

40. (C) Let $y = \frac{d}{dx}\left[\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)\right]$

Put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$

$$\therefore\ y = \frac{d}{dx}\left[\sin^{-1}\left(\sin\alpha\sqrt{1-\sin^2\beta} - \sin\beta\sqrt{1-\sin^2\alpha}\right)\right]$$

$$= \frac{d}{dx}\left[\sin^{-1}[\sin(\alpha-\beta)]\right]$$

$$= \frac{d}{dx}(\alpha-\beta)$$

$$= \frac{d}{dx}(\sin^{-1}x - \sin^{-1}\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}$$

41. (B) $f(x-y), f(x)\, f(y)$ and $f(x+y)$ are in AP

$\Rightarrow 2f(x)\, f(x) = f(x-y) + f(x+y)$ ...(i)

when $x = 0,\ y = 0$

$2f(0)\, f(0) = f(0) + f(0)$

$\Rightarrow f(0) = 1$ $[\because f(0) \neq 0]$

when $x = 0,\ y = -x$ then from (i)

$2f(0)\, f(-x) = f(x) + f(-x)$

$\Rightarrow f(-x) = f(x)$

$\therefore f(x) + f(-x) = 0$

Hence $f'(2) + f'(-2) = 0$

& $f'(3) + f'(-3) = 0$

42. (B) We have

$$x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

$$\begin{bmatrix}2x\\3x\end{bmatrix} + \begin{bmatrix}-y\\y\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

$$\begin{bmatrix}2x-y\\3x-y\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

Now,

$2x - y = 10$ ...(i)

$3x + y = 5$ ...(ii)

Solving eq^n (i) and (ii) we get

$x = -3,\ y = -4$

43. (D) $1^3, 2^3$

Only two factors are perfect cube

44. (C) We know that

$$\left[\frac{8}{5}\right] = [1.6] = 1$$

$$\left[\frac{-8}{5}\right] = [-1.6] = -2$$

$$\left[\frac{-26}{7}\right] = [-3.7] = -4$$

Now, A.T.Q.,

$1 - 2 + 4 = 3$

45. (B) Here $(x-3)$ and $\left(x - \frac{1}{3}\right)$ is factor of $ax^2 + 5x + b$

$\Rightarrow 3, \frac{1}{3}$ are zeros

We know that

Sum of zeros $= \frac{-B}{A}$

$\Rightarrow 3 + \frac{1}{3} = \frac{-5}{a}$

$\Rightarrow a = \frac{-3}{2}$

Product of zeros $= \frac{C}{A}$

$\Rightarrow 1 = \frac{b}{a}$

$\Rightarrow b = \frac{-3}{2}$

46. (B) We have

$A^2 = A \Rightarrow A^3 = A^2 = A$

Now,

$(A - I)^3 + (A + I)^3 - 7A$

$\Rightarrow A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3AI^2 + 3A^2I - 7A$

$\Rightarrow A - I - 3A + 3A + A + I + 3A + 3A - 7A$

$\Rightarrow A$

47. (B) We know that

$\sin x_i \le 1 \qquad \forall\, i = 1, 2, ...n$

$\sin x_1 + 2\sin x_2 + ...n \sin x_n \le 1 + 2 + 3... ....+ n$

$\sin x_1 + 2\sin x_2 + ...n \sin x_n \le \frac{n(n+1)}{2}$

$\text{Max}(\sin x_1 + 2\sin x_2 + ...n \sin x_n) = \frac{n(n+1)}{2}$

48. (A) We have LHD

$\lim_{h\to0}\frac{f(k)-f(k-h)}{h}$

$\lim_{h\to0}\frac{k\sin(k\pi)-(k-h)\sin(k-h)\pi}{h}$

$\lim_{h\to0}\frac{-(k-1)\sin(k-h)\pi}{h}$

We know

$\sin(k\pi) = 0$

$\sin(k\pi - \pi h) = (-1)\ k-1\ \sin h\pi$

$\lim_{h\to0}\frac{-(k-1)(-1)^{k-1}\sin h\pi}{h\pi}\times\pi$

$(-1)^k\,(k-1)\pi$

49. (A) Consider, $f(n) = \sin^n\theta + \cos^n\theta$

$\therefore \frac{f(3)-f(5)}{f(5)-f(7)} = \frac{\sin^3\theta+\cos^3\theta-\sin^5\theta-\cos^5\theta}{\sin^5\theta-\sin^5\theta+\cos^7\theta-\cos^7-\theta}$

$= \frac{\sin^3\theta+\cos^5\theta-\sin^3\theta-\cos^5\theta}{\sin^5\theta-\sin^7\theta+\cos^5\theta-\cos^7-\theta}$

$= \frac{\sin^3\theta(1-\sin^2\theta)+\sin^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta+\cos^5\theta(1-\cos^2-\theta)}$

$= \frac{\sin^3\theta\cos^2\theta+\cos^3\theta\sin^2\theta}{\sin^5\theta\sin^2\theta+\sin^5\theta\sin^2-\theta}$

$= \frac{\sin^2\theta\cos^2\theta(\sin^3\theta+\cos\theta)}{\sin^2\theta\cos^2\theta(\sin^3\theta+\cos^3\theta)}$

$= \frac{\sin\theta+\cos\theta}{\sin^3\theta+\cos^2\theta}$

$= \frac{f(1)}{f(3)}$

50. (C) Consider, $(1 + \tan1°) = 1 + \tan(45° - 44°)$

$= 1 + \frac{\tan45°-\tan44°}{1+\tan45°\tan44°}$

$\left[\because \tan A - \tan B = \frac{\tan A-\tan B}{1+\tan A\tan B}\right]$

$= 1 + \frac{1-\tan44°}{1+\tan44°}$

$= \frac{1+\tan44°+1-\tan44°}{1+\tan44°}$

$= \frac{2}{1+\tan44°}$

$\therefore \left(\frac{2}{1+\tan44°}\right)\left(\frac{2}{1+\tan43°}\right)\left(\frac{2}{1+\tan42°}\right)$

$......(1+\tan43°)(1+\tan44°)2 = 2^n$

$\Rightarrow 2\times2\times.........23 \text{ times} = 2^n$

$\Rightarrow 2^{23} = 2^n$

$\Rightarrow n = 23$

51. (A) Consider, $\cos^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}k$

$\Rightarrow \cos^{-1}\left(\frac{3}{5}\times\frac{12}{13}-\sqrt{1-\frac{9}{25}}\sqrt{1-\frac{144}{169}}\right) = \cos^{-1}k$

$\left[\because \cos^{-1}x+\cos^{-1}y = \cos^{-1}(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right]$

$\Rightarrow \cos^{-1}\left(\frac{36}{65}-\sqrt{\frac{16}{25}}\sqrt{\frac{25}{169}}\right) = \cos^{-1}k$

$\Rightarrow \cos^{-1}\left(\frac{36}{65}-\frac{20}{65}\right) = \cos^{-1}k$

$\Rightarrow \cos^{-1}\frac{16}{65} = \cos^{-1}k$

$\Rightarrow k = \frac{16}{65}$

52. (A) Consider, $\sin 2x + 2\sin x - \cos x - 1 = 0$

$\Rightarrow 2\sin x\cos x + 2\sin x - \cos x - 1 = 0$

$[\because \sin 2x = 2\sin x\cos x]$

$\Rightarrow 2\sin x(\cos x + 1) - 1(\cos x + 1) = 0$

$\Rightarrow (\cos x + 1)(2\sin x - 1) = 0$

$\Rightarrow \cos x + 1 = 0 \text{ or } 2\sin x - 1 = 0$

$\Rightarrow \cos x = -1 \text{ or } \sin x = \frac{1}{2}$

$\Rightarrow \cos x = \cos\pi \text{ or } \sin x = \sin\frac{\pi}{6} \text{ or } \sin x$

$= \sin\frac{5\pi}{6}$

$x = \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

53. (C) Consider, $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \cos x & \cos x \\ \sin x + 2\cos x & \sin x & \cos x \\ \sin x + 2\cos x & \cos x & \sin x \end{vmatrix} = 0$

[$C_1 \to C_1 + C_2 + C_3$]

$\Rightarrow (\sin x + 2\cos x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow (\sin x + 2\cos x)$

$\begin{vmatrix} 0 & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

[$R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$]

$\Rightarrow (\sin x + 2\cos x)(\cos x - \sin x)^2 = 0$

$\Rightarrow \sin x = -2\cos x$ or $\cos x = \sin x$

$\Rightarrow \tan x = -2$ or $\tan x = 1$

$x = \frac{\pi}{4}\left[\because -\frac{\pi}{4} \le x \le \frac{\pi}{4}\right]$

54. (D) Area of triangle using Heron's formula is given by $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$\therefore$ A = $\sqrt{35(35-17)(35-25)(35-28)}$

$\Rightarrow \frac{1}{2} \times 17 \times h = 210$

$\Rightarrow h = \frac{420}{17}$

55. (C) Consider, $\cos A + \cos B + \cos C = \frac{3}{2}$

We know that, $\cos 60° = \frac{1}{2}$

For A = B = C = 60°

$\cos 60° + \cos 60° + \cos 60° = \frac{3}{2}$

56. (B) Taking an example of an equilateral triangle, we have

$\Delta = \frac{a^2\sqrt{3}}{4}$

$\Rightarrow a^2 = \frac{4\Delta}{\sqrt{3}}$

$\Rightarrow 3a^2 = \frac{4\Delta}{\sqrt{3}}(3)$

$\Rightarrow 3a^2 = 4\sqrt{3}\,\Delta$

Therefore, the minimum value of the sum of squares of sides is $4\sqrt{3}\,\Delta$.

57. (A) If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{j}$ and $c\hat{i} + c\hat{j} + b\hat{k}$

$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

[$C_1 \to (C_1 - C_2)$]

$\begin{vmatrix} 0 & a & c \\ 1 & 0 & 1 \\ 0 & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab$

58. (C) Consider, $\vec{b} = 3\hat{i} + 6\hat{j} + 6\hat{k}$

$\vec{b} = 3(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{b} = 3(\vec{a})$

59. (D) Consider $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{2}} - 1}{(1+x)^{\frac{1}{2}} - 1}$

$= \lim_{x\to 0} 1$

$= 1$

60. (A) Consider, $\lim_{x\to 0}\frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$

$= \lim_{x\to 0}\frac{(a^{\tan -1} - 1) - (a^{\sin x} - 1)}{\tan x - \sin x}$

$= \lim_{x\to 0}\frac{\left(\frac{a^{\tan x} - 1}{\tan x}\right)\tan x - \left(\frac{a^{\sin x} - 1}{\sin x}\right)\sin x}{\tan x - \sin x}$

$= \lim_{x\to 0}\frac{(\log_e a)\tan x - (\log_e a)\sin x}{\tan x - \sin x}$

$\left[\because \lim_{x\to 0}\frac{a^x - 1}{x} = \log_e a\right]$

$= \lim_{x\to\infty}\log_e a\left(\frac{\tan x - \sin x}{\tan x - \sin x}\right)$

$= \log_e a$

61. (D) Consider, $1 + 16x^2y = \tan(x - 2y)$
diff. w.r.t. x

$$\Rightarrow 32xy + 16x^2\frac{dy}{dx} = \sec^2(x-2y)\left(1-2\frac{dy}{dx}\right)$$

at point $\left(\frac{\pi}{4}, 0\right)$

$$\Rightarrow 0 + 16\left(\frac{\pi}{4}\right)^2\frac{dy}{dx} = \sec^2\left(\frac{\pi}{4}-0\right)\left(1-2\frac{dy}{dx}\right)$$

$$\Rightarrow \pi^2\frac{dy}{dx} = 2 - 4\frac{dy}{dx}$$

$$\Rightarrow (\pi^2 + 4)\frac{dy}{dx} = 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\pi^2+4}$$

62. (B) The greatest coefficient in the expansion is same as the coefficient of middle term. The Coefficient of middle term in

$\left(x+\frac{1}{x}\right)^{2n} = {}^{2n}C_n$

$$\frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!}$$

63. (B) Total number of outcomes during throwing a pair of dice is $6^2 = 36$
Number of favor events = (1, 6), (2, 5), (3, 4), (5, 6), (6, 1), (4, 3), (5, 2), (6, 5) = 8

Required probability = $\frac{8}{36} = \frac{2}{9}$

64. (A) Required probability = $\frac{{}^8C_6}{{}^{10}C_6}$

$= \frac{28}{210}$

$= \frac{2}{15}$

65. (D) Consider, $y + \sqrt{1+y^2} = e^x$

$\Rightarrow \sqrt{1+y^2} = e^x - y$

$\Rightarrow 1 + y^2 = e^2x + y^2 - 2ye^x$

$\Rightarrow 2ye^x = e^{2x} - 1$

$\Rightarrow 2y = e^x - e^{-x}$

$\Rightarrow y = e^x - e^{-x}$

$\Rightarrow y = \frac{e^x - e^{-x}}{2}$

66. (C) The number of the ways in which the expert opinion can be expressed is 4^5 = 1024

67. (D) Given, $\lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = 4$

$f'(0) = 4\left[\because f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}\right]$

$g'(x) = (2x + 2) f(x) + (x^2 + 2x + 3) f'(x)$

$g'(0) = 2f(0) + 3f'(0)$

$= 2(5) + 3(4)$

$= 10 + 12$

$= 22$

68. (B) Here, we are given the focus S(1, –1), directrix $x - y - 3 = 0$ and eccentricity $\frac{1}{2}$

Let P(x, y) be any point, then according to definition of ellipse we have

SP = ePM

$$\Rightarrow \sqrt{(x-1)^2+(y+1)^2} = \frac{1}{2}\left(\frac{x-y-3}{\sqrt{1^2+(-1)^3}}\right)$$

$$\Rightarrow (x-1)^2 + (y+1)^2 = \frac{1}{8}(x-y-3)^2$$

$\Rightarrow 8(x^2 - 2x + 1 + y^2 + 2y + 1) = x^2 + y^2 + 9 - 2xy + 6y - 6x$

$\Rightarrow 8x^2 - 16x + 8y^2 + 16y + 16 = x^2 + y^2 + 9 - 2xy + 6y - 6x$

$\Rightarrow 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$

69. (D) Consider, $I = \int_0^{\pi/4}\sqrt{\tan x} + \sqrt{\cot x}\, dx$

$$= \int_0^{\pi/4}\sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}}\, dx$$

$$= \int_0^{\pi/4}\frac{\sin x + \cos x}{\sqrt{\cos x \sin x}}\, dx$$

= Let $\sin x - \cos x = t$

$\Rightarrow (\cos x + \sin x)dx = dt$

Also, $(\sin x - \cos x)^2 = t^2$

$\Rightarrow 1 - 2\sin x\cos x = t^2$

$\Rightarrow \sin x\cos x = \frac{1-t^2}{2}$

$\Rightarrow \sqrt{\sin x\cos x} = \frac{\sqrt{1-t^2}}{\sqrt{2}}$

$$\therefore \int_0^{\pi/4}\frac{\sin x+\cos x}{\sqrt{\cos x\sin x}}\,dx = \int_{-1}^{0}\frac{1}{\sqrt{1-t^2}}\,dx$$

$= \sqrt{2}\sin^{-1}t$

$= \sqrt{2}\sin^{-1}(\sin x - \cos x)$

$= \sqrt{2}\sin^{-1}(\sin x - \cos x)$

$$\therefore I = \int_0^{\pi/4}\sqrt{\tan x}+\sqrt{\cot x}\,dx = \sqrt{2}\left[\sin^{-1}(\sin x - \cos x)\right]_0^{\pi/4}$$

$$= \sqrt{2}\left(\frac{\pi}{2}\right) = \frac{\pi}{\sqrt{2}}$$

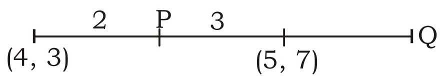
70. (D) Let us suppose, the perpendiculars lines be the x and y-axis.
Now, the sum of the distances from the point P(x, y) is 1. i.e., $|x|+|y| = 1$
This is the locus of the point P which would be the square whose sides are
$x + y = 1; -x + y = 1; x - y = 1; -x - y = 1$

71. (D) Consider, $s_n = u + \frac{a}{2}(2n-1)$
Substitute $n = 8$, $s_n = 114$, $u = 0$ in $s_n = u + \frac{a}{2}(2n-1)$
$\Rightarrow 114 = 0 + \frac{a}{2}(16 - 1)$
$\Rightarrow 114 = 7.5a$
$\Rightarrow a = 15.2$ m/sec^2

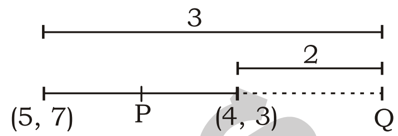
72. (A) $f(x) = \sin\left(x+\frac{\pi}{5}\right) + \cos\left(x+\frac{\pi}{5}\right)$
$= \sqrt{2}\sin\left(x+\frac{\pi}{5}+\frac{\pi}{4}\right)$
$f(x)$ is maximum when $x = \frac{\pi}{5}+\frac{\pi}{4} = \frac{\pi}{2}$
$x = \frac{\pi}{20}$

73. (D) S = 0.4 + 0.44 + 0.444 +......n terms
S = 4(0.1 + 0.11 + 0.111 +......n terms)
$S = 4\left(\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+........n \text{ terms}\right)$
$S = \frac{4}{9}\left(\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+........n \text{ terms}\right)$
$S = \frac{4}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+.....n \text{ terms}\right]$
$S = \frac{4}{9}\left[(1+1+...n \text{ terms})-\left(\frac{1}{10}+\frac{1}{100}+...n \text{ terms}\right)\right]$
$S = \frac{4}{9}\left[n-\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$
$S = \frac{4}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

74. (A) Firstly take internally



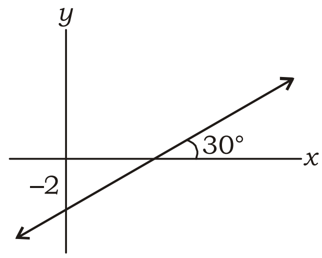
Co-ordinate of point P
$= \left(\frac{4\times3+5\times2}{2+3}, \frac{3\times3+7\times2}{2+3}\right)$
$= \left(\frac{22}{5}, \frac{23}{5}\right)$
Now, Externally



Co-ordinate of point Q =
$\left(\frac{4\times3-5\times2}{3-2}, \frac{3\times3-7\times2}{3-2}\right)$
= (2, –5)
Now, distance between (PQ) =
$\sqrt{\left(2-\frac{22}{5}\right)^2+\left(-5-\frac{23}{5}\right)^2}$
$= \frac{12\sqrt{17}}{5}$

75. (B) $(m^2 - mn)y = (mn + n^2)x + n^3$ ...(i)
$y = \frac{(mn+n^2)}{(m^2-mn)}x + \frac{n^3}{(m^2-mn)}$
Slope of equation $S_1 = \frac{(mn+n^2)}{(m^2-mn)}$
$(mn + m^2)y = (mn - n^2)x + m^2$ ...(ii)
$y = \left(\frac{mn-n^2}{mn+m^2}\right)x + \frac{m^3}{mn+m^2}$
Slope of equation $S_2 = \frac{mn-n^2}{mn+m^2}$
Angle between lines $= \tan^{-1}\left(\frac{S_1 - S_2}{1+S_1\times S_2}\right)$
$= \tan^{-1}\left(\frac{\frac{mn+n^2}{m^2-mn}-\frac{mn-n^2}{mn+m^2}}{1+\frac{mn+n^2}{m^2-mn}.\frac{mn-n^2}{mn+m^2}}\right)$
After solving this
Angle $= \tan^{-1}\left(\frac{4m^2n^2}{m^4-n^4}\right)$

76. (D)



Equation of line is

$y = mx + c$

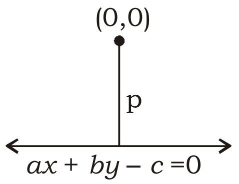
$y = \tan 30°\, x - 2$

$y = \frac{1}{\sqrt{3}} x - 2$

$\sqrt{3}\, y = x - 2\sqrt{3}$

$x - \sqrt{3}\, y - 2\sqrt{3} = 0$

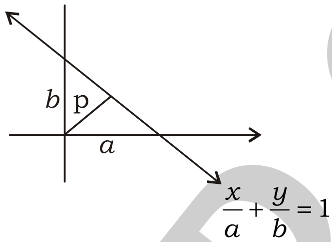
77. (C) **Statement I**



$p = \frac{|-c|}{\sqrt{a^2 + b^2}} \Rightarrow p^2 = \frac{c^2}{a^2 + b^2}$

It is true.

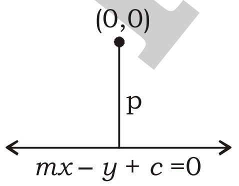
Statement II



$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

It is true.

Statement III



$p = \frac{|c|}{\sqrt{m^2 + 1}}$

$\Rightarrow p^2 = \frac{c^2}{m^2 + 1} \Rightarrow \frac{1}{p^2} = \frac{m^2 + 1}{c^2}$

It is false.

78. (D) $f(x) = |x + 1|$

going through options

(A) $f(x^2) = |x^2 + 1|$

$\{f(x)\}^2 = (x + 1)^2$

$\Rightarrow f(x^2) \neq \{f(x)\}^2$

(B) $f|x| = ||x| + 1|$

$|f(x)| = ||x + 1|| = |x + 1|$

$\Rightarrow f|x| \neq |f(x)|$

(C) $f(x + y) = |x + y + 1|$

$f(x) + f(y) = |x + 1| + |y + 1|$

$f(x + y) \neq f(x) + f(y)$

Now, option D is correct.

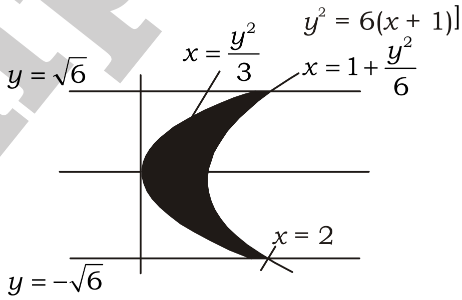
79. (A) $y = f(x) = \frac{x^2}{1 + x^2}$

Clearly $y \geq 0$,

$x^2 < 1 + x^2$

So range is [0, 1)

80. (C)



Solving $y^2 = 6(x + 1)$ and $y^2 = 3x$

we get $x = 2$ and $y = \pm\sqrt{6}$

Area $= \int_{-\sqrt{6}}^{\sqrt{6}} \left(1 + \frac{y^2}{6} - \frac{y^2}{3}\right) dy$

$= 2\int_{0}^{\sqrt{6}} \left(1 - \frac{y^2}{6}\right) dy$

$= 2\left[y - \frac{y^3}{18}\right]_0^{\sqrt{6}}$

$= \frac{2 \times 2\sqrt{6}}{3} = \frac{4\sqrt{6}}{3}$

81. (A)

Bag 1	Bag 2
4 copper coins	6 copper coins
3 silver coins	2 silver coins

$\frac{1}{2} \times \frac{4}{7}$ or $\frac{1}{2} \times \frac{6}{8}$

$\frac{2}{7} + \frac{3}{8} = \frac{16 + 21}{56} = \frac{33}{56}$

82. (B) $\int \sin 4x \cos 7x \, dx = A\cos 3x + B\cos 11x$

L.H.S

$\Rightarrow \int \sin 4x \cos 7x \, dx = \frac{1}{2}\int 2\sin 4x \cos 7x \, dx$

$= \frac{1}{2}\int [\sin(4x+7x) + \sin(4x-7x)]\, dx$

$= \frac{1}{2}\int \sin 11x - \sin 3x \, dx$

$= \frac{1}{2}\left[\frac{-\cos 11x}{11} + \frac{\cos 3x}{3}\right]$

$\Rightarrow \frac{-\cos 11x}{22} + \frac{\cos 3x}{6}$

compair L.H.S and R.H.S

$A = \frac{1}{6}$, $B = \frac{-1}{22}$

83. (D) $x^2 - 5x + 16 = 0$

$\alpha + \beta = 5$, $\alpha\beta = 16$

and $x^2 + px + q = 0$

$\left(\alpha^2 + \beta^2 + \frac{\alpha\beta}{2}\right) = -p$, $(\alpha^2 + \beta^2).\frac{\alpha\beta}{2} = q$

Now,

$\alpha + \beta = 5$

Square both side

$\alpha^2 + \beta^2 + 2\alpha\beta = 25$

$\alpha^2 + \beta^2 = 2\times16 = 25$

$\alpha^2 + \beta^2 = -7$

Again

$\alpha^2 + \beta^2 + \frac{\alpha\beta}{2} = -p$

$\Rightarrow -7 + 8 = -p$

$\Rightarrow p = -1$

and (a2 + . $\frac{\alpha\beta}{2}$ = q

$\Rightarrow -7 \times 8 = q$

$\Rightarrow q = -56$

84. (A)

85. (D)

Class	f	C
0 – 10	17	17
10 – 20	19	36
20 - 30	**21**	**57**
30 – 40	23	80
40 – 50	20	100

$N = 100$, $\frac{N}{2} = 50$

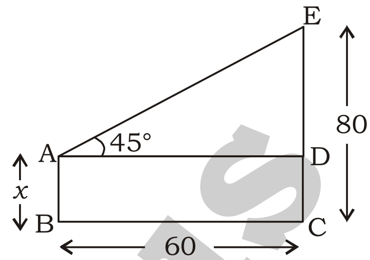
$l_1 = 20$, $l_2 = 30$, $f = 21$, $C = 36$

Median $= l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$

$= 20 + \frac{50-36}{21} \times (30 - 20)$

$= 20 + \frac{14}{21} \times 10 = 26\frac{2}{3}$

86. (A)



Let, 'EC' be the first tree an 'AB' be the second tree.

Height of first tree is '80 m' and let height of second tree be 'x'

Then , AB = x and ED = $(80 - x)$

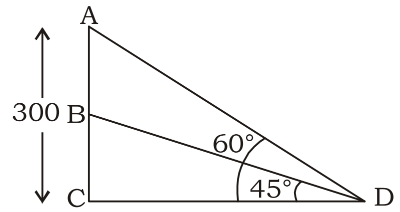
In ΔEAD,

$\tan 45° = \frac{ED}{AD}$

$1 = \frac{80 - x}{60}$

$x = 20$ m

87. (A)



Let, the height fo first plane be 'AC' which is equal to 300 m. Height of second plane be 'BC'.

From triangle ACD,

$\tan 60° = \frac{300}{CD}$

$\sqrt{3} = \frac{300}{CD}$

$CD = \frac{300}{\sqrt{3}}$ ...(i)

From triangle BCD,

$\tan 45° = \frac{BC}{CD}$

CD = BC ...(ii)

From equation (i) and (ii)

$BC = \frac{300}{\sqrt{3}}$

$BC = 100\sqrt{3}$

88. (B) $\begin{bmatrix} A = \text{Hindi} \\ B = \text{English} \end{bmatrix}$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$\Rightarrow 60 = 45 + 33 - x$
$\Rightarrow x = 18$

89. (A) $\sqrt{5+12i} = \sqrt{5+2\times 6i}$
$= \sqrt{5+2\times 3\times 2i}$
$= \sqrt{(3)^2+(2i)^2+2\times 3\times 2i}$
$= \sqrt{(3+2i)^2} = \pm 3 + 2i$

90. (D) $x^2 + kx - b = 0$
Let roots α, β
$\alpha + \beta = -k$, $\alpha\beta = -b$
$\alpha^2 + \beta^2 = 2b$ [given]
Now,
$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$(-k)^2 = 2b - 2b$
$k = 0$

91. (D) $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$
$[\because 1 + \omega + \omega^2 = 0]$
$= (-2\omega^2)^7 = (-2)^7\omega^{14} = -128\,\omega^2$

92. (B) Write the nth term of the given series and simplify it to get its lowest form.
Then, apply, $S_n = \sum T_n$

Given series is $\frac{1^3}{1} + \frac{1^3-2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$
$\ldots.\infty$

Let T_n be the n^{th} term of the given series.

$\therefore T_n = \frac{1^3+2^3+3^3+\ldots\ldots+n^3}{1+3+5+\ldots.+\text{upto } n \text{ terms}}$

$= \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{n^2} = \frac{(n+1)^2}{4}$

$S_9 = \sum_{n=1}^{9}\frac{(n+1)^2}{4} = \frac{1}{4}(2^2 + 3^2 + \ldots.+10^2) + 1^2 - 1^2]$

$= \frac{1}{4}\left[\frac{10(10+1)(20+1)}{6} - 1\right] = \frac{384}{4} = 96$

93. (B) Since, $x_1, x_2, \ldots., x_n$ are any real numbers.

$\therefore \frac{x_1^2+x_2^2+\ldots..+x_n^2}{n} \geq \left(\frac{x_1+x_2+\ldots..+x_n}{n}\right)^2$

[using m^{th} power theorem]

$\Rightarrow n\sum_{i=1}^{n} x_i^2 \geq \left(\sum_{i=1}^{n} x_i\right)^2$

94. (C) There are two possible cases
Case I Five 1's, one 2's, one 3's

Number of numbers = $\frac{7!}{5!} = 42$

Case II Four 1's, three 2's

Number of numbers = $\frac{7!}{4!3!} = 35$

$\therefore$ Total number of numbers = 42 + 35 = 77

95. (B) $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + \ldots..+ ({}^{21}C_{10} - {}^{10}C_{10})$
$= ({}^{21}C_1 - {}^{21}C_2 + \ldots.+ {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots \ldots.+ {}^{10}C_{10})$

$= \frac{1}{2}({}^{21}C_1 - {}^{21}C_2 + \ldots..+ {}^{21}C_{20}) - (2^{10} - 1)$

$= \frac{1}{2}({}^{21}C_1 - {}^{21}C_2 + \ldots..+ {}^{21}C_{21} - 1) - (2^{10} - 1)$

$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1) = 2^{20} - 1 - 2^{10} + 1$

$= 2^{20} - 2^{10}$

96. (A) Let $\phi(x) = f(x) - g(x) = \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$

Now, to check one-one.
Take any straight line parallel to X-axis which will intersect $\phi(x)$ only at one point.
$\Rightarrow \phi(x)$ is one-one.
To check onto

As $f(x) = \begin{cases} x, x \in Q \\ -x, x \notin Q \end{cases}$, which shows

$y = x$ and $y = -x$ for rational and irrational values
$\Rightarrow y \in$ real numbers.
$\therefore$ Range = Co domain $\Rightarrow$ onto
Thus, $f - g$ is one-one and onto.

97. (B) Given, $f(x) = [\tan^2 x]$
Now, $-45° < x < 45°$
$\Rightarrow \tan(-45°) < \tan x < \tan(45°)$
$\Rightarrow -\tan 45° < \tan x < \tan(45°)$
$\Rightarrow -1 < \tan x < 1$
$\Rightarrow 0 < \tan^2 x < 1$
$\Rightarrow [\tan^2 x] = 0$
i.e. $f(x)$ is zero for all values of x from $x = -45°$ to $45°$.
Thus, $f(x)$ exists when $x \to 0$ and also it is continuous at $x = 0$.
Also, $f(x)$ is differentiable at $x = 0$ and have a value of zero.

98. (B) Given $P(x) = a_0 + a_1x^2 + a_2x^4 + ... + a_nx^{2n}$
where, $a_n > a_{n-1} > a_{n-2} > ... > a_2 > a_1 > a_0 > 0$
$\Rightarrow P'(x) = 2a_1x + 4a_2x^3 + ... + 2na_nx^{2n-1}$
$= 2x(a_1 + 2a_2x^2 + ... + na_nx^{2n-2})$ ...(i)
where, $(a_1 + 2a_2x^2 + 3a_3x^4 + ... + na_nx2^{n-2}) > 0, \forall x \in R$.

Thus, $\begin{cases} P'(x) > 0, \text{when } x > 0 \\ P'(x) < 0, \text{when } x < 0 \end{cases}$

i.e. $P'(x)$ changes sign from (–ve) to (+ve) at $x = 0$.
$\therefore$ $P(x)$ attains minimum at $x = 0$.
Hence, it has only one minimum at $x = 0$.

99. (C) $I_1 = \int_{1-k}^{k} xf[x(1-x)]dx$

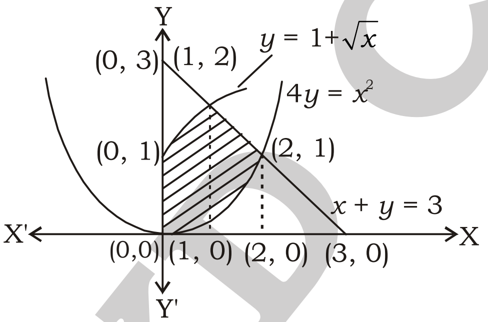
$\Rightarrow I_1 = \int_{1-k}^{k} (1-x)f[(1-x)x]dx$

$= \int_{1-k}^{k} f[(1-x)dx] - \int_{1-k}^{k} xf(1-x)\, dx$

$\Rightarrow I_1 = I_2 - I_1 \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$

100. (D) Required area

$= \int_0^1 (1+\sqrt{x})\, dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$



$= \left[x + \frac{x^{3/2}}{3/2}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2$

$= \left(1 + \frac{2}{3}\right) + \left(6 - 2 - 3 + \frac{1}{2}\right) - \left(\frac{8}{12}\right)$

$= \frac{5}{3} + \frac{3}{2} - \frac{2}{3} = 1 + \frac{3}{2} = \frac{5}{2}$ sq units

101. (D) Given differential equation is
$y(1 + xy)\, dx = x\, dy$
$\Rightarrow y\, dx + xy^2 dx = x\, dy$

$\Rightarrow \frac{x\,dy + y\,dx}{y^2} = x\, dx$

$\Rightarrow -\frac{(y\,dx - x\,dy)}{y^2} = x\, dx \Rightarrow -d\left(\frac{x}{y}\right) = x\, dx$

On integrating both sides, we get

$-\frac{x}{y} = \frac{x^2}{2} + C$

$\because$ It passes through (1, –1).

$\therefore\ 1 = \frac{1}{2} + C \Rightarrow C = \frac{1}{2}$

Now, from eq(i) $-\frac{x}{y} = \frac{x^2}{2} + \frac{1}{2}$

$\Rightarrow x^2 + 1 = -\frac{2x}{y}$

$\Rightarrow y = -\frac{2x}{x^2+1}$

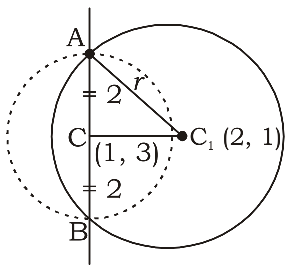
$\therefore f\left(-\frac{1}{2}\right) = \frac{4}{5}$

102. (D) Let, the vertices of triangle be $A(1, \sqrt{3})$, $B(0,0)$ and $C(2, 0)$. Here, AB = BC = CA = 2.
Therefore, it is an equilateral triangle. So, the incentre coincides with centroid.

$\therefore\ I = \left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right)$

$\Rightarrow I = \left(1, \frac{1}{\sqrt{3}}\right)$

103. (C) Here, radius of smaller circle, AC = $\sqrt{1^2 + 3^2 - 6} = 2$
Clearly, from the figure the radius of bigger circle
$r^2 = 2^2 + [(2-1)^2 + (1-3)^2]$
$r^2 = 9 \Rightarrow r = 3$



104. (B) Let equation of line L_1 be $y = mx$. Intercepts made by L_1 and L_2 on the circle will be equal i.e. L_1 and L_2 are at the same distance from the centre of the circle; Centre of the given circle is (1/2, –3/2). Therefore,

$$\frac{|1/2-3/2-1|}{\sqrt{1+1}} = \left|\frac{\frac{m}{2}+\frac{3}{2}}{\sqrt{m^2-1}}\right| \Rightarrow \frac{2}{\sqrt{2}} = \frac{|m+3|}{2\sqrt{m^2+1}}$$

$\Rightarrow 8m^2 + 8 = m^2 + 6m + 9$

$\Rightarrow 7m^2 - 6m - 1 = 0$ $(7m + 1)(m - 1) = 0$

$\Rightarrow m = -\frac{1}{7}$, $m = 1$

Thus, two chords are $x + 7y = 0$ and $x - y = 0$.

105. (A) Let the tangent to parabola be $y = mx + \frac{a}{m}$, if it touches the other curve, then D = 0, to get the value of m.

For parabola, $y^2 = 4x$

Let $y = mx + \frac{1}{m}$ be tangent line and it touches the parabola $x^2 = -32y$

$\therefore x^2 = -32\left(mx + \frac{1}{m}\right)$

$\Rightarrow x^2 + 32mx + \frac{32}{m} = 0$

$\because (32m)^2 - 4\left(\frac{32}{m}\right) = 0 \Rightarrow m^3 + \frac{1}{8}$

$\therefore m = \frac{1}{2}$

106 (B) Given, A = $\sin^2\theta + (1 - \sin^2\theta)^2$

$\Rightarrow A = \sin^4\theta - \sin^2\theta + 1$

$\Rightarrow A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\Rightarrow 0 \le \left(\sin^2\theta - \frac{1}{2}\right)^2 \frac{1}{4}$ $[\because 0 \le \sin^2\theta \le 1]$

$\therefore \frac{3}{4} \le A \le 1$

107. (B) Since, $\tan\theta < 0$.

$\therefore$ Angle θ is either in the second or fourth quadrant.

Then, $\sin\theta > 0$ or < 0

$\therefore \sin\theta$ may be $\frac{4}{5}$ or $-\frac{4}{5}$

108. (B) Given $f(x) = \frac{3x-2}{5}$

Let = $y = \frac{3x-2}{5}$

$\Rightarrow 3x - 2 = 5y \Rightarrow x = \frac{5y+2}{3}$

$\Rightarrow f^{-1}(x) = \frac{5y+2}{3}$

109. (D) $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Unit vector in direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$= \frac{3i-2j+6k}{\sqrt{3^2+(-2)^2+6^2}} = \frac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$

110. (C) For continuity of the function $x = 2$

$\lim_{h\to 0} f(2-h) = f(2) = \lim_{h\to 0} f(2+h)$

Now, $f(2 - h) = 2(2 - h) + 1 = 5 - 2h$

$\therefore \lim_{h\to 0} f(2-h) = 5$

Also, $f(2 + h) = 3(2 + h) - 1 = 5 + 3h$

$\lim_{h\to 0} f(2+h) = 5$

So, for continuity $f(2) = 5$.

$\therefore k = 5$

111. (A) Equation of the plane passing through (3, 4, 1) is

$a(x - 3) + b(y - 4) + c(z - 1) = 0$ ...(i)

Since this plane passes through (0, 1, 0) also

$\therefore a(0 - 3) + b(1 - 4) + c(0 - 1) = 0$

or $-3a - 3b - c = 0$

or $-3a + 3b + c = 0$ ...(ii)

Since (i) is parallel to

$\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$

$\therefore 2a + 7b + 5c = 0$ ...(iii)

From (ii) and (iii)

$\frac{a}{15-7} = \frac{b}{2-15} = \frac{c}{21-6}$ = k

$\Rightarrow a = 8k, b = -13k, c = 15k$

Putting in (i), we have

$8k(x - 3) - 13k(y - 4) + 15k(z - 1) = 0$

$\Rightarrow 8(x - 3) - 13(y - 4) + 15(z - 1) = 0$

$\Rightarrow 8x - 13y + 15z + 13 = 0$

Which is the required equation of the plane.

112. (B) Given that Mean = 13 and Mode = 7

We know that

Mode = 3Median – 2Mean

$\Rightarrow$ 7 = 3Median – 2 × 13

$\Rightarrow$ 3Median = 33 $\Rightarrow$ Median = 11

113. (C) 1 0 1 1 1 . 0 . 0 1 1

$1\times2^0=1$

$1\times2^1=2$

$1\times2^2=4$

$0\times2^3=0$

$1\times2^4=16$

23

$0 = 0\times2^{-1}$

$\frac{1}{4} = 1\times2^{-2}$

$\frac{1}{8} = 1\times2^{-3}$

$\frac{1}{4}+\frac{1}{8}=\frac{3}{8}=0.375$

$(10111)_2 = (23)_{10}$, $(0.011)_2 = (0.375)_{10}$

Hence $(10111.011)_2 = (23.375)_{10}$

114. (C) digits 0, 1, 3, 5, 8, 9, 6

6	6	5

= 6 × 6 × 5 = 180

115. (C) 101, 103,..................999

Now, $T_n = a + (n-1)d$

$999 = 101 + (n-1)\times2$

$898 = (n-1)\times2$

$449 = n-1 \Rightarrow n = 450$

Now, $S = \frac{n}{2}(2a + (n-1)d)$

$S = \frac{450}{2}(2\times101 + 449\times2)$

$S = 450 \times 550 = 247500$

116. (B) Given that $\theta = 130°$

$y = \sin\theta + \cos\theta$

$y = \sin130 + \cos130$

$y = \sin130 + \cos(90+40)$

$y = \sin130 - \sin40$

We know that 130 > 40

sin130 > sin40

Hence $y > 0$

117. (C) $z = \frac{3+i}{(2-i)^2}$

$z = \frac{3+i}{3-4i}\times\frac{3+4i}{3+4i}$

$z = \frac{5+15i}{25} = \frac{1+3i}{5}$

118. (B) Word "STATEMENT"

The total no. of arrangement $= \frac{9!}{3!2!} = \frac{9!}{12}$

No. of arrangement when T's come together $= \frac{7!}{2!} = \frac{7!}{2}$

No. of arrangement when T's don't come together $= \frac{9!}{12} - \frac{7!}{2}$

$= 6\times7! - \frac{7!}{2} = \frac{11\times7!}{2}$

119. (C) A = $\{x : x$ is multiple of 2$\}$

A = {2, 4, 6, 8, 10,......}

B = $\{x : x$ is multiple of 3$\}$

B = {3, 6, 9, 12, 15,......}

C = $\{x : x$ is a multiple of 6$\}$

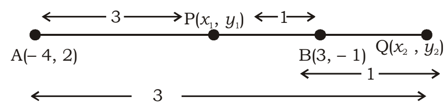
= {6, 12, 18,......}

Now, $(A\cap C)$ = {6, 12, 18,.........} = C

and $(B\cap C)$ = {6, 12, 18,.........} = C

Hence $(A\cap C)\cup(B\cap C) = C$

120. (C)



$x_1 = \frac{3\times3+1\times(-4)}{3+1} = \frac{5}{4}$

and $y_1 = \frac{3\times(-1)+1\times2}{3+1} = \frac{-1}{4}$

Co-ordinate of P $= \left(\frac{5}{4},\frac{-1}{4}\right)$

$x_2 = \frac{3\times3 - 1\times(-4)}{3-1} = \frac{13}{2}$

and $y_2 = \frac{3\times(-1) - 1\times2}{3-1} = \frac{-5}{2}$

Co-ordinate of Q $= \left(\frac{13}{2},\frac{-5}{2}\right)$

Now,

Distance Between P and Q

$= \sqrt{\left(\frac{5}{4}-\frac{13}{2}\right)^2 + \left(\frac{-1}{4}+\frac{5}{2}\right)^2}$

$= \sqrt{\left(\frac{-21}{4}\right)^2 + \left(\frac{9}{4}\right)^2} = \sqrt{\frac{522}{16}} = \frac{3\sqrt{58}}{4}$

NDA (MATHS) MOCK TEST - 176 (Answer Key)

1.	(A)	21.	(D)	41.	(B)	61.	(D)	81.	(A)	101.	(D)
2.	(B)	22.	(B)	42.	(B)	62.	(B)	82.	(B)	102.	(D)
3.	(D)	23.	(A)	43.	(D)	63.	(B)	83.	(D)	103.	(C)
4.	(B)	24.	(C)	44.	(C)	64.	(A)	84.	(A)	104.	(B)
5.	(B)	25.	(D)	45.	(B)	65.	(D)	85.	(D)	105.	(A)
6.	(C)	26.	(A)	46.	(B)	66.	(C)	86.	(A)	106.	(B)
7.	(D)	27.	(A)	47.	(B)	67.	(D)	87.	(A)	107.	(B)
8.	(C)	28.	(A)	48.	(A)	68.	(D)	88.	(B)	108.	(B)
9.	(D)	29.	(D)	49.	(A)	69.	(D)	89.	(A)	109.	(D)
10.	(B)	30.	(C)	50.	(C)	70.	(D)	90.	(D)	110.	(C)
11.	(C)	31.	(A)	51.	(A)	71.	(D)	91.	(D)	111.	(A)
12.	(B)	32.	(B)	52.	(A)	72.	(A)	92.	(B)	112.	(B)
13.	(B)	33.	(A)	53.	(C)	73.	(D)	93.	(B)	113.	(C)
14.	(D)	34.	(D)	54.	(D)	74.	(A)	94.	(C)	114.	(C)
15.	(A)	35.	(C)	55.	(C)	75.	(B)	95.	(B)	115.	(C)
16.	(B)	36.	(D)	56.	(B)	76.	(D)	96.	(A)	116.	(B)
17.	(A)	37.	(B)	57.	(A)	77.	(C)	97.	(B)	117.	(C)
18.	(C)	38.	(C)	58.	(C)	78.	(D)	98.	(B)	118.	(B)
19.	(A)	39.	(C)	59.	(D)	79.	(A)	99.	(C)	119.	(C)
20.	(B)	40.	(C)	60.	(A)	80.	(C)	100.	(D)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777