NDA MATHS MOCK TEST - 178 (SOLUTION)

1. (B) Consider, $f_n(x) = \frac{1}{n}(\cos^n x + \sin^n x)$

$\therefore f_4(x) - f_6(x) = \frac{1}{4}(\cos^4 x + \sin^4 x) - \frac{1}{6}(\cos^6 x + \sin^6 x)$

$= \frac{1}{4}[(\cos^2 x + \sin^2 x)^2 - 2\cos^2 x\sin^2 x)] - \frac{1}{6}[(\cos^2 x + \sin^2 x)(\cos^4 x + \sin^4 x - \cos^2 x\sin^2 x)]$

$$\left[\begin{array}{l}\because a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2 \\ a^3 + b^3 = (a+b)(a^2 - ab + b^2)\end{array}\right]$$

$= \frac{1}{4}(1 - 2\cos^2 x\sin^2 x) - \frac{1}{6}[(\cos^2 x + \sin^2 x)^2 - 2\cos^2 x\sin^2 x - \cos^2 x\sin^2 x]$

$= \frac{1}{4} - \frac{1}{2}\cos^2 x\sin^2 x - \frac{1}{6}(1 - 3\cos^2 x\sin^2 x)$

$= \frac{1}{4} - \frac{1}{6}\cos^2 x\sin^2 x - \frac{1}{6} + \frac{1}{2}\cos^2 x\sin^2 x$

$= \frac{1}{4} - \frac{1}{6}$

$= \frac{1}{12}$

2. (C) Consider, $\cos 3x\cos 2x\cos x = \frac{1}{4}$

$\Rightarrow 4\cos 3x\cos 2x\cos x - 1 = 0$
$\Rightarrow (2\cos 3x\cos x)\ 2\cos 2x - 1 = 0$
$\Rightarrow (\cos 4x + \cos 2x)\ 2\cos 2x - 1 = 0$
$[\because 2\cos A\cos B = \cos(A + B) + \cos(A - B)]$
$\Rightarrow 2\cos 4x\cos 2x + 2\cos^2 2x - 1 = 0$
$\Rightarrow 2\cos 4x\cos 2x + \cos 4x = 0$
$[\because \cos 2A = 2\cos^2 A - 1]$
$\Rightarrow \cos 4x(2\cos 2x + 1) = 0$
$\Rightarrow \cos 4x = 0$ or $2\cos 2x + 1 = 0$

$\Rightarrow \cos 4x = \cos\frac{\pi}{2}$ or $\cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow 4x = \frac{\pi}{2}$ or $2x = \frac{2\pi}{3}$

$\Rightarrow x = \frac{\pi}{8}$ or $x = \frac{\pi}{3}$

3. (A) Consider, $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow x^3 - 3x^2 + 3x + 7 + 1 - 1 = 0$
$\Rightarrow (x - 1)^3 = -8$
$\Rightarrow x - 1 = -2, 2\omega, -2\omega^2$
$\Rightarrow x = -1, 1 - 2\omega, 1 - 2\omega^2$
$\therefore \alpha = -1, \beta = 1 - 2\omega$ and $\gamma = 1 - 2\omega^2$
Substituting $\alpha = -1, \beta = 1 - 2\omega$ and $\gamma = 1 - 2\omega^2$ in $\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$, we get $3\omega^2$

4. (B) Consider, $\arg\left(\frac{z+i}{z-i}\right) = \frac{\pi}{4}$

$\Rightarrow \arg(z + i) - \arg(z - i) = \frac{\pi}{4}$

$\Rightarrow \arg(x + iy + i) - \arg(x + iy - i) = \frac{\pi}{4}$

$[z = x + iy]$

$\Rightarrow \arg(x + i(y + 1)) - \arg(x + i(y - 1)) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y+1}{x}\right) - \tan^{-1}\left(\frac{y+1}{x}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{y+1}{x} - \frac{y-1}{x}}{1 + \frac{y+1}{x}\frac{y-1}{x}}\right) = \frac{\pi}{4}$

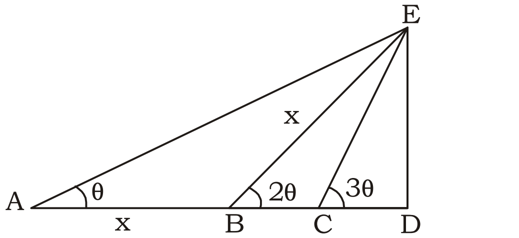
$\Rightarrow \frac{\frac{2}{x}}{1 + \frac{y^2 - 1}{x^2}} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\frac{2}{x}}{\frac{x^2 + y^2 - 1}{x^2}} = 1$

$\Rightarrow \frac{2x}{x^2 + y^2 - 1} = 1$

$\Rightarrow x^2 + y^2 - 2x - 1 = 0$
It is the equation of a circle with radius $\sqrt{2}$.
Therefore, the perimeter of the circle is $2\sqrt{2}\pi$

5. (A) For the given statement, we can draw the figure like this



From the figure, we have $\angle AEB = \angle BEC = \theta$

Hence, BE is the bisector of triangle AEC.

Now, using bisector theorem, we have

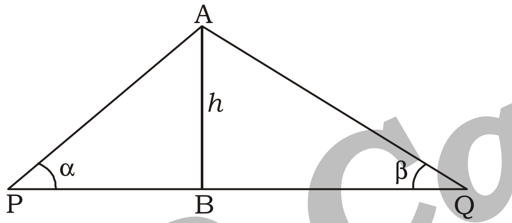
$$\frac{AE}{EC} = \frac{AB}{BC}$$

$$\Rightarrow \frac{\frac{h}{\sin\theta}}{\frac{h}{\sin 3\theta}} = \frac{AB}{BC}$$

[Using sin property in ΔAED and ΔECD]

$$\Rightarrow \frac{AB}{BC} = \frac{\sin 3\theta}{\sin\theta}$$

6. (B)



In ΔABP, $\tan\alpha = \frac{h}{BP}$

$BP = h\cot\alpha$ ...(i)

In ΔABQ, $\tan\beta = \frac{h}{BQ}$

$BQ = h\cot\beta$ ...(ii)

Adding (i) and (ii), we get

$BP + BQ = h\cot\alpha + h\cot\beta$

$\Rightarrow d = h(\cot\alpha + \cot\beta)$

$$\Rightarrow h = \frac{d}{\cot\alpha + \cot\beta}$$

7. (B) Consider, $y = \log_e x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(1,0)} = 1$$

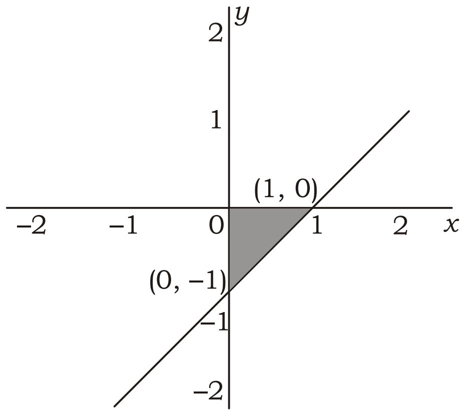
The equation of tangent to the curve $y = \log_e x$ at (1, 0) is given by

$$y - 0 = \left(\frac{dy}{dx}\right)_{(1,0)} (x - 1)$$

$\Rightarrow y = x - 1$

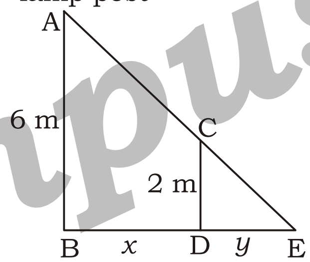
$\Rightarrow x - y = 1$

The equation of tangent to the curve $y = \log_e x$ intersecting the coordinate axis at (1, 0) and (0, –1).



Hence, the area of triangle formed by the coordinate axes is $\frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ units^2

8. (B) lamp post



By AA similarity, $\Delta ABE \sim \Delta CDE$

$$\therefore \frac{AB}{BE} = \frac{CD}{DE}$$

$$\Rightarrow \frac{6}{x+y} = \frac{2}{y}$$

$\Rightarrow 3y = x + y$

$\Rightarrow 2y = x$

$$\Rightarrow 2\frac{dy}{dt} = \frac{dx}{dt}$$

$$= \frac{1}{2}(5)$$

= 2.5 km/hour

9. (C) Consider, $f(x) = a\log|x| + bx^2 + x$

$$\Rightarrow f'(x) = \frac{a}{x} + 2bx + 1$$

For $x = -1$,

$a + 2b = 1$ ...(i)

For $x = 2$,

$a + 8b = -2$ ...(ii)

solving (i) and (ii), we get

$a = 2$

$$b = -\frac{1}{2}$$

10. (C) The relation between the roots of cubic polynomical is $\alpha\beta + \beta\gamma + \alpha\gamma$

$= \frac{\text{coeffecient of} x}{\text{coeffecient of } x^3}$

Here, the coefficient of x is 0.

Therefore, $\alpha\beta + \beta\gamma + \alpha\gamma = 0$ ...(i)

Consider, $\begin{vmatrix} \alpha\beta & \beta\gamma & \alpha\gamma \\ \beta\gamma & \alpha\gamma & \alpha\beta \\ \alpha\gamma & \alpha\beta & \beta\gamma \end{vmatrix}$

$= \begin{vmatrix} \alpha\beta+\beta\gamma+\alpha\gamma & \beta\gamma & \alpha\gamma \\ \alpha\beta+\beta\gamma+\alpha\gamma & \alpha\gamma & \alpha\beta \\ \alpha\beta+\beta\gamma+\alpha\gamma & \alpha\beta & \beta\gamma \end{vmatrix}$ $[C_1 \to C_1 + C_2 + C_3]$

= 0 [Using(i)]

11. (A) Let us suppose the first term and common ratio of the geometric progression be A and R.

Now, $a = AR^{p-1}$

$\log a = \log(AR^{p-1})$

$\log a = \log A + (p-1)\log R$

Similarly, $\log b = \log A + (q-1)\log R$ and $\log c = \log A + (r-1)\log R$

Consider, $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log A+(p-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log A+(r-1)\log R & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log A+(p-1)\log R & p-1 & 1 \\ \log A+(q-1)\log R & q-1 & 1 \\ \log A+(r-1)\log R & r-1 & 1 \end{vmatrix}$

$[C_2 \to C_2 - C_3]$

$= \begin{vmatrix} 0 & p-1 & 1 \\ 0 & q-1 & 1 \\ 0 & r-1 & 1 \end{vmatrix}$

= 0

12. (A) Consider, $A^2 = 2A - I$

$\Rightarrow A^3 = 2A^2 - IA$

$\Rightarrow A^3 = 2(2A - I) - A$

$\Rightarrow A^3 = 3A - 2I$

$\Rightarrow A^n = nA - (n-1)I$

13. (C) $(1 + x^2)^5 = {}^5C_0 + {}^5C_1 (x^2) + {}^5C_2 (x^2)^2 + {}^5C_3 (x^2)^4 + {}^5C_5(x^2)^5$

$= 1 + 5x^2 + 10x^4 + 10x^6 + 5x^8 + x10$

$(1+ x)^4 = {}^4C_0 + {}^4C_1 x + {}^4C_2 x^2 + {}^4C_3 x^3 + {}^4C_4 x^4$

$= 1 + 4x + 6x^2 + 4x^3 + x^4$

Therefore, to find the coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x)^4$ we will have to multiply the coefficient which makes the power of x to 5

= 40 + 20

= 60

14. (D) Probability of getting a defective bulb = $\frac{10}{100} = \frac{1}{10}$

Probability of getting a non defective bulb

$= 1 - \frac{1}{10} = \frac{9}{10}$

The probability that out of a sample of 5 bulbs none is defective is ${}^5C_0 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5$

15. (D) A number is divisible by both 2 and 3 if it divisible by 6.

The number divisible by 6 and 6, 12, 18,..... 96 = 16

Now, required probability $= \frac{{}^{16}C_3}{{}^{100}C_3}$

$= \frac{560}{161700} = \frac{4}{1155}$

16. (B) Consider, $\frac{dy}{dx} = \sin(10x + 6y)$

$\Rightarrow \frac{dy}{dx} = \sin(10x + 6y)$

Let $10x + 6y = t$

$\Rightarrow 10 + 6\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{6}\left(\frac{dt}{dx} - 10\right)$

$\therefore \frac{1}{6}\left(\frac{dt}{dx} - 10\right) = \sin t$

$\Rightarrow \frac{dt}{dx} - 10 = 6\sin t$

$\Rightarrow \frac{dt}{dx} = 6\sin t + 10$

$\Rightarrow \int \frac{dt}{6\sin t + 10} = \int dx$

Solving this, we will get

$5\tan(5x + 3y) = 4\tan(4x + k) - 3$

17. (D) The line $y = mx + c$ is tangent to hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2m^2 - b^2 = c^2$

Consider, $ax + by = 1$

$y = -\frac{ax}{b} + \frac{1}{b}$

Substitute $m = -\frac{a}{b}$ and $c = \frac{1}{b}$ in $a^2m^2 - b^2 = c^2$, we get

$a^2\left(\frac{-a}{b}\right)^2 - b^2 = \left(\frac{1}{b}\right)^2$

$\Rightarrow \frac{a^4}{b^2} - b^2 = \frac{1}{b^2}$

$\Rightarrow \frac{a^4 - b^4}{b^2} = \frac{1}{b^2}$

$\Rightarrow a^4 - b^4 = 1$

$\Rightarrow (a^2 - b^2)(a^2 + b^2) = 1$

$\Rightarrow a^2 - b^2 = \frac{1}{a^2 + b^2}$

$\Rightarrow a^2 - b^2 = \frac{1}{e^2a^2}$

18. (A) Given that $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

Now, $\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta.\tan\phi}$

$\Rightarrow \tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}$

$\Rightarrow \tan(\theta + \phi) = \frac{5/6}{5/6}$

$\Rightarrow \tan(\theta + \phi) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4}$

19. (A) $\cos A = \frac{3}{4}$

$\Rightarrow 1 - 2\sin^2\frac{A}{2} = \frac{3}{4}$

$\Rightarrow 2\sin^2\frac{A}{2} = \frac{1}{4}$

$\Rightarrow \sin^2\frac{A}{2} = \frac{1}{8}$

Now, $\sin\frac{A}{2}.\sin\frac{3A}{2}$

$\Rightarrow \sin\frac{A}{2}\left(3\sin\frac{A}{2} - 4\sin^3\frac{A}{2}\right)$

$[\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta]$

$\Rightarrow 3\sin^2\frac{A}{2} - 4\sin^4\frac{A}{2}$

$\Rightarrow 3 \times \frac{1}{8} - 4 \times \left(\frac{1}{8}\right)^2$

$\Rightarrow \frac{3}{8} - \frac{1}{16} = \frac{5}{16}$

20. (D) $(1 + \tan\alpha.\tan\beta)^2 + (\tan\alpha - \tan\beta)^2 - \sec^2\alpha.\sec^2\beta$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + 2\tan\alpha.\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha.\tan\beta - \sec^2\alpha.\sec^2\beta$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + \tan^2\alpha + \tan^2\beta - (1 + \tan^2\alpha)(1 + \tan^2\beta)$

$\Rightarrow 1 + \tan^2\alpha.\tan^2\beta + \tan^2\alpha + \tan^2\beta - 1 - \tan^2\alpha - \tan^2\beta - \tan^2\alpha.\tan^2\beta$

$\Rightarrow 0$

21. (A) $\cos 46°.\cos 47°...............\cos 135° = 0$

$[\because \cos 90° = 0]$

22. (D) $\cos\alpha + \cos\beta + \cos\gamma = 0$

$\cos\alpha = 0, \cos\beta = 0, \cos\gamma = 0$

$\Rightarrow \alpha = 90°, \beta = 90°, \gamma = 90°$

Now, $\sin\alpha + \sin\beta + \sin\gamma$

$\Rightarrow \sin 90° + \sin 90° + \sin 90° = 1+1+1 = 3$

23. (A) $\sin^{-1}\frac{2p}{1+p^2} - \cos^{-1}\frac{1-p^2}{1+p^2} = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = \tan^{-1}\frac{2x}{1-x^2}$

$\left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$

$\Rightarrow 2[\tan^{-1}p - \tan^{-1}q] = \tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}\frac{p-q}{1+pq} = 2\tan^{-1}x$

On comparing

$x = \frac{p-q}{1+pq}$

24. (D) **Statement 1**

$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

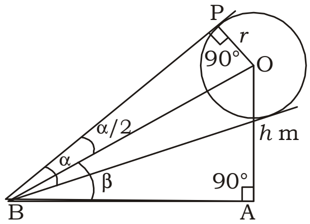
Statement 1 is incorrect.

Statement 2

$\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, when $x = y$

Statement 2 is incorrect.

25. (A)



Let AO = h

In ΔPOB

$\sin\frac{\alpha}{2} = \frac{PO}{OB}$

$\Rightarrow \sin\frac{\alpha}{2} = \frac{r}{OB} \Rightarrow OB = r.\text{cosec}\frac{\alpha}{2}$

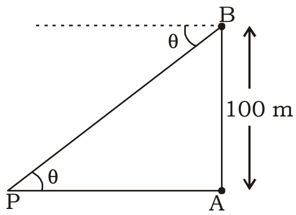
In ΔAOB

$\sin\beta = \frac{OA}{OB}$

$\Rightarrow \sin\beta = \frac{h}{r.\text{cosec}\frac{\alpha}{2}}$

$\Rightarrow h = r.\sin\beta.\text{cosec}\frac{\alpha}{2} \Rightarrow h = \frac{r.\sin\beta}{\sin\frac{\alpha}{2}}$

26. (C)



Let $\theta = \tan^{-1}\frac{5}{12} \Rightarrow \tan\theta = \frac{5}{12}$

In ΔABP

$\tan\theta = \frac{AB}{AP}$

$\Rightarrow \frac{5}{12} = \frac{100}{AP} \Rightarrow AP = 240$ m

The distance between the boat and the lighthouse = 240 m

27. (D) Equation $x^2 + \alpha x - \beta = 0$

Roots are α and β,

then $\alpha + \beta = -\alpha$

$\Rightarrow 2\alpha + \beta = 0$ ...(i)

$\alpha.\beta = -\beta \Rightarrow \alpha = -1$

from eq(ii)

$2(-1) + \beta = 0 \Rightarrow \beta = 2$

Another equation $= -x^2 + \alpha x + \beta$

$= -x^2 - x + 2$

$= -x^2 - x - \frac{1}{4} + \frac{1}{4} + 2$

$= -\left(x + \frac{1}{2}\right)^2 + \frac{9}{4}$

Greatest value of the equation $= \frac{9}{4}$

28. (B) Equation $|1 - x| + x^2 = 5$

Now, $1 - x + x^2 = 5$

$b^2 - 4ac = \sqrt{(-1)^2 - 4 \times (-4)} = \sqrt{17}$

Roots are irrational.

and $-(1 - x) + x^2 = 5$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow (x - 2)(x + 3) = 0$

$\Rightarrow x = 2, -3$

Roots are rational.

Hence equation has rational root and an irrational root.

29. (A) Let α_1, β_1 are the roots of $x^2 + px + q = 0$ and α_2, β_2 are roots of $x^2 + lx + m = 0$.

$\alpha_1 + \beta_1 = -p,\ \alpha_1.\beta_1 = q$

$\alpha_2 + \beta_2 = -l,\ \alpha_2.\beta_2 = m$

Given that $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$

by Componendo & Dividendo Rule

$= \frac{\alpha_1 + \beta_1}{\alpha_1 - \beta_1} = \frac{\alpha_2 + \beta_2}{\alpha_2 - \beta_2}$

$= \frac{(\alpha_1 + \beta_1)^2}{(\alpha_1 + \beta_1)^2 - 4\alpha_1.\beta_1} = \frac{(\alpha_2 + \beta_2)^2}{(\alpha_2 + \beta_2)^2 - 4\alpha_2.\beta_2}$

$\Rightarrow \frac{p^2}{p^2 - 4q} = \frac{l^2}{l^2 - 4m}$

$\Rightarrow p^2l^2 - 4p^2m = p^2l^2 - 4l^2q$

$\Rightarrow p^2m = l^2q$

30. (C) Equation $x^2 + bx + c = 0$

Let roots are α and β.

$\alpha + \beta = -b$ and $\alpha.\beta = c$

A.T.Q

$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$

$\Rightarrow \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\Rightarrow -b = \frac{b^2 - 2c}{c^2}$

$\Rightarrow -bc^2 = b^2 - 2c$

$\Rightarrow 2c = b^2 + bc^2$

$\Rightarrow 2c = b(b + c^2)$

$\Rightarrow \frac{2}{b} = \frac{b + c^2}{c}$

$\Rightarrow \frac{2}{b} = c + \frac{b}{c}$

$c, \frac{1}{b}, \frac{b}{c}$ are in A.P.

Hence $\frac{1}{c}, b, \frac{c}{b}$ are in H.P.

31. (D) Equation $x^2 - 2kx + k^2 - 4 = 0$

Now, $x = \dfrac{-(-2k) \pm \sqrt{(-2k)^2 - 4\times 1(k^2-4)}}{2}$

$\Rightarrow x = \dfrac{2k \pm \sqrt{4k^2 - 4k^2 + 16}}{2}$

$\Rightarrow x = \dfrac{2k \pm 4}{2} \Rightarrow x = k \pm 2$

A.T.Q,

$-3 < k \pm 2 < 5$

Now, $-3 < k + 2 < 5$ or $-3 < k - 2 < 5$

$\Rightarrow -3 - 2 < k < 5 - 2$ or $-3 + 2 < k < 5 + 2$

$\Rightarrow -5 < k < 3$ or $-1 < k < 7$

Hence $-1 < k < 3$

32. (C) $2x^2 + 3x - \alpha = 0$ has roots -2 and β,

then $-2 + \beta = \dfrac{-3}{2} \Rightarrow \beta = \dfrac{1}{2}$

and $-2.\beta = \dfrac{-\alpha}{2}$

$\Rightarrow -2 \times \dfrac{1}{2} = \dfrac{-\alpha}{2} \Rightarrow \alpha = 2$

33. (B) $B = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Co-factors of B-

$C_{11} = (-1)^{1+1}\begin{vmatrix} 4 & 0 \\ 1 & 0 \end{vmatrix} = 0$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = 2 - 4 = -2$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} = 0$, $C_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$C_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = -(3 - 2) = -1$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix} = 0$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 0 \\ 2 & 0 \end{vmatrix} = 0$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} = 12 - 4 = 8$

$C = \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & -1 \\ 0 & 0 & 8 \end{bmatrix}$

$AdjB = C^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 8 \end{bmatrix}$

34. (B) A is an orthogonal matrix,
then $A' = A^{-1}$

35. (C) We know that
$(A + B)' = A' + B'$
and $(AB)' = B'A'$
Hence statement 1 and 3 are correct.

36. (A) $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|A| = \cos\theta(\cos\theta) - \sin\theta(-\sin\theta)$

$= \cos^2\theta + \sin^2\theta = 1$

Co-factors of A -

$C_{11} = (-1)^{1+1}\begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = \cos\theta$

$C_{12} = (-1)^{1+2}\begin{vmatrix} -\sin\theta & 0 \\ 0 & 1 \end{vmatrix} = \sin\theta$

$C_{13} = (-1)^{1+3}\begin{vmatrix} -\sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix} = 0$

$C_{21} = (-1)^{2+1}\begin{vmatrix} \sin\theta & 0 \\ 0 & 1 \end{vmatrix} = -\sin\theta$

$C_{22} = (-1)^{2+2}\begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = \cos\theta$

$C_{23} = (-1)^{2+3}\begin{vmatrix} \cos\theta & \sin\theta \\ 0 & 0 \end{vmatrix} = 0$

$C_{31} = (-1)^{3+1}\begin{vmatrix} \sin\theta & 0 \\ \cos\theta & 0 \end{vmatrix} = 0$

$C_{32} = (-1)^{3+2}\begin{vmatrix} \cos\theta & 0 \\ -\sin\theta & 0 \end{vmatrix} = 0$

$C_{33} = (-1)^{3+3}\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

$C = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AdjA = C^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now, $A^{-1} = \dfrac{Adj\ A}{|A|}$

$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

37. (B) $A = \begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix}$

Now, $A^2 = \begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix} \begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix}$

$\Rightarrow A^2 = \begin{vmatrix} -2\times(-2)+2\times 2 & -2\times 2+2\times(-2) \\ 2\times(-2)-2\times 2 & 2\times 2-2\times(-2) \end{vmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$\Rightarrow A^2 = -4\begin{vmatrix} -2 & 2 \\ 2 & -2 \end{vmatrix}$

$\Rightarrow A^2 = -4A$

38. (D) Given that $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Statement I

$f(\theta)\times f(\phi) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta.\cos\phi - \sin\theta.\sin\phi & -\cos\theta.\sin\phi - \sin\theta.\cos\phi & 0 \\ \sin\theta.\cos\phi + \cos\theta.\sin\phi & -\sin\theta.\sin\phi + \cos\theta.\cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(\theta)\times f(\phi) = \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(\theta)\times f(\phi) = f(\theta + \phi)$

Statement 1 is correct.

Statement 2

$|f(\theta)\times f(\phi)| = \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|f(\theta)\times f(\phi)| = \cos(\theta+\phi).\cos(\theta+\phi) + \sin(\theta+\phi).\sin(\theta+\phi)$

$|f(\theta)\times f(\phi)| = \cos^2(\theta+\phi) + \sin^2(\theta+\phi) = 1$

Statement 2 is correct.

Statement 3

$f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(x) = \cos x.\cos x + \sin x.\sin x$

$f(x) = \cos^2 x + \sin^2 x = 1$

$f(-1) = 1$

here $f(x) = f(-x)$

Statement 3 is correct.

39. (C) Given that $a + b + c = 0$

Now, $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} a+b+c-x & a+b+c-x & a+b+c-x \\ c & b-x & a \\ b & a & c-x \end{vmatrix}$

$\Rightarrow (a+b+c-x)\begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$\Rightarrow a + b + c - x = 0$

$\Rightarrow 0 - x = 0 \Rightarrow x = 0$

40. (C) 1. α, β are complementary angles, then $\alpha + \beta = 90°$

Now, $\begin{vmatrix} \cos^2\frac{\alpha}{2} & \sin^2\frac{\alpha}{2} \\ \sin^2\frac{\beta}{2} & \cos^2\frac{\beta}{2} \end{vmatrix}$

$\Rightarrow \cos^2\frac{\alpha}{2}.\cos^2\frac{\beta}{2} - \sin^2\frac{\alpha}{2}.\sin^2\frac{\beta}{2}$

$\Rightarrow \frac{1+\cos\alpha}{2}\times\frac{1+\cos\beta}{2} - \frac{1-\cos\alpha}{2}\times\frac{1-\cos\beta}{2}$

$\Rightarrow \frac{1}{4}(1+\cos\alpha+\cos\beta+\cos\alpha.\cos\beta) - \frac{1}{4}(1-\cos\alpha-\cos\beta+\cos\alpha.\cos\beta)$

$\Rightarrow \frac{1}{4}[2\cos\alpha + 2\cos\beta]$

$\Rightarrow \frac{1}{2}[\cos\alpha + \cos\beta]$

$\Rightarrow \frac{1}{2}\times 2\cos\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow \cos\left(\frac{90}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right)$ $(\because \alpha + \beta = 90)$

$\Rightarrow \cos 45.\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{\sqrt{2}}.\cos\left(\frac{\alpha-\beta}{2}\right)$

2. Maximum value of the determinant

$= \frac{1}{\sqrt{2}}$

Since both statements are correct.

41. (B) $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} > 0$

$\Rightarrow a(bc-1) - 1(c-1) + 1(1-b) > 0$
$\Rightarrow abc - a - c + 1 + 1 - b > 0$
$\Rightarrow abc - (a+b+c) + 2 > 0$
$\Rightarrow abc + 2 > a + b + c$
We know that
A.M ≥ G.M

$\frac{a+b+c}{3} \geq (abc)^{1/3}$

$a + b + c \geq 3(abc)^{1/3}$

$\Rightarrow a + b + c + 2 > 3(abc)^{1/3}$
Let $(abc)^{1/3} = x$
$\Rightarrow x^3 + 2 > 3x$
$\Rightarrow x^3 - 3x + 2 > 0$
$\Rightarrow (x+2)(x-1)^2 > 0$
$\Rightarrow x > -2$
$\Rightarrow (abc)^{1/3} > -2$
$\Rightarrow abc > -8$
Hence abc is greater than – 8. [$\because a + b + c = 0$]

42. (C)

43. (B) $T_{p+q} = a + (p+q-1)d$
$T_{p-q} = a + (p-q-1)d$
Now, $T_{p+q} + T_{p-q} = 2a + (2p-2)d$
$\Rightarrow T_{p+q} + T_{p-q} = 2[a + (p-1)d]$
$\Rightarrow T_{p+q} + T_{p-q} = 2T_p$
Hence the sum of $(p+q)^{th}$ and $(p-q)^{th}$ terms of an AP is equal to twice the p^{th} term.

44. (D) $S_n = nP + \frac{n(n-1)Q}{2}$

$S_{n-1} = (n-1)P + \frac{(n-1)(n-2)Q}{2}$

Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = nP + \frac{n(n-1)Q}{2} - (n-1)P - \frac{(n-1)(n-2)Q}{2}$

$\Rightarrow T_n = P + \frac{Q}{2}(n^2 - n - n^2 + n + 2n - 2)$

$\Rightarrow T_n = P + \frac{Q}{2}(2n-2)$

$\Rightarrow T_n = P + Q(n-1)$
$\Rightarrow T_{n-1} = P + Q(n-2)$
Common difference = $T_n - T_{n-1}$
$= P + Q(n-1) - P - Q(n-2)$
$= Q(n - 1 - n + 2) = Q$

45. (A) $\frac{1}{\log_3 e} + \frac{1}{\log_3 e^2} + \frac{1}{\log_3 e^4} + \ldots\ldots$ upto infinite terms

$\Rightarrow \frac{1}{\log_3 e} + \frac{1}{2\log_3 e} + \frac{1}{4\log_3 e} + \ldots$upto infinite terms

$\Rightarrow \frac{1}{\log_3 e}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\text{.upto infinite terms}\right)$

$\Rightarrow \frac{1}{\log_3 e} \times \frac{1}{1 - \frac{1}{2}}$

$\Rightarrow (\log_e 3) \times \frac{1}{1/2}$

$\Rightarrow 2\log_e 3 = \log_e 9$

46. (B) $S_n = n^2 - 2n$
$S_{n-1} = (n-1)^2 - 2(n-1)$
$S_{n-1} = n^2 + 1 - 2n - 2n + 2$
$S_{n-1} = n^2 - 4n + 3$
$T_n = S_n - S_{n-1}$
$T_n = (n^2 - 2n) - (n^2 - 4n + 3)$
$T_n = 2n - 3$
$T_5 = 2 \times 5 - 3 = 7$

47. (B) p, q, r are in G.P.,
then $q^2 = pr$ ....(i)
and a, b, c are in G.P.,
then $b^2 = ac$ ...(ii)
From eq(i) and eq(ii)
$b^2 \times q^2 = pr \times ac$
$(bq)^2 = ap \times cr$
Hence ap, bq, cr also are in G.P.

48. (D) S = 0.5 + 0.55 + 0.555 + ... upto n terms

$S = \frac{5}{9}$ [0.9 + 0.99 + 0.999 + upto n terms]

$S = \frac{5}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \ldots\text{.upto } n \text{ terms}\right]$

$S = \frac{5}{9}(1 + 1 + 1 + \ldots\ldots \text{ upto } n \text{ term})$

$-\frac{5}{9}\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{100} + \ldots\ldots\text{upto } n \text{ terms}\right)$

$S = \frac{5}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$

$S = \frac{5}{9}\left(n - \frac{\frac{1}{10}}{\frac{9}{10}}\left(1 - \frac{1}{10^n}\right)\right)$

$S = \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$

49. (C) A.T.Q.,

$\frac{p+q+r}{3} = 5$

$\Rightarrow p + q + r = 15$ ...(i)

and $\frac{s+t}{2} = 10$

$\Rightarrow s + t = 20$ ...(ii)

From eq(i) and eq(ii)

$p + q + r + s + t = 15 + 20$

$\Rightarrow p + q + r + s + t = 35$

Average of all the five numbers $= \frac{35}{5} = 7$

50. (C)

51. (D)

52. (B)

53. (C) Given

$$f(x) = 2[x] + \cos x = \begin{cases} \cos x, & 0 \le x < 1 \\ 2 + \cos x, & 1 \le x < 2 \\ 4 + \cos x, & 2 \le x < 3 \end{cases}$$

Since, $\cos x < 1$ and $2 + \cos x > 1$

$\therefore f(x)$ never given the value one

Hence, $f(x)$ is into

If $0 < \alpha < \pi - 3$, then $f(\pi - \alpha) = f(\pi + \alpha)$

54. (C) We observe that $f(1) = 3$ and $f(-1) = 3$

$\therefore$ 1 –1 but $f(1) = f(-1)$

So, f is not a one-one f^n

Clearly, $1, -1 \in Z$ such that $g(1) = 1$ and $g(-1) = (-1)^4 = 1$

i.e. $1 \neq -1$ but $g(1) = g(-1)$

So g is not a one-one fn.

Let $x, y \in R$ be such that

$h(x) = h(y)$

$\Rightarrow x^3 + 4 = y^3 + 4$

$\Rightarrow x^3 = y^3$ $x = y$

$\therefore h : R \to R$ is a one-one f^n.

55. (C) We have $f(x) = g(x)$

$\Rightarrow 2x^2 - 1 = 1 - 3x$

$\Rightarrow 2x^2 + 3x - 2 = 0$

$\Rightarrow (x + 2)(2x - 1) = 0$

$\Rightarrow x = -2, \frac{1}{2}$

Thus, $f(x)$ and $g(x)$ are equal on the set $\left\{-2, \frac{1}{2}\right\}$

56. (A) $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$

$$= \frac{1 - \cos 2x}{2} + \frac{1 - \cos\left(2x + 2\frac{\pi}{3}\right)}{2} + \frac{1}{2}\left[2\cos x \cos\left(x + \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[1 - \cos 2x + 1 - \cos\left(2x + 2\frac{p}{3}\right) + \cos\left(2x + \frac{\pi}{3}\right) + \cos\frac{\pi}{3}\right]$$

$$= \left[\frac{5}{2} - \left\{\cos 2x + \cos\left(2x + \frac{2\pi}{3}\right)\right\} + \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$= \left[\frac{5}{2} - \left\{\cos 2x + \cos\left(2x + \frac{2\pi}{3}\right)\right\} + \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$= \left[\frac{5}{2} - 2\cos\left(2x + \frac{\pi}{3}\right)\cos\frac{\pi}{3} + \cos\left(2x + \frac{\pi}{3}\right)\right]$$

$$= \frac{5}{4} \ \forall x$$

$\therefore$ g of $= g(f(x)) = g\left(\frac{5}{4}\right) = \frac{5}{4} \times \frac{4}{5} = 1$

Hence, go $f(x) = 1 \ \forall x$

57. (D) $\vec{r} = \vec{a} - \vec{b}$

$= -2\hat{i} + 3\hat{j} - \hat{k}, \vec{b}$

$= |\vec{r}| = \sqrt{4 + 1 + 16} = \sqrt{21}$

$\vec{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}}$

$= \frac{-2}{\sqrt{21}}\hat{i} + \frac{1}{\sqrt{21}}\hat{j} + \frac{4}{\sqrt{21}}\hat{k}$

58. (C) Given

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$; $\vec{a} \times \vec{c} \times \vec{b}$

and $\vec{a} . \vec{c} = 3$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ ...(i)

Then

$\vec{a} . \vec{c} = 3$

$\Rightarrow (\hat{i} + \hat{j} + \hat{k}).(x\hat{i} + y\hat{j} + z\hat{k}) = 3$

$\Rightarrow x + y + z = 3$ ...(ii)

Also

$\vec{a} \times \vec{c} = \vec{b} = 3$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$\Rightarrow (z - y)\hat{i} - (z - x)\hat{j} + (y - x)\hat{k} = \hat{j} - \hat{k}$

$\Rightarrow z - y = 0$ ...(iii)

$\Rightarrow x - z = 1$ ...(iv)

$\Rightarrow y - x = -1$ ...(v)

Solving eq^n and we get

$x = \frac{5}{3}, y = \frac{2}{3}$ and $z = \frac{2}{3}$

Substituting in eq^n (i) and we get

$\vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

59. (A) $y = \frac{2^x}{1+2^x}$

$2^x = \frac{y}{1-y}$

taking log both sides

$\log_2 2^x = \log_2 \frac{y}{1-y}$

$x \log_2 2 = \log_2 \frac{y}{1-y}$

$x = \log_2 \frac{y}{1-y}$

60. (D) y is well defined when $\log_{10}(1 - x)$ 0 and $x + 2 \geq 0$, Hence $-2 \leq x < 0$

61. (B) For continuity of $f(x)$ at $x = -\frac{\pi}{2}$ and $\frac{\pi}{2}$, we have

$\lim_{x\to -\frac{\pi}{2}-} f(x) = 2 = \lim_{x\to -\frac{\pi}{2}+} f(x) = -A + B = f\left(-\frac{\pi}{2}\right) = 2$

and $\lim_{x\to \frac{\pi}{2}-} f(x) = 2 = \lim_{x\to \frac{\pi}{2}+} f(x) = 0 = f\left(\frac{\pi}{2}\right) = 2$

$\Rightarrow -A + B = 2$ and $A + B = 0$ $\therefore$ $A = -1$, $B = 1$

62. (C) $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = \{(\vec{a}\times\vec{b}.\vec{c}) - (\vec{a}\times\vec{b}.\vec{b})\vec{c}\}.(\vec{c}\times\vec{a})$

$= [\vec{a}\ \vec{b}\ \vec{c}]\vec{b}.(\vec{c}\times\vec{a}) = [\vec{a}\ \vec{b}\ \vec{c}]^2 = 25$

63. (C) $\lim_{x\to\frac{\pi}{2}}\left\{2x\tan x - \frac{\pi}{\cos x}\right\} = \lim_{x\to\frac{\pi}{2}}\left\{\frac{2x\sin x - \pi}{\cos x}\right\}$

is $\frac{0}{0}$ form Use L' Hospital Rule, we get result -2.

64. (D) Let sides AB, BC and AC be c, a, b respectively in ΔABC.

Area of triangle $= \frac{1}{2}\ bc \sin A$

$\Rightarrow 10\sqrt{3} = \frac{1}{2}\ 5.8 \sin A$

$\Rightarrow \sin A = \frac{\sqrt{3}}{2}$

$\therefore$ A = 60° or 120°

65. (C) Equation of curves

$c_1 : y = x^2$

$c_2 : 9x^2 + 16y^2 = 25$

Let m_1 and m_2 be the slope of the tangents to these curve at the point of intersection (1, 1)

$\Rightarrow m_1 = 2$ and $m_2 = -\frac{9}{16}$

So $\theta_1 = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \theta_1 = \tan^{-1}\frac{41}{2}$

Similarly at the point of inersection

(–1, 1) $\theta_2 = \tan^{-1}\left|\frac{-2-\frac{9}{16}}{1-\frac{18}{16}}\right| = \tan^{-1}\frac{41}{2}$

66. (A) Since,

$-\frac{\pi}{2} \leq \tan^{-1}\frac{1}{x} \leq \frac{\pi}{2}$

So $\lim_{x\to 0} f(x) = 0 f(0)$

$\Rightarrow f$ is continuous

but $\lim_{x\to 0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \tan^{-1}\frac{1}{x}$ does not exit, so not differentiable at $x = 0$.

Continuous at $x = 0$ but not differentiable at $x = 0$

67. (A) Required chance $= \frac{5!}{\left(\frac{6!}{2!}\right)} = \frac{1}{3}$

68. (A) Given

L : 3 sinA + 4 cosB = 6

M : 4 sinB + 3 cosA = 1

In ΔABC,

adding L^2 and M^2 we get

$\sin(A + B) = \frac{1}{2}$

$\therefore \sin C = \sin(180° - \overline{A + B}) = \frac{1}{2}$

$\therefore$ C = 30° or 150°

Discard C = 150° because for this value of C, A will be less than 30°.

Hence 3 sinA + 4 cosB $< \frac{3}{2} + 4 < 6$ a contradiction

$\therefore$ C = 30°

69. (B) $c - \frac{1}{a}\sin{-1}\frac{a}{|x|}$

Put $x = \frac{1}{t}$ so

$I = \int\frac{dx}{x\sqrt{x^2 - a^2}}$ reduces to $-\frac{1}{a}\int\frac{dt}{\sqrt{\left(\frac{1}{a}\right)^2 - t^2}}$

Hence $I = c - \frac{1}{a}\sin^{-1}\frac{a}{|x|}$

70. (A) $\int (7x-2)\sqrt{3x+2}\,dx = 7\int\left(x-\frac{2}{7}\right)\sqrt{3x+2}\;dx$

$= \frac{7}{3}\int\left(3x-\frac{6}{7}\right)\sqrt{3x+2}\;dx$

$= \frac{7}{3}\int\left(3x+2-2-\frac{6}{7}\right)\sqrt{3x+2}\;dx$

$= \frac{7}{3}\int\left((3x+2)-\frac{20}{7}\right)\sqrt{3x+2}\;dx$

$= \frac{7}{3}\int\left((3x+2)^{3/2}-\frac{20}{7}(3x+2)^{1/2}\right)dx$

$= \frac{7}{3}\left\{\frac{(3x+2)^{5/2}}{\frac{5}{2}}\right\} - \frac{20}{3}\left\{\frac{(3x+2)^{3/2}}{\frac{3}{2}}\right\} + c$

$= \frac{14}{15}(3x+2)^{5/2} - \frac{40}{3}(3x+2)^{3/2} + c$

71. (B) I = $\int \tan x \tan 2x \tan 3x\;dx$ ...(i)

we have $\tan 3x = \tan(2x + x)$

$\tan 3x = \frac{\tan 2x + \tan x}{1-\tan 2x \tan x}$

$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

$\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$

Put in eq^n (i)

I = $\int(\tan 3x - \tan 2x - \tan x)dx$

$= -\frac{1}{3}\log_e|\cos 3x| - \frac{1}{2}\log_e|\cos 2x| - \log_e|\cos x| + c$

72. (C) Let I = $\int\sqrt{\sec x - 1}\;dx$

I = $\int\sqrt{\frac{1-\cos x}{\cos x}}\;dx$

I = $\int\sqrt{\frac{(1-\cos x)(1+\cos x)}{\cos x(1-\cos x)}}\;dx$

I = $\int\sqrt{\frac{\sin^2 x}{\cos x(1+\cos x)}}\;dx$

I = $\int\frac{\sin x}{\sqrt{\cos^2 x + \cos x}}\;dx$

Put $\cos x = t$

$\Rightarrow -\sin x\,dx = dt$

I = $-\int\frac{dt}{\sqrt{t^2+t}} = -\int\frac{dt}{\sqrt{t^2+t+\frac{1}{4}-\frac{1}{4}}}$

I = $-\int\frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$

$\left[\text{here}\int\frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

I = $-\log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right| + c$

I = $-\log\left|\left(\cos x+\frac{1}{2}\right)+\sqrt{\left(\cos x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right| + c$

73. (D) Let I = $\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$

Put $\log x = t$

$\Rightarrow x = e^t$

$\Rightarrow dx = e^t\,dt$

I = $\int\left\{\log t+\frac{1}{t^2}\right\}e^t\;dt$

I = $\int\left\{\log t+\frac{1}{t}-\frac{1}{t}+\frac{1}{t^2}\right\}e^t dt$

I = $\int\left\{\log t+\frac{1}{t}\right\}e^t dt+\int\left\{\frac{-1}{t}+\frac{1}{t^2}\right\}e^t dt$

I = $\int e^t\log t dt+\int e^t.\frac{1}{t}dt+\int e^t\left(-\frac{1}{t}\right)dt+\int e^t\left(\frac{1}{t^2}\right)dx$

I = $(\log t).e^t - \int\frac{1}{t}e^t dt+\int e^t.\frac{1}{t}dt+\left(-\frac{1}{t}\right)e^t$

$-\int\frac{1}{t^2}.e^t dt+\int e^t\frac{1}{t^2}dt+c$

I = $e^t(\log t) - \frac{1}{t}e^t + c$

I = $x\log(\log x) - \frac{x}{\log x} + c$

74. (C) Given

L : $\sin a + \sin b = \frac{1}{\sqrt{2}}$

M : $\cos a + \cos b = \frac{\sqrt{6}}{2}$

So $L^2 + M^2$ implies $\cos(a-b) = 0$ While LM (using $\cos(a-b) = 0$) given $\sin(a+b)$ $= \frac{\sqrt{3}}{2}$

75. (A) Required equation $x^2-\left(-\frac{1}{\alpha}-\frac{1}{\beta}\right)x+\left(-\frac{1}{\alpha}\right)\left(-\frac{1}{\beta}\right) = 0$

Where $\alpha + \beta = -3$ and α, β are roots of $x^2 + 3x + 5 = 0$

$\Rightarrow 5x^2 - 3x + 1 = 0$

76. (A) Given diff. Eq. can be written as

$$y\frac{dy}{dx} - \frac{1}{2(x+1)}y^2 = -\frac{x}{2(x+1)}$$

Let $y^2 = t$ so $2y\frac{dy}{dx} = \frac{dt}{dx}$

Hence eq. reduces to $\frac{dt}{dx} - \frac{1}{(x+1)}t = -\frac{x}{(x+1)}$ where I.F. $= e^{-\int\frac{1}{1+x}dx} = \frac{1}{(x+1)}$

Hence solution t.IF. $= \int Q.IF.dx + c$

$\Rightarrow y^2 = (1 + x)\log\frac{c}{1+x} - 1$

77. (C) Obviously p, q satisfy the equation $5x^2 - 7x - 3 = 0$

Hence $p + q = \frac{7}{5}$, $pq = -\frac{3}{5}$

Given $\alpha = 5p - 4q$ and $\beta = 5q - 4p$.

The required equation $x^2 - (\alpha + \beta) + \alpha\beta = 0$

$\Rightarrow 5x^2 - 7x - 439 = 0$

78. (B) Let $\sin^{-1}x = \theta$, given $3\sin^{-1}[x(3 - 4x^2)]$

$\Rightarrow 3\theta = \sin^{-1}[\sin\theta(3 - 4\sin^2\theta)]$

$-\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}$, Hence $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$ $\therefore$ $-\frac{1}{2}$

$\sin\theta \le \frac{1}{2}$ i.e $-\frac{1}{2} \le x \le \frac{1}{2}$

79. (B) Required ellipse $\sqrt{(x+1)^2 + (y+1)^2}$

$= e\left(\frac{x-y+3}{\sqrt{2}}\right)$ where $e = \frac{1}{2}$

$(x + 1)^2 + (y - 1)^2 = \frac{1}{8}(x - y + 3)^2$

80. (D) $a = ib = \cos(\log i^{4i}) = \cos\left[4i\left\{\log|i| + i\frac{\pi}{2}\right\}\right]$

$= 1$ $\therefore$ $a = 1$, $b = 0$

81. (B) $y = \sqrt{2x - x^2}$

so $\frac{dy}{dx} = \frac{1-x}{\sqrt{1-(x-1)^2}}$ $\begin{cases} > 0 \text{ for } 0 < x < 1 \\ < 0 \text{ for } x \in (1,2) \end{cases}$

So f increase in (0, 1) and decrease in (1, 2).

82. (C) Let $S_n = 1 + 4 + 13 + 40 + 121 + 364 + ...T_{n-1} + T_n$

Rewrite $S_n = 1 + 4 + 13 + 40 + 121 + 364 + ...(T_n - T_{n-1}) - T_n$

$\Rightarrow T_n = 1.\frac{3^n - 1}{3 - 1}$ and $T_n = \frac{3^n - 1}{2}$

Alternative : put options directly.

83. (C) $(0.2)^x = 2$

Taking log on both sides

$\log(0.2)^x = \log 2$

$x\log(0.2) = 0.3010$, [since $\log 2 = 0.3010$]

$x\log\left(\frac{2}{10}\right) = 0.3010$

$x[\log 2 - \log 10] = 0.3010$

$x[\log 2 - 1] = 0.3010$, [since $\log 2 = 0.3010$]

$x[-0.699] = 0.3010$

$x = \frac{0.3010}{-0.699}$

$x = -0.4306....$

$x = -0.4$(nearest tenth)

84. (A) Here, the number of observations is even, i.e., 8.

Arranging the data in asceding order, we get 21, 22, 24, 25, 27, 30, 33, 34

Therefore, median $= \left(\frac{n}{2}\right)^{th}$

$$\frac{\left\{\left(\frac{n}{2}\right)^{th}\text{observation} + \left(\frac{n}{2}+1\right)^{th}\text{observation}\right\}}{2}$$

$= \left(\frac{8}{2}\right)^{th}$ observation $+ \left(\frac{8}{2}+1\right)^{th}$ obervation

$= 4^{th}$ observation + (4 + 1)th observation

$= \frac{\{25+27\}}{2}$

$= \frac{52}{2}$

$= 26$

85. (D) Mean $= \frac{(f_1x_1 + f_2x_2 + f_3x_3 + f_4x_4 + f_5x_5)}{(f_1 + f_2 + f_3 + f_4 + f_5)}$

$= \frac{(40\times8 + 42\times6 + 34\times15 + 36\times14 + 46\times7)}{(8+6+15+14+7)}$

$= \frac{(320 + 252 + 510 + 504 + 322)}{50}$

$= \frac{1908}{50}$

$= 38.16$

86. (A)

8	2980	
8	372	4
8	46	4
8	5	6
	0	5

Hence $2980_{10} = 5644_8$

87. (A)

88. (B) (I) The card is king a queen :
Number of kings in a deck of 52 cards = 4
Number of queen in a deck of 52 cards = 4
Total number of king or queen in a deck of 52 cards = 4 + 4 = 8
P(the card is a king or queen)
= Number of king or queen/Total number of playing cards

$= \frac{\text{Number of king or queen}}{\text{Total number of playing cards}}$

$= \frac{8}{52}$

$= \frac{2}{13}$

(II) The card is either a red card or an ace:
Total number of red card or an ace in a deck of 52 cards = 28
P(the card is either a red card or an ace)

$= \frac{\text{Number of cards which is either a red card or an ace}}{\text{Total number of playing cards}}$

$= \frac{28}{52}$

$= \frac{7}{13}$

(III) The card is not a king:
Number of kings in a deck of 52 cards = 4
P(the card is a king)

$= \frac{\text{Number of kings}}{\text{Total number of playing cards}}$

$= \frac{4}{52}$

$= \frac{1}{13}$

P(the card is not a king)
= 1 – P(the card is a king)

$= \frac{1-1}{13}$

$= \frac{(13-1)}{13}$

$= \frac{12}{13}$

(IV) The card is a five or lower:
Number of cards is a five or lower = 16
P(the card is a five or lower)

$= \frac{\text{Number of card is a five or lower}}{\text{Total number of playing cards}}$

$= \frac{16}{52}$

$= \frac{4}{13}$

89. (C) $\cos 7\frac{1}{2}$ lies in the first quadrant

Therefore, $\cos 7\frac{1}{2}$ is positive

For all values of the angle A we know that, $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
Therefore, $\cos 15° = \cos(45° - 30°)$
$\cos 15° = \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Again for all values of the angle A we know that, $\cos A = 2\cos^2\frac{A}{2} - 1$

$\Rightarrow 2\cos^2\frac{A}{2} = 1 + \cos A$

$\Rightarrow 2\cos^2 7\frac{1}{2} = 1 + \cos 15°$

$\Rightarrow 2\cos^2 7\frac{1}{2} = \frac{1+\cos 15°}{2}$

$\Rightarrow 2\cos^2 7\frac{1}{2} = \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{2}$

$\Rightarrow 2\cos^2 7\frac{1}{2} = \frac{2\sqrt{2}+\sqrt{3}+1}{4\sqrt{2}}$

$\Rightarrow 2\cos^2 7\frac{1}{2} = \sqrt{\frac{4+\sqrt{6}+\sqrt{2}}{8}}$,

[Since cos71/2 is positive]

$\Rightarrow 2\cos^2 7\frac{1}{2} = \sqrt{\frac{4+\sqrt{6}+\sqrt{2}}{2\sqrt{2}}}$

Therefore, $\cos 7\frac{1}{2} = \sqrt{\frac{4+\sqrt{6}+\sqrt{2}}{2\sqrt{2}}}$

90. (B) The given parabola is $y^2 = 12x$
Now, Let $(k, 2k)$ be the co-ordinates of the required point $(k \neq 0)$
Since the point lies $(k, 2k)$ on the parabola $y^2 = 12x$,
Therefore, we get,
$(2k)^2 = 12k$
$\Rightarrow 4k^2 = 12k$
$\Rightarrow k = 3$(since, $k \neq 0$)
Therefore, the co-ordinates of the required point are (3, 6)

91. (A) Let P(x, y) be any point on the required ellipse and PM be the perpendicular from P upon the directrix $3x + 4y - 5 = 0$
Then by the definition,

$\frac{SP}{PM} = e$

$\Rightarrow SP = e.PM$

$\Rightarrow \sqrt{(x-1)^2+(y-2)^2} = \frac{1}{2}\left|\frac{3x+4y-5}{\sqrt{3^2+4^2}}\right|$

$\Rightarrow (x-1)^2 + (y-2)^2 = \frac{1}{4}.\frac{(3x+4y-5)^2}{25}$, [Squaring both sides]

$\Rightarrow 100(x^2 + y^2 - 2x - 4y + 5) = 9x^2 + 16y^2 + 24xy - 30x - 40y + 25$

$\Rightarrow 91x^2 + 84y^2 - 24xy - 170x - 360x + 475 = 0$, which is the required equation of the ellipse.

92. (C) The given equation is of the hyperbola is

$\frac{x^2}{9} - \frac{y^2}{25} = 1$

We know that the point $P(x_1, y_2)$ lies outside, on or inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ according as $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1 < 0, =$ or > 0

According to the given problem,

$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$

$= \frac{6^2}{9} - \frac{(-5)^2}{25} - 1$

$= \frac{26}{9} - \frac{25}{25} - 1$

$= 4 - 1 - 1$

$= 2 > 0$

Therefore, the point (6, –5) lies inside the hyperbola $\frac{x^2}{9} - \frac{y^2}{25} = 1$

93. (D) Let the given points be A(3, 0), B(6, 4) and C(–1, 3). Then we have,

AB2 = $(6-3)^2 + (4-0)^2 = 9 + 16 = 25$

BC2 = $(-1-6)^2 + (3-4)^2 = 49 + 1 = 50$

and $CA^2 = (3+1)^2 + (0-3)^2 = 16 + 9 = 25$

From the above results we get,

$AB^2 = CA^2$ i.e., AB = CA,

Which proves that the triangle ABC is isosceles

Again, $AB^2 + AC^2 = 25 + 25 = 50 = BC^2$

Which shows that the triangle ABC is right-angled

Therefore, the triangle formed by joining the given points is a right-angled isosceles triangle

94. (A) $a^{2-x}.b^{5x} = a^{x+3}.b^{3x}$

Therefore, $\frac{b^{5x}}{b^{3x}} = \frac{a^{x+3}}{a^{2-x}}$

or, $b^{5x-3x} = a^{x+3-1+x}$

or, $b^{2x} = a^{2x+1}$ or, $b^{2x} = a^{2x}.a$

or, $\left(\frac{b}{a}\right)^{2x} = a$

or, $\log\left(\frac{b}{a}\right)^{2x} = \log a$

(taking logarithm both sides)

or, $2x\log\left(\frac{b}{a}\right) = \log a$

or, $x\log\left(\frac{b}{a}\right) = \left(\frac{1}{2}\right)\log a$

95. (B) The given complex quantity is $(2-3i)(-1+7i)$

Let $z_1 = 2 - 3i$ and $z_2 = -1 + 7i$

Therefore, $|z_1| = \sqrt{2^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$

and $|z_2| = \sqrt{(-1)^2+7^2} = \sqrt{1+49} = \sqrt{13}$

Therefore, the required modulus of the given complex quantity = $|z_1z_1| = |z_1||z_1| = \sqrt{13}.5\sqrt{2} = 5\sqrt{26}$

96. (D) The given complex number $\frac{i}{1-i}$

Now, multiply the numerator and denominator by the conjugate or the denominator i.e, $(1 + i)$, we get

$\frac{i(1+i)}{(1+i)(1+i)}$

$= \frac{(1+i^2)}{(1-i^2)}$

$= \frac{i-1}{2}$

$= -\frac{1}{2} + i.\frac{1}{2}$

We see that in the z-plane the point $z = -\frac{1}{2} + i.\frac{1}{2} = \left(-\frac{1}{2},\frac{1}{2}\right)$ lies in the second quadrant. Hence, if amp $z = \theta$ then,

$\tan\theta = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$, where $= \frac{\pi}{2} < \theta \le n$

Thus, $\tan\theta = -1 = \tan\left(n - \frac{\pi}{n}\right) = \tan\frac{3\pi}{4}$

Therefore, required argument of $\frac{i}{1-i}$ is $\frac{3\pi}{4}$

97. (A) A.M. $\geq$ G.M. $\geq$ H.M.

98. (B) $I = \int_0^{\pi/2}(2\log\sin x - \log\sin 2x)\,dx$

$I = \int_0^{\pi/2}\{2\log\sin x - \log(2\sin x\cos x)\}\,dx$

$I = \int_0^{\pi/2}\{2\log\sin x - \log 2 - \log\sin x - \log\cos x)\}\,dx$

$I = \int_0^{\pi/2}\{\log\sin x - \log 2 - \log\cos x\}\,dx$

$I = \int_0^{\pi/2}\log\sin x\,dx - \log 2\int_0^{\pi/2}dx - \int_0^{\pi/2}\log\cos x dx$

$I = \int_0^{\pi/2}\log\sin x\,dx - \log 2[x]_0^{\pi/2} - \int_0^{\pi/2}\log\cos\left(\frac{\pi}{2} - x\right)dx$

$I = \int_0^{\pi/2}\log\sin x\,dx - \frac{\pi}{2}\log 2 - \int_0^{\pi/2}\log\sin x\,dx$

$I = -\frac{\pi}{2}\log 2$

99. (A) $\int_0^x f(t)dt = x + \int_x^1 t\,f(t)\,dt$

[using Leibniz's Rule]

$\Rightarrow \frac{d}{dx}\left(\int_0^x f(t)\,dt\right) = \frac{d}{dx}\left(x + \int_x^1 t(t)\,dt\right)$

$f(x) = 1 + 0 - x\,f(x)$

$f(x) = 1 - x\,f(x)$

$f(x) = \frac{1}{1+x}$

$\Rightarrow f(1) = \frac{1}{2}$

100. (A) We have,

$\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$

$= \frac{12\sqrt{3} + 8\sqrt{2} + 9\sqrt{3} - 6\sqrt{2}}{27 - 8}$

$= \frac{21\sqrt{3} + 2\sqrt{2}}{19}$

$= \frac{21 \times 1.732 + 2 \times 1.414}{19}$

$= \frac{36.372 + 2.828}{19}$

$= \frac{39.2}{19} = 2.063$

101. (A) Let $y = \sqrt{\frac{(x-3)(x^2+4)}{(3x^2+4x+5)}}$

Taking log on both sides, we have

$\log y = \frac{1}{2}[\log(x-3) + \log(x^2+4) - \log(3x^2 + 4x + 5)]$

Now, diff. w.r.to x,

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$

$\frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$

$= \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left[\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$

102. (D) $\{0\} \rightarrow$ Singleton set and $x^2 + 1 = 0$

$x^2 = -1$

x is a complex number

while $\{x : x^2 + 1 = 0, x \in R\}$

So, it is a null set

103. (B) $f(-x) = \log[-x + \sqrt{1+x^2}\,]$

$f(x) + f(-x) = \log[x + \sqrt{1+x^2}\,]$

$\log[-x + \sqrt{1+x^2}\,]$

$\log[1 + x^2 - x^2] = \log 1 = 0$

$\Rightarrow f(-x) = -f(x)$

So, $f(x)$ is an odd function of x.

104. (C) Let $f(x) = (3\cos x + 4\sin x) + 5$

we know that,

$-\sqrt{a^2+b^2} \leq a\cos x + b\sin x \leq \sqrt{a^2+b^2}$

$\Rightarrow -\sqrt{3^2+4^2} \leq 3\cos x + 4\sin x \leq \sqrt{3^2+4^2}$

$\Rightarrow -5 \leq 3\cos x + 4\sin x \leq 5$

$\Rightarrow -5 + 5 \leq 3\cos x + 4\sin x + 5 \leq 5 + 5$

$\Rightarrow 0 \leq (3\cos x + 4\sin + 5) \leq 10$

$\Rightarrow 0 \leq f(x) \leq 10$

105. (A) Given that

$x^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{1} = -b$ ...(i)

$\alpha\beta = \frac{c}{1} = c$ ...(ii)

$\therefore \alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta} = \frac{-b}{c}$

106. (C) Given that,
the sum of an infinite G.P. = x

$\Rightarrow \frac{a}{1-r} = x$
when, a = first term
and r = common ratio

$\frac{2}{1-r} = x$ ...(i)
[given that, $a = 2$ and $|r| < 1$]
$|r| < 1$
$-1 < r < 1$
$1 > -r > -1$
$1 + 1 > 1 - r > 1 - 1$
$0 < 1 - r < 2$
$(1 - r) < 2$
$\frac{1}{1-r} > \frac{1}{2}$
$\frac{1}{1-r} > 1$
$x > 1$

107. (D)

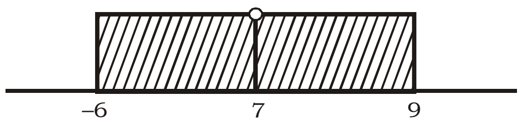
108. (D) $\frac{\cosh x + \cosh y}{\sinh x - \sinh y}$

$\Rightarrow \frac{2\cosh\frac{x+y}{2}.\cosh\frac{x-y}{2}}{2\cosh\frac{x+y}{2}.\sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$

109. (D) $n(S) = 6 \times 6 = 36$
E = {(6,3), (3, 6), (5, 4), (4, 5)}; $n(E) = 4$

Probability = $\frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

110. (B) function $f(x) = \frac{\sqrt{\log_e(55+3x-x^2)}}{x-7}$
$\log_e(55 + 3x - x^2) \geq 0$, $x - 7 \neq 0$
$\Rightarrow 55 + 3x - x^2 \geq 1$, $x \neq 7$
$\Rightarrow x^2 - 3x - 54 \leq 0$
$\Rightarrow (x + 6)(x - 9) \leq 0$



$x \in [-6, 9] - \{7\}$

111. (C) Let $y = \sin\left(x - \frac{\pi}{6}\right) + \cos\left(x - \frac{\pi}{6}\right)$

$\Rightarrow \frac{dy}{dx} = \cos\left(x - \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right)$

for maximum and minima

$\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = 0$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \sin\left(x - \frac{\pi}{6}\right)$

$\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{2} - x + \frac{\pi}{6} \Rightarrow x = \frac{5\pi}{12}$

112. (C) $f(x) = \begin{cases} 5x^2 - 7 & 1 \leq x < 3 \\ 2x + \lambda & 3 \leq x < 6 \end{cases}$ is continuous
at $x = 3$,then

$\lim_{x\to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\Rightarrow \lim_{x\to 3} 5x^2 - 7 = \lim_{x\to 3} 2x + \lambda$

$\Rightarrow 5 \times 9 - 7 = 2 \times 3 + \lambda$

$\Rightarrow 38 = 6 + \lambda \Rightarrow \lambda = 32$

113. (B) $(A \cap B) \cup (B \cap C) \cup (C \cap A) \cup (A \cap B \cap C)$

114. (C) $x = \frac{2at}{1-t^2}$ ...(i)

$\Rightarrow \frac{dx}{dt} = \frac{(1-t^2)2a - 2at(-2t)}{(1-t^2)^2}$

$\Rightarrow \frac{dx}{dt} = 2a\left[\frac{1-t^2+2t^2}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dx}{dt} = \frac{2a(1+t^2)}{(1-t^2)^2}$

and $y = \frac{a(1+t^2)}{(1-t^2)}$ ...(ii)

$\Rightarrow \frac{dy}{dt} = a\left[\frac{(1-t^2)2t - (1+t^2)(-2t)}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = a\left[\frac{2t - 2t^3 + 2t + 2t^3}{(1-t^2)^2}\right]$

$\Rightarrow \frac{dy}{dt} = \frac{4at}{(1-t^2)^2}$

Now,

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{4at}{(1-t^2)^2} \times \frac{(1-t^2)^2}{2a(1+t^2)}$

$\Rightarrow \frac{dy}{dx} = \frac{2t}{1+t^2}$ ...(iii)

from eq.(i) and eq.(ii)

$\frac{x}{y} = \frac{2at}{1-t^2} \times \frac{1-t^2}{a(1+t^2)}$

$\Rightarrow \frac{x}{y} = \frac{2t}{1+t^2}$

from eq.(iii)

$$\frac{dy}{dx} = \frac{2t}{1+t^2} = \frac{x}{y}$$

115. (A) $f'(x) = x^3 + \dfrac{3}{2x^4}$

On integrating both side

$$\Rightarrow f(x) = \frac{x^4}{4} + \frac{3}{2}\,\frac{x^{-4+1}}{-4+1} + C$$

116. (C) Differential equation

$$\frac{d^2y}{dx^2} = x.e^{-2x}$$

On integrating

$$\frac{dy}{dx} = \int x.e^{-2x}\,dx$$

$$\frac{dy}{dx} = x.\int e^{-2x}dx - \int\left\{\frac{d}{dx}(x)\int e^{-2x}dx\right\}dx$$

$$\frac{dy}{dx} = x.\,\frac{e^{-2x}}{-2} - \int 1.\frac{e^{-2x}}{-2}\,dx + c$$

$$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx + c$$

$$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{2}\,\frac{e^{-2x}}{-2} + c$$

$$\frac{dy}{dx} = \frac{-1}{2}x.e^{-2x} + \frac{1}{4}e^{-2x} + c$$

Again, integrating

$$y = \frac{-1}{2}\int x.e^{-2x}\,dx - \frac{1}{4}.\int e^{-2x}dx + c\int 1.dx + d$$

$$y = -\frac{1}{2}\left[\frac{-x}{2}e^{-2x} - \frac{1}{4}e^{-2x}\right] - \frac{1}{4}\times\frac{e^{-2x}}{-2} + cx + d$$

$$y = \frac{1}{4}x.e^{-2x} + \frac{1}{8}.e^{-2x} + \frac{1}{8}.e^{-2x} + cx + d$$

$$y = \frac{1}{4}x.e^{-2x} + \frac{1}{4}.e^{-2x} + cx + d$$

117. (B) Let $y = \sin(\tan x^2)$ and $z = x^2$

$\Rightarrow\ y = \sin(\tan z)$

On differentiating both side w.r.t. 'z'

$$\Rightarrow \frac{dy}{dz} = \cos(\tan z).\sec^2 z$$

$$\Rightarrow \frac{dy}{dz} = \cos(\tan x^2).\sec^2 x^2$$

118. (D) Given that $f(x) = \dfrac{1}{g(x)}$, $g(x) = \dfrac{1}{x}$

then $f(x) = x$

L.H.S. $= f(f(f(f(f(g(x))))))$

$$= f\left(f\left(f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)\right)\right)$$

$$= f\left(f\left(f\left(f\left(\frac{1}{x}\right)\right)\right)\right)$$

$$= f\left(f\left(f\left(\frac{1}{x}\right)\right)\right) = f\left(f\left(\frac{1}{x}\right)\right)$$

$$= f\left(\frac{1}{x}\right) = \frac{1}{x}$$

R.H.S. $= g(g(g(g(g(f(x))))))$

$$= g(g(g(g(g(x)))))$$

$$= g\left(g\left(g\left(g\left(\frac{1}{x}\right)\right)\right)\right)$$

$$= g(g(g(x))) = g\left(g\left(\frac{1}{x}\right)\right)$$

$$= g(x) = \frac{1}{x}$$

L.H.S. = R.H.S

Hence option(D) is correct.

119. (C) Given that, $\bar{x} = 20$, $\bar{y} = 20$, $\sigma_x = 4, \sigma_y = 2$ and $r_{xy} = 0.6$

regression equation of x on y -

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$\Rightarrow x - 20 = 0.6 \times \frac{4}{2}(y - 80)$$

$\Rightarrow x - 20 = 1.2(y - 80)$

$\Rightarrow x - 20 = 1.2y - 96$

$\Rightarrow x = 1.2y - 76$

120. (C) $n(S) = {}^9C_3 = 84$

$n(E) = {}^4C_1 \times {}^5C_2 + {}^4C_2 \times {}^5C_1 \times {}^4C_3 \times {}^5C_0$

$n(E) = 4 \times 10 + 6 \times 5 + 4 \times 1 = 74$

Probability $P(E) = \dfrac{n(E)}{n(S)} = \dfrac{74}{84} = \dfrac{37}{42}$

NDA (MATHS) MOCK TEST - 178 (Answer Key)

1.	(B)	21.	(A)	41.	(B)	61.	(B)	81.	(B)	101.	(A)
2.	(C)	22.	(D)	42.	(C)	62.	(C)	82.	(C)	102.	(D)
3.	(A)	23.	(A)	43.	(B)	63.	(C)	83.	(C)	103.	(B)
4.	(B)	24.	(D)	44.	(D)	64.	(D)	84.	(A)	104.	(C)
5.	(B)	25.	(A)	45.	(A)	65.	(C)	85.	(D)	105.	(A)
6.	(A)	26.	(C)	46.	(B)	66.	(A)	86.	(A)	106.	(C)
7.	(B)	27.	(D)	47.	(B)	67.	(B)	87.	(A)	107.	(D)
8.	(B)	28.	(B)	48.	(D)	68.	(A)	88.	(B)	108.	(D)
9.	(C)	29.	(A)	49.	(C)	69.	(B)	89.	(C)	109.	(D)
10.	(C)	30.	(C)	50.	(C)	70.	(A)	90.	(B)	110.	(B)
11.	(A)	31.	(D)	51.	(D)	71.	(B)	91.	(A)	111.	(C)
12.	(A)	32.	(C)	52.	(B)	72.	(C)	92.	(C)	112.	(C)
13.	(C)	33.	(B)	53.	(C)	73.	(D)	93.	(D)	113.	(B)
14.	(D)	34.	(B)	54.	(C)	74.	(C)	94.	(A)	114.	(C)
15.	(D)	35.	(C)	55.	(C)	75.	(A)	95.	(B)	115.	(A)
16.	(B)	36.	(A)	56.	(A)	76.	(A)	96.	(D)	116.	(C)
17.	(D)	37.	(B)	57.	(D)	77.	(C)	97.	(B)	117.	(B)
18.	(A)	38.	(D)	58.	(C)	78.	(B)	98.	(B)	118.	(D)
19.	(A)	39.	(C)	59.	(A)	79.	(B)	99.	(A)	119.	(C)
20.	(D)	40.	(C)	60.	(D)	80.	(D)	100.	(A)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777