

**NDA MATHS MOCK TEST - 180 (SOLUTION)**

1. (D)  $y = \frac{\sin x + \cos x}{\sin x - \cos x}$

diff. w.r.t.  $x$

$$\frac{dy}{dx} = \frac{(\sin x - \cos x) \frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x) \frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$\frac{dy}{dx} = \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$\frac{dy}{dx} = \frac{-[\sin^2 x + \cos^2 x - 2\sin x \cos x] - [\sin^2 x + \cos^2 x + 2\sin x \cos x]}{(\sin x - \cos x)^2}$$

$$\frac{dy}{dx} = \frac{-2}{(\sin x - \cos x)^2}$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=0} = \frac{-2}{(0-1)^2} = -2$$

2. (B) We know that  
then If Points are collinear  
Area of  $\Delta = 0$

$$\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$k(3k-1) + 3k(1-2k) + 3(2k-3k) = 0$$

$$3k^2 - k + 3k - 6k^2 + 6k - 9k = 0$$

$$-3k^2 - k = 0$$

$$-k(3k+1) = 0$$

$$\Rightarrow k = 0, k = -\frac{1}{3}$$

3. (D) Let root  $2\alpha, 3\alpha$

$$2\alpha + 3\alpha = \frac{-2b}{3a}$$

$$5\alpha = \frac{-2(b)}{3(a)}$$

$$\alpha = \frac{-2b}{15a}$$

Now,

$$2\alpha \cdot 3\alpha = \frac{c}{3a}$$

$$6\alpha^2 = \frac{c}{3a}$$

$$6\left(\frac{-2b}{15a}\right)^2 = \frac{c}{3a}$$

$$6 \times \frac{4b^2}{15 \times 15 \times a^2} = \frac{c}{3a}$$

$$8b^2 = 25ac$$

4. (A)  $|z+4| \leq 3$

$$-(z+4) \leq 3$$

$$z+4 \geq -3$$

$$z+1 \leq -6$$

$$-(z+1) \leq 6$$

$$|z+1| \leq 6$$

5. (C) We have,

$$-3 \leq 4 - \frac{7x}{2} \leq 18$$

$$\Rightarrow -3 - 4 \leq -\frac{7x}{2} \leq 18 - 4$$

$$\Rightarrow -7 \leq -\frac{7x}{2} \leq 14$$

$$\Rightarrow 7 \geq \frac{7x}{2} \geq -14$$

$$\Rightarrow 2 \geq x \geq -4$$

6. (B) We know that  $f(-x) = -f^n(x) \rightarrow f_n$  is odd  
 $f(-x) = f(x) \rightarrow f^n$  is even

$$f(x) = x \frac{a^x + 1}{a^x - 1}$$

$$f(-x) = (-x) \frac{a^{(-x)} + 1}{a^{(-x)} - 1}$$

$$f(-x) = (-x) \frac{1+a^x}{1-a^x}$$

$$f(-x) = x \frac{1+a^x}{a^x - 1}$$

$$\Rightarrow f(-x) = f(x)$$

$$\Rightarrow f(x) \text{ is even } f^n$$

7. (C) Number of reflexive relation =  $2^{n^2-n}$

$$= 2^{4^2-4}$$

$$= 2^{16-4}$$

$$= 2^{12}$$

8. (A)  $\left(\frac{1+i}{1-i}\right)^{398} = x + iy$

$$\Rightarrow \left[\frac{(1+i)(1+i)}{(1-i)(1-i)}\right]^{398} = x + iy$$

$$\Rightarrow \left[\frac{(1+i)^2}{1+1}\right]^{398} = x + iy$$

$$\Rightarrow \left[\frac{1+i^2+2i}{2}\right]^{398} = x + iy$$

$$\Rightarrow [i]^{398} = x + iy$$

$$\Rightarrow [i^2]^{199} = x + iy$$

$$\Rightarrow (-1)^{199} = x + iy$$

$$\Rightarrow -1 + 0i = x + iy$$

$$\Rightarrow x = -1, y = 0$$

9. (D)

10. (A) Let point  $\left(\frac{y^2}{4}, y\right)$

$$\text{Distance (D)} = \sqrt{(x-2)^2 + (y-1)^2}$$

$$D = \sqrt{\left(\frac{y^2}{4} - 2\right)^2 + (y-1)^2}$$

$$D = \sqrt{\frac{y^4}{16} + 16 - y^2 + y^2 + 1 - 2y}$$

$$D = \sqrt{\frac{y^4}{16} + 5 - 2y}$$

$$D^2 = \frac{y^4}{16} + 5 - 2y = (\text{let } A)$$

$$A = \frac{y^4}{16} + 5 - 2y$$

diff. w.r.t.  $y$

$$\frac{dA}{dy} = \frac{d}{dy} \left( \frac{y^4}{16} - 2y + 5 \right)$$

$$= \frac{4y^3}{16} - 2 = \frac{y^3}{4} - 2$$

again diff. w.r. to  $y$

$$\frac{d^2y}{dy^2} = \frac{3y^2}{4} > 0$$

$$\Rightarrow \frac{y^3}{4} - 2 = 0$$

$$\Rightarrow y^3 = 8$$

$$\Rightarrow y = 2$$

Now,

$$y^2 = 4x$$

$$(2)^2 = 4x \Rightarrow x = 1$$

11. (A)  $S = y^2 - 8x$

$$S_{(2,1)} = (1)^2 - 8 \times 2$$

$$S_{(2,1)} = -15 < 0$$

$\Rightarrow$  Point is inside parabola

$\Rightarrow$  There is no tangent on parabola by point  $(2, 1)$

12. (B) Latus rectum  $l = \frac{2b^2}{a}$

major axis =  $2a$

$$\Rightarrow l = \frac{1}{2} \times 2a = a$$

Now,

$$\Rightarrow l = \frac{2b^2}{a} = a$$

$$\Rightarrow 2b^2 = a^2$$

[since, we know that  $b^2 = a^2(1 - e^2)$ ]

$$\Rightarrow 2[a^2(1 - e^2)] = a^2$$

$$\Rightarrow 1 - e^2 = \frac{1}{2}$$

$$\Rightarrow e^2 = \frac{1}{2}$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

13. (B) We have  $\int \frac{dx}{\sqrt{5x^2 - 2x}} = \int \frac{dx}{\sqrt{5\left(x^2 - \frac{2x}{5}\right)}}$

$$= \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{\left(x - \frac{1}{5}\right)^2 - \left(\frac{1}{5}\right)^2}}$$

Put  $x - \frac{1}{5} = t$ . Then  $dx = dt$

$$\text{Therefore, } \int \frac{dx}{\sqrt{5x^2 - 2x}} = \frac{1}{\sqrt{5}} \int \frac{dt}{\sqrt{t^2 - \left(\frac{1}{5}\right)^2}}$$

$$= \frac{1}{\sqrt{5}} \log \left| t + \sqrt{t^2 - \left(\frac{1}{5}\right)^2} \right| + C$$

$$= \frac{1}{\sqrt{5}} \log \left| x - \frac{1}{5} + \sqrt{x^2 - \frac{2x}{5}} \right| + C$$

14. (D) Consider  $\frac{x^2}{(x^2+1)(x^2+4)}$  and Put  $x^2 = y$ .

$$\text{Then } \frac{x^2}{(x^2+1)(x^2+4)} = \frac{y}{(y+1)(y+4)}$$

$$\text{Write } \frac{y}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$$

So that  $y = A(y+4) + B(y+1)$

Comparing coefficients of  $y$  and constant terms on both sides, we get  $A + B = 1$  and  $4A + B = 0$ , which give

$$A = -\frac{1}{3} \text{ and } B = \frac{4}{3}$$

$$\text{Thus, } \frac{x^2}{(x^2+1)(x^2+4)} = -\frac{1}{3(x^2+1)} + \frac{4}{3(x^2+4)}$$

$$\text{Therefore, } \int \frac{x^2 dx}{(x^2+1)(x^2+4)} = -\frac{1}{3} \int \frac{dx}{x^2+1}$$

$$+ \frac{4}{3} \int \frac{dx}{x^2+4}$$

$$= -\frac{1}{3} \tan^{-1}x + \frac{4}{3} \times \frac{1}{2} \tan^{-1} \frac{x}{2} + C$$

$$= -\frac{1}{3} \tan^{-1}x + \frac{2}{3} \tan^{-1} \frac{x}{2} + C$$

15. (A) The A.M. =  $\frac{(2+4+8+\dots+2^n)}{n}$

$$= \frac{(2^1+2^2+2^3+\dots+2^n)}{n}$$

$$= \frac{2(1+2+2^2+\dots+2^{n-1})}{n}$$

$$= \frac{2(2^n-1)}{n(2-1)} = \frac{2(2^n-1)}{n}$$

$$\text{A.M.} = \frac{2(2^n-1)}{n}$$

16. (C) We are given that : A.M. = 10, G.M. = 8.  
Also we know that (A.M.)(H.M.) = (G.M.)<sup>2</sup>

$$\text{or } (10)(\text{H.M.}) = 8^2 = 64 \text{ or H.M.} = \frac{64}{10} = 6.4$$

17. (D) Coefficient of variation =  $\frac{\sigma}{X} \times 100$

$$= \frac{120}{800} \times 100 = 15\%$$

18. (B)  $P(A) = 1 - P(\bar{A}) = 1 - \frac{2}{3} = \frac{1}{3}$

$$\text{Since, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}$$

$$\Rightarrow P(B) = \frac{2}{3}$$

19. () Let us arrange the items in ascending order :

3, 4, 6, 9, 10, 11, 18, 22

Median : M = average of

$$\left\{ \binom{n}{2}^{\text{th}} \text{ and } \binom{n}{2+1}^{\text{th}} \right\} \text{ terms}$$

$$= \text{Average of } \left(\frac{8}{2}\right)^{\text{th}} \text{ and } \left(\frac{8}{2}+1\right)^{\text{th}} \text{ terms}$$

$$= \text{Average of 4th and 5th terms} = \frac{9+10}{2}$$

$$= \frac{19}{2} = 9.5$$

20. (C) Possibilities of sum of the dice is more than 10 (5, 6) (6, 5) (6, 6) = 3

Possibilities of sum of the dice is divisible by 3 (1, 2) (1, 5) (2, 1) (2, 4) (3, 3) (3, 6) (4, 2) (4, 5) (5, 1) (5, 4) (6, 3) (6, 6) = 12

$$\text{Required ratio} = \frac{3}{36} : \frac{12}{36} = 1 : 4$$

21. (D) Required probability =  $\frac{4}{52} \times \frac{5}{51} = \frac{5}{663}$

22. (C)  $P(E) = \frac{n(E)}{n(S)}$

$$n(S) = 14C_3$$

$$n(E) = 5C_1 \text{ and } 3C_1 \text{ and } 6C_1$$

$$P(E) = \frac{5C_1 \text{ and } 3C_1 \text{ and } 6C_1}{14C_3}$$

$$= \frac{45}{182}$$

23. (A) Total probability  $n(S) = 12C_2$   
Required probability =  $1 - P(\text{none is pink})$   
Probability of getting none is pink balls,

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{8C_2}{12C_2} = \frac{14}{33}$$

Required probability =  $1 - P(\text{None is pink})$

$$= 1 - \left(\frac{14}{33}\right)$$

$$= \frac{19}{33}$$

24. (B) Let the number of balls transferred =  $x$   
Now,

$$\frac{{}^x C_2}{{}^{(4+x)} C_2} = \frac{3}{14}$$

$$\Rightarrow \frac{x(x-1)}{(4+x)(3+x)} = \frac{3}{14}$$

$$\Rightarrow 14x^2 - 14x = 3x(12 + 4x + 3x + x^2)$$

$$\Rightarrow 14x^2 - 14x = 36 + 21x + 3x^2$$

$$\Rightarrow 11x^2 - 35x - 36 = 0$$

$$\Rightarrow (x-4)(11x+9) = 0$$

$$\Rightarrow x = 4, \frac{-9}{11} \text{ (rejected)}$$

25. (A) Let  $u = \ln(x + \sin x)$  and  $v = x + \cos x$

$$\text{Now, } \frac{du}{dx} = \frac{1}{(x + \sin x)} (1 + \cos x)$$

$$\text{and } \frac{dv}{dx} = 1 - \sin x$$

Now, we can find derivative of  $u$  w.r.t.v,

$$\frac{du}{dx} = \frac{(1 + \cos x)}{(x + \sin x)}$$

$$\frac{dv}{dx} = 1 - \sin x$$

$$\Rightarrow \frac{du}{dv} = \frac{(1 + \cos x)}{(x + \sin x)(1 - \sin x)}$$

26. (C)  $S = \sqrt{t^2 + 1}$  Differentiating on both sides w.r.t.  $t$ , we get

$$\frac{ds}{dt} = \frac{1}{2\sqrt{t^2 + 1}} \times 2t = \frac{t}{\sqrt{t^2 + 1}}$$

Differentiating again both sides w.r.t.  $t$ , we get

$$\frac{d^2s}{dt^2} = \frac{\sqrt{t^2 + 1}(1) - t \times \frac{1}{2\sqrt{t^2 + 1}}(2t)}{(\sqrt{t^2 + 1})^2}$$

$$= \frac{t^2 + 1 - t^2}{(\sqrt{t^2 + 1})^2 \sqrt{t^2 + 1}} = \frac{1}{(\sqrt{t^2 + 1})^3} = \frac{1}{S^3}$$

27. (D) Given that,  $y = \cos t$   
and  $x = \sin t$

$$\text{Then, } \frac{dy}{dt} = -\sin t$$

$$\text{and } \frac{dx}{dt} = \cos t$$

$$\text{Now, } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{-\sin t}{\cos t} = \frac{-x}{y}$$

28. (A)  $\because x = k(\theta + \sin\theta)$  and  $y = k(1 + \cos\theta)$ ,

$$\frac{dx}{d\theta} = k(1 + \cos\theta)$$

$$\text{and } \frac{dy}{d\theta} = -k \sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-k \sin\theta}{k(1 + \cos\theta)}$$

$$= \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} = -\tan \frac{\theta}{2}$$

$$\therefore \left(\frac{dy}{dx}\right)_{\text{at } \theta = \frac{\pi}{2}} = -\tan \frac{\pi}{4} = -1$$

29. (D) Let  $y = \log x$   
On diff. w.r.t.  $x$   
from 1 to  $n$  times, we get

$$y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3} \dots$$

$$y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$$

30. (A) Given curve,  
 $y = \sin^{-1}(\sin^2 x)$   
On differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - (\sin^2 x)^2}} \cdot \frac{d}{dx}(\sin^2 x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \sin x \cdot \cos x}{\sqrt{1 - \sin^4 x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x}{\sqrt{1 - \sin^4 x}}$$

$$\left(\frac{dy}{dx}\right)_{x=0} = \frac{\sin 0}{\sqrt{1 - \sin 0}} = \frac{0}{\sqrt{1 - 0}} = 0$$

$\therefore$  Slope of the curve = 0

Hence, slope of the tangent to the given curve = Slope of that curve = 0

31. (C) Equation of tangent to the curve  
 $y = e^{2x}$  at  $(0, 1)$  is

$$(y - 1) = \left(\frac{dy}{dx}\right)_{(0,1)} (x - 0)$$

$$\Rightarrow (y - 1) = 2(x - 0)$$

$$\Rightarrow y - 1 = 2x$$

$$\therefore y = 2x + 1$$

Since, the tangent meets X-axis.

So, put  $y = 0$  in equation of tangent, we get

$$0 = 2x + 1 \Rightarrow x = -\frac{1}{2}$$

Hence, at  $\left(-\frac{1}{2}, 0\right)$ , the tangent to the curve at  $(0, 1)$  meet the X-axis.

32. (A) Consider,

$$\int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right) dx = \int (\sec^2 x - \operatorname{cosec}^2 x) dx$$

$$= \tan x - (-\cot x) + C$$

$$= \tan x + \cot x + C$$

$$= \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) + C$$

$$= \left(\frac{1}{\sin x \cdot \cos x}\right) + C$$

$$= 2 \operatorname{cosec} 2x + C$$

33. (C) Consider,  $\int \frac{1}{1+e^x} dx = \int \frac{e^{-x}}{e^{-x}+1} dx$

$$= -\log(1+e^x) + C = -\log\left(\frac{1+e^x}{e^x}\right) + C$$

$$\left[ \because \int \frac{f'(x)}{f(x)} dx = \log f(x) + C \right]$$

$$= -\{\log(1+e^x) - \log e^x\} + C$$

$$= x - \log(1+e^x) + C$$

34. (B)  $I = \int \sec^2 x (\sec^2 x) \tan x dx$

$$= \int \sec^2 x (1 + \tan^2 x) \tan x dx$$

$$= \int (1+t^2)t dt$$

where  $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$= \int (t+t^3) dt = \frac{1}{2} t^2 + \frac{1}{4} t^4 + C$$

$$= \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + C$$

35. (B)  $I = \int \sqrt{e^x - 1} dx$

Putting  $e^x - 1$  and  $e^x dx = 2t dt$

i.e.  $dx = \frac{2t}{(t^2+1)} dt$ , we get

$$I = \int \frac{2t^2}{(t^2+1)} dt = \int \left( 2 - \frac{2}{t^2+1} \right) dt$$

$$= 2t - 2 \tan^{-1} t + C$$

$$= 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + C$$

36. (C) Let  $I = \int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

$$= \int_0^{\pi/2} \frac{\sec^2 x dx}{a^2 + b^2 \tan^2 x}$$

[ $\because$  divide numerator and denominator  $\cos^2 x$ ]

Let  $\tan x = t \Rightarrow \sec^2 x dx = dt$

When  $x = 0$ , then  $t = 0$

and when  $x = \frac{\pi}{2}$ , then  $t = \infty$

$$I = \frac{1}{b^2} \int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$$

$$= \frac{1}{b^2} \frac{1}{\left(\frac{a}{b}\right)} \left[ \tan^{-1} \left( \frac{bt}{a} \right) \right]_0^{\infty}$$

$$\left[ \because \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C \right]$$

$$= \frac{1}{ab} [\tan^{-1}(\infty) - \tan^{-1}(0)]$$

$$= \frac{1}{ab} \left[ \frac{\pi}{2} - 0 \right] = \frac{\pi}{2ab}$$

37. (D)  $\frac{1}{e} \leq x \leq 1 \Rightarrow \log x \leq 0$

$$\Rightarrow |\log x| = -\log x$$

$$1 < x \leq e \Rightarrow \log x \geq 0$$

$$\Rightarrow |\log x| = \log x$$

$$I = \int_{1/e}^1 |\log x| dx + \int_1^e |\log x| dx$$

$$= \int_{1/e}^1 -\log x dx + \int_1^e \log x dx$$

$$= [-x \log x - x]_{1/e}^1 + [(x \log x - x)]_1^e$$

$$= [x(1 - \log x)]_{1/e}^1 + [x(\log x - 1)]_1^e$$

$$= \left[ 1 - \frac{1}{e}(1+1) + 1 \right] = 2 \left( 1 - \frac{1}{e} \right)$$

38. (C)  $I = \int_{\alpha}^{\left(\frac{\pi}{2}-\alpha\right)} \frac{1}{1+\sqrt{\cot x}} dx$

$$I = \int_{\alpha}^{\left(\frac{\pi}{2}-\alpha\right)} \frac{\sqrt{\sin x}}{(\sqrt{\sin x} + \sqrt{\cos x})} dx$$

Using prop. V with  $a + b = \frac{\pi}{2}$ , we get:

$$I = \int_{\alpha}^{\left(\frac{\pi}{2}-\alpha\right)} \frac{\sqrt{\cos x}}{(\sqrt{\cos x} + \sqrt{\sin x})} dx \quad \dots(ii)$$

$$\therefore 2I = \int_{\alpha}^{\left(\frac{\pi}{2}-\alpha\right)} dx = [x]_{\alpha}^{\left(\frac{\pi}{2}-\alpha\right)}$$

$$\Rightarrow 2I = \left( \frac{\pi}{2} - \alpha - \alpha \right)$$

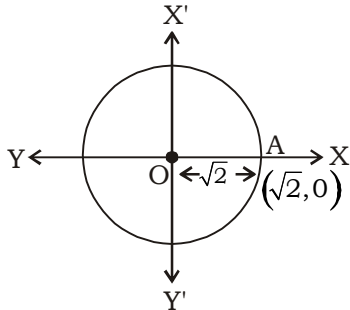
$$\Rightarrow 2I = \left( \frac{\pi}{2} - 2\alpha \right)$$

$$\Rightarrow I = \left( \frac{\pi}{4} - \alpha \right)$$

Remember

If  $f(-x) = -f(x)$ , then  $\int_a^a f(x) dx = 0$

40. (B) Given curve is the equation of circle,  
 $x^2 + y^2 = (\sqrt{2})^2$



∴ Required area  
 $= 4 \times \int_{x=0}^{x=\sqrt{2}} y \, dx$   
 $= 4 \int_0^{\sqrt{2}} \sqrt{2-x^2} \, dx$   
 $= 4 \left[ \frac{x}{2} \sqrt{2-x^2} + \frac{2}{2} \sin^{-1} \frac{x}{\sqrt{2}} \right]_0^{\sqrt{2}}$   
 $= 4 \left( \frac{\pi}{2} - 0 \right) = 2\pi \text{ sq units}$

41. (C)  $y \frac{dy}{dx} + x = a \Rightarrow y \, dy + x \, dx = a \, dx$   
 On integrating both sides, we get  
 $\int y \, dy + \int x \, dx = \int a \, dx$   
 $\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} = ax$   
 $\Rightarrow x^2 + y^2 - 2ax = 0$   
 which represents a set of circles

42. (A) The given differential equation is

$$\frac{dy}{dx} + \sqrt{1-y^2} = 0$$

$$\Rightarrow \int \frac{1}{\sqrt{1-y^2}} \, dy + \int \frac{1}{\sqrt{1-x^2}} \, dx = 0$$

$$\Rightarrow \sin^{-1} y + \sin^{-1} x = C$$

43. (A) The given differential equation is  
 $(1+e^x)y \, dy = e^x \, dx$   
 On integrating both sides, we get

$$\int y \, dy = \int \left( \frac{e^x}{1+e^x} \right) dx$$

$$\Rightarrow \frac{y^2}{2} = \log(1+e^x) + \log C$$

$$\Rightarrow y^2 = 2 \log[C(1+e^x)]$$

$$\Rightarrow y^2 = \log[C^2(1+e^x)^2]$$

44. (A) (A)  $|-x| = |x|$ . The inequality  $|x| \leq x$  is

correct for  $x \geq 0$

(B) The inequality is not fulfilled for  $x < 0$ .  
 For  $x \geq 0$ ,  $|x| = x$ , and the slack inequality  $x^2 \geq x^2$  is correct.

(C)  $\sqrt{x^2} = |x|$ ;  $|x| \leq -x$  for  $x \leq 0$ .

(D)  $\sqrt{2x^2} = |x| \sqrt{2}$ ;  $x\sqrt{2} > |x| \sqrt{2}$  or  $x > |x|$  is incorrect for any  $x$ .

45. (A) Let  $P(a, b, c) = a^3 + b^3 + c^3 - 3abc$   
 and  $Q(a, b, c) = (a+b+c)^3$

We know that

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)[(a+b+c)^2 - 3(ab+bc+ca)]$$

We are given that  $ab+bc+ca = 0$

$$\therefore a^3 + b^3 + c^3 - 3abc = (a+b+c)(a+b+c)^2 = (a+b+c)^3$$

$$P = (a, b, c) = (a+b+c)^3$$

Thus, from (i) and (ii), required H.C.F. is  $(a+b+c)^3$

46. (B) Given,  $1 + \sin x + \sin^2 x + \sin^3 x + \dots \infty$   
 $= 4 + 2\sqrt{3}$

$$\Rightarrow 1 - \sin x = \frac{1}{4 + 2\sqrt{3}}$$

$$\therefore S_{\infty} = \frac{a}{1-r}. \text{ Here } a = 1, r = \sin x$$

Also  $|\sin x| < 1$  as  $|\sin x| = 1$  will make the sum of infinite G.P. infinite

$$\Rightarrow \sin x = 1 - \frac{1}{4 + 2\sqrt{3}} = \frac{4 + 2\sqrt{3} - 1}{4 + 2\sqrt{3}}$$

$$= \frac{3 + 2\sqrt{3}}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$= \frac{12 + 8\sqrt{3} - 6\sqrt{3} - 12}{16 - 12} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{\pi}{3} \text{ or } \frac{2\pi}{3} \text{ as } 0 < x < \pi$$

47. (C) We have,

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21) = 19\hat{j} + 19\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

48. (D) The repeated guessing of correct from multiple choice question are Bernoulli trials.

Let  $X$  represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer

$$\text{is, } p = \frac{1}{3}$$

$$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Clearly,  $X$  has a binomial distribution

with  $n = 5$  and  $p = \frac{1}{3}$

$$\therefore P(X = x) = {}^n C_x q^{n-x} p^x$$

$$= {}^5 C_x \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^x$$

$p(\text{guessing more than 4 correct answers})$

$$= p(X \geq 4)$$

$$= p(X = 4) + p(X = 5)$$

$$= {}^5 C_4 \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^4 + {}^5 C_5 \left(\frac{1}{3}\right)^5$$

$$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$$

$$= \frac{10}{243} + \frac{1}{243}$$

$$= \frac{11}{243}$$

49. (A)  $S_1 = \frac{n(n+1)}{2}$ ,  $S_2 = \frac{n(n+1)(2n+1)}{6}$ ,

$$S_3 = \frac{n^2(n+1)^2}{4}$$

$$9S_2^2 = 9 \left\{ \frac{1}{6} n(n+1)(2n+1) \right\}^2$$

$$= 9 \left\{ \frac{1}{36} n^2(n+1)^2(2n+1)^2 \right\}$$

$$= \frac{1}{4} n^2(n+1)^2(4n^2 + 4n + 1) = \frac{1}{4} n^2(n+1)^2$$

$$(4n(n+1) + 1)$$

$$= \frac{1}{4} n^2(n+1)^2(1+8) \left( \frac{1}{2} n(n+1) \right) = S_3 (1+8S_1)$$

50. (B)  $f(a) = \log\left(\frac{1-a}{1+a}\right)$ ,  $f(b) = \log\left(\frac{1-b}{1+b}\right)$

$$f\left(\frac{a+b}{1+ab}\right) = \log\left(\frac{1-\frac{a+b}{1+ab}}{1+\frac{a+b}{1+ab}}\right)$$

$$\Rightarrow f\left(\frac{a+b}{1+ab}\right) = \log\left(\frac{1-ab-a-b}{1+ab+a+b}\right)$$

$$= \log\left(\frac{(1-a)+b(a-1)}{(1+a)+b(a+1)}\right) = \log\left(\frac{(1-a)(1-b)}{(1+a)(1+b)}\right)$$

$$= \log\left(\frac{1-a}{1+a}\right) + \log\left(\frac{1-b}{1+b}\right)$$

$$(\because \log ab = \log a + \log b)$$

$$= f(a) + f(b)$$

51. (C)  $f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha \cdot \left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}\right) + 1}$

$$= \frac{\frac{\alpha^2 x}{x+1}}{\frac{\alpha x + x + 1}{x+1}} = \frac{\alpha^2 x}{\alpha x + x + 1}$$

$$\text{Given, } f(f(x)) = x \Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x$$

$$\Rightarrow \alpha^2 x = \alpha x^2 + x^2 + x$$

$$\Rightarrow \alpha^2 - 1 = (\alpha + 1)x$$

$$\Rightarrow (\alpha - 1)(\alpha + 1) - (\alpha + 1)x = 0$$

$$\Rightarrow (\alpha + 1)(\alpha - 1 - x) = 0$$

$$\Rightarrow \alpha + 1 = 0 \Rightarrow \alpha = -1 \quad [\because \alpha - 1 - x \neq 0]$$

52. (A)  $0.\overline{234} = 0.2343434\dots$   
 $= 0.2 + 0.034 + 0.00034 + 0.0000034 + \dots + \infty$

$$= \frac{2}{10} + \frac{34}{1000} + \frac{34}{100000} + \frac{34}{10000000} + \dots + \infty$$

$$= \frac{2}{10} + \frac{34}{10^3} \left[ 1 + \frac{1}{10^2} + \dots + \infty \right]$$

$$= \frac{2}{10} + \frac{34}{10^3} \left( \frac{1}{1 - \frac{1}{10^2}} \right)$$

$$= \frac{2}{10} + \frac{34}{1000} \times \frac{100}{99} = \frac{198 + 34}{990} = \frac{232}{990} = \frac{116}{495}$$

53. (D)  $a, b, c$  are in A.P.  
 $\Rightarrow 2b = a + c$  ... (i)  
 $x, y, z$  are in G.P.  
 $\Rightarrow y = xr, z = xr^2$ , where  $r$  is the common ratio  
 $\therefore x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$   
 $= x^{b-c} \cdot (xr)^{c-a} \cdot (xr^2)^{a-b}$   
 $= x^{b-c} \cdot x^{c-a} \cdot r^{c-a} \cdot x^{a-b} \cdot r^{2a-2b}$   
 $= x^{b-c+c-a+a-b} \cdot r^{c-a+2a-2b}$   
 $= x^0 \cdot r^{c+a-2b} = x^0 \cdot r^{2b-2b} = x^0 \cdot r^0 = 1$

54. (C) The area of the parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$   
 Adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(1+21) - \hat{j}(1-6)$$

$$+ \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Hence, the area of the given parallelogram is  $15\sqrt{2}$  square units

55. (A) Here  $a = 19, d = 18\frac{1}{5} - 19 = -\frac{4}{5}$   
 Let the  $n$ th term be the first negative term. Then,  
 $T_n < 0 \Rightarrow a + (n-1)d < 0$   
 $\Rightarrow 19 + (n-1)\left(-\frac{4}{5}\right) < 0$   
 $\Rightarrow 19 - \frac{4}{5}n + \frac{4}{5} < 0$   
 $\Rightarrow \frac{99}{5} - \frac{4}{5}n < 0 \Rightarrow n > \frac{99}{5} \times \frac{5}{4} > \frac{99}{4}$   
 $= 24\frac{3}{4} \Rightarrow n = 25$

56. (A) The given vectors are  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$   
 and  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$   
 $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$   
 $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$   
 $\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$   
 $|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$   
 Hence, the unit vector in the direction of  $(\vec{a} + \vec{b})$  is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{(\hat{i} + \hat{k})}{\sqrt{2}} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

57. (C)  $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2} = \log_{(1-x)} 10 + \sqrt{x+2}$

$$\text{Let } F(x) = \log_{(1-x)} 10 \text{ and } g(x) = \sqrt{x+2}$$

$F(x)$  is defined when  $(1-x) > 0$  and  $1-x \neq 1$

$$\Rightarrow x < 1 \text{ and } x \neq 0$$

$$\Rightarrow x \in (-\infty, 1) - \{0\}$$

$g(x)$  is defined when  $x+2 \geq 0$

$$\Rightarrow x \geq -2$$

$$\Rightarrow x \in [-2, \infty)$$

From (i) and (ii), the domain of  $f(x) = F(x) + g(x)$  is common domain of  $F(x)$  and  $g(x)$ , i.e.,  $(-\infty, 1) - \{0\} \cap [-2, \infty) = [-2, 1) - \{0\}$

58. (B) Let  $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz} = k$

$$\text{Then, } \frac{a-x}{px} = k \Rightarrow \frac{a-x}{kx} = p \Rightarrow p = \frac{1}{k} \left( \frac{a}{x} - 1 \right)$$

$$\text{Similarly, } q = \frac{1}{k} \left( \frac{a}{y} - 1 \right), r = \frac{1}{k} \left( \frac{a}{z} - 1 \right)$$

Now,  $p, q, r$  are in A.P.

$$\Rightarrow \frac{1}{k} \left( \frac{a}{x} - 1 \right), \frac{1}{k} \left( \frac{a}{y} - 1 \right), \frac{1}{k} \left( \frac{a}{z} - 1 \right) \text{ are in A.P.}$$

$$\Rightarrow \frac{a}{x} - 1, \frac{a}{y} - 1, \frac{a}{z} - 1 \text{ are in A.P.}$$

$$\Rightarrow \frac{a}{x}, \frac{a}{y}, \frac{a}{z} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \text{ are in A.P.}$$

$$\Rightarrow x, y, z \text{ are in H.P.}$$

59. (A) It is given that,

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 2 \text{ and } \vec{a} \cdot \vec{b} = \sqrt{6}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Now, we know that

$$\therefore \sqrt{6} = \sqrt{3} \times 2 \times \cos \theta$$

$$\Rightarrow \cos \theta = \frac{\sqrt{6}}{\sqrt{3} \times 2}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

Hence, the angle between the given vectors and is  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{4}$



60. (A)  $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$   
 $\vec{x} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{a} = 12$   
 $\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$   
 $\Rightarrow |\vec{x}|^2 - 1 = 12$   
 $||\vec{a}| = 1 \text{ as } \vec{a} \text{ is unit vector}$   
 $\Rightarrow |\vec{x}|^2 = 13$   
 $\Rightarrow |\vec{x}| = \sqrt{13}$

61. (D) Let d be the common difference for the A.P. ;  $a_1, a_2, a_3, \dots$

Then,  $\frac{S_p}{S_q} = \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$

$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$   
 $\Rightarrow q[2a_1 + (p-1)d] = p[2a_1 + (q-1)d]$   
 $\Rightarrow 2a_1q + pqd - qd = 2a_1p + pqd - pd$   
 $\Rightarrow pd - qd = 2a_1p - 2a_1q$   
 $\Rightarrow d(p - q) = 2a_1(p - q)$   
 $\Rightarrow d = 2a_1$

Now,  
 $\frac{\text{Term } 6}{\text{Term } 21} = \frac{a_6}{a_{21}} = \frac{a_1 + (6-1)d}{a_1 + (21-1)d} = \frac{a_1 + 5 \times 2a_1}{a_1 + 20 \times 2a_1}$

$= \frac{11a_1}{41a_1} = \frac{11}{41}$

62. (D)  $A = \frac{1}{2} \begin{vmatrix} h & h & 1 \\ 1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix}$   
 $= \frac{1}{2} [h(1-3) - h(1-2) + 1(3+2)]$

$= \frac{1}{2} [-2h + h + 1] = \frac{1-h}{2}$

Area =  $|A| = \left| \frac{1-h}{2} \right| = 2$

$\Rightarrow h - 1 = \pm 4$   
 $\Rightarrow h = 5, -3$

63. (C) Let point  $(h, k)$   
 According to question

$\frac{3h + 4k - 11}{\sqrt{3^2 + 4^2}} = \frac{20h - 21k - 15}{\sqrt{20^2 + (-21)^2}}$

$\Rightarrow \frac{3h + 4k - 11}{5} = \frac{20h - 21k - 15}{29}$

On solving

$43h - 221k + 244 = 0$

Hence locus of point

$43x - 221y + 244 = 0$

64. (C) Word "INTEGRATION"

The required Permutation

$= \frac{11!}{2!2!2!} = \frac{39916800}{8} = 4989600$

65. (C) We know that

$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

$\frac{r \cdot {}^nC_r}{{}^nC_{r-1}} = n-r+1$

Now,  $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \dots + \frac{nC_n}{C_{n-1}}$

$\Rightarrow (n-1+1) + (n-2+1) + (n-3+1) + \dots + (n-n+1)$

$\Rightarrow n + (n-1) + (n-2) + \dots + 1$

$\Rightarrow \frac{n(n+1)}{2}$

66. (A)  $1 \ 1 \ 0 \ 0$   
 $\begin{matrix} \leftarrow 0 \times 2^3 = 0 & \leftarrow 0.101 \\ \leftarrow 0 \times 2^2 = 0 & \leftarrow \\ \leftarrow 1 \times 2^1 = 2 & \leftarrow \\ \leftarrow 1 \times 2^0 = 1 & \leftarrow \\ \hline & \frac{5}{8} = 0.625 \end{matrix}$

67. (B)  $\lim_{x \rightarrow \infty} \frac{2x^3 + 5x^2 - 7}{3x^2 - 5x^3 + 3x - 2}$

$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^3 \left( 2 + \frac{5}{x} - \frac{7}{x^3} \right)}{x^3 \left( -5 + \frac{3}{x} + \frac{3}{x^2} - \frac{2}{x^3} \right)}$

$\Rightarrow \frac{2+0+0}{-5+0+0} = \frac{-2}{5}$

68. (C)  $z = \frac{3+4i}{(1+i)^2}$

Conjugate of  $z = \frac{3-4i}{(1-i)^2}$

$= \frac{3-4i}{1+i^2-2i}$

$= \frac{3-4i}{-2i} \times \frac{i}{i}$

$= \frac{3i-4i^2}{-2i^2} = \frac{3i+4}{2}$

69. (A)  $f(x) = \begin{cases} 2x^2 - 5, & -1 < x \leq 3 \\ x - \lambda, & 3 < x \leq 7 \end{cases}$  is

continuous at  $x = 3$ , then

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

$$\Rightarrow 2 \times 3^2 - 5 = 3 - \lambda \Rightarrow \lambda = -10$$

70. (A)  $A = \{x \in \mathbb{R}, x^2 + 3x - 28 \leq 0\}$

$$x^2 + 3x - 28 \leq 0$$

$$(x + 7)(x - 4) \leq 0$$

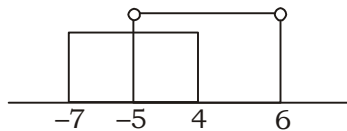
$$-7 \leq x \leq 4$$

and  $B = \{x \in \mathbb{R}, x^2 - x - 30 < 0\}$

$$x^2 - x - 30 < 0$$

$$(x - 6)(x + 5) < 0$$

$$-5 < x < 6$$



**Statement I**

$$(A \cup B) = \{x \in \mathbb{R}, -7 \leq x < 6\}$$

Statement I is correct.

**Statement II**

$$(A \cup B) = \{x \in \mathbb{R}, -5 < x \leq 4\}$$

Statement II is incorrect.

71. (B)  $f(x) = \begin{cases} \frac{1 - \cos 6x}{x^2}, & x < 0 \\ a, & x = 0 \text{ is continuous} \\ \frac{3\sqrt{x}}{\sqrt{9 + \sqrt{x}} - 3}, & x > 0 \end{cases}$

at  $x = 0$ , then

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos 6x}{x^2} = \lim_{x \rightarrow 0} \frac{3\sqrt{x}}{\sqrt{9 + \sqrt{x}} - 3} = a$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin^2 3x}{9x^2} \times 9 = \lim_{x \rightarrow 0} \frac{3\sqrt{x}(\sqrt{9 + \sqrt{x}} + 3)}{9 + \sqrt{x} - 9} = a$$

$$\Rightarrow \lim_{x \rightarrow 0} 18 \times \left(\frac{\sin 3x}{3x}\right)^2 = \lim_{x \rightarrow 0} \frac{3\sqrt{x}(\sqrt{9 + \sqrt{x}} + 3)}{\sqrt{x}} = a$$

$$\Rightarrow 18 = 3 \times (3 + 3) = a$$

$$\Rightarrow a = 18$$

72. (B) Conic

$$9x^2 - 16y^2 + 12x - 24y + 59 = 0$$

$$\Rightarrow (9x^2 + 12x + 4) - (16y^2 + 24y + 9) + 64 = 0$$

$$\Rightarrow (3x + 2)^2 - (4y + 3)^2 + 64 = 0$$

$$\Rightarrow 9 \left(x + \frac{2}{3}\right)^2 - 16 \left(y + \frac{3}{4}\right)^2 = -64$$

$$\frac{\left(x + \frac{2}{3}\right)^2}{\frac{64}{9}} - \frac{\left(y + \frac{3}{4}\right)^2}{4} = -1$$

let  $x + \frac{2}{3} = X, y + \frac{3}{4} = Y$

$$\Rightarrow \frac{X^2}{\frac{64}{9}} - \frac{Y^2}{4} = -1$$

$$a^2 = \frac{64}{9}, b^2 = 4$$

Now,  $e = \sqrt{1 + \frac{a^2}{b^2}}$

$$e = \sqrt{1 + \frac{64}{9 \times 4}} \Rightarrow e = \frac{5}{3}$$

foci(X, Y) = (0, ±be)

$$X = 0, Y = \pm be$$

$$x + \frac{2}{3} = 0, y + \frac{3}{4} = \pm 2 \times \frac{5}{3}$$

$$x = -\frac{2}{3}, y + \frac{3}{4} = \frac{10}{3} \text{ and } y + \frac{3}{4} = -\frac{10}{3}$$

$$y = \frac{31}{12} \text{ and } y = -\frac{49}{12}$$

foci are  $\left(\frac{-2}{3}, \frac{31}{12}\right)$  and  $\left(\frac{-2}{3}, \frac{-49}{12}\right)$

73. (B) Equation  $x^2 + ax + b = 0$

let roots  $\alpha, k\alpha$

$$\alpha + k\alpha = -a \Rightarrow \alpha(1 + k) = -a \quad \dots(i)$$

$$\alpha \cdot k\alpha = b \Rightarrow \alpha^2 k = b \quad \dots(ii)$$

and equation

$$x^2 + mx + n = 0$$

let roots  $\beta, k\beta$

$$\beta + k\beta = -a \Rightarrow \beta(1 + k) = -a \quad \dots(iii)$$

$$\beta \cdot k\beta = b \Rightarrow \beta^2 k = n \quad \dots(iv)$$

from eq(i) and eq(ii)

$$\frac{\alpha}{\beta} = \frac{a}{m} \quad \dots(v)$$

from eq(ii) and eq(iv)

$$\frac{\alpha^2}{\beta^2} = \frac{b}{n}$$

$$\left(\frac{\alpha}{\beta}\right)^2 = \frac{b}{n}$$

[from eq (v)]

$$\alpha^2 n = m^2 b$$

74. (B)  $S = 7 \times 7^{\frac{1}{2}} \times 7^{\frac{1}{4}} \times 7^{\frac{1}{8}} \times \dots \dots \dots \infty$   
 $S = 7^{(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \dots \dots \infty)}$   
 $S = 7^{\frac{1}{1 - \frac{1}{2}}} \Rightarrow S = 7^2 = 49$

75. (A) Given that  $P(A) = \frac{1}{3}$ ,  $P(B) = \frac{1}{4}$   
 $P(A \cup B) = \frac{4}{9}$   
 $P(A \cap B) = P(A) + P(B) - P(A \cup B)$   
 $P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{4}{9} = \frac{5}{36}$   
 Now,  $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{5}{36}}{\frac{1}{4}} = \frac{5}{9}$

76. (A)  $S = \{1, 2, 3, \dots, 29, 30\}$ ;  $n(S) = 30$   
 $X = \{4, 8, 12, 16, 20, 24, 28\}$ ;  $n(X) = 7$   
 $Y = \{2, 4, 6, 8, \dots, 28, 30\}$ ;  $n(Y) = 15$   
 1.  $P(X) = \frac{n(X)}{n(S)} = \frac{7}{30}$   
 Statement 1 is correct.  
 2.  $P(Y) = \frac{n(Y)}{n(S)} = \frac{15}{30} = \frac{1}{2}$   
 Statement 2 is incorrect.

77. (B) Let  $z = x + iy$   
 Now,  $\text{Re}(z^2 - 2i) = 4$   
 $\Rightarrow \text{Re}[(x + iy)^2 - 2i] = 4$   
 $\Rightarrow \text{Re}[x^2 - y^2 + 2xyi - 2i] = 4$   
 $\Rightarrow x^2 - y^2 = 4$   
 It is a rectangular hyperbola.

78. (C)  $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = \frac{1}{\infty} \times (-1 \text{ to } 1) = 0$   
 [ $\because -1 \leq \sin x \leq 1$ ]

79. (C)  $y = e^x (b \sin x + a \cos x)$  ... (i)  
 On differentiating both side w. r. t. 'x'  
 $\Rightarrow \frac{dy}{dx} = e^x (b \cos x - a \sin x) + e^x (b \sin x + a \cos x)$   
 $\Rightarrow \frac{dy}{dx} = e^x (b \cos x - a \sin x) + y$  ... (ii)  
 Again, differentiating  
 $\Rightarrow \frac{d^2y}{dx^2} = e^x (-b \sin x - a \cos x) + e^x (b \cos x$

$-a \sin x) + \frac{dy}{dx}$   
 $\Rightarrow \frac{d^2y}{dx^2} = -e^x (b \sin x + a \cos x) + \frac{dy}{dx} - y + \frac{dy}{dx}$   
 $\Rightarrow \frac{d^2y}{dx^2} = -y + 2 \frac{dy}{dx} - y$   
 $\Rightarrow \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$

80. (B)  $z = \frac{1+i}{1+2i} + \frac{2-2i}{3-i}$   
 $z = \frac{(1+i)(1-2i)}{(1+2i)(1-2i)} + \frac{(2-2i)(3+i)}{(3-i)(3+i)}$   
 $z = \frac{3-i}{5} + \frac{8-4i}{10} = \frac{7-3i}{5}$   
 $\bar{z} = \frac{7+3i}{5}$   
 Now,  $(z^2 - z\bar{z}) = z(z - \bar{z})$   
 $\Rightarrow (z^2 - z\bar{z}) = \frac{7-3i}{5} \left( \frac{7-3i}{5} - \frac{7+3i}{5} \right)$   
 $\Rightarrow (z^2 - z\bar{z}) = \frac{7-3i}{5} \left( \frac{-6i}{5} \right)$   
 $\Rightarrow (z^2 - z\bar{z}) = \frac{-42i + 18i^2}{25}$   
 $\Rightarrow (z^2 - z\bar{z}) = \frac{-42i - 18}{25} = -\frac{18 + 42i}{25}$

81. (B) Given that  $\vec{a} = 2\hat{i} + 6\hat{j} - 3\hat{k}$  and  
 $\vec{b} = 8\hat{i} - 12\hat{j} + 9\hat{k}$

Projection of  $\vec{a}$  on  $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$   
 $= \frac{|2 \times 8 + 6 \times (-12) - 3 \times 9|}{\sqrt{8^2 + (-12)^2 + 9^2}}$   
 $= \frac{|16 - 72 - 27|}{17} = \frac{83}{17}$

82. (C)

83. (B) 
$$\begin{bmatrix} 1 & 2 & -4 \\ 2 & -1 & 0 \\ -3 & -4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 7 \\ 16 \end{bmatrix}$$

Using elementary method

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 2 & -1 & 0 & 7 \\ -3 & -4 & -2 & 16 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 + 3R_1$$

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 2 & -14 & 64 \end{array} \right]$$

$$R_3 \rightarrow R_3 + \frac{2}{5}R_2$$

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 16 \\ 0 & -5 & 8 & -25 \\ 0 & 0 & \frac{-54}{5} & 54 \end{array} \right]$$

Now,  $x + 2y - 4z = 16$  .....(i)

$5y + 8z = 25$  .....(ii)

$-\frac{54}{5}z = 54$  .....(iii)

On solving

$x = 2, y = -3, z = -5$

Hence 
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -5 \end{bmatrix}$$

84. (C)  $(\log_2 x)(\log_3 9) = \log_5 y$

$(\log_2 x)(\log_3 3^2) = \log_5 y$

$2(\log_2 x)(\log_3 3) = \log_5 y$

$(\log_2 x^2) = \log_5 y$  or  $(\log_2 x)(\log_3 3) = \frac{1}{2} \log_5 y$

$x^2 = 2$  and  $y = 5$  or  $(\log_2 x) = \log_5 \sqrt{y}$

$x = \sqrt{2}$  and  $y = 5$  or  $x = 2$  and  $\sqrt{y} = 5$

$x = \sqrt{2}$  and  $y = 5$  or  $x = 2$  and  $y = 25$

85. (A)  $x = a \sec \alpha \cdot \cos \beta, y = b \tan \alpha, z = c \sec \alpha \cdot \sin \beta$

Now,  $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}$

$\Rightarrow \frac{a^2 \sec^2 \alpha \cdot \cos^2 \beta}{a^2} - \frac{b^2 \tan^2 \alpha}{b^2} + \frac{c^2 \sec^2 \alpha \cdot \sin^2 \beta}{c^2}$

$\Rightarrow \sec^2 \alpha \cdot \cos^2 \beta - \tan^2 \alpha + \sec^2 \alpha \cdot \sin^2 \beta$

$\Rightarrow \sec^2 \alpha \cdot \cos^2 \beta + \sec^2 \alpha \cdot \sin^2 \beta - \tan^2 \alpha$

$\Rightarrow \sec^2 \alpha (\cos^2 \beta + \sin^2 \beta) - \tan^2 \alpha$

$[\because \sin^2 A + \cos^2 A = 1]$

$\Rightarrow \sec^2 \alpha - \tan^2 \alpha$   $[\because \sec^2 A - \tan^2 A = 1]$

86. (B) 
$$\frac{1 + \cos(B-C)\cos A}{1 + \cos(B-A)\cos C}$$

$\Rightarrow \frac{1 + \cos(B-C) \cdot \cos[180 - (B+C)]}{1 + \cos(B-A) \cdot \cos[180 - (B+A)]}$

$\Rightarrow \frac{1 - \cos(B-C) \cdot \cos(B+C)}{1 - \cos(B-A) \cdot \cos(B+A)}$

We know that

$\cos(\alpha + \beta) \cdot \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$

$\Rightarrow \frac{1 - \cos^2 B + \sin^2 C}{1 - \cos^2 B + \sin^2 A}$

$\Rightarrow \frac{\sin^2 B + \sin^2 C}{\sin^2 B + \sin^2 A}$

$\Rightarrow \frac{b^2 + c^2}{b^2 + a^2}$  [ by Sine Rule ]

87. (C) Equations

$x - y + 2z = 4$

$2x + y - 3z = 5$

$x + y + z = 13$

$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 5 \\ 13 \end{bmatrix}$

Using elementary method

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & -3 & 5 \\ 1 & 1 & -1 & 13 \end{array} \right]$$

$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - R_1$

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & -3 & 5 \\ 1 & 1 & -1 & 13 \end{array} \right]$$

$R_3 \rightarrow R_3 - \frac{2}{3}R_2$

$$[A/B] = \left[ \begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 3 & -7 & -3 \\ 0 & 0 & \frac{11}{3} & 11 \end{array} \right]$$

$x - y + 2z = 4$  .....(i)

$3y - 7z = -3$  .....(ii)

$\frac{11}{3}z = 11$  .....(iii)

On solving

$x = 4, y = 6, z = 3$

88. (C)  $f(x) = ax + \frac{\sqrt{a}}{x}$

On differentiating both side w.r.t. 'x'

$$f'(x) = a - \frac{\sqrt{a}}{x^2}$$

$$f'(a) = a - \frac{\sqrt{a}}{a^2}$$

$$f(a) = a - \frac{1}{a^{3/2}} = \frac{a^{5/2} - 1}{a^{3/2}}$$

89. (C)  $I = \int \frac{x^4 - 1}{x^2 \sqrt{x^4 + x^2 + 1}} dx$

$$I = \int \frac{x^2 - \frac{1}{x^2}}{x \sqrt{x^2 + 1 + \frac{1}{x^2}}} dx$$

$$I = \int \frac{x - \frac{1}{x^3}}{\sqrt{x^2 + \frac{1}{x^2} + 1}} dx$$

Let  $x^2 + \frac{1}{x^2} + 1 = t$

$$\left(2x - \frac{2}{x^3}\right) dx = dt$$

$$\left(x - \frac{1}{x^3}\right) dx = \frac{1}{2} dt$$

$$I = \frac{1}{2} \int \frac{1}{\sqrt{t}} dt$$

$$I = \frac{1}{2} \frac{t^{1/2}}{1/2} + C$$

$$I = \sqrt{t} + C \Rightarrow I = \sqrt{x^2 + \frac{1}{x^2} + 1} + C$$

90. (A)  $I_n = \int_0^{\pi/4} \tan^n x dx$

$$I_n = \int_0^{\pi/4} (\tan x)^{n-2} \tan^2 x dx$$

$$I_n = \int_0^{\pi/4} (\tan x)^{n-2} (\sec^2 x - 1) dx$$

$$I_n = \int_0^{\pi/4} (\tan x)^{n-2} \sec^2 x dx - \int_0^{\pi/4} (\tan x)^{n-2} dx$$

$$I_n = \left[ \frac{(\tan x)^{n-2+1}}{n-2+1} \right]_0^{\pi/4} - I_{n-2}$$

$$I_n + I_{n-2} = \left[ \frac{(1)^{n-1}}{n-1} - 0 \right]$$

$$I_n + I_{n-2} = \frac{1}{n-1}$$

91. (D) Given that

$$|\vec{a}| = \frac{7}{4}, |\vec{b}| = 8 \text{ and } \vec{a} \times \vec{b} = 5\hat{i} - 3\hat{j}$$

$$+ 8\hat{k}$$

Now,

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta$$

$$\Rightarrow \sqrt{5^2 + (-3)^2 + 8^2} = \frac{7}{4} \times 8 \times \sin\theta$$

$$\Rightarrow \sqrt{98} = 7 \times 2 \sin\theta$$

$$\Rightarrow 7\sqrt{2} = 7 \times 2 \sin\theta$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$$

92. (B)

age	x	f	f × x
20-30	25	5	125
30-40	35	6	210
40-50	45	8	360
50-60	55	9	495
60-70	65	2	130
		$\sum f = 30$	1320

$$\text{Mean age} = \frac{\sum f \times x}{\sum f}$$

$$\text{Mean age} = \frac{1320}{30} = 44$$

93. (C) In the expansion of  $\left(\frac{x}{5} - \frac{5}{x}\right)^{10}$

$$\text{Middle term} = \left(\frac{10}{2} + 1\right)^{\text{th}} = 6^{\text{th}}$$

$$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{x}{5}\right)^5 \left(-\frac{5}{x}\right)^5 = -252$$

94. (A) Given that  $f(x) = \frac{1}{\sqrt{\log_e(43+x-x^2)}}$

$$\log_e(43+x-x^2) > 0$$

$$\Rightarrow 43+x-x^2 > 1$$

$$\Rightarrow x^2 - x - 42 < 0$$

$$\Rightarrow (x-7)(x+6) < 0$$

$$+ \quad \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \quad +$$

$$x \in (-6, 7)$$

95. (B)  $y = x^y$   
 taking log both side  
 $\Rightarrow \log y = y \log x$   
 On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \left( \frac{1}{y} - \log x \right) \frac{dy}{dx} = \frac{y + x \log x}{x}$$

$$\Rightarrow \frac{1 - y \log x}{y} \cdot \frac{dy}{dx} = \frac{y + x \log x}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(y + x \log x)}{x(1 - y \log x)}$$

96. (A) We know that

$$\sinh A + \sinh B = 2 \sinh \frac{A+B}{2} \cdot \cosh \frac{A-B}{2}$$

$$\text{and } \cosh A - \cosh B = 2 \sinh \frac{A+B}{2} \cdot \sinh \frac{A-B}{2}$$

Now,  $\frac{\sinh x + \sinh y}{\cosh x - \cosh y}$

$$\Rightarrow \frac{2 \sinh \frac{x+y}{2} \cdot \cosh \frac{x-y}{2}}{2 \sinh \frac{x+y}{2} \cdot \sinh \frac{x-y}{2}} = \coth \frac{x-y}{2}$$

97. (C)  $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$

We know that

$$\lim_{x \rightarrow a} [f(x)]^{g(x)} = e^{\lim_{x \rightarrow a} g(x)[f(x)-1]}, \text{ if } [1^\infty] \text{ form}$$

$$\Rightarrow e^{\lim_{x \rightarrow \frac{\pi}{2}} \tan x (\sin x - 1)}$$

$$\Rightarrow e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x (\sin x - 1)}{\cos x}} \quad \left[ \frac{0}{0} \right] \text{ form}$$

by L - Hospital's Rule

$$\Rightarrow e^{\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x \cdot \cos x + (\sin x - 1) \cos x}{-\sin x}}$$

$$\Rightarrow e^{\frac{\sin \frac{\pi}{2} \cdot \cos \frac{\pi}{2} + \left( \sin \frac{\pi}{2} - 1 \right) \cos \frac{\pi}{2}}{\sin \frac{\pi}{2}}}$$

$$\Rightarrow e^0 = 1$$

98. (D) Three-digit odd numbers  
 101, 103, 105, 107.....999

Now,  $l = a + (n - 1)d$   
 $999 = 101 + (n - 1) \times 2$   
 $898 = (n - 1) \times 2 \Rightarrow n = 450$

$$\text{Sum} = \frac{n}{2} [2a + (n - 1)d]$$

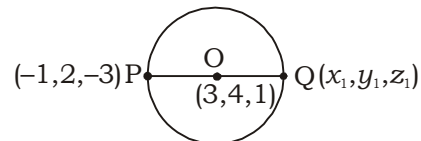
$$= \frac{450}{2} [2 \times 101 + (450 - 1) \times 2]$$

$$= 450 [101 + 449]$$

$$= 450 \times 550 = 247500$$

99. (D)

100. (D) equation of sphere  
 $x^2 + y^2 + z^2 - 6x - 8y - 2z + 6 = 0$   
 $u = -3, u = -4, w = -1$   
 centre = (3, 4, 1)  
 Let co-ordinate of Q =  $(x_1, y_1, z_1)$



co-ordinate of mid-point of line PQ

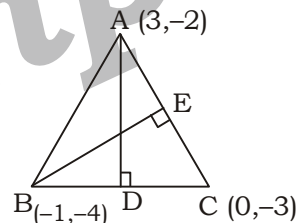
$$3 = \frac{x_1 - 1}{2} \Rightarrow x_1 = 7$$

$$4 = \frac{y_1 + 2}{2} \Rightarrow y_1 = 6$$

$$1 = \frac{z_1 - 3}{2} \Rightarrow z_1 = 5$$

Hence end point of its diameter = (7, 6, 5)

101. (C)



$$\text{slope of line BC} = \frac{-3 + 4}{0 + 1} = 1$$

$$\text{slope of line AD} = \frac{-1}{1} = -1$$

equation of line AD

$$y + 2 = -1(x - 3)$$

$$x + y = 1 \quad \dots(i)$$

$$\text{slope of line AC} = \frac{-3 + 2}{0 - 3} = \frac{-1}{-3} = \frac{1}{3}$$

$$\text{slope of line BE} = \frac{-1}{1/3} = -3$$

equation of line BE

$$y + 4 = -3(x + 1)$$

$$3x + y = -7 \quad \dots(ii)$$

from eq (i) and eq(ii)

$$x = -4, y = 5$$

Intersection point of line BE and AD is orthocenter.

Hence orthocentre = (-4, 5)

102. (A)

103. (B) Point  $(2k-1, k^r)$ Now,  $2k-1 = k^r$  $\Rightarrow k = 1$  for  $r \in \mathbb{R}$ 104. (B)  $8 \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}$ 

$$\Rightarrow \frac{4 \times 2 \sin \frac{2\pi}{15} \cdot \cos \frac{2\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$\Rightarrow \frac{4 \times \sin \frac{4\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}}{\sin \frac{2\pi}{15}}$$

$$\Rightarrow \frac{2}{\sin \frac{2\pi}{15}} \times 2 \sin \frac{4\pi}{15} \cdot \cos \frac{4\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}$$

$$\Rightarrow \frac{2}{\sin \frac{2\pi}{15}} \times \sin \frac{8\pi}{15} \cdot \cos \frac{8\pi}{15} \cdot \cos \frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{\sin \frac{2\pi}{15}} \times \sin \frac{16\pi}{15} \cdot \cos \frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2 \sin \frac{2\pi}{15}} \times 2 \sin \frac{16\pi}{15} \cdot \cos \frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2 \sin \frac{2\pi}{15}} \cdot \sin \frac{32\pi}{15}$$

$$\Rightarrow \frac{1}{2 \sin \frac{2\pi}{15}} \cdot \sin \left( 2\pi + \frac{2\pi}{15} \right)$$

$$\Rightarrow \frac{1}{2 \sin \frac{2\pi}{15}} \cdot \sin \frac{2\pi}{15}$$

$$= \frac{1}{2}$$

105. (B)  ${}^{23}C_4 + \sum_{r=1}^4 {}^{22+r}C_3$ 

$$\Rightarrow {}^{23}C_4 + {}^{23}C_3 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$$

We know that

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

$$\Rightarrow {}^{24}C_4 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$$

$$\Rightarrow {}^{25}C_4 + {}^{25}C_3 + {}^{26}C_3$$

$$\Rightarrow {}^{26}C_4 + {}^{26}C_3 = {}^{27}C_4$$

106. (B)  $\cos(x-iy) = A + iB$ 

$$\Rightarrow \cos x \cdot \cos iy + \sin x \cdot \sin iy = A + iB$$

We know that

$$\cos iA = \cosh A \text{ and } \sin iA = i \sinh A$$

$$\Rightarrow \cos x \cdot \cosh y + i \sin x \cdot \sinh y = A + iB$$

On comparing

$$A = \cos x \cdot \cosh y, B = \sin x \cdot \sinh y$$

107. (A) line  $6x - 8y = 4$  ... (i)

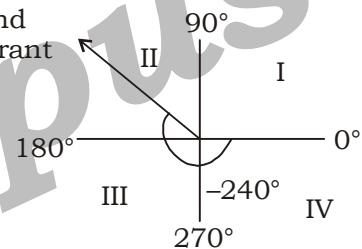
$$\text{and } 16y - 12x = 9$$

$$\Rightarrow 6x - 8y = \frac{-9}{2} \quad \dots (ii)$$

$$\text{Distance} = \frac{4 + \frac{9}{2}}{\sqrt{6^2 + (-8)^2}} = \frac{17}{2 \times 10}$$

$$= \frac{17}{20}$$

108. (B) Second quadrant



109. (A)

$$110. (C) A = \begin{bmatrix} 3 & 0 & 3 \\ 5 & -8 & 2 \\ -2 & 3 & -1 \end{bmatrix}$$

$$|A| = 3(8-6) + 3(15-16) = 6-3=3$$

Now,  $(\text{Adj}A) = |A|^{n-2}A$  where  $n$  is order.

$$\text{adj}(\text{adj}A) = |A|^{3-2}A$$

$$= |A| \cdot A = 3A$$

$$\det(\text{adj}(\text{adj}A)) = |3A| = 3^3 |A| = 3 \times 3 = 9$$

111. (B)

112. (B) We Know that

$$\omega^2 = \frac{-1-i\sqrt{3}}{2}$$

$$\text{Now, } (-1-i\sqrt{3})^{51} = 2^{51} \left( \frac{-1-i\sqrt{3}}{2} \right)^{51}$$

$$= 2^{51} (\omega^2)^{51}$$

$$= 2^{51} \times \omega^{3 \times 51}$$

$$= 2^{51} \quad [\because \omega^3 = 1]$$

113. (C) equation  $bx^2 + cx + a = 0$

$$\alpha + \beta = \frac{-c}{b}$$

$$\alpha\beta = \frac{a}{b}$$

Now,  $(b\alpha - a)(b\beta - a)$

$$\Rightarrow b^2\alpha\beta - ab\beta - ab\alpha + a^2$$

$$\Rightarrow b^2 \times \alpha\beta - ab(\alpha + \beta) + a^2$$

$$\Rightarrow b^2 \times \frac{a}{b} - ab \times \left(\frac{-c}{b}\right) + a^2$$

$$\Rightarrow ab + ac + a^2$$

$$\Rightarrow a(a + b + c)$$

114. (B)

115. (B)  $dy = y \tan x \, dx$

$$\Rightarrow \frac{dy}{y} = \tan x \, dx$$

On integrating both side

$$\log y = \log \sec x + \log c$$

$$y = c \cdot \sec x$$

its passes through the point (0,1)

$$1 = c \cdot \sec 0$$

$$\Rightarrow c = 1$$

from eq(i)

$$y = \sec x$$

116. (D)  $\cot(-1740) = -\cot(1740)$

$$= -\cot(360 \times 5 - 60)$$

$$= \cot 60 = \frac{1}{\sqrt{3}}$$

117. (D) 
$$\begin{vmatrix} a^2 & b+c & a \\ b^2 & c+a & b \\ c^2 & a+b & c \end{vmatrix}$$

$$C_2 \rightarrow C_2 + C_3$$

$$\Rightarrow \begin{vmatrix} a^2 & a+b+c & a \\ b^2 & a+b+c & b \\ c^2 & a+b+c & c \end{vmatrix}$$

$$\Rightarrow (a+b+c) \begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow (a+b+c) \begin{vmatrix} a^2 & 1 & a \\ b^2 - a^2 & 0 & b - a \\ c^2 - a^2 & 0 & c - a \end{vmatrix}$$

$$\Rightarrow (a+b+c)(b-a)(c-a) \begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ c+a & 0 & 1 \end{vmatrix}$$

$$R_3 \rightarrow R_3 - R_2$$

$$\Rightarrow (a+b+c)(b-a)(c-a) \begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ c-b & 0 & 0 \end{vmatrix}$$

$$\Rightarrow (a+b+c)(b-a)(c-a)(c-b) \begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$\Rightarrow (a+b+c)(a-b)(b-c)(c-a)[-1(-1)]$$

$$= (a+b+c)(a-b)(b-c)(c-a)$$

118. (B)  $\tan \theta = \frac{-4}{3}$

$\Rightarrow \theta \in \text{II quad or IV quad}$

$$\therefore 0 < \sin \theta < 1 \text{ or } -1 < \sin \theta < 0$$

$$\therefore \sin \theta \text{ may be } \frac{4}{5} \text{ or } -\frac{4}{5}$$

119. (B) The given equation is

$$\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$$

$$\Rightarrow 2 \sin 2x \cos x - 3 \sin 2x = 2 \cos 2x \cos x - 3 \cos 2x$$

$$\Rightarrow \sin 2x(2 \cos x - 3) = \cos 2x(2 \cos x - 3)$$

$$\Rightarrow \sin 2x = \cos 2x \quad \left( a \cos x \neq \frac{3}{2} \right)$$

$$\Rightarrow \tan 2x = 1$$

$$\Rightarrow 2x = n\pi + \frac{\pi}{4}$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$$

120. (B)  $\frac{\cos^2 6A}{\cos^2 2A} - \frac{\sin^2 6A}{\sin^2 2A} = \left( \frac{\cos 6A}{\cos 2A} \right)^2 - \left( \frac{\sin 6A}{\sin 2A} \right)^2$

$$\Rightarrow \left( \frac{4 \cos^3 2A - 3 \cos 2A}{\cos 2A} \right)^2 - \left( \frac{3 \sin 2A - 4 \sin^3 2A}{\sin 2A} \right)^2$$

$$\Rightarrow (4 \cos^2 2A - 3)^2 - (3 - 4 \sin^2 2A)^2$$

$$\Rightarrow 16 \cos^4 2A + 9 - 24 \cos^2 2A - 9 - 16 \sin^4 2A + 24 \sin^2 2A$$

$$\Rightarrow 16(\cos^4 2A - \sin^4 2A) - 24(\cos^2 2A - \sin^2 2A)$$

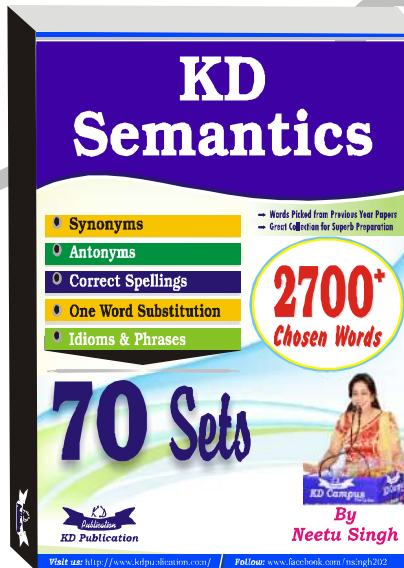
$$\Rightarrow 16(\cos^2 2A - \sin^2 2A) - 24(\cos^2 2A - \sin^2 2A)$$

$$\Rightarrow (\cos^2 2A - \sin^2 2A)(16 - 24) = -8 \cos 4A$$



**NDA (MATHS) MOCK TEST - 180 (Answer Key)**

- |         |         |         |         |          |          |
|---------|---------|---------|---------|----------|----------|
| 1. (D)  | 21. (D) | 41. (C) | 61. (D) | 81. (B)  | 101. (C) |
| 2. (B)  | 22. (C) | 42. (A) | 62. (A) | 82. (C)  | 102. (A) |
| 3. (D)  | 23. (A) | 43. (A) | 63. (C) | 83. (B)  | 103. (B) |
| 4. (A)  | 24. (B) | 44. (A) | 64. (C) | 84. (C)  | 104. (B) |
| 5. (C)  | 25. (A) | 45. (A) | 65. (C) | 85. (A)  | 105. (B) |
| 6. (B)  | 26. (C) | 46. (B) | 66. (A) | 86. (B)  | 106. (B) |
| 7. (C)  | 27. (D) | 47. (C) | 67. (B) | 87. (C)  | 107. (A) |
| 8. (A)  | 28. (A) | 48. (D) | 68. (C) | 88. (C)  | 108. (B) |
| 9. (D)  | 29. (D) | 49. (A) | 69. (A) | 89. (C)  | 109. (A) |
| 10. (A) | 30. (A) | 50. (B) | 70. (A) | 90. (A)  | 110. (C) |
| 11. (A) | 31. (C) | 51. (C) | 71. (B) | 91. (D)  | 111. (B) |
| 12. (B) | 32. (A) | 52. (A) | 72. (B) | 92. (B)  | 112. (B) |
| 13. (B) | 33. (C) | 53. (D) | 73. (B) | 93. (C)  | 113. (C) |
| 14. (D) | 34. (B) | 54. (C) | 74. (B) | 94. (A)  | 114. (B) |
| 15. (A) | 35. (B) | 55. (A) | 75. (A) | 95. (B)  | 115. (B) |
| 16. (C) | 36. (C) | 56. (A) | 76. (A) | 96. (A)  | 116. (D) |
| 17. (D) | 37. (D) | 57. (C) | 77. (B) | 97. (C)  | 117. (D) |
| 18. (B) | 38. (C) | 58. (B) | 78. (C) | 98. (D)  | 118. (B) |
| 19. (A) | 39. (*) | 59. (A) | 79. (C) | 99. (D)  | 119. (B) |
| 20. (C) | 40. (B) | 60. (A) | 80. (B) | 100. (D) | 120. (B) |



**Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003**

**Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock**

**Note:- If you face any problem regarding result or marks scored, please contact 9313111777**