HSSC MOCK TEST – 171 (SOLUTION)

6. (C) Cost of x meter wire = ₹d

Cost of 1 meter wire = $\frac{d}{x}$

Cost of y meter wire = ₹$\frac{d}{x}$y

7. (A) $\frac{3}{4}$ = 7.5 and $\frac{5}{6}$ = 0.833

$\frac{4}{5}$ = .8

9. (C) $f(x) = \frac{1}{\sqrt{29-x^2}} \Rightarrow f'(x) = \frac{x}{(29-x^2)^{3/2}}$

Now, $\lim_{x\to 2} \frac{f(2)-f(x)}{x^3-8}$ $\left[\frac{0}{0}\right]$form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 2} \frac{-f'(x)}{3x^2}$

$\Rightarrow \lim_{x\to 2} \frac{\frac{-x}{(29-x^2)^{3/2}}}{3x^2}$

$\Rightarrow \frac{\frac{-2}{(29-4)^{3/2}}}{3\times 4}$

$\Rightarrow \frac{-2}{12\times 125} = \frac{-1}{750}$

10. (D) According to question,
a + b = 32
and, ab = 256
a = 16 and b = 16

19. (A) $\frac{1}{3} + \frac{1}{2} + \frac{1}{x} = 4$

$\Rightarrow \frac{2x+3x+6}{6} = 4$

$\Rightarrow 5x = 18$

$\Rightarrow x = \frac{18}{5}$

26. (D) ATQ,
The distance from the origin (0, 0, 0) to line 2x – 3y + 4x = 6
= 2x – 3y + 4x – 6

$= \frac{|2\times 0-3\times 0+4\times 0-6|}{\sqrt{2^2+3^2+4^2}} = \frac{6}{\sqrt{29}}$

29. (A) $\frac{\cos(x+y)}{\cos(x-y)} = \frac{a-b}{a+b}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\cos(x+y)+\cos(x-y)}{\cos(x+y)-\cos(x-y)} = \frac{a-b+a+b}{a-b-a-b}$

$\Rightarrow \frac{2\cos\left(\frac{x+y+x-y}{2}\right).\cos\left(\frac{x+y-x+y}{2}\right)}{2\sin\left(\frac{x+y+x-y}{2}\right).\sin\left(\frac{x-y-x-y}{2}\right)} = \frac{2a}{-2b}$

$\Rightarrow \frac{2\cos x.\cos y}{-2\sin x.\sin y} = \frac{-a}{b}$

$\Rightarrow \cot x.\cot y = \frac{a}{b}$

$\Rightarrow \frac{\cot y}{\tan x} = \frac{a}{b}$

32. (B) Series = $1.3^2 + 3.5^2 + 5.7^2 + \ldots\ldots$
$T_n = (2n-1)(2n+1)^2$
$T_n = 8n^3 + 4n^2 - 2n - 1$
$S_n = \sum T_n$
$S_n = 8\sum n^3 + 4\sum n^2 - 2\sum n - \sum 1$

$S_n = 8\times\frac{n^2(n+1)^2}{4} + 4\times\frac{n}{6}(n+1)(2n+1) -$

$2\times\frac{n(n+1)}{2} - n$

$S_n = 2n^2(n+1)^2 + \frac{2}{3}n(n+1)(2n+1) - n(n+1) - n$

$S_n = \frac{n}{3}[6n(n+1)^2 + 2(n+1)(2n+1) - 3(n+1) - 3]$

$S_n = \frac{n}{3}[6n(n^2+1+2n) + 2(2n^2+3n+1) - 3n - 3 - 3]$

$S_n = \frac{n}{3}[6n^3 + 16n^2 + 9n - 4]$

34. (D) $x = 3 + 2\sqrt{2}$

$x + \frac{1}{x} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2}$

$\Rightarrow x + \frac{1}{x} = 6$

$\Rightarrow \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 6 - 2$

$\Rightarrow \sqrt{x} - \frac{1}{\sqrt{x}} = 2$

35. (A) In the expansion of $\left(x^3-\frac{2}{x}\right)^{11}$

$T_{r+1} = {}^{11}C_r (x^3)^{11-r}\left(\frac{-2}{x}\right)^r$

$= {}^{11}C_r\, x^{33-4r}(-2)^r$

Hence $33 - 4r = -3 \Rightarrow r = 9$

The coefficient of $x^{-3} = {}^{11}C_9(-2)^9$

$= \frac{11!}{9!2!}\times 2^9$

Again, $33 - 4r = 5 \Rightarrow r = 7$

The coefficient of $x^5 = {}^{11}C_7(-2)^7$

$= -\frac{11!}{7!4!}\times 2^7$

The required ratio $= \frac{-\frac{11!}{9!2!}\times 2^9}{-\frac{11!}{7!4!}\times 2^7}$

$= \frac{2}{3} = 2:3$

37. (B) $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$ ...(i)

On multiply by x

$\Rightarrow x(1+x)^n = C_0x + C_1x^2 + C_2x^3 + \ldots + C_nx^{n+1}$

On differentiating both side w.r.t 'x'

$\Rightarrow x \times n(1+x)^{n-1} + (1+x)^n.1 = C_0 + 2C_1x + 3C_2x^2 + \ldots + (n+1)C_nx^n$

$\Rightarrow nx(1+x)^{n-1} + (1+x)^n = C_0 + 2C_1x + 3C_2x^2 + \ldots + (n+1)C_nx^n$ ...(ii)

$x \to \frac{1}{x}$ in eq(i)

$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots + \frac{C_n}{x^{n+1}}$ ...(iii)

from eq(ii) and eq(iii)

coefficient of x^0 in $\left(1+\frac{1}{x}\right)^n$ $[nx(1+x)^{n+1} + (1+x)^n]$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow$ coefficient of x^0 in

$\left[\frac{(1+x)^n}{x^n}\times nx(1+x)^{n-1} + \frac{(1+x)^n}{x^n}\times(1+x)^n\right]$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow$ coefficient of x^{n-1} in $[n(1+x)^{2n-1}]$ + coefficient of x^n in $(1+x)^{2n} = C_0^2 + 2C_1^2 + 3C_2^2 + \ldots\ldots + (n+1)C_n^2$

$\Rightarrow n \times {}^{2n-1}C_{n-1} + {}^{2n}C_n = C_0^2 + 2C_1^2 + \ldots + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{2n!}{n!n!} = C_0^2 + 2C_1^2 \ldots + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{2n(2n-1)!}{n(n-1)!n!} = C_0^2 + 2C_1^2 + \ldots$

$\ldots + (n+1)C_n^2$

$\Rightarrow \frac{(2n-1)!}{(n-1)!n!}[n+2] = C_0^2 + 2C_1^2 + \ldots + (n+1)C_n^2$

Hence $C_0^2 + 2C_1^2 + \ldots\ldots + (n+1)C_n^2$

$= (n+2)\,{}^{2n-1}C_{n-1}$

39. (B) **Statement I**

Prime numbers

2, 3, 5, 7, 11, 13, 17, 19, 23

The required sum $= 2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 + 23^2$

$= 4 + 9 + 25 + 49 + 121 + 169 + 289 + 361 + 529$

$= 1556$

Statement I is incorrect.

Statement II

Odd natural numbers

1, 3, 5, 7, 9, 11, 13

The required sum $= 1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3$

$= 1 + 27 + 125 + 343 + 729 + 1331 + 2197$

$= 4753$

Statements II is correct.

40. (C) Ellipse $\frac{x^2}{9}+\frac{y^2}{4} = 1$

$a^2 = 9 \Rightarrow a = 3$, $b^2 = 4 \Rightarrow b = 2$

The sum of the focal distance

= length of major- axis

$= 2a = 2 \times 3 = 6$

45. (D) Numbers = 10, 15, 20, 25 and 30

Required average $= \frac{10+15+20+25+30}{5}$

$= 20$

53. (C) $(1-y^2)\frac{dx}{dy} + yx = ay$

$= \frac{dx}{dy} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

I.F $= \int_e \frac{y}{1-y^2}dy$

$= e^{\frac{-1}{2}\ln(1-y^2)}$

$= \frac{1}{\sqrt{1-y^2}}$

56. (D) 138, 161, 185, 210, **236**
+23 +24 +25 +26

63. (D) $f(x) = x^2 - 6x + 3$
$f'(x) = 2x - 6$
For maximum $2x - 6$
$2x - 6 \geq 0$
$x \geq 3$
$x \in [3, \infty)$

70. (A) According to question,
$x^2 + (x + 2)^2 + (x + 4)^2 = 200$
$\Rightarrow x^2 + x^2 + 4 + 4x + x^2 + 16 + 8x = 200$
$\Rightarrow 3x^2 + 12x + 20 = 200$
$\Rightarrow x^2 + 4x - 60 = 0$
$\Rightarrow x^2 + 10x - 6x - 60 = 0$
$\Rightarrow x(x + 10) - 6(x + 10) = 0$
$\Rightarrow x = 6$
$\therefore$ Required numbers 6, 8 and 10.

HSSC MOCK TEST – 171 (ANSWER KEY)

1.	(C)	11.	(C)	21.	(B)	31.	(*)	41.	C)	51.	(D)	61.	(D)	71.	(C)	81.	(D)
2.	(A)	12.	(D)	22.	(A)	32.	(B)	42.	B)	52.	(D)	62.	(C)	72.	(C)	82.	(A)
3.	(A)	13.	(A)	23.	(A)	33.	(D)	43.	C)	53.	(C)	63.	(D)	73.	(D)	83.	(D)
4.	(B)	14.	(B)	24.	(B)	34.	(D)	44.	D)	54.	(C)	64.	(A)	74.	(A)	84.	(C)
5.	(B)	15.	(B)	25.	(D)	35.	(A)	45.	D)	55.	(D)	65.	(B)	75.	(B)	85.	(B)
6.	(C)	16.	(B)	26.	(D)	36.	(B)	46.	D)	56.	(D)	66.	(C)	76.	(B)	86.	(C)
7.	(A)	17.	(B)	27.	(A)	37.	(B)	47.	D)	57.	(A)	67.	(D)	77.	(A)	87.	(B)
8.	(B)	18.	(B)	28.	(A)	38.	(B)	48.	C)	58.	(A)	68.	(C)	78.	(A)	88.	(C)
9.	(C)	19.	(A)	29.	(A)	39.	(B)	49.	A)	59.	(C)	69.	(A)	79.	(D)	89.	(A)
10.	(D)	20.	(D)	30.	(A)	40.	(C)	50.	B)	60.	(C)	70.	(A)	80.	(C)	90.	(B)