# NDA MATHS MOCK TEST - 182 (SOLUTION)

1. (B) $y = x^y$

taking log both side

$\Rightarrow \log y = y \log x$

On differentiating both side w.r.t.'$x$'

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y \times \frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y + x\log x}{x}$$

$$\Rightarrow \frac{1 - y\log x}{y}.\frac{dy}{dx} = \frac{y + x\log x}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(y + x\log x)}{x(1 - y\log x)}$$

2. (A) We know that

$$\sinh A + \sinh B = 2\sinh\frac{A+B}{2}.\cosh\frac{A-B}{2}$$

and $\cosh A - \cosh B = 2\sinh\frac{A+B}{2}.\sinh\frac{A-B}{2}$

Now, $\frac{\sinh x + \sinh y}{\cosh x - \cosh y}$

$$\Rightarrow \frac{2\sinh\frac{x+y}{2}.\cosh\frac{x-y}{2}}{2\sinh\frac{x+y}{2}.\sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$$

3. (C) $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$

We know that

$\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$, if $[1^\infty]$ form

$$\Rightarrow e^{\lim_{x\to\pi/2} \tan x(\sin x - 1)}$$

$$\Rightarrow e^{\lim_{x\to\pi/2}\frac{\sin x(\sin x - 1)}{\cos x}} \quad \left[\frac{0}{0}\right] \text{form}$$

by L - Hospital's Rule

$$\Rightarrow e^{\lim_{x\to\pi/2}\frac{\sin x.\cos x + (\sin x - 1)\cos x}{-\sin x}}$$

$$\Rightarrow e^{\frac{\sin\frac{\pi}{2}.\cos\frac{\pi}{2} + \left(\sin\frac{\pi}{2} - 1\right)\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}}$$

$\Rightarrow e^0 = 1$

4. (C) 1 0 1 0 1

$1\times2^0 = 1$
$0\times2^1 = 0$
$1\times2^2 = 4$
$0\times2^3 = 0$
$1\times2^4 = 16$
21

0 . 1 1

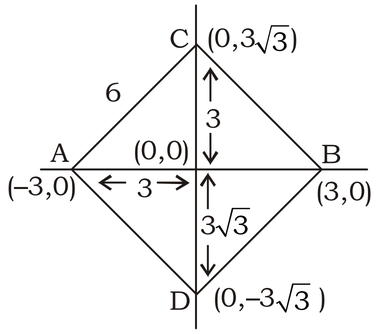
$\frac{1}{2} = 1\times2^{-1}$

$\frac{1}{4} = 1\times2^{-2}$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$

Hence $(10101.11)_2 = (21.75)_{10}$

5. (C)




Hence third vertex of an equilateral triangle = $\left(0,\pm3\sqrt{3}\right)$

6. (D)

7. (A) Given that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{7}$

$$P(\overline{A}) = \frac{2}{3},\ P(\overline{B}) = \frac{4}{7}$$

The Probability = $P\left(A\cap\overline{B}\right) + P\left(\overline{A}\cap B\right)$

$= P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$= \frac{1}{3}\times\frac{4}{7} + \frac{2}{3}\times\frac{3}{7} = \frac{10}{21}$

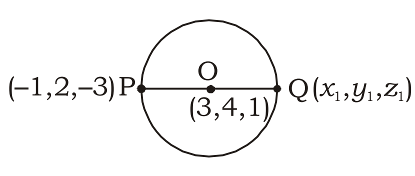
8. (D) equation of sphere

$x^2 + y^2 + z^2 - 6x - 8y - 2z + 6 = 0$

$u = -3,\ u = -4,\ w = -1$

centre = (3, 4, 1)

Let co-ordinate of Q = $(x_1, y_1, z_1)$




co-ordinate of mid-point of line PQ

$3 = \frac{x_1 - 1}{2} \Rightarrow x_1 = 7$

$4 = \frac{y_1 + 2}{2} \Rightarrow y_1 = 6$

$1 = \frac{z_1 - 3}{2} \Rightarrow z_1 = 5$

Hence end point of its diameter = (7,6,5)

9. (D) Let A = $\begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

$|A| = 3(1-2) - 1(0-4) = -3 + 4 = 1$

Hence A is an elementary matrix.

10. (B) The total no. of arrangement = $\frac{7!}{2!} = \frac{7!}{2}$

No. of arrangement when N's comes together = 6!

The required Probability = $\frac{6!}{7!/2} = \frac{2}{7}$

11. (C) $f(z) = \frac{5+z^2}{1-z}$

given that $z = 1 - 2i$

$f(z) = \frac{5+(1-2i)^2}{1-1+2i}$

$f(z) = \frac{2-4i}{2i} \times \frac{i}{i}$

$f(z) = \frac{2i-4i^2}{-2}$

$f(z) = \frac{2i+4}{-2} = -2 - i$

Now, $|f(z)| = \sqrt{(-2)^2+(-1)^2}$

$|f(z)| = \sqrt{4+1} = \sqrt{5}$

12. (B) $8\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{4\times 2\sin\frac{2\pi}{15}.\cos\frac{2\pi}{15}}{\sin\frac{2\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{4\times\sin\frac{4\pi}{15}}{\sin\frac{2\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{2}{\sin\frac{2\pi}{15}}\times 2\sin\frac{4\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{2}{\sin\frac{2\pi}{15}}\times\sin\frac{8\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{\sin\frac{2\pi}{15}}\times\sin\frac{16\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}\times 2\sin\frac{16\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\frac{32\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\left(2\pi+\frac{2\pi}{15}\right)$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\frac{2\pi}{15} = \frac{1}{2}$

13. (C) Vectors $\vec{x} = a\hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{y} = -\hat{i} + b\hat{j} - \hat{k}$ and $\vec{z} = 3\hat{i} + \hat{j} + c\hat{k}$ are prependicular to eachother,

then $\vec{x}.\vec{y} = 0$

$\Rightarrow -a - 2b - 2 = 0$

$\Rightarrow a + 2b + 2 = 0$ ...(i)

$\vec{y}.\vec{z} = 0$

$\Rightarrow -3 + b - c = 0$

$\Rightarrow b - c = 3$ ...(ii)

and $\vec{x}.\vec{z} = 0$

$\Rightarrow 3a - 2 + 2c = 0$

$\Rightarrow 3a + 2c = 2$ ...(iii)

On solving eq(i), (ii) and (iii)

$a = 5,\ b = \frac{-7}{2},\ c = \frac{-13}{2}$

14. (B) Let $a + ib = \sqrt{-2+2\sqrt{35}i}$

On squaring both side

$(a^2 - b^2) + 2abi = -2 + 2\sqrt{35}\,i$

On comparing

$a^2 - b^2 = -2$ and $2ab = 2\sqrt{35}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 4 + 4 \times 35$

$\Rightarrow (a^2 + b^2)^2 = 144$

$\Rightarrow a^2 + b^2 = 12$ ...(ii)

from eq(i) and eq(ii)

$2a^2 = 10,\ 2b^2 = 14$

$a = \pm\sqrt{5},\ b = \pm\sqrt{7}$

Hence $\sqrt{-2+2\sqrt{35}i} = \pm\left(\sqrt{5}+\sqrt{7}i\right)$

15. (D) $i^{501} + i^{502} + i^{503} + i^{504} + i^{505}$

$\Rightarrow i^{501}(1 + i + i^2 + i^3 + i^4)$

$\Rightarrow i^{3\times 167}(1 + i - 1 - i + 1)$

$\Rightarrow 1 \times 1 = 1$

16. (A) We know that
$\sin ix = i \sinh y$

Now, $\sinh\left(\frac{i\pi}{3}\right) = -i \sin\left[i\left(\frac{i\pi}{3}\right)\right]$

$= -i \sin\left(\frac{-\pi}{3}\right)$

$= i \sin\frac{\pi}{3} = \frac{\sqrt{3}i}{2}$

17. (D) $\sin ix - i\cos ix = -i(\cos ix + i \sin ix)$
$= -i.e^{i(ix)} = -i.e^{-x}$

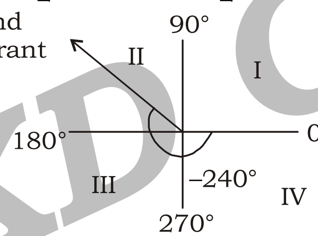
18. (B) lines $x - 3y = -4$ ...(i)
$2x - y = 7$ ...(ii)
$4x - 5y = 11$ ...(iii)
Intersecting point of line (i) and (ii) is (5, 3).
Let the equation of line which is perpendicular to the line (iii)
$5x + 4y = c$ ...(iv)
its passes through the point (5, 3)
$5 \times 5 + 4 \times 3 = c \Rightarrow c = 37$
from eq (iv)
$5x + 4y = 37$

19. (B) $b\,(c\cos A - a\cos C)$

$\Rightarrow b\left[c.\frac{b^2+c^2-a^2}{2bc} - a.\frac{a^2+b^2-c^2}{2ab}\right]$

$\Rightarrow \frac{b^2+c^2-a^2}{2} - \frac{a^2+b^2-c^2}{2} \Rightarrow c^2 - a^2$

20. (B) Second quadrant




21. (A)

22. (B) **Statement I**
45 < 53
sin45 < sin53 and cos45 > cos53
sin45 < sin53 sin45 > cos53
then cos53 < sin45 < sin53
Hence cos53 – sin53 is negative.
Statement I is incorrect.
**Statement II**
23 < 45
sin23 < sin45 and cos23 > cos45
cos 23 > sin45
then sin23 < sin45 < cos23
Hence sin 23 – cos23 is negative.
Statement II is correct.

23. (A) $I = \int_0^{\pi} \frac{\sqrt{\sin\frac{x}{2}}}{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}\,dx$ ...(i)

Prop. IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi} \frac{\sqrt{\sin\frac{\pi-x}{2}}}{\sqrt{\sin\frac{\pi-x}{2}} + \sqrt{\cos\frac{\pi-x}{2}}}$

$I = \int_0^{\pi} \frac{\sqrt{\cos\frac{x}{2}}}{\sqrt{\cos\frac{x}{2}} + \sqrt{\sin\frac{x}{2}}}\,dx$ ...(ii)

from eq(i) and eq(ii)

$2\,I = \int_0^{\pi} \frac{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}\,dx$

$2I = \int_0^{\pi} 1.dx$

$2I = [x]_0^{\pi} \Rightarrow 2I = \pi \Rightarrow I = \frac{\pi}{2}$

24. (D) $y = 2^{\sin x}$
On differentiating both side w.r.t.'$x$'

$\Rightarrow \frac{dy}{dx} = 2^{\sin x}.\log 2.\cos x$

$\Rightarrow \frac{dy}{dx} = 2^{\sin x}.\cos x.\log 2$

25. (D) R = {(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)} on a set A = {1,2,3}
<u>Reflexive:-</u>
1R1, 2R2, 3R3
R is reflexive.
<u>Symmetric:-</u>
1R2, 2R1, 1R3, 3R1
R is symmetric.
<u>Transitive:-</u>
1R2, 2R3 1R3
R is transitive.
Hence R is an equivalence relation.

26. (B) $x - iy = \begin{vmatrix} 3i & -2 & i \\ 1 & -i & 0 \\ 1 & 4i & -2i \end{vmatrix}$

Now, $x + iy = \begin{vmatrix} -3i & -2 & -i \\ 1 & i & 0 \\ 1 & -4i & 2i \end{vmatrix}$

$\Rightarrow x + iy = -3i(2i^2) + 2(2i) - i(-4i - i)$
$\Rightarrow x + iy = 6i + 4i - i(-5i)$
$\Rightarrow x + iy = 10i - 5$

27. (D) A = {1,2,3}, B = {2,3,4}, C = {3,4,5}

$(B \cap C)$ = {2,3,4} $\cap$ {3,4,5} = {3,4}

Now, $A \times (B \cap C)$ = {1,2,3} × {3,4}

The number of elements in $A \times (B \cap C)$ = 3 × 2 = 6

28. (A) | 7 | 10 | 10 | = 7 × 10 × 10 = 700

29. (C) $\begin{bmatrix} 8 & -2 & 3 \\ 4 & 0 & 5 \\ 6 & -1 & \lambda \end{bmatrix}$ is not an invertible matix, then |A| = 0

$\Rightarrow \begin{vmatrix} 8 & -2 & 3 \\ 4 & 0 & 5 \\ 6 & -1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 8 \times 5 + 2(4\lambda - 30) + 3(-4) = 0$

$\Rightarrow 40 + 8\lambda - 60 - 12 = 0$

$\Rightarrow 8\lambda = 32 \Rightarrow \lambda = 4$

30. (B) We Know that

$\omega^2 = \dfrac{-1-i\sqrt{3}}{2}$

Now, $(-1-i\sqrt{3})^{51} = 2^{51}\left(\dfrac{-1-i\sqrt{3}}{2}\right)^{51}$

$= 2^{51}(\omega^2)^{51}$

$= 2^{51} \times \omega^{3\times 34}$

$= 2^{51}$ $\quad [\because \omega^3 = 1]$

31. (C) $\begin{vmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{vmatrix} = -\sin^2\theta - \cos^2\theta$

$= -(\cos^2\theta + \sin^2\theta) = -1$

32. (C) $\begin{vmatrix} 2 & 2 & 2 \\ 2 & 2+\sin\theta & 2 \\ 2+\cos\theta & 2 & 2 \end{vmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 2 & 2 & 2 \\ 0 & \sin\theta & 0 \\ \cos\theta & 0 & 0 \end{vmatrix}$

$\Rightarrow 2 \times 0 - 2 \times 0 + 2(-\sin\theta.\cos\theta)$

$\Rightarrow -\sin 2\theta$

Maximum value of $-\sin 2\theta = 1$

33. (C) equation $bx^2 + cx + a = 0$

$\alpha + \beta = \dfrac{-c}{b}$

$\alpha.\beta = \dfrac{a}{b}$

Now, $(b\alpha - a)(b\beta - a)$

$\Rightarrow b^2\alpha\beta - ab\beta - ab\alpha + a^2$

$\Rightarrow b^2 \times \alpha.\beta - ab(\alpha + \beta) + a^2$

$\Rightarrow b^2 \times \dfrac{a}{b} - ab \times \left(\dfrac{-c}{b}\right) + a^2$

$\Rightarrow ab + ac + a^2 \Rightarrow a(a + b + c)$

34. (B)

35. (A) $I = \displaystyle\int_0^1 \frac{e^{\cot^{-1}x}}{1+x^2}\,dx$

Let $\cot^{-1}x = t$ $\quad$ when $x \to 0$, $t \to \dfrac{\pi}{2}$

$-\dfrac{1}{1+x^2}\,dx = dt$ $\quad$ $x \to 1$, $t \to \dfrac{\pi}{4}$

$\dfrac{1}{1+x^2}\,dx = -dt$

$I = \displaystyle\int_{\pi/2}^{\pi/4} -e^t\,dt$

$I = -\left[e^t\right]_{\pi/2}^{\pi/4}$

$I = -[e^{\pi/4} - e^{\pi/2}]$

$I = e^{\pi/2} - e^{\pi/4}$

36. (D) $\begin{vmatrix} a^2 & b+c & a \\ b^2 & c+a & b \\ c^2 & a+b & c \end{vmatrix}$

$C_2 \to C_2 + C_3$

$\Rightarrow \begin{vmatrix} a^2 & a+b+c & a \\ b^2 & a+b+c & b \\ c^2 & a+b+c & c \end{vmatrix}$

$\Rightarrow (a+b+c)\begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow (a+b+c)\begin{vmatrix} a^2 & 1 & a \\ b^2-a^2 & 0 & b-a \\ c^2-a^2 & 0 & c-a \end{vmatrix}$

$\Rightarrow (a+b+c)(b-a)(c-a)\begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ c+a & 0 & 1 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow (a+b+c)(b-a)(c-a)\begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ c-b & 0 & 0 \end{vmatrix}$

$\Rightarrow (a+b+c)(b-a)(c-a)(c-b)\begin{vmatrix} a^2 & 1 & a \\ b+a & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow (a+b+c)(a-b)(b-c)(c-a)[-1(-1)]$

$= (a+b+c)(a-b)(b-c)(c-a)$

37. (D)

38. (C) In the explansion of $\left(\frac{x^2}{4}-\frac{2}{x}\right)^7$

$T_{r+1} = \left(\frac{x^2}{4}\right)^{7-r}\left(\frac{-2}{x}\right)^r$

$= {}^7C_r \left(\frac{1}{4}\right)^{7-r} (-2)^r x^{14-3r}$

Now, $14 - 3r = 2$

$\Rightarrow 3r = 12 \Rightarrow r = 4$

Coefficient of $x^2 = {}^7C_4 \left(\frac{1}{4}\right)^3 (-2)^4$

$= 35 \times \frac{1}{64} \times 2^4 = \frac{35}{4}$

39. (B) Digits 0,1,2,3,4,5,6,7,8,9

no. of -digits numbers

(i) when last digit is '0'

| 8 | 7 | 4 | 1 |
|---|---|---|---|

$= 8\times7\times4\times1 = 224$

(2,4,6,8) ; 0

(ii) when last digit is '2'

| 7 | 7 | 5 | 1 |
|---|---|---|---|

$= 7\times7\times5\times1 = 245$

(1,3,5,7,9) ; 2

'0' cannot put here

(iii) when last two digits is '04'

| 8 | 7 | 1 | 1 |
|---|---|---|---|

$= 8\times7\times1\times1 = 56$

0 4

(iv) when last digit is '4'

| 7 | 7 | 4 | 1 |
|---|---|---|---|

$= 7\times7\times4\times1 = 196$

(2,4,6,8) ; 4

'0' cannot put here

(v) when last digit is '6'

| 7 | 7 | 5 | 1 |
|---|---|---|---|

$= 7\times7\times5\times1 = 245$

(1,3,5,7,9) ; 6

'0' cannot put here

(vi) when last two digits is '08'

| 8 | 7 | 1 | 1 |
|---|---|---|---|

$= 8\times7\times1\times1 = 56$

0 8

(vii) when last digit is '8'

| 7 | 7 | 4 | 1 |
|---|---|---|---|

$= 7\times7\times4\times1 = 196$

(2,4,6,8) ; 8

'0' cannot put here

Hence total number of arrangement

= 224 + 245 + 56 + 196 + 245 + 56 + 196

= 1218

40. (C) We know that

$e = 2.718$ , $\pi = 3.14$

then, $e^2 - 3 = 4.387$, $\pi^2 - 5 = 4.859$

$[e] = 2$, $[\pi] = 3$, $[e^2-3] = 4$, $[\pi^2-5] = 4$

Now, $\begin{vmatrix} [e] & [\pi] & [e^2-3] \\ [\pi] & [\pi^2-5] & [e] \\ [e^2-3] & [\pi^2-5] & [\pi] \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 4 & 3 \end{vmatrix}$

$\Rightarrow 2(12-8) - 3(9-8) + 4(12-16) = -11$

41. (B) $\lim_{x\to0}\frac{\sin x - \tan x}{x^3}$

$\Rightarrow \lim_{x\to0}\frac{\tan x(\cos x - 1)}{x^3}$

$\Rightarrow -\lim_{x\to0}\frac{\tan x}{x}\,\frac{2\sin^2\frac{x}{2}}{x^2}$

$\Rightarrow -\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{4\times\frac{x^2}{4}} = \frac{-2}{4} = \frac{-1}{2}$

42. (C) $y = \cos^{-1}\left(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right)$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\sqrt{x} = \sin\phi \Rightarrow \phi = \sin^{-1}\sqrt{x}$

$\Rightarrow y = \cos^{-1}\left[\cos\theta.\sqrt{1-\sin^2\phi} + \sin\phi.\sqrt{1-\cos^2\theta}\right]$

$\Rightarrow y = \cos^{-1}[\cos\theta.\cos\phi + \sin\phi.\sin\theta]$

$\Rightarrow y = \cos^{-1}[\cos(\theta-\phi)]$

$\Rightarrow y = \theta - \phi$

$\Rightarrow y = \cos^{-1}x - \sin^{-1}\sqrt{x}$

On differentiating both side w.r.t.'$x$'

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

43. (A) $y = \left(\frac{1}{x}\right)^x$

taking log both side

$\Rightarrow \log y = x \log\left(\frac{1}{x}\right)$

$\Rightarrow \log y = -\,x \log x$

On differentiating both side w.r.t.'$x$'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\,x \times \frac{1}{x} - \log x.1$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -1 - \log x$

$\Rightarrow \frac{dy}{dx} = -y\,(1 + \log x)$

44. (C)

45. (B) Given that $f(x) = (x-2)^2 + 6$

$f(a) \Rightarrow f(4) = (4-2)^2 + 6 = 10$

$f(b) \Rightarrow f\left(\frac{11}{2}\right) = \left(\frac{11}{2} - 2\right)^2 + 6 = \frac{73}{4}$

$f'(x) = 2(x-2)$

$f'(c) = 2(c-2)$

Now, $f'(c) = \frac{f(b) - f(a)}{b-a}$

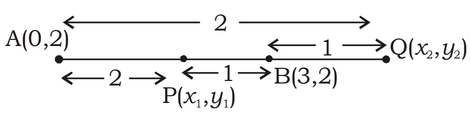
$\Rightarrow 2(c-2) = \frac{\frac{73}{4} - 10}{\frac{11}{2} - 4}$

$\Rightarrow 2(c-2) = \frac{\frac{33}{4}}{\frac{3}{2}}$

$\Rightarrow 2(c-2) = \frac{11}{2}$

$\Rightarrow c - 2 = \frac{11}{4} \Rightarrow c = \frac{19}{4}$

46. (C)




$x_1 = \frac{2\times 3 + 1\times 0}{2+1}$, $y_1 = \frac{2\times 2 + 1\times 2}{2+1}$

$x_1 = 2,\qquad y_1 = 2$

$P(x_1, y_1) = (2, 2)$

$x_2 = \frac{2\times 3 - 1\times 0}{2-1}$, $y_2 = \frac{2\times 2 - 1\times 2}{2-1}$

$x_2 = 6,\qquad y_2 = 2$

$Q(x_2, y_2) = (6, 2)$

Distance between P and Q

$= \sqrt{(2-6)^2 + (2-2)^2}$

$= \sqrt{16+0} = 4$

47. (A) $I = \int e^{\sin^{-1}x}\left[\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}}\right]dx$

Let $\sin^{-1}x = t \Rightarrow x = \sin t$

$dx = \cos t\, dt$

$I = \int e^t\left[\frac{\sin t + \cos t}{\cos t}\right]\cos t\, dt$

$I = \int e^t\left[\sin t + \cos t\right]dt$

We know that

$\int e^x\,[f(x) + f'(x)]\,dx = e^x f(x) + c$

$I = e^t \sin t + c$

$I = x.\,e^{\sin^{-1}x} + c$

48. (A) equation of line which passes through the point (2,–3)

$y + 3 = m(x-2)$ ...(i)

On differentiating both side w.r.t.'$x$'

$\Rightarrow \frac{dy}{dx} = m$

from equation (i)

$\Rightarrow y + 3 = \frac{dy}{dx}(x-2)$

$\Rightarrow y = (x-2)\frac{dy}{dx} - 3$

49. (C) Differential equation

$\frac{dy}{dx} = y^2(e^x - 1)$

$\Rightarrow \frac{dy}{y^2} = (e^x - 1)\,dx$

On integrating both side

$\Rightarrow \frac{y^{-2+1}}{-2+1} = e^x - x + c$

$\Rightarrow \frac{-1}{y} = e^x - x + c \Rightarrow y\,(e^x - x + c) + 1 = 0$

50. (C) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} x+a & b & c \\ -x & x & 0 \\ -x & 0 & x \end{vmatrix} = 0$

$\Rightarrow (x + a)x^2 - b(-x^2) + cx^2 = 0$

$\Rightarrow x^3 + ax^2 + bx^2 + cx^2 = 0$

$\Rightarrow x^2(x + a + b + c) = 0$

$x = 0,\ x + a + b + c = 0$

$x = -(a + b + c)$

51. (B) $7x - 6y + 20 = 0$

and $7x - 6y - 12 = 0$

The required line

$7x - 6y + 4 = 0$

52. (C) $a, A_1, A_2, b$

$d = \frac{b-a}{3}$

$A_1 = a + d$

$A_1 = a + \frac{b+a}{3} = \frac{2a+b}{3}$

$A_2 = a + 2d$

$A_2 = a + 2d$

$= a + \frac{2b-2a}{3} = \frac{a+2b}{3}$

and $a, G_1, G_2, b$

$r = \left(\frac{b}{a}\right)^{1/3}$

$a_1 = ar$

$a_1 = a\left(\frac{b}{a}\right)^{1/3} = a^{2/3}.\ b^{1/3}$

$a_2 = ar^2$

$= a.\left(\frac{b}{a}\right)^{2/3} = a^{1/3}.\ b^{2/3}$

Now, $\frac{A_1+A_2}{G_1G_2} = \frac{\frac{2a+b}{3}+\frac{a+2b}{3}}{a^{2/3}.b^{1/3}.a^{1/3}.b^{2/3}}$

$= \frac{a+b}{ab}$

53. (C) Digits 0,1,3,5,6,7,8

| 6 | 7 | 7 | 7 |
|---|---|---|---|

$= 6\times7\times7\times7 = 2058$

'0' cannot put here

54. (C) quadratic equation

$px^2 + qx + c = 0$

$\alpha + \beta = \frac{-q}{p}$ ...(i)

and quadratic equation

$ax^2 + bx + c = 0$

$\alpha + k + \beta + k = \frac{-b}{a}$

$\Rightarrow \frac{-q}{p} + 2k = \frac{-b}{a}$ [from eq(i)]

$\Rightarrow 2k = \frac{q}{p} - \frac{b}{a} \Rightarrow k = \frac{1}{2}\left(\frac{q}{p} - \frac{b}{a}\right)$

55. (A) No. of ways $= {}^{15-1}C_{11-1}$

$= {}^{14}C_{10} = 1001$

56. (C) $\lim_{x\to\infty} \frac{1^2+2^2+3^2+\ldots+n^2}{n(1+2+3+\ldots+n)}$

$\Rightarrow \lim_{x\to\infty} \frac{\frac{n}{6}(n+1)(2n+1)}{n\times\frac{n(n+1)}{2}}$

$\Rightarrow \lim_{x\to\infty} \frac{(2n+1)}{3n}$

$\Rightarrow \lim_{x\to\infty} \frac{n\left(2+\frac{1}{n}\right)}{3n} = \frac{2}{3}$

57. (A) $\vec{a} = -\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -4\hat{i} + 3\hat{k}, \vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

$= (-1)(-4) + 1\times0 + (-2)\times3 = -2$

$\vec{a}.\vec{c} = (-1)(-2) + 1(-1) + (-2)\times5 = -9$

We know that

$(\vec{x}\times\vec{y})\times\vec{z} = (\vec{x}.\vec{z})\,\vec{y} - (\vec{x}.\vec{y})\,\vec{z}$

Now, $(\vec{a}\times\vec{c})\times\vec{b} = \lambda\vec{c} + \mu\vec{b}$

$\Rightarrow (\vec{a}.\vec{b})\,\vec{c} - (\vec{a}.\vec{c})\,\vec{b} = \lambda\vec{c} + \mu\vec{b}$

$\Rightarrow -2\vec{c} + 9\vec{b} = \lambda\vec{c} + \mu\vec{b}$

On comparing

$\lambda = -2,\ \mu = 9$

Hence $(\lambda, \mu) = (-2, 9)$

58. (B) Word "SUCCESS"

The required permutaion $= \frac{7!}{3!2!} = 420$

59. (A) In the explansion of $(1+x)^{38}$

$T_{r+9} = T_{(r+8)+1} = {}^{38}C_{r+8}\ x^{r+8}$

and $T_{3r-5} = T_{(3r-6)+1} = {}^{38}C_{3r-6}\ x^{3r-6}$

According to question

${}^{38}C_{r+8} = {}^{38}C_{3r-6}$

Now, $r + 8 + 3r - 6 = 38$

$\Rightarrow 4r + 2 = 38$

$\Rightarrow 4r = 36 \Rightarrow r = 9$

60. (D) Let first four terms of an A.P. are $a-d$, $a$, $a+d$ and $a+2d$

According to question

$a - d + a + a + d = 57$

$\Rightarrow 3a = 57 \Rightarrow a = 19$

and $a - d + a + a + d + a + 2d = 92$

$\Rightarrow 4a + 2d = 92$

$\Rightarrow 4 \times 19 + 2d = 92$

$\Rightarrow 76 + 2d = 92 \Rightarrow d = 8$

Now, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 19 + 9 \times 8]$

$\Rightarrow S_{10} = 10\times(19 + 36) = 550$

61. (C) Let $y = e^{\sqrt{x}} \tan^{-1}\sqrt{x}$ and $z = \sqrt{x}$

$y = e^{z}.\tan^{-1} z$

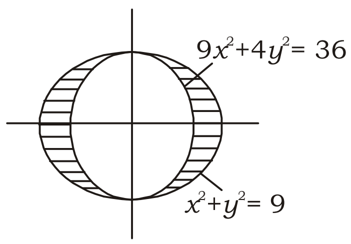
On differentiating both side w.r.t.'$z$'

$$\frac{dy}{dz} = e^{z}.\frac{1}{1+z^2} + \tan^{-1}z.\ e^{z}$$

$$\frac{dy}{dz} = \frac{e^{z}}{1+z^2}\left[1+(1+z^2)\tan^{-1}z\right]$$

$$\frac{dy}{dz} = \frac{e^{\sqrt{x}}}{1+x}\left[1+(1+x)\tan^{-1}\sqrt{x}\right]$$

62. (B)




An ellipse

$9x^2 + 4y^2 = 36 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$

and circle

$x^2 + y^2 = 9$

Area of ellipse $= \pi ab$

$= \pi \times 2 \times 3 = 6\pi$

Area of circle $= \pi r^2$

$= \pi \times (3)^2 = 9\pi$

The required Area $= 9\pi - 6\pi = 3\pi$ sq.unit

63. (B) We know that

$$\lim_{x\to 0}\frac{\cos ax - \cos bx}{x^2} = \frac{b^2-a^2}{2}$$

Now, $$\lim_{x\to 0}\frac{\cos 6x - \cos 8x}{x^2} = \frac{8^2-6^2}{2}$$

$$= \frac{28}{2} = 14$$

64. (C) Short method :-

$$\frac{d}{dx}\left[\left((f(x)^{g(x)}\right)\right] = (f(x))^{g(x)}\left[\frac{g(x)}{f(x)}.f'(x)+\log(f(x)).g'(x)\right]$$

Now, $$\frac{d}{dx}[(\log x)^x] = (\log x)^x\left[\frac{x}{\log x}.\frac{1}{x}+\log(\log x).1\right]$$

$$\Rightarrow \frac{d}{dx}[(\log x)^x] = (\log x)^x\left[\frac{x}{\log x}+\log(\log x)\right]$$

65. (A) Standand differential equation

$$\frac{dy}{dx} = \frac{a\,f(x,y)+b}{c\,f(x,y)+d}$$

where $f(x,y)$ is linear equation in $x$ and $y$.

Solution of differential equation

$$\frac{\begin{vmatrix} x & y \\ c & a \end{vmatrix}}{\begin{vmatrix} c & a \\ a & b \end{vmatrix}} f(c,a)+\log|f(x,y).f(c,a)+f(d,b)| = c$$

Now, Given differential equation

$$\frac{dy}{dx} = \frac{-(x-y)+1}{2(x-y)-1}$$

Compare with standand diff. equation

$f(x,y) = x - y$, $a = -1$, $b = 1$, $c = 2$, $d = -1$

$f(2,-1) = 2 + 1 = 3$ and $f(-1,1) = -1-1 = -2$

Solution of differential equation

$$\frac{\begin{vmatrix} x & y \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}} f(2,-1)+\log|f(x,y).f(2,-1)+f(-1,1)| = c$$

$$\Rightarrow \frac{-x-2y}{2-1}(3) + \log|(x-y)\times 3 - 2| = c$$

$\Rightarrow -3x - 6y + \log|3x - 3y - 2| = c$

$\Rightarrow 3x + 6y + c = \log|3x - 3y - 2|$

66. (C) $\log_2(x^2 - 5x + 28) < 6$

$\Rightarrow x^2 - 5x + 28 < 2^6$

$\Rightarrow x^2 - 5x + 28 < 64$

$\Rightarrow x^2 - 5x - 36 < 0$

$\Rightarrow (x - 9)(x + 4) < 0$

$x \in (-4, 9)$

67. (C) We know that

If $S_p = q$, $S_q = p$, then $S_{p+q} = -(p + q)$

Now, $S_{32} = 64$, $S_{64} = 32$

then $S_{(32+64)} = -(32 + 64) \Rightarrow S_{96} = -96$

68. (D) $f(x) = \dfrac{[x-2]}{[x-1]}$

**At $x = 1$**

L.H.L. $= \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h)$

$= \lim_{h \to 0} \dfrac{[1-h-2]}{[1-h-1]}$

$= \lim_{h \to 0} \dfrac{[-1-h]}{[0-h]}$

$= \dfrac{-2}{-1} = 2$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h)$

$= \lim_{h \to 0} \dfrac{[1+h-2]}{[1+h-1]}$

$= \lim_{h \to 0} \dfrac{[-1+h]}{[0+h]}$

$= \dfrac{-1}{0} = \infty$

L.H.L $\neq$ R.H.L.

$f(x)$ is not continous at $x = 1$.

**At $x = 2$**

L.H.L. $= \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2 - h)$

$= \lim_{h \to 0} \dfrac{[2-h-2]}{[2-h-1]}$

$= \lim_{h \to 0} \dfrac{[0-h]}{[1-h]}$

$= \dfrac{-1}{0} = \infty$

R.H.L. $= \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2 + h)$

$= \lim_{h \to 0} \dfrac{[2+h-2]}{[2+h-1]}$

$= \lim_{h \to 0} \dfrac{[0+h]}{[1+h]}$

$= \dfrac{0}{1} = 0$

L.H.L. $\neq$ R.H.L.

$f(x)$ is not continuous at $x = 2$.

69. (A) Let A $= \begin{bmatrix} 1 & 0 & -1 \\ 3 & -2 & 4 \\ 5 & -1 & 3 \end{bmatrix}$

Co-factors of A –

$C_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 4 \\ -1 & 3 \end{vmatrix}$, $C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix}$, $C_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -2 \\ 5 & -1 \end{vmatrix}$

$= -2 \qquad = 11 \qquad = 7$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & -1 \\ -1 & 3 \end{vmatrix}$, $C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 5 & 3 \end{vmatrix}$, $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix}$

$= 1 \qquad = 8 \qquad = 1$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & -1 \\ -2 & 4 \end{vmatrix}$, $C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix}$, $C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix}$

$= -2 \qquad = -7 \qquad = -2$

$C = \begin{bmatrix} -2 & 11 & 7 \\ 1 & 8 & 1 \\ -2 & -7 & -2 \end{bmatrix}$

Adj A $= C^T = \begin{bmatrix} -2 & 1 & -2 \\ 11 & 8 & -7 \\ 7 & 1 & -2 \end{bmatrix}$

70. (C) $\log\left(a + \sqrt{a^2 + x^2}\right) + \log\left[\dfrac{1}{a + \sqrt{a^2 + x^2}}\right]$

$\Rightarrow \log\left(a + \sqrt{a^2 + x^2}\right)\left(\dfrac{1}{a + \sqrt{a^2 + x^2}}\right)$

$[\because \log m + \log n = \log mn]$

$\Rightarrow \log 1 = 0$

71. (C) $y = ax \sin\left(\dfrac{1}{x} + b\right)$

On differentiating both side w.r.t.'$x$'

$\Rightarrow y_1 = ax.\cos\left(\dfrac{1}{x} + b\right)\left(\dfrac{-1}{x^2}\right) + a\sin\left(\dfrac{1}{x} + b\right).1$

$\Rightarrow y_1 = \dfrac{-a}{x}\cos\left(\dfrac{1}{x} + b\right) + a\sin\left(\dfrac{1}{x} + b\right)$

$\Rightarrow xy_1 = -a\cos\left(\dfrac{1}{x} + b\right) + ax\sin\left(\dfrac{1}{x} + b\right)$

$\Rightarrow xy_1 = -a\cos\left(\dfrac{1}{x} + b\right) + y$

Again, differentiating

$\Rightarrow xy_2 + y_1 = -a(-1)\sin\left(\dfrac{1}{x} + b\right)\left(\dfrac{-1}{x^2}\right) + y_1$

$\Rightarrow xy_2 + y_1 = \dfrac{-a}{x^2}\sin\left(\dfrac{1}{x} + b\right) + y_1$

$\Rightarrow x^4y_2 + x^3y_1 = -ax\sin\left(\dfrac{1}{x} + b\right) + x^3y_1$

$\Rightarrow x^4y_2 = -y \Rightarrow x^4y_2 + y_1 = 0$

72. (A) Line $3x - 4y - 7$

$m_1 = \frac{3}{4}$

and line $3x + 5y = 9$

$m_2 = \frac{-3}{5}$

Now, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{3}{4}+\frac{3}{5}}{1+\frac{3}{4}\left(\frac{-3}{5}\right)}\right|$

$\Rightarrow \tan\theta = \left(\frac{\frac{27}{20}}{\frac{11}{20}}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{27}{11}\right)$

73. (B) The required probability P(E) = 0

74. (A) Given that $\theta = 30°$

$m = \tan 30° = \frac{1}{\sqrt{3}}$

equation of line

$y = mx + c$

$y = \frac{1}{\sqrt{3}}x + c$ ....(i)

its passes through the point (–3,1)

$1 = \frac{1}{\sqrt{3}} \times (-3) + c \Rightarrow c = \sqrt{3} + 1$

from eq(i)

$y = \frac{1}{\sqrt{3}}x + \sqrt{3} + 1$

$\Rightarrow \sqrt{3}\,y - x = 3 + \sqrt{3}$

75. (C) S = 0.2 + 0.22 + 0.222+.......$n$ terms

$S = \frac{2}{9}(0.9 + 0.99 + 0.999 + \ldots\ldots n \text{ terms})$

$S = \frac{2}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\ldots n \text{ terms}\right]$

$S = \frac{2}{9}(1 + 1 + \ldots\ldots n \text{ terms})$

$-\frac{2}{9}\left[\frac{1}{10}+\frac{1}{100}+\ldots n \text{ terms}\right]$

$S = \frac{2}{9}n - \frac{2}{9} \times \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}$

$S = \frac{2}{9}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

76. (B) Differential equation

$\frac{dy}{dx} + yx = e^{\frac{-x^2}{2}}$

On comparing with standand linear differential equation

$P = x$, $Q = e^{\frac{-x^2}{2}}$

I.F. $= e^{\int P.dx} = e^{\int x\,dx} = e^{\frac{x^2}{2}}$

Solution of diffterential equation

$y \times \text{I.F.} = \int Q \times \text{I.F.}\,dx$

$\Rightarrow y \times e^{\frac{x^2}{2}} = \int e^{\frac{-x^2}{2}} \times e^{\frac{x^2}{2}}\,dx$

$\Rightarrow y \times e^{\frac{x^2}{2}} = \int 1\,dx$

$\Rightarrow y \times e^{\frac{x^2}{2}} = x + c$

77. (C) $\overrightarrow{OA} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\overrightarrow{OB} = -3\hat{i} + 2\hat{j} - 4\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AB} = -3\hat{i} + 2\hat{j} - 4\hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$

$\overrightarrow{AB} = -4\hat{i} + 4\hat{j} - 7\hat{k}$

Direction Cosine of line $\overrightarrow{AB} = <\frac{-4}{9}, \frac{4}{9}, \frac{-7}{9}>$

78. (A) In ΔABC, A(–4,2), B(–2,9) and C(0,–2) co-ordinates of centroid

$= \left(\frac{-4-2+0}{3}, \frac{2+9-2}{3}\right) = (-2, 3)$

79. (C) $(1 + \sin x - \cos x)^2$

$\Rightarrow 1 + \sin^2 x + \cos^2 x + 2\sin x - 2\sin x.\cos x - 2\cos x$

$\Rightarrow 1 + 1 + 2\sin x - 2\sin x.\cos x - 2\cos x$

$\Rightarrow 2(1 + \sin x) - 2\cos x(1 + \sin x)$

$\Rightarrow 2(1 + \sin x)(1 - \cos x)$

80. (C) $\sin^{-1}\left[\sin\left(\frac{\pi}{4}\right)\right] = \sin^{-1}\left[\sin\left(2\pi + \frac{\pi}{4}\right)\right]$

$= \sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$

81. (C) Median

82. (B) Differential equation

$$\frac{dy}{dx} = x^2.e^{-x}$$

$$\Rightarrow \int dy = \int x^2.e^{-x}\, dx$$

$$\Rightarrow y = x^2\int e^{-x}dx - \int\left\{\frac{d}{dx}x^2.\int e^{-x}dx\right\}dx + c$$

$$\Rightarrow y = -x^2.e^{-x} - \int 2x(-e^{-x})\,dx + c$$

$$\Rightarrow y = -x^2.e^{-x} + 2\left[x.\int e^{-x}dx - \int\left\{\frac{d}{dx}(x).\int e^{-x}dx\right\}dx + c\right]$$

$$\Rightarrow y = -x^2.e^{-x} + 2\left[-x.e^{-x} - \int 1.(-e^{-x}dx\right] + c$$

$$\Rightarrow y = -x^2.e^{-x} - 2x.e^{-x} + 2\int e^{-x}dx + c$$

$$\Rightarrow y = -x^2.e^{-x} - 2x.e^{-x} - 2e^{-x} + c$$

83. (C) Let $y = \log(\log(\log x)$
On differentiating both side w.r.t.'$x$'

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\log(\log x)}\,\frac{1}{\log x}\,\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x\log x.\log(\log x)}$$

84. (B) P(23, 12) = $x$

$$\Rightarrow \frac{23!}{(23-12)!} = x \Rightarrow x = \frac{23!}{11!}$$

and 3!× (23,11) = $y$

$$\Rightarrow 6 \times \frac{23!}{11!\,12!} = y$$

$$\Rightarrow 6 \times \frac{x}{12!} = y$$

$$\Rightarrow \frac{x}{2\times 11!} = y \Rightarrow x = 2y \times 11!$$

85. (C) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha.\beta = \frac{c}{a}$

Now, $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 + 2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}}{\frac{c}{a}}$$

$$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} = \frac{b^2 - 2ac}{ac}$$

86. (C) $\lim_{x\to 0} x\sin\frac{1}{x} = p$ $\left[\because -1 \le \sin\frac{1}{x} \le 1\right]$

$\Rightarrow 0 \times (-1 \text{ to } 1) = p \Rightarrow p = 0$

and $\lim_{x\to 0}\frac{\tan x}{x} = q$

$\Rightarrow 1 = q$

Hence $p = 0$ , $q = 1$

87. (C) In ΔABC,

$$4bc\ \sin\left(\frac{A\text{-}B\text{-}C}{2}\right)$$

$$\Rightarrow 4bc\sin\left[\frac{A-(\pi-A)}{2}\right] \quad [\because A+B+C=\pi]$$

$$\Rightarrow 4bc\sin\left[\frac{2A-\pi}{2}\right]$$

$$\Rightarrow 4bc\sin\left[A-\frac{\pi}{2}\right]$$

$$\Rightarrow -4bc\sin\left(\frac{\pi}{2}-A\right)$$

$$\Rightarrow -4bc\cos A$$

$$\Rightarrow -4bc \times \frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow -2(b^2+c^2-a^2) = 2(a^2-b^2-c^2)$$

88. (D) $n(S) = {}^{13}C_4 = 715$

$n(E) = {}^6C_1 \times {}^7C_3 + {}^6C_2 \times {}^7C_2 + {}^6C_3 \times {}^7C_1$

$= 6 \times 35 + 15 \times 21 + 20 \times 7 = 665$

The required probability P(E) = $\frac{n(E)}{n(S)}$

$$P(E) = \frac{665}{715} = \frac{133}{143}$$

89. (C) $\tan\left(\tan^{-1}\frac{3}{7} + \frac{\pi}{4}\right)$

$$\Rightarrow \tan\left[\tan^{-1}\frac{3}{7} + \tan^{-1}(1)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{3}{7}+1}{1-\frac{3}{7}\times 1}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{10}{7}}{\frac{4}{7}}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{5}{2}\right)\right] = \frac{5}{2}$$

90. (A) **Statement I**

$\tan\theta = x$ and $\frac{1}{x} = \cot\theta$

Now, $x + \frac{1}{x} = \tan\theta + \cot\theta$

$\Rightarrow x + \frac{1}{x} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{2(\sin^2\theta + \cos^2\theta)}{2\cos\theta.\cos\theta}$

$\Rightarrow x + \frac{1}{x} = \frac{2}{\sin 2\theta}$

$\Rightarrow x + \frac{1}{x} = 2\ \text{cosec}2\theta$

Statement I is correct.

**Statement II**

$x - \frac{1}{x} = 2\tan\theta$

On squaring both side

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 4\tan^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\tan^2\theta + 2\tan^2\theta + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\tan^2\theta + 2\sec^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(\tan^2\theta + \sec^2\theta)$

Statement II is incorrect.

91. (D) Given that $f(x) = \frac{[x]-6}{[x-7]}$

**At $x = 7$**

L.H.L. $= \lim_{x\to 7^-} f(x) = \lim_{x\to 0} f(7-h)$

$= \lim_{x\to 0} \frac{[7-h]-6}{[7-h-7]}$

$= \lim_{x\to 0} \frac{6-6}{[0-h]}$

$= \frac{0}{-1} = 0$

R.H.L. $= \lim_{x\to 7^+} f(x) = \lim_{x\to 0} f(7+h)$

$= \lim_{x\to 0} \frac{[7+h]-6}{[7+h-7]}$

$= \lim_{x\to 0} \frac{7-6}{[0+h]} = \frac{1}{0} = \infty$

$f(x)$ is not continuous at $x = 7$.

**At $x = 6$**

L.H.L. $= \lim_{x\to 6^-} f(x) = \lim_{x\to 0} f(6-h)$

$= \lim_{x\to 0} \frac{[6-h]-6}{[6-h-7]}$

$= \lim_{x\to 0} \frac{5-6}{[-1-h]}$

$= \frac{-1}{-2} = \frac{1}{2}$

R.H.L. $= \lim_{x\to 6^+} f(x) = \lim_{x\to 0} f(6+h)$

$= \lim_{x\to 0} \frac{[6+h]-6}{[6+h-7]}$

$= \lim_{x\to 0} \frac{6-6}{[-1+h]} = \frac{0}{-1} = 0$

L.H.L. ≠ R.H.L.

$f(x)$ is not continuous at $x = 6$.

92. (C) $\lim_{x\to\infty} \frac{3x^2 - 4x + 1}{2x - 4x^2 - 5}$ $\left[\frac{\infty}{\infty}\right]$ Form

$\Rightarrow \lim_{x\to\infty} \frac{x^2\left(3 - \frac{4}{x} + \frac{1}{x^2}\right)}{x^2\left(-4 + \frac{2}{x} - \frac{5}{x}\right)}$

$\Rightarrow \lim_{x\to\infty} \frac{3-0+0}{-4+0-0} = \frac{-3}{4}$

93. (D) $\frac{\tan 176.\tan 64 - 1}{\tan 176 + \tan 64}$

$\Rightarrow \frac{\tan(90+86).\tan(90-26) - 1}{\tan(90+86) + \tan(90-26)}$

$\Rightarrow \frac{-\cot 86.\cot 26 - 1}{-\cot 86 + \cot 26}$

$\Rightarrow \frac{\cot 26.\cot 86 + 1}{\cot 86 - \cot 26}$

$\Rightarrow \cot(26-86)$ $\left[\because \cot(A-B) = \frac{\cot A.\cot B + 1}{\cot B - \cot A}\right]$

$\Rightarrow \cot(-60) = -\cot 60 = -\frac{1}{\sqrt{3}}$

94. (A) $a + 2d$, $a + 4d$ and $a + 7d$ are in G.P., then $(a + 4d)^2 = (a + 2d)(a + 7d)$

$\Rightarrow a^2 + 16d^2 + 8ad = a^2 + 2ad + 7ad + 14d^2$

$\Rightarrow 16d^2 + 8ad - 9ad - 14d^2 = 0$

$\Rightarrow 2d^2 - ad = 0$

$\Rightarrow d(2d - a) = 0$

$d \neq 0,\ 2d = a$

Now,

Common ratio $= \dfrac{a+4d}{a+2d}$

$= \dfrac{2d+4d}{2d+2d}$

$= \dfrac{6d}{4d} = \dfrac{3}{2}$

95. (A) Function $\cos^{-1}[\log_7 3x]$

Now, $-1 \le \log_7 3x \le 1$

$\Rightarrow 7^{-1} \le 3x \le 7^1$

$\Rightarrow \dfrac{1}{7} \le 3x \le 7$

$\Rightarrow \dfrac{1}{21} \le x \le \dfrac{7}{3}$

96. (B) $x = \dfrac{at}{1-t^2}$ ....(i)

On differentiating both side w.r.t.'$t$'

$\Rightarrow \dfrac{dx}{dt} = \dfrac{a(1-t^2).1 - at(-2t)}{(1-t^2)^2}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{a(1+t^2)}{(1-t^2)^2}$

and $y = \dfrac{2a(1+t^2)}{1-t^2}$ ....(i)

On differentiating both side w.r.t.'$t$'

$\Rightarrow \dfrac{dy}{dt} = \dfrac{2a(1-t^2).2t - 2a(1+t^2)(-2t)}{(1-t^2)^2}$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{8at}{(1-t^2)^2}$

Now, $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{8at}{(1-t^2)^2} \times \dfrac{(1-t^2)^2}{a(1+t^2)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{8t}{1+t^2}$ ....(iii)

from eq(i) and (ii)

$\dfrac{x}{y} = \dfrac{at}{1-t^2} \times \dfrac{1-t^2}{2a(1+t^2)}$

$\Rightarrow \dfrac{x}{y} = \dfrac{t}{2(1+t^2)} \Rightarrow \dfrac{t}{1+t^2} = \dfrac{2x}{y}$ ...(iv)

from eq(iii) and eq(iv)

$\dfrac{dy}{dx} = 8 \times \dfrac{2x}{y}$ ....(v)

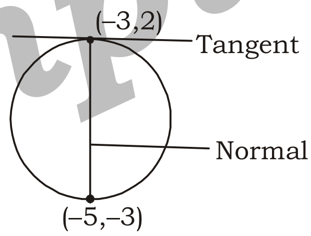
Again, differentiating

$\Rightarrow \dfrac{d^2y}{dx^2} = 16\left[\dfrac{y.1 - x.\dfrac{dy}{dx}}{y^2}\right]$

$\Rightarrow \dfrac{d^2y}{dx^2} = 16\left[\dfrac{y - x \times \dfrac{16x}{y}}{y^2}\right]$

$\Rightarrow \dfrac{d^2y}{dx^2} = 16\left[\dfrac{y^2 - 16x^2}{y^3}\right]$

97. (A)




When end points of a diameter are given, then

equation of circle

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$\Rightarrow (x + 3)(x + 5) + (y - 2)(y + 3) = 0$

$\Rightarrow x^2 + 8x + 15 + y^2 + y - 6 = 0$

$\Rightarrow x^2 + y^2 + 8x + y + 9 = 0$

98. (D) $\Delta \neq 0,\ a = b,\ h = 0$

99. (C) A is $(x - 2)\times(x - 4)$ matrix and B is $(y + 1)\times(9 - y)$ matrix

Both AB and BA exist, then

$x - 4 = y + 1 \Rightarrow x - y = 5$ ...(i)

and $x - 2 = 9 - y \Rightarrow x + y = 11$ ...(ii)

On solving

$x = 8,\ y = 3$

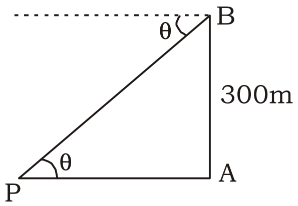
100. (A) Plane $2x - 5y - 14z = 16$ and point $(-1, 2, -3)$

Distance $= \dfrac{2(-1) - 5(2) - 14(-3) - 16}{\sqrt{2^2 + (5)^2 + (-14)^2}}$

$= \dfrac{-2 - 10 + 42 - 16}{\sqrt{4 + 25 + 196}} = \dfrac{14}{15}$

101. (B) U = {1,2,3,4,5,6,7,8,9}
A = {4,6,7}, B = {4,3,9}, C = {4,6,9}
$(A\cap C)$ = {4,6,7} $\cap$ {4,6,9} = {4,6}
$(B\cap C)$ = {4,3,9} $\cap$ {4,6,9} = {4,9}
Now, $(A\cap C) - (B\cap C)$ = {4,6} – {4,9}
= {6}

102. (C)




Given that AB = 300 m

Let $\theta = \sin^{-1}\left(\frac{5}{13}\right) \Rightarrow \sin\theta = \frac{5}{13}$

and $\tan\theta = \frac{5}{12}$

**In ΔABP :-**

$\tan\theta = \frac{AB}{AP}$

$\Rightarrow \frac{5}{12} = \frac{300}{AP} \Rightarrow AP = 720$ m

Hence Distance between boat and the lighthouse AP = 720 m

103. (D) $\sin\theta = \frac{7}{25}$ and $\sin\phi = \frac{3}{5}$

$\cos\theta = \frac{24}{25}$, $\cos\phi = \frac{4}{5}$

$\cos(\theta+\phi) = \cos\theta.\cos\phi - \sin\theta.\sin\phi$

$\cos(\theta+\phi) = \frac{24}{25}\times\frac{4}{5} - \frac{7}{25}\times\frac{3}{5}$

$\cos(\theta+\phi) = \frac{96-21}{125} = \frac{75}{125} = \frac{3}{5}$

Now, $\cos\left(\frac{\theta+\phi}{2}\right) = \sqrt{\frac{1+\cos(\theta+\phi)}{2}}$

$= \sqrt{\frac{1+\frac{3}{5}}{2}}$

$= \sqrt{\frac{8}{5\times 2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

104. (D) equation $ax^2 + bx + c = 0$
One root is $3 - 7i$ and other root is $3 + 7i$.

Now, Sum of roots = $\frac{-b}{a}$

$\Rightarrow 3 - 7i + 3 + 7i = \frac{-b}{a}$

$\Rightarrow 6 = \frac{-b}{a} \Rightarrow 6a + b = 0$

105. (B) $\sin 18 = \frac{\sqrt{5}-1}{4}$

106. (A) Vectors $\vec{a} = 2\vec{i} + (1-2\lambda)\vec{j} + 3\vec{k}$ and $\vec{b} = (2+\lambda)\vec{i} + 2\vec{j} - 4\vec{k}$ are perpendicular,

then $\vec{a}.\vec{b} = 0$

$\Rightarrow 2\times(2+\lambda) + (1-2\lambda)\times 2 + 3(-4) = 0$

$\Rightarrow 4 + 2\lambda + 2 - 4\lambda - 12 = 0$

$\Rightarrow -6 - 2\lambda = 0 \Rightarrow \lambda = -3$

107. (D) 2 sin10.sin30.sin50.sin70

$\Rightarrow 2\sin10.\sin50.\sin70 \times \frac{1}{2}$

$\Rightarrow 2\times\frac{1}{4}\sin(3\times 10)\times\frac{1}{2}$

$\left[\because \sin\theta.\sin(60-\theta).\sin(60+\theta) = \frac{1}{4}\sin 3\theta\right]$

$\Rightarrow 2\times\frac{1}{4}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$

108. (B) $\frac{\theta^\circ}{\theta^c} = \frac{180}{\pi}$ ...(i)

and $\theta^\circ \times \theta^c = 80\pi$

from eq(i)

$\theta^\circ \times \theta^\circ \times \frac{\pi}{180} = 80\pi$

$\Rightarrow (\theta^\circ)^2 = 180 \times 80$

$\Rightarrow (\theta^\circ)^2 = 14400 \Rightarrow \theta^\circ = 120$

109. (A) Line $\frac{x-2}{2} = \frac{y-3}{5} = \frac{z+5}{14}$

and plane $2x - y + 2z = 4$

Let angle between line and plane is θ, then

$\sin\theta = \frac{2\times 2 + 5(-1) + 14\times 2}{\sqrt{2^2+5^2+14^2}\sqrt{2^2+(-1)^2+2^2}}$

$\Rightarrow \sin\theta = \frac{27}{15\times 3}$

$\Rightarrow \sin\theta = \frac{3}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{5}\right)$

110. (A)

111. (D) Equation of straight line which makes equal intercept on both axes

$\frac{x}{a}+\frac{y}{a} = 1$

$x + y = a$ ....(i)

its passes through the point (–1,2)

$\Rightarrow -1 + 2 = a \Rightarrow a = 1$

From eq(i)

$x + y = 1$

112. (B) Given that $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/4}$ ...(i)

and $x^m - y^m = 9$

On differentiating both side w.r.t. '$x$'

$mx^{m-1} - my^{m-1}\frac{dy}{dx} = 0$

$\Rightarrow my^{m-1}\frac{dy}{dx} = mx^{m-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{x}{y}\right)^{m-1}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right)^{1-m}$

On comparing with eq(i)

$\frac{1}{4} = 1 - m \Rightarrow m = \frac{3}{4}$

113. (C) Ratio of angles = 1 : 7 : 2

Let angles = $x$, $7x$, $2x$

Now, $x + 7x + 2x = 180$

$\Rightarrow 10x = 180 \Rightarrow x = 18$

Angles = 18, 126, 36

Now, $\frac{a}{\sin A} = \frac{b}{\sin C} = \frac{c}{\sin C}$

$\frac{\text{largest side}}{\text{smallest side}}\left(\frac{b}{a}\right) = \frac{\sin B}{\sin A}$

$= \frac{\sin 126}{\sin 18}$

$= \frac{\cos 36}{\sin 18}$

$= \frac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}-1}{4}} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$

114. (A) $(A \cup B \cup C) - (A \cap C) - (A \cap B) - (B \cap C)$

115. (D) $f(z) = \frac{4 - z^2}{1 + z}$

put $z = 1 + i$

$f(z) = \frac{4 - (1+i)^2}{1+1+i}$

$f(z) = \frac{4-2i}{2+i} \times \frac{2-i}{2-i}$

$f(z) = \frac{6-8i}{5}$

Now, $|f(z)| = \frac{\sqrt{6^2+8^2}}{5} = \frac{10}{5} = 2$

116. (C) $f(x) = \begin{cases} 5x - x^2 + 1, & 2 \le x < 3 \\ -3x + \lambda, & 3 \le x < 4 \end{cases}$

is continuous at $x = 3$,

then $\lim_{x \to 3^-} f(x) = f(3)$

$\Rightarrow 5 \times 3 - 3^2 + 1 = -3 \times 3 + \lambda$

$\Rightarrow 7 = -9 + \lambda = \lambda = 16$

117. (D) The required no. of triangles = ${}^{11}C_3 - {}^{6}C_3$

$= 165 - 20 = 145$

118. (A)

| Class | $f$ | $c$ |
|---|---|---|
| 0 - 4 | 13 | 13 |
| 4 - 8 | 18 | 31 |
| 8 -12 | 20 | 51 ← Median class |
| 12-16 | 23 | 74 |
| 16-20 | 26 | 100 |

$N = 100, \frac{N}{2} = 50$

$l_1 = 8,\ l_2 = 12,\ f = 20,\ c = 31$

Median = $l_1 + \frac{\frac{N}{2} - C}{f} \times (l_2 - l_1)$

Median = $8 + \frac{50-31}{20} \times (12 - 8)$

Median = $8 + \frac{19}{20} \times 4 = 11.8$

119. (C) $u^2 + v^2 + w^2 - d > 0$

120. (C)

# NDA (MATHS) MOCK TEST - 182 (Answer Key)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | (B) | 21. | (A) | 41. | (B) | 61. | (C) | 81. | (C) | 101. | (B) |
| 2. | (A) | 22. | (B) | 42. | (C) | 62. | (B) | 82. | (B) | 102. | (C) |
| 3. | (C) | 23. | (A) | 43. | (A) | 63. | (B) | 83. | (C) | 103. | (D) |
| 4. | (C) | 24. | (D) | 44. | (C) | 64. | (B) | 84. | (B) | 104. | (D) |
| 5. | (C) | 25. | (D) | 45. | (B) | 65. | (A) | 85. | (B) | 105. | (B) |
| 6. | (D) | 26. | (B) | 46. | (C) | 66. | (C) | 86. | (C) | 106. | (A) |
| 7. | (A) | 27. | (D) | 47. | (A) | 67. | (C) | 87. | (C) | 107. | (D) |
| 8. | (D) | 28. | (A) | 48. | (A) | 68. | (D) | 88. | (D) | 108. | (B) |
| 9. | (D) | 29. | (C) | 49. | (C) | 69. | (A) | 89. | (C) | 109. | (A) |
| 10. | (B) | 30. | (B) | 50. | (C) | 70. | (C) | 90. | (A) | 110. | (A) |
| 11. | (C) | 31. | (C) | 51. | (B) | 71. | (C) | 91. | (D) | 111. | (D) |
| 12. | (B) | 32. | (C) | 52. | (C) | 72. | (A) | 92. | (C) | 112. | (B) |
| 13. | (C) | 33. | (C) | 53. | (C) | 73. | (B) | 93. | (D) | 113. | (C) |
| 14. | (B) | 34. | (B) | 54. | (C) | 74. | (D) | 94. | (A) | 114. | (A) |
| 15. | (D) | 35. | (A) | 55. | (C) | 75. | (C) | 95. | (A) | 115. | (D) |
| 16. | (A) | 36. | (D) | 56. | (C) | 76. | (B) | 96. | (B) | 116. | (C) |
| 17. | (D) | 37. | (D) | 57. | (A) | 77. | (C) | 97. | (A) | 117. | (D) |
| 18. | (B) | 38. | (C) | 58. | (B) | 78. | (A) | 98. | (D) | 118. | (A) |
| 19. | (B) | 39. | (B) | 59. | (A) | 79. | (C) | 99. | (C) | 119. | (C) |
| 20. | (B) | 40. | (C) | 60. | (D) | 80. | (C) | 100. | (A) | 120. | (C) |





*Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003*

*Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock*

*Note:- If you face any problem regarding result or marks scored, please contact 9313111777*