NDA MATHS MOCK TEST - 190 (SOLUTION)

1. (C) $(1+x^2)^4(1+x)^6$

$\Rightarrow [{}^4C_0 + {}^4C_1(x^2)^1 + {}^4C_2(x^2)^2 + {}^4C_3(x^2)^3 + {}^4C_4(x^2)^4]\,[{}^6C_0 + {}^6C_1x + {}^6C_2x^2 + {}^6C_3x^3 + {}^6C_4x^4 + {}^6C_5x^5 + {}^6C_6x^6]$

Coefficient of $x^4 = {}^4C_0.{}^6C_4 + {}^4C_1.{}^6C_2 + {}^4C_2.{}^6C_0$

$= 1\times\frac{6!}{4!2!} + 4\times\frac{6!}{2!4!} + \frac{4!}{2!2!}\times 1$

$= 1\times15 + 4\times15 + 6\times1 = 81$

2. (C) $\frac{\left(\sin\frac{\pi}{16}+i\cos\frac{\pi}{16}\right)^8}{\left(\sin\frac{\pi}{16}-i\cos\frac{\pi}{16}\right)^8} \Rightarrow \frac{\left[i\left(\cos\frac{\pi}{16}-i\sin\frac{\pi}{16}\right)\right]^8}{\left[-i\left(\cos\frac{\pi}{16}+i\sin\frac{\pi}{16}\right)\right]^8}$

$\Rightarrow \frac{i^8\left(\cos\frac{\pi}{16}-i\sin\frac{\pi}{16}\right)^8}{(-i)^8\left(\cos\frac{\pi}{16}+i\sin\frac{\pi}{16}\right)^8}\;\frac{\left(\cos\frac{\pi}{16}-i\sin\frac{\pi}{16}\right)^8}{\left(\cos\frac{\pi}{16}+i\sin\frac{\pi}{16}\right)^8}$

Apply Demoivre's theorem

$\Rightarrow \frac{\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}} \Rightarrow \frac{0-i}{0+i} = -1$

3. (C) "TEARS"

Total words starting with A = 4! = 24
Total words starting with E = 4! = 24
Total words starting with R = 4! = 24
Total words starting with S = 4! = 24
Total words starting with TA = 3! = 6
The required numbers = 24 × 4 + 6
= 96 + 6 = 102

4. (B) Distance between foci $= \sqrt{(3+5)^2+(4-4)^2}$

$\Rightarrow 2ae = 8 \Rightarrow 2a\times\frac{4}{3} = 8 \Rightarrow a = 3$

Now, $b^2 = a^2(e^2-1)$

$\Rightarrow b^2 = 9\left(\frac{16}{9}-1\right) \Rightarrow b^2 = 9\times\frac{7}{9} \Rightarrow b^2 = 7$

and coordinate of centre $= \left(\frac{3-5}{2}, \frac{4+4}{2}\right)$

$= (-1, 4)$

Equation of hyperbola

$\frac{(x+1)^2}{9} - \frac{(y-4)^2}{7} = 1$

5. (A) Possibilities of getting sum of the dice is divisible by 3{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)} = 12

Possibilities of getting sum of the dice is divisible by 4{(1, 3),(2, 2), (2, 6), (3,1), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6) = 9

The required difference $= \frac{12}{36} - \frac{9}{36}$

$= \frac{3}{36} = \frac{1}{12}$

6. (D) $\lim_{x\to 0}\frac{\cos 2x - 1}{\sin 2x}$ $\left[\frac{0}{0}\right]$ form

By L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{-2\sin 2x}{2\cos 2x} = \lim_{x\to 0}(-\tan 2x) = 0$

7. (C) $\int_0^{\pi/2}\log|\tan x + \cot x|\,dx$

$\Rightarrow \int_0^{\pi/2}\log\left|\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right|dx$

$\Rightarrow \int_0^{\pi/2}\log\left|\frac{\sin^2 x+\cos^2 x}{\sin x.\cos x}\right|dx$

$\Rightarrow \int_0^{\pi/2}\log\left|\frac{1}{\sin x.\cos x}\right|dx$

$\Rightarrow -\int_0^{\pi/2}\log\sin x\;dx - \int_0^{\pi/2}\log\cos x\;dx$

$\Rightarrow -\int_0^{\pi/2}\log\sin x\;dx - \int_0^{\pi/2}\log\sin x\;dx$ [by PropIV]

$\Rightarrow -2\int_0^{\pi/2}\log\sin x\;dx$

$\Rightarrow -2\left(\frac{-\pi}{2}\log 2\right) = \pi\log 2$

$\left[\because \int_0^{\pi/2}\log\sin x\,dx = \frac{-\pi}{2}\log 2\right]$

8. (A) $5\cos^2\theta + 7\sin^2\theta = 6$

$\Rightarrow 5\cos^2\theta + 7(1-\cos^2\theta) = 6$

$\Rightarrow 5\cos^2\theta + 7 - 7\cos^2\theta = 6$

$\Rightarrow 1 = 2\cos^2\theta$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{4} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}$

9. (A) Differential equation

$(1 + x^2)\frac{dy}{dx} + 2xy = 3x$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{3x}{1+x^2}$

Compare with general equation

P = $\frac{2x}{1+x^2}$, Q = $\frac{3x}{1+x^2}$

I.F. = $e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$

Solution of differential equation

$\Rightarrow y \times \text{I.F.} = \int Q \times \text{I.F}\,dx + c$

$\Rightarrow y(1+x^2) = \int \frac{3x}{1+x^2} \times (1+x^2)\,dx + c$

$\Rightarrow y(1+x^2) = \int 3x\,dx + c$

$\Rightarrow y + yx^2 = \frac{3x^2}{2} + c$

$\Rightarrow 2y + 2yx^2 = 3x^2 + c$

$\Rightarrow 2yx^2 - 3x^2 + 2y = c$

10. (C) The required no. = $2^4 - 2 = 16 - 2 = 14$

11. (D) Asympotes of the given hyperbola

$y = \pm\frac{b}{a}x$

angle between asymptotes = $2\tan^{-1}\left(\frac{b}{a}\right)$

12. (A) $\lim_{n\to\infty}\left[\frac{1}{1.2} + \frac{1}{2.3} + ... + \frac{1}{n(n+1)}\right]$

$\Rightarrow \lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + ... + \left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$

$\Rightarrow \lim_{n\to\infty}\left[1-\frac{1}{n+1}\right] \Rightarrow \lim_{n\to\infty}\left[\frac{n}{n+1}\right]$

$\Rightarrow \lim_{n\to\infty}\left[\frac{n}{n\left(1+\frac{1}{n}\right)}\right] \Rightarrow \lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}$

$\Rightarrow \frac{1}{1+0} = 1$

13. (B) Let $y = \cos^{-1}(\sin x) + \tan^{-1}(\cot x)$

$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2}-x\right)\right] + \tan^{-1}\left[\tan\left(\frac{\pi}{2}-x\right)\right]$

$y = \frac{\pi}{2} - x + \frac{\pi}{2} - x$

$y = \pi - 2x$

On differentiating both sides w.r.t. 'x'

$\frac{dy}{dx} = -2$

14. (A) $I = \int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(i)

Prop.IV $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$I = \int_0^{\pi/2}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}\,dx$

$I = \int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,\text{dx}$

$2I = \int_0^{\pi/2} 1\,dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

15. (B) We know that

$\lim_{x\to\infty}[f(x)]^{g(x)} = e^{\lim_{x\to\infty} g(x)[f(x)-1]}$

Now, $\lim_{x\to\infty}\left(\frac{x^2+6x+5}{x^2-3x+4}\right)^x \Rightarrow e^{\lim_{x\to\infty} x\left[\frac{x^2+6x+5}{x^2-3x+4}-1\right]}$

$\Rightarrow e^{\lim_{x\to\infty} x\left[\frac{x^2+6x+5-x^2+3x-4}{x^2-3x+4}\right]} \Rightarrow e^{\lim_{x\to\infty} x\left[\frac{9x+1}{x^2-3x+4}\right]}$

$\Rightarrow e^{\lim_{x\to\infty}\frac{x^2\left(9+\frac{1}{x}\right)}{x^2\left(1-\frac{3}{x}+\frac{4}{x^2}\right)}} \Rightarrow e^{\left(\frac{9+0}{1-0+0}\right)} = e^9$

16. (C) $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{10}{4!} + ...\infty$

$T_n = \frac{n(n+1)}{2.n!} \Rightarrow T_n = \frac{n(n+1)}{2.n(n-1)!}$

$\Rightarrow T_n = \frac{n+1}{2(n-1)!} \Rightarrow \frac{n-1+2}{2(n-1)!}$

$\Rightarrow T_n = \frac{n-1}{2(n-1)!} + \frac{2}{2(n-1)!}$

$\Rightarrow T_n = \frac{1}{2(n-2)!} + \frac{1}{(n-1)!}$

Now, $S_n = \sum T_n$

$\Rightarrow \sum\left[\frac{1}{2(n-2)!}+\frac{1}{(n-1)!}\right]$

$\Rightarrow \frac{1}{2}\sum\frac{1}{(n-2)!}+\sum\frac{1}{(n-1)!}$

$\Rightarrow \frac{1}{2}e + e = \frac{3}{2}e$

17. (B) cos24° + cos65° + cos115° + cos204° + cos240°

$\Rightarrow$ cos24° + cos65° + cos(180° – 65°) + cos(180° + 24°) + cos(270° – 30°)

$\Rightarrow$ cos24°+ cos65° – cos65° – cos24° – sin30°

$\Rightarrow -\sin 30° = -\frac{1}{2}$

18. (D) Let $z = (5-6i)^2$

$\Rightarrow z = 25 + 36i^2 - 60i$

$\Rightarrow z = 25 - 36 - 60i$

$\Rightarrow z = -11 - 60i$

Conjugate of $z = \frac{-1}{11+60i}\times\frac{11-60i}{11-60i}$

$\Rightarrow \frac{-(11-60i)}{121-3600i^2} \Rightarrow \frac{-(11-60i)}{121+3600}$

$\Rightarrow \frac{-(11+60i)}{3721}$

19. (B) $C(n, r+1) + 2C(n, r) + C(n, r-1)$

$\Rightarrow {}^nC_{r+1} + 2.{}^nC_r + {}^nC_{r-1}$

$\Rightarrow {}^nC_{r+1} + {}^nC_r + {}^nC_r + {}^nC_{r-1}$

We know that

$\Rightarrow {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$\Rightarrow {}^{n+2}C_{r+1} \Rightarrow C(n+2, r+1)$

20. (A) $\begin{vmatrix}0 & a & b\\ a & 0 & c\\ b & c & 0\end{vmatrix}^2 \Rightarrow [0 - a(-bc) + b(ac-0)]^2$

$\Rightarrow [abc + abc]^2 \Rightarrow [2abc]^2 = 4a^2b^2c^2$

21. (B) $y = \cot^{-1}(\operatorname{cosec}x - \cot x)$

$y = \cot^{-1}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$

$y = \cot^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

$y = \cot^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}}\right)$

$y = \cot^{-1}\left(\tan\frac{x}{2}\right)$

$y = \cot^{-1}\left[\cot\left(\frac{\pi}{2}-\frac{x}{2}\right)\right]$

$y = \frac{\pi}{2}-\frac{x}{2}$

On differentiating both sides w.r.t.'x'

$\frac{dy}{dx} = \frac{-1}{2}$

22. (C) $\sqrt{x^2+y^2}+\sqrt{y^2-x^2} = c$

On differentiating both sides w.r.t 'x'

$\Rightarrow \frac{1}{2\sqrt{x^2+y^2}}\left(2x+2y\frac{dy}{dx}\right)+\frac{1}{2\sqrt{y^2-x^2}}\left(2y\frac{dy}{dx}-2x\right) = 0$

$\Rightarrow \frac{x}{\sqrt{x^2+y^2}}+\frac{y}{\sqrt{x^2+y^2}}\frac{dy}{dx}+\frac{y}{\sqrt{y^2-x^2}}\frac{dy}{dx}-\frac{x}{\sqrt{y^2-x^2}} = 0$

$\Rightarrow y\frac{dy}{dx}\left(\frac{1}{\sqrt{x^2+y^2}}+\frac{1}{\sqrt{y^2-x^2}}\right)$

$= x\left(\frac{1}{\sqrt{y^2-x^2}}+\frac{1}{\sqrt{x^2+y^2}}\right)$

$\Rightarrow y\frac{dy}{dx}\left(\frac{\sqrt{y^2-x^2}+\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}\sqrt{y^2-x^2}}\right)$

$= x\left(\frac{\sqrt{x^2+y^2}-\sqrt{y^2-x^2}}{\sqrt{y^2-x^2}\sqrt{x^2+y^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{x}{y}\left(\frac{\sqrt{x^2+y^2}-\sqrt{y^2-x^2}}{\sqrt{x^2+y^2}+\sqrt{y^2-x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{x}{y}\left(\frac{\sqrt{x^2+y^2}-\sqrt{y^2-x^2}}{\sqrt{x^2+y^2}+\sqrt{y^2-x^2}}\times\frac{\sqrt{x^2+y^2}-\sqrt{y^2-x^2}}{\sqrt{x^2+y^2}+\sqrt{y^2-x^2}}\right)$

$\Rightarrow \frac{dy}{dx}=\frac{x}{y}\left[\frac{(x^2+y^2)+(y^2-x^2)-2\sqrt{x^2+y^2}.\sqrt{y^2-x^2}}{(x^2+y^2)-(y^2+x^2)}\right]$

$\Rightarrow \frac{dy}{dx}=\frac{x}{y}\left[\frac{2y^2-2\sqrt{y^4-x^4}}{2x^2}\right]$

$\Rightarrow \frac{dy}{dx}=\frac{1}{xy}\left[y^2-\sqrt{y^4-x^4}\right]$

23. (A) Differential equation

$$\frac{dy}{dx} + e^{x+y} + x^2e^y = 0$$

$$\Rightarrow \frac{dy}{dx} = -e^{x+y} - x^2e^y$$

$$\Rightarrow \frac{dy}{dx} = -e^x.e^y - x^2e^y$$

$$\Rightarrow \frac{dy}{dx} = -e^y(e^x + x^2)$$

$$\Rightarrow e^{-y}\,dy = -(e^x + x^2)dx$$

On integrating

$$\Rightarrow -e^{-y} = -\left(e^x + \frac{x^3}{3}\right) - c$$

$$\Rightarrow e^{-y} = e^x + \frac{x^3}{3} + c$$

24. (B) Ellipse $4x^2 + 9y^2 - 18y - 16 = 0$

$$\Rightarrow 4x^2 + 9(y^2 - 2y + 1 - 1) - 16 = 0$$

$$\Rightarrow 4x^2 = 9(y-1)^2 - 9 - 16 = 0$$

$$\Rightarrow 4x^2 = 9(y-1)^2 = 25$$

$$\Rightarrow \frac{x^2}{25/4} + \frac{(y-1)^2}{25/9} = 1$$

$$a^2 = \frac{25}{4},\ b^2 = \frac{25}{9}$$

Now, $e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1-\frac{25/9}{25/4}}$

$$\Rightarrow e = \sqrt{1-\frac{4}{9}} \Rightarrow e = \frac{\sqrt{5}}{3}$$

25. (C) No. of permutations = $\frac{8!}{2!2!} = 10080$

26. (D) $\frac{1}{\log_2 e} + \frac{1}{\log_2 e^2} + \frac{1}{\log_2 e^4} + \ldots\infty$

$$\Rightarrow \frac{1}{\log_2 e} + \frac{1}{2\log_2 e} + \frac{1}{4\log_2 e} + \ldots\infty$$

$$\Rightarrow \frac{1}{\log_2 e}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots.\infty\right] \Rightarrow \log_e 2\left[\frac{1}{1-\frac{1}{2}}\right]$$

$$\Rightarrow \frac{1}{1/2}\log_e 2 \Rightarrow 2\log_e 2 = \log_e 4$$

27. (C) Given that

$$\left|z - \frac{2}{z}\right| = 1$$

We know that

$$|a+b| \le |a| + |b|$$

Now, $|z| = \left|\left(z-\frac{2}{z}\right) + \frac{2}{z}\right| \le \left|z - \frac{2}{z}\right| + \left|\frac{2}{z}\right|$

$$\Rightarrow |z| \le 1 + \frac{2}{|z|} \Rightarrow |z|^2 \le |z| + 2$$

$$\Rightarrow |z|^2 - |z| - 2 \le 0$$

$$\Rightarrow [|z| - 2]\,[|z| + 1] \le 0$$

$$\Rightarrow -1 \le |z| \le 2$$

Hence maximum value of $|z| = 2$

28. (B) In the expansion of $\left(\frac{x^2}{4} - \frac{3}{x}\right)^9$

$$T_r = T_{(r-1)+1} = {}^9C_{r-1}\left(\frac{x^2}{4}\right)^{9-(r-1)}\left(\frac{-3}{x}\right)^{r-1}$$

$$T_r = {}^9C_{r-1}\left(\frac{1}{4}\right)^{10-r}(-3)^{r-1}x^{21-3r}$$

Now, $21 - 3r = 3$

$$\Rightarrow 3r = 18 \Rightarrow r = 6$$

29. (C) $\lim_{x\to\infty}\frac{\sqrt{4x^2+3} - \sqrt{9x^2+2}}{(2x+5)}$

$$\Rightarrow \lim_{x\to\infty}\frac{x\left[\sqrt{4+\frac{3}{x^2}} - \sqrt{9+\frac{2}{x^2}}\right]}{x\left(2+\frac{5}{x}\right)}$$

$$\Rightarrow \frac{\sqrt{4+0} - \sqrt{9+0}}{2+0} \Rightarrow \frac{2-3}{2} = \frac{-1}{2}$$

30. (B) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \frac{x-1}{x-2}\times\frac{x+1}{x+2}}\right] = \frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{x^2 - x + 2x - 2 + x^2 + x - 2x - 2}{(x^2-4) - (x^2-1)} = 1$$

$$\Rightarrow \frac{2x^2 - 4}{-3} = 1$$

$$\Rightarrow 2x^2 - 4 = -3$$

$$\Rightarrow 2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

31. (C) The required distance

$$= \left|\frac{6\times(-1)-3\times2-2\times3+11}{\sqrt{6^2\times(-3)^2+(-2)^2}}\right|$$

$$= \left|\frac{-6-6-6+11}{\sqrt{36+9+4}}\right| = \left|\frac{-7}{7}\right| = 1$$

32. (C) $\int_0^4 |x-3|\,dx = \int_0^3 |x-3|\,dx + \int_3^4 |x-3|\,dx$

$$\Rightarrow \int_0^3 -(x-3)dx + \int_3^4 (x-3)dx$$

$$\Rightarrow \left[-\left(\frac{x^2}{2}-3x\right)\right]_0^3 + \left[\left(\frac{x^2}{2}-3x\right)\right]_3^4$$

$$\Rightarrow -\left(\frac{9}{2}-9\right) + 0 + (8-12) - \left(\frac{9}{2}-9\right)$$

$$\Rightarrow -\left(\frac{9}{2}-9\right) - 4$$

$\Rightarrow -9 + 18 - 4 = 5$

33. (A) Lines $\frac{x}{3}+\frac{y}{4} = 1$ and $\frac{x}{2}+\frac{y}{3} = 1$

intersection point = (–6, 12)

equation of straight line which is parallel to the line $3x - 5y + 6 = 0$

$3x - 5y = c$

It passes through the point (–6, 12)

$\Rightarrow 3 \times (-6) - 5 \times 12 = c$

$\Rightarrow -18 - 60 = c \Rightarrow c = -78$

The required equation

$3x - 5y = -78 \Rightarrow 3x - 5y + 78 = 0$

34. (C) $y = \cos(\ln x)$ ...(i)

On differentiating both sides w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = -\sin(\ln x).\frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx} = -\sin(\ln x)$$

Again, differentiating

$$\Rightarrow x.\frac{d^2y}{dx^2}+\frac{dy}{dx}.1 = -\cos(\ln x).\frac{1}{x}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx} = -y \quad \text{[from eq(i)]}$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = 0$$

35. (B) Equation

$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

$\Rightarrow 2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$

$\Rightarrow 2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

$\Rightarrow (\cos x - \sqrt{3})(2\cos x + \sqrt{3}) = 0$

$\Rightarrow 2\cos x = -\sqrt{3}$, $\cos x \neq \sqrt{3}$

$$\Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow \cos x = \cos\frac{5\pi}{6}$$

$$\Rightarrow x = 2n\pi \pm \frac{5\pi}{6}$$

36. (B) $3^x + 3^y = 3^{x+y}$

On differentiating both sides w.r.t.'x'

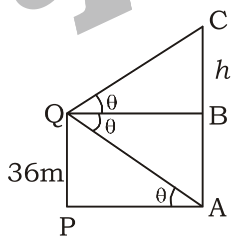
$$3^x\log3 + 3^y\log3\frac{dy}{dx} = 3^{x+y}\log3\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow 3x + 3y\frac{dy}{dx} = 3^{x+y} + 3^{x+y}\frac{dy}{dx}$$

$$\Rightarrow (3^y - 3^{x+y})\frac{dy}{dx} = 3^{x+y} - 3^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{3^x(3^y-1)}{3^y(1-3^x)} \Rightarrow \frac{dy}{dx} = \frac{-3^x(3^y-1)}{3^y(3^x-1)}$$

37. (C)



Let BC = h

and $\angle$BQC = $\angle$BQA = θ

In Δ PQA :-

$$\tan\theta = \frac{36}{PA} \quad ...(i)$$

In Δ BQC:-

$$\tan\theta = \frac{BC}{QB}$$

$$\Rightarrow \tan\theta = \frac{h}{PA} \quad ...(ii)$$

from eq(i) and eq(ii)

$$\frac{36}{PA} = \frac{h}{PA} \Rightarrow h = 36$$

Height of the tower = 36 + 36 = 72 m

38. (B) Hyperbola $16x^2 - 9y^2 - 32x + 36y - 56 = 0$

$\Rightarrow 16x^2 - 32x - 9y^2 + 36y - 56 = 0$

$\Rightarrow 16(x-1)^2 - 16 - 9(y-2)^2 + 36 - 56 = 0$

$\Rightarrow 16(x-1)^2 - 9(y-2)^2 = 36$

$$\Rightarrow \frac{(x-1)^2}{9/4} - \frac{(y-2)^2}{4} = 1$$

$$a^2 = \frac{9}{4} \Rightarrow a = \frac{3}{2}$$

Vertices (X, Y) = $(\pm a, 0)$

$$X = \pm a \Rightarrow x - 1 = \pm\frac{3}{2}$$

$$\Rightarrow x - 1 = \frac{3}{2} \text{ and } x - 1 = -\frac{3}{2}$$

$$\Rightarrow x = \frac{5}{2} \text{ and } x = -\frac{1}{2}$$

$Y = 0 \Rightarrow y - 2 = 0 \Rightarrow y = 2$

Hence vertices are $\left(\frac{5}{2}, 2\right)$ and $\left(-\frac{1}{2}, 2\right)$.

39. (B) $\dfrac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$

$$\Rightarrow \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4} \times \frac{\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}}{\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}}$$

$$\Rightarrow \lim_{x\to 0} \frac{1+\sqrt{1+x^4} - 2}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$$

$$\Rightarrow \lim_{x\to 0} \frac{\sqrt{1+x^4} - 1}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)} \times \frac{\sqrt{1+x^4}+1}{\sqrt{1+x^4}+1}$$

$$\Rightarrow \lim_{x\to 0} \frac{1+x^4-1}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

$$\Rightarrow \lim_{x\to 0} \frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

$$\Rightarrow \frac{1}{\left(\sqrt{1+\sqrt{1+0}} + \sqrt{2}\right)\left(\sqrt{1+0}+1\right)}$$

$$\Rightarrow \frac{1}{\left(\sqrt{2}+\sqrt{2}\right)\times 2} \Rightarrow \frac{1}{2\sqrt{2}\times 2} = \frac{1}{4\sqrt{2}}$$

40. (A) $\int_0^{\pi} |\cos x|^3\, dx \Rightarrow \int_0^{\pi/2} \cos^3 x\, dx - \int_{\pi/2}^{\pi} \cos^3 x\, dx$

$$\Rightarrow \int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4}\right) dx - \int_{\pi/2}^{\pi} \left(\frac{\cos 3x + 3\cos x}{4}\right) dx$$

$$\Rightarrow \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_0^{\pi/2} - \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_{\pi/2}^{\pi}$$

$$\Rightarrow \frac{1}{4}\left[\left(\frac{1}{3}\sin\frac{3\pi}{2} + 3\sin\frac{\pi}{2}\right) - \left(\frac{1}{3}\sin 0 + 3\sin 0\right)\right]$$

$$\Rightarrow -\frac{1}{4}\left[\left(\frac{1}{3}\sin 3\pi + 3\sin\pi\right) - \left(\frac{1}{3}\sin\frac{3\pi}{2} + 3\sin\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \frac{1}{4}\left[\left(-\frac{1}{3}+3\right) - 0\right] - \frac{1}{4}\left[\left(0 - \left(-\frac{1}{3}+3\right)\right)\right]$$

$$\Rightarrow \frac{1}{4}\times\frac{8}{3} + \frac{1}{4}\times\frac{8}{3} = \frac{4}{3}$$

41. (D) Equation $x^2 + 2x + 2 = 0$

$\Rightarrow (x+1)^2 + 1 = 0$

$\Rightarrow (x+1)^2 - (i)^2 = 0$

$\Rightarrow (x + 1 - i)(x + 1 + i) = 0$

$\Rightarrow x = i - 1, -1 - i$

$\alpha = i - 1$ and $\beta = -1 - i$

Now, $\alpha^{17} + \beta^{17}$

$\Rightarrow (\alpha^2)^8 + (\beta^2)^8.\beta$

$\Rightarrow [(i-1)^2]^8 (i-1) + [(-1-i)^2]^8 . (-1-i)$

$\Rightarrow (-2i)^8 (i-1) + (2i)^8 (-1-i)$

$\Rightarrow 256(i-1) + 256(-1-i)$

$\Rightarrow 256i - 256 - 256 - 256i = -512$

42. (B) Curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ Intersect Orthogonally, then

$$\frac{1}{a_1} - \frac{1}{b_1} = \frac{1}{a_2} - \frac{1}{b_2}$$

$$\Rightarrow \frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{b_1} - \frac{1}{b_2}$$

43. (B) Given that a : b = 9 : 4

$$\text{A.M} = \frac{a+b}{2} = \frac{9+4}{2} = \frac{13}{2}$$

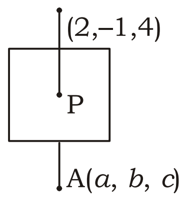
$$\text{G.M} = \sqrt{ab} = \sqrt{9\times 4} = 3 \times 2 = 6$$

$$\text{Now, } \frac{\text{A.M}}{\text{G.M}} = \frac{13}{2\times 6}$$

$\Rightarrow$ A.M : G.M = 13 : 12

44. (B) $\dfrac{a-2}{3} = \dfrac{b+1}{-1} = \dfrac{c-4}{2} = \lambda$

$\Rightarrow a = 3\lambda + 2,\ b = -\lambda - 1,\ c = 2\lambda + 4$



$$P = \left(\frac{a+2}{2}, \frac{b-1}{2}, \frac{c+4}{2}\right) = \left(\frac{3\lambda}{2}+2, -\frac{\lambda}{2}-1, \lambda+4\right)$$

P on the plane $3x - y + 2z + 6 = 0$

$\Rightarrow 3\left(\frac{3\lambda}{2}+2\right)-\left(\frac{-\lambda}{2}-1\right) + 2\,(\lambda + 4) + 6 = 0$

$\Rightarrow \frac{9\lambda}{2} + 6 + \frac{\lambda}{2} + 1 + 2\lambda + 8 + 6 = 0$

$\Rightarrow 7\lambda + 21 = 0 \Rightarrow \lambda = -3$

then, $a = 3\lambda + 2 = -9 + 2 = -7$

$b = -\lambda - 1 = 3 - 1 = 2$

$c = 2\lambda + 4 = 2 \times (-3) + 4 = -2$

The required equation

$\frac{x+7}{4} = \frac{y-2}{2} = \frac{z+2}{-5}$

45. (C) $\left[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\Rightarrow \left(\vec{a}\times\vec{b}\right).\left[\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)\right] = \lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\Rightarrow \left(\vec{a}\times\vec{b}\right)\ \left[\left\{\left(\vec{b}\times\vec{c}\right).\vec{a}\right\}\vec{c} - \left\{\left(\vec{b}\times\vec{c}\right).\vec{c}\right\}.\vec{a}\right] =$

$\lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\left[\because \left(\vec{a}\times\vec{b}\right)\times\vec{c} = (\vec{a}.\vec{c})\vec{b} - \left(\vec{a}.\vec{b}\right)\vec{c}\right]$

$\Rightarrow \left(\vec{a}\times\vec{b}\right)\left[\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]\vec{c} - 0\right] = \lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\Rightarrow \left[\vec{a}\quad \vec{b}\quad \vec{c}\right]\left[\left(\vec{a}\times\vec{b}\right).\vec{c}\right] = \lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\Rightarrow \left[\vec{a}\quad \vec{b}\quad \vec{c}\right]\left[\vec{a}\quad \vec{b}\quad \vec{c}\right] = \lambda\left[\vec{a}\quad \vec{b}\quad \vec{c}\right]^2$

$\Rightarrow \lambda = 1$

46. (B) $\int \frac{1}{x^2\left(x^3+1\right)^{2/3}}\,dx \Rightarrow \int \frac{1}{x^2.x^2\left(1+\frac{1}{x^3}\right)^{2/3}}\,dx$

$\Rightarrow \int \frac{1}{x^4\left(1+\frac{1}{x^3}\right)^{2/3}}\,dx$

Let $1 + \frac{1}{x^3} = t \Rightarrow \frac{-3}{x^4}\,dx = dt$

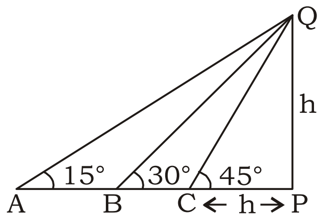
$\Rightarrow \frac{1}{x^4}\,dx = \frac{-1}{3}\,dt$

$\Rightarrow \frac{-1}{3}\int \frac{dt}{t^{2/3}} \Rightarrow \frac{-1}{3}\times\frac{t^{-\frac{2}{3}+1}}{\frac{-2}{3}+1} + c$

$\Rightarrow \frac{-1}{3}\times\frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c \Rightarrow -t^{\frac{1}{3}} + c \Rightarrow -\left(1+\frac{1}{x^3}\right)^{\frac{1}{3}} + c$

47. (A) Let PQ = h m,

then CP = h m [$\because \angle$PCQ = 45°]



In ΔBPQ:-

$\tan 30° = \frac{PQ}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BC+h}$

$\Rightarrow BC = h\left(\sqrt{3}-1\right)$

In ΔAPQ:-

$\tan 15° = \frac{PQ}{AP} \Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{AC+h}$

$\Rightarrow AC + h = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)h$

$\Rightarrow AC + h = \left(2+\sqrt{3}\right)h \Rightarrow AC = h\left(\sqrt{3}+1\right)$

Now, $\frac{BC}{AC} = \frac{h(\sqrt{3}-1)}{h(\sqrt{3}+1)}$

Hence, BC : AC = $\left(\sqrt{3}-1\right):\left(\sqrt{3}+1\right)$

48. (C) $\frac{x-2}{2} = \frac{y+1}{-3} = \frac{z-2}{4} = \lambda$

P $(2\lambda + 2, -3\lambda - 1, 4\lambda + 2)$

lies on the plane $x + 2y - 5z + 16 = 0$

then, $2\lambda + 2 + 2(-3\lambda - 1) - 5(4\lambda + 2) + 16 = 0$

$\Rightarrow 2\lambda + 2 - 6\lambda - 2 - 20\lambda - 10 + 16 = 0$

$\Rightarrow -24\lambda + 6 = 0 \Rightarrow \lambda = \frac{1}{4}$

Point P $= \left(2\times\frac{1}{4}+2, -3\times\frac{1}{4}-1, 4\times\frac{1}{4}+2\right)$

$= \left(\frac{5}{2}, \frac{-7}{4}, 3\right)$

Let Q $= \left(\frac{1}{2}, \frac{5}{4}, -3\right)$

The required distance

$= \sqrt{\left(\frac{5}{2}-\frac{1}{2}\right)^2 + \left(-\frac{7}{4}-\frac{5}{4}\right)^2 + (3+3)^2}$

$= \sqrt{4+9+36} = 7$

49. (B) Equation of the sphere
$(x + 3)^2 + (y - 2)^2 + (z - 4)^2 = 4^2$
$\Rightarrow x^2 + 9 + 6x + y^2 + 4 - 4y + z^2 + 16 - 8z = 16$
$\Rightarrow x^2 + y^2 + z^2 + 6x - 4y - 8z + 13 = 0$

50. (A) $n(S) = 6 \times 6 = 36$
$E = \{(6, 4), (4, 6), (5, 5)\}$; $n(E) = 3$
The required Probability = $\frac{3}{36} = \frac{1}{12}$

51. (C) $\frac{2b^2}{a} = 6 \Rightarrow b^2 = 3a$ ...(i)
and $2b = \frac{1}{2}(2ae) \Rightarrow 2b = ac$
On squaring
$\Rightarrow 4b^2 = a^2e^2$
$\Rightarrow 4 \times 3a = a^2\left(1+\frac{3a}{a^2}\right)$ [from eq(i)]
$\Rightarrow 12a = a^2 + 3a \Rightarrow a^2 = 9a \Rightarrow a = 9$
from eq(i)
$b^2 = 3 \times 9 = 27$
eccentricity $e = \sqrt{1+\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1+\frac{27}{81}}$
$\Rightarrow e = \sqrt{1+\frac{1}{3}} \Rightarrow e = \frac{2}{\sqrt{3}}$

52. (B) $a + d$, $a + 4d$ and $a + 8d$ are in G.P,
then $(a + 4d)^2 = (a + d)(a + 8d)$
$\Rightarrow a^2 + 16d^2 + 8ad = a^2 + ad + 8ad + 8d^2$
$\Rightarrow 8d^2 = ad \Rightarrow \frac{a}{d} = 8$
Now, common ratio = $\frac{a+4d}{a+d}$
$= \frac{\frac{a}{d}+4}{\frac{a}{d}+1} = \frac{8+4}{8+1} = \frac{12}{9} = \frac{4}{3}$

53. (B) $16x^2 + 7y^2 = 112 \Rightarrow \frac{x^2}{7} + \frac{y^2}{10} = 1$
here $a^2 = 7$, $b^2 = 16$
Now, $e = \sqrt{1-\frac{a^2}{b^2}} \Rightarrow e = \sqrt{1-\frac{7}{16}}$
$\Rightarrow e = \sqrt{\frac{9}{16}} = \frac{3}{4}$

54. (C)

55. (B) The equation of the given circle $2x^2 + 2y^2 - 7x - 9y - 13 = 0$ ...(i)
The length of tangent drawn from point $(3, -4)$
$= \sqrt{(3)^2 + (-4)^2 - \frac{7}{2}(3) - \frac{9}{2}(-4) - \frac{13}{2}} = \sqrt{26}$ unit

56. (A) Let us consider PT_1 and PT_2 be the length of the tangents from $P(f, g)$ to the circles $x^2 + y^2 = 6$ and $x^2 + y^2 + 3x + 3y = 0$ respecively, then
$PT_1 = \sqrt{f^2 + g^2 - 6}$ and $PT_2 = \sqrt{f^2 + g^2 + 3f + 3g}$
Now, according to question $(PT_1) = 2(PT_2)$
$\Rightarrow (PT_1)^2 = 4(PT_2)^2$
$\Rightarrow f^2 + g^2 - 6 = 4[f^2 + g^2 + 3f + 3g]$
$\Rightarrow 3f^2 + 3g^2 + 12f + 12g + 6 = 0$
$\Rightarrow f^2 + g^2 + 4f + 4g + 2 = 0$

57. (C) Let us consider (PT_1) and (PT_2) are the lengths of the tangents drawn from point $(1, 2)$ to the circles $x^2 + y^2 + x + y - 4 = 0$ and $3x^2 + 3y^2 - x - y - \lambda = 0$ respectively, then
$PT_1 = \sqrt{(1)^2 + (2)^2 + 1 + 2 - 4} = 2$
$PT_2 = \sqrt{(1)^2 + (2)^2 - \frac{1}{3} - \frac{2}{3} - \frac{\lambda}{3}} = \sqrt{4 - \frac{\lambda}{3}}$
Now, $\frac{(PT_1)}{(PT_2)} = \frac{4}{3} \Rightarrow 3(PT_1) = 4(PT_2)$
On squaring both sides
$\Rightarrow 9(PT_1)^2 = 16(PT_2)^2$
$\Rightarrow 9 \times (2)^2 = 16\left(4 - \frac{\lambda}{3}\right) \Rightarrow \lambda = \frac{21}{4}$

58. (C) The equation of the circle passing through (0, 0), (1, 0) and (0, 1) is
$x^2 + y^2 - x - y = 0$
If it passes through the point (t, t), then
$t^2 + t^2 - t - t = 0$
$\Rightarrow 2t^2 - 2t = 0 \Rightarrow 2t(t-1) = 0$
$\Rightarrow t = 1$

59. (A) Equation of circle $x^2 + y^2 + 4x - 4y + 4 = 0$
Let us consider the eqution of line which has equal intercept on coordinate axes $x + y = a$
If equation (ii) is the tangent of circle (i) then perpendicular drawn from centre to this will be radius
centre $(-2, +2)$
So $\left|\frac{-2+2-a}{\sqrt{(1)^2+(1)^2}}\right| = \sqrt{(-2)^2(2)^2 - 4}$
$\Rightarrow \frac{a}{\sqrt{2}} = 2 \Rightarrow a = 2\sqrt{2}$
Hence the equation $x + y = 2\sqrt{2}$

60. (A) The equation of a chord joining points having eccentric angle α and β is given as

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

If it passes the focus (ae, 0), then

$$e\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\Rightarrow e=\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}=\frac{2\sin\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha-\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right).\cos\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow e=\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$$

61. (A) The ellipse equation $\frac{x^2}{4}+\frac{y^2}{1}=1$

Straight line equation $y = 4x + c$

We know that line $y = mx + c$ touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

If $c^2 = a^2m^2 + b^2$, so

Here $a^2 = 4$, $b^2 = 1$, $m = 4$

$\therefore\ c^2 = 4(4)^2 + 1$

$\Rightarrow c^2 = 65 \Rightarrow c = \pm\sqrt{65}$

Hence there are two values of c.

62. (A) The ellipse equation $\frac{x^2}{6}+\frac{y^2}{2}=1$

$$\Rightarrow \frac{x^2}{(\sqrt{6})^2}+\frac{y^2}{(\sqrt{2})^2}=1$$

Let θ be the eccentric angle of the point P, then the coordinate of the point P($\sqrt{6}\cos\theta$, $\sqrt{2}\sin\theta$)

The centre of the ellipse is at the origin

It is given that OP = 2

$\Rightarrow \sqrt{(\sqrt{6}\cos\theta)^2+(\sqrt{2}\sin\theta)^2}=2$

$\Rightarrow 6\cos^2\theta + 2\sin^2\theta = 4$

$\Rightarrow 3\cos^2\theta + \sin^2\theta = 2$

$\Rightarrow 3(1-\sin^2\theta) + \sin^2\theta = 2$

$\Rightarrow 2\sin^2\theta = 1$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \pm\frac{\pi}{4}$

63. (B) The ellipse equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

The equation of the normal at(x_1, y_1) to ellipse is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

Here $x_1 = ae$ and $y_1=\frac{b^2}{a}$

So, the equation of the normal at positive end of the latus rectum is

$$\frac{a^2x}{ae}-\frac{b^2y}{b^2/a}=a^2-b^2$$

$\Rightarrow \frac{ax}{e}-ay=a^2-a^2(1-e^2)$

$\Rightarrow x - ey - e^3a = 0$

64. (A) Asymptotes of the given hyperbola are

$y=\pm\frac{b}{a}x$

Therefore angle between them $= 2\tan^{-1}\left(\frac{b}{a}\right)$

65. (C) Let $p(x, y)$ be any point on the conic, then SP = e.PM

$\Rightarrow \sqrt{(x-1)^2+(y+1)^2}=\sqrt{2}\left(\frac{x-y+1}{\sqrt{2}}\right)$

On solving

$\Rightarrow 2xy - 4x + 4y + 1 = 0$

66. (C) For given ellipse $a^2 = 16$

So, eccentricity $e=\sqrt{1-\frac{b^2}{a^2}} \Rightarrow \sqrt{1-\frac{b^2}{16}}$

$\Rightarrow \frac{\sqrt{16-b^2}}{4}$

So the foci. of the ellipse are (ae, 0) i.e. $\left(\pm\sqrt{16-b^2},0\right)$

Now, for the hyperbola $a^2=\left(\frac{12}{5}\right)^2$,

$b^2=\left(\frac{9}{5}\right)^2$

The eccentricity $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{81}{144}}=\frac{5}{4}$

The foci of hyperbola ($\pm ae$, 0)

i.e. (± 3, 0)

Since the foci of ellipse and hyperbola coincide so $\sqrt{16-b^2}=3 \Rightarrow b^2 = 7$

67. (C) Since the difference of the focal distances of any point on a hyperbola is constant equal to its transverse axis, therefore the locus of P is a hyperbola.

68. (A) $\int \frac{dx}{\sqrt{\sin^3 x.\cos x}} \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^3 x}}$

Put tan$x$ = t ⇒ sec²$x.dx$ = dt

$$\Rightarrow \int \frac{dt}{\sqrt{t^3}} \Rightarrow \int t^{\frac{-3}{2}} dt \Rightarrow \frac{t^{\left(\frac{-3}{2}+1\right)}}{\left(\frac{-3}{2}+1\right)} + c$$

$$\Rightarrow -2.t^{-\frac{1}{2}} + c \Rightarrow -\frac{2}{\sqrt{\tan x}} + c$$

69. (B) $\int \frac{\cos x + x\sin x}{x(x+\cos x)} dx$

$$\Rightarrow \int \frac{(x+\cos x) - x + x\sin x}{x(x+\cos x)} dx$$

$$\Rightarrow \int \left\{\frac{1}{x} - \frac{(1-\sin x)}{(x+\cos x)}\right\} dx$$

$$\Rightarrow \int \frac{1}{x} dx - \int \frac{(1-\sin x)}{(x+\cos x)} dx$$

$$\Rightarrow \log x - \log(x + \cos x) + c$$

$$\Rightarrow \log\left(\frac{x}{x+\cos x}\right) + c$$

70. (A) $\int \frac{(\cos^3 x + \cos^5 x)}{(\sin^2 x + \sin^4 x)} dx$

$$\Rightarrow \int \frac{\cos x(\cos^2 x + \cos^4 x)}{(\sin^2 x + \sin^4 x)} dx$$

$$\Rightarrow \int \frac{\cos x(1-\sin^2 x) + (1-\sin^2 x)^2}{(\sin^2 x + \sin^4 x)} dx$$

Put sin$x$ = t ⇒ cos$x$ dx = dt

$$\Rightarrow \int \frac{[(1-t^2) + (1-t^2)^2]}{(t^2+t^4)} dt$$

$$\Rightarrow \int \frac{(t^4+t^2) + 2(t^2+1) - 6t^2}{t^2(t^2+1)} dt$$

$$\Rightarrow \int \left[1 + \frac{2}{t^2} - \frac{6}{(t^2+1)}\right] dt \Rightarrow t - \frac{2}{t} - 6\tan^{-1} t + c$$

$$\Rightarrow \sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$$

71. (B) $\int \frac{1}{x(x^n+1)} dt \Rightarrow \int \frac{x^{n-1}}{x^n(x^n+1)} dt$

Put $x^n = t \Rightarrow n.x^{n-1}\, dx = dt$

$$\Rightarrow x^{n-1} dx = \frac{1}{n}.dt$$

$$\Rightarrow \int \frac{\frac{1}{n}.dt}{t(t+1)} \Rightarrow \frac{1}{n}\int \frac{1}{t(t+1)} dt \Rightarrow \frac{1}{n}\int\left[\frac{1}{t} - \frac{1}{(t+1)}\right] dx$$

$$\Rightarrow \frac{1}{n}[\log t - \log(t+1)] + c$$

$$\Rightarrow \frac{1}{n}.\log\frac{t}{(t+1)} + c \Rightarrow \frac{1}{n}\log\frac{x^n}{(x^n+1)} + c$$

72. (B) If vertices of a parallelogram are z_1, z_2, z_3, z_4, then as diagonals bisect each other

$$\therefore \frac{z_1+z_3}{2} = \frac{z_2+z_4}{2} \Rightarrow z_1 + z_3 = z_2 + z_4$$

73. (B) $(1 + \omega)^7 = A + B\omega$

$\Rightarrow (-\omega^2)^7 = A + B\omega$ $(\because 1 + \omega + \omega^2 = 0)$

$\Rightarrow -\omega^{14} = A + B\omega$

$\Rightarrow -\omega^2 = A + B\omega$ $(\because \omega^3 = 1)$

$\Rightarrow 1 + \omega = A + B\omega \Rightarrow A = 1, B = 1$

74. (D) $|z| = |\omega|$ and arg $z = \pi -$ arg ω

Let $\omega = re^{i\theta}$, then $z = re^{i(\pi-\theta)}$

$\Rightarrow z = re^{i\pi}.e^{-i\theta}$

$= (re^{-i\theta})(\cos\pi + i\sin\pi) = \overline{\omega} \times (-1) = -\overline{\omega}$

75. (B) Probability of hitting the target = 1 – probability of no one hitting the target

$$= 1 - \left(\frac{3}{4}\right)\times\left(\frac{2}{5}\right) = \frac{7}{10}$$

76. (A) Total Balls = 18

Required probability = $\frac{{}^4C_3}{{}^{18}C_3} = \frac{1}{204}$

77. (A) $n(S) = {}^{15}C_2 = 105$

$n(E) = {}^5C_2 + {}^7C_2 = 10 + 21 = 31$

Now, $P(E) = \frac{n(E)}{n(S)} = \frac{31}{105}$

78. (C) tan$x$ + sec$x$ = 2cos$x$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow \sin x + 1 = 2\cos^2 x \Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2}, -1$$

$$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \in [0, 2\pi]$$

But for $x = \frac{3\pi}{2}$, given eq. is not defined,

∴ Only 2 solutions.

79. (B) $\sec 2x - \tan 2x \Rightarrow \frac{1-\sin 2x}{\cos 2x}$

$\Rightarrow \frac{1-\cos 2\left(\frac{\pi}{4}-x\right)}{\sin 2\left(\frac{\pi}{4}-x\right)}$

$\Rightarrow \frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)} = \tan\left(\frac{\pi}{4}-x\right)$

80. (C) $\sin\left\{(\omega^{10}+\omega^{23})\pi - \frac{\pi}{4}\right\} \Rightarrow \sin\left\{(\omega+\omega^2)\pi - \frac{\pi}{4}\right\}$

$\Rightarrow \sin\left\{-\pi - \frac{\pi}{4}\right\} \Rightarrow \sin\left\{-\frac{5\pi}{4}\right\} = \frac{1}{\sqrt{2}}$

81. (B) Parabola $9y^2 - 16x - 12y - 12 = 0$

$\Rightarrow y^2 - \frac{4}{3}y = \frac{16}{9}x + \frac{4}{3}$

$\Rightarrow \left(y-\frac{2}{3}\right)^2 = \frac{16}{9}x + \frac{4}{3} + \frac{4}{9}$

$\Rightarrow \left(y-\frac{2}{3}\right)^2 = \frac{16}{9}x + \frac{16}{9}$

$\Rightarrow \left(y-\frac{2}{3}\right)^2 = \frac{16}{9}(x+1)$

Axis of parabola

$y - \frac{2}{3} = 0 \Rightarrow 3y = 2$

82. (B) The parabola equation $y^2 = 4ax$ ...(i)

$\Rightarrow 2y.\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y}$

At point $P(x_1, y_1)$

the length of sub-tangent $= \frac{y_1}{\left(\frac{dy}{dx}\right)_p}$

$= \frac{y_1}{\frac{2a}{y_1}} = \frac{y_1^2}{2a}$

Length of subnormal $= y_1.\left(\frac{dy}{dx}\right)$

$= y_1.\frac{2a}{y_1} = 2a$

Now it is clear that subtangent $\left(\frac{y_1^2}{2a}\right)$, ordinate (y_1) and subnormal $(2a)$ are in G.P. Series.

83. (B) The equation of tangent at point $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_1\right)$ to the parabola $y^2 = 4ax$ are

$t_1 y = x + at_1^2$ ...(i)

and $t_2 y = x + at_2^2$ ...(ii)

These lines are perpendicular if

$m_1 m_2 = -1 \Rightarrow \frac{1}{t_1}.\frac{1}{t_2} = -1 \Rightarrow t_1.t_2 = -1$

84. (C) Curve $2y = 3 - x^2 \Rightarrow 2.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = -x$

At point (1, 1), $\frac{dy}{dx} = -1$

Slope of normal $= -\frac{1}{(-1)} = 1$

equation of normal at point (1, 1)

$y - 1 = 1(x - 1)$

$\Rightarrow x - y = 0$

85. (C) Curve $y^2 = x$ ...(i)

$2y.\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

Slope = tan45° = 1

Now, $\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

Put the value of y in eq.(i)

$x = \frac{1}{4}$

The required point $= \left(\frac{1}{4}, \frac{1}{2}\right)$

86. (A) Curve $x = a(\theta - \sin\theta) \Rightarrow \frac{dx}{d\theta} = a(1+\cos\theta)$

$y = a(1-\cos\theta) \Rightarrow \frac{dy}{d\theta} = a\sin\theta$

At $\theta = \frac{\pi}{2}$, Point $\left(\frac{a\pi}{2}, a\right)$

Now, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\sin\theta}{1+\cos\theta}$

$$\frac{dy}{dx}\left(at\,\theta=\frac{\pi}{2}\right) = \frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}} = \frac{1}{1+0} = 1$$

Length of normal = $y_1\sqrt{1+\left(\frac{dy}{dx}\right)^2_{\left(at\,\theta=\frac{\pi}{2}\right)}}$

$= a\sqrt{1+1} = a\sqrt{2}$

87. (D) $\begin{bmatrix}1 & 2\\ -3 & 4\end{bmatrix} + 2X = \begin{bmatrix}4 & 3\\ 2 & -6\end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix}4 & 3\\ 2 & -6\end{bmatrix} - \begin{bmatrix}1 & 2\\ -3 & 4\end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix}3 & 1\\ 5 & -10\end{bmatrix}$

$\Rightarrow X = \begin{bmatrix}3/2 & 1/2\\ 5/2 & -5\end{bmatrix}$

88. (B) $y = x^x$
taking log both sides
$\Rightarrow \log y = x.\log x$
On differentiating both sides w.r.t.'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + (\log x).1$

$\Rightarrow \frac{dy}{dx} = y(1+\log x)$

$\Rightarrow \frac{dy}{dx} = x^x(1+\log x)$

89. (D) $6\cos^2\theta - 2\cos2\theta - 3 = 0$
$\Rightarrow 6\cos^2\theta - 2(2\cos^2\theta - 1) - 3 = 0$
$\Rightarrow 6\cos^2\theta - 4\cos^2\theta - 1 = 0$

$\Rightarrow 2\cos^2\theta = 1 \Rightarrow \cos^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2 \Rightarrow \theta = 45°$

Now, $\sec^2 3\theta = \sec^2 135° \Rightarrow \sec^2(90+45)$

$\Rightarrow (-\text{cosec } 45)^2 \Rightarrow (\sqrt{2})^2 = 2$

90. (D) $f(x) = \begin{cases}\frac{a\sin2x - b\cos x}{\frac{\pi}{2}-x}, & x > \frac{\pi}{2}\\ 4, & x = \frac{\pi}{2}\\ \frac{2b\cos x}{\frac{\pi}{2}-x}, & x < \frac{\pi}{2}\end{cases}$ is

continuous function, then

$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} f(x) = \lim_{x\to\left(\frac{\pi}{2}\right)^+} f(x) = f\left(\frac{\pi}{2}\right)$$

Now, $= \lim_{x\to\left(\frac{\pi}{2}\right)^-} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{2b\cos x}{\frac{\pi}{2}-x} = 4$

by L-Hospital Rule

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{-2b\sin x}{-1} = 4$

$\Rightarrow 2b\sin\frac{\pi}{2} = 4$

$\Rightarrow 2b = 4 \Rightarrow b = 2$

and $\lim_{x\to\left(\frac{\pi}{2}\right)^+} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{a\sin2x - b\cos x}{\frac{\pi}{2}-x} = f\left(\frac{\pi}{2}\right)$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{2a\cos x + b\sin x}{-1} = 4$

$\Rightarrow \frac{2a\cos\pi + b\sin\pi}{-1} = 4$

$\Rightarrow \frac{-2a+0}{-1} = 4 \Rightarrow a = 2$

Hence $= a - b = 2 - 2 = 0$

91. (C) Given that vertices are (–1, 2), (–2,–5) and (6, a)
A.T.Q,

Area $= \frac{1}{2}\begin{vmatrix}-1 & 2 & 1\\ -2 & -5 & 1\\ 6 & a & 1\end{vmatrix}$

$\Rightarrow 4 = \frac{1}{2}[-1(-5-a) - 2(-2-6) + 1(-2a+30)]$

$\Rightarrow 8 = [5 + a + 16 - 2a + 30]$
$\Rightarrow 8 = -a + 51 \Rightarrow a = 43$

92. (D) $\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{B+C}{2}\right).\sin\left(\frac{C+A}{2}\right)$

$\Rightarrow \sin\left(\frac{180-C}{2}\right).\sin\left(\frac{180-A}{2}\right).\sin\left(\frac{180-B}{2}\right)$

$\Rightarrow \cos\frac{C}{2}.\cos\frac{A}{2}.\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2} \times 2\cos\frac{C}{2}.\cos\frac{A}{2}.\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}\left[\cos\left(\frac{C+A}{2}\right)+\cos\left(\frac{C-A}{2}\right)\right].\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}\left[\cos\left(\frac{180-B}{2}\right)+\cos\left(\frac{C-A}{2}\right)\right].\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}\left[\sin\frac{B}{2}+\cos\frac{C-A}{2}\right]\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}.\sin\frac{B}{2}.\cos\frac{B}{2}+\frac{1}{2}\sin\frac{C+A}{2}.\cos\frac{C-A}{2}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times 2\sin\frac{B}{2}.\cos\frac{B}{2} + \frac{1}{2} \times \frac{1}{2}$

$\times\left[2\sin\frac{C+A}{2}.\cos\frac{C-A}{2}\right]$

$\Rightarrow \frac{1}{4}\sin B + \frac{1}{4}$

$\left[\sin\left(\frac{C+A}{2}+\frac{C-A}{2}\right)+\sin\left(\frac{C+A}{2}-\frac{C-A}{2}\right)\right]$

$\Rightarrow \frac{1}{4}\sin B + \frac{1}{4}[\sin C + \sin A]$

$\Rightarrow \frac{1}{4}[\sin A + \sin B + \sin C]$

93. (A) A is the transpose of B.

94. (C) The required no. of terms = ${}^{n+2}C_2$

$= \frac{(n+2)!}{2!n!} = \frac{(n+2)(n+1)n!}{2 \times n!}$

$= \frac{(n+1)(n+2)}{2}$

95. (B) Let $z = \left(\frac{1-2i}{2+i}\right)^2$

$\Rightarrow z = \frac{1+4i^2-4i}{4+i^2+4i}$

$\Rightarrow z = \frac{1-4-4i}{4-1+4i}$

$\Rightarrow z = \frac{-3-4i}{3+4i}$

$\Rightarrow z = \frac{-1(3+4i)}{3+4i} = -1$

Conjugate of $z = \bar{z} = -1$

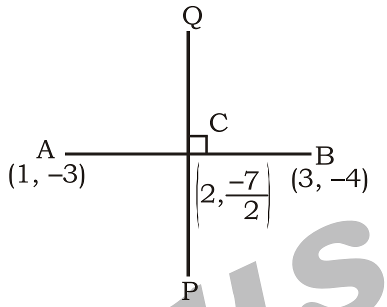
96. (A) Function is one-one but onto.

97. (C)

2	51	1
2	25	1
2	12	0
2	6	0
2	3	1
2	1	1
	0	

$(51)_{10} = (110011)_2$

98. (C)



mid-point of line joining

the points = $\left(\frac{1+3}{2},\frac{-3-4}{2}\right) = \left(2,\frac{-7}{2}\right)$

Slope of line AB (m_1) = $\frac{-4+3}{3-1} = \frac{-1}{2}$

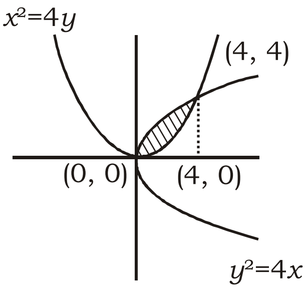
Slope of line PQ (m_2) = $\frac{-1}{\frac{-1}{2}} = 2$

Equation of line PQ

$y + \frac{7}{2} = 2(x-2)$

$\Rightarrow 4x - 2y = 11$

99. (C)



$y_1 \Rightarrow y = 2\sqrt{x}$ and $y_2 \Rightarrow y = \frac{x^2}{4}$

The required Area = $\int_0^4 (y_1 - y_2)\,dx$

$= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx \quad = \left[2 \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{4 \times 3}\right]_0^4$

$= \left[\frac{4}{3} \times (4)^{\frac{3}{2}} - \frac{1}{12}(4)^3\right] = \frac{37}{3} - \frac{16}{3}$

$= \frac{16}{3}$ sq. unit

100. (A) Given that $x^2 + y^2 = 8$

Let A = x^2y^2

$\Rightarrow A = x^2 (8 - x^2)$

$\Rightarrow A = 8x^2 - x^4$

$\Rightarrow \frac{dA}{dx} = 16x - 4x^3$

$\Rightarrow \frac{d^2A}{dx^2} = 16 - 12x^2$

for maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 16x - 4x^3 = 0$

$\Rightarrow 4x(4 - x^2) = 0$

$\Rightarrow x = 0, 2, -2$

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=0} = 16 - 2 \times 0 = 16$ (minima)

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=2} = 16 - 12 \times 2^2 = -32$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{at\,x=-2} = 16 - 12(-2)^2 = -32$ (maxima)

Function minimum at $x = 0$, $y = 2\sqrt{2}$

Minimum value of $x^2y^2 = 0$

101. (C) We know that

$\sin ix = \frac{e^x - e^{-x}}{-2i}$ and $\cos ix = \frac{e^x + e^{-x}}{2}$

Now, $\cos ix - i\sin ix = \frac{e^x + e^{-x}}{2} - i \times \frac{e^x - e^{-x}}{-2i}$

$\Rightarrow \cos ix - i\sin ix = \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = e^x$

102. (B) Word "STATEMENT"

The total no. of arrangement $= \frac{9!}{3!2!} = \frac{9!}{12}$

No. of arrangement when T's come together $= \frac{7!}{2!} = \frac{7!}{2}$

No. of arrangement when T's don't come together $= \frac{9!}{12} - \frac{7!}{2} = 6 \times 7! - \frac{7!}{2} = \frac{11 \times 7!}{2}$

103. (C) $y = \text{cosec}(\cot^{-1}x)$ ...(i)

On differentiating both sides w.r.t. 'x'

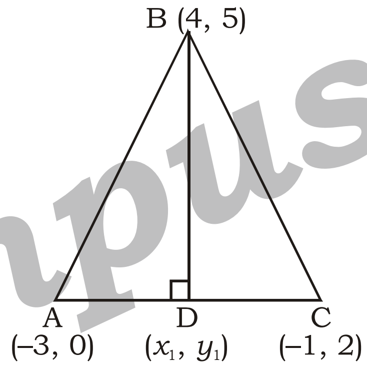
$\Rightarrow \frac{dy}{dx} = -\text{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\frac{-1}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \text{cosec}(\cot^{-1}x).\frac{x}{1+x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{yx}{1+x^2}$ [from eq (i)]

$\Rightarrow (1 + x^2)dy = yx\,dx$

104. (C) Let D = (x_1, y_1)



Slope of line $AC(m_1) = \frac{2-0}{-1+3} = 1$

Slope of line BD $(m_2) = \frac{y_1 - 5}{x_1 - 4}$

Now, $m_1 \times m_2 = -1$

$\Rightarrow 1 \times \frac{y_1 - 5}{x_1 - 4} = -1$

$\Rightarrow x_1 + y_1 = 9$ ...(i)

Equation of line (AC)

$y - 2 = \frac{2-0}{-1+3}(x + 1)$

$\Rightarrow y - 2 = x + 1$

$\Rightarrow x - y = -3$

Point D(x_1, y_1) lies on the line AC

$x_1 - y_1 = -3$ ...(ii)

from eq. (i) and eq. (ii)

$x_1 = 3, y_1 = 6$

Co-ordinate of foot of altitude = (3, 6).

105. (C) Let circumcentre of $\Delta ABC(P) = (x_1, y_1)$
$AP = BP = CP$
Now, $AP^2 = BP^2$
$\Rightarrow (x_1+3)^2 + (y_1-0)^2 = (x_1-4)^2 + (y_1-5)^2$
$\Rightarrow x_1^2 + 9 + 6x_1 + y_1^2 = x_1^2 + 16 - 8x_1 + y_1^2 + 25 - 10y_1$
$\Rightarrow 9 + 6x_1 = 16 - 8x_1 + 25 - 10y_1$
$\Rightarrow 14x_1 + 10y_1 = 32$
$\Rightarrow 7x_1 + 5y_1 = 16$ ...(i)
Now, $AP^2 = CP^2$
$\Rightarrow (x_1+3)^2 + (y_1-0)^2 = (x_1+1)^2 + (y_1-2)^2$
$\Rightarrow x_1^2 + 9 + 6x_1 + y_1^2 = x_1^2 + 1 + 2x_1 + y_1^2 + 4 - 4y_1$
$\Rightarrow 9 + 6x_1 = 1 + 2x_1 + 4 - 4y_1$
$\Rightarrow 4x_1 + 4y_1 = -4$
$\Rightarrow x_1 + y_1 = -1$ ...(ii)
from eq(i) and eq(ii)
$x_1 = \frac{21}{2}$ and $y_1 = \frac{-23}{2}$
Hence circumcentre of ΔABC
$= \left(\frac{21}{2}, \frac{-23}{2}\right)$

106. (B) Centroid of ΔABC
$= \left[\frac{-3+4-1}{3}, \frac{0+5+2}{3}\right] = \left(0, \frac{7}{3}\right)$

107. (B) $a + 46d = 434$ ...(i)
$a + 433d = 47$ ...(ii)
from eq(i) and eq(ii)
$d = -1$ and $a = 480$
let n^{th} term is 0.
then $0 = a + (n-1)d$
$\Rightarrow 0 = 480 + (n-1)(-1)$
$\Rightarrow n - 1 = 480 \Rightarrow n = 481$

108. (D) Matrix A $\rightarrow y \times (y-7)$
Matrix B $\rightarrow x \times (9-x)$
Both AB and BA exist,
then $y - 7 = x \Rightarrow x - y = -7$ ...(i)
and $y = 9 - x \Rightarrow x + y = 9$ ...(ii)
from eq(i) and eq(ii)
$x = 1$ and $y = 8$

109. (A) Equation
$ax^2 + cx - b = 0$
Roots are cot(B/2) and cot(C/2).
then $\cot\frac{B}{2} + \cot\frac{C}{2} = \frac{-c}{a}$
and $\cot\frac{B}{2}.\cot\frac{C}{2} = \frac{-b}{a}$
Now, $\cot\left(\frac{B}{2}+\frac{C}{2}\right) = \frac{\cot\frac{B}{2}.\cot\frac{C}{2} - 1}{\cot\frac{B}{2} + \cot\frac{C}{2}}$
$\Rightarrow \cot\left(\frac{180-A}{2}\right) = \frac{-\frac{b}{a} - 1}{\frac{-c}{a}}$
$\Rightarrow \tan\frac{A}{2} = \frac{-b-a}{-c}$
We know that A = 90°
$\tan 45° = \frac{b+a}{c} \Rightarrow c = a + b$

110. (B) We know that
A.M. $\geq$ G.M. $\geq$ H.M.
Hence $\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$
$\Rightarrow \frac{2ab}{a+b} \leq \sqrt{ab} \leq \frac{a+b}{2}$

111. (C) $y = 9 - 9^{1/3} + 9^{2/3}$
$\Rightarrow y - 9 = 9^{2/3} - 9^{1/3}$ ...(i)
$\Rightarrow (y-9)^3 = (9^{2/3} - 9^{1/3})^3$
$\Rightarrow y^3 - 729 - 3 \times y \times 9(y-9)$
$= 9^2 - 9 - 3 \times 9^{2/3} \times 9^{1/3}(9^{2/3} - 9^{1/3})$
$\Rightarrow y^3 - 729 - 27y^2 + 243y = 81 - 9 - 27(y-9)$
from eq(i)
$\Rightarrow y^3 - 27y^2 + 270y - 1044 = 0$
$\Rightarrow y^3 - 27y^2 + 270y - 44 = 1044 - 44$
$\Rightarrow y^3 - 27y^2 + 280y - 44 = 1000$

112. (D) Direction ratio (3, –1, –2) and (2, y, –3)
Angle between lines
$\cos\theta = \frac{3\times2 + (-1)\times y + (-2)\times(-3)}{\sqrt{9+1+4}\sqrt{4+y^2+9}}$
$\Rightarrow \cos\frac{\pi}{2} = \frac{6-y+6}{\sqrt{14}\sqrt{y^2+13}}$
$\Rightarrow 0 = \frac{12-y}{\sqrt{14}\sqrt{y^2+13}}$
$\Rightarrow 12 - y = 0 \Rightarrow y = 12$

113. (B) degree = 2

114. (C) Let $y = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + \ldots\ldots\ldots\ldots}}}$
$\Rightarrow y = \sqrt{3+2y} \Rightarrow y^2 = 3 + 2y$
$\Rightarrow y^2 - 2y - 3 = 0 \Rightarrow (y-3)(y+1) = 0$
$\Rightarrow y = -1, 3$
Hence $\sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + \ldots\ldots\ldots}}} = 3$

115. (D) $\begin{vmatrix} 1+y & 1 & 1 \\ 1 & 1+z & 1 \\ 1 & 1 & 1+x \end{vmatrix} = k$

$\Rightarrow (1+y)\left[(1+z)(1+x)-1\right]$

$-1\left[1+x-1\right]+1(1-1-z)=k$

$\Rightarrow (1+x)(1+y)(1+z)\ -1-y-x-z=k$

$\Rightarrow 1+y+z+yz+x+xy+xz+xyz$

$-1-y-x-z=k$

$\Rightarrow \dfrac{xy+yz+zx+xyz}{xyz}=\dfrac{k}{xyz}$

$\Rightarrow z^{-1}+x^{-1}+y^{-1}+1=\dfrac{k}{xyz}$

given that $x^{-1}+y^{-1}+z^{-1}=0$

$\Rightarrow 0+1=\dfrac{k}{xyz} \Rightarrow k=xyz$

116. (C) given that

$\dfrac{x^2}{2}+\dfrac{y^2}{18}=1$

$a=\sqrt{2}$, $b=\sqrt{18}$

Area of an ellipse = πab

$=\pi\sqrt{2}\times\sqrt{18}=6\pi$ sq. unit

117. (B) $z=\dfrac{1-2i}{1-i}-\dfrac{3-i}{1+2i}$

$z=\dfrac{(1-2i)(1+i)}{(1-i)(1+i)}-\dfrac{(3-i)(1-2i)}{(1+2i)(1-2i)}$

$z=\dfrac{3-i}{2}-\dfrac{1-7i}{3}$

$z=\dfrac{7+11i}{6}$

Now,

$z^2+z\bar{z}=\left(\dfrac{7+11i}{6}\right)^2+\left(\dfrac{7+11i}{6}\right)\left(\dfrac{7-11i}{6}\right)$

$=-\dfrac{72+154i}{36}+\dfrac{60}{36}=\dfrac{-6+77i}{18}$

118. (A) curve $\sqrt{x}+\sqrt{y}=\sqrt{2} \Rightarrow y=\left(\sqrt{2}-\sqrt{x}\right)^2$

curve cut the x-axis *i.e.* $y=0$, $x=2$

Area = $\int_0^2 y.dx$

Area = $\int_0^2 \left(\sqrt{2}-\sqrt{x}\right)^2 dx$

Area = $\int_0^2 \left(2+x-2\sqrt{2}\sqrt{x}\right)dx$

Area = $\left[2x+\dfrac{x^2}{2}-2\sqrt{2}\dfrac{x^{3/2}}{3/2}\right]_0^2$

Area = $2\times2+\dfrac{2\times2}{2}-\dfrac{4}{3}\sqrt{2}\,(2)^{3/2}-0$

Area = $4+2-\dfrac{4}{3}\times4=\dfrac{2}{3}$ sq. unit

Short Method:-

Curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Area = $\dfrac{a^2}{6}$

given that $\sqrt{x}+\sqrt{y}=\sqrt{2}$

Area = $\dfrac{(2)^2}{6}=\dfrac{2}{3}$ sq. unit

119. (C) Given that A= $\tan^{-1}3 \Rightarrow \tan A = 3$

and C = $\tan^{-1}2 \Rightarrow \tan C = 2$

Now, tan (A+C) $=\dfrac{\tan A+\tan C}{1-\tan A.\tan C}$

$\Rightarrow \tan(180-B)=\dfrac{2+3}{1-2\times3}$

$\Rightarrow -\tan B=\dfrac{5}{-5} \Rightarrow \tan B=1 \Rightarrow B=45^\circ$

120. (A) Given that

$\log_5 2$, $\log_5(3^x-1)$ and $\log_5(5\times3^x-13)$ are in A.P,

then $2\log_5(3^x-1)=\log_5 2+\log_5(5\times3^x-13)$

$\Rightarrow \log_5\left(3^x-1\right)^2=\log_5\left\{2\left(5\times3^x-13\right)\right\}$

$\Rightarrow \left(3^x\right)^2+1-2\times3^x=10\times3^x-26$

$\Rightarrow \left(3^x\right)^2-12\times3^x+27=0$

$\Rightarrow \left(3^x-9\right)\left(3^x-3\right)=0$

$3^x=9 \Rightarrow x=2$ or $3^x=3 \Rightarrow x=1$

NDA (MATHS) MOCK TEST - 190 (Answer Key)

1.	(C)	21.	(B)	41.	(D)	61.	(A)	81.	(B)	101.	(C)
2.	(C)	22.	(C)	42.	(B)	62.	(A)	82.	(B)	102.	(B)
3.	(C)	23.	(A)	43.	(B)	63.	(B)	83.	(B)	103.	(C)
4.	(B)	24.	(B)	44.	(B)	64.	(A)	84.	(C)	104.	(C)
5.	(A)	25.	(C)	45.	(C)	65.	(C)	85.	(C)	105.	(C)
6.	(D)	26.	(D)	46.	(B)	66.	(C)	86.	(A)	106.	(B)
7.	(C)	27.	(C)	47.	(A)	67.	(C)	87.	(D)	107.	(B)
8.	(A)	28.	(B)	48.	(C)	68.	(A)	88.	(B)	108.	(D)
9.	(A)	29.	(C)	49.	(B)	69.	(B)	89.	(D)	109.	(A)
10.	(C)	30.	(B)	50.	(A)	70.	(A)	90.	(D)	110.	(B)
11.	(D)	31.	(C)	51.	(C)	71.	(B)	91.	(C)	111.	(C)
12.	(A)	32.	(C)	52.	(B)	72.	(B)	92.	(D)	112.	(D)
13.	(B)	33.	(A)	53.	(B)	73.	(B)	93.	(A)	113.	(B)
14.	(A)	34.	(C)	54.	(C)	74.	(D)	94.	(C)	114.	(C)
15.	(B)	35.	(B)	55.	(B)	75.	(B)	95.	(B)	115.	(D)
16.	(C)	36.	(B)	56.	(A)	76.	(A)	96.	(A)	116.	(C)
17.	(B)	37.	(C)	57.	(C)	77.	(A)	97.	(C)	117.	(B)
18.	(D)	38.	(B)	58.	(C)	78.	(C)	98.	(C)	118.	(A)
19.	(B)	39.	(B)	59.	(A)	79.	(B)	99.	(C)	119.	(C)
20.	(A)	40.	(A)	60.	(A)	80.	(C)	100.	(A)	120.	(A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777