QUANTITATIVE ABILITY - 73 (SOLUTION)

1. (B) As the average weight of A decreased after the student left, his weight must be more than the average weight of A. As the average weight of B decreased after the student joined, his weight must be less than the average weight of B. So, his weight must be between 40 kg and 60 kg.

2. (D) Distance covered by P, t h after starting from X = at

When Q overtakes P, then he would have covered (a + b) (t - p) = at - ap+ bt - bp and this equals at = at - ap+ bt - bp = at

$$t = \frac{p(a+b)}{b} \quad \text{....(i)}$$

Let R start q h after Q started.

Distance covered by R when he overtakes P would be (a + 2b) (t - p - q) = at ....(ii)

Substituting the value of t from equation (i) and simplifying, we get

$$q = \frac{pa}{a+2b}$$

3. (C) Let the speeds of the cars leaving P and Q b p km/h and q km/h, respectively.

Then, px = qy

and pz = qx

On dividing equation (i) by equation (ii), we get

$$\frac{x}{z} = \frac{y}{x}$$

$$x = \sqrt{yz}$$

4. (A) If the height is decreased by x cm, then Decrease in the volume

$$= \left(\frac{1}{3}\right)\left[\pi r^2 h - \pi r^2 (h - x)\right] = \frac{1}{3}\pi r^2 x$$

If the radius decreased by x cm, then Decrease in volume $= \left(\frac{1}{3}\right)\left[\pi r^2 h - \pi (r - x)^2 h\right]$

$$= \left(\frac{1}{3}\right)\pi\left[\pi r^2 h - \pi (r - x)^2 h\right] = \left(\frac{1}{3}\right)\pi\left[r^2 h - \left(r^2 - 2xr + x^2\right)h\right]$$

$$= \left(\frac{1}{3}\right)\pi\left[2xrh - x^2 h\right]$$

Combining the above results,

$$\pi r^2 x = \pi\left[2xrh - x^2 h\right]$$

Cancelling π and x both sides, we get

$r^2 = 2rh - xh$

$$\therefore \quad x = \frac{-r^2 + 2rh}{h}$$

5. (C) $\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$

$$\frac{q}{1+pq+1}+\frac{r}{r+qr+1}+\frac{p}{p+pr+1}$$

$$=\frac{q}{q+\frac{1}{r}+1}+\frac{r}{r+\frac{1}{p}+1}+\frac{p}{p+pr+1}$$

$$=\frac{qr}{qr+1+r}+\frac{pr}{pr+p+1}+\frac{p}{p+pr+1}$$

$$=\frac{qr}{\frac{1}{p}+1+r}+\frac{pr}{pr+p+1}+\frac{p}{p+pr+1}$$

$$=\frac{pqr}{1+p+pr}+\frac{pr}{1+p+pr}+\frac{p}{1+p+pr}$$

$$=\frac{pqr+pr+p}{1+p+pr}=\frac{p(qr+r+1)}{1+p+pr}=\frac{p\left(\frac{1}{p}+r+1\right)}{1+p+pr}$$

$$=\frac{1+p+pr}{1+p+pr}=1 \qquad (\because pqr=1)$$

6. (D) Here, $10 < n < 1000$

Let n be the two-digit number.

Then,

$n = 10a + b$,

$P_n = ab$

$S_n = a + b$

$ab + a + b = 10a + b$

$ab = 9a$

$b = 9$

So, there are $\boxed{9}$ two digit numbers i.e. 19, 29, 39,..., 99.

Again, let n be the three-digit number.

Then, $n = 100a + 10b + c$,

$P_n = abc$, $S_n = a + b + c$

$\therefore$ $abc + a + b + c = 100a + 10b + c$

$abc = 99a + 9b$

$bc = 99 + 9\frac{b}{a}$

But the maximum value tor be = 81 and RHS is more than 99.

So, no three-digit number is possible.

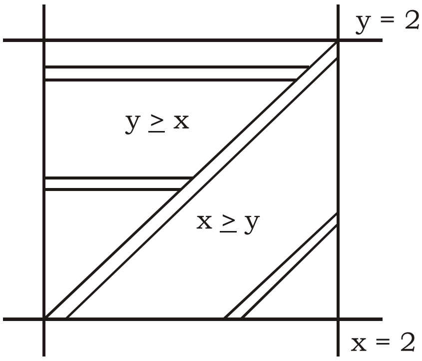
Hence, required number of integers is 9.

7. (C) Let $x \geq 0$, $y \geq 0$ anc $x \geq y$

Then, $|x + y| + |x - y| = 4$

$x + y + x - y = 4$

$x = 2$



And in case $x \geq 0$, $y \geq 0$, $x \leq y$

$x + y + y - x = 4$

$y = 2$

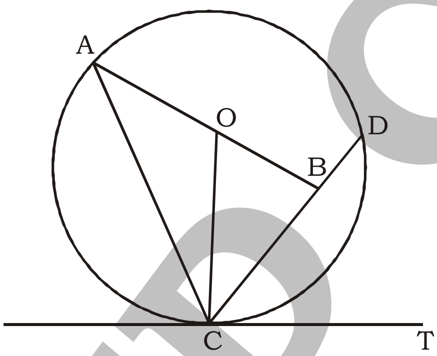
Area in the first quadrant is 4.

By symmetry, total area = 4 × 4 = 16 sq. units

8. (D) Statements I and II are wrong, since when p is prime number so it does not have any factor. So, when all factors (or numbers) before p do not involve in the product, so it is not divisible by p or any prime number greater than p. Statement III is wrong, since 1 × 2 × 3 × 4 × 5 × 6 is divisible by 5. Since, x in values prime number less than (p - 1).

Hence Statement (iv) is correct.

9. (C)



$\angle OCT = 90°$, $\angle DCT = 45°$ and $\angle OCB = 45°$

Also,

$\angle COB = 45°$ (ΔBOC is a right angled triangle)

$\angle AOC = 180° - 45° = 135°$

Here, CD = 10

$\therefore$ BC = 5 cm = OB

Then, in ΔBOC,

$OC = 5\sqrt{2}$ (using Pythagoras theorem)

$OC = OA = 5\sqrt{2}$

In ΔAOC,

$AC^2 = OA^2 + OC^2 - 2OA.OC.\cos 135° = 2(OA)^2 - 2(OA)^2.\cos 135°$

$= 2\left(5\sqrt{2}\right)^2 - 2\left(5\sqrt{2}\right)^2 \times \frac{-1}{\sqrt{2}} = 100 + \frac{100}{\sqrt{2}}$

$AC^2 = 170.70$

$AC = 13$ cm

$\therefore$ Perimeter of ΔAOC = AC + OC + AO $= 13 + 5\sqrt{2} + 5\sqrt{2}$

= 13 + 10 × 1.1414 = 27 cm(approx)

10. (A) $22^3 + 23^3 + 24^3 + ... + 87^3 + 88^3$

On rearranging, $(22^3 + 88^3) + (23^3 + 87^3) + (24^3 + 86^3) + ... + (54^3 + 56^3) + 55^3$

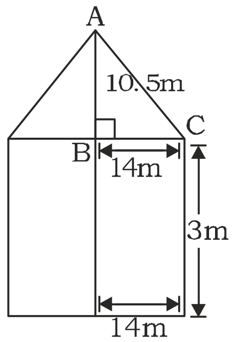
Now, we know that $a^n + b^n$ is divisible by (a + b), when n is odd number.

Therefore, all the terms except 55^3 is divisible by 110.

Now, the remainder when 55^3 is divided by 110 is 55.

Hence, the required remainder when whole expression is divided by 110 is 55.

11. (C)



In ΔABC,

$AC = \sqrt{10.5^2 + 14^2}$

$AC = 17.5m$

$l = 17.5\ m$

Total surface area = $\pi rl + 2\pi rh = \frac{22}{7} \times 14 \times 17.5 + 2 \times \frac{22}{7} \times 14 \times 3 = 1034\ m^2$

The cost of painting = 1034 × 2 = ₹ 2068

12. (A)

Let length be x and breadth be y.

$(x + 14)(y - 6) = xy$

$xy - 6x + 14y - 84 = xy$

$14y - 6x = 84$ ....(i)

$(x - 14)(y + 10) = xy$

$xy + 10x - 14y - 140 = xy$

$10x - 14y = 140$ ....(ii)

Adding equation (i) and (ii), we get

$4x = 224$

$x = 56$

Put the value of x in equation (i),

$14y - 6 \times 56 = 84$

$14y = 420$

$y = 30$

13. (A) Cost of raw material = 4x

Cost of labour = 3x

Cost of miscellaneous = 2x

Total cost = 4x + 3x +2x = 9x

New cost = $\frac{4x \times 110}{100} + \frac{3x \times 108}{100} + \frac{2x \times 95}{100} = 9.54x$

Percentage rise = $\frac{9.54x - 9x}{9x} \times 100 = 6\%$

14. (A) Let the number be x

$3-x : 5-x :: 6-x : 7-x$

$(3-x)(7-x) = (5-x)(6-x)$

$21 - 3x - 7x + x^2 = 30 - 5x - 6x + x^2$

$21 - 10x + x^2 = 30 - 11x + x^2$

$x = 9$

15. (B) $a : b = \frac{2}{9} : \frac{1}{3}, b : c = \frac{2}{7} : \frac{5}{14}, d : c = \frac{7}{10} : \frac{3}{5}$

a : b = 2 : 3 b : c = 4 : 5

d : c = 7 : 6 c : d = 6 : 7

Then,

a : b : c : d = 48 : 72 : 90 : 105 = 16 : 24 : 30 : 35

16. (D) Given, Total earning of A + B + C = ₹ 76000 ....(i)

Percentage of their saving are 30%, 25% and 20% respectively.

Let, savings of A, B and C be $4x$, $5x$ and $6x$ respectively

Now, 30% of A = $4x$

$30 \times \frac{A}{100} = 4x$

$A = \frac{40}{3}x$ ....(i)

Also, 25% of B = $5x$

$25 \times \frac{B}{100} = 5x$

$B = 20x$ ....(iii)

Also, 20% of C = $6x$

$20 \times \frac{C}{100} = 6x$

$C = 30x$ ....(iv)

On using (ii), (iii) & (iv) in (i), we get

$\frac{40x}{3} + 20x + 30x = 76,000$

$x = 1200$

$A = \frac{40x}{3} = \frac{40}{3} \times 1200 = ₹\ 16000$

$B = 20x = 20 \times 1200 = ₹\ 24000$

$C = 30x = 30 \times 1200 = ₹\ 36000$

$\therefore$ (A + B) – C = (40000 – 36000) = ₹ 4000

17. (C) Let money be P.

ATQ,

$\frac{P \times 15 \times 5}{100} - \frac{P \times 12 \times 4}{100} = 1890$

$\frac{27P}{100} = 1890$

$P = \frac{1890 \times 100}{27}$

P = ₹ 7000

18. (A) Let initial amount = ₹ x

ATQ,

$\frac{x}{3} \times \frac{7 \times 2}{100} + \frac{2}{5} \times \frac{x \times 10 \times 2}{100} + \frac{4 \times x \times 12 \times 2}{15 \times 100} = 1430$

$\frac{14x}{300} + \frac{4x}{50} + \frac{8x}{125} = 1430$

$\frac{7x}{150} + \frac{2x}{25} + \frac{8x}{125} = 1430$

$\frac{35x + 60x + 48x}{750} = 1430$

$143x = 1430 \times 750$

$x = \frac{1430 \times 750}{143} = ₹\ 7500$

19. (C) Let cost of 100 Articles is ₹100 ($\because$ 1Article = ₹1)

50% of 100 Article — Rest (50%)

50 Articles → 20% Profit → ₹60

50 Articles → 40% Profit → ₹70

₹60 + ₹70 = 130

If 100 articles $\xrightarrow{25\%\text{Profit}}$ $\frac{S.P}{₹125}$ → Diff = 5 unit = 100

1 unit = ₹ 20

20. (A) Let C. P. = ₹ x

$$S.P. = \frac{x \times 108}{100}$$

Again C. P. = $\frac{x \times 80}{100}$

$$S.P. = \frac{80x}{100} \times \frac{140}{100} = \frac{112x}{100}$$

ATQ,

$$\frac{112x}{100} - \frac{108x}{100} = 640$$

x = ₹ 16000

21. (A) According to the Question

Price → 100 → 120
× ×
Sale → 100 → 85

= 10000 → 10200

+2% increase

22. (D) According to the question,

He should purchase = $\frac{400}{320 \times 50\%}$ = 5 shirts

23. (A) Let work done by a man in a day be x and work done by a woman be y

From question,

$$4x + 6y = \frac{1}{8} \quad \text{............(i)}$$

$$3x + 7y = \frac{1}{10} \quad \text{............(ii)}$$

On solving (i) & (ii), we get

$$x = \frac{11}{400} \text{ and } y = \frac{1}{400}$$

Required ratio = $\frac{x}{y} = \frac{11}{400} \div \frac{1}{400}$ = 11 : 1

24. (B) Given,

$$\frac{1}{B} = \frac{2}{A}$$

A = 2B ....(i)

Also, given $\frac{1}{C} = \frac{3}{A}$

A = 3C ....(ii)

Also, given $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = \frac{1}{2}$ ....(iii)

On using (i) in (iii), we get

$$\frac{1}{2B}+\frac{1}{B}+\frac{3}{3C}=\frac{1}{2}$$

$$\frac{1}{2B}+\frac{1}{B}+\frac{3}{2B}=\frac{1}{2} \qquad \text{[using (i) \& (ii)]}$$

$$\frac{1+2+3}{2B}=\frac{1}{2}$$

2B = 12

B = 6 days

25. (A) $\frac{M_1 \times D_1}{W_1}=\frac{M_2 D_2}{W_2}$

$$\frac{100\times16}{\frac{1}{7}}=\frac{M_2\times80}{\frac{6}{7}}$$

$$M_2=\frac{100\times16\times6}{80}$$

$$M_2=120$$

Required labourers = 120 – 100 = 20

26. (A) Given,

A + B +C = 14,400............................(i)

Let savings of A, B & C are $8x$, $9x$ and $20x$ respectively.

Also given percentage of expenditure of A, B & C are 80%, 85% & 75% respectively.

∴ Percentage of savings of A, B & C be 20%, 15% & 25% respectively.

Now, 20% of A = $8x$

$$\frac{20\times A}{100}=8x$$

A = $40x$..........................(ii)

Again, 15% of B = $9x$

$$\frac{15\times B}{100}=9x$$

B = $60x$...........................(iii)

Again, 25% of C = $20x$

$$\frac{25\times C}{100}=20x$$

C = $80x$ ....(iv)

On using (ii), (iii) and (iv) in (i), we get

$40x + 60x + 80x = 14,400$

$x = 80$

A = $40x = 40 \times 80$ = ₹ 3,200

B = $60x = 60 \times 80$ = ₹ 4,800

C = $80x = 80 \times 80$ = ₹ 6,400

27. (D) New ratio of fares (1st, 2nd and 3rd) = $8 \times \frac{5}{6} : 6 \times \frac{11}{12} : 3 \times 1 = 80 : 66 : 36$

Ratio of passengers = 9 : 12 : 26

Ratio of amount collected = 40 × 9 : 12 × 33 : 26 × 18

Amount collected from 1st class fares = $\frac{90}{306} \times 1088$ = ₹ 320

28. (A) Speed = 15 km per hour = $15 \times \frac{5}{18} = \frac{25}{6}$ m / s.

Water flow out in one second = $0.2 \times 0.15 \times \frac{25}{6}$ m^3

Volume of tank = $150 \times 100 \times 3\ m^3$

Time taken = $\frac{150 \times 100 \times 3 \times 6}{.2 \times .15 \times 25}$ = 100 hours.

29. (D) Given,

$\frac{1}{A} = \frac{1}{8}$ ....(i)

Also given, $\frac{1}{A} - \frac{1}{B} = \frac{1}{20}$

$\frac{1}{B} = \frac{1}{A} - \frac{1}{20}$

$\frac{1}{B} = \frac{1}{8} - \frac{1}{20}$

$\frac{1}{B} = \frac{3}{40}$

B takes $\frac{40}{3}$ min to empty the tank

Also given, rate of water flowing out = $6kl$

∴ Capacity of tank = $\frac{40}{3} \times 6kl = 80\ kl$

30. (C) Speed = $\frac{350 \times 60}{1000} = 21\ km / hr$

Total time taken = $\frac{84}{21} + 13 \times 6$

4 + 78 min = 5 hours + 18 min.

31. (A) Let total coaches be N

Decrease in the speed = x

$x \propto \sqrt{N}$

$x = K\sqrt{N}$ [K = constant]

$4 = K\sqrt{4}$

K = 2

$24 = K\sqrt{N}$

$24 = 2\sqrt{N}$

$\sqrt{N} = 24/2$

N = 144 coaches

Number of coaches that can be exactly pulled by the engine = 144 – 1 = 143 coaches

32. (C) Let minors be x

Consumption by adults = 8 × 15 = 120

Total Consumption = (x + 8) × 10.8

Average consumption by minors = $\frac{(8+x)10.8-120}{x} = 6$

x = 7

33. (C) Sum of 8 numbers = 20 × 8 = 160

Let the sixth number be x.

ATQ,

$\left(15\frac{1}{2}\right)\times 2+\left(21\frac{1}{3}\right)\times 3+x+x+4+x+7=160$

31+ 64 + 3x + 11 = 160

3x = 160 – 106

$x = \frac{54}{3} = 18$

8th Number = x + 7 = 18 + 7 = 25

34. (D) $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$

B = 2 mc

A = $(1+ m^2)$

C = $c^2 - a^2$

Roots are equal

∴ D = 0

$B^2 - 4AC = 0$

$(2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$-c^2 + a^2 + a^2m^2 = 0$

$c^2 = a^2(1+m^2)$

35. (D) Given,

$l x^2 + nx + n = 0$................(i)

$\alpha / \beta = p / q$ ....(ii)

Equation (i) $\Rightarrow$ $\alpha + \beta = -n / l$ ....(iii)

$\alpha \ \beta = n / l$ ....(iv)

Equation (ii) $\Rightarrow$ $\sqrt{\alpha / \beta} = \sqrt{p / q}$ ....(v)

$\sqrt{\beta / \alpha} = \sqrt{q / p}$ ....(vi)

$\therefore$ $\sqrt{p / q} + \sqrt{q / p} + \sqrt{n / l}$

$= \sqrt{\alpha / \beta} + \sqrt{\beta / \alpha} + \sqrt{\alpha \ \beta}$ (using (v), (vi) & (iv))

$= \dfrac{\sqrt{\alpha}}{\sqrt{\beta}} + \dfrac{\sqrt{\beta}}{\sqrt{\alpha}} + \dfrac{\sqrt{\alpha \ \beta}}{1} = \dfrac{(\alpha + \beta) + (\alpha \ \beta)}{\sqrt{\alpha}.\sqrt{\beta}}$

$= \dfrac{-n / l + n / l}{\sqrt{\alpha}.\sqrt{\beta}} = 0 / \sqrt{\alpha}.\sqrt{\beta} = 0$

36. (B) $3x^2 + 2x + 1 = 0$

$a + b = -\dfrac{2}{3}$

$ab = \dfrac{1}{3}$

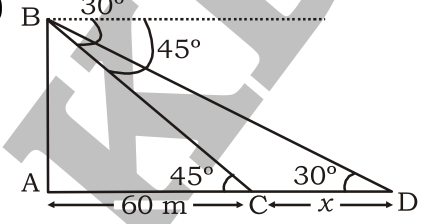
Product of roots $= \dfrac{1-\alpha}{1+\alpha} \times \dfrac{1-\beta}{1+\beta} = 3$

Sum of roots $= \dfrac{1-\alpha}{1+\alpha} + \dfrac{1-\beta}{1+\beta} = 2$

Required equation = x^2 – (sum of the roots) x + product of roots = 0

$x^2 - 2x + 3 = 0$

37. (A)



Let height of the tower = AB

In $\triangle ABC$,

$\tan 45° = \dfrac{AB}{AC}$

AB = 60 m. [$\because$ tan 45 = 1]

In $\triangle ADB$,

$\tan 30^\circ = \frac{60}{60+x}$

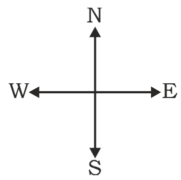
$\frac{1}{\sqrt{3}} = \frac{60}{60+x}$

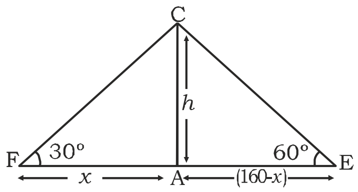
$x = 60(\sqrt{3}-1) = 60 \times (1.73 - 1)$

$x = 60 \times 0.73 = 43.8$ m

$\therefore$ Required Speed $= \frac{43.8}{5} \times \frac{18}{5} = 31.5$ km/hr

38. (A)





In $\triangle AFC$,

$\tan 30^\circ = \frac{h}{x}$

$x = \sqrt{3}$ h ……(i)

In $\triangle AEC$,

$\tan 60^\circ = \frac{h}{160-x} = \sqrt{3}$

$\sqrt{3}$ $(160-x) = h$

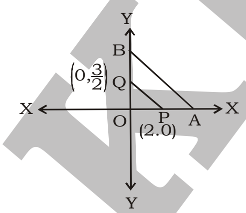
$\sqrt{3}$ $(160 - \sqrt{3}h) = h$ [From (i)]

$4h = 160\sqrt{3}$

$h = \frac{160\sqrt{3}}{3}$ ft $= 40\sqrt{3}$ ft

39. (B) OP = 2

OQ = $\frac{3}{2}$

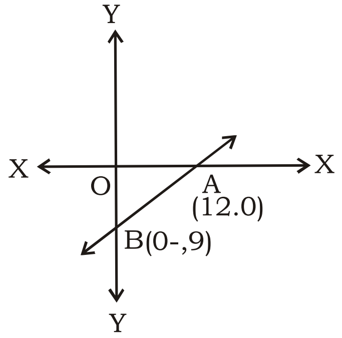


$\therefore$ PQ $= \sqrt{OP^2 + OQ^2} = \sqrt{2^2 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{4 + \frac{4}{9}} = \sqrt{\frac{25}{4}}$

$= \frac{5}{2} = 2.5$ cm

40. (A)



Putting $x = 0$ in $9x - 12y = 108$

we get, $y = -9$

Putting $y = 0$ in $9x - 12y = 108$,

we get, $x = 12$

OA = 12, OB = 9

AB = $\sqrt{OA^2 + OB^2}$ = $\sqrt{12^2 + 9^2}$

= $\sqrt{144 + 81}$ = $\sqrt{225}$ = 15 units

41. (A) $\left(x + \frac{1}{x}\right)^2 = 3$

$x + \frac{1}{x} = \sqrt{3}$

On cubing both sides,

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 3\sqrt{3}$

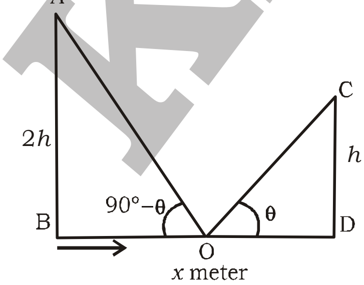
$x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0$

$x^6 + 1 = 0$

$\therefore$ $x^{206} + x^{200} + x^{90} + x^{84} + x^{18} + x^{12} + x^6 + 1$

$= x^{200}(x^6+1) + x^{84}(x^6+1) + x^{12}(x^6+1) + (x^6+1) = 0$

42. (A) CD = h metre, AB = 2h metre



OB = OD = $\frac{x}{2}$ metre

From Δ OCD,

$\tan\theta = \frac{h}{\frac{x}{2}} = \frac{2h}{x}$ __________(i)

From Δ OAB,

$\tan(90^\circ - \theta) = \frac{AB}{BO}$

$\cot\theta = \frac{2h}{\frac{x}{2}} = \frac{4h}{x}$ _________(ii)

Multiplying both the equations,

$\tan\theta.\ \cot\theta = \frac{2h}{x} \times \frac{4h}{x}$

$x^2 = 8h^2$

$h = \frac{x}{2\sqrt{2}}$ meter

43. (C) $\tan 2\theta.\ \tan 3\theta = 1$

$\tan 3\theta = \frac{1}{\tan 2\theta} = \cot 2\theta$

$\tan 3\theta = \tan(90^\circ - 2\theta)$

$3\theta = 90^\circ - 2\theta$

$5\theta = 90^\circ$

$\theta = 18^\circ$

$2\cos^2 \frac{5\theta}{2} - 1 = 2\cos^2 45^\circ - 1 = 2\times\frac{1}{2} - 1 = 0$

44. (B) $\sin 17^\circ = \frac{x}{y}$

$\cos 17^\circ = \sqrt{1-\sin^2 17^\circ} = \sqrt{1-\frac{x^2}{y^2}} = \sqrt{\frac{y^2-x^2}{y^2}} = \sqrt{\frac{y^2-x^2}{y^2}}$

$\therefore$ $\sec 17^\circ = \frac{y}{\sqrt{y^2-x^2}}$

$\sin 73^\circ = \sin(90^\circ - 17^\circ) = \cos 17^\circ$

$\sec 17^\circ - \sin 73^\circ = \frac{y}{\sqrt{y^2-x^2}} - \frac{\sqrt{y^2-x^2}}{y}$

$= \frac{y^2 - y^2 + x^2}{y\sqrt{y^2-x^2}} = \frac{x^2}{y\sqrt{y^2-x^2}}$

45. (C) Votes got by Rahul Gandhi = (100-10)% of $\frac{4}{5}$ of total voters

$= 90\%$ of $\frac{4}{5}$ of total voters $= \frac{9}{10}\times\frac{4}{5}$ of total voters

$= \frac{18}{25}$ of total voters = 216 voters ....(i)

Now, Votes got by Varun Gandhi = (100-20)% of $\left(1-\frac{4}{5}\right)th$ of the total voters

$= 80\%$ of $\frac{1}{5}th$ of total voters $= \frac{4}{5}\times\frac{1}{5}$ of total voters

$= \frac{4}{25}$ of total voters $= \frac{216}{18}\times 4 = 48$ voters

So, total number of votes polled = (216 + 48) votes = 264 votes

46. (B) Net discount given by A $=\left(5+25-\frac{5\times 25}{100}\right)\% = 28.75\%$

Net discount given by B $= \left(16+12-\frac{16\times 12}{100}\right)\% = 26.08\%$

A is giving more discount

It is more profitable to purchase the fan from A.

47. (C) 4 + 44 + 444 +........... to n terms

= 4 (1 + 11 + 111 +.... to n terms)

$= \frac{4}{9}$ (9 + 99 + 999 + ... to n terms)

$= \frac{4}{9}$ [(10 – 1)+(100 – 1) + (1000 – 1)+........ to n terms]

$= \frac{4}{9}$ [(10 +10^2 + 10^3 +......... to n terms) – n]

$= \frac{4}{9}$ [10(1 + 10 +10^2 + ... to n terms) – n]

$= \frac{40}{9}\left(\frac{10^n-1}{9}\right) - \frac{4}{9}n$ [$\because$ 1 +10 +10^2 + ... to n terms $= \frac{10^n-1}{9}$]

$= \frac{40}{81}(10^n - 1) - \frac{4}{9}n$

48. (A) Cash price of refrigerator = $1500 + \left(1020 \times \frac{10}{11}\right) + \left(1003 + \frac{100}{121}\right) + \left(990 \times \frac{1000}{1331}\right)$

$= 1500 + \left\{\frac{(10200 \times 121) + (100300 \times 11) + 990000}{1331}\right\}$

$= 1500 + \left(\frac{1234200 + 1103300 + 990000}{1331}\right)$

$= 1500 + \frac{3327500}{1331} = 1500 + 2500 = 4000$

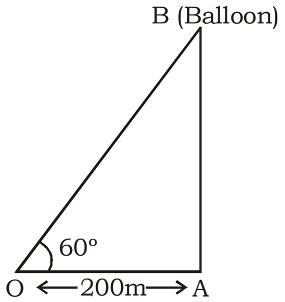
Alternative Method:-

Cash price of refrigerator = $1500 + \frac{1020}{1.1} + \frac{1003}{(1.1)^2} + \frac{990}{(1.1)^3}$

$= 1500 + \frac{1020}{1.1} + \frac{1003}{1.21} + \frac{990}{1.331}$

= 1500 + 2500 = 4000

49. (A)



In the given figure, after leaving the point A, balloon reach to point B vertically upward in 1.5 min

Here, O $\rightarrow$ the observer

So, $\angle BOA = 60°$ (Observer)

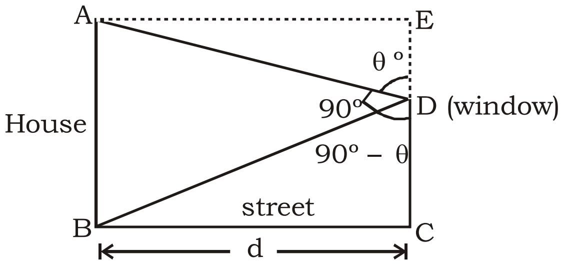
$\tan 60° = \frac{AB}{OA}$

$AB = OA \tan 60° = 200 \times \sqrt{3}$ m

So, speed of the balloon = $\frac{\text{distance}}{\text{time}} = \frac{\text{AB}}{\text{time to reach from A to B}}$

$= \frac{200\sqrt{3}m}{\frac{1.5}{60}\text{sec}}$ = 3.87 m/sec.

50. (C)



Here,

AB $\rightarrow$ height of the house

CD $\rightarrow$ height of the window

ATQ,

$\angle ADB = 90°$

Also,

here, line AD makes as angle $\theta°$ with the vertical line DE.

$\angle ADE = \theta°$

$\angle BDC = 90° - \theta$

In ΔBCD ,

$$\tan(90° - \theta) = \frac{BC}{CD} = \frac{d}{CD}$$

$$\cot\theta = \frac{d}{CD}$$

$$CD = \frac{d}{\cot\theta} = d\tan\theta$$

In ΔADE ,

$$\tan\theta = \frac{AE}{DE} = \frac{d}{DE}$$

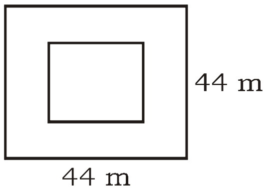
$$DE = \frac{d}{\tan\theta} = d\cot\theta$$

So, the height of the house,

$$AB = CD + DE = d(\tan\theta + \cot\theta)$$

$$= d\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = d\left(\frac{1}{\cos\theta \times \sin\theta}\right) = d\sec\theta\,\text{cosec}\,\theta$$

51. (C)



Total area of the square field = $(44 \times 44)\text{m}^2 = 1936\text{m}^2$
At the rate of ₹ 1 per sq. mtr; the total cost would be ₹ 1936
But the total cost = ₹ 3536
Difference = ₹ 3536 – ₹ 1936 = ₹ 1600
₹ 1600 would be the extra cost on the flower bed and as the extra cost on the flower bed is ₹ 1 per sq. mtr.
Area of flower bed = 1600 sq. mtr.
Side of flower bed = $\sqrt{1600}$ m² = 40 m

So, width of the gravel path = $\frac{44-40}{2}$ = 2 metre

52. (D) $3^{x^2 - xy + y^2} = 81 = 3^4$

$x^2 - xy + y^2 = 4$ … (i)

And $2^{x^3+y^3} = 256 = 2^8$

$x^3 + y^3 = 8$ … (ii)

Dividing (ii) by (i),

$x + y = 2$

53. (B) The given equation = $c + \frac{d-y}{y} = e - 1 + \frac{f}{y}$

$c + \frac{d}{y} - \frac{y}{y} = e - 1 + \frac{f}{y}$

$c + \frac{d}{y} - 1 = e - 1 + \frac{f}{y}$

$\frac{d}{y} - \frac{f}{y} = e - c$

$\frac{d-f}{y} = e - c$

$y = \frac{d-f}{e-c}$

54. (B) The given equation= $bx^2 - ax + \log_2 m^y = 0$

Now,

Sum of the roots = $x_1 + x_2$

$= \frac{-a}{b} = \frac{a}{b}$ ………. (i)

The given relation = $x_1^2 - x_2^2 = a^2$

$(x_1 + x_2)(x_1 - x_2) = a^2$

$\frac{a}{b}(x_1 - x_2) = a^2$ [From equation (i)]

$x_1 - x_2 = ab$ ………… (ii)

From equation (i) and equation (ii)

$$x_1 = \frac{1}{2}\left[\frac{a}{b} + ab\right] = \frac{a(b^2+1)}{2b}$$

$$x_2 = \frac{1}{2}\left[\frac{a}{b} - ab\right] = \frac{a(1-b^2)}{2b}$$

Hence, the roots are $\frac{a}{2b}(b^2 + 1)$ and $\frac{a}{2b}(1 - b^2)$.

55. (C) The given expression = $(\sqrt{k+l})^2 + (\sqrt{m})^2 + (\sqrt{n})^2 + 2\sqrt{m}\sqrt{k+l} - 2\sqrt{n}\sqrt{m} - 2\sqrt{n}\sqrt{k+l}$

$= \left[\sqrt{k+l} + \sqrt{m} - \sqrt{n}\right]^2$

So, the square root of the given expressions = $\pm\left[\sqrt{k+l} + \sqrt{m} - \sqrt{n}\right]$

56. (A) Part of the cistern filled in 3 min = $\frac{3}{12} + \frac{3}{16} = \frac{21}{48} = \frac{7}{16}$

Let remaining $\frac{9}{16}$ part was filled in x min

Then, $\frac{x}{12} \times \frac{7}{8} + \frac{x}{16} \times \frac{5}{6} = \frac{9}{16}$

$x\left(\frac{7+5}{96}\right) = \frac{9}{16}$

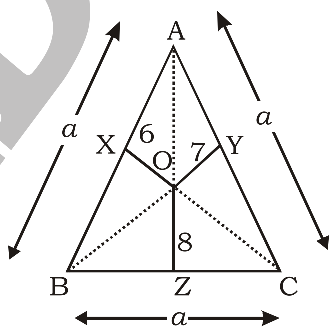
$x = \frac{9}{16} \times \frac{96}{12} = 4.5$ min

57. (A) Joining point O to three vertices A, B and C.
Now,
Area of (ΔOBC + ΔOCA + ΔOAB) = area of ΔABC.

$\frac{1}{2}(a \times 8 + a \times 7 + a \times 6) = \frac{\sqrt{3}}{4}a^2 = \frac{21a}{2}$

$a = \frac{42}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore \quad a = 14\sqrt{3}$



Hence, area of triangle ABC = $\frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}(14\sqrt{3})^2$ m² = 254.6 m²

58. (B) Ratio of investment of Sita, Gita and Rita

= 5000 × 3 + 7000 × 9) : (4000 × 1 + 3000 × 11) : (7000 × 11)

= 78000 : 37000 : 77000 = 78 : 37 : 77

$\therefore$ Share of Rita in profit = $\frac{77}{78+37+77}$ × 1218 = ₹ 488.47

59. (A) Let PQ be the ladder such that its top Q is on the wall OQ and bottom P is on the ground. The ladder is pulled away from the wall through a distance *a*, so that its top Q slides and takes position Q'.

Clearly, PQ = P'Q'.

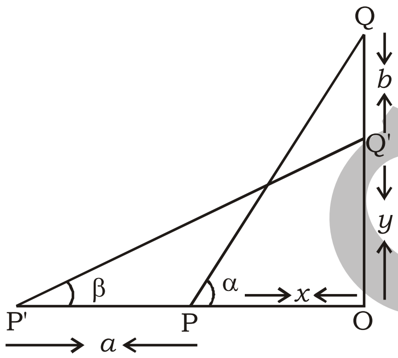
In $\Delta^{'s}$ POQ and P'OQ', we have

$$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}$$

$$\sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$$

$$\sin\alpha = \frac{b+y}{PQ}, \cos\alpha = \frac{x}{PQ}$$

$$\sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$$



$$\sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ} \text{ and } \cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$$

$$\sin\alpha - \sin\beta = \frac{b}{PQ} \text{ and } \cos\beta - \cos\alpha = \frac{a}{PQ}$$

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{b}{a}$$

$$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

60. (B) Number of tourists from USA = $\frac{5760}{32} \times \frac{16}{14} \times 100 = 20571$

61. (D) Number of tourists from China of age more than 60 years = 50000 × $\frac{14.25}{100} \times \frac{10}{100}$ = 712

62. (A) Number of tourists from Japan of age group (16-30) = $\frac{7500}{15} \times 12.75 \times \frac{25}{100}$

Number of tourists from USA of age group (0-15) = $\frac{7500}{15} \times 16 \times \frac{12}{100}$

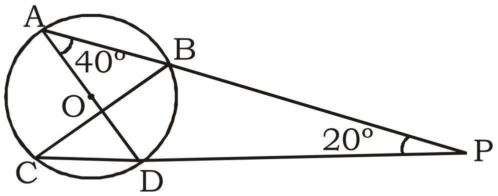
$\therefore$ Required ratio = $\frac{12.75 \times 25}{16 \times 12}$ = 425 : 256

63. (D)

64. (C) Number of tourists from rest of the countries = $\frac{7980}{14.25} \times 28 = 15680$

$\therefore$ Number of tourists from rest of countries of age group (16-30) = $15680 \times \frac{25}{100} = 3920$

65. (B)



In ΔADP,

Exterior $\angle ADC$ = Interior ($\angle A + \angle P$) = 40° + 20° = 60°

$\angle ABC = \angle ADC = 60°$ [Angles in the same segment]

Since AD is the diameter

$\angle ABD = 90°$

So, $\angle DBC = \angle ABD - \angle ABC$

= 90° – 60° = 30°

66. (A) Investment of A = 50000 × 12 = ₹ 600000

Investment of B = 60000 × (12 – x)

Investment of C = 70000 × (12 – x)

ATQ,

$\frac{600000}{60000 \times (12 - x)} = \frac{20}{18}$

$180 = 240 - 20x$

$x = 3$

67. (C)

	A	B	C
Efficiency	3 :	2 :	6
Number of days	2 :	3 :	1

Number of days taken by A = 12

Number of days taken by B = 18

Number of days taken by C = 6

1 day's work of (A + B) = $\frac{5}{36}$

1 day's work of (B + C) = $\frac{8}{36}$

1 day's work (C + A) = $\frac{9}{36}$

Day	1	2	3	4	5	6
	5/36	8/36	9/36	5/36	8/36	1/36

(Days 1–5 total: 35/36)

In 5 days total work done = $\frac{35}{36}$

Now, the rest of work $\left(i.e. \frac{1}{36}\right)$ is done by AC.

Number of days taken by AC for the rest of the work = $\frac{\frac{1}{36}}{\frac{9}{36}}$

There, total time taken to complete the work = 5 + $\frac{1}{9}$ = $5\frac{1}{9}$ days

68. (A) Filling done by all 3 pipes in 3 min = $\frac{3}{20} + \frac{3}{10} + \frac{3}{30} = \frac{11}{20}$

Filling done by 2nd pipe in 3 min = $\frac{3}{10}$ So, required ratio = $\frac{\frac{3}{10}}{\frac{11}{20}} = \frac{6}{11}$

69. (B) Average Speed = $\frac{\text{Total distance}}{\text{Total time}}$

ATQ,

$53 + \frac{1}{3} = \frac{200}{\frac{50}{40} + \frac{150}{x}}$

$\frac{160}{3} = \frac{200 \times 40x}{50x + 6000}$

$8000x + 960000 = 24000x$

$16000x = 960000$

$x = 60$ Km/h

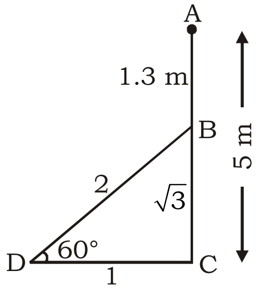
70. (A) Rest part of milk = $1 - \frac{40}{400} = \frac{9}{10}$

Required pure milk = $40 \times \left(\frac{9}{10}\right)^6$

$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$

= 21.2576 litres $\approx$ 21.25 litres

71. (B)



$\sqrt{3} \to 5 - 1.3$

$\sqrt{3} \to 3.7$

$2 \to \frac{3.7}{\sqrt{3}} \times 2 = \frac{7.4\sqrt{3}}{3}$

Length of the ladder = 4.27 m

Distance of the ladder from the foot of the pole = $\frac{3.7}{\sqrt{3}}$

$= \frac{3.7 \times 1.73}{3} = 2.14\text{ m}$

72. (3) Number of employees in Teaching profession = $26800 \times \frac{15}{100} = 4020$

Number of employees in Medical profession = $26800 \times \frac{27}{100} = 7236$

Total number of employees = 4020 + 7336 =11256

Number of employees in Management profession = $26800 \times \frac{17}{100}$ = ₹ 4556

∴ Required difference = 11256 – 4556 = 6700

Quicker Method:

Required difference = (15 + 27 – 17)% of 26800 = 25% of 26800 = 6700

73. (4) Total number of employees in Management profession = $26800 \times \frac{17}{100} = 4556$

Number of female employees in Management profession = $4556 \times \frac{3}{4} = 3417$

∴ Required number of male employees in Management profession = 4556 – 3417 = 1139

74. (2) Total number of employees from Film Production = $26800 \times \frac{19}{100} = 5092$

Now, number of employees from Film Production who went on strike = $5092 \times \frac{25}{100} = 1273$

∴ Number of employees who have not participated in strike = 5092 – 1273 = 3819

Quicker Method:

Required number of employees who have not participated in strike = $26800 \times \frac{75}{100} = 3819$

75. (4) Required number of employees who participated in both Engineering and Industries professions = $26800 \times \left(\frac{9+13}{100}\right) = 268 \times 22 = 5896$

76. (1) Total number of teachers = $26800 \times \frac{15}{100} = 4020$

Number of teachers who are not permanent = $4020 \times \frac{3}{5} = 804 \times 3 = 2412$

∴ Number of teachers who are permanent = 4020 – 2412 = 1608

77. (3) Average = $\frac{210+204+231+231}{4} = 219$

78. (1) Total number of girls = 70 + 117 + 54 + 129 + 136 + 176 = 682

79. (4) Required difference = 225 – 225 = 0

80. (4) Let the total number of students be x.

∴ Boys = $\frac{44x}{100}$ and girls = $\frac{56x}{100}$

ATQ,

$\frac{56x}{100} - \frac{44x}{100} = 30$

$x = \frac{3000}{12} = 250$

∴ Boys = $\frac{44}{100} \times 250 = 110$

Similarly,

Total students = $\frac{132 \times 100}{40} = 330$

Girls = $\frac{30 \times 330}{100} = 99$

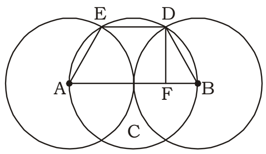
∴ Ratio = $\frac{110}{99} = \frac{10}{9}$

81. (4) Students from $F_{1986} = 375$

Students from $C_{1986} = 250$

Required% = $\frac{375}{250} \times 100 = 150\%$

82. (B)



ABDE will be trapezium

AB = 4 units

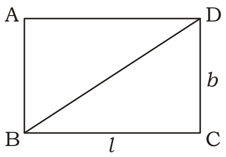
DE = $\frac{1}{2}$AB = 2 units

FB = 1 unit, BD = 2 units.

DF = $\sqrt{2^2 - 1^2}$ = $\sqrt{3}$ units

$\therefore$ Area of ABDE = $\frac{1}{2}$(AB + DE) × DF = $\frac{1}{2}$(4 + 2) × $\sqrt{3}$ = $3\sqrt{3}$ sq. units

83. (D)



BD = length of diagonal = speed × time = $\frac{52}{60}$ × 15 = 13 metre

BD = $\sqrt{l^2 + b^2}$

$l^2 + b^2 = 169$ ....(i)

Again,

$(l + b) = \frac{68}{60} \times 15 = 17$ ....(ii)

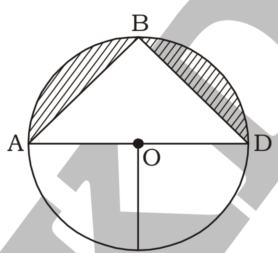
$(l + b)^2 = l^2 + b^2 + 2\,lb$

$17^2 = 169 + 2\,lb$

$2\,lb = 289 - 169 = 120$

$lb = \frac{120}{2} = 60$ m^2

84. (C)



Let Radius of circle = a units

Area of semi circle = $\frac{\pi a^2}{2}$ sq. units

Area of triangle ABD = $\frac{1}{2} \times a \times 2a = a^2$

$\therefore$ Area of shaded region = $\frac{\pi a^2}{2} - a^2 = a^2\left(\frac{\pi}{2} - 1\right)$ sq. units

85. (C) Since, point of intersection of medians is "centroid".

$\therefore$ co-ordinates of centroid = $\left(\frac{0+5+7}{3}, \frac{6+3+3}{3}\right) = \left(\frac{12}{3}, \frac{12}{3.}\right)$ = (4, 4)

86. (B) Pankaj → 20 days

Let the total work = 20 units

Then 25% = $\frac{1}{4}$

Remaining work = 20 × $\frac{3}{4}$ = 15 units

15 units done by Neha in 10 days

20 units (Total work) done by Neha = $\frac{10}{15} \times 20 = \frac{40}{3}$ days

Pankaj → 20 days — 2

Neha → $\frac{40}{3}$ days — 3

(40) Total work

$\therefore$ Time required for Pankaj and Neha to complete the work = $\frac{40}{5}$ = 8 days

87. (D) Let the distance of total journey = LCM of (8, 6) = 24 units

$\therefore$ $\frac{3}{8}$ of the journey = $\frac{3}{8} \times 24$ = 9 units

$\frac{5}{6}$ of the journey = $\frac{5}{6} \times 24$ = 20 units i.e. it covered 20 – 9 = 11 units of distance in 4.30 p.m. – 11 a.m. = $5\frac{1}{2}$ hours = $\frac{11}{2}$ hours

Speed of person = $\frac{11}{11/2}$ = 2 km/hr

$\frac{3}{8}$ of the journey will be covered in = $\frac{9}{2} = 4\frac{1}{2}$ hours

Starting time = 11 a.m – $4\frac{1}{2}$ hours = 6.30 a.m

88. (A) Required books in each stack = HCF of each type of books = HCF of 84, 90 and 12 = 6

89. (B)

	Alcohol : Water		Alcohol : Water
1st Glass	2 : 1	×5	10 : 5
2nd Glass	3 : 2	×3	9 : 6
			19 : 11

90. (B) $\frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$

$= \frac{(a-b)^3}{(a-b)(b-c)(c-a)} + \frac{(b-c)^3}{(a-b)(b-c)(c-a)} + \frac{(c-a)^3}{(a-b)(b-c)(c-a)}$

$\left[\begin{array}{l} (a-b)+(b-c)+(c-a)=0 \\ \text{So,} \\ (a-b)^3+(b-c)^3+(c-a)^3 = 3(a-b)(b-c)(c-a) \end{array}\right]$

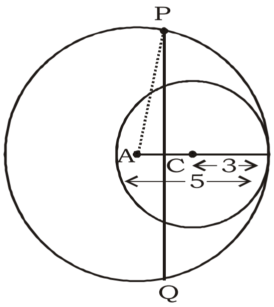
$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)} = \frac{3(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$

91. (C) Population of a town on 1st January 2001 = 500000

Percentage increase in population = 4%

So, population of a town on 1st January 2004 = $500000 \times \left(1+\frac{4}{100}\right)^3 = 562432$

92. (B)



AB = (5 – 3) cm = 2 cm

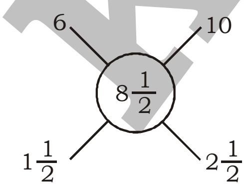
AC = BC = $\frac{1}{2}$AB = 1 cm

AP = 5 cm [radius of bigger circle]

So, PC = $\sqrt{(5)^2-(1)^2} = \sqrt{24} = 2\sqrt{6}$ cm

$\therefore$ PQ = $2 \times 2\sqrt{6} = 4\sqrt{6}$ cm

93. (D) From the rule of alligation



Ratio between 1st and 2nd sum = 3 : 5

2nd sum = $\frac{5}{3} \times 7500 =$ ₹ 12500

94. (C) Let the C.P. of each article be ₹ x

ATQ,

$$\frac{50x \times 120}{100} + \frac{50x \times 140}{100} - \frac{100x \times 125}{100} = 100$$

$60x + 70x - 125x = 100$

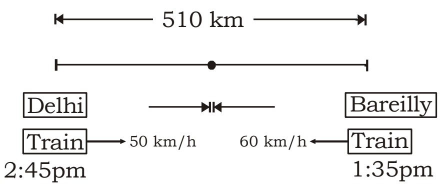
$5x = 100$

$x =$ ₹ 20

95. (C) 25% (stolen) + 10% (Dropped) $\Rightarrow 35\% = \frac{7}{20}$, $50\% = \frac{1}{2}$

Sum – Remain
20 – 13
2 – 1
40 – 13
↓ ×130 ↓ ×130
5200 1690

96. (C)



Let R is a point where both the trains meet.

Till 2 : 45 pm the distance covered by the second train= $\frac{70}{60} \times 60 = 70$ km

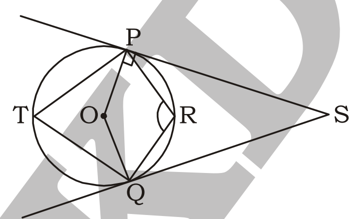
Remaining distance = 510 – 70 = 440 km

Now relative speed of both trains = 50 + 60 =110 km/h

Required time of meeting = $\frac{440}{110}$ = 4 hours

Distance from Delhi to meeting point R = 4 × 50 = 200 km

97. (D)



$\angle OPS = \angle OQS = 90°$

$\angle PSQ = 20°$ (Given)

$\angle POQ = 160°$

$\angle PSQ + \angle POQ = 180°$

PTQ = 80°

PRQT is a cyclic quadrilateral

$\therefore$ $\angle PRQ = 180° - 80° = 100°$

98. (C) According to the question,

$$n \times \frac{90}{100} \times \frac{80}{100} \times \frac{75}{100} = 270$$

$$n = \frac{270 \times 10 \times 10 \times 100}{9 \times 8 \times 75}$$

n = 500 chocolates

Short trick:-

$10\% = \frac{1}{10}$, $20\% = \frac{1}{5}$, $25\% = \frac{1}{4}$

ATQ,

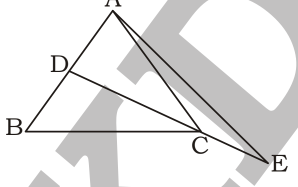
Quantity	Remaining
10	9
5	4
4	3
200	108
↓ ×2.5	↓ ×2.5
500	270

99. (B) Discount offered by Ravi = $25 + 5 - \frac{25 \times 5}{100} = 28.75\%$

Discount offered by Vivek = $16 + 12 - \frac{16 \times 12}{100} = 26.08\%$

Buying from Ravi is more preferable.

100. (C)



ΔABC is equilateral,

$\angle BCD = \angle DCA = 30°$ CD bisects $\angle ACB$

ACE = 180° – 30° = 150°

AC = CE

$\angle CAE = \angle CEA = \frac{30}{2} = 15°$

QUANTITATIVE ABILITY - 73 (ANSWER KEY)

1.	(B)	26.	(A)	51.	(C)	76.	(1)
2.	(D)	27.	(D)	52.	(D)	77.	(3)
3.	(C)	28.	(A)	53.	(B)	78.	(1)
4.	(A)	29.	(D)	54.	(B)	79.	(4)
5.	(C)	30.	(C)	55.	(C)	80.	(4)
6.	(D)	31.	(A)	56.	(A)	81.	(4)
7.	(C)	32.	(C)	57.	(A)	82.	(B)
8.	(D)	33.	(C)	58.	(B)	83.	(D)
9.	(C)	34.	(D)	59.	(A)	84.	(C)
10.	(A)	35.	(D)	60.	(B)	85.	(C)
11.	(C)	36.	(B)	61.	(D)	86.	(B)
12.	(A)	37.	(A)	62.	(A)	87.	(D)
13.	(A)	38.	(A)	63.	(D)	88.	(A)
14.	(A)	39.	(B)	64.	(C)	89.	(B)
15.	(B)	40.	(A)	65.	(B)	90.	(B)
16.	(D)	41.	(A)	66.	(A)	91.	(C)
17.	(C)	42.	(A)	67.	(C)	92.	(B)
18.	(A)	43.	(C)	68.	(A)	93.	(D)
19.	(C)	44.	(B)	69.	(B)	94.	(C)
20.	(A)	45.	(C)	70.	(A)	95.	(C)
21.	(A)	46.	(B)	71.	(B)	96.	(C)
22.	(D)	47.	(C)	72.	(3)	97.	(D)
23.	(A)	48.	(A)	73.	(4)	98.	(C)
24.	(B)	49.	(A)	74.	(2)	99.	(B)
25.	(A)	50.	(C)	75.	(4)	100.	(C)