

NDA (MATHS) MOCK TEST - 37 (SOLUTION)

1. (B) $y = \ln(e^{mx} + e^{-mx})$

On differentiating it w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{e^{mx} + e^{-mx}} \cdot \frac{d}{dx} (e^{mx} + e^{-mx})$$

$$= \frac{1}{e^{mx} + e^{-mx}} (me^{mx} - me^{-mx})$$

$$\therefore \left(\frac{dy}{dx}\right)_{at\ x=0} = m \left(\frac{1-1}{1+1}\right) = 0$$

2. (B) $f(x) = x^2 - x^{-2}$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^2} - \left(\frac{1}{x}\right)^{-2}$$

$$= \frac{1}{x^2} - \frac{1}{x^2}$$

$$= \frac{1}{x^2} - x^2$$

$$= -(x^2 - x^{-2}) = -f(x)$$

3. (C) We know that, in a GP the product of two terms equidistant from the beginning and end is a constant and is equal to the product of first term and last term, i.e., if $a_1, a_2, a_3, \dots, a_{(n-2)}, a_{n-1}, a_n$ are in GP, then

$$a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = \dots$$

Given that,

$$S_2 S_u = S_p S_8 \Rightarrow (p+8) = (2+11)$$

4. (A) Remainder $\Rightarrow = 5$

$$182 \div 2 = 91 + 0$$

$$91 \div 2 = 45 + 1$$

$$45 \div 2 = 22 + 1$$

$$22 \div 2 = 11 + 0$$

$$11 \div 2 = 5 + 1$$

$$5 \div 2 = 2 + 1$$

$$2 \div 2 = 1 + 0$$

$$1$$

$$\therefore (182)_{10} = (10110110)_2$$

5. (B)
$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \frac{1}{abc} \begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix}$$

$$\frac{abc}{abc} \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

6. (B) Radius of the circle = $\sqrt{2}$

$$\therefore \text{area of the circle} = \pi(\sqrt{2})^2 = 2\pi \text{ sq. units}$$

7. (B) $\tan 105^\circ = \tan(60^\circ + 45^\circ)$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ}$$

$$\left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \right]$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$

8. (B) Let $\alpha = \omega$ and $\beta = \omega^2$

$$\text{Then, } xyz = (a+b)(a\omega + b\omega^2)(a\omega^2 + b\omega) = a^3 + b^3$$

9. (C) $I = \int_0^{\pi/2} \frac{\sin 2\theta}{\cos 2\theta + \sin 2\theta} d\theta$

$$\text{also } I = \int \frac{\sin\left(\frac{\pi}{2} - 2\theta\right)}{\sin\left(\frac{\pi}{2} - 2\theta\right) + \cos\left(\frac{\theta}{2} - 2\theta\right)}$$

$$= \int_0^{\pi/2} \frac{\cos 2\theta}{\cos 2\theta + \sin 2\theta} d\theta$$

$$\text{Now } I + I = \int_0^{\pi/2} \frac{\sin 2\theta + \cos 2\theta}{\sin 2\theta + \cos 2\theta} d\theta$$

$$= \int_0^{\pi/2} 1 d\theta$$

$$= [\theta]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$$

10. (D) $\frac{dy}{dx} = 1 - e^x$ is positive, if $e^x < 1$.

$$\Rightarrow x < 0 \Rightarrow x \in (-\infty, 0)$$

So, the interval $(-\infty, -1)$ is part of interval $(-\infty, 0)$.

11. (A) Factories both numerator and denominator.

$$\lim_{x \rightarrow \pi/4} \frac{(1 - \cot x)(1 + \cot x + \cot^2 x)}{(1 - \cot x)(2 + \cot x + \cot^2 x)}$$

$$= \frac{1+1+1}{2+1+1} = \frac{3}{4}$$

12. (B) $\therefore x^2 - 2x + \sin^2 \theta = 0$

$$\therefore x = \frac{2 \pm \sqrt{4 - 4\sin^2 \theta}}{2}$$

$\Rightarrow x = 1 \pm \cos \theta$

$\therefore -1 \leq \cos \theta \leq 1$

$\therefore 0 \leq 1 \pm \cos \theta \leq 2 \Rightarrow 0 \leq \cos \theta \leq 2 \Rightarrow x \in [0, 2]$

13. (B) Required probability = P (Indian wins first and third test) + P (India wins second and third test)

$$= \frac{1}{2} \left(1 - \frac{1}{2}\right) \left(\frac{1}{2}\right) + \left(1 - \frac{1}{2}\right) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

14. (C) 100101 + 101 is

$$\begin{array}{r} 100101 \\ +101 \\ \hline 101010 \end{array}$$

100101 + 101 + 1101 is

$$\begin{array}{r} 101010 \\ +1101 \\ \hline 110111 \end{array}$$

$\therefore 100101 + 101 + 1101 + 100$ is

$$\begin{array}{r} 110111 \\ 100 \\ \hline 111011 \end{array}$$

15. (D) $f[f(x)] = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\frac{\alpha x}{x+1} + 1} = \frac{\alpha^2 x}{\alpha x + x + 1}$

$\therefore f[f(x)] = x$

$\therefore \frac{\alpha^2 x}{\alpha x + x + 1} = x \Rightarrow \alpha^2 x = \alpha x^2 + x^2 + x$

By inspection, we get $\alpha = -1$

16. (B)

17. (C) Applying $R_1 \rightarrow R_1 - R_2$ and $R_1 \rightarrow R_2 - R_3$ and take $(x-1)$ common from R_1 and R_2 ,

$$\Delta = (x-1)^2 \begin{vmatrix} x+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} = (x-1)^3$$

$\therefore k = 3$

18. (B) $y = Ae^{3x} + Be^{5x}$ (i)

$y_1 = 3Ae^{3x} + 5Be^{5x}$ (ii)

$y_2 = 9Ae^{3x} + 25Be^{5x}$ (iii)

Eliminating A and B from the above three equations, we get

$$\begin{vmatrix} e^{3x} & e^{5x} & -y \\ 3e^{3x} & 5e^{5x} & -y_1 \\ 9e^{3x} & 25e^{5x} & -y_2 \end{vmatrix} = 0 \Rightarrow -e^{3x} \times e^{5x}$$

$$\begin{vmatrix} 1 & 1 & y \\ 3 & 5 & y_1 \\ 9 & 25 & y_2 \end{vmatrix} = 0$$

Expanding, we get $30y - 16y_1 + 2y_2 = 0$

of $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

19. (B) We know that,

$$\tan \frac{A-B}{2} = \sqrt{\frac{1 - \cos(A-B)}{1 + \cos(A-B)}} = \sqrt{\frac{1 - \frac{31}{32}}{1 + \frac{31}{32}}}$$

$$= \frac{1}{\sqrt{63}} \Rightarrow \frac{a-b}{a+b} \cdot \cot \frac{C}{2} = \frac{1}{\sqrt{63}}$$

$$\left(\because \tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}\right)$$

$$\Rightarrow \frac{1}{9} \cot \frac{C}{2} = \frac{1}{\sqrt{63}} \Rightarrow \tan \frac{C}{2} = \frac{\sqrt{7}}{3}$$

Now, $\cos C = \frac{1 - \tan^2 \frac{C}{2}}{1 + \tan^2 \frac{C}{2}} = \frac{1 - \frac{7}{9}}{1 + \frac{7}{9}} = \frac{1}{8}$

20. (B) $\therefore c^2 = a^2 + b^2 - 2ab \cos C$

$$\therefore c^2 = 25 + 16 - 40 \times \frac{1}{8} = 36 \Rightarrow c = 6$$

21. (B) We know that, $\sin A = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$

$$\Rightarrow (\sin A) \tan^2 \frac{A}{2} - 2 \tan \frac{A}{2} + \sin A = 0$$

$$\Rightarrow \frac{2 \pm \sqrt{4 - 4 \sin^2 A}}{2 \sin A} = \frac{2 \pm 2 \cos A}{2 \sin A} = \frac{1 \pm \cos A}{\sin A}$$

$$\tan \frac{A}{2} = \frac{1 \pm \sqrt{1 - \sin^2 A}}{\sin A}$$

Eq. (i) gives two values of $\tan \frac{A}{2}$, when $\sin A$ is given but ($\sin A \neq 0$)

22. (D) $R = [x : x \text{ is a set of all children of a same father}]$

Reflexive Let p be the children of same father.

Hence, pRp is a reflexive.

Symmetry Let p and q be the children of same father.

So, q and p be the children of same father.

And q and p be the children of same father.

Hence, R is symmetric.

Transitive Let p and q be the children of same father. And q and r be the children of same father.

So, p and r be the children of same father R .

Hence, R is transitive.

Since, R have all three properties such that reflexive, symmetry and transitive, so R is an equivalence relation.

23. (B) $1 = \int \frac{x(1-x)}{\sqrt{1-x^2}} dx$

$$= \int \frac{x}{\sqrt{1-x^2}} dx + \int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2} \int \frac{2x}{\sqrt{1-x^2}} dx + \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

$$= -\frac{1}{2} \times 2\sqrt{1-x^2} + \int \sqrt{1-x^2} dx$$

$$- \int \frac{1}{\sqrt{1-x^2}} dx$$

$$= \left(\frac{x}{2} - 1\right) \sqrt{1-x^2} - \frac{1}{2} \sin^{-1} x + C$$

24. (D) Apply $C_1 + C_2$, thus making two zero in C_1 and expanding we get

$$\Delta = (\omega^2 + 2\omega) \begin{vmatrix} \omega^2 & -\omega \\ \omega & -\omega^2 \end{vmatrix}$$

$$= (-1 + \omega) (-\omega^4 + \omega^2) \quad (\because \omega^2 + \omega = -1)$$

$$= (-1 + \omega) (-\omega + \omega^2) \quad (\because \omega^3 = 1)$$

$$= \omega^2 - \omega^3 - 2\omega^2 = 1 + \omega - 2\omega^2$$

$$= -\omega^2 - 2\omega^2 = -3\omega^2$$

25. (B) every element of A can be image to ten elements of the set.

\therefore Total number of mapping = 10^{10}

26. (C)

27. (C) $\because y^2 = P(x)$ (i)

$$2y \frac{dy}{dx} = P'(x) \quad \text{..... (ii)}$$

$$\text{and } 2y \frac{d^2y}{dx^2} + 2 \left(\frac{dy}{dx}\right)^2 = P''(x)$$

$$\Rightarrow 2y \frac{d^2y}{dx^2} = P''(x) - 2 \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2y^3 \frac{d^2y}{dx^2} = y^2 P''(x) - 2 \left(y \frac{dy}{dx}\right)^2$$

$$\Rightarrow 2y^3 \frac{d^2y}{dx^2} = P(x) P''(x) - 2 \left\{ \frac{P'(x)}{2} \right\}^2$$

[using Eqs. (i) and (ii)]

$$\Rightarrow 2y^3 \frac{d^2y}{dx^2} = P(x) P''(x) - \frac{1}{2} \{P'(x)\}^2$$

$$\Rightarrow 2 \frac{d}{dx} \left\{ y^3 \frac{d^2y}{dx^2} \right\} = P(x) P''(x) + P'(x) P''(x)$$

$$- \frac{1}{2} \times 2' (x) P''(x)$$

$$= P(x) P''(x)$$

28. (A)

29. (C)

30. (D) Suppose and number p is placed in envelope number q , then card number q must be placed in a wrong envelope.

Hence, at least two cards must be palced in wrong envelope if all of them are not kept in their corresponding envelopes.

31. (C) We know that

$$(AB)^n = A^n B^n \text{ is true only when } AB = BA$$

32. (A)

33. (A) By cosine rule,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos 60 = \frac{(3)^2 + c^2 - (4)^2}{2 \times 3 \times c}$$

$$\Rightarrow \frac{1}{2} = \frac{9 + c^2 - 16}{2 \times 3 \times c}$$

$$\Rightarrow 3c = c^2 - 7 = c^2 - 3c - 7 = 0$$

34. (C)

35. (A) Given, $z = 1 + i \tan \alpha$, where $\pi < \alpha < \frac{3\pi}{2}$.

$$\Rightarrow |z| = \sqrt{1 + \tan^2 \alpha} \Rightarrow |z| = \sqrt{\sec^2 \alpha}$$

$$\alpha \quad \left(\because \pi < \alpha < \frac{3\pi}{2} \right)$$

36. (B) Given that,

$$(x+1)^2 - 1 = 0$$

$$\Rightarrow (x+1)^2 - (1)^2 = 0$$

$$\Rightarrow (x+1+1)(x+1-1) = 0$$

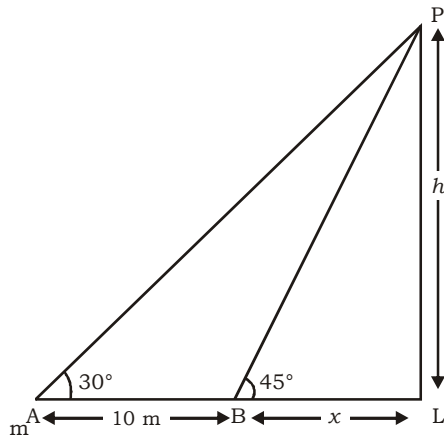
$$[\because a^2 - b^2 = (a-b)(a+b)]$$

$$\Rightarrow (x+2)(x) = 0$$

$$\therefore x = 0, -2$$

Hence, $(x+1)^2 - 1 = 0$ has two real roots.

37. (D)
38. (A)
39. (B) Let $BL = x$ m and $PL = h$ m



In ΔPBL

$$\tan 45^\circ = \frac{h}{x} = 1$$

Now, in ΔPAL ,

$$\tan 30^\circ = \frac{h}{10+x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3} h = 10 + x$$

$$\Rightarrow \sqrt{3} h = 10 + h$$

$$\Rightarrow (\sqrt{3} - 1)h = 10$$

$$\therefore h = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{10(\sqrt{3}+1)}{3-1} = \frac{10(\sqrt{3}+1)}{2}$$

$$= 5(\sqrt{3} + 1) = (5\sqrt{3} + 5) \text{ m}$$

40. (b) $\therefore \Delta\Delta' = \begin{vmatrix} a_1 & b_1 & c_1 & A_1 & B_1 & C_1 \\ a_2 & b_2 & c_2 & A_2 & B_2 & C_2 \\ a_3 & b_3 & c_3 & A_3 & B_3 & C_3 \end{vmatrix}$

$$= \begin{vmatrix} \Sigma a_1 A_1 & 0 & 0 \\ 0 & \Sigma a_2 A_2 & 0 \\ 0 & 0 & \Sigma a_3 A_3 \end{vmatrix} = \begin{vmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{vmatrix} = \Delta^3$$

$\therefore \Delta' = \Delta^2$

41. (B) $y = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right) = \tan^{-1} \left[\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{1 + 2 \cos^2 \frac{x}{2} - 1} \right]$

$$= \tan^{-1} \left(\tan \frac{x}{2} \right)$$

$$\Rightarrow y = \frac{x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

42. (C) If r is radius of the circle, then $\pi r^2 = 154$

$$\therefore r^2 = \frac{154 \times 7}{22} = 49 \quad \left(\text{taking } \pi = \frac{22}{7} \right)$$

$$\Rightarrow r = 7$$

Also, solving the equations of two given diameters, we get the coordinates of the centre as $(1, -1)$.

Hence, the equation of the circle is

$$(x-1)^2 + (y+1)^2 = 7^2 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y = 47$$

43. (C) Given that, (α, β) are the roots of the equation $x^2 + x + 2 = 0$, then

$$\alpha + \beta = -1 \quad \dots (i)$$

$$\text{and } \alpha \cdot \beta = 2 \quad \dots (ii)$$

Now, we have $\frac{\alpha^{10} + \beta^{10}}{\alpha^{-10} + \beta^{-10}} = (\alpha\beta)^{10} = (2)^{10}$

$$\text{[from eq. (ii)]}$$

$$= 1024$$

44. (A) The number of diagonals which can be drawn by joining the angular points of a polygon of 100 sides = ${}^{100}C_2 - 100$

$$= \frac{100!}{2!98!} - 100 = \frac{100 \times 99 \times 98!}{2 \times 98!} - 100$$

$$= 50 \times 99 - 100 = 4950 - 100$$

$$= 4850$$

45. (A) of centroid

$$= \left\{ \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right\}$$

$$= \left\{ \frac{2(x_1 + x_2 + x_3)}{6}, \frac{2(y_1 + y_2 + y_3)}{6} \right\}$$

$$\left\{ \frac{x_1 + x_2 + x_2 + x_3 + x_3 + x_1}{6}, \frac{y_1 + y_2 + y_2 + y_3 + y_3 + y_1}{6} \right\}$$

$$= \left\{ \frac{\frac{x_1 + x_2}{2} + \frac{x_2 + x_3}{2} + \frac{x_3 + x_1}{2}}{3}, \frac{\frac{y_1 + y_2}{2} + \frac{y_2 + y_3}{2} + \frac{y_3 + y_1}{2}}{3} \right\}$$

$$= \left\{ \frac{4+3+2}{3}, \frac{2+3+2}{3} \right\} = \left(3, \frac{7}{3} \right)$$

46. (A)

47. (C) $\therefore f(x) = kx^3 - 9x^2 + 9x + 3$

On differentiating w.r.t. x , we get

$$f'(x) = 3kx^2 - 18x + 9$$

For a function to be monotonically increasing.

$$\Delta = b^2 - 4ac < 0$$

$$\Rightarrow 36 - 12k < 0 \Rightarrow k > 3$$

48. (B) $3e^x \tan y \, dx + (1 + e^x) \sec^2 y \, dy = 0$

$$\Rightarrow \int \frac{3e^x}{1+e^x} \, dx + \int \frac{\sec^2 y}{\tan y} \, dy = 0$$

$$\Rightarrow 3 \log(1 + e^x) + \log \tan y = \log C$$

$$\Rightarrow \log (1 + e^x)^3 \tan y = \log C$$

$$(1 + e^x)^3 \tan y = C$$

49. (C) Given equation of parabola can be rewritten

$$\text{as } (x + 3)^2 = -\frac{2}{5}(y + 7) \text{ or } X^2 = 4AY$$

$$4A = -\frac{2}{5}$$

$$A = -\frac{1}{10}$$

Focus is $x = 0, Y = A$

$$\text{or } x + 3 = 0, y + 7 = -\frac{1}{10}$$

$$\therefore \left(-3, -\frac{71}{10}\right)$$

50. (*) Middle term = $\frac{4}{2} + 1 = 3\text{rd}$

$$\text{Coefficient of } T_3 = \text{Coefficient of } T_{2+1}$$

$$= {}^4C_2 \cdot 2^2 \times 3^2 = 360$$

51. (C) $x + y = 20$ and $z = xy^3$ is maximum

$$z = y^3(20 - y) = 20y^3 - y^4$$

$$\frac{dz}{dy} = 60y^2 - 4y^3 = 0$$

$$\therefore 4y^2(15 - y) = 0$$

$$\therefore y = 0, 15$$

$$\text{Now, } \frac{d^2z}{dy^2} = 120y - 12y^2 = 12y(10 - y)$$

$$\text{At } y = 15, \frac{d^2z}{dy^2} = 12 \times 15(10 - 15) < 0$$

\therefore Two parts are (15, 5)

52. (A) $\log \tan 89^\circ = \log \cot 1^\circ = -\log \tan 1^\circ$

\therefore Given expression becomes

$$\log \tan 1^\circ + \log \tan 2^\circ + \dots + \log \tan 44^\circ + \log \tan 45^\circ - \log \tan 44^\circ - \dots - \log \tan 2^\circ - \log \tan 1^\circ = \log \tan 45^\circ = \log 1 = 0$$

53. (A) Let $I = \int_0^a x f(x) dx$ (i)

By using property,

$$\int_0^a f(x) dx = \int_0^a f(a - x) dx$$

$$\therefore I = \int_0^a (a - x)f(a - x) dx$$

$$= \int_0^a (a - x)f(x) dx \text{ (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = a \int_0^a f(x) dx \Rightarrow I = \frac{a}{2} \int_0^a f(x) dx$$

54. (D) Total number of numbers in a factory = worker + owner = $9 + 1 = 10$

Now, the total daily income of workers of a factory including that of the owner = $110 \times 10 = 1100$ and the total daily income of workers of a factory excluding that of the owner = $(10 - 1) \times 76 = 9 \times 76 = 684$

Hence, the daily income of the owner = ₹ $(1100 - 684) = ₹ 416$

55. (A) $1 - \cos A = 2 \sin^2 \left(\frac{A}{2}\right)$

$$1 + \cos A = 2 \cos^2 \left(\frac{A}{2}\right)$$

Applying componendo and dividendo rule.

$$\frac{1 - \cos \theta}{1 + \cos \theta} = \frac{a(1 - \cos \phi) - b(1 - \cos \phi)}{a(1 + \cos \phi) + b(1 + \cos \phi)}$$

$$\Rightarrow \tan^2 \frac{\theta}{2} = \frac{a - b}{a + b} \tan^2 \frac{\phi}{2}$$

$$\therefore \tan \frac{\theta}{2} = \sqrt{\frac{a - b}{a + b}} \tan \frac{\phi}{2}$$

56. (D) $\left(\frac{1 + 2i}{2 + i} \times \frac{2 - i}{2 - 1}\right)^2 = \left(\frac{2 - i + 4 - 2i^2}{5}\right)^2$

$$\left(\frac{4 + 3i}{5}\right)^2$$

$$\frac{16 - 9 + 24i}{25} = \frac{7 + 24i}{25}$$

$$\text{Conjugate} = \frac{7}{25} - \frac{24}{5}i$$

57. (B) Selection of 3 points from given 14 points can be made in ${}^{14}C_3 = 364$ (i)

But selection of 3 points from the points on one line cannot give any triangle. Such selections are

$${}^3C_3 + {}^5C_3 + {}^6C_3 = 1 + 10 + 20 = 31 \text{ (ii)}$$

Hence, total number of triangle that can be formed = $364 - 31 = 333$

58. (A)

59. (C) Arrangement is $\times M \times C \times T \times$, first we place 3 consonant in $3!$ ways and then 3 vowels.

At four ' \times ' places (2 between them and 2 on sides) in which on vowel E is repeated can be placed in ${}^4P_3 / 2!$ ways.

Hence, required number = $3 \cdot {}^4P_3 / 2! = 72$

60. (B) Here, $P(A) = p, P(B) = q, P(\bar{A}) = 1 - p, P(\bar{B}) = 1 - q$

The probability that one person is alive

$$= P(A \text{ dies and } B \text{ lives}) + P(B \text{ dies and } A \text{ lives})$$

$$= p(1-q) + q(1-p)$$

$$= p - pq + q - qp = p + q - 2pq$$

61. (A) $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^3 + 2x - 3} \Rightarrow \frac{dy}{y^2 - y - 2}$

$$= \frac{dx}{x^2 + 2x - 3}$$

$$\Rightarrow \frac{1}{3} \left[\frac{1}{(y-2)} - \frac{1}{(y+1)} \right] dy$$

$$= \frac{1}{4} \left[\frac{1}{(x-1)} - \frac{1}{(x+3)} \right] dx$$

$$\therefore \frac{1}{3} \log \left| \frac{y-2}{y+1} \right| = \frac{1}{4} \log \left| \frac{x-1}{x+3} \right| + C$$

62. (C) Let $r = xi + yj + zk$, then

$$r \times a = b \times a \Rightarrow (r - b) \times a = 0$$

$$\therefore z = -1, x - y = 2$$

$$\text{and } r \times b = a \times b \Rightarrow (r - a) \times b = 0$$

$$\therefore y = 1, x + 2z = 1$$

$$\therefore x = 3, y = 1 \text{ and } z = -1$$

$$\therefore r = 3i + j - k$$

63. (D) According to the question,

$$1400 = \frac{28 \times 1400}{100} + \frac{35 \times 1400}{100}$$

$$+ \frac{12 \times 1400}{100} + \frac{8 \times 1400}{100} + 105 + \text{Transport}$$

$$\Rightarrow 1400 = 392 + 490 + 168 + 112 + 105 + \text{Transport}$$

$$\therefore \text{Transport} = ₹ 133 \text{ crores}$$

64. (D) Given, $\frac{dr}{dt} = 3 \text{ cm/s}$

Since, area of circle (A) = πr^2

On differentiating it w.r.t.t, we get

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi \times 10 \times 3 \quad (\because r = 10 \text{ cm})$$

$$= 60\pi \text{ cm}^2/\text{s}$$

65. (D) $\frac{1+2i}{1-(1+i^2-2i)} = \frac{1+2i}{1-1+1+2i} = \frac{1+2i}{1+2i} = 1$

$$\left| \frac{1+2i}{1-(1-i)^2} \right| = |1| = 1$$

66. (D) $\therefore A =$ Event of getting an even sum

$$= [(1, 1), (1, 3), (3, 1), (2, 2), (1, 5), (5, 1), (2, 4), (4, 2), (4, 2), (3, 3), (2, 6), (6, 2), (3, 5), (5, 3), (4, 4), (4, 6), (6, 4), (5, 5), (6, 6)]$$

and $B =$ Event of getting sum less than 5

$$= \{(1, 1), (2, 1), (1, 2), (1, 3), (3, 1), (2, 2)\}$$

$$A \cap B = \{(1, 1), (1, 3), (3, 1), (2, 2)\}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{18}{36} + \frac{6}{36} - \frac{4}{36} = \frac{5}{9}$$

67. (*)

68. (A) $y = \tan^{-1} x - x$

On differentiating w.r.t.x, we get

$$\frac{dy}{dx} = \frac{1}{1+x^2} - 1 = \frac{-x^2}{1+x^2}$$

$$\therefore \frac{dy}{dx} < 0, \forall \in \mathbb{R}$$

Hence, y is a decreasing function

69. (C) let $I = \int a^x e^x dx$

$$= a^x \int e^x dx - \int \left(\frac{d}{dx} (a^x) \right) \int e^x dx dx$$

$$= a^x e^x - \int a^x \log a e^x dx$$

$$a^x e^x - \log a \int a^x e^x dx$$

$$a^x e^x - \log a \times I$$

$$(1 + \log a) I = a^x e^x$$

$$I = \frac{a^x e^x}{1 + \log a} = \frac{a^x e^x}{\log ae}$$

70. (D) $n = 9, d = -\frac{1}{6}, a = \frac{1}{2} \Rightarrow S_n = \frac{3}{2}$

71. (D) Given $(1, 3, 2)_{1 \times 3} \begin{pmatrix} 1 & 3 & 0 \\ 3 & 0 & 2 \\ 2 & 0 & 1 \end{pmatrix}_{3 \times 3} \begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix}_{3 \times 1} = (0)_{1 \times 1}$

$$\Rightarrow (1 + 9 + 4 \cdot 3 + 0 + 0 \cdot 0 + 6 + 2)_{1 \times 3} \begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix}_{3 \times 1}$$

$$= (0)_{1 \times 1}$$

$$\Rightarrow (14 \ 3 \ 8) \begin{pmatrix} 0 \\ 3 \\ x \end{pmatrix}_{3 \times 1} = 0_{1 \times 1}$$

$$\Rightarrow (0 + 9 + 8x) = (0) \Rightarrow (8x + 9) = 0$$

On comparing,

$$8x + 9 = 0 \Rightarrow x = -\frac{9}{8}$$

72. (A) $\frac{dy}{dx} = -\frac{1}{1 + \cos 2x} \cdot \frac{1}{2\sqrt{(\cos 2x)}} (-2 \sin 2x)$

Now, put $\pi = \frac{\pi}{6}$, $\cos 2x = \frac{1}{2}$ and $\sin 2x = \frac{\sqrt{3}}{2}$

$$\therefore \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -\frac{1}{\left(1+\frac{1}{2}\right)} \cdot \frac{1}{2\sqrt{\frac{1}{2}}} (-2) \sqrt{\frac{3}{2}} = \sqrt{\frac{2}{3}}$$

73. (B) Given differential equation is

$$\log \left(\frac{dy}{dx}\right) + x = 0$$

$$\Rightarrow \log \left(\frac{dy}{dx}\right) = -x \Rightarrow \frac{dy}{dx} = e^{-x}$$

On integrating both sides, we get

$$y = -e^{-x} + C$$

Which is the required general solution.

74. (B) Breaking the given integral into partial fractions, we get

$$\frac{1}{(x-2)^2(x-3)} = \frac{1}{(x-2)^2} - \frac{1}{x-2} + \frac{1}{x-3}$$

$$\therefore \int \frac{dx}{(x-2)^2(x-3)} = -\int (x-2)^{-2} dx$$

$$- \int \frac{dx}{x-2} + \int \frac{dx}{x-3}$$

$$= -\frac{(x-2)^{-1}}{-1} - \log(x-2) + \log(x-3) + C_3$$

$$= \frac{1}{x-2} - \log \frac{x-2}{x-3} + C_3$$

$$\therefore C_1 = 1, C_2 = -1$$

75. (A) $\therefore 2^x + 3^y = 17$ (i)

$$\text{and } 2^{x+2} - 3^{y+1} = 5 \text{ (ii)}$$

$$\Rightarrow 4 \cdot 2^x - 3 \cdot 3^y = 5$$

Solving Eqs. (i) and (ii).

$$2^x = 8 \text{ and } 3^y = 9$$

$$\Rightarrow x = 3 \text{ and } y = 2$$

76. (D) Let a GP series is a, ar, ar^2, \dots

where, a = First term and r = Common ratio

$$\text{Sum of infinite series (S)} = \frac{a}{1-r}$$

$$\text{or } 6 = \frac{a}{(1-r)} \text{ or } a = 6(1-r)$$

$$\text{Sum of first two term (S}_2\text{)} = a + ar$$

$$\text{or } \frac{9}{2} = a(1+r)$$

by Eq. (i), put the value of a

$$\frac{9}{2} = 6(1-r)(1+r) \text{ or } \frac{3}{4} = 1-r^2$$

$$\text{or } r = +\frac{1}{2}$$

$$\text{Case I if } r = 1/2, \text{ then } a = 6\left(1-\frac{1}{2}\right) \text{ or } a = 3$$

$$\text{Case II if } r = -1/2, \text{ then } a = 6\left(1+\frac{1}{2}\right) \text{ or } a$$

$$= 3 \times 3 = 9$$

77. (C)

$$78. (C) \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 4A}}} =$$

$$\sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 4A)}}}$$

$$= \sqrt{2 + \sqrt{2 + 2\cos 2A}} = \sqrt{2(1 + \cos A)} = 2 \cos \frac{A}{2}$$

79. (D) Given that $x = a + bt - ct^2$ and $y = at + bt^2$

$$\text{Acceleration in } x \text{ direction} = \frac{d^2x}{dt^2} = -2c$$

$$\text{and acceleration in } y \text{ direction} = \frac{d^2y}{dt^2} = 2b$$

\therefore Resultant acceleration

$$= \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = \sqrt{(-2c)^2 + (2b)^2}$$

$$= 2\sqrt{b^2 + c^2}$$

80. (C) Given, the distance between the points

$$(7, 1, -3) \text{ and } (4, 5, \lambda) = 13$$

$$\Rightarrow \sqrt{(4-7)^2 + (5-1)^2 + (\lambda+3)^2} = 13$$

$$\Rightarrow \sqrt{(-3)^2 + (-4)^2 + (\lambda+3)^2} = 13$$

$$\Rightarrow \sqrt{9+16 + (\lambda+3)^2} = 13$$

$$\Rightarrow \sqrt{25 + (\lambda+3)^2} = 13$$

On squaring both sides, we get

$$25 + (\lambda+3)^2 = 169$$

$$\Rightarrow 25 + \lambda^2 + 9 + 6\lambda - 169 = 0$$

$$\Rightarrow \lambda^2 + 6\lambda - 135 = 0$$

$$\Rightarrow \lambda^2 + 15\lambda - 9\lambda - 135 = 0$$

$$\Rightarrow \lambda(\lambda+15) - 9(\lambda+15) = 0$$

$$\Rightarrow (\lambda+15)(\lambda-9) = 0$$

$$\lambda = 9, -15$$

$$81. (A) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x}$$

$$= \lim_{x \rightarrow 0} 2 \left(\frac{\sin x}{x}\right)^2 = 2 \cdot 1 \cdot 1 = 2$$

82. (C) The coefficients of three successive terms are ${}^nC_{r-1}$, nC_r , ${}^nC_{r+1}$.

$$\Rightarrow \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{1}{7} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_{r+1}} = \frac{7}{42} = \frac{1}{6}$$

Simplifying $\frac{r}{n-r+1} = \frac{1}{7}$ and $\frac{r+1}{n-r} = \frac{1}{6}$

$$\Rightarrow n - 8r = -1 \text{ and } n - 7r = 6$$

Solving $n = 55$

83. (B) The centre and radius of circle $(x - \alpha)^2 + (y - \beta)^2 = 9$ are (α, β) and 3, respectively.

Since, (α, β) lies on the straight line $y = x$
 $\therefore \alpha = \beta$

Now, the circle touches the circle $x^2 + y^2 = 1$ externally.

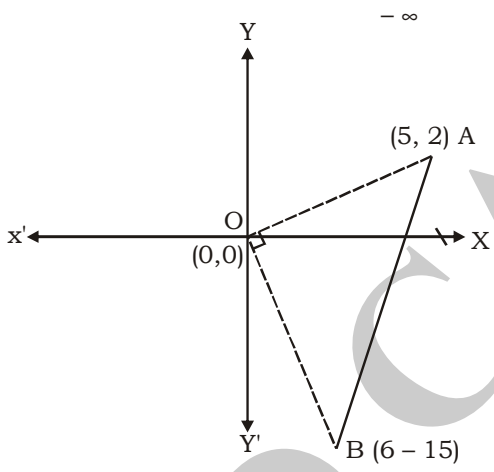
$$\alpha^2 + \beta^2 = 3 + 1 \Rightarrow \alpha^2 + \beta^2 = 4$$

$$\Rightarrow 2\alpha^2 = 4$$

$$\Rightarrow \alpha = \pm \sqrt{2}$$

$$\Rightarrow \alpha = \pm \sqrt{2} \text{ and } \beta = \pm \sqrt{2}$$

84. (C)



Slope of A = $\frac{2}{5}$

Slope of B = $\frac{-15}{6} = \frac{-5}{2}$

$$\therefore m_1 \cdot m_2 = \frac{2}{5} \times \frac{-5}{2} = -1$$

i.e., angle between OA and OB is $\pi/2$.

Hence, the line segment AB subtend right angle at origin O.

85. (C) Any point on the given line is $(5r - 3, 2r + 1, 3r - 4)$. If it is the foot of the perpendicular from $(0, 2, 3)$, then $5(5r - 3 - 0) + 2(2r + 1 - 2) + 3(3r - 4 - 3) = 0$

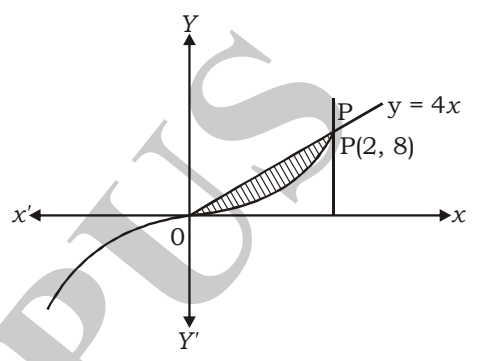
$$\Rightarrow 38r = 38 \Rightarrow r = 1$$

So, foot the perpendicular is $(2, 3, -1)$

86. (C) $y = x^3$ is a curve known as semi-cubical parabola. If $x \rightarrow -x$ and $y \rightarrow -y$ the equation does not change. It is symmetrical in 1st and 3rd quadrants. The line $y = 4x$ meets it at $4x = x^3$

$$\therefore x = 0, 2, -2 \Rightarrow y = 0, 8, -8$$

$$\therefore \text{Area in 1st quadrant} = \int_0^2 (y_1 - y_2) dx$$



$$A = \int_0^2 (4x - x^3) dx$$

$$= \left[2x^2 - \frac{x^4}{4} \right]_0^2 = 4$$

87. (C) Statement I A matrix is only an arrangement of numbers, it has no definite value. e.g., $[7] \neq 7$

Statement II Let $\Delta_1 = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}_{3 \times 3} = (2 - 3) = -3$

and $\Delta_2 = \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}_{2 \times 2} = 3 - 6 = -3$

Hence, two determinants of different orders may have the same value.

88. (A) $f(x) = x \left(\frac{a^x - 1}{a^x + 1} \right)$

Put $x = -x$, we get

$$f(-x) = (-x) \left(\frac{a^{-x} - 1}{a^{-x} + 1} \right)$$

$$= -x \left(\frac{\frac{1 - a^x}{a^x}}{\frac{1 + a^x}{a^x}} \right) = (-x) \left(\frac{1 - a^x}{1 + a^x} \right)$$

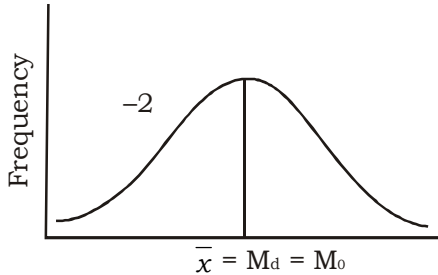
$$= x \left(\frac{a^x - 1}{a^x + 1} \right) = f(x)$$

So, $f(x)$ is an even function.

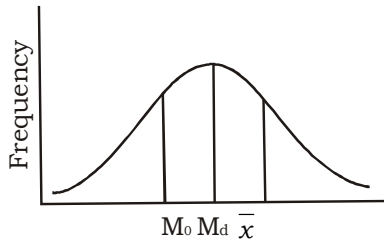
89. (D) frequency curve may be symmetrical, positive skew and negative skew.

For symmetry \Rightarrow Mean = Median = Mode

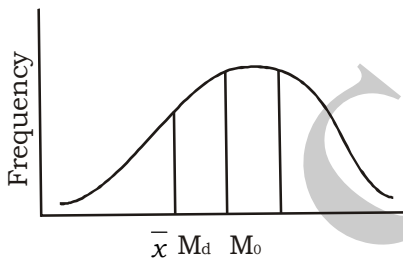
$$\Rightarrow \bar{x} = M_d = M_0$$



For positive skew, Mean > Median > Mode



For negative skew, Mean < Median < Mode



90. (B) $\therefore S = 4\pi r^2$

$$\Rightarrow \frac{dS}{dt} = \frac{8\pi r dr}{dt} \text{ and } V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = \frac{4\pi r^2}{8\pi r} \cdot \frac{dS}{dt}$$

$$= \frac{1}{2}r \cdot \frac{dS}{dt}$$

91. (B) First we arrange the given data in ascending order, we get 3, 6, 6, 7, 7, 7, 8, 9, 9, 10, 10, 10, 12

Total terms, $n = 13$ (odd)

$$\therefore \text{Median} = \left(\frac{n+1}{2}\right)\text{th term}$$

$$= \left(\frac{13+1}{2}\right)\text{th term} = 7\text{th term} = 8$$

92. (A) Given equations.

$$8x - 9y = 20 \quad \dots\dots (i)$$

$$\text{and } 7x - 10y = 9 \quad \dots\dots (ii)$$

On multiplying Eq. (i) by 10 and Eq. (ii) by 9 and then subtracting Eq. (ii) from Eq. (i), we get

$$80x - 90y = 200$$

$$63x - 90y = 81$$

$$\begin{array}{r} - \quad + \quad - \\ \hline 17x = 119 \end{array} \Rightarrow x = 7$$

$$\text{and } 10y = 7(7) - 9 = 49 - 9$$

$$\Rightarrow 10y = 40$$

$$\Rightarrow y = 4$$

$$\therefore 2x - y = 2(7) - 4 = 14 - 4 = 10$$

93. (A) Let $I = \int (x \cos x + \sin x) dx$

$$= \int x \cos x dx + \int \sin x dx$$

$$= (x \sin x - \int \sin x dx) + \int \sin x dx = x \sin x + C$$

94. (C) Given, $\frac{1}{\sin x} \frac{d^2y}{dx^2} = \text{cosec } x - 2 \sin x$

$$\Rightarrow \frac{d^2y}{dx^2} = 1 - 2 \sin^2 x = \cos 2x$$

On integrating both sides w.r.t. x , we get

$$\frac{dy}{dx} = \frac{\sin 2x}{2} + C_1$$

Now, again integrating both sides w.r.t. x , we

$$\text{get } y = -\frac{\cos 2x}{4} + C_1 x + C_2$$

95. (B) Here, $n(S) = 52$,

$$n(E_1) = 1, n(E_2) = 1, n(E_1 \cap E_2) = \phi$$

$$\therefore (E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{52} + \frac{1}{52} - 0 = \frac{1}{26}$$

96. (A) Given, $(2x + 3y + 4) + \lambda(6x - y + 12) = 0$

$$2x + 6\lambda x + 3y - \lambda y + 4 + 12\lambda = 0$$

$$2x(3\lambda + 1) + y(3 - \lambda) + 4 + 12\lambda = 0$$

Since, line (i) is parallel to Y-axis,

So, the coefficient of y must be zero.

$$\therefore 3 - \lambda = 0 \Rightarrow \lambda = 3$$

97. (B) let α be a root of $x^2 - x + k = 0$. The, 2α is a root of

$$x^2 - x + 3k = 0$$

$$\therefore 4\alpha^2 - 2\alpha + 3k = 0 \text{ and } \alpha^2 - \alpha + k = 0$$

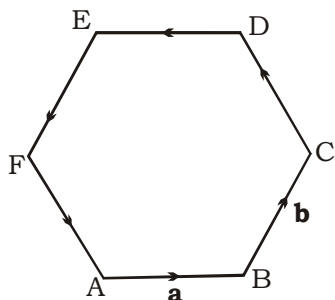
$$\Rightarrow \frac{\alpha^2}{-2k + 3k} = \frac{\alpha}{3k - 4k} = \frac{1}{-4 + 2}$$

$$\Rightarrow \alpha^2 = -\frac{k}{2} \text{ and } \alpha = \frac{k}{2}$$

$$\text{Now, } \alpha^2 = (\alpha)^2 \Rightarrow \left(-\frac{k}{2}\right) = \left(\frac{k}{2}\right)^2$$

$$\Rightarrow k^2 + 2k = 0 \Rightarrow k = 0 \text{ or } -2$$

98. (B)



$$AC = AB + BC = a + b$$

$$AD = 2BC = 2b$$

$$\therefore FA = DC = AC - AD$$

99. (A) $\int \sec^n x \tan x \, dx$

$$= \int \sec^{n-1} x \cdot \sec x \tan x \, dx$$

$$= \frac{(\sec)^{n-1+1}}{n-1+1} = \frac{1}{n} \sec^n x + c$$

100. (A) $\frac{\frac{1}{3} \log_2 17}{\frac{1}{2} \log_3 23} - \frac{\frac{2}{3} \log_2 17}{\log_3 23} = 0$

$$\Rightarrow \frac{\frac{2}{3} \log_2 17}{\log_3 23} - \frac{\frac{2}{3} \log_2 17}{\log_3 23} = 0$$

101. (D) Given, $2a = 3(2b)$

$$\therefore \frac{b^2}{a^2} = \frac{1}{9}$$

$$\Rightarrow c^2 = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}}$$

$$\therefore e = \frac{2\sqrt{2}}{3}$$

102. (B) We know that two matrices A and B are defined for addition, if they are of the same type, Thus, if A be $m \times n$, then B should also be $m \times n$ order. Again, since AB is also defined therefore number of columns in A i.e, n should be equal to number of rows in B i.e, m. Hence, $n = m$ and in that case both A and B will be square matrices of order equal to $m = n$.

103. (C) $y = \sin^{-1} \frac{4x}{1+4x^2} = \sin^{-1} \frac{2 \tan \theta}{1 + \tan^2 \theta}$, where

$$\tan \theta = 2x$$

$$\sin^{-1} \sin 2\theta$$

$$= 2\theta$$

$$= 2 \tan^{-1} 2x$$

$$\frac{dy}{dx} = \frac{2}{1+(2x)^2} \times 2 = \frac{4}{1+4x^2}$$

104. (B) Given differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} = 0$$

$$\Rightarrow \left(3 \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} - 4\right) = \left(\frac{d^3y}{dx^3}\right)^{2/3}$$

On cubing both sides,

$$\left(\frac{d^3y}{dx^3}\right)^2 = \left(3 \frac{d^2y}{dx^2} - 5 \frac{dy}{dx} - 4\right)^2$$

Degree = 2

105. (D) Given, $\left(x^2 - \frac{1}{x}\right)^9$

General term,

$$T_{r+1} = {}^nC_r (x^2)^{9-r} \cdot \left(-\frac{1}{x}\right)^r$$

$$= {}^nC_r x^{18-2r} (-1)^r x^{-r}$$

$$= {}^nC_r x^{18-3r} (-1)^r$$

For independent term,

$$\text{Put } 18 - 3r = 0$$

$$\Rightarrow 3r = 18$$

$$\Rightarrow r = 6$$

$$\therefore T_{(6+1)} = {}^9C_6 x^{(18-18)} \cdot (-1)^6$$

$$T_7 = {}^9C_6 \cdot 1 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84$$

106. (A) $\int_{-x/2}^{x/2} |\sin x| \, dx = \int_{-x/2}^0 (-\sin x) \, dx +$

$$\int_0^{x/2} (\sin x) \, dx$$

$$= -[-\cos x]_{\pi/2}^0 - [\cos x]_0^{\pi/2}$$

$$= [\cos 0 - \cos(-\pi/2)] - (\cos \pi/2 - \cos 0)$$

$$= (1 - 0) - (0 - 1) = 1 + 1 = 2$$

107. (B) $\therefore \cos(\sin^{-1} x) = \frac{1}{2}$

$$\Rightarrow \sin^{-1} x = \cos^{-1} \left(\frac{1}{2}\right)$$

$$\Rightarrow \sin^{-1} x = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \therefore \tan(\cos^{-1} x) &= \tan\left(\cos^{-1} \frac{\sqrt{3}}{2}\right) \\ &= \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \end{aligned}$$

Hence, $\tan(\cos^{-1} x)$ have two values.

108. (D) If each item of a data is increased or decreased by the same constant, then standard deviation of the data remains unchanged, i.e., SD is 6.

109. (B) $\log_{81} 243 = \log_{(3^4)} (3)^5 = \frac{5}{4} \log_3 3$

$$= \frac{5}{4} \times 1 = \frac{5}{4} = 1.25 \quad \left(\because \log_a a^m = \frac{m}{n}\right)$$

110. (C) $\int_{\ln 2}^x \frac{1}{2e^x - 1} dx = \ln \frac{3}{2}$

Let $I = \int_{\ln 2}^x \frac{1}{2e^x - 1} dx$

Put $e^x - 1 = t \Rightarrow dx = \frac{dt}{1+t}$

$$I = \int_{\ln 2}^x \frac{1}{t(1+t)} dt = \int_{\ln 2}^x \left[\frac{1}{t} - \frac{1}{1+t} \right] dt$$

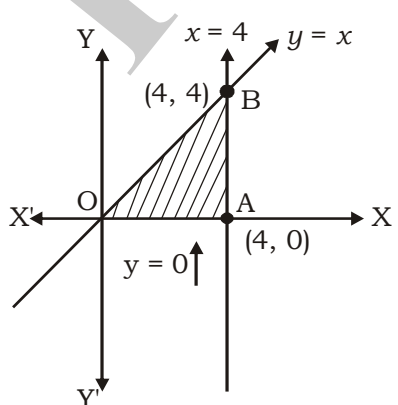
$$= [\ln t - \ln(1+t)]_{\ln 2}^x = [\ln(e^x - 1)] - [\ln e^x]_{\ln 2}^x$$

$$= \left[\ln \left(\frac{e^x - 1}{e^x} \right) \right]_{\ln 2}^x = \ln \left(\frac{e^x - 1}{e^x} \right) - \ln \frac{1}{2}$$

$$= \ln 2 \left(\frac{e^x - 1}{e^x} \right) \Rightarrow 2 \left(\frac{e^x - 1}{e^x} \right) = \frac{3}{2} \text{ (Given)}$$

$$\Rightarrow e^x = 4 \Rightarrow x = \ln 4$$

111. (B) \therefore Required Area = area (Δ OAB)

$$= \frac{1}{2} \times 4 \times 4 = 8 \text{ sq units}$$


112. (B) If the values of a set are measured in cm, then the variance has unit cm^2 .

113. (B) Let required ratio be $\lambda : 1$. Then, the coordinates of point which divides the line joining $(-1, 1)$ and $(5, 7)$ in the ratio $\lambda : 1$, is

$$\left(\frac{5\lambda - 1}{\lambda + 1}, \frac{7\lambda + 1}{\lambda + 1} \right)$$

But it lies on $x + y = 4$

$$\therefore \frac{5\lambda - 1}{\lambda + 1} + \frac{7\lambda + 1}{\lambda + 1} = 4$$

$$\Rightarrow 12\lambda = 4\lambda + 4, \Rightarrow \lambda = 1/2$$

\therefore Required ratio = $1 : 2$

114. (D) By the definition of the greatest integer function, $[x] = -1$ when $-1 \leq x < 0$ and $[x] = 0$ when $0 \leq x < 1$. Hence, by the definition of the greatest integer function

$$f(x) = \frac{\sin(-1)}{-1} = \sin 1 \text{ when } -1 \leq x < 0 \dots (i)$$

$$\text{and } f(x) = \frac{\sin 0}{0} = \frac{0}{0}$$

When $0 \leq x < 1$ (ii)

$$\therefore \text{Lf}(0-0) = \lim_{h \rightarrow 0} \sin 1 = \sin 1$$

$$\text{and } \text{Rf}(0+0) = \lim_{h \rightarrow 0} 0 = 0$$

Since, $f(0-0) \neq f(0+0)$, then the limit of $f(x)$ at $x = 0$ does not exist.

115. (D) $\therefore x + iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$$\Rightarrow x + iy = 6i(3i^2 + 3) + 3i(4i + 20) + 1(12 - 60i)$$

$$= -18i + 18i - 12 + 60i + 12 - 60i = 0$$

116. (A)

117. (A) On taking log both sides, we get $p \log x + q \log y = (p + q) \log(x + y)$

$$\Rightarrow p \frac{1}{x} + q \frac{1}{y} \frac{dy}{dx} = (p + q) \frac{1}{x + y} \cdot \left(1 + \frac{dy}{dx} \right)$$

$$\frac{p}{x} - \frac{p + q}{x + y} = \left(\frac{p + q}{x + y} - \frac{q}{y} \right) \frac{dy}{dx}$$

$$\Rightarrow \frac{py - qx}{x(x + y)} = \frac{py - qx}{y(x + y)} \frac{dy}{dx}$$

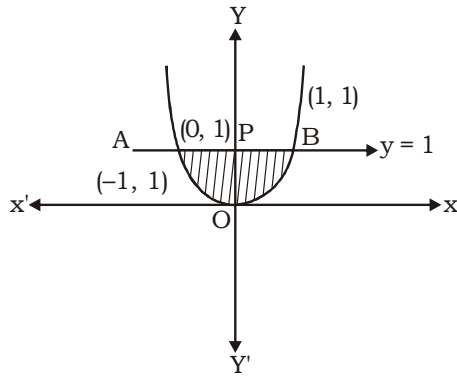
$$\frac{dy}{dx} = \frac{y}{x}$$

118. (D)

119. (D) Given equation of parabola and line are

$$x^2 = y \quad \dots (i)$$

$$\text{and } y = 1 \quad \dots (ii)$$



On solving Eqs. (i) and (ii), we get

$$x^2 = 1 \Rightarrow x = \pm 1$$

\therefore Required area = $2 \times$ Area of OPBO

$$= 2 \int_0^1 x \, dy = \int_0^1 \sqrt{y} \, dy = 2 \left[\frac{2y^{2/3}}{3} \right]_0^1$$

$$= \frac{4}{3}(1 - 0) = \frac{4}{3} \text{ sq units}$$

120. (B) Given that, $\alpha = k$ and $\gamma = 2i + 3j + 4k$

Since, β is perpendicular to both α and γ .

$$\text{i.e., } \beta = \pm (\alpha \times \gamma) = + \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= + i(0 - 3) - j(0 - 2) + k(0 - 0)$$

$$= + (-3i + 2j)$$



K D Campus Pvt. Ltd

2007, OUTRAM LINES, 1ST FLOOR, OPPOSITE MUKHERJEE NAGAR POLICE STATION, DELHI-110009

NDA (MATHS) MOCK TEST - 37 (Answer Key)

- | | | | | | |
|---------|---------|---------|---------|----------|----------|
| 1. (B) | 21. (B) | 41. (B) | 61. (A) | 81. (A) | 101. (D) |
| 2. (B) | 22. (D) | 42. (C) | 62. (C) | 82. (C) | 102. (B) |
| 3. (C) | 23. (B) | 43. (C) | 63. (D) | 83. (B) | 103. (C) |
| 4. (A) | 24. (D) | 44. (A) | 64. (D) | 84. (C) | 104. (B) |
| 5. (B) | 25. (B) | 45. (A) | 65. (D) | 85. (C) | 105. (D) |
| 6. (B) | 26. (C) | 46. (A) | 66. (D) | 86. (C) | 106. (A) |
| 7. (B) | 27. (C) | 47. (C) | 67. (*) | 87. (C) | 107. (B) |
| 8. (B) | 28. (A) | 48. (B) | 68. (A) | 88. (A) | 108. (D) |
| 9. (C) | 29. (C) | 49. (C) | 69. (C) | 89. (D) | 109. (B) |
| 10. (D) | 30. (D) | 50. (*) | 70. (D) | 90. (B) | 110. (C) |
| 11. (A) | 31. (C) | 51. (C) | 71. (D) | 91. (B) | 111. (B) |
| 12. (B) | 32. (A) | 52. (A) | 72. (A) | 92. (A) | 112. (B) |
| 13. (B) | 33. (A) | 53. (A) | 73. (B) | 93. (A) | 113. (B) |
| 14. (C) | 34. (C) | 54. (D) | 74. (B) | 94. (C) | 114. (D) |
| 15. (D) | 35. (A) | 55. (A) | 75. (A) | 95. (B) | 115. (D) |
| 16. (B) | 36. (B) | 56. (D) | 76. (D) | 96. (A) | 116. (A) |
| 17. (C) | 37. (D) | 57. (B) | 77. (C) | 97. (B) | 117. (A) |
| 18. (B) | 38. (A) | 58. (A) | 78. (C) | 98. (B) | 118. (D) |
| 19. (B) | 39. (B) | 59. (C) | 79. (D) | 99. (A) | 119. (D) |
| 20. (B) | 40. (B) | 60. (B) | 80. (C) | 100. (A) | 120. (B) |

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003