NDA (MATHS) MOCK TEST - 43 (SOLUTION)

1. (C) If A and B are finite sets, then
$n(A - B) = n(A) - n(A \cap B)$

2. (A) Given, A = {1, 4, 9, 16, 25, 36, 49, 64, 81}
and B = {2, 4, 6,....}
Now, $A \cap B$ = {4, 16, 36, 64}
$\therefore$ The cardinality of $(A \cap B)$
= Number of elements in $(A \cap B)$ = 4

3. (B) Let n (A) = m, n(B) = n
The total possible subsets of A and B are 2^m and 2^n, respectively.
According to the given,
$2^m - 2^n = 56$
$\Rightarrow 2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$
$\Rightarrow n = 3, m - n = 3 \Rightarrow m = 6, n = 3$

4. (A) $\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(AB) - n(BC) - n(CA) + n(ABC)$
= 80 (number of families reading atleast one newspapers A, B and C)
$\therefore$ Total number of families = 100
So, 20 families do not read any newspaper.

5. (D) $\because x + iy = \begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix}$

$\Rightarrow x + iy = 6i(3i^2 + 3) + 3i(4i + 20) + 1(12 - 60i) = -18i + 18i - 12 + 60i + 12 - 60i = 0$
$\Rightarrow x - iy = 0$

6. (A) $\because x = \dfrac{3+5i}{2}$

$\therefore x^3 = \dfrac{27 + 125i^3 + 225i^2 + 135i}{8}$

$= \dfrac{27 - 125i - 225 + 135i}{8}$

$= \dfrac{-198 + 10i}{8} = \dfrac{-99 + 5i}{4}$

and $x^2 = \dfrac{9 + 25i^2 + 30i}{4} = \dfrac{9 - 25 + 30i}{4}$

$= \dfrac{-8 + 15i}{2}$

Now, $= 2x^3 + 2x^2 - 7x + 72$

$= \left(\dfrac{-99 + 5i}{2}\right) + (-8 + 15i) - \dfrac{7(3+5i)}{2} + 72$

$= -\dfrac{99}{2} + \dfrac{5i}{2} - 8 + 15i - \dfrac{21}{2} - \dfrac{35}{2}i + 72$

$= \left(-\dfrac{99}{2} - 8 - \dfrac{21}{2} + 72\right) + \left(\dfrac{5}{2} + 15 - \dfrac{35}{2}\right)i$

$= \dfrac{-99 - 16 - 21 + 144}{2} = \dfrac{8}{2} = 4$

7.(A) $\dfrac{1}{-a+ib} = \left(\dfrac{-a}{a^2+b^2} - i\dfrac{b}{a^2+b^2}\right)$ ($\because$ A + iB form)

Equation of line which passes through the point (a, b) and the point $\left(\dfrac{-a}{a^2+b^2}, \dfrac{-b}{a^2+b^2}\right)$,

$(y - b) = \dfrac{\dfrac{-b}{a^2+b^2} - b}{\dfrac{-a}{a^2+b^2} - a}(x - a) = \dfrac{b}{a}(x - a)$

$\Rightarrow ay = bx$

$\therefore$ a straight line is passing through the points represented by the complex numbers a + ib and $\dfrac{1}{-a+ib}$, which passes through the origin.

8. (C) Let $z = \cos\theta + i\sin\theta$

Now, on rotating through an angle $\dfrac{\pi}{2}$, z becomes $\cos\left(\dfrac{\pi}{2} + \theta\right) + i\sin\left(\dfrac{\pi}{2} + \theta\right)$

$= -\sin\theta + i\cos\theta = i^2\sin\theta + i\cos\theta$
$= i(\cos\theta + i\sin\theta) = iz$

9. (C) Since, r and s are the roots of $Ax^2 + Bx + C = 0$, then $r + s = -\dfrac{B}{A}$ and $rs = \dfrac{C}{A}$

Now, the roots of $x^2 + px + q = 0$ be r^2 and s^2.
$\therefore r^2 + s^2 = -p$ and $r^2s^2 = q$
$\Rightarrow (r + s)^2 - 2rs = -p$

$\Rightarrow \dfrac{B^2}{A^2} - \dfrac{2C}{A} = -p$

$\Rightarrow \dfrac{B^2 - 2AC}{A^2} = -P$

$\Rightarrow p = \dfrac{2AC - B^2}{A^2}$

10. (D) Since, α and β are the roots of the equation $x^2 - 2x - 1 = 0$, then
$\alpha + \beta = 2$ and $\alpha\beta = -1$
$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$
$\Rightarrow 4 = \alpha^2 + \beta^2 - 2$

$\Rightarrow \alpha^2 + \beta^2 = 6$

$\Rightarrow (\alpha + \beta)^2 = 6^2 \Rightarrow = \alpha^4 + \beta^4 + 2 = 36$

$\Rightarrow \alpha^4 + \beta^4 = 34$

Now, $\alpha^2\beta^{-2} + \alpha^{-2}\beta^2 = \dfrac{\alpha^2}{\beta^2} + \dfrac{\beta^2}{\alpha^2}$

$= \dfrac{\alpha^4 + \beta^4}{(\alpha\beta)^2} = \dfrac{34}{(-1)^2} = 34$

11. (A) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta\ (\alpha+\beta)$

$= (4)^3 - 3\times\dfrac{3}{2}(4) = 64 - 18 = 46$

12. (A) $x^2 - \left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right)x + \dfrac{1}{\alpha}\cdot\dfrac{1}{\beta} = 0$

$\Rightarrow x^2 - \left(\dfrac{\alpha+\beta}{\alpha\beta}\right)x + \dfrac{1}{\alpha\beta} = 0$

$\Rightarrow x^2 - \left(\dfrac{4}{\frac{3}{2}}\right) + \left(\dfrac{1}{\frac{3}{2}}\right) = 0$

$\Rightarrow 3x^2 - 8x + 2 = 0$

13. (C) Let a and d be the first term and common difference of the AP.

$\therefore$ a + 58d = 449 ...... (i)

and a + 448d = 59 ...... (ii)

On solving Eqs. (i) and (ii), we get

a = 507 and d = – 1

Now, assume that nth term will be zero.

$\therefore$ 0 = 507 + (n – 1) (–1)

$\Rightarrow$ 507 = n – 1

$\Rightarrow$ n = 508

14. (C) Since, the given series $\log_a x$, $\log_b x$ and $\log_c x$ are in HP,

$\Rightarrow \dfrac{\log x}{\log a}, \dfrac{\log x}{\log b}$ and $\dfrac{\log x}{\log c}$ are in HP.

$\Rightarrow \dfrac{\log a}{\log x}, \dfrac{\log b}{\log x}$ and $\dfrac{\log c}{\log x}$ are in AP.

$\Rightarrow \log_x a$, $\log_x b$ and $\log_x c$ are in AP.

$\Rightarrow$ a, b and c are in GP.

15. (A) Given, series is $1\cdot3^2 + 2\cdot5^2 + 3\cdot7^2 + \ldots \infty$

This is an arithmetic geometric series whose nth term is equal to

$T_n = n(2n + 1)^2 = 4n^3 + 4n^2 + n$

$\therefore S_n = \sum_1^n T_n = \sum_1^n (4n^3 + 4n^2 + n)$

$= 4\sum_1^n n^3 + 4\sum_1^n n^2 + \sum_1^n n$

$= 4\left[\dfrac{n}{2}(n+1)\right]^2 + \dfrac{4}{6}n\,(n + 1)\,(2n + 1) + \dfrac{n}{2}(n + 1)$

$= n(n + 1)\left[n^2 + n + \dfrac{4}{6}(2n+1) + \dfrac{1}{2}\right]$

$= \dfrac{n}{6}(n + 1)(6n^2 + 14n + 7)$

16. (C) First five terms of a geometric pergression are a, ar, ar^2, ar^3 and ar^4.

$\therefore$ Mean $= \dfrac{a + ar + ar^2 + ar^3 + ar^4}{5}$

$= \dfrac{a(1 + r + r^2 + r^3 + r^4)}{5}$

$= \dfrac{a\left(\dfrac{r^5-1}{r-1}\right)}{5}$

$= \dfrac{a(r^5-1)}{5(r-1)}$

17. (C) $4^{\log_3 2^3} + 9^{\log_2 2^2} = 10^{\log_x 83} \Rightarrow 4^{1/2} + 9^2 = 10^{\log_x 83}$

$\Rightarrow 2 + 81 = 10^{\log_x 83} \Rightarrow 83 = 10^{\log_x 83} \Rightarrow x = 10$

18. (C) $\dfrac{\log_{\sqrt{\alpha\beta}} H}{\log_{\sqrt{\alpha\beta\gamma}} H} = \dfrac{\log_H \sqrt{\alpha\beta\gamma}}{\log_H \sqrt{\alpha\beta}}$

$\left(\because \log_a b = \dfrac{1}{\log_b a}\right)$

$= \log_{\sqrt{\alpha\beta}} \sqrt{\alpha\beta\gamma}$

$= \dfrac{1}{2}\log_{\sqrt{\alpha\beta}} (\alpha\beta\gamma)$

$= \dfrac{1/2}{1/2}\log_{\alpha\beta} (\alpha\beta\gamma)$ $\left(\because \log_{a^m} b = \dfrac{1}{m}\log_a b\right)$

$= \log_{\alpha\beta} \alpha\beta\gamma$

19. (A) $\because$ $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix}$

$= \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

20. (B) Let A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow$ $|A| = \cos^2\alpha + \sin^2\alpha = 1$

adj(A) = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{|A|}$ adj (A) = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

21. (C) Given that, A = $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{(15-8)}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$ and AC = $\begin{bmatrix} 19 & 24 \\ 37 & 46 \end{bmatrix}$

$\Rightarrow$ $A^{-1}AC = A^{-1}\begin{bmatrix} 19 & 24 \\ 37 & 46 \end{bmatrix}$

$\Rightarrow$ $C = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 19 & 24 \\ 37 & 46 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 95-74 & 120-92 \\ -76+111 & -96+138 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 21 & 28 \\ 35 & 42 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

22. (B) $A + A^T$ is a square matrix.
Now, $(A + A^T)^T = A^T + (A^T)^T = A^T + A$
Hence, $A + A^T$ is symmetric matrix.

23. (D) Here A = $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$,

B = $\begin{bmatrix} l \\ m \\ n \end{bmatrix}$ and X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\therefore$ $|A| = -2\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix}$

$= -2(4-1) - (-2-1) + 1(1+2)$
$= -6 + 3 + 3 = 0$

Now, adj (A) = $\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} i \\ m \\ n \end{bmatrix} = 3\begin{bmatrix} l+m+n \\ l+m+n \\ l+m+n \end{bmatrix} = 3\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore$ (adj A) · B = 0
So, the given system of equations has an infinitely many solutions.

24. (C) The homogeneous linear system of equations is consistent i.e, possesses non-trivial solutions (one or many). If

$\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 1 & k & 5 \\ k & -12 & -14 \end{vmatrix} = 0$

$\Rightarrow 2(-14k + 60) - 3(-14 - 5k) + 5(-12 - k^2) = 0$
$\Rightarrow$ $5k^2 + 13k - 102 = 0$
$\Rightarrow$ $(5k - 17)(k + 6) = 0$
$\Rightarrow$ $k = -6, \frac{17}{5}$

25. (B) Given $a^{-1} + b^{-1} + c^{-1} = 0$

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

$\Rightarrow$ Expand with respect to R_1
$\Rightarrow$ $(1 + a)(1 + b)(1 + c) - 1\} - 1\{1 + c - 1\} + 1(1 - 1 - b)\} = \lambda$
$\Rightarrow$ $(1 + a)\{b + c + bc\} - c - b = \lambda$
$\Rightarrow$ $b + c + bc + ab + ac + abc - c - b = \lambda$
$\Rightarrow$ $bc + ab + ac + abc = \lambda$
$\Rightarrow$ $abc\left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right\} + abc = \lambda$
$\Rightarrow$ $abc\{(a^{-1} + b^{-1} + c^{-1}) + 1\} = \lambda$
$\Rightarrow$ $abc(0 + 1) = \lambda$ [from Eq. (i)]
$\Rightarrow$ $\lambda = abc$

26. (B) The given system of equations has infinitely many solutions, then

$\frac{2}{2a} = \frac{3}{a+b} = \frac{7}{28}$

$\Rightarrow$ a = 4
and 12 = a + b and a = 4
$\Rightarrow$ b = 8 $\Rightarrow$ b = 2a

27. (D) Let $\Delta = \begin{vmatrix} 1 & x & y+z \\ 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$

On applying $C_3 \to C_3 + C_2$

$\Rightarrow$ $\Delta = \begin{vmatrix} 1 & x & x+y+z \\ 1 & y & x+y+z \\ 1 & z & x+y+z \end{vmatrix} = (x+y+z)\begin{vmatrix} 1 & x & 1 \\ 1 & y & 1 \\ 1 & z & 1 \end{vmatrix}$

$= (x + y + z) \times 0$ ($\because$ two columns are idential)
$= 0$

28. (C) We have, $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^{n-1} & 3^{n-1} & 5^{n-1} \end{vmatrix}$

$$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} 2^{r-1} & \sum_{r=1}^{n} 2\cdot 3^{r-1} & \sum_{r=1}^{n} 4\cdot 5^{r-1} \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix}$$

$$\Rightarrow \sum_{r=1}^{n} D_r = \begin{vmatrix} 2^n - 1 & 3^n - 1 & 5^n - 1 \\ x & y & z \\ 2^n - 1 & 3^n - 1 & 5^n - 1 \end{vmatrix} = \sum_{r=1}^{n} D_r = 0$$

($\because$ two rows aresame)

29. (D) Given, $(3 - 2x)(1 + 3x)^{-3}$
$= (3 - 2x)(1 - 9x + 54x^2 - 270x^3 + \ldots..)$
= coefficient of $x^3 = -810 - 108 = -918$

30. (B) $\left(\frac{1-x}{1+x}\right)^2 = (1-x)^2 (1+x)^{-2} = (1-2x+x^2)(1+x)^{-2}$
$= (1 - 2x + x^2)(1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots)$
$\therefore$ Coefficient of x^4 in $\left(\frac{1-x}{1+x}\right)^2 = 5 + 8 + 3 = 16$

31. (A) Given that, $(1 + ax)^n = 1 + 8x + 24x^2 + \ldots$
$\Rightarrow 1 + \frac{n}{1}ax + \frac{n(n-1)}{1\cdot 2}a^2x^2 + \ldots.. = 1 + 8x + 24x^2 + \ldots$
On comparing the coefficients of x and x^2, we get
$\Rightarrow na = 8, \frac{n(n-1)}{1\cdot 2}a^2 = 24$
$\Rightarrow na(n-1)a = 48$
$\Rightarrow 8(8 - a) = 48 \Rightarrow 8 - a = 6 \Rightarrow a = 2 \Rightarrow n = 4$

32. (A) Statement I is true but statement II is false, because coefficient of (r + 1)th term in the expansion of $(1 + x)^n$ th is $(-1)^r\ {}^nC_r$.

33. (B) Given that, C(n, 12) = C(n, 8)
$\Rightarrow {}^nC_{12} = {}^nC_8$
$\Rightarrow n = 12 + 8 = 20$ ($\because {}^nC_x \Rightarrow {}^nC_y \Rightarrow x + y = n$)
So, ${}^{22}C_n = C(22,n) = {}^{22}C_{20}$
$= \frac{22!}{2!20!} = 231$

34. (B) Possibilities of words formed the letters of word 'JOKE' are JOKE, KOJE, KEJO, JEKO, EJOK, EKOJ,OKEJ, OJEK
Thus, required number of words = 8.

35. (C) Required number of triangles formed
${}^{12}C_3 - {}^7C_3$
$= \frac{12!}{3!9!} - \frac{7!}{3!4!} = \frac{12\cdot 11\cdot 10}{3\cdot 2\cdot 1} - \frac{7\cdot 6\cdot 5}{3.2\cdot 1}$
$= 220 - 35 = 185$

36. (B) Let total number of teams that participated in the championship = n
Then, ${}^nC_2 = 153 \Rightarrow \frac{n(n-1)}{2} = 153 \Rightarrow n(n-1) = 306$
n = 18

37. (D) Required probability $= \frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{{}^6C_1 \times {}^6C_1 \times {}^6C_1}$
$= \frac{6\cdot 5\cdot 4}{6\cdot 6\cdot 6} = \frac{5}{9}$

38. (B) The total number of three-digit numbers using the digits 0, 2, 4, 6 and 8 = 5 × 5 × 4 = 100
$\because$ Favourable events = {222, 444, 666, 888}
Now, the total number of numbers in which all the three digits are the same = 4
$\therefore$ Required probability $= \frac{4}{100} = \frac{1}{25}$

39. (B) $\because P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$
and $P(\bar{B}) = \frac{1}{2}$
$P(B) = 1 - P(\bar{B}) = 1 - \frac{1}{2}$
$= \frac{1}{2}$
We know that,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$
$\Rightarrow \frac{5}{6} = 1 - P(\bar{A}) + \frac{1}{2} - \frac{1}{3}$
$\Rightarrow P(\bar{A}) = 1 + \frac{1}{2} - \frac{1}{3} - \frac{5}{6}$
$= \frac{6+3-2-5}{6} = \frac{2}{6} = \frac{1}{3}$

40. (D) The evernts when flipping a coin and head occurs = {HT, HH}
The events when flipping a coin and tail occurs = $\{T_1, T_2, T_3, T_4, T_5, T_6\}$
Total events = $\{HT, HH, T_1, T_2, T_3, T_4, T_5, T_6\}$

Favourable events of getting one head and one tail = {HT}

$\therefore$ Required probability = $\frac{1}{8}$

41. (A) Given, X + Y = 15
The total number of ordered pairs
= (5, 10), (6, 9), (7, 8), (8, 7), (9, 6), (10, 5)
$\therefore$ n(S) = 6
In each above pairs exactly one is even number.
$\therefore$ n(E) = 6

$\therefore$ Required probability = $\frac{n(E)}{n(S)} = \frac{6}{6} = 1$

42. (A) $(0.1011)_2 = 1 \times \frac{1}{2} + 0 \times \frac{1}{2^2} + 1 \times \frac{1}{2^3} + 1 \times \frac{1}{2^4} =$
0.5 + 0.125 + 0.0625 = 0.6875

43. (D) The smallest five digit binary number is 10000.
The greatest four digit binary number is 1001.
Now, the difference between them
= $(10000)_2 - (1001)_2 = (111)_2$
Which is the greatest three digit binary integer.

44. (C) We know that, $\sec^2\theta + \cos^2\theta \geq 2, \forall\ 0 < \theta < \frac{\pi}{2}$

$\because$ AM $\geq$ GM

$$\left(\sec^2\theta + \frac{1}{\sec^2\theta}\right) \geq 2\left(\sec^2\theta \cdot \frac{1}{\sec^2\theta}\right)^{1/2}$$

$\Rightarrow (\sec^2\theta + \cos^2\theta) \geq 2$
$\therefore$ y $\geq$ 2

45. (A) Given, $\cot\theta = 2\cos\theta$
$\Rightarrow \cos\theta\,(1 - 2\sin\theta) = 0$

For $\frac{\pi}{2} < \theta < \pi$, $\cos\theta \neq 0$

$\therefore 1 - 2\sin\theta = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{5\pi}{6}$

46. (B) We know that,

$60'' = 1' \Rightarrow 30'' = \frac{1'}{2}$

$35'30'' = \left(35 + \frac{1}{2}\right)' = \left(\frac{71}{2}\right)'$

and 60' = 1°

$\therefore \left(\frac{71}{2}\right)' = \left(\frac{71}{120}\right)^\circ$

$\therefore 114°\ 35'30'' = \left(114 + \frac{71}{120}\right)^\circ$

We know that, 2π rad = 360°

$\Rightarrow \left(\frac{13751}{120}\right)^\circ = \frac{2\pi}{360°} \times \frac{13751}{120}$ rad

$= \frac{2 \times 22 \times 13751}{7 \times 360 \times 120}$ rad

= 2.0008069 rad

$\Rightarrow$ 114° 35' 30'' = 2 rad (approx)

47. (B) We have, $\sin A = n \sin B \Rightarrow \frac{n}{1} = \frac{\sin A}{\sin B}$

On applying componendo and dividendo

$\Rightarrow \frac{n-1}{n+1} = \frac{\sin A - \sin B}{\sin A + \sin B}$

$$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$$

$= \tan\frac{A-B}{2}\cot\frac{A+B}{2}$

$\Rightarrow \frac{n-1}{n+1}\tan\left(\frac{A+B}{2}\right) = \tan\frac{A-B}{2}$

48. (C) (b – c) A + (c – a) sin B + (a – b) sin C
= (b – c) ak + (c – a) bk + (a – b) kc
= k [ab – ac + bc – ab + ac – bc] = 0

49. (C) Given, sides a, b and c of a ΔABC are in AP. Then,
2b = a + c .... (i)

$\because \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{b^2 + c^2 - (2b - c)^2}{2bc}$

[$\because$ from Eq. (i), a = 2b – c $\Rightarrow a^2 = (2b - c)^2$]

$\Rightarrow \cos A = \frac{b^2 + c^2 - 4b^2 - c^2 + 4bc}{2bc}$

$\Rightarrow \cos A = \frac{4bc - 3b^2}{2bc} = \frac{4c - 3b}{2c}$

$\Rightarrow \cos A = \frac{4c - 3b}{3c}$

50. (A) Circumradius, $R = \frac{abc}{4\Delta}$ ..... (i)

Here, 2s = a + b + c = 13 + 14 + 15 = 42
$\Rightarrow$ s = 21
$\Delta^2 = s(s - a)(s - b)(s - c) = 21 \cdot 8 \cdot 7 \cdot 6$

$\Delta = 84$

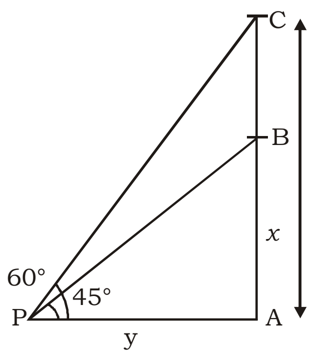
$\therefore\ R = \frac{13\cdot 14\cdot 15}{4\cdot 84} = \frac{65}{8}$

51. (A) $\because A + B + C = \pi$

$A + B = \pi - C \Rightarrow \left(\frac{A+B}{2}\right) = \left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\frac{C}{2}$

52. (C) Let the height of the lower plane from the ground = x and PA = y



Now in ΔABP,

$\tan 45° = \frac{x}{y} = \frac{AB}{AP} = 1$

$\Rightarrow x = y$

Again in ΔAPC, ... (i)

$\tan 60° = \frac{AC}{AP} = \frac{300}{y} = \sqrt{3}$

$\Rightarrow y = \frac{300}{\sqrt{3}}$

$\Rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{300\sqrt{3}}{\sqrt{3}}$ [from Eq. (i)]

$\Rightarrow x = 100\sqrt{3}\ m$

53. (C) $\tan^{-1}\frac{1}{3} - \tan^{-1}\frac{1}{4} = \tan^{-1} x$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{3}\times\frac{1}{4}}\right) = \tan^{-1} x$

$\Rightarrow \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1} x$

$\Rightarrow x = \frac{1}{13}$

54. (B) $\cos\left\{\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}\right\}$

$= \cos\cos^{-1}\left\{\frac{4}{5}\cdot\frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2}\cdot\sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$

$(\because \cos^{-1} x + \cos^{-1} y = \cos^{-1}\{xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\})$

$= \frac{48}{65} - \sqrt{1-\frac{16}{25}}\cdot\sqrt{1-\frac{144}{169}}$

$= \frac{48}{65} - \sqrt{\frac{9}{25}}\cdot\sqrt{\frac{25}{169}}$

$= \frac{48}{65} - \frac{3}{5}\cdot\frac{5}{13} = \frac{48}{65} - \frac{3}{13} = \frac{48-15}{65} = \frac{33}{65}$

55. (B) We know,

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ \forall\ x \in (-\infty, \infty)$

56. (B) The equation sin–1 $(3x - 4x^3) = 3\sin^{-1} x$ is true for all values of x lying in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$. ($\because$ by property)

57. (A) Given, that, $f(x) = \sin^{-1}[\log_2 (x/2)]$
Domain of $\sin^{-1} x$ is $x \in [-1, 1]$

$\Rightarrow -1 \le \log_2\left(\frac{x}{2}\right) \le 1 \Rightarrow 2^{-1} \le \frac{x}{2} \le 2^1$

$\Rightarrow \frac{1}{2} \le \frac{x}{2} \le 2 \Rightarrow 1 \le x \le 4$

$\therefore\ x \in [1, 4]$

58. (A) $|\sin x|$ and $|\cos x|$ has period π. Here, $f(x)$ is an even function and sin$x$, cos$x$ are complementary.

Thus, period of $f(x) = \frac{1}{2}$ |LCM of π and π| = $\frac{\pi}{2}$

59. (B) We have, X = {1, 2 ; 3} and Y = {0, 1} and f : X $\to$ Y is defined by f = {(1, 1), (2, 1), (3, 0)} Here, f shows the property of onto not one-to one.

60. (B) $f(x) = \sqrt{\log\frac{1}{|\sin x|}} \Rightarrow \sin x \ne 0$

$\Rightarrow x \ne n\pi + (-1)^n 0 \Rightarrow x \ne n\pi$.
All real values of x except $\{n\pi\}$
i.e., Domain of $f(x) = R - \{n\pi, n \in I\}$

61. (D) $\lim_{\alpha\to\beta}\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}$, applying L' Hospital's rule

$= \lim_{\alpha\to\beta}\frac{2\sin\alpha\cos\alpha-0}{2\alpha-0}$

$= \lim_{\alpha\to\beta}\frac{\sin 2\alpha}{2\alpha} = \frac{\sin 2\beta}{2\beta}$

62. (C) Since, $f(x)$ is continuous at $x = \pi/2$.

$\therefore f\left(\frac{\pi}{2}\right) = \lim_{x\to\pi/2} f(x)$

$\Rightarrow \lambda = \lim_{x\to\pi/2}\frac{1-\sin x}{\pi-2x}$ $\left(\frac{0}{0}\,form\right)$

Applying L' Hospital's rule,

$\Rightarrow \lambda = \lim_{x\to\pi/2}\frac{-\cos x}{-2} \Rightarrow \lambda = 0$

63. (A) $\lim_{x\to\infty}[\sqrt{a^2x^2+ax+1}-\sqrt{a^2x^2+1}]$

After rationaliztion,

$= \lim_{x\to\infty}\frac{a}{\sqrt{a^2+\frac{a}{x}+\frac{1}{x^2}}+\sqrt{a^2+\frac{1}{x^2}}}$

$= \frac{a}{\sqrt{a^2}+\sqrt{a^2}} = \frac{a}{2a} = \frac{1}{2}$

64. (A) $\because f(x) = \sin^2 x^2$

$\Rightarrow f'(x) = 2\sin x^2\cdot\cos x^2\cdot\frac{d}{dx}(x^2)$

$\therefore f'(x) = 2\sin x^2\cdot\cos x^2\cdot 2x$

$= 4x\sin x^2\cos x^2$

65. (A) $\because x = \sin t - t\cos t$

On differentiating wrt t, we get

$\frac{dx}{dt} = \cos t - \cos t + t\sin t = t\sin t$

and $y = t\sin t + \cos t$

$\therefore \frac{dy}{dt} = t\cos t + \sin t - \sin t$

$= t\cot t$

Hence, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{t\cos t}{t\sin t}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{t=\frac{x}{2}} = \cot\frac{\pi}{2} = 0$

66. (A) Let $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$= \cos^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

and $u = \cos^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\therefore \frac{dy}{du} = \frac{du}{du}$ $(\because y = u)$

$= 1$

67. (C) We have, $y = 3x - \frac{\cos x}{2}$

On differentiating wrt y, we get

$1 = 3\frac{dx}{dy} + \frac{\sin x}{2}\cdot\frac{dx}{dy}$ .... (i)

$\Rightarrow \left(3+\frac{\sin x}{2}\right)\frac{dx}{dy} = 1$

$\Rightarrow \frac{dx}{dy} = \left(\frac{1}{3+\frac{\sin x}{2}}\right)$ [from Eq. (i)]

From Eq. (i), $1 = 3\frac{dx}{dy} + \frac{\sin x}{2}\cdot\frac{dx}{dy}$

Again differentiating wrt y, we get

$0 = \frac{3d^2x}{dy^2} + \frac{\cos x}{2}\left(\frac{dx}{dy}\right)^2 + \frac{\sin x}{2}\frac{d^2x}{dy^2}$

$= \left(3+\frac{\sin x}{2}\right)\frac{d^2x}{dy^2} + \frac{\cos x}{2}\cdot\frac{1}{\left(3+\frac{\sin x}{2}\right)^2}$

$= \left(3+\frac{\sin x}{2}\right)\frac{d^2x}{dy^2} + \frac{2\cos x}{(6+\sin x)^2}$

$\Rightarrow \left(3+\frac{\sin x}{2}\right)\frac{d^2x}{dy^2} = -\frac{2\cos x}{(6+\sin x)^2}$

$\frac{d^2x}{dy^2} = -\frac{2\cos x}{(6+\sin x)^2}\cdot\frac{1}{\left(3+\frac{\sin x}{2}\right)}$

68. (B) $f(x)$ defined as $f(x) = \begin{cases}\log x, & x>0\\ \log(-x) & x<0\end{cases}$

$f'(x) = \begin{cases}1/x, & x>0\\ 1/x, & x<0\end{cases} \Rightarrow f'(x) = \frac{1}{x},\ x\neq 0$

69. (A) Given, $y = \tan^{-1} x - x$

On differentiating wrt x, we get

$$\frac{dy}{dx} = \frac{1}{1+x^2} - 1 = \frac{-x^2}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} < 0, \forall\ x \in R$$

70. (C) $f(x) = k \sin x + \frac{1}{3}\sin 3x$

$$f'(x) = k \cos x + \frac{3}{3}\cos 3x$$

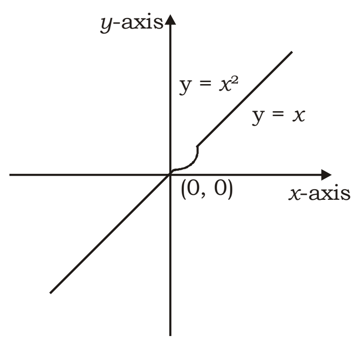
Put $f'(x) = 0$, for maxima

$k \cos x + \cos 3x = 0$

At $x = \frac{\pi}{3}$, $k \cos \frac{\pi}{3} + \cos \pi = 0$

$$\Rightarrow k\left(\frac{1}{2}\right) = 1 \Rightarrow k = 2$$

71. (A) $g(x) = \min(x, x^2)$



$\therefore$ $g(x)$ is an increasing function.

72. (D) We have,

$y = f(e^x)$ ...... (i)

On differentiating Eq. (i) wrt x, we get

$$\frac{dy}{dx} = f'(e^x)\, e^x \cdot$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = f''(e^x)e^x \cdot e^x + f'(e^x) \cdot e^x$$

$$\Rightarrow \frac{d^2y}{dx^2} = f''(e^x)\, e^{2x} + f'(e^x)\, e^x$$

73. (D) We know, that, the area of the largest rectangular field to be enclosed with 200 m of fencing is possible, if length and breadth of the rectangular field are equal.

$\therefore$ $2(x + x) = 200$

$$\Rightarrow x = \frac{200}{4} = 50 \text{ m}$$

$\therefore$ Area of the largest rectangular field $= 50 \times 50 = 2500 \text{ m}^2$

74. (A) Let $I = \int 13^x\, dx = \frac{13^x}{\log 13} + C$

$$\left(\because \int a^x dx = \frac{a^x}{\log_e a}\right)$$

75. (B) We have, $I = \int e^{\log(\tan x)}\, dx$

$= \int \tan x\, dx$ $\quad (\because e^{\log x} = x)$

$= \log(\sec x) + C$

76. (B) We have, $I = \int \frac{\sin x}{\sqrt{\sin^2 x - \sin^2 \alpha}}\, dx$

$$= \int \frac{\sin x}{\sqrt{\cos^2\alpha - \cos^2 x}}\, dx$$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore\ I = -\int \frac{dt}{\sqrt{\cos^2\alpha - t^2}} = \cos^{-1}\left(\frac{t}{\cos\alpha}\right) + C$$

$\Rightarrow I = \cos^{-1}(\cos x \sec \alpha) + C$

77. (C) Let $I = \int \frac{(x+3)e^x}{(x+4)^2}\, dx = \int \frac{(x+4-1)dx}{(x+4)^2}$

$$= \int \left(\frac{x+4}{(x+4)^2} - \frac{1}{(x+4)^2}\right) e^x\, dx$$

$$\Rightarrow I = \int e^x \frac{1}{x+4}\, dx - \int \frac{e^x \cdot 1}{(x+4)^2}\, dx$$

$$= \frac{e^x}{x+4} + \int e^x \frac{1}{(x+4)^2}\, dx - \int \frac{e^x}{(x+4)^2}\, dx + C$$

$$\therefore\ I = \frac{e^x}{x+4} + C$$

78. (B) $\int \sin x \log(\tan x)\, dx$

$$= -\cos x \log(\tan x) - \int (-\cos x) \cdot \frac{1}{\tan x} \cdot \sec^2 x\, dx$$

$$= -\cos x \log(\tan x) + \int \frac{1}{\sin x}\, dx$$

$$= -\cos x \log(\tan x) + \int \operatorname{cosec} x\, dx$$

$$= -\cos x \log(\tan x) + \log\left(\tan\frac{x}{2}\right) + C$$

79. (A) Let $I = \int_0^{\pi/2} |\cos x - \sin x|\, dx$

$$= \quad I = \int_0^{\pi/4} \{-(\sin x - \cos x)\}\, dx +$$

$$\int_{\pi/4}^{\pi/2}(\sin x - \cos x)\,dx$$

$= [\cos x + \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$

$\Rightarrow$ I = $\left\{2\left(\frac{1}{\sqrt{2}}\right)-1\right\}+\left\{-1+2\left(\frac{1}{\sqrt{2}}\right)\right\}$

$\Rightarrow$ I = $2\left(\sqrt{2}-1\right)$

80. (C) Let I = $\int_{-2}^{2}(px^2+qx+s)\,dx$

$\because$ qx is an odd function, therfore its integral value is zero.

$\therefore$ I = $2\int_0^2 (px^2+s)\,dx$

For finding a numerical value of I, it is necessary to know the values of p and s only

81. (B) I = $\int_0^{2\pi}(\sin x + |\sin x|)\,dx$

$= \int_0^{\pi}(\sin x + \sin x)\,dx + \int_0^{2\pi}(\sin x - \sin x)dx$

$= \int_0^{\pi} 2\sin x\,dx + \int_0^{2\pi} 0\,dx = 2\,[-\cos x]_0^{\pi} + 0$

$= -2(\cos\pi - \cos 0) = -2(-1-1) = 4$

82. (A) Given, $f(x) = a + bx + cx^2$

$\therefore$ $\int_0^1 f(x)\,dx = \int_0^1 (a + bx + cx^2)\,dx$ ..... (i)

$= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3}\right]_0^1$

$= a + \frac{b}{2} + \frac{c}{3}$ ...... (ii)

Here, $f(0) = a$, $f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

and $f(1) = a + b + c$ [from. Eq. (i)]

Now, $\frac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$

$= \frac{a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)+a+b+c}{6}$

$= a + \frac{b}{2} + \frac{c}{3}$ .... (iii)

From Eqs. (ii) and (iii), we get

$$\int_0^1 f(x)\,dx = \frac{f(0)+4f\left(\frac{1}{2}\right)+f(1)}{6}$$

83. (A) We know that,

$\int_{-3}^{9} f(x)\,dx = \int_{-3}^{2} f(x)\,dx + \int_{2}^{9} f(x)\,dx$ .... (i)

by (property)

$\left\{\because \int_a^b f(x) = \int_a^c f(x)dx + \int_c^a f(x)dx\right\}$

where $a \le c \le b$

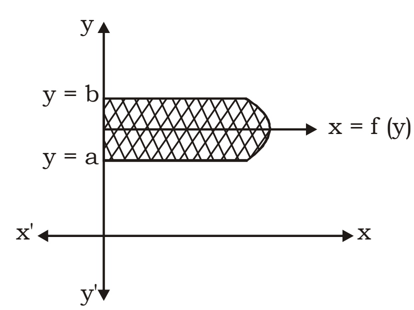
But $\int_{-3}^{9} f(x)\,dx = \frac{-5}{6}$ and $\int_{-3}^{2} f(x)\,dx = \frac{7}{3}$

From Eq. (i)

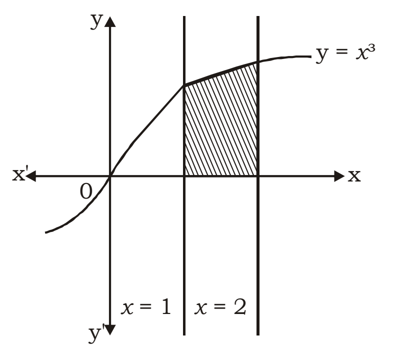
$-\frac{5}{6} = \frac{7}{3} + \int_2^9 f(x)\,dx$

$\int_2^9 f(x)\,dx = \frac{-5}{6} - \frac{7}{3} = \frac{-5-14}{6} = \frac{-19}{6}$

84. (C) $\therefore$ Required area = $\int_{y=a}^{y=b} x\,dy$



85. (B) Required area = $\int_1^2 x^3\,dx$



$= \left[\frac{x^4}{4}\right]_1^2 = \frac{1}{4}(16-1)$

$= \frac{15}{4}$ sq units

86. (B) The given equation can be rewritten as

$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$

From above, it is clear that the degree of equation is 2.

87. (D) Here, y = A cos ωt + B sin ωt

$$\frac{dy}{dt} = - A\,\omega \sin \omega t + B\omega \cos \omega t$$

$$\Rightarrow \frac{d^2y}{dt^2} = - A\,\omega^2 \cos wt - B\,\omega^2 \sin \omega t$$

$= \omega^2 (A \cos \omega t + B \sin \omega t)$

$= - \omega^2 y$

$$\Rightarrow \frac{d^2y}{dt^2} + \omega^2 y = 0$$

88. (C) A differential equation which is of the form.

$$\frac{dy}{dx} + Py = a$$

and $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ is called a linear equation.

(a) $\frac{d^2y}{dx^2} + 4y = 0$ is linear equation.

(b) $x\frac{dy}{dx} + y = x^3 \Rightarrow \frac{dy}{dx} + \frac{y}{x} = x^2$ is a linear equation.

(c) $\cos^2 x \frac{dy}{dx} + y = \tan x$

$\Rightarrow \frac{dy}{dx} + \sec^2 x \cdot y = \tan x \times \sec^2 x$ is also a linear equation.

While (c) $(x - y)^2 \frac{dy}{dx} = 9$ is not a linear equation.

89. (B) $y\frac{dy}{dx} = K - x$

$\Rightarrow y\,dy = (K - x)\,dx$

$$\Rightarrow \frac{y^2}{2} = Kx - \frac{x^2}{2} + \frac{C}{2}$$

$\Rightarrow x^2 + y^2 - 2Kx - C = 0$

Which represents a family of circle whose centre lies on the x-axis .

90. (B) Given equation,

$$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{2}\right)dy = -2\cos\left(\frac{x}{2}\right)dx$$

On integrating both sides, we get

$$\int \operatorname{cosec}\left(\frac{y}{2}\right)dy = -\int 2\cos\left(\frac{x}{2}\right)dx + C$$

$$\Rightarrow \frac{\log\tan\left(\frac{y}{4}\right)}{\frac{1}{2}} = -\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$$\Rightarrow \log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$$

91. (D) ∵ 1000° = 2 ×360° + 280°

∴ From above it is clear that the revolving line will be in the fourth quadrant.

92. (C) The vetices of the triangle are P (2, 7), Q(4, –1), R(–2, 6)

∴ $PQ = \sqrt{(4-2)^2 + (-1-7)^2} = \sqrt{4+64} = \sqrt{68}$

$QR = \sqrt{(-2-4)^2 + (6+1)^2} = \sqrt{36+49} = \sqrt{85}$

and $RP = \sqrt{(-2-2)^2 + (6-7)^2} = \sqrt{16+1}$

$= \sqrt{17}$

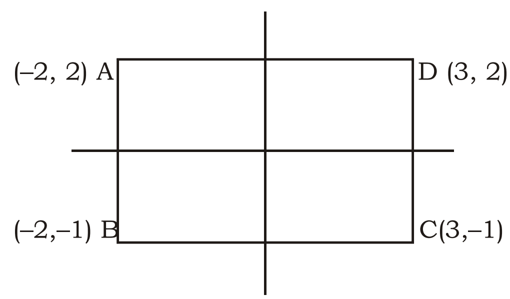
∴ $QR^2 = RP^2 + PQ^2$

⇒ 85 = 17 + 68

⇒ 85 = 85

ΔPQR is right angled.

93. (C) Let the points A, B, C and D are (–2, 2), (–2, –1), (3, –1) and (3, 2), respectively.



Then, AB = 3, BC = 5, CD = 3, DA = 5

So, it is rectangle.

94. (D) We know, if coordinate axes are rotated, then

P = (x cos θ – y sin θ, x sin θ + y cos θ).

It is rotated at an angle 135° i.e., θ = 135° and the new point be

P = [(4 cos(90° + 45°) + 3 sin (90° + 45°), 4 sin (90° + 45°)– 3 cos (90° + 45°)]

= (– 4 sin 45° + 3 cos 45°, 4 cos 45° + 3 sin 45°)

$$= \left[-4\cdot\left(\frac{1}{\sqrt{2}}\right) + 3\cdot\frac{1}{\sqrt{2}}, 4\cdot\frac{1}{\sqrt{2}} + 3\cdot\frac{1}{\sqrt{2}}\right]$$

$$= \left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

95. (A) Given equation is compared with $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$.
Now, $a_1a_2 + b_1b_2 = (1)(\sqrt{3}) + (-\sqrt{3})(1) = 0$
$\therefore$ Lines are perpendicular.
$\therefore$ $\theta = 90°$

96. (B) Since, slope of line $x\cos\theta + y\sin\theta = 2$ is $-\cot\theta$ and slope of the line $x - y = 3$ is 1. Also, these lines are perpendicular to each other
$\therefore$ $(-\cot\theta)(1) = -1$
$\Rightarrow \cot\theta = 1 = \cot\theta\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$

97. (D) The given equation of straight line is is
$x + 2by - 2p = 0$ ..... (i)
Lenght of perpendicular from origin to line (i) = P
$\therefore \left|\frac{0+0-2p}{\sqrt{1+4b^2}}\right| = p$
$\Rightarrow \frac{2p}{\sqrt{1+4b^2}} = p \Rightarrow 4 = 1 + 4b^2$
$\Rightarrow 4b^2 = 3 \Rightarrow b = \frac{\sqrt{3}}{2}$

98. (B) The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets the x-axis in two points on opposite of the orgin, if $c < 0$ (by property).

99. (C) Centre of the circe is (2, 3) Obivously, the line $3x + 2y = 12$ passes through the centre of the circle.
Hence, it is a diameter of the circle.

100. (B) Let C (h, k) be the centre of circle
$\therefore$ AC = BC
$\Rightarrow \sqrt{(h-2)^2+(k-3)^2} = \sqrt{(h-4)^2+(k-5)^2}$
$\Rightarrow h^2 - 4h + 4 + k^2 - 6k + 9 = h^2 - 8h + 16 + k^2 - 10k + 25$
$\Rightarrow 4h + 4k = 28$ ..... (i)
also, centre lies on a given line,
$\therefore k - 4h + 3 = 0$ ..... (ii)
On solving Eqs. (i) and (ii), we get
h = 2, k = 5
Also, radius r = AC
$= \sqrt{(2-2)^2+(5-3)^2} = 2$
$\therefore$ Equation of circle
$(x-2)^2 + (y-5)^2 = 2^2$
$\Rightarrow x^2 + y^2 - 4x - 10y + 25 = 0$

101.(B) $\therefore$ equation of required circle is
$(x-4)^2 + (y-6)^2 = (\sqrt{3})^2$
$\Rightarrow x^2 + y^2 - 8x - 12y + 16 + 36 = 3$
$\Rightarrow x^2 + y^2 - 8x - 12y + 49 = 0$

102. (D) Let the point on the parabola is (x_1, y_1), then focal distance
$= a + x_1$
$\Rightarrow 2 + x_1 = 4$ $(\because a = 2)$
$\Rightarrow x_1 = 2$
On putting this value in $y^2 = 8x$
$\Rightarrow y_1^2 = 8 \times 2$
$\Rightarrow y_1 = \pm 4$

103.(D) The equation of curve is
$4x^2 - 9y^2 = 1$
$\Rightarrow \frac{x^2}{1/4} - \frac{y^2}{1/9} = 1$ .... (i)
This is an equation of a hyperbola and the equation of conjugate axes is y-axis i.e. $x = 0$
On putting $x = 0$ in Eq. (i), we get
$y^2 = -\frac{1}{9}$ or $y = \frac{1}{3}i$, i.e., imaginary points
Hence, no point of intersection exists.

104. (B) The equation of curve is
$2x^2 - 8x + y^2 - 2y + 1 = 0$
$\Rightarrow 2(x^2 - 4x + 4 - 4) + (y^2 - 2y + 1 - 1) + 1 = 0$
$\Rightarrow 2[(x-2)^2 - 4] + (y-1)^2 = 0$
$\Rightarrow 2(x-2)^2 + (y-1)^2 = 8$
$\Rightarrow \frac{(x-2)^2}{4} + \frac{(y-2)^2}{8} = 1$ ..... (i)
This equation is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Here, $a^2 = 4$ and $b^2 = 8$
$\because e = \sqrt{\frac{b^2-a^2}{b^2}}$
$\therefore e = \sqrt{\frac{8-4}{8}}$
$\Rightarrow e = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

105.(A) I. Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Then, foci are S = and S' = – (– ae, 0)
Equation of tangent at any point P is
$y = mx + \sqrt{a^2m^2 + b^2}$
Now, length of perpendicular from foci are
$L_1 = \frac{mae + \sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}$
and $L_2 = \frac{-mae + \sqrt{a^2m^2+b^2}}{\sqrt{1+m^2}}$
$\Rightarrow L_1 \times L_2 = \frac{a^2m^2 + b^2 - m^2a^2e^2}{1+m^2}$

$= \dfrac{a^2m^2(1-e^2)+b^2}{1+m^2}$

$= \dfrac{m^2b^2+b^2}{1+m^2}$ [$\because$ $b^2 = a^2(1-e^2)$]

$= \dfrac{b^2(1+m^2)}{1+m^2} = b^2$

II. Let the mid-point of the focal chord of the given ellipse be (h,k). Then, its equation is

$\dfrac{hx}{a^2} + \dfrac{ky}{b^2} = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2}$

Since, it passes through the focus i.e., (ae,0)

$\therefore \dfrac{hae}{a^2} = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2}$

$\Rightarrow \dfrac{he}{a} = \dfrac{h^2}{a^2} + \dfrac{k^2}{b^2}$

$\therefore$ Locus of mid-point is $\dfrac{ex}{a} = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$

Hence, only statement I is correct.

106.(B) We have, a = i – 2j + k and b = 4i – 4j + 7k

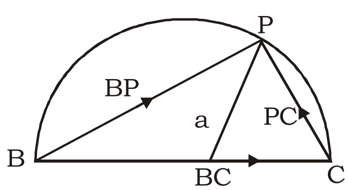
$\therefore$ Projection of a on b is given by

$= \dfrac{a \cdot b}{|b|} = \dfrac{(1-2j+k)\cdot(4i-4j+7k)}{\sqrt{16+16+49}}$

$= \dfrac{19}{\sqrt{81}} = \dfrac{19}{9}$

107.(D) Since, P is a point on circumference of a semi-circle of radius a which is bounded by the diameter BC.

In ΔPBC,



BP · PC = |BP|· |PC| cos 90°

BP · PC = 0

108. (B) Since, p and q are collinear, then p = λq

$\Rightarrow$ $(x-2)$ a + b = λ $(x+1)$ a – λb

On equating the coefficients,

$x - 2 = \lambda(x+1)$ and $-\lambda = 1$

$\Rightarrow x - 2 = -(x+1)$

$\Rightarrow 2x = 1 \Rightarrow x = \dfrac{1}{2}$

109. (D) Let A = i + j + k, B = 2i + 4j – 5k and C = bi + 2j + 3k

$\therefore$ B + C = 2i + 4j – 5k + bi + 2j + 3k

= (2 + b) i + 6j – 2k

Unit vector parallel to B + C

$n = \dfrac{(2+b)i+6j-2k}{\sqrt{(2+b)^2+6^2+(-2)^2}}$

$n = \dfrac{(2+b)i+6j-2k}{\sqrt{b^2+4b+44}}$

Now, (i + j + k) · n = 1 (according to questions)

$\Rightarrow 2 + b + 6 - 2 = \sqrt{b^2+4b+44}$

$\Rightarrow (b+6)^2 = b^2 + 4b + 44$

$\Rightarrow b^2 + 36 + 12b = b^2 + 4b + 44$

$\Rightarrow 8b = 8$

$\Rightarrow b = 1$

110.(C) We know that, in a parallelogram, diagonals bisect each other. Mid-point of OQ = Mid-point of PR

$\therefore \left(\dfrac{0+m}{2}, \dfrac{0+n}{2}, \dfrac{0+r}{2}\right) = \left(\dfrac{1+3}{2}, \dfrac{1+4}{2}, \dfrac{1+5}{2}\right)$

$\Rightarrow$ m = 4, n = 5, r = 6

Hence, m + n + r = 4 + 5 + 6 = 15

111.(B) For the sphere,

Coefficient of x = coefficient of y = coefficient of z

$\Rightarrow$ a = b = c

So, $ax^2 + by^2 + cz^2 - 6x = 0$

$\Rightarrow x^2 + y^2 + z^2 - \dfrac{6x}{a} = 0$

$\therefore$ Centre = $\left(\dfrac{3}{a}, 0, 0\right)$

Given that, radius = 1

$\sqrt{\left(\dfrac{3}{a}\right)^2 + 0 + 0} = 1$

$\dfrac{3}{a} = 1 \Rightarrow a = 3$

$\therefore$ Centre = (1, 0, 0)

112.(A) Let P(x_1, y_1, z_1) be the point.

Then, distance of P from x-axis = $\sqrt{y_1^2 + z_1^2}$

In yz plane, $x = 0$

Given that distance of P (x_1, y_1, z_1) from $x = 0$ is $\dfrac{x_1}{\sqrt{1}}$

Distance of P from x–axis = 3 × distance of P from yz-plane

$\sqrt{y_1^2 + z_1^2} = 3x_1$

On squaring bothsides, we get

$y_1^2 + z_1^2 = 9x_1^2$

Thus, path of P(x_1, y_1, z_1) is

$y^2 + z^2 = 9x^2$

113.(A) Equation of given sphere is
$x^2 + y^2 + z^2 - 4x + 6y - 8z - 71 = 0$, whose centre is (2, –3, 4) and

radius = $\sqrt{2^2(-3)^2+4^2+71}$ = $\sqrt{100}$
= 10 units

Now, CA = $\sqrt{(1-2)^2+(-1+3)^2+(2-4)^2}$
= $\sqrt{(-1)^2+(2)^2+(-2)^2}$
= $\sqrt{1+4+4}$ = $\sqrt{9}$ = 3

and CB = $\sqrt{(2-2)^2+(-3+3)^2+(4-4)^2}$
= $\sqrt{0+0+0}$ = 0

This shows that points A and B are inside the sphere.

114.(B) Let $x_1, x_2, x_3, \ldots, x_n$ be n observations. Then,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\therefore$ New mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}+10\right)$

$$= \frac{1}{\alpha}\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) + \frac{1}{n}\cdot(10n)$$

$$= \frac{1}{\alpha}\bar{x} + 10 = \frac{\bar{x}+10\alpha}{\alpha}$$

115.(B) Let the number of boys in class (n_1) = x
and let the number of girls in class (n_2) = y (given)

The mean weight of all students $(\bar{w}_{12})$ = 60 (given)

Mean weight of boys $(\bar{w}_1)$ = 70 kg (given)

Mean wight of girls $(\bar{w}_2)$ = 55 kg

$$\bar{w}_{12} = \frac{\bar{w}_1 n_1 + \bar{w}_2 n_2}{n_1 + n_2}$$

$\Rightarrow 60 = \frac{70x+55y}{x+y}$

$\Rightarrow 60x + 60y = 70x + 55y$

$\Rightarrow 10x = 5y$

$\Rightarrow \frac{x}{y} = \frac{1}{2} \Rightarrow x : y = 1 : 2$

116. A) $a_1 x + b_1 y = 0$, $a_2 x + b_2 y = 0$ has a non-zero solution only,

when $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}$ is singular i.e., $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$

So, correct answer is (a).

117.(A) Both A and R are true and R is the correct explanation of A.

118.(D) $|x|$ is continuous at $x = 0$ it can be easily seen from the graph, but it is not differentiable at $x = 0$

119.(A) Both A and R are true and R is the correct explanation of A.

120. (A) (i) A . B = A . C
A . (B – C) = 0
$\Rightarrow$ Either A = 0, A is perpendicular to B – C or B = C
(ii) A × B = A × C
A × (B – C) = 0
$\Rightarrow$ Either A = 0, A parallel to B – C or B = C
From, both conditions it implies that B = C

NDA (MATHS) MOCK TEST - 43 (Answer Key)

1.	(C)	21.	(C)	41.	(A)	61.	(D)	81.	(B)	101.	(B)
2.	(A)	22.	(B)	42.	(A)	62.	(C)	82.	(A)	102.	(D)
3.	(B)	23.	(D)	43.	(D)	63.	(A)	83.	(A)	103.	(D)
4.	(A)	24.	(C)	44.	(C)	64.	(A)	84.	(C)	104.	(B)
5.	(D)	25.	(B)	45.	(A)	65.	(A)	85.	(B)	105.	(A)
6.	(A)	26.	(B)	46.	(B)	66.	(A)	86.	(B)	106.	(B)
7.	(A)	27.	(D)	47.	(B)	67.	(C)	87.	(D)	107.	(D)
8.	(C)	28.	(C)	48.	(C)	68.	(B)	88.	(C)	108.	(B)
9.	(C)	29.	(D)	49.	(C)	69.	(A)	89.	(B)	109.	(D)
10.	(D)	30.	(D)	50.	(A)	70.	(C)	90.	(B)	110.	(C)
11.	(A)	31.	(A)	51.	(A)	71.	(A)	91.	(D)	111.	(B)
12.	(A)	32.	(A)	52.	(B)	72.	(D)	92.	(C)	112.	(A)
13.	(C)	33.	(B)	53.	(C)	73.	(D)	93.	(C)	113.	(A)
14.	(C)	34.	(B)	54.	(B)	74.	(A)	94.	(D)	114.	(B)
15.	(A)	35.	(C)	55.	(B)	75.	(B)	95.	(A)	115.	(B)
16.	(C)	36.	(A)	56.	(B)	76.	(B)	96.	(B)	116.	(A)
17.	(C)	37.	(D)	57.	(A)	77.	(C)	97.	(D)	117.	(A)
18.	(C)	38.	(B)	58.	(A)	78.	(B)	98.	(B)	118.	(D)
19.	(A)	39.	(B)	59.	(B)	79.	(A)	99.	(C)	119.	(A)
20.	(B)	40.	(D)	60.	(B)	80.	(C)	100.	(B)	120.	(A)

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003