NDA MOCK TEST-47 (SOLUTION)

1. (B) I. We know that, $\sin\theta \in [-1, 1]$; $\theta \in$
i.e the value of $\sin\theta$. also lies between – 1 to 1.
II. We also know that, $\cos \in [-1, 1]$; $\theta \in$
i.e the value of $\cos\theta$, also lies between – 1 to 1.

2. (A) Case I, Let $a + b \geq 0 \Rightarrow |a + b| = a + b$ (By definition)
$\Rightarrow |a + b| \leq |a| + |b|$ [$\because a \leq |a|$ # $a \in$]
$\therefore |a + b| < |a| + |b|$
Case II, Let $a + b \leq 0 \Rightarrow |a + b| = -(a + b)$
$= |a + b| = (-a) + (-b)$
$|a + b| \leq |a| + |b|$ [$\because -a \leq |a|$ # $a \in$]
$\Rightarrow |a + b| \leq |a| + |b|$
Hence, $|a + b| \leq |a| + |b|$.

3. (A) We have
$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$
Since,
$n(A \times B) = 4$, the number of subsets of $A \times B$ is 2^4.
Therefore, the number of relations from A into B will be $2^4 = 16$

4. (B) The given differential equation is
$(x + y)(dx - dy) = dx + dy$
$\Rightarrow (x + y - 1)\,dx = (x + y + 1)\,dy$
$$\Rightarrow \frac{dy}{dx} = \frac{x , y - 1}{x , y , 1} \quad \ldots(1)$$
Let $x + y = v$ ... (2)
so that as usual
$$\frac{dy}{dx} = \frac{dv}{dx} - 1 \ldots (3)$$
Using (2), (3), (1) becomes
$$\frac{dv}{dx} - 1 = \frac{V - 1}{V , 1}$$
$$\Rightarrow \frac{dv}{dx} = \frac{2V}{V , 1}$$
$$\Rightarrow 2dx = \left(1 , \frac{1}{v}\right) dv$$
$\therefore$ integrating
$2x + c = v + \log v$
$\Rightarrow x - y + c = \log(x + y)$

5. (C) The coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms of the expansion $(1 + x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$, respectively.
Since they are equal so ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$
Therefore, either $r - 6 = 2r - 2$ or $r - 6 = 34 - (2r - 2)$
[using the fact that if ${}^{n}C_r = {}^{n}C_p$, then either $r = p$ or $r = n - p$]
So, we get $r = -4$ or $r = 14$
r being a natural number, $r = -4$ is not possible so, $r = 14$

6. (A) $595 = 2\times25 + 91$ (dividing 595 by 252)
$252 = 2\times91 + 70$ (dividing 252 by 91)
$91 = 1\times70 + 21$ (dividing 91 by 70)
$70 = 3\times21 + 7$ (dividing 70 by 21)
$21 = 7\times3$
Therefore g.c.d of 595 and 252 is $d = 7$
from (1)
$d = 7 = 70 - 3\times21$
$= 70 - (391 - 1\times70)$
$= 70 - 3\times91 + 3\times70$
$= 4\times70 - 3\times91$
$= 4(252 - 2\times91) - 3\times91$
$= 4\times252 - 11\times91$
$= 4\times252 - 11(595 - 2\times252)$
$= 4\times252 - 11\times595 + 22\times252$
$= 26\times252 - 11\times595$
$= 252m + 595n$
Here $m = 26$, $n = -11$

7. (B) $\because a = 7 \bmod 5$
$\therefore 5 / a - 7$
$a - 7 = 5k$, where k is any integer
Putting $k = 0, 1, 2, 3, \ldots -1, -2, -3\ldots$ successively,
we get
$a - 7 = 0, 5, 10, 15, \ldots, -5, -10, -15\ldots$
$a = 7, 12, 17, 22, \ldots, 2, -3, -8, \ldots$
Hence all integers are congruent to 7 mod 5.

8. (C) We know that
$2^1 = 2 \bmod 7$
$2^2 = 4 \bmod 7$ ...(i)
$2^3 = 8 \bmod 7$
$\Rightarrow 2^3 = 1 \bmod 7$
$\Rightarrow (2^3)^6 = 1 \bmod 7$
$2^{18} = 1 \bmod 7$ ...(ii)
multiplying (1) and (2), we get
$2^{20} = 4 \bmod 7$.
$\therefore$ 4 is the remainder when 220 is divided by 7.

9. (C) Probability = RBRB + BRBR
$$= \left(\frac{5}{12} \propto \frac{7}{11} \propto \frac{4}{10} \propto \frac{6}{9}\right) \times 2$$
$$= \frac{7}{99} \times 2$$
$$= \frac{14}{99} = 0.14$$

10. (B) $$P = \frac{{}^{13}C_2 \propto {}^{39}C_1}{{}^{52}C_4} + \frac{{}^{13}C_4}{{}^{52}C_4}$$
$$= \left(\frac{78 \propto 39}{270725} , \frac{715}{270725}\right)$$
$$= \frac{3757}{270725} = 0.013$$

11. (A) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$R_2 - R_1 \to R_2$
$R_3 - R_1 \to R_3$

$\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

$\Rightarrow (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c-a \end{vmatrix}$

Expanding by C_1

$(b-c)(c-a) \left[1 \begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}\right]$

$\Rightarrow (b-c)(c-a)(c+a-b-a)$
$\Rightarrow (b-c)(c-a)(c-b)$
$\Rightarrow (a-b)(b-c)(c-a)$

12. (D) **Note :** - The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular matrix. Then b is equal to

Solution :

The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is singular, if

$\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix} = 0$

$\Rightarrow -1(60+12) + 2(30+6) + b(-20+20) = 0$
$-72 + 72 + 0 \times b = 0$

$\Rightarrow$ The given matrix is singular for any only value of b.

13. (B) Using the determinant

$0 = \frac{1}{2} \begin{vmatrix} 1 & x & y \\ 1 & 1 & 2 \\ 1 & 3 & 6 \end{vmatrix}$

$\Rightarrow 0 = \frac{1}{2}\left[1\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} - 1\begin{vmatrix} x & y \\ 3 & 6 \end{vmatrix} + 1\begin{vmatrix} x & y \\ 1 & 2 \end{vmatrix}\right]$

$0 = \frac{1}{2}(6 - 6 - 6x + 3y + 2x - y)$

$0 = -4x + 2y$
$\Rightarrow 2x - y = 0$

14. (A) $\Rightarrow -\int_{-\theta/2}^{0} \sin x \, dx + \int_{0}^{\theta/2} \sin x \, dx$

$\Rightarrow -\left[-\cos x\right]_{-\theta/2}^{0} + \left[-\cos x\right]_{0}^{\theta/2}$

$\Rightarrow \left[\cos x\right]_{-\theta/2}^{0} + \left[-\cos x\right]_{0}^{\theta/2}$

$\Rightarrow 1 + 0 - 0 + 1$
$= 2$

15. (C) $\tan\left[\frac{1}{2}\sin^{-1}\frac{2\tan^2\rho}{1+\tan^2\rho} + \frac{1}{2}\cos^{-1}\frac{1-\tan^2\rho}{1+\tan^2\rho}\right]$

Putting $a = \tan\theta$

$= \tan\left[\frac{1}{2}\cdot 2\rho + \frac{1}{2}2\rho\right] = \tan 2\theta$

$= \frac{2\tan\rho}{1-\tan^2\rho} = \frac{2a}{(1-a^2)}$

16. (A) Let $x = \frac{\theta}{8}$ · then $2x = \frac{\theta}{4}$

Now, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\tan\frac{\theta}{4} = \frac{2\tan\frac{\theta}{8}}{1-\tan^2\frac{\theta}{8}}$

Let $y = \tan\frac{\theta}{8}$, then $1 - \frac{2y}{1-y^2}$

or $y^2 + 2y - 1 = 0$

Therefore, $y = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

Since, $\frac{\theta}{8}$ lies in first quadrant,

$y = \tan\frac{\theta}{8}$ is positive

Hence $\tan\frac{\theta}{8} = \sqrt{2} - 1$

17.(C) Dividing the numerator and the denominator by the highest power, x^5, we find that

$\lim_{x \to -\infty} \frac{95x^3 + 57x + 30}{x^5 - 1000}$

$= \lim_{x \to -\infty} \frac{95x^3 + 57x + 30}{x^5 - 1000} \cdot \frac{\frac{1}{x^5}}{\frac{1}{x^5}}$

$$= \lim_{x \downarrow -f} \frac{\frac{95}{x^2} , \frac{57}{x^4} , \frac{30}{x^5}}{1 - \frac{1000}{x^5}}$$

$$= \frac{0 , 0 , 0}{1 - 0} = 0$$

18. (C) Let objects 1, 2, 3, 4, 5 be placed in places marked 1, 2, 3, 4, 5 respectively. Then the number of derangements in which none of the object occupied its original position is given by

$$5! \left[1 - \frac{1}{1!} , \frac{1}{2!} - \frac{1}{3!} , \frac{1}{4!} - \frac{1}{5!}\right]$$

= 60 – 20 + 5 – 1 = 44

Also total numbers of arrangements = 5! = 120

Hence required probability = $\frac{44}{120} = \frac{11}{30}$

19. (B)

x	y	x^2	y^2	xy
4	2	16	4	8
2	4	4	16	8
3	2	9	4	6
4	4	16	16	16
2	4	4	16	8
Σ 15	16	49	56	46

$$r = \frac{x\text{T}xy - \text{T}x\text{T}y}{\sqrt{|(n\text{T}x^2 -)\text{T}x*^2||(n\text{T}y^2 -)\text{T}y*^2|}}$$

$$= \frac{5 \propto 46 - 15 \propto 16}{\sqrt{|()5 \propto 49 - 225*)5 \propto 56 - 256*|}}$$

$$= \frac{230 - 240}{\sqrt{)245 - 225*)280 - 256*}}$$

$$= \frac{-10}{\sqrt{20 \propto 24}}$$

$$= \frac{-10}{\sqrt{5 \propto 4 \propto 6 \propto 4}} = \frac{-10}{4\sqrt{30}} = \frac{-5}{2 \propto 5.77}$$

= – 0.4566

20. (D) $\frac{d}{dx} \sec \tan^{-1} x = \sec \tan^{-1} x \cdot \tan \tan^{-1} x \cdot \frac{1}{)1 , x^2*}$

$$= \sec \theta \cdot \frac{x}{1 , x^2} \quad \text{, where } \tan^{-1} x = \theta$$

$$= \frac{x\sqrt{1 , \tan^2 \rho}}{)1 , x^2*} = \frac{x}{\sqrt{1 , x^2}}$$

21. (B) Taking log both sides

log y = log x . log x = (log x)^2

Differentiate w.r.t x

$$\frac{1}{y}\frac{dy}{dx} = 2 \log x \cdot \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{2y}{x} \log x$$

$$\Rightarrow \frac{dy}{dx} = x^{\log x - 1} \cdot 2 \log x$$

22. (D) Domain of $\cot^{-1}x$ is R, the set of Reals.

$\frac{x}{\sqrt{x^2 - .x^2\perp}}$ is defined if $x^2 \neq [x^2]$

i.e x^2 is not integer.

since, $x^2 = [x^2]$, if x^2 is an integer,

Hence $x^2 \neq$ non – negative integer.

i.e O or positive integer

Hence domain = R – $\left|\sqrt{x}, x \times 0, x \notin \right.$~.

23. (B) Let $\vec{n} = x\hat{i} + y\hat{j} + z\hat{k}$

then $x^2 + y^2 + z^2 = 1$ ...(i)

$\vec{n}.k = \cos\frac{\theta}{4} \Rightarrow Z = \frac{1}{\sqrt{2}}$ ...(ii)

Also $\vec{n} + \hat{i} + \hat{j} = (x + 1)\hat{i} + (y + 1)\hat{j} +$ Z$\hat{k}$ is a unit vector

Therefore, $(x + 1)^2 + (y + 1)^2 + z^2 = 1$ ...(iii)

Solving (i), (ii) and (iii) we get

$$x = \frac{-1}{2}, y = \frac{-1}{2}, z = \frac{1}{\sqrt{2}}$$

$$\therefore \vec{n} = \frac{-1}{2}\hat{i} - \frac{1}{2}\hat{j} + \frac{1}{\sqrt{2}} k$$

24. (C) $\because$ 1, ω and ω² are the three cube roots of unity.

$\therefore$ 1 + ω + ω² = 0 and ω³ = 1

$$\frac{a\xi^6 , b\xi^4 , c\xi^2}{b , c\xi^{10} , a\xi^8} = \frac{a , b\xi , c\xi^2}{b , c\xi , a\xi^2}$$

$$= \frac{\xi)a , b\xi , c\xi^2*}{\xi)b , c\xi , a\xi^2*}$$

$$= \frac{\xi)a , b\xi , c\xi^2*}{)a\xi^3 , b\xi , c\xi^2*}$$

$$= \frac{\xi)a , b\xi , c\xi^2*}{)a , b\xi , c\xi^2*} = \omega$$

25. (D) Since, $3 < 4 \Rightarrow \frac{3}{(-5)} > \frac{4}{(-5)}$

on multiplying or dividing an inequality by a negative number on both sides its sign changes.

26. (A) $(1 + \omega)(1 + 2\omega)(1 + 3\omega)(1 + 5\omega)$

$\Rightarrow [1 + 2\omega + \omega + 2\omega^2][1 + 5\omega + 3\omega + 15\omega^2]$

$\Rightarrow [1 + \omega + 2(\omega + \omega^2)][1 + 8\omega + 15\omega^2]$

$\Rightarrow [1 + \omega + 2(-1)][1 + 8\omega + 15\omega^2]$

$\Rightarrow [\omega - 1][1 + 8\omega + 15\omega^2]$

$\Rightarrow [\omega + 8\omega^2 + 15\omega^3 - 1 - 8\omega - 15\omega^2]$

$\Rightarrow [-1 - 7\omega - 7\omega^2 + 15]$

$\Rightarrow [14 - 7\omega - 7\omega^2]$

$\Rightarrow 14 - 7(\omega + \omega^2)$

$\Rightarrow 14 - 7(-1)$

$\Rightarrow 21$

27. (C) Selection of 1 boy and 3 girls in ${}^5C_1 \times {}^4C_3 = 20$ way

Selection of 4 girls and no boys in ${}^5C_0 \times {}^4C_4 = 1$ way

$\therefore n(E)$ = total no. of ways = 21

$n(s) = {}^9C_4 = \frac{9!}{4!5!} = 9 \times 7 \times 2$

$\therefore P(E) = \frac{20 + 1}{9 \times 7 \times 2} = \frac{1}{6}$

28. (B) $y = x^2 - 2x + 7$ ...(i)

Differentiate w.r.t x

$\frac{dy}{dx} = 2x - 2$

Slope of line $2x - y + 9 = 0$

$-y = -9 - 2x$

$y = 2x + 9$

$m = 2$

$\because$ slope of tangent = slope of line

$\Rightarrow 2x - 2 = 2$

$x = 2$

Value of x put in equation (i)

$y = x^2 - 2x + 7$

$y = 4 - 4 + 7$

$y = 7$

$\therefore$ equation of tangent is $(y - 7) = 2(x - 2)$

$y - 7 = 2x - 4$

$y - 2x = 3$

29. (C) Notice that $\frac{1}{x-2}$ increase without bound as x approaches 2 from the right and $\frac{1}{x-2}$ decrease without bound as x approaches 2 from the left.

i.e $\lim_{x \to 2^+} \frac{1}{x-2} = +\infty$

and $\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$

we also have $\lim_{x \to 2} 3x - 5 = 1$ and it follows that

$\lim_{x \to 2^-} \frac{3x-5}{x-2} = -\infty$

30. (B) $\int e^x \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) dx$

$\int e^x \left(\frac{2 - 2\sin x \cos x}{2\sin^2 x}\right) dx$

$\Rightarrow \int e^x (\text{cosec}^2 x - \cot x) dx$

$\Rightarrow -\left[\int e^x \cot x \, dx - \int e^x \text{cosec}^2 x \, dx\right]$

$\Rightarrow \int e^x \text{cosec}^2 x \, dx - \cot x \int e^x dx + \int (D)\cot x \int e^x dx \, dx$

$= \int e^x \text{cosec}^2 x - \cot x \int e^x dx - \int e^x \text{cosec}^2 x dx$

$= -e^x \cot x + c$

31. (D) $\tan^{-1}\left[\frac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 + x^2}}\right]$

Put $x^2 = \cos 2\theta$

$2\theta = \cot^{-1} x^2$

$\theta = \frac{1}{2}\cos^{-1} x^2$

$\tan^{-1} = \left[\frac{\sqrt{1 + \cos 2\rho} - \sqrt{1 - \cos 2\rho}}{\sqrt{1 + \cos 2\rho} + \sqrt{1 - \cos 2\rho}}\right]$

$\tan^{-1}\left[\frac{\sqrt{2\cos^2\rho} - \sqrt{2\sin^2\rho}}{\sqrt{2\cos^2\rho} + \sqrt{2\sin^2\rho}}\right]$

$\tan^{-1}\left[\frac{(\sqrt{2})(\sqrt{\cos^2\rho} - \sqrt{\sin^2\rho})}{(\sqrt{2})(\sqrt{\cos^2\rho} + \sqrt{\sin^2\rho})}\right]$

$\tan^{-1}\left[\frac{\cos\rho - \sin\rho}{\cos\rho + \sin\rho}\right]$

divide by $\cos\theta$

$\tan^{-1}\left[\tan\left(\frac{\theta}{4} - \rho\right)\right]$

$\Rightarrow \frac{\theta}{4} - \theta$

$\Rightarrow \frac{\theta}{4} - \frac{\cos^{-1} x^2}{2}$

32. (D) Given $z^2 + z + 1 = 0$
$\therefore (z-1)(z^2 + z + 1) = 0, \quad \therefore z^3 = 1$
If n is not multiple of 3, then we can write $n = 3m + r$, where $m \in$ I and $r = 1$ or 2.
then $2n = 6m + 2r$
if $r = 1$, then $2r = 2$
$\therefore z^n + z^{2n} = (z^3)^m . z^r + (z^3)^{2m} . z^{2r}$
$= z^r + z^{2r} = z + z^2 = -1$
If $r = 2$, then $2r = 4$
$\therefore z^n + z^{2n} = (z^3)^m . z^r + (z^3)^{2m} . z^{2r} = z^2 + z^4$
$= z^2 + z = -1$
Hence, $z^n + z^m = -1$

33. (C) $x + y = 2(t^2 + 1)$, $x - y = 2t$
Eliminating t, we get
$$x + y = \frac{1}{2}(x-y)^2 + 2$$
or $x^2 - 2xy + y^2 - 2x - 2y + 4 = 0$
for which, $D \neq 0$ and $h^2 = ab$
Hence the given curve represent a parabola.

34. (A) The lines are
$$x = 1,\ y = 2 \text{ i.e } \frac{x-1}{0} = \frac{y-2}{0} = \frac{z}{1}$$
$$\text{and } y = -1,\ z = 0 \text{ i.e } \frac{x}{1} = \frac{y+1}{0} = \frac{z}{0}$$
$\therefore$ If θ is the angle between them, then
$\cos\theta = 0.1 + 0.0 + 1.0 = 0$
i.e $\theta = 90°$

35. (A) Coordinate of the given point p are (2, 3, –1) op is normal to the required plane so direction rations of the normal to the plane are 2, 3, –1.
Equation of the plane through P(2, 3, –1) is $a(x-2) + b(y-3) + c(z+1) = 0$
Since, the direction ratios of the normal to the plane are 2, 3, –1
so, we have $\frac{a}{2} = \frac{b}{3} = \frac{c}{-1}$ and hence the equation of the required plane is
$2(x-2) + 3(y-3) - 1(z+1) = 0$
or $2x + 3y - z = 14$.

36. (A) The projection of the line
$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{3} \quad \ldots\text{(i)}$$
on the plane $x + y + z - 1 = 0$ ...(ii)
is the line of intersection of the plane (ii) and the plane perpendicular to the plane (ii) and containing the line (i).
Now, equation of the plane through line (i) can be taken as
$A(x-1) + B(y-1) + c(z-3) = 0$ ...(iii)
where 2.A + 1.B + 3.C = 0 ...(iv)
If plane (iii) is perpendicular to the plane (ii) then
1.A + 1.B + 1.C = 0 ...(v)
(iv) and (v) $\Rightarrow \frac{A}{-2} = \frac{B}{1} = \frac{c}{1} = k$ (say)
$\therefore A = -2x,\ B = k,\ c = k$
Putting in (iii), we get
$2x - y - z + 3 = 0$ ...(vi)
Hence the required projection is given by planes (ii) and (vi).

37.(A) Given $x^2 + y^2 = 2xy$
Difference w.r.t x
$$2x + 2y\frac{dx}{dy} = 2\left(x\frac{dy}{dx} + y\right)$$
$$\therefore \frac{dy}{dx} = 1$$

38. (B) We have
$$Rf'(1) = \lim_{h\to 0}\frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0}\frac{[(1+h)^3 - 1] - 0}{h} = 3$$
$$Lf'(1) = \lim_{h\to 0}\frac{f(1-h) - f(1)}{-h}$$
$$= \lim_{h\to 0}\frac{[(1-h) - 1] - 0}{-h} = 1$$
$\therefore Rf'(1) \neq Lf'(1)$
$\Rightarrow f(x)$ is not differentiable at $x = 1$
Now, $f(1+0) = \lim_{h\to 0} f(1+h) = 0$
$f(1-0) = \lim_{h\to 0} f(1-h) = 0$
$\therefore f(1+0) = f(1-0) = f(0)$
$\Rightarrow f(x)$ is continuous at $x = 1$
Hence $x = 1$, $f(x)$ is continuous and not differentiable.

39. (D) $y = (x+1)^{\frac{1}{3}} - (x-1)^{\frac{1}{3}}$ on [0, 1]
$$\frac{dy}{dx} = \frac{1}{3}\left[\frac{1}{(x+1)^{\frac{2}{3}}} - \frac{1}{(x-1)^{\frac{2}{3}}}\right] \text{ m to, 1L}$$
$$\left[\because \frac{dy}{dx} \text{ does not exist at } x = 1\right]$$
$$= \frac{1}{3}\frac{(x-1)^{\frac{2}{3}} - (x+1)^{\frac{2}{3}}}{(x+1)^{\frac{2}{3}}\cdot(x-1)^{\frac{2}{3}}}$$

$\frac{dy}{dx}$ = 0 at x = 0 which is end point so there is no critical point in [0, 1]

Also $y_{x=0} = 2$ and $y_{x=1} = 1$

$\therefore y_{\text{greatest}} = 2$

40. (B) The given equation of line can be rewritten as

$$\frac{x}{5} - \frac{y}{3} = 1$$

and $y = \frac{3x-15}{5}$

Required area = $\int_1^3 y\,dx$

$$= \int_1^3 \frac{3x-15}{5}\,dx$$

$$= \frac{1}{5}\int_1^3 (3x-15)\,dx$$

$$= \frac{1}{5}\left[\frac{3x^2}{2} - 15x\right]_1^3$$

$$= \frac{1}{5}\left[\frac{27}{2} - 45 - \frac{3}{2} + 15\right]$$

$$= \frac{1}{5}\left[\frac{24}{2} - 30\right] = \frac{1}{5}(12-30)$$

$= \frac{-18}{5} = \frac{18}{5}$ sq. units [neglecting negative sign]

41. (B) $r = \frac{\text{cov}(x \cdot y)}{\sigma_x \sigma_y} \Rightarrow \sigma_y = \frac{\text{cov}(x \cdot y)}{r.\sigma_x}$

$$= \frac{16}{0.5 \times 4} = 8$$

42. (A) Arranging the data as

$\alpha - \frac{7}{2}, \alpha - 3, \alpha - \frac{5}{2}, \alpha - 2, \alpha - \frac{1}{2}, \alpha + \frac{1}{2},$ $\alpha + 4, \alpha + 5$

median = $\frac{1}{2}\left(\beta - 2, \beta - \frac{1}{2}\right) = \alpha - \frac{5}{4}$

43. (B) **Note :-** Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30°. $|(\vec{a} \times \vec{b}) \times \vec{c}| =$

Solution :

$\vec{a} \cdot \vec{c} = |\vec{c}| \Rightarrow 3|\vec{c}|\cos\theta = |\vec{c}|$

$\Rightarrow \cos\theta = \frac{1}{3}$

$|\vec{c} - \vec{a}| = 2\sqrt{2} \Rightarrow \vec{c}^2 + \vec{a}^2 - 2\vec{c} \cdot \vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 6|\vec{c}|\cos\theta = 8 \because |\vec{a}| = 3$

$\Rightarrow (|\vec{c}| - 1)^2 = 0$

$\Rightarrow |\vec{c}| = 1$

$\Rightarrow |\vec{a} \times \vec{b}| = |(2i + j - 2k) \times (i \times j)| = 3$

$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}| \cdot |\vec{c}| \sin 30°$

$= \frac{3}{2}$

44. (D) $|(a \times b) \cdot c| = |a||b||c|$

$\Leftrightarrow |ab \sin\theta\, \hat{n} \cdot c| = abc$

$\Leftrightarrow |(ab \sin\theta) 1.c \cos\phi 1 = abc$

$\Leftrightarrow |\sin\theta||\cos\phi| = 1$

$\Leftrightarrow \theta = \frac{\theta}{2}$ and $\phi = 0$

$\Leftrightarrow a$ is $\perp$ to b and c is || to $\hat{n}$

$\Leftrightarrow a$ is $\perp$ to b and c is $\perp$ to both a and b

$\Leftrightarrow a, b, c$ are mutually perpendicular

$\Leftrightarrow a.b = b.c = c.a = 0$

45. (C) The given differential equation can be written as

$$\frac{dy}{dx} - \frac{1}{2x} \cdot y = \frac{3}{2x}$$

which is linear

$\therefore$ I.F $e^{\int -\frac{1}{2x}dx} = e^{\frac{-1}{2}\log x} = x^{\frac{-1}{2}}$

$\therefore$ Solution is $y.x^{\frac{-1}{2}} = c + \int \frac{3}{2x} x^{\frac{-1}{2}}\,dx$

$yx^{\frac{-1}{2}} = c - 3x^{\frac{-1}{2}}$

or $(y + 3)^2 = c^2x$

which represents parabola.

46. (C) $\sin^2x + \sin^2y = 1$

$\Rightarrow \sin^2x = 1 - \sin^2y$

$\Rightarrow \sin^2x = \cos^2y$

$\Rightarrow \sin x = \cos y$

$\Rightarrow \sin x = \sin (90° - y)$

$\Rightarrow x = 90° - y$

$\Rightarrow x + y = 90°$

$\cot (x + y) = \cot 90° = 0$

47. (B) $S = 64t - 16t^2$

$\Rightarrow \frac{ds}{dt} = 64 - 32t$

For maximum height, $\frac{ds}{dt} = 0$

Thus, $64 - 32t = 0$

$\Rightarrow 32t = 64$

t = 2s

48. (A) $\begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

$\Rightarrow k\begin{vmatrix} 1 & (b+c) & (b^2+c^2) \\ 1 & (c+a) & (c^2+a^2) \\ 1 & (a+b) & (a^2+b^2) \end{vmatrix} = (a-b)(b-c)(c-a)$

By applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 + R_3$ we get

$\Rightarrow k\begin{vmatrix} 0 & (b-c) & (b^2-c^2) \\ 0 & (c-a) & (c^2-a^2) \\ 1 & (a+b) & (a^2+b^2) \end{vmatrix} = (a-b)(b-c)(c-a)$

$\Rightarrow k(b-a)(c-b)\begin{vmatrix} 0 & 1 & (b+c) \\ 0 & 1 & (c+a) \\ 1 & (a+b) & (a^2+b^2) \end{vmatrix}$

$= (a-b)(b-c)(c-a)$

$\Rightarrow k(b-a)(c-b) \,.\, 1[c+b-b-a] = (a-b)(b-c)(c-a)$

$\Rightarrow k(b-a)(c-b)(c-a) = (a-b)(b-c)(c-a)$

$\Rightarrow k(a-b)(b-c)(c-a) = (a-b)(b-c)(c-a)$

$\therefore k = 1$

49. (B) $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$\text{Adj}A = \begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

Now, $A(\text{Adj } A) = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} . \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 12-2 & -6+6 \\ 4-4 & -2+12 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

50. (A)

Marks	No. of Students y (f)	$\frac{1}{x}$	$f\ \frac{1}{x}$
20	4	0.0500	0.2000
21	2	0.0476	0.0952
22	7	0.0454	0.3178
23	1	0.0435	0.0435
24	3	0.0417	0.1251
25	1	0.0400	0.0400
	18		0.8216

$\text{H.M} = \dfrac{N}{\Sigma f\left(\frac{1}{x}\right)} = \dfrac{18}{0.8216} = 21.91$

51. (B) (1) A relation R on a set A is said to be a reflexive realtion, if $(a, a) \in R, \ \forall\ a \in R$

$\therefore R$ is reflexive.

(2) A relation R on a set A is said to be a symmetric relation, if $(a, b) \in R \Rightarrow (b, a) \in R, \ \forall\ a, b \in R$

Since, (b, a) A

$\therefore R$ is not symmetric

(3) A relation R on a set A is said to be a transitive relation if $(a, b) \in R, (b, c) \in R \Rightarrow (c, a) \in R$

$\therefore R$ is transitive.

52. (D) $\text{Adj A} = \begin{bmatrix} a & 0 \\ -1 & b \end{bmatrix} \ \therefore \ A = \begin{bmatrix} b & 0 \\ -1 & a \end{bmatrix}$

since, $|A| = ab$

$A^{-1} = \dfrac{AdjA}{|A|} = \dfrac{1}{ab}\begin{bmatrix} a & 0 \\ -1 & b \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} \frac{1}{b} & 0 \\ \frac{-1}{ab} & \frac{1}{a} \end{bmatrix}$

Hence, $|A^{-1}| = \dfrac{1}{ab}$

53. (B) Let $p(h, k)$ be the mid-point of the chord op through vectex (0, 0), then P is $(2h, 2k)$ which lie on the parabola $y^2 = 4x$

$\therefore 4x^2 = 8h$

Hence the locus of p is $y^2 = 2x$.

54. (C) The given equation $x^2 + 2y^2 - 2x + 3y + 2 = 0$, can be written as $\dfrac{(x-1)^2}{\frac{1}{8}} + \dfrac{\left(y+\frac{3}{4}\right)^2}{\frac{1}{16}} = 1$

Which represents an ellipse for which

$a^2 = \frac{1}{8},\ b^2 = \frac{1}{16}$

$\therefore b^2 = 3^2$ and centre is $(h, k) = (-1, -2)$

$e^2 = 1 - \dfrac{b^2}{a^2} = \dfrac{16}{25} \Rightarrow e = \dfrac{4}{5}$

$\therefore$ coordinates of the foci are

$(h, k \pm ae)$, i.e $(-1, -2 \pm 4)$

i.e $(-1, 2) = (-1, -6)$

55. (D) $f'(x) = -(x+1)e^{-x}$

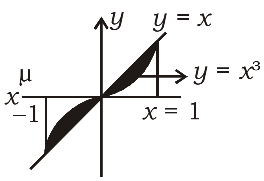
obviously $f'(x) < 0$, when $x > -1$

and $f'(x) > 0$, when $x < -1$

Hence, $f(x)$ in decreasing in $(-1, \infty)$ and increasing in $(-\infty, -1)$

56. (A) Clearly $f(1) = e^{o} + 1 - 2 = 1 + 1 - 2 = 0$
So 1 is a root of equation
Let $\alpha > 1$ be also a root. then $f(\alpha) = 0$
$\therefore$ By rolle's theorem $f'(x) = 0$ for at least one x between 1 and α.
Now $f'(x) = e^{x-1} + 1 = 0$
$\Rightarrow e^{x-1} = -1$
Which is not possible as $x \in]1, \alpha[$, i.e $x > 1$
$\therefore$ There exist no root > 1
Similarly we can see that no root exists less than1
Thus, there is only one root.

57. (D) Both curves $y = x^3$ and $y = x$ are symmetrical in opposite quadrants and points of intersection are $x = \pm 1$ and 0.
$\therefore$ required area



$\therefore$ required area

$= 2\left|\left(\int y_2 dx - \int y_1 dx \right)\right|$

$= 2 \int)x - x^3 * \, dx = \frac{1}{2}$

58. (B) $\frac{)1 ,\ i*x - 2i}{3 ,\ i} + \frac{)2 - 3i*y ,\ i}{3 - i} = i$

$\frac{)3 - i*|()1 ,\ i*x - 2i*, \ )3 ,\ i*|()2 - 3i*y ,\ i|}{9 ,\ 1} = i$

$\Rightarrow 4x + 9y = 3,\ 2x - 7y = 13$
$\Rightarrow x = 3,\ y = -1$

59. (D) Given expression
$= 1 - 1 + 1 - 1 + \ldots + (-1)^n$
Which can't be determined, unless n is known.

60. (A)

	Product	Integral Part
0.25 × 2 = 1.5		0 ↓ Result
0.5 × 2 = 1.0		1

$\therefore (0.25)_{10} = (0.10)_2$

61. (C) $63 + (n-1).2 = 3 + (n-1).] \Rightarrow 60 = 7n - 7 - 2n + 2$
$\Rightarrow 5n = 65 \Rightarrow n = 13$

62. (C) $\frac{\frac{n}{2}|(2a ,\ )n - 1*d)|}{\frac{n}{2}|(2A ,\ )n - 1*D)|} = \frac{3n ,\ 4}{5n ,\ 6}$

$\Rightarrow \frac{2a ,\ )n - 1*d}{2A ,\ )n - 1*D} = \frac{3n ,\ 4}{5n ,\ 6}$

$\Rightarrow \frac{a ,\ \frac{n-1}{2} d}{A ,\ \frac{n-1}{2} D} = \frac{3n ,\ 4}{5n ,\ 6}$

we have to find 5th term

i.e $\frac{a ,\ 4d}{A ,\ 4D}$, put $\frac{n-1}{2} = 4$

$\Rightarrow n = 9 \Rightarrow \frac{a ,\ 4d}{A ,\ 4D} = \frac{3 \propto 9 \propto 4}{5 \propto 9 ,\ 6} = \frac{31}{51}$

63. (D) Let point be $(x, 0)$

$\therefore \left| \frac{4x ,\ 3 \propto 0 - 12}{\sqrt{16 ,\ 9}} \right| = 8$

$\Rightarrow 4x - 12 = \pm 40 \Rightarrow 4x = 52, -28$
$\Rightarrow x = 13, -7$

64. (A) slope of the line through A (5, 0) is

$m = \tan 30^o = \frac{1}{\sqrt{3}}$

When rolated 15° clockwise
Then line marks angles 15° with x-axis
$\therefore$ slope of the line $= \tan 15^o = \tan (60^o - 45^o)$

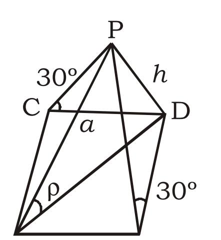
$= \frac{\sqrt{3} - 1}{1 ,\ \sqrt{3}} = \frac{)\sqrt{3} - 1*^2}{3 - 1} = 2 - \sqrt{3}$

$\therefore$ Equation of the line is $y - 0 = 2 - \sqrt{3}\,(x - 5)$

$\Rightarrow (2 - \sqrt{3})x - y - 5(2 - \sqrt{3}) = 0.$

65. (A) Let circle be $(x - h)^2 + (y - k)^2 = 9$ ...(i)
Circle (i) passes through the point (7, 3).
$\therefore (7 - h)^2 + (3 - k)^2 = 9$...(ii)
Also centre (h, k) lies m line $y = x - 1$
$\Rightarrow k = h - 1$ ...(iii)
from (ii) & (iii), we have $(7 - h)^2 + (4 - h)^2 = 9$
$\Rightarrow 49 - 14h + h^2 + 16 - 8h + h^2 = 9$
$\Rightarrow h^2 - 11h + 28 = 0$
$\Rightarrow 2h^2 - 22h + 56 = 0$
$(h - 7)(h - 4) = 0$
$\Rightarrow h = 7$ or $h = 4$
from (iii), when $h = 7$, $k = 6$
substituting in (1), we get circle as $(x - 7)^2 + (y - 6)^2 = 9$

66. (B)



Let a be the length of a side of square plot ABCD and h, the height of the pole standing at D.

Since elevations of p from A or C is 30° and that from B is θ.

$\therefore$ In ΔPCD, $\tan 30° = \frac{h}{a}$

i.e $\frac{h}{a} = \frac{1}{\sqrt{3}}$

and in ΔPBD

$\tan\theta = \frac{PD}{BD} = \frac{h}{a\sqrt{2}} = \frac{1}{\sqrt{6}}$

$\therefore$ BD $= \sqrt{AB^2 + AD^2} = a\sqrt{2}$

67. (A) $\frac{dy}{d\rho} = -a\sin\theta$

$\frac{dx}{d\rho} = a(1 + \cos\theta)$

$\frac{dy}{dx} = \frac{\frac{dy}{d\rho}}{\frac{dx}{d\rho}} = \frac{-a\sin\rho}{a(1+\cos\rho)} = -\tan\frac{\rho}{2}$

$\therefore \frac{d^2y}{dx^2} = \frac{-1}{2}\sec^2\frac{\rho}{2}\left(\frac{d\rho}{dx}\right)$

$= \frac{-1}{2}\sec^2\frac{\rho}{2}\cdot\left(\frac{1}{a(1+\cos\rho)}\right)$

$= \frac{-1}{4a}\sec^4\frac{\rho}{2}$

At $\theta = \frac{\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{-1}{4a}\sec^4\frac{\theta}{4} = \frac{-1}{a}$

68. (B) Let r be the radius and θ the angle of the sector.

$\therefore$ Perimeter $= 2r +$ arc AB $= 2r + r\theta$
$= 20$ cm (given)

$\Rightarrow \theta = \frac{(20-2r)}{r}$

The area of sector

$A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\,\frac{(20-2r)}{r}$

$A = 10r - r^2$

$\frac{dA}{dt} = 10 - 2r = 0$, for max or min of A

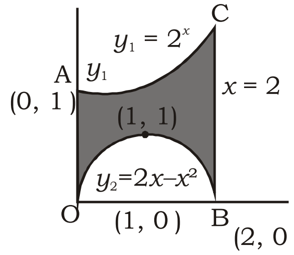
$\Rightarrow r = 5$cm

$\frac{d^2A}{d^2r} = -2$, which is –ve

$\therefore$ A is max. when $r = 5$ cm

$\therefore$ max $A = 10\times5 - 5^2 = 25$ sq. cm.

69. (A)



The two curves $y = 2^x$ and $y = 2x - x^2$ do not intersect between $x = 0$ and $x = 2$

$\therefore$ Required area

$= \int_0^2 y_1 dx - \int_0^2 y_2 dx$

$= \int_0^2 (2^x - 2x + x^2)dx$

$= \frac{3}{\log 2} - \frac{4}{3}.$

70. (C) Given integral

$= \int_0^{\frac{\theta}{2}} \frac{dx}{1+\cos\left(\frac{1}{2}\theta - x\right)}$

$= \frac{1}{2}\int_0^{\frac{\theta}{2}} \sec^2\left(\frac{1}{4}\theta - \frac{1}{4}x\right)dx$

$= \left[-\tan\left(\frac{1}{4}\theta - \frac{1}{2}x\right)\right]_0^{\frac{\theta}{2}} = 1$

71. (C) Given,

$y = \tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right) + \tan^{-1}\left(\frac{3+2\log x}{1-3x2\log x}\right)$

$= \tan^{-1}1 - \tan^{-1}(2\log x) + \tan^{-1}3 + \tan^{-1}(2\log x)$

$y = \tan^{-1}1 + \tan^{-1}3$

$\therefore$ $y' = 0$ and $y'' = 0$

72.(A) Given integral

$I = \frac{1}{2}\int_0^{\theta}[\sin(m+n)x - \sin(m-n)x]\,dx$

$= -\frac{1}{2}\left[\frac{\cos(m+n)x}{m+n} - \frac{\cos(m-n)x}{m-n}\right]_0^{\theta}$

$= -\frac{1}{2}\left[\left\{\frac{(-1)^{m+n}}{m+n} - \frac{(-1)^{m-n}}{m-n}\right\} - \left\{\frac{1}{m+n} - \frac{1}{m-n}\right\}\right]$

since, $n - m$ is odd, $\therefore$ $n + m$ must be odd, so $(-1)^{m+n} = (-1)^{m-n} = -1$

Also since

$|m| \neq |n|$, $m+n \neq 0$, $m-n \neq 0$

$\therefore I = \frac{1}{m+n} - \frac{1}{m-n}$

$= \frac{m-n-m-n}{m^2-n^2} = \frac{2n}{n^2-m^2}$

73. (C) Here $n = 3$
∴ Number of subsets = $2^3 = 8$.

74. (B) A = $\{x^2 : x \in \}$

75. (B) p : 100 is divisible by 3 ; 'F'
q : 100 is divisible by 11 ; 'F'
r : 100 is divisible by 5 ; 'T'
∴ p, q and r is 'F' i.e false

76. (B) Given,
N = 200, $\overline{x} = 48$ and $\sigma = 3$ and it is required to find the value of Σx and Σx^2

Now, $\overline{x} = \frac{\Sigma x}{200}$

$\Rightarrow \Sigma x = N\overline{x} = 200 \times 48 = 9600$

Also, $\sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

Substituting the value, we get

$\sigma^2 = \frac{\Sigma x^2}{200} - \left(\frac{9600}{200}\right)^2$

$\Rightarrow \Sigma x^2 = 4{,}62{,}600$

77. (A) Center is (2, 3)

(8, 4) —— (2, 3) —— (β, χ)

$\frac{8 + \beta}{2} = 2$; $\frac{4 + \chi}{2} = 3$

$\Rightarrow \alpha = -4, \beta = 2$ i.e $(-4, 2)$

78. (C) Given $\frac{2b^2}{a} = b \Rightarrow a = 2b$ ...(i)

$c = \sqrt{a^2 - b^2} = \sqrt{3}\, b$

∴ Eccentricity = $\frac{c}{a} = \frac{\sqrt{3}b}{a} = \frac{\sqrt{3}}{2}$ (∵ from (i))

79. (D) $\vec{a}, \vec{b}, \vec{c}$ are L.D vectors, so

$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \beta & \chi \end{vmatrix} = 1 - \beta = 0$

$\Rightarrow \beta = 1$

Also, $|c| = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$

Thus $\alpha = \pm 1, \beta = 1$.

80. (A) ∵ p and q are the roots of $x^2 - px + q = 0$.
∴ $p + q = p$, $pq = q$
$\Rightarrow q(p - 1) = 0$
$\Rightarrow q = 0, p = 1$

81. (C) $P\left(\frac{\overline{A}}{\overline{B}}\right) = \left(\frac{\overline{A} \cup \overline{B}}{P(\overline{B})}\right) = \frac{P(\overline{A \supset B})}{P(\overline{B})}$

$= \frac{1 - P(A \supset B)}{P(\overline{B})}$

82. (D) Let A, A, I, N be arranged.
Now there are 5 place for 'S' to be arranged
∴ ways for arrangement of 'S'

$= {}^5C_4 \frac{4!}{2!} = 60$

And total number of ways of arrangement of 8 letters = $\frac{8!}{4!2!} = \frac{8.7.6.5}{2}$

∴ P(no two S occur together)

$= \frac{60 \times 2}{8 \times 7 \times 6 \times 5} = \frac{1}{14}$

83. (C) Solving $y = 0$ and $y = 4 + 3x - x^2$
We get $x = -1, 4$. curve does not intersect x-axis between $x = -1$ and $x = 4$

∴ Area = $\int_{-1}^{4} (4 + 3x - x^2)\,dx$

$= \frac{125}{6}$ sq. units.

84. (A) Let $z = x + iy$, then
$z = \overline{z} \Rightarrow x + iy = x - iy \Rightarrow y = 0$
$\Rightarrow z$ is real.

85. (D) $\frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} = \frac{(1 - i\sqrt{3})^2}{4} = \frac{-2 - 2\sqrt{3}i}{4}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

∴ $\arg\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = -(\pi - \tan^{-1}\sqrt{3})$

$= \frac{-2\theta}{3}$

86. (B) Consider $\frac{5 - 2x}{3} \le \frac{x}{6} - 5$

multiplying throughout by 6, we get
$10 - 4x \le x - 30 \Rightarrow 40 \le 5x \Rightarrow 8 \le x$
$\Rightarrow x \ge 8$.

87. (A) Let Q(α, β) be image of $p(-8, 12)$ in the line $4x + 7y + 13 = 0$. Then (i) R is mid-point of PQ = 3 and (ii) PQ line.

Coordinate of mid-point $R\left(\frac{\beta-8}{2}, \frac{\chi , 12}{2}\right)$

As this point lies on line.

$\Rightarrow 4\left(\frac{\beta-8}{2}\right) + 7\left(\frac{\chi , 12}{2}\right) + 13 = 0$

$\Rightarrow 4\alpha - 32 + 7\beta + 84 + 26 = 0$

$\Rightarrow 4\alpha + 7\beta + 78 = 0$ ...(i)

Also PQ line $\Rightarrow \frac{\chi , 12}{\beta , B} \times \frac{-4}{7} = -1$

$\Rightarrow 4\beta - 48 = 7\alpha + 56$

$\Rightarrow 7\alpha - 4\beta + 104 = 0$ ...(ii)

Solving (i) and (ii), we get $\alpha = -16$, $\beta = -2$

Hence image is (–16, –2).

88. (C) Third term from end = $(n - 3 + 2)^{th}$ term

i.e $(n-1)^{th}$ from beginning

$\therefore$ coefficient = ${}^nC_{n-2} = {}^nC_2 = 45$

$\Rightarrow n(n-1) = 90$

$\Rightarrow n = 10$

$T_6 = {}^{10}C_5 \left(y^{\frac{1}{3}}\right)^5 \left(x^{\frac{1}{3}}\right)^5 = -252\ x^{\frac{5}{3}}\ y^{\frac{5}{2}}$.

89. (A) $T_n = \frac{n^2}{)n , 1*!} = \frac{n^2 - 1 , 1}{)n , 1*!}$

$= \frac{)n-1*)n , 1*}{)n , 1*!} + \frac{1}{)n , 1*!}$

$= \frac{)n-1*}{n!} + \frac{1}{)n , 1*!}$

$= \frac{n}{n!} - \frac{1}{n!} + \frac{1}{)n , 1*!}$

$= \frac{1}{)n-1*!} - \frac{1}{n!} + \frac{1}{)n , 1*!}$

$S_n = \left(\frac{1}{0!}, \frac{1}{1!}, \frac{1}{2!}, \ldots\right) - \left(\frac{1}{1!}, \frac{1}{2!}, \frac{1}{3!}, \ldots\right)$

$+ \left(\frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \ldots\right)$

$= e - (e-1) + (e-2)$

$= e - 1$

90. (B) ${}^nP_r = 336$, ${}^nc_r = 56$

We know that ${}^nP_r = r!\,{}^nc_r \Rightarrow 336 = 56r!$

$\Rightarrow r! = 6 = 3! \Rightarrow r = 3$

Consider ${}^nc_r = 56 \Rightarrow {}^nc_3 = 56$

$\Rightarrow \frac{n)n-1*)n-2*}{3!} = 56$

$\Rightarrow n(n-1)(n-2) = 56 \times 6 = 8 \times 7 \times 6$

$n(n-1)(n-2) = 8(8-1)(8-2)$

$\Rightarrow n = 8$

Hence $n = 8$, $r = 3$

91. (A) $f(x) = \alpha x^2 + \beta x^2 + r$

obviously $f(x)$ is continuous in a closed interval $[a, b]$ and differentiable in open interval $]a, b[$.

$\therefore$ by Lagrange's mean value theorem, there exist a point c, such that $a < c < b$, where

$f'(c) = \frac{f)b* - f)a*}{b-a}$

$\Rightarrow 2\alpha c + \beta = \frac{\beta)b^2 - a^2*, \chi)b - a*}{b-a}$

$\Rightarrow 2\alpha c + \beta = \alpha(b + a) + \beta$

$c = \frac{a , b}{2}$

92. (D) Here $\frac{dr}{dt} = 4$ cm/s

Area = A = πr^2

$\therefore \frac{dA}{dt} = \left(2\theta r \frac{dr}{dt}\right)_{r>10} = 80\pi$ cm²/s

But in (B) the result is 80π cm/s which is wrong unit so not true.

93. (D) $f(x) = |x| + |x-1|$

$= \begin{cases} -2x , 1 & : -1 / x = 0 \\ 1 & : 0 / x = 1 \\ 2x - 1 & : 1 / x / 2 \end{cases}$

$f'(x) = \begin{cases} -2 : -1 \le x < 0 \\ \text{not exist} : x = 0 \\ 0 : 0 < x < 1 \\ \text{not exist} : x = 1 \\ 2 : 1 < x \le 2 \end{cases}$

Hence $f(x)$ is decreasing in $[-1, 0[$ and increasing in $]1, 2]$,

i.e, neither increasing nor decreasing in $[-1, 2]$.

94. (C) $\log_2 x$ is real if $x > 0$ so we should have

$\log_3 \log_{\left(\frac{4}{\theta}\right)} (\tan^{-1} x)^{-1} > 0$

$\because$ Base 3 > 1 so $\log_{\frac{4}{\theta}} (\tan^{-1} x)^{-1} > 1$

Now the base $\frac{4}{\theta} > 1$ so

$(\tan^{-1} x)^{-1} > \frac{4}{\theta}$

or $\tan^{-1} x < \frac{4}{\theta}$

so $0 < x < 1$

Hence required domain is (0, 1)

95. (B) We have $\frac{dy}{dx} = \frac{1}{|x|} \cdot \frac{d}{dx}|x|$ ...(i)

We have $|x| = \begin{cases} x : x \geq 0 \\ -x : x < 0 \end{cases}$ ...(ii)

$\therefore \frac{d|x|}{dx} = \begin{cases} 1 : x \geq 0 \\ -1 : x < 0 \end{cases}$ ...(iii)

from (i) and (iii), we have

$\frac{dy}{dx} = \frac{1}{x} \cdot 1$ $(x \geq 0)$

and $\frac{dy}{dx} = \frac{1}{(-x)} \cdot (-1) = \frac{1}{x}$, $(x < 0)$

Hence $\frac{dy}{dx} = \frac{1}{x}$

96. (C) Required probability
= 1 – P(red balls)

$= 1 - \frac{{}^7C_2}{{}^9C_2}$

$= 1 - \frac{7 \times 6}{9 \times 8} = \frac{5}{12}$

97. (C) $\int_{-1}^{1} |1 - x|\, dx = \int_{-1}^{1} (1 - x) dx$

$\because 1 - x \geq 0$, when $-1 \leq x \leq 1$

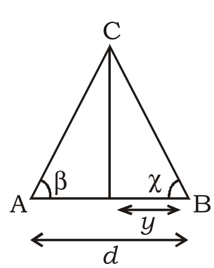
$= \left[x - \frac{1}{2}x^2\right]_{-1}^{1} = 2$

98. (D) $A \cup B = A \cap C$ ...(i)
$A \cap B = A \cap C$ ...(ii)
from (i) and (ii)
$A \cup B = A \cap B$
since A = B
Again from (ii) $A \cap B = A \cap B$
since $B = C$
Hence $A = B = C$

99. (B) $\begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1 + P \end{vmatrix} = 20$

since, $2[5(1 + P) - 0] = 20$
$\Rightarrow 10(1 + P) = 20$
$\Rightarrow P = 1$

100. (*) from Δ CAD and CDB



$\frac{d - y}{h} = \cot\alpha$ ...(i)

$\frac{y}{h} = \cot\beta$ ...(ii)

from (i) & (ii)

$\frac{d}{h} - \cot\beta = \cot\alpha \Rightarrow \frac{d}{h} = \cot\alpha + \cot\beta$

$\therefore \frac{h}{d} = \frac{1}{\cot\beta + \cot\chi} = \frac{\cot(\beta + \chi)}{\cot\beta\cot\chi - 1}$

101. (C) $\left[\left(\frac{d^4y}{dx^4}\right)^3\right]^{\frac{2}{3}} - 7x\left(\frac{d^3y}{dx^3}\right) = 8$

$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 - 7x\left(\frac{d^3y}{dx^3}\right) = 8$

Hence, order = 4
Degree = 2

102. (C) $[F(x)^2] = \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$f(x^2) + 2 = x^2 + \frac{1}{x^2} + 2$

$[f(x)]^2 = \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$= x^3 + \frac{1}{x^3} + 3f(x)$

Now, $f(x^3) + 3f(x) = x^3 + \frac{1}{x^3} + 3f(x)$

Hence, both the statements are true.

103. (D) Let a and d be the first term and common difference of an AP.
According to question,
$P.T_p = q.T_q$
$\Rightarrow p[a + (p - 1)d] = q[a + (q - 1)d]$
$\Rightarrow pa + (p^2 - p)d] = qa + (q^2 - q)d$
$\Rightarrow (p - q)a = (q^2 - p^2 + p - q)d$
$\Rightarrow (p - q)a = (p - q)(-p - q + 1)d$
$\Rightarrow a = -(p + q - 1)d$
Now, $T_{p+q} = a + (p + q - 1)d$
$= -(p + q - 1)d + (p + q - 1)d$
$= 0$

104. (C) P (ace) = $\frac{4}{52} = \frac{1}{13}$

$P(\text{King}) = \frac{4}{52} = \frac{1}{13}$

P(2 areas and one king) = $^3C_2\left(\frac{1}{13}\right)^2\left(\frac{1}{13}\right)$

$= \frac{3}{(13)^3}$

105. (B) Total ways = $^{80}C_2$
Favourable ways = $^{20}C_2$

$P = \frac{^{20}C_2}{^{80}C_2}$

$= \frac{19}{316}$

106. (C) $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} = \frac{\cot 54^\circ}{\tan(90^\circ - 54^\circ)} + \frac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)}$

$= \frac{\cot 54^\circ}{\cot 54^\circ} + \frac{\tan 20^\circ}{\tan 20^\circ}$

$\Rightarrow 1 + 1 = 2$

107. (D) $\lim_{x \to 0} \frac{5^x 2^x - 5^x - 2^x + 1}{x^2}$

$\lim_{x \to 0} \frac{5^x(2^x - 1) - 1(2^x - 1)}{x^2}$

$\Rightarrow \lim_{x \to 0} \frac{5^x - 1}{x} \cdot \frac{2^x - 1}{x}$

$\Rightarrow \log_e 5 \log_e 2$

108. (A) Number of ways to choose 8 players from 12 players = $^{12}C_8 = \frac{12!}{8!4!} = 495$

and number of ways to choose a captain and a vice-captain
$= {^8C_1} \times {^7C_1}$
$= 8 \times 7 = 58$
Hence, required number of
$= 495 \times 56 = 27720$

109. (B) Let $y = x^x$
$\log y = x \cdot \log x \qquad x > 0$
Differentiate w.r.t x

$\frac{1}{y}\frac{dy}{dx} = (1 + \log x)$

$\therefore \frac{dy}{dx} = x^x(1 + \log x)$

$\therefore \frac{dy}{dx} = 0 \Rightarrow \log x = -1$

$\Rightarrow x = e^{-1} = \frac{1}{e}$

$\therefore$ stationary point is $x = \frac{1}{e}$

$\frac{d^2y}{dx^2} = x^x(1 + \log x)^2 + x^x \cdot \frac{1}{x}$

when $x = \frac{1}{e}$, $\frac{d^2y}{dx^2} = \left(\frac{1}{e}\right)^{\frac{1}{e}-1} > 0$

$\therefore$ y is minimum at $x = \frac{1}{e}$ and

minimum value $= \left(\frac{1}{e}\right)^{\frac{1}{e}} = e^{-\frac{1}{e}}$

110. (B) $(gof)(x) = g(f(x)) = g(e^x)$
$= \ln e^x = x \ln e = x$

$\therefore \frac{dy}{dx} = 1.$

111. (D) By the result
mode = 3 median – 2 mean
assuming, mean > median > mode
mean – mode = 3(mean – median)
$\therefore$ 63 = 3 (mean – median)
i.e mean – median = 21

112. (C) $(1 + i)^{2n} = (l - i)^{2n}$

$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1 \Rightarrow (i)^{2n} = 1$

$\therefore n = 2$ is the smallest positive integer.

113. (B) $A^2 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

114. (C) Any point on the given line is
$(5r - 3, 2r + 1, 3r - 4)$
If it is the foot of the perpendicular from (1, 2, 3), then
$5(5r - 3 - 0) + 2(2r + 1 - 2) + 3(3r - 4 - 3) = 0$
i.e $38r = 38$
i.e $r = 1$
$\therefore$ foot of perpendicular is (2, 3, –1)

115. (B) $\cos A = \frac{6^2 + 10^2 - 14^2}{2 \times 6 \times 10} = \frac{36 + 100 - 196}{120}$

$= \frac{-60}{120} = \frac{-1}{2}$

$\Rightarrow A = 120^\circ$

116. (A) $(623)^{\frac{1}{4}} = (625-2)^{\frac{1}{4}}$

$= 5\left(1-\frac{2}{625}\right)^{\frac{1}{4}}$

$= 5\left(1-\frac{2}{4\times 625}\right)$

$= 5\,(1-0.0008) = 4.996$

117. (C) $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$

Given,

$\lim_{x\to 1} f(x) = f(1)$

$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{h\to 0} f(1-h) = \lim_{h\to 0} f(1+h) = f(1)$

$\Rightarrow \lim_{h\to 0} \{a + b(1-h)\} = \lim_{h\to 0} \{b - a(1+h)\} = 4$

$\Rightarrow a + b = b - a = 4 \Rightarrow a = 0,\ b = 4$

118. (A) $I = \int \frac{\log x^2}{x}\,dx = 2\int \frac{\log x}{x}\,dx$

$= 2\cdot\frac{1}{2}(\log x)^2 + C$

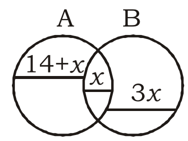
$\Rightarrow (\log x)^2 + C$

119. (B) $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$A^2 = I$

120. (C) Given $n(A-B) = 14 + x$



$n(B-A) = 3x$

$n(A\cap B) = x$

$n(A) = n(B)$

$\Rightarrow n(A-B) + n(A\cap B) = n(B-A) + n(A\cap B)$

$\Rightarrow 14 + x + x = 3x + x$

$\Rightarrow 14 = 2x$

$x = 7$

NDA MOCK TEST-47 (ANSWER KEY)

1.	(B)	21.	(B)	41.	(B)	61.	(C)	81.	(C)	101.	(C)
2.	(A)	22.	(D)	42.	(A)	62.	(C)	82.	(D)	102.	(C)
3.	(A)	23.	(B)	43.	(B)	63.	(D)	83.	(C)	103.	(D)
4.	(B)	24.	(C)	44.	(D)	64.	(A)	84.	(A)	104.	(C)
5.	(C)	25.	(D)	45.	(C)	65.	(A)	85.	(D)	105.	(B)
6.	(A)	26.	(A)	46.	(C)	66.	(B)	86.	(B)	106.	(C)
7.	(B)	27.	(C)	47.	(B)	67.	(A)	87.	(A)	107.	(D)
8.	(C)	28.	(B)	48.	(A)	68.	(B)	88.	(C)	108.	(A)
9.	(C)	29.	(C)	49.	(B)	69.	(A)	89.	(A)	109.	(B)
10.	(B)	30.	(B)	50.	(A)	70.	(C)	90.	(B)	110.	(B)
11.	(A)	31.	(D)	51.	(B)	71.	(C)	91.	(A)	111.	(D)
12.	(D)	32.	(D)	52.	(D)	72.	(A)	92.	(D)	112.	(C)
13.	(B)	33.	(C)	53.	(B)	73.	(C)	93.	(D)	113.	(B)
14.	(A)	34.	(A)	54.	(C)	74.	(B)	94.	(C)	114.	(C)
15.	(C)	35.	(A)	55.	(D)	75.	(B)	95.	(B)	115.	(B)
16.	(A)	36.	(A)	56.	(A)	76.	(B)	96.	(C)	116.	(A)
17.	(C)	37.	(A)	57.	(D)	77.	(A)	97.	(C)	117.	(C)
18.	(C)	38.	(B)	58.	(B)	78.	(C)	98.	(D)	118.	(A)
19.	(B)	39.	(D)	59.	(D)	79.	(D)	99.	(B)	119.	(B)
20.	(D)	40.	(B)	60.	(A)	80.	(A)	100.	(*)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*