NDA MATHS MOCK TEST - 51 (SOLUTION)

1. (C) Sum of roots $= (m + n) + (m - n)$
$= 2m$
Product of roots $= (m + n)(m - n)$
$= m^2 - n^2$
$\therefore$ Quadratic equation is
x^2 – (Sum of roots)x + Product of roots = 0
$x^2 - 2mx + (m^2 - n^2) = 0$

2. (C) Since α and β are the roots of
$x^2 + px - q = 0$
$\therefore \alpha + \beta = -p,\ \alpha\beta = -q$
Again since γ, δ are the roots of
$x^2 - px + r = 0$
$\therefore \gamma + \delta = p, \gamma\delta = r$

$(\beta + \gamma)(\beta + \delta) = \beta^2 + \beta\delta + \gamma\beta + \gamma\delta$
$= \beta^2 + \beta(\delta + \gamma) + \gamma\delta$
$= \beta^2 + \beta(p) + \gamma\delta$
$[\therefore \gamma + \delta = p \text{ and } \gamma\delta = r]$
$= \beta^2 + \beta(-\alpha - \beta) + r$
$[\because P = -(\alpha + \beta)]$
$= \beta^2 + (-\beta)(\alpha + \beta) + r$
$= \beta^2 - \alpha\beta - \beta^2 + r$
$= -\alpha\beta + r$
$= -(-q) + r$
$= q + r$
Hence, $(\beta + \gamma)(\beta + \delta) = q + r$

3. (B) Since, the sum of cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$
and the sum of squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
$\therefore$ The sum of cubes of first 20 natural numbers
$= \left[\frac{20(20+1)}{2}\right]^2$
$= \left(\frac{20 \times 21}{2}\right)^2$
$= (10 \times 21)^2$
$= 44100$
and the sum of squares of first 20 natural numbers $= \frac{20(20+1)(2 \times 20 + 1)}{6}$
$= \frac{20 \times 21 \times 41}{6} = 2870$

4. (B) **Statement I**
LHS $= (\omega^{10} + 1)^7 + \omega$
$= [(\omega^3)^3\omega + 1]^7 + \omega \qquad [\because \omega^3 = 1]$
$= (\omega + 1)^7 + \omega$
$= (-\omega^2)^7 + \omega$
$[\because 1 + \omega + \omega^2 = 0 \therefore 1 + \omega = -\omega^2]$
$= -\omega^{14} + \omega = -(\omega^3)^4\omega^2 + \omega$
$= -\omega^2 + \omega = (1 + \omega) + \omega = 1 + 2\omega \neq 0$
$\therefore$ Statement 1 is false.
Statement 2
LHS $= (\omega^{105} + 1)^{10}$
$= (\omega^3)^{35} + 1]^{10} \qquad [\because \omega^2 = 1]$
$= (1 + 1)^{10}$
$= 2^{10} = p^{10}$ which is true for prime numbers 2.
So, Statement 1 is false and Statement 2 is true.

5. (C) Given series is
$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots\ldots$
Since, it is geometric progression,
Here, First term $a = 1$
Common ratio $r = -\frac{1}{2} < 1$
$\therefore$ The sum of first eight terms of the series
i.e. $S_8 = \frac{a(1 - r^8)}{(1 - r)}$

[by formula, $S_n = \frac{a(1-r^n)}{1-r}$, where $r < 1$]

$$= \frac{1\left[1-\left(-\frac{1}{2}\right)^8\right]}{1-\left(-\frac{1}{2}\right)} = \frac{1-\frac{1}{256}}{1+\frac{1}{2}} = \frac{\frac{255}{256}}{\frac{3}{2}}$$

$$= \frac{255}{256} \times \frac{2}{3} = \frac{85}{128}$$

6. (D) There are 8 letters in word 'BASEBALL' in which 2B, 2A, 2L, 1S and 1E.
So, the number of permutations that can be formed from all the letters of the word 'BASEBALL'

$$= \frac{8!}{2!2!2!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 2 \times 1}$$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 42 \times 120 = 5040$

7. (D) R = [x : x is a set of all children of a same father]
(i) **Reflexive** Let p be the child of same father.
$\therefore$ pRp is a reflexive.
(ii) **Symmetry** Let p and q be the children of same father.
$\therefore$ q and p be the children of same father.
$\therefore$ R is symmetric.
(iii) **Transitive** Let p and q be children of same father and q and r be the children of same father.
$\therefore$ p and r be the children of same father R.
$\therefore$ R is transitive.
$\because$ R have all three properties such that reflexive, symmetry and transitive, so R is an equivalence relation.

8. (B) Since, the roots of the quadratic equation $3x^2 - 5x + p = 0$ are real and unequal.
$\therefore$ Discriminant > 0.
$\Rightarrow$ $b^2 - 4ac > 0$
$\Rightarrow (-5)^2 - 4(3)(p) > 0$ (here, $b = -5$, $a = 3$, $c = p$)
$\Rightarrow 25 - 12p > 0 \Rightarrow 25 > 12p$
$\Rightarrow 12p < 25 \Rightarrow p < \frac{25}{12}$

9. (D) $(1011)_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$= 8 + 0 + 2 + 1 = 11$

10. (A) For integer part of 57.375 i.e. $(57)_{10}$

2	57	
2	28	1
2	14	0
2	7	0
2	3	1
	1	1

$\therefore (57)_{10} = (111001)_2$
For after decimal part of 57.375 i.e., $(0.375)_{10}$
Now,

				Binary
0.375	×	2	= 0.75	0
0.75	×	2	= 1.5	1
0.5	×	2	= 1.0	1

$(0.375)_{10} = (0.011)_2$
$\therefore (57.375)_{10} = (111001.011)_2$

11. (B) $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

$$\Rightarrow \frac{\log x}{\log 3} \times \frac{\log 2x}{\log x} \times \frac{\log y}{\log 2x} = \frac{\log x^2}{\log x}$$

$$\left(\because \log_b a = \frac{\log a}{\log b}\right)$$

$$\Rightarrow \frac{\log y}{\log 3} = \frac{2\log x}{\log 2x} \quad [\because \log a^b = b\log a]$$

$\Rightarrow \log y = 2\log 3$
$\Rightarrow \log y = \log 3^2$ [$\because \log m = \log n \Rightarrow m = n$]
$\Rightarrow \log y = \log 9$
$\Rightarrow y = 9$

12. (A) Given relation is
R = {(1, 2), (1, 3), (2, 1), (1, 1), (2, 2), (3, 3), (2, 3)}
and P = {1, 2, 3}
(i) **Reflexive** If aRa, $\forall a \in P$.
Then, R is reflexive.
In R, $1R1$, $2R2$ and $3R3$ where $1, 2, 3 \in P$.
$\therefore$ R is reflexive.
(ii) **Symmetry** If $aRb \Rightarrow bRa$, where $a, b \in P$.
Then R is symmetry.
In R, $1R3 \not\Rightarrow 3R1$ and $2R3 \not\Rightarrow 3R2$
$\therefore$ R is not symmetry.
(iii) **Transitive** If aRb and $bRc \Rightarrow aRc$ where $a, b, c \in P$.
Then, R is transitive.
In R, $1R2$ and $2R3 \Rightarrow 1R3$
and $1R2$ and $2R1 \Rightarrow 1R1$
$\therefore$ R is transitive.

13. (D) Given $\sum_{n=1}^{13}(i^n + i^{n+1})$

$= (i + 1)\sum_{n=1}^{13}(i^n) = (i + 1)(i + i^2 + ... + i^{12})$

$= (i + 1)\left\{\frac{i(1 - i^{13})}{(1 - i)}\right\} = \frac{(i^2 + i)\{1 - (i^2)^6 i\}}{(1 - i)}$

$[\because i^2 = -1]$

$= \frac{(i - 1)}{(1 - i)}(1 - i) = (i - 1)$

Explanation (Q.No. 14-15):

Let the first term of an AP is a and common difference is d.

Given, $S_{10} = 120$ and $S_{20} = 440$

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2a + (10 - 1)d]$

$\Rightarrow \quad 120 = 5(2a + 9d)$

$\Rightarrow 2a + 9d = 24 \quad ...(i)$

and $\quad S_{20} = \frac{20}{2}[2a + 10 - 1)d]$

$\Rightarrow \quad 440 = 10(2a + 19d)$

$\Rightarrow \quad 2a + 19d = 44 \quad ...(ii)$

On subtracting eqn (i) from eqn. (ii), we get

$10d = 20$

$\Rightarrow \quad d = 2$

On putting the value of d in Eq. (i), we get

$2a + 9(2) = 24$

$\Rightarrow 2a + 18 = 24$

$\Rightarrow \quad 2a = 6$

$\Rightarrow \quad a = 3$

14. (B)

15. (B)

16. (B) Since, a non-empty set A has an elements therefore its power set contains 2^n elements because power set have same element as number of subsets of set A.

17. (B) Given,

A = {$x \in W$, the set of whole numbers and $x < 3$}

= {0, 1, 2}

B = {$x \in N$, the set of natural numbers and $2 \le x < 4$}

= {2, 3}

C = {3, 4}

$A \cup B = \{0, 1, 2, 3\}$

$(A \cup B) \times C = \{0, 1, 2, 3\} \times \{3, 4\}$

$= \{(0, 3), (0, 4), (1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

Required number of elements containing by $(A \cup B) \times C$ is 8.

18. (C) $\frac{\sqrt{2} + i}{\sqrt{2} - i} = \frac{\sqrt{2} + i}{\sqrt{2} - i} \times \frac{\sqrt{2} + i}{\sqrt{2} + i}$

$= \frac{(\sqrt{2} + i)^2}{(\sqrt{2})^2 + (i)^2} = \frac{2 + i^2 + 2\sqrt{2}i}{2 - i^2}$

$= \frac{2 - 1 + 2\sqrt{2}i}{2 - (-1)} = \frac{1 + 2\sqrt{2}i}{3}$

$\therefore \frac{\sqrt{2} + i}{\sqrt{2} - i} = \frac{1 + 2\sqrt{2}i}{3}$

$\Rightarrow \frac{\sqrt{2} + i}{\sqrt{2} - i} = \frac{1}{3} + \frac{2\sqrt{2}}{3}i$

$\Rightarrow \left|\frac{\sqrt{2} + i}{\sqrt{2} - i}\right| = \left|\frac{1}{3} + \frac{2\sqrt{2}}{3}i\right|$

$= \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2\sqrt{2}}{3}\right)^2}$

$= \sqrt{\frac{1}{9} + \frac{8}{9}} = \sqrt{\frac{9}{9}} = 1$

Alternate method

We know that

If Z_1 and Z_2 are two complex numbers.

Then, $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}$, Provided $Z_2 \neq 0$

$\therefore \left|\frac{\sqrt{2} + i}{\sqrt{2} - i}\right| = \frac{|\sqrt{2} + i|}{|\sqrt{2} - i|} = \frac{\sqrt{2 + 1}}{\sqrt{2 + 1}} = \frac{\sqrt{3}}{\sqrt{3}} = 1$

19. (A) The number of diagonals which can be drawn by joining the angular points of a polygon of 100 sides = ${}^{100}C_2 - 100$

$= \frac{100!}{2!98!} - 100$

$= \frac{100 \times 99 \times 98!}{2 \times 98!} - 100$

$= 50 \times 99 - 100$

$= 4950 - 100$

$= 4850$

20. (A) Let the angles of triangles are a, $a + d$ and $2 + 2d$.

Given, $a = 30°$

$\because \quad a + a + d + a + 2d = 180°$

$\therefore \quad 3a + 3d = 180°$

$\Rightarrow \quad 3 \times 30° + 3d = 180°$

$\Rightarrow \quad 90° + 3d = 180°$

$\Rightarrow \quad 3d = 90°$

$\Rightarrow \quad d = 30°$

$\therefore$ Angles of triangle are 30°, 60° and 90°.

Hence, greatest angle = 90° = $\frac{\pi}{2}$

21. (D) If each element in a row of a determinant is multiplied by the same factor r, then the value of the determinant is multiplied by r.

22. (C) We know that by the property of diagonal matrix.

At A = Diagonal (a_1, a_2, a_3)

Then, A^{-1}= Inverse of A

= Diagonal (a_1^{-1}, a_2^{-1}, a_3^{-1})

= Diagonal $\left(\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}\right)$

Hence, the inverse of diagonal matrix is a diagonal matrix.

23. (C) The transpose of any matrix A is obtained by interchange the row into corresponding column. So, B is the transpose of A.

24. (B) $\begin{bmatrix} x \\ x \\ y \end{bmatrix} + \begin{bmatrix} y \\ y \\ z \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+y+z \\ x+y+0 \\ y+z+0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$

$\Rightarrow \quad x + y + z = 10 \quad \ldots$ (i)

$x + y = 5 \quad \ldots$ (ii)

$y + z = 5$

From Eqs. (i) and (iii), we get

$\Rightarrow \quad x + (5) = 10 \Rightarrow x = 5$

On putting the vlaue of x in Eq. (ii), we get

$5 + y = 5$

$y = 0$

25. (D) From option (D), we have

C = A $\cos\alpha$ + B $\sin\alpha$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cos\alpha + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \sin\alpha$

$= \begin{bmatrix} \cos\alpha & 0 \\ 0 & \cos\alpha \end{bmatrix} + \begin{bmatrix} 0 & \sin\alpha \\ -\sin\alpha & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

= C

Hence, (D) is the correct option.

26. (C)

27. (D) $\begin{vmatrix} 1+\omega & \omega^2 & \omega \\ 1+\omega^2 & \omega & \omega^2 \\ \omega+\omega^2 & \omega & \omega^2 \end{vmatrix}$

Apply $C_1 + C_2 \rightarrow C_1$.

$= \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ 1+\omega^2+\omega & \omega & \omega^2 \\ \omega+\omega^2+\omega & \omega & \omega^2 \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega^2 & \omega \\ 0 & \omega & \omega^2 \\ -1+\omega & \omega & \omega^2 \end{vmatrix} \qquad \because 1+\omega+\omega^2 = 0$

$= (-1+\omega)\begin{vmatrix} \omega^2 & \omega \\ \omega & \omega^2 \end{vmatrix}$

$= (-1+\omega)(\omega^4 - \omega^2)$

$= (-1+\omega)(\omega^3.\omega - \omega^2)$

$= (-1+\omega)(\omega - \omega^2)$

$= -\omega + \omega^2 + \omega^2 - \omega^3$

$= -\omega + 2\omega^2 - 1$

$= -(1+\omega) + 2\omega^2$

$= 3\omega^2$

28. (C) In the word GARDEN, all the letters are different.

$\therefore$ The no. of ways to arrange the letters of this word = 6! = 720

$\because$ Vowels are in alphabetical order.

$\therefore$ The no. of arrangments = $\frac{6!}{2} = \frac{720}{2} = 360$

29. (C) $x + 2 = 0 \Rightarrow x = -2$

Again,

$x^2 + 2x = 0$

$\Rightarrow x(x + 2) = 0$

$\Rightarrow x = 0, -2$

Also,

$x^2 + x - 2 = 0$

$\Rightarrow\ x^2 + 2x - x - 2 = 0$

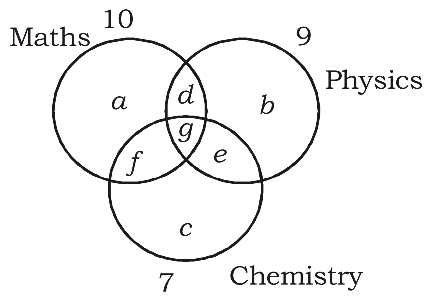
$\Rightarrow\ x(x + 2) - (x + 2) = 0$

$\Rightarrow\ (x - 1)(x + 2) = 0$

$\Rightarrow\ x = 1, -2$

From the above solutions, we conclude that for $x = -2$, V = R = S.

30. (B)



Given:-

$f = 0,\ d = 4,\ g = 0$

$\because\ a + d + f + g = 10$

$a + 4 + 0 + 0 = 10$

$\Rightarrow\ a = 6$

Again

$b + d + g + e = 9$

$b + 4 + 0 + e = 9$

$b + e = 5$

Also, $a + d + b + c + e = 20$

$\Rightarrow\ 6 + 4 + 5 + c = 20$

$\Rightarrow\ +$

$c = 20 - 15 = 5$

Now,

$c + e = 7$

$5 + e = 7$

$e = 7 - 5 = 2$

31. (A) Since, α and γ be the roots of $Ax^2 - 4x + 1 = 0$

$\therefore\ \alpha + \gamma = \dfrac{4}{A}$ and $\alpha\gamma = \dfrac{1}{A}$

And β and δ be the roots of $Bx^2 - 6x + 1 = 0$

$\therefore\ \beta + \delta = \dfrac{6}{B}$ and $\beta\delta = \dfrac{1}{B}$

Also, $\alpha, \beta,$ and δ are in HP.

$\therefore\ \dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}$ and $\dfrac{1}{\delta}$ are in AP.

$$\Rightarrow \frac{1}{\beta} - \frac{1}{\alpha} = \frac{1}{\delta} - \frac{1}{\gamma}$$

$$\Rightarrow \frac{1}{\beta} - \frac{1}{\delta} = \frac{1}{\alpha} - \frac{1}{\gamma}$$

$$\Rightarrow \frac{\delta - \beta}{\alpha\beta} = \frac{\gamma - \alpha}{\alpha\gamma}$$

$$\Rightarrow \frac{\sqrt{(\delta + \beta)^2 - 4\alpha\delta}}{\beta\delta} = \frac{\sqrt{(\gamma + \alpha)^2 - 4\alpha\gamma}}{\alpha\gamma}$$

$$\Rightarrow \frac{\sqrt{\frac{36}{B^2} - \frac{4}{B}}}{\frac{1}{B}} = \frac{\sqrt{\frac{16}{A^2} - \frac{4}{A}}}{\frac{1}{A}}$$

$\Rightarrow\ \sqrt{36 - 4B} = \sqrt{16 - 4A}$

$\Rightarrow\ 36 - 4B = 16 - 4A$

$\Rightarrow\ 4A - 4B = -20$

$\Rightarrow\ A - B = -5$

$\Rightarrow\ -A + B = 5$

It is possible if A = 3 and B = 8.

32. (C) The given system of equation is

$kx + y + z = k - 1$

$x + ky + z = k - 1$

$x + y + kz = k - 1$

$$\therefore\ A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$$

$$B = \begin{bmatrix} k & -1 \\ k & -1 \\ k & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}$$

Expanding along R_1

$= k(k^2 - 1) - 1(k - 1) + 1(1 - k)$

$= k^3 - k - k + 1 + 1 - k$

$= k^3 - 3k + 2$

The given system of equations has no solution, if $|A| = 0$

$\Rightarrow\ k^3 - 3k + 2 = 0$

$\Rightarrow\ (k - 1)^2(k + 2) = 0$

$\Rightarrow\ k = 1$ or $k = -2$

33. (B) We know that the largest side has the greatest angle opposite it.

$\therefore\ a = 6$ cm, $b = 10$ cm and $c = 14$ cm

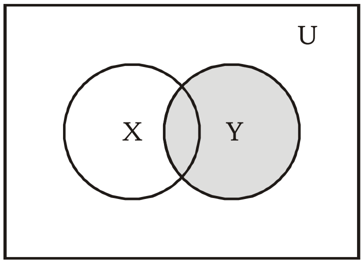
$\therefore\ \cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$ [By cosine rule]

$$= \frac{36+100-196}{2\times 6\times 10}$$

$$= -\frac{1}{2} = \cos 120^\circ$$

$\angle C \quad = 120^\circ$

34. (C) From the figure, it is clear that



$(X - Y)' = X' \cup Y$

Alternative:-

$(X - Y)' = (X \cap Y')'$

$= X' \cup (Y')'$

$= X' \cup Y$

35. (C) We know that area of ΔABC whose sides are a, b and c are

$$\Delta = \frac{c^2 \sin A . \sin B}{2 \sin C}$$

$$= \frac{a^2 \sin B . \sin C}{2 \sin A}$$

$$= \frac{b^2 \sin C . \sin A}{2 \sin B}$$

where A + B + C = 180°

So, finding the area of ΔABC, angles A, B and side C are required.

36. (C) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

If AB = BA

$$\Rightarrow \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix} \Rightarrow a = b$$

From the above it is clear that there exist infinitely B such that AB = BA.

37. (D) $M = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{bmatrix}$

$$\text{Now } |M| = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{bmatrix} = k(3-8) = -5k$$

If $k \neq 0$, then inverse of M exists.

Thus, statement A implies B as well as B implies A.

38. (A) $\because 2^x + 3^y = 17$

and $2^{x+2} - 3^{y+1} = 5$

$2^x . 4 - 3 . 3^y = 5$

From Eqs. (i) and (ii),

$2^x = 8$ and $3^y = 9$

$x = 3$ and $y = 2$

39. (D) $\because$ P(32, 6) = k C(32, 6)

$$\Rightarrow \frac{32!}{26!} = k \times \frac{32!}{6!.26!}$$

$\Rightarrow$ k = 6! = 720

40. (D) $\because \dfrac{\sqrt{3}+i}{1+\sqrt{3}i} = \dfrac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$

$$= \frac{\sqrt{3}-3i+i+\sqrt{3}}{1+3}$$

$$= \frac{2\sqrt{3}-2i}{4} = \frac{\sqrt{3}-i}{2}$$

41. (A) $\because (0.1101)_2 = 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-4}$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = \frac{13}{16}$$

$= (0.8125)10$

Hence, $(0.8125)_{10} = (0.1101)_2$

Alternative Method

0.8125 × 2	= 1.625	1
0.625 × 2	= 1.25	1
0.25 × 2	= 0.50	0
0.5 × 2	= 1.0	1

$\Rightarrow (0.8125)_{10} = (0.1101)_2$

42. (B) $\begin{vmatrix} x & y & y+z \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

$(\because R_1 \to R_1 + R_2 + R_3)$

$$\Rightarrow (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

$$\Rightarrow (x+y+z)\begin{vmatrix}1 & 0 & 0\\ z & -z+y & x+y-2z\\ x & z-x & z-x\end{vmatrix}=0$$

($\because C_2 \to C_2 - C_1$, $C_3 \to C_3 - 2C_1$)

Expand with respect to R_1

$$\Rightarrow (x+y+z)\begin{vmatrix}-z+y & x+y-2z\\ z-x & z-x\end{vmatrix}=0$$

$\Rightarrow (x+y+z)(-z+y-x-y+2z) = 0$

$\Rightarrow$ $x + y = -z$ or $z = x$

43. (A) Let $\Delta = \begin{vmatrix}k & b+c & b^2+c^2\\ k & c+a & c^2+a^2\\ k & a+b & a^2+b^2\end{vmatrix}$

Transpose of whole determinant

$$= k\begin{vmatrix}1 & 1 & 1\\ b+c & c+a & a+b\\ b^2+c^2 & a+b & a^2+b^2\end{vmatrix}$$

$$= k\begin{vmatrix}1 & 0 & 0\\ b+c & a-b & a-c\\ b^2+c^2 & a^2-b^2 & a^2-c^2\end{vmatrix}$$

($\because C_2 \to C_2 - C_1 : C_3 \to C_3 - C_1$)

$$= k\begin{vmatrix}1 & 0 & 0\\ b+c & a-b & a-c\\ b^2+c^2 & (a-b)(a+b) & (a-c)(a+c)\end{vmatrix}$$

Expand with respect R_1

$$= k(a-b)(b-c)\begin{vmatrix}1 & 1\\ a+b & a+c\end{vmatrix}$$

$= k(a-b)(a-c)(a+c-a-b)$

$= k(a-b)(b-c)(c-a)$

But $\Delta = (a-b)(b-c)(c-a)$

On comparing

Thus, $k = 1$

44. (C) Total number of proper subsets of a finite set with n elements $= 2^n - 1$.

(by propery)

45. (A) Since, $(x + a)$ is factor of

$x^2 + px + q$ and $x^2 + lx + m$

$\therefore$ $a^2 - ap + q = 0$

$a^2 - la + m = 0$

$\Rightarrow (l-p)a = m - q$

$\Rightarrow a = \dfrac{m-q}{l-p}$ $(l \neq p)$

46. (C) We know that,

$[(A \cup B) \cap C]' = (A \cup B)' \cup C'$

$= (A' \cap B') \cup C'$

$= A' \cap B' \cup C'$

(by De Morgan's Law)

47. (A) $\tan(-1575°) = -\tan(4 \times 360° + 135°)$

$= -\tan 135°$

$= -\tan(90° + 45°)$

$= \cot 45° = 1$

48. (C) $\because$ $\text{cosec}^2\theta = 3\sqrt{3}\cot\theta - 5$

$\Rightarrow 1 + \cot^2\theta - 3\sqrt{3}\cot\theta + 5 = 0$

($\because \text{cosec}^2\theta = 1 + \cot^2\theta$)

$\Rightarrow \cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$

$$\cot\theta = \frac{3\sqrt{3} \pm \sqrt{27-24}}{2}$$

$$= \frac{3\sqrt{3} \pm \sqrt{3}}{2} = 2\sqrt{3}, \sqrt{3}$$

$\Rightarrow \cot\theta \neq 2\sqrt{3}$, $\cot\theta = \sqrt{3} = \cot\dfrac{\pi}{6}$

$\Rightarrow \theta = \dfrac{\pi}{6}$

49. (D) $\sqrt{1-x^2} + \sqrt{1-y^2} = a$

On differentiating w.r.t. x, we get

$$\frac{1}{2\sqrt{1-x^2}}(-2x) + \frac{1(-2y)}{2\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}\sqrt{\frac{1-y^2}{1-x^2}}$$

50. (D) Given, $x = \log t$ and $y = t^2 - 1$

$\Rightarrow 2x = \log t^2$

$\Rightarrow 2x = \log(y+1) \Rightarrow e^{2x} = y + 1$

On differentiating w.r.t. x, twicely, we get

$$e^{2x}2 = \frac{dy}{dx}$$

$$\Rightarrow 4e^{2x} = \frac{d^2y}{dx^2}$$

At $t = 1$, $x = 0$

$$\frac{d^2y}{dx^2} = 4e^{2(0)} = 4$$

51. (D) An injective function means one-one.

In option (D), $f(x) = -x$

For every values of x, we get a different value of f.

Hence, it is injective.

52. (*) Let $u = \log_x 5$ and $v = \log_5 x$

Then, $\frac{du}{dx} = \frac{-\log 5}{(\log x)^2} \cdot \frac{1}{x}$

and $\frac{dv}{dx} = \frac{1}{x \log 5}$

$$\frac{dv}{dx} = \frac{du/dx}{dv/dx} = \frac{\frac{-\log 5}{(\log x)^2} \times \frac{1}{x}}{\left(\frac{1}{\log 5}\right) \times \frac{1}{x}}$$

$$= -\left(\frac{\log 5}{\log x}\right)^2$$

$= -(\log_x 5)^2$

53. (B) Given, $v = s + 1$

$\Rightarrow \frac{ds}{dt} = s + 1 \quad \left(\because v = \frac{ds}{dt}\right)$

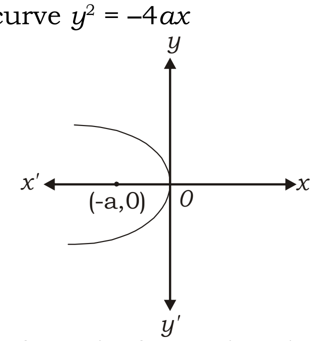
$\Rightarrow \int \frac{dS}{s+1} = \int dt$

$\Rightarrow \log(s + 1) = t$

At $s = 9$ m, $t = \log(10)$S

$\Rightarrow t = (\log 10)$S

54. (C) Given curve $y^2 = -4ax$

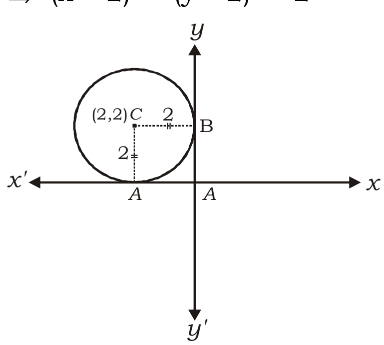


It is curve from the figure that the curve lies in the second and third quadrants.

55. (C) Given equation is

$x^2 + 4x + 3 + y^2 - 4y = 0$

$\Rightarrow (x + 2)^2 + (y - 2)^2 = 2^2$



Here, we see that the circle touches both the axis.

56. (B) $\therefore \cos 60^0 = \left|\frac{1 \times 1 + 1 \times (-1) + 1 \times n}{\sqrt{1^2 + 1^2 + 1^2} \times \sqrt{1^2 + 1^2 + n^2}}\right|$

$$\left(\because \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|\right)$$

$\Rightarrow \frac{1}{2} = \frac{n}{\sqrt{3}\sqrt{2 + n^2}} \Rightarrow 3(2 + n^2) = 4n^2$

$\Rightarrow n^2 = 6 \Rightarrow n = \pm\sqrt{6} \Rightarrow n = \sqrt{6}$

57. (A) Given, $ax \cos\phi + by \sin\phi - ab = 0$

At point $\left(\sqrt{b^2 - a^2}, 0\right)$

$$d_1 = \left|\frac{a\sqrt{b^2 - a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}\right|$$

At point $(-\sqrt{b^2 - a^2}.0)$

$$d_2 = \left|\frac{-a\sqrt{b^2 - a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}\right|$$

$$\therefore d_1d_2 = \left|\frac{[a^2(b^2 - a^2)\cos^2\phi - a^2b^2]}{a^2\cos^2\phi + b^2\sin^2\phi}\right|$$

$$= \left|-\frac{a^2(-b^2\sin^2\phi - a^2\cos^2\phi)}{a^2\cos^2\phi + b^2\sin^2\phi}\right|$$

$= a^2$

58. (C) Mid-point of (p,q) and $(q,-p)$ is

$\left(\frac{p+q}{2}, \frac{q-p}{2}\right)$ which is given $\left(\frac{r}{2}, \frac{s}{2}\right)$

$\therefore \frac{p+q}{2} = \frac{r}{2}$ and $\frac{q-p}{2} = \frac{s}{2}$

Now, length of segment $= \sqrt{(p-q)^2 + (q+p)^2}$

$= \sqrt{s^2 + r^2}$

59. (B) Equation of plane passing through (1, –2, 4) and the direction consines of whose normal (2, 1, 2) is

$2(x - 1) + 1(y + 2) + 2(z - 4) = 0$

$2x + y + 2z - 8 = 0$

Required distance

$$= \left|\frac{2(3) + 1(2) + 2(3) - 8}{\sqrt{4 + 1 + 4}}\right|$$

$$\left(\therefore \text{distance} = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|\right)$$

$= \frac{6}{3} = 2$

60. (D) Equation of plane passing through (1,–3,1) and the direction cosines of whose normal (1,–3,1) is

$1(x - 1) - 3(y + 3) + 1(z - 1) = 0$

$\Rightarrow x - 3y + z - 11 = 0$

$\Rightarrow \frac{x}{11} - \frac{y}{11/3} + \frac{z}{11} = 0$ (intercept from)

The above plane intercept the *x*-axis at a distance of 11.

61. (A) $\because l^2 + m^2 + n^2 = 1$

i.e., $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 1$

$\Rightarrow \cos^2\alpha + \cos^2\alpha + \cos^2\theta + 1$

$[\because (\alpha = \beta), (\gamma = \theta)]$ .... (i)

Also, $\sin^2\theta = 2\sin^2\alpha$ (given)

$\Rightarrow 1 - \cos^2\theta = 2(1 - \cos^2\alpha)$

$\Rightarrow \cos^2\theta = 2\cos^2\alpha - 1$

$\therefore$ From Eq. (i),

$2\cos^2\alpha + (2\cos^2\alpha - 1) = 1$

$\Rightarrow 4\cos^2\alpha = 2 \Rightarrow \cos^2\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$

62. (C) let the point (x_1, y_1) be equidistant from the given points.

$\therefore \sqrt{[x_1 - (m+n)]^2 + [y_1 - (n-m)]^2}$

$= \sqrt{[x_1 - (m-n)]^2 + [y_1 - (n+m)]^2}$

$\Rightarrow x_1^2 + (m+n)^2 - 2x_1(m+n) + y_1^2 + (n-m)^2 - 2y_1(n-m)$

$= x_1^2 + (m-n)^2 - 2x_1(m-n) + y_1^2 + (n+m)^2 - 2y_1(n+m)$

$\Rightarrow 2x_1(m - n - m - n) + 2y_1(n + m - n + m) = 0$

$\Rightarrow -4x_1n + 4y_1m = 0 \Rightarrow my_1 = nx_1$

Hence, locus of the point is

$nx = my$

63. (C) Given the centre of sphere to be (6, –1, 2)

$\therefore$ Radius = Perpendicular distance to the plane from the centre

$\therefore$ Radius $= \left[\frac{2(6) - 1(-1) + 2(2) - 2}{\sqrt{4+1+4}}\right] = \frac{15}{3} = 5$

$\therefore$ Equation of sphere is

$(x-6)^2 + (y+1)^2 + (z-2) = 5^2$

$\Rightarrow x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

64. (A) The intersection of the given plane is

$x - y + 2z - 1 + \lambda(x + y - z - 3) = 0$

$\Rightarrow x(1+\lambda) + y(\lambda-1) + z(2-\lambda) - 3\lambda - 1 = 0$

Direction ratios of normal to the above plane is $(1+\lambda, \lambda-1, 2-\lambda)$

Since, the line formed intersected by planes and the normal of the plane are perpendicular, then

by taking option (a)

$-1(1+\lambda) + 3(\lambda - 1) + 3(2 - \lambda) = 0$

$\Rightarrow -1 - \lambda + 3\lambda - 3 + 4 - 2\lambda = 0$

$0 = 0$

65. (B) Given ellipse is $\frac{x^2}{169} + \frac{y^2}{25} = 1$

$e = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$ $\left(\because e = \sqrt{1 - \frac{b^2}{a^2}}\right)$

Also, ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$e = \sqrt{1 - \frac{b^2}{a^2}}$ ($\because$ According to the question)

$\frac{12}{13} = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{144}{169} = \frac{25}{169} \Rightarrow \frac{a}{b} = \frac{13}{5}$

66. (C) The projection of b on a $= \frac{a.b}{|\hat{a}|} = \hat{a}.b$

$[\because |\hat{a}| = 1]$

67. (A) By taking option (a).

Condition of perpendicularity a.b = 0

$\pm\frac{(3i + 4j)}{5}.(4i - 3 + k) = \frac{1}{5}(12 - 12) = 0$

68. (D) Let $r_1 = bi - aj$

Condition of perpendicularity a.b = 0

Now, $r_1.r = (bi - aj).(ai + bj)$

$= ab - ab = 0$

69. (D) Given, a = 2i – 3j + 4k

Also, b = ma

= m(2i – 3j + 4k)

As b is a unit vector.

Now, $|2i - 2j + 4k| = \sqrt{4 + 9 + 16} = \sqrt{29}$

Therefore, m should be $\frac{1}{\sqrt{29}}$ $(\because |b| = 1)$

70. (A) Since, $(\lambda a + b).(a - \lambda b) = 0$ (given)

$\Rightarrow \lambda|a|^2 + (1 - \lambda^2)a.b - \lambda|b|^2 = 0$

$\Rightarrow (1 - \alpha^2)|a||b|\cos 60^0 = 0$ $(\because |a| = |b|)$

$\Rightarrow \lambda = \pm 1$ or $\lambda = 1$ (given $\theta = 60^0$)

71. (C) Since, $|a + b|^2 + |a - b|^2 = 2(|a|^2 + |b|^2)$ (by paralleogram law)

$\Rightarrow |a + b|^2 + 7^2 = 2(3^2 + 4^2)$

$\Rightarrow |a + b|^2 = 1$

$\Rightarrow |a + b| = 1$

72. (B) Let $d_1 = 3i + 6j - 2k$

and $d_2 = 4i - j + 3k$

Now, $d_1.d_2 = 3(4) + 6(-1) - 2(3) = 0$

Hence, $|d_1| = \sqrt{4^2 + 1^2 + 3^2} = \sqrt{26}$

$\Rightarrow |d_1| \neq |d_2|$

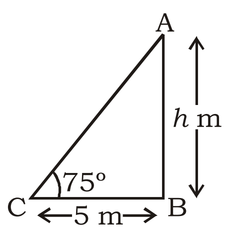
Hence, given quadrilateral is a rhombus.

73. (C) $\frac{\sin x}{1 + \cos} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1 + 2\cos^2\frac{x}{2} - 1}$

$= \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$

$= \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$

74. (C) Let 'h' be the height of the flag post.



In ΔABC,

$\tan 75° = \frac{AB}{BC} = \frac{h}{5}$

$\Rightarrow \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} = \frac{h}{5}$

$\Rightarrow \frac{1 + \sqrt{3}}{\sqrt{3} - 1} = \frac{h}{5}$

$\Rightarrow \frac{h}{5} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3})^2 - (1)^2}$

$\Rightarrow \frac{h}{5} = \left(\frac{3 + 1 + 2\sqrt{3}}{3 - 1}\right)$

$= 5(2 + \sqrt{3})$

$(\because \sqrt{3} = 1.732)$

$= 5 \times 3.732$

$= 19$ m (Approx.)

75. (D) $A = P(\{1, 2\}) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

From above, it is clear that

$\{1, 2\} \in A$

76. (C) When we take 12 wrongly in place of 8, then geometric mean = 6

$\Rightarrow (x_1.x_2.12)^{1/3} = 6$

$\Rightarrow x_1.x_2.12 = 216$

$\Rightarrow x_1.x_2 = 18$ ... (i)

Now, we take the right observation 8 in place of 12, then the geometric mean

$= (x_1.x_2.8)^{1/3}$

$= (18.8)^{1/3}$

$= 2\sqrt[3]{18}$

77. (D) $\because A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, adj (A)$= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}^T$

$= \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

and $|A| = 4 - 6 = -2$

$\therefore A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\left(\because A^{-1} = \frac{adj(A)}{|A|}\right)$

$\Rightarrow [b_{ij}] = \frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow b_{22} = -\frac{1}{2}$

78. (B) $\because 4\sin^2 x + 4\cos x - 1 = 0$

$\Rightarrow 4 - 4\cos^2 x + 4\cos x - 1 = 0$

$\Rightarrow -4\cos^2 x + 4\cos x + 3 = 0$

$\Rightarrow 4\cos^2 x - 4\cos x - 3 = 0$

$\Rightarrow 4\cos^2 x - 6\cos x + 2\cos x - 3 = 0$

$\Rightarrow (2\cos x - 3)(2\cos x + 1) = 0$

$\Rightarrow \cos x = \frac{3}{2}$ (which is not possible)

$\cos x = -\frac{1}{2}$

$\Rightarrow \cos A = -\frac{1}{2} = \cos 240°$

[$\because$ A lies in IIIrd quadrant]

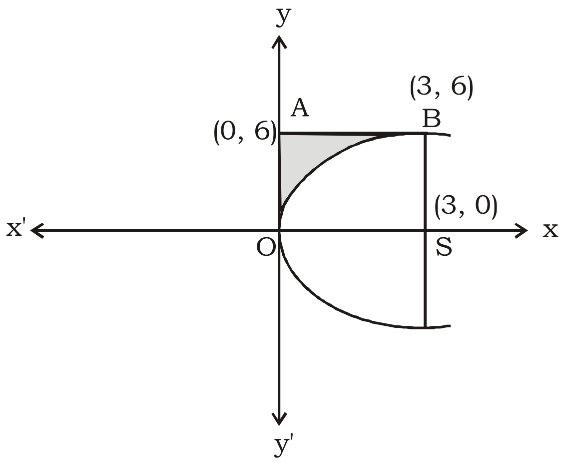
$\Rightarrow A = 240°$

79. (C) Equation of curve is

$y^2 = 12x$

At $y = 6$, $36 = 12x$

$\Rightarrow x = 3$



$\therefore$ Required area

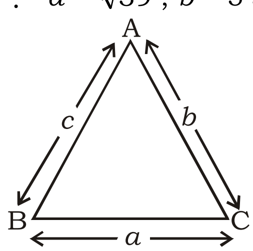
$= \int_0^3 (6 - \sqrt{12x})\,dx$

$= [6x]_0^3 - \sqrt{12}\left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^3$

$= [6 \times 3] - \frac{\sqrt{12} \times 2 \times \sqrt{27}}{3}$

$= 18 - 12 = 6$ sq unit

80. (B) $\because$ $a = \sqrt{39}$, $b = 5$ and $c = 7$



By cosine rule,

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$= \frac{25 + 49 - 39}{2 \times 5 \times 7}$

$= \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow A = \frac{\pi}{3}$

81. (A) $\because \frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(1-1-2i)}$

$= \frac{1+2i}{1+2i} = 1$

$\therefore \left|\frac{1+2i}{1-(1-i)^2}\right| = 1$

82. (D) Direction ratio of line AB

$= 2k - k,\ 0 - 1,\ 2 + 1$

$= (< k, -1, 3>)$

and direction ratio of line BC

$= 2 + 2k - 2k,\ k - 0,\ 1 - 2$

$= (<2, k, -1>)$

$\because (2)(k) + (-1)(k) + (3)(-1) = 0$

$\Rightarrow 2k - k - 3 = 0 \Rightarrow k = 3$

83. (C) $\int \frac{1}{1+e^x}\,dx = \int \frac{e^{-x}}{e^{-x}+1}\,dx$

$\left(\because \int \frac{f'(x)}{f(x)}\,dx = \log f(x) + C\right)$

$= -\log(1 + e^{-x}) + C$

$= -\log\left(\frac{1+e^x}{e^x}\right) + C$

$= -\{\log(1 + e^x) - \log e^x\} + C$

$= x - \log(1 + e^x) + C$

84. (C) The function $f(x) = x \operatorname{cosec} x$

LHL $= f(0^-) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} - h \operatorname{cosec}(-h)$

$= \lim_{h\to 0} \frac{h}{\sin h} = 1$

RHL $= f(0^+) = f(0+h)$

$= \lim_{h\to 0} h \operatorname{cosec} h$

$= \lim_{h\to 0} \frac{h}{\sin h} = 1$

and $f(0)$ = not defined.

So, the function $f(x)$ is continuous for all values of x. Except at $x = n\pi$ where n is an integer.

85. (D) $\because ax\frac{dy}{dx} + 2ay = xy\frac{dy}{dx}$

$\Rightarrow ax\frac{dy}{dx} - xy\frac{dy}{dx} = -2ay$

$\Rightarrow (xy - ax)\frac{dy}{dx} = 2ay$

$\Rightarrow \frac{dy}{dx} = \frac{2ay}{xy - ax}$

$\Rightarrow \frac{(y-a)}{y} dy = \frac{2a}{x} dx$

$\Rightarrow \int\left(1 - \frac{a}{y}\right) dy = \int\left(\frac{2a}{x} dx\right)$

$\Rightarrow y - a \log y = 2a \log x + a \log C$

$\Rightarrow y = a \log x^2 y C$

$\Rightarrow x^2 y = ke^{y/a}$

where $k = \frac{1}{C}$

86. (A) Let vector $b = xi + yi + zk$
and $a = 2i + j - k$
Given that $a.b = 3$ ...... (i)
$\Rightarrow (xi + yj + zk).(2i + j - k) = 3$
$\Rightarrow 2x + y - z = 3$
$\because$ Vectors a and b are collinear, *i.e.*, Angle between both the vectors should be 0°.
Then, $a.b = |a||b| \cos 0$

$\Rightarrow a.b = \sqrt{4+1+1}\sqrt{x^2+y^2+z^2} \times 1$

$\Rightarrow a.b = \sqrt{6}\sqrt{x^2+y^2+z^2}$ ... (ii)

From Eqs. (i) and (ii),

$\Rightarrow 3 = \sqrt{6}\sqrt{x^2+y^2+z^2}$

$\Rightarrow \frac{3}{2} = x^2+y^2+z^2$ ...(iii)

Hence, $b = \left(1, \frac{1}{2}, -\frac{1}{2}\right)$ will satisfy Eq. (iii)

87. (D) $\frac{1+i}{1-i} = \frac{(1+i)^2}{1+1} = \frac{1+i^2+2i}{2} = i$

$\therefore \left(\frac{1+i}{1-i}\right)^n = i^n = 1$

Which is possible for $n = 4$

88. (C) If $a = xi + yj + zk$
and $b = k$, c from a right handed system.

$\therefore c = (a \times b) = \begin{vmatrix} i & j & k \\ x & y & z \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow c = i(y-0) - j(x-0) + k(0-0)$

$\Rightarrow c = yi - xj$

89. (C) $\because x = t^2, y = t^3$

$\Rightarrow \frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2}{2t} = \frac{3}{2}t$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2} . \frac{dt}{dx} = \frac{3}{2} . \frac{1}{2t}$

$= \frac{3}{4t}$

90. (D) $\because \tan^3 x$ is an odd function.

$\therefore \int_{-\pi/4}^{\pi/4} \tan^3 x\, dx = 0$

$\because \left\{\int_{-a}^{a} f(x)\, dx = \right.$

$2\int_0^a f(x)dx$, if $f(-x), f(x)$, *i.e.*, even function
0 if $f(-x) = -f(x)$, *i.e.*, odd function

$\Rightarrow f(x) = \tan^3(-x)$
$\Rightarrow = -\tan^2 x = -f(x)$

91. (C) We know that in a parallelogram, diagonals bisect each other. Mid-point of OQ = Mid-point of PR.

$\therefore \left(\frac{0+m}{2}, \frac{0+n}{2}, \frac{0+r}{2}\right) \equiv \left(\frac{1+3}{2}, \frac{1+4}{2}, \frac{1+5}{2}\right)$

$\Rightarrow m = 4, n = 5, r = 6$
Hence, $m + n + r = 4 + 5 + 6 = 15$

92. (B) $3e^x \tan y\, dx + (1 + e^x) \sec^2 y\, dy = 0$

$\Rightarrow \int \frac{3e^x}{1+e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$

$\Rightarrow 3 \log(1 + e^x) + \log \tan y = \log C$
$\Rightarrow \log(1 + e^x)^3 \tan y = \log C$
$\Rightarrow (1 + e^x)^3 \tan y = C$

93. (C) We know that the locus of the points, the difference of whose distances from two points being constant, is a hyperbola.
[*by definition of hyperbola*]

94. (C) $\because y^2 = 4a(x - a)$ ... (i)
On differentiating w.r.t. x, we get
$2yy' = 4a$

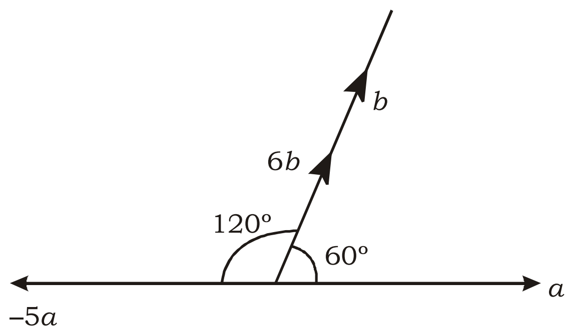
$\Rightarrow a = \frac{yy'}{2}$

On putting the value of a in Eq. (i), we get

$y^2 = 4\left(\frac{yy'}{2}\right)\left(x - \frac{yy'}{2}\right)$

$\Rightarrow yy'(yy' - 2x) + y^2 = 0$

95. (B) From the figure, it is clear that the angle between 6b and –5a is 120° or $\frac{2\pi}{3}$.



96. (B) The given equation can be rewritten as

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

From above it is clear that the degree of equation is 2.

97. (B) $\because \sin A = \frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

If A is not known but sin A is known, then 2 values of $\tan\frac{A}{2}$ can be calculated, because above equation is a quadratic equation in $\tan\frac{A}{2}$.

98. (C) $\text{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) = -\frac{\pi}{3}$

$\because$ The range of $\text{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(\frac{\pi}{2}, \pi\right]$

and $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\because$ The range of $\sec^{-1} x$ is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

$\therefore$ Both statements I and II are correct.

99. (C) Both the statements I and II are correct, by property of correlation coefficient.

100. (B) If the values of a set are measured in cm, then the unit of variance is cm^2.

101. (D) Required probability

$$= \frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{{}^6C_1 \times {}^6C_1 \times {}^6C_1}$$

$$= \frac{6 \bullet 5 \bullet 4}{6 \bullet 6 \bullet 6}$$

$$= \frac{5}{9}$$

102. (C) The equation $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle, if $a = b$ and $h = 0$.
Then, the equation becomes the general equation of a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

103. (A) $\because P(A \cup B) = 0.5,\ P(\overline{B}) = 0.8,\ P\left(\frac{A}{B}\right) = 0.4$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(B) \times P\left(\frac{A}{B}\right) = P(A \cap B)$

$[\because P(B) = 1 - P(\overline{B})]$

$\Rightarrow P(A \cap B) = 0.4 \times (1 - 0.8)$
$= 0.4 \times 0.2$
$= 0.08$

104. (A) $\because \cos^2\alpha + \cos^2\beta + \cos^2\lambda = 1$

$\Rightarrow 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\lambda = 2$

$\Rightarrow 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\lambda - 1 = 2 - 3$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\lambda = -1$

and now,

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\lambda = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\lambda = 2$

Hence, only statement I is correct.

105. (B) Since, point (3, 7, 1) satisfies the equation of plane $2x + 3y - 6z = 21$
Hence, (3, 7, 1) lies on the plane.

106. (D) The equation of planes are $x + y + 2z = 3$ and $-2x + y - z = 11$.
We know that, the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by –

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

Here, $a_1 = 1, b_1 = 1, c_1 = 2, a_2 = -2, b_2 = 1, c_2 = -1$

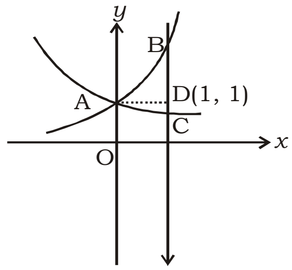
$$\Rightarrow \cos\theta = \left|\frac{1 \times (-2) + 1 \times 1 + 2 \times (-1)}{\sqrt{1+1+4}\sqrt{4+1+4}}\right|$$

$$= \left|\frac{-2+1-2}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

107. (C) The equation of curves are $y = e^x$ and $y = e^{-x}$.



$\therefore \quad e^x = \frac{1}{e^x}$

$\Rightarrow \quad e^{2x} = e^0 \Rightarrow x = 0$

$\therefore$ Required area $= \int_0^1 (e^x - e^{-x})dx$

$= [e^x + e^{-x}]_0^1$

$= e + e^{-1} - e^0 - e^0$

$= \left(e + \frac{1}{e} - 2\right)$ sq unit

108. (D) The equation of curve is

$$4x^2 - 9y^2 = 1$$

$\Rightarrow \quad \frac{x^2}{\frac{1}{4}} - \frac{y^2}{\frac{1}{9}} = 1$

This is an equation of a hyperbola and the equation of conjugate axis is y-axis *i.e.*, $x = 0$.

Put $x = 0$ in Eqn. (i),

$y^2 = -\frac{1}{9}$

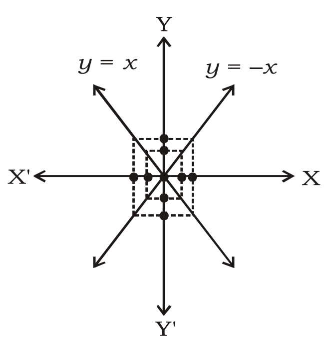
or

$y = \frac{1}{3} i$, an imaginary points

Hence, no point of intersection exists.

109. (A) $\because$ The lines $y = x$ and $y = -x$ lie at the same distances in coordinate axes.

$\therefore \quad y = \pm x \Rightarrow x \pm y = 0$



So, $x \pm y = 0$ is the locus of a point which moves equidistant from the coordinates axes.

110. (B) $\int e^x \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$

Let $f(x) = \sqrt{x}$

$= f'(x) = \frac{1}{2\sqrt{x}}$

$[\because \int e^x.[f(x) + f'(x)]dx = e^x f(x) + C]$

$= e^x.\sqrt{x} + C$

111. (C) p = Magnitude of $3i - 2j = \sqrt{9+4} = \sqrt{13}$

q = Magnitude of $2i + 2j = \sqrt{4+4+1} = 3$

r = Magnitude of $4i - j + k = \sqrt{16+1+1}$

$= \sqrt{18} = 3\sqrt{2}$

and S = Magnitude of $2i + 2j + 3k$

$= \sqrt{4+4+9} = \sqrt{17}$

$\therefore \quad r > s > p > q.$

112. (B) $\because c = 2$, A = 120° and $a = \sqrt{6}$.

$\therefore \quad \frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{\sqrt{6}}{\sin 120^\circ} = \frac{2}{\sin C}$

$\Rightarrow \sin C = \frac{2 \times \sqrt{3}}{\sqrt{6} \times 2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin C = \sin 45^\circ = \angle C = 45^\circ$

113. (D)

Class Interval	*f*	*cf*
0.5 – 5.5	3	3
5.5 – 10.5	7	10
10.5 – 15.5	6	16
15.5 – 20.5	5	21
	21	50

N = 21

$\frac{N}{2} = \frac{21}{2} = 10.5$

$\because$ Median class is 10.5 – 15.5.

$\therefore$ Median $= 10.5 + \frac{10.5 - 10}{6} \times 5$

$= 10.5 + 0.417 = 10.917$

Thus, median is not combined in the modal class and the distribution is not bell-shaped because in this distribution

Mean $\neq$ Median $\neq$ Mode

for the next two (2) items that follow:-

Class Interval	f	cf	x	fx
0-10	5	5	5	25
10-20	10	15	15	150
20-30	20	35	25	500
30-40	5	40	35	175
40-50	10	50	45	450
	50	145	125	1300

$\therefore \frac{N}{2} = \frac{50}{2} = 25$

114. (C) Median group is 20-30

$\Rightarrow$ Median $= 20 + \frac{25-15}{20} \times 10$

$= 20 + 5 = 25$

115. (B) Mean $= \frac{\sum fx}{\sum f} = \frac{1300}{50} = 26$

116. (A) Since, the given matrix is

$$A = \begin{bmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{bmatrix} = 0$$

This matrix is singular.

$\therefore \ |A| = 0$

$$\Rightarrow \begin{vmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{vmatrix} = 0$$

apply $R_2 + R_3 \rightarrow R_2$

$$\Rightarrow \begin{vmatrix} 2-x & 1 & 1 \\ 0 & -x & -x \\ -1 & -3 & -x \end{vmatrix}$$

$\Rightarrow (2-x)(x^2-3x) + 1(x) + 1(-x) = 0$

$\Rightarrow (2-x)(x)(x-3) = 0$

$\Rightarrow x = 2, 0, 3$

Hence, solution set S = {0, 2, 3}.

117. (C) $\because f(x) = \cos 2x - \sin 2x$

$[\because f(x) = a\cos x + b\sin x$

$-\sqrt{a^2+b^2} \le f(x) \le \sqrt{a^2+b^2}\ ]$

$-\sqrt{1+1} \le \cos 2x - \sin 2x \le \sqrt{1+1}$

$-\sqrt{2} \le \cos 2x - \sin 2x \le \sqrt{2}$

So, Range of $f(x)$ is $[-\sqrt{2}, \sqrt{2}]$.

118. (A) The given differential equation is

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\Rightarrow \int \frac{1}{\sqrt{1-y^2}} dy + \int \frac{1}{\sqrt{1-x^2}} dx = 0$$

$\Rightarrow \sin^{-1}y + \sin^{-1}x = C$

119. (C) $\because z = \left(1+\cos\frac{\pi}{5}\right) + i\sin\frac{\pi}{5}$

$= 2\cos^2\frac{\pi}{10} + i\,2\sin\frac{\pi}{10}\cos\frac{\pi}{10}$

$= 2\cos\frac{\pi}{10}\left[\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right]$

$= 2\cos\frac{\pi}{10}.e^{i\pi/10}$

$\left\{\begin{matrix}\because e^{i\theta} = \cos\theta + i\sin\theta \\ |e^{i\theta}| = 1\end{matrix}\right\}$

$\Rightarrow |z| = \left|2\cos\frac{\pi}{10}.e^{i\pi/10}\right|$

$= 2\cos\frac{\pi}{10}$

120. (C) $\because$ $xi + yj + zk$ is a unit vector.

and $x^2 + y^2 + z^2 = 1$ (given)

$\Rightarrow x : y : z = \sqrt{3} : 2 : 3$

$\therefore (\sqrt{3}k)^2 + (2k)^2 + (3k)^2 = 1$

$\Rightarrow 3k^2 + 4k^2 + 9k^2 = 1$

$\Rightarrow k^2 = \frac{1}{16}$

$\Rightarrow k = \frac{1}{4}$

Hence, $z = 3k = 3 \times \frac{1}{4} = \frac{3}{4}$

NDA MATHS MOCK TEST - 51 (ANSWER KEY)

1. (C)	31. (A)	61. (A)	91. (C)
2. (C)	32. (C)	62. (C)	92. (B)
3. (B)	33. (B)	63. (C)	93. (C)
4. (B)	34. (C)	64. (A)	94. (C)
5. (C)	35. (C)	65. (B)	95. (B)
6. (D)	36. (C)	66. (C)	96. (B)
7. (D)	37. (D)	67. (A)	97. (B)
8. (B)	38. (A)	68. (D)	98. (C)
9. (D)	39. (D)	69. (D)	99. (C)
10. (A)	40. (D)	70. (A)	100. (B)
11. (B)	41. (A)	71. (C)	101. (D)
12. (A)	42. (B)	72. (B)	102. (C)
13. (D	43. (A)	73. (C)	103. (A)
14. (B)	44. (C)	74. (C)	104. (A)
15. (B)	45. (A)	75. (D)	105. (B)
16. (B)	46. (C)	76. (C)	106. (D)
17. (B)	47. (A)	77. (D)	107. (C)
18. (C)	48. (C)	78. (B)	108. (D)
19. (A)	49. (D)	79. (C)	109. (A)
20. (A)	50. (D)	80. (B)	110. (B)
21. (D)	51. (D)	81. (A)	111. (C)
22. (C)	52. (*)	82. (D)	112. (B)
23. (C)	53. (B)	83. (C)	113. (D)
24. (B)	54. (C)	84. (C)	114. (C)
25. (D)	55. (C)	85. (D)	115. (B)
26. (C)	56. (B)	86. (A)	116. (A)
27. (D)	57. (A)	87. (D)	117. (C)
28. (C)	58. (C)	88. (C)	118. (A)
29. (C)	59. (B)	89. (C)	119. (C)
30. (B)	60. (D)	90. (D)	120. (C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*