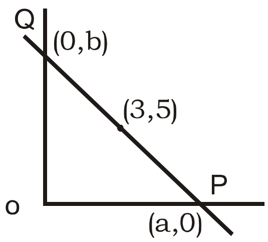
NDA MATHS MOCK TEST - 55 (SOLUTION)

1. (D) Let (0,b) & (a,0) are two points on y-axis and x-axis respectively.

$\frac{a+0}{2} = 3, \quad \frac{b+0}{2} = 5$



a=6, b=10

$(\Delta OPQ) = \frac{1}{2} \times OP \times OQ$

$= \frac{1}{2} \times 6 \times 10 = 30 =$ sq.unit.

2. (A) The given line is x-y-$\sqrt{2}$
Centre of the circle (0,b)

$\therefore$ b, radius of the circle $\left|\frac{0\times 1 - b\times 1 - \sqrt{2}}{\sqrt{1^2+(-1)^2}}\right|$

b = $\left|\frac{-b-\sqrt{2}}{\sqrt{2}}\right|$

b = $\frac{b+\sqrt{2}}{\sqrt{2}} \Rightarrow \sqrt{2}\,b = b+\sqrt{2} \Rightarrow (\sqrt{2}-1)b = \sqrt{2}$

b = $\frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} = \frac{2+\sqrt{2}}{2-1} = 2+\sqrt{2}$

3. (A)
4. (D)
5. (C)
6. (B)
7. (B)
8. (A)

9. (A) $y = 3x$ ______(1)
$y = 6x$ ______(2)
$y = 9$ ______(3)

$(3,9), \left(\frac{3}{2}, 9\right), (0,0)$

$ar(\Delta OAB) = \frac{1}{2} \times 9 \times \left(3 - \frac{3}{2}\right)$

$= \frac{1}{2} \times 9 \times \frac{3}{2} = \frac{27}{4}$ sq. unit.

10. (B) Centroid= $\left[\frac{3+\frac{3}{2}+0}{3}, \frac{9+9+0}{3}\right]$

$= \left[\frac{9}{6}, \frac{18}{3}\right]$

$= \left[\frac{3}{2}, 6\right]$

11. (C) $f'(x) = 2(x-1)(x+1)(x-2)^3 + (x-1)^2 (x-2)^3 + (x-1)^2(x+1)3(x-2)^2$

$f'(x) = (x-1)(x-2)^2 + \left[\begin{matrix} 2(x+1)(x-2) + (x-1) \\ (x-2) + 3(x-1)(x+1) \end{matrix}\right]$

$= (x-1)(x-2)^2 \left[\begin{matrix} 2(x^2-2x+x-2) \\ +x^2-2x-x+2+3x^2-3 \end{matrix}\right]$

$= (x-1)(x-2)^2 \left[\begin{matrix} 2x^2-2x-4+x^2 \\ -3x+2+3x^2-3 \end{matrix}\right]$

$= (x-1)(x-2)^2 \left[6x^2-5x-5\right]$

x = 1, 2, $\frac{5 \pm \sqrt{25+120}}{12}$

x = 1, 2, $\frac{5 \pm 12}{12}$ =1, 2, $\frac{17}{12}, \frac{-7}{12}$

$f'(x) = (x-2)^2 (6x^2-5x-5) + (x-1)\,2(x-2)(6x^2-5x-5) + (x-1)(x-2)^2(12x-5)$

$f''(1) = (1-2)^2(6-5-5) + (1-1)2(1-2)(6-5-5) + (1-1)(1-2)^2(12-5)$

$= 1 \times -4 + 0 + 0 < 0$

$f''(2) = (2-2)^2(24-10-5) + (2-1)2(2-2)(24-10-5) + (2-1)(2-2)^2(12\times 2-5)$

$= 0$

$$f''\left(\frac{17}{12}\right) < 0$$

$$f''\left(\frac{-7}{12}\right) > 0$$

12. (B)

13. (D) $f'(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$f'(0) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} =$$

$$P(-1) = \begin{vmatrix} 6 & 0 \\ 1 & P^2 \end{vmatrix} = -P \times 6P^2 = -6P^3$$

14. (D) $f'(0) = p\begin{vmatrix} 6x & -\sin x & -\cos x \\ 6 & -1 & 0 \\ 1 & p & p^2 \end{vmatrix}$

$$f''(0) = p\begin{vmatrix} 0 & 0 & -1 \\ 6 & -1 & 0 \\ 1 & p & p^2 \end{vmatrix} = 0$$

$$\Rightarrow -p(6p+1) = 0$$

$$\Rightarrow p = \frac{-1}{6} = 0$$

15. (C) $\cos A + \cos B + \cos C = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$

or, $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$,

when $\angle A = \angle B = \angle C = 60^\circ$

$\therefore \sin\frac{A}{2}\sin\frac{b}{2}\sin\frac{c}{2} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

16. (D) $\cos\left(\frac{60^\circ + 60^\circ}{2}\right)\cos\left(\frac{60^\circ + 60^\circ}{2}\right)$

$\cos\left(\frac{60^\circ + 60^\circ}{2}\right)$

$= \cos 60^\circ \cos 60^\circ \cos 60^\circ$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

17. (D) $x^2 + bx + c = 0$

$\tan\alpha + \tan\beta = \frac{-b}{1}$

$\tan\alpha.\tan\beta = \frac{c}{1}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$= \frac{-b}{1-c} = b(c-1)^{-1}$.

18. (B) $\sin(\alpha+\beta)\sec\alpha.\sec\beta$

$= \frac{\sin\alpha.\cos\beta + \cos\alpha.\sec\beta}{\cos\alpha\cos\beta}$

$= \frac{\sin\alpha.\cos\beta}{\cos\alpha.\cos\beta} + \frac{\cos\alpha.\sin\beta}{\cos\alpha.\cos\beta}$

$= \tan\alpha + \tan\beta$

19. (A) $(x-1)^2 + (y-3)^2 = r^2$ ____(1)

centre (1,3)

$x^2 + y^2 - 8x + 2y + 8 = 0$ ____(2)

$x^2 + y^2 + 2gx + 2zy + 6 = 0$

centre (-g,-f) , centre (4,-1)

$\sqrt{(1-4)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

20. (D) $c_1 : x^2 + y^2 - 2x - 6y + 10 = r^2$

$c_2 : x^2 + y^2 - 8x + 2y + 8 = 0$

$r_2 = \sqrt{g^2 + f^2 - c}$

$r_2 = \sqrt{4^2 + (-1)^2 - 8} = \sqrt{16+1-8} = \sqrt{9} = 3$

mid point of 'o'$=\left(\frac{5}{2}, 1\right)$

$\because$ OM < r

$\sqrt{\left(\frac{5}{2}-1\right)^2 + (1-3)^2}$ < r

$\sqrt{\frac{9}{4} + 4}$ <r

$\frac{5}{2} < r$ = r > 2

21. (D) Given lines are

$x + y + 1 = 0$ ___(1)

$3x + 2y + 1 = 0$ ___(2)

(2)$\times 1$ ___(1) $\times 2$

$3x + 2y + 1 - 2x - 2y - 2 = 0$

$\Rightarrow x - 1 = 0$

$\therefore y = -2$

eq^n of the line through (1,-2) and paralled to x-axis is $y+2=0$

22. (B) eq^n of the line through (1,-2) and parallel to y-axis

$$y+2=\frac{1}{0}(x-1)$$

$$\Rightarrow x-1=0$$

23. (A) Let $z=a+ib$

Now, $\left|\frac{z-4}{z-8}\right|=1 \Rightarrow \frac{|z-4|}{|z-8|}=1$

$$\Rightarrow \frac{|a-4+ib|}{|a-8+ib|}=1$$

$$\Rightarrow \frac{\sqrt{(a-4)^2+b^2}}{\sqrt{(a-8)^2+b^2}}=1 \text{ or, } (a-4)^2+b^2$$

$$=(a-8)^2+b^2$$

$$\Rightarrow a^2-8a+16+b^2=a^2-16b+64+b^2$$

$$\Rightarrow 8a=48$$

$$\therefore a=6$$

again, $\left|\frac{z}{z-2}\right|=\frac{3}{2}$

$$\Rightarrow \frac{|z|}{|z-2|}=\frac{3}{2} \Rightarrow \frac{\sqrt{a^2+b^2}}{\sqrt{(a-2)^2+b^2}}=\frac{3}{2}$$

$$4\left(\sqrt{a^2+b^2}\right)=9\left[(a-2)^2+b^2\right]$$

$$\Rightarrow 4a^2+4b^2=9a^2-36a+36+b^2$$

$$\Rightarrow 5a^2-36a+36+5b^2=0$$

$$\Rightarrow 5(6)^2-36\times 6+36+5b^2=0$$

$$\Rightarrow 180-216+36+5b^2=0$$

$$5b^2=0$$

$$b=0$$

$$\therefore z=6+0i$$

$$|z|=\sqrt{6^2+0^2}=6$$

24. (D) $\left|\frac{z-6}{z+6}\right|=\left|\frac{6+0i-6}{6+0i+6}\right|=\left|\frac{0}{12}\right|=0$

25. (C) At $x=0$

$$\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)$$

$$=\lim_{h\to 0}\left[(0-h)+\pi\right]=\pi$$

L.H.L = R.H.L

$$=\lim_{x\to 0^+} f(x)=\lim_{x\to 0} f(0+h)=\lim_{x\to 0}\pi\cos(0+h)=\pi$$

$$=f(0)=\pi\cos 0=\pi$$

L.H.L = R.H.L = $f(0)$

$\Rightarrow f(x)$ is continuous at $x=0$

Now at $x=\frac{\pi}{2}$

L.H.L = $\lim_{x\to \frac{\pi}{2}^-} f(x)=\lim_{h\to 0} f\left(\frac{\pi}{2}-h\right)$

$$=\lim_{h\to 0}\pi\cos\left(\frac{\pi}{2}-h\right)=\lim_{h\to 0}\pi\sinh=0$$

R.H.L = $\lim_{x\to \frac{\pi}{2}^+} f(x)=\lim_{h\to 0} f\left(\frac{\pi}{2}+h\right)$

$$=\lim_{h\to 0}\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2$$

$$=\lim_{h\to 0} h^2=0$$

$f\left(\frac{\pi}{2}\right)=\pi\cos\frac{\pi}{2}=0$, $f(x)$ is also

continuous at x = $\frac{\pi}{2}$

26. (D) LHD =

$$\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{(0-h)+\pi-\pi}{-h}=1$$

R.H.D =

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\pi\cos(0+h)-\pi\cos 0}{h}$$

$$=\lim_{h\to 0}\frac{\pi\cosh-\pi}{h}=\pi\lim_{h\to 0}\left(\frac{\cosh-1}{h}\right)$$

= not defined

$\therefore$ L.H.D $\neq$ R.H.D

$\Rightarrow f(x)$ is not differentiate at $x=0$

At , $x=\frac{\pi}{2}$

L.H.D = $\lim_{h\to 0}\frac{f\left(\frac{\pi}{2}-h\right)-f\left(\frac{\pi}{2}\right)}{-h}$

$$=\lim_{h\to 0}\frac{\pi\cos\left(\frac{\pi}{2}-h\right)-\pi\cos\frac{\pi}{2}}{-h}$$

$= \lim_{h\to 0} \frac{\pi \sinh - 0}{-h} = -\pi$

R.H.D = $\lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right)+f\left(\frac{\pi}{2}\right)}{h}$

$= \lim_{h\to 0} \frac{\left(\frac{\pi}{2}+h+\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2}{h}$

$\lim_{h\to 0} \frac{h^2 - 0}{h} = 0$

L.H.D $\neq$ R.H.D

$= f$(x) is not differentiate at $x = \frac{\pi}{2}$.

27. (C) $\alpha + \beta = -b, b > 0$

$\alpha \times \beta = c < 0, c < 0$

I. $\beta < -\alpha$

$\Rightarrow \alpha + \beta < 0$

$\Rightarrow -b < 0$

$\Rightarrow b > 0$

which is true since b > 0 (given)

II. $\beta < |\alpha|$, which is also true

28. (B) I . $\alpha + \beta + \alpha\beta > 0$

$-b + c > 0$

$b - c < 0$

which is not true since c < 0

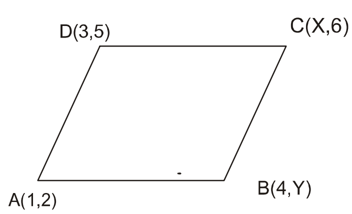
II. $\alpha^2\beta + \beta^2\alpha > 0$

$\alpha\beta(\alpha+\beta) > 0$

$c(-b) > 0$

which is true $[\because c < 0]$

29. (C)



$\because ABCD$ is a ||gm

$\Rightarrow$ mid point of AC = midpoint of BD

$\left[\frac{x+1}{2}, \frac{6+2}{2}\right] = \left[\frac{3+4}{2}, \frac{5+y}{2}\right]$

$\Rightarrow \frac{x+1}{2} = \frac{7}{2}$

or, $x = 6$

also, $4 = \frac{5+y}{2}$

$y = 3$

Now, $AC^2 - BD^2$

$(6-1)^2 + (6-2)^2 - (4-3)^2 + (3-5)^2$

$= 25 + 16 - (1+4)$

$= 41 - 5$

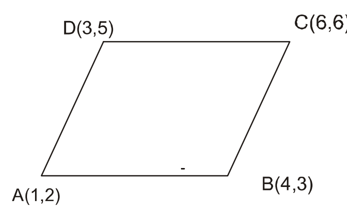
$= 36$

30. (A) point of intersection of diagonals

$= \left[\frac{6+1}{2}, \frac{6+2}{2}\right]$

$= \left[\frac{7}{2}, 4\right]$

31. (D)



Area of ||gm ABCD = 2 $\times$ area of $\triangle$ ABC
=

$2 \times \frac{1}{2}\left[1(3-6) + 4(6-2) + 6(2-3)\right]$

$= [-3 + 16 - 6]$

$= [16 - 9] = 7$

32. (D)
33. (B)
34. (C)
35. (C)
36. (A)
37. (B) $p_1 = 2x - y + 3z - 2 = 0$

$p_2 = x + y - z - 1 = 0$

Equation of the plane through the intersection of p_1 and p_2 is

$(2x - y + 3z - 2) + \alpha(x + y - z - 1) = 0$ __(1)

also, (1) passes through (1,0,1)

$\Rightarrow (2 - 0 + 3 - 2) + \alpha(1 + 0 - 1 - 1)$

$\Rightarrow 3 + \alpha(-1) = 0$

$\therefore \alpha = 3$

$\therefore$ (1) becomes

$$(2x-y+3z-2)+3(x+y-z-1)=0$$

$$5x+2y-5=0$$

38. (C) $r=\left|\dfrac{5\times0+2\times0-5}{\sqrt{5^2+2^2}}\right|=\dfrac{5}{\sqrt{29}}$

39. (C) $f(x)=\left|x^2-5x+6\right|$

$$\left|(x-3)(x-2)\right|$$

$f(x)=(x-3)(x-2)$ if x > 3

$$f''(x)=2x-5$$

$$f''(4)=2\times4-5=3$$

40. (B) $f(x)=-(x-3)(x-2)$ if x < 3

$$-\left(x^2-5x+6\right)$$

$$f'(x)=(2x-5)$$

$$f''(x)=-2$$

$$f''(2.5)=-2$$

41. (A) Let the eq^n of the circle passing through origin be

$x^2+y^2+2gx+2fy=0$ ___(1)

(1) passes through (a,b), (-b,-a)

$\Rightarrow a^2+b^2+2ga+2fb=0$ ___(A)

& $b^2+a^2-2gb-2fa$ ___(B)

Now,

$$b\left[a^2+b^2+2ga+2fb=0\right]$$
$$+a\left[b^2+a^2-2ga-2fa=0\right]$$

$$2f\left(b^2-a^2\right)=-b\left(a^2+b^2\right)-a\left(a^2+b^2\right)-$$
$$\left(a^2+b^2\right)(a+b)$$

$$2f(a+b)(a-b)=\left(a^2+b^2\right)(a+b)$$

$$f=\frac{a^2+b^2}{2(a-b)}$$

again,

$$a\left[a^2+b^2+2ga+2fb=0\right]$$
$$+b\left[b^2+a^2-2ga-2fa=0\right]$$

$$2g\left(a^2-b^2\right)=-a\left(a^2+b^2\right)-b\left(a^2+b^2\right)$$
$$-\left(a^2+b^2\right)(a+b)$$

$$2g(a+b)(a-b)=-\left(a^2+b^2\right)(a+b)$$

$$g=\frac{-\left(a^2+b^2\right)}{2(a-b)}$$

$$f+g=\frac{a^2+b^2}{2(a-b)}-\frac{a^2+b^2}{2(a-b)}=0$$

42. (B) eq^n of the circle is

$x^2+y^2-\dfrac{\left(a^2+b^2\right)}{a-b}x+\dfrac{\left(a^2+b^2\right)}{a-b}y=0$ _____(1)

intercept made by circle (1) on x-axis

$$=\frac{a^2+b^2}{a-b}$$

intercept made by circle (1) on y- axis

$$=\left(\frac{-(a^2+b^2)}{a-b}\right)$$

sum of the squares of intercepts =

$$\left(\frac{a^2+b^2}{a-b}\right)^2+\left(\frac{-(a^2+b^2)}{a-b}\right)^2=2\left(\frac{a^2+b^2}{a-b}\right)^2$$

43. (B)

44. (A)

45. (D) $y=cx+c^2-3c^{\frac{3}{2}}+2$

$$\frac{dy}{dx}=c$$

$$\Rightarrow y=x\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2-3\left(\frac{dy}{dx}\right)^{\frac{3}{2}}+2$$

$$\Rightarrow y-x\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2-2=-3\left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

$$\Rightarrow\left[y-x\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2-2\right]^2=9\left(\frac{dy}{dx}\right)^3$$

order = 1
degree = 4

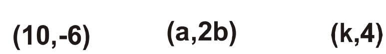
46. (C)

$0.3125\times2=0.6250$	0
$0.6250\times2=1.2500$	1
$0.2500\times2=0.5000$	0
$0.5000\times2=1.0000$	1

$$(0.3125)_{10}=(0.0101)_2$$

47. (C)

48. (A)



$(a, 2b)$ isthe mid point of $(10, -6)$ and $(k, 4)$

$\Rightarrow \frac{10+k}{2} = a$ and $\frac{-6+4}{2} = 2b \Rightarrow \frac{-1}{2} = b$

Again,

$a - 2b = 7$

$a - 2 \times \frac{-1}{2} = 7$

$a = 6$

Now, $\frac{10+K}{2} = 6$

$k = 2$

49. (B) I. $\because \Delta ABC$ is equilateral,

$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$

$3\tan(120^\circ)\tan 60^\circ = 1$

$3 \times -\cot 30^\circ \times \tan 60^\circ = 1$

$-3 \times \sqrt{3}.\sqrt{3} = 1$

$-9 \neq 1$. which is not true.

II. $\tan\left(\frac{A}{2} + C\right) < \tan A$

$\tan\left(\frac{78}{2} + 36\right) < \tan 78^\circ$

$\tan(39 + 36^\circ) < \tan 78^\circ$

$\tan(75^\circ) < \tan 78^\circ$

which is true.

III. $\tan\left(\frac{A+B}{2}\right)\sin\frac{C}{2} < \cos\frac{C}{2}$

$A + B + C = 180^\circ$,

$\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

$\tan\left(\frac{\pi}{2} - \frac{c}{2}\right)\sin\frac{c}{2} < \cos\frac{c}{2}$

$\cot\frac{C}{2}\sin\frac{C}{2} < \cos\frac{C}{2}$

$\cot\frac{C}{2}\sin\frac{C}{2} < \cos\frac{C}{2}$

$\cos\frac{c}{2} < \cos\frac{c}{2}$,which is not true

50. (C) $2\sin 24^\circ \sin 12^\circ \times 2\sin 60^\circ \cos 36^\circ$ =A

A $= \frac{4\sqrt{3}}{2}\sin 12^\circ \sin 24^\circ \cos 36^\circ$

B $= 2\cos 36^\circ \sin 24^\circ \times 2\sin 60^\circ \sin 12^\circ$

B $= \frac{4\sqrt{3}}{2}\sin 12^\circ \sin 24^\circ \sin 36^\circ$

$\frac{A}{B} = 1$

51. (A) $\bar{x} = \frac{10+9+21+16+24}{5} = 16$

$M.D.(\bar{X}) = \frac{\sum|x_i - \bar{x}|}{n}$

$= \frac{|10-16|+|9-16|+|21-16|+|16-16|+|24-16|}{5}$

$= \frac{6+7+5+0+8}{5} = \frac{26}{5} = 5.2$

52. (B) E =sum on three faces is $\angle 5$

then, $E = \{(1,1,1),(1,1,2)(1,2,1),(2,1,1)\}$

E' = contains $216 - 4 = 212$ elements

$p(E') = \frac{212}{216} = \frac{53}{54}$

$\therefore p(E') = \frac{212}{216} = \frac{53}{54}$

53. (D) $P\{(A \cap B') \cup (A' \cap B)\}$

$= P(A \cap B') + (A' + B)$

$= P(A)P(B') + P(A')P(B)$

$= \frac{1}{3} \times \frac{1}{4} + \frac{2}{3} \times \frac{3}{4}$

$= \frac{1}{12} + \frac{1}{12} = \frac{7}{12}$

54. (C) Variance= $\sigma^2 = \frac{\sum_{i=1}^{20} X_i^2}{n} - \left(\frac{\sum_{i=1}^{20} X_i^2}{n}\right)^2$

$= \frac{84000}{20} - \left(\frac{1000}{20}\right)^2$

$= 4200 - 2500 = 1700$

55. (A) P(queen of spade)= $\frac{1}{52}$

56. (C) Let E = sum of two faces is less than 4

then E= $\{(1,1)(1,2)(2,1)\}$

P(E)= $\frac{33}{36} = \frac{11}{12}$

57. (B) $P(\overline{A}\cap B)=P(\overline{A})P(B)$

$0.3=0.8P(B)$

$P(B)=\frac{3}{8}$

Now, $p\left(\frac{A}{A\cup B}\right)=\frac{P[A\cap(A\cup B)]}{P(A\cup B)}$

$=\frac{P(A)}{P(A)+P(B)-P(A\cap B)}$

$=\frac{P(A)}{P(A)+P(B)-P(A)P(B)}$

$=\frac{\frac{1}{5}}{\frac{1}{5}+\frac{3}{8}-\frac{1}{5}\times\frac{3}{8}}=\frac{\frac{1}{5}}{\frac{8+15-3}{40}}=\frac{1}{5}\times\frac{40}{20}=\frac{2}{5}$

58. (D) $n\overline{x}=x^1+x^2+......+x^n$

$\frac{n\overline{x}-x_2+\lambda}{n}=\frac{x_1+\lambda+x_3+....+x_n}{n}$

$\frac{n\overline{x}-x_2+\lambda}{n}$ is the new mean.

59. (D) $x=\frac{3+5+1+6+5+9+5+2+8+6}{10}$

$\Rightarrow\frac{50}{10}=5$

Data in increasing order 1, 2, 3, 4, 5, 5, 5, 6, 6, 8, 9

$y=\frac{5+5}{2}=5$

$z=5$

Hence, $x=y=z$

60. (B)

61. (B) $A=\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}$

$A^2=\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}\neq -A$

$A^3=A^2.A=\begin{pmatrix}2 & -2\\ -2 & 2\end{pmatrix}\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}$

$=\begin{pmatrix}-2-2 & 2+2\\ 2+2 & -2-2\end{pmatrix}=\begin{pmatrix}-4 & 4\\ 4 & -4\end{pmatrix}$

$=4\begin{pmatrix}-1 & 1\\ 1 & -1\end{pmatrix}=4A$

which is true.

62. (D) 1. $\begin{vmatrix}41 & 1 & 5\\ 79 & 7 & 9\\ 29 & 5 & 3\end{vmatrix}$

Apply , c_1-8c_3

$\Rightarrow\begin{vmatrix}41-8\times5 & 1 & 5\\ 79-8\times9 & 7 & 9\\ 29-8\times3 & 5 & 3\end{vmatrix}=\begin{vmatrix}1 & 1 & 5\\ 7 & 7 & 9\\ 5 & 5 & 3\end{vmatrix}$

$\because c_1-c_2$

$\begin{vmatrix}41 & 1 & 5\\ 79 & 7 & 9\\ 29 & 5 & 3\end{vmatrix}=0$

2. $\begin{vmatrix}1 & a & b+c\\ 1 & b & c+a\\ 1 & c & a+b\end{vmatrix}$

Apply,

$c_2+c_3=c_3\begin{vmatrix}1 & a & a+b+c\\ 1 & b & a+b+c\\ 1 & c & a+b+c\end{vmatrix}$

$=(a+b+c)=\begin{vmatrix}1 & a & 1\\ 1 & b & 1\\ 1 & c & 1\end{vmatrix}=0(\because c_1=c_3)$

3. $\begin{vmatrix}0 & c & b\\ -c & 0 & a\\ -b & -a & 0\end{vmatrix}$

$0(0+a^2)-c(0+ab)+b(ac-0)$

$0-abc+ab=0$

Hence 1, 2, 3 are true.

63. (A) $y-\sqrt{3}x-5=0\Rightarrow y=\sqrt{3}x+5$

$\therefore m_1=\sqrt{3}$,

$\sqrt{3}y-x+6=0\Rightarrow\frac{1}{\sqrt{3}}x-6=y$

$\therefore m_2=\frac{1}{\sqrt{3}}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}\sqrt{3}}\right| = \frac{\frac{3-1}{\sqrt{3}}}{2} = \frac{1}{\sqrt{3}}$$

$\theta = 30^\circ$

64. (D) $\because 1 + \omega_1 + \omega_2 = 0$ also, $\omega_1\omega_2 = 1$

$\Rightarrow (\omega_1 + \omega_2)^2 = 1$

$\Rightarrow \omega_1^2 + \omega_2^2 + 2\omega_1\omega_2 = 1$

$\Rightarrow \omega_1^2 + \omega_2^2 = 1 \Rightarrow 1 - 2 = -1$

Now, $(\omega_1 - w_2)^2$

$= \omega_1^2 + \omega_2^2 - 2\omega_1\omega_2$

$= -1 - 2 \times 1$

$= -3$

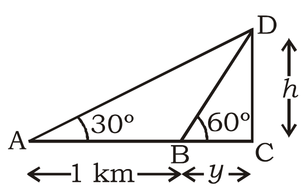
65. (A) $\therefore$ $x = \sin\theta\cos\theta$ and $y = \sin\theta + \cos\theta$

$\therefore$ $y^2 - 2x = (\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 2\sin\theta\cos\theta$

$= \sin^2\theta + \cos^2\theta = 1$

66. (B) In ΔBDC,



$\tan 60^\circ = \frac{CD}{BC}$

$\Rightarrow \sqrt{3} = \frac{h}{y}$

$\Rightarrow h = \sqrt{3}y$

In ΔADC,

$\tan 30^\circ = \frac{CD}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{1+y}$

$\Rightarrow 1 + y = h\sqrt{3}$

$\Rightarrow 1 + y = 3y$ [From Eq. (i)]

$\Rightarrow y = \frac{1}{2}$

$h = \frac{\sqrt{3}}{2}$ km [From Eq. (i)]

67. (D) $\sin^4 x - \cos^4 x = p$ (given)

$\Rightarrow (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = p$

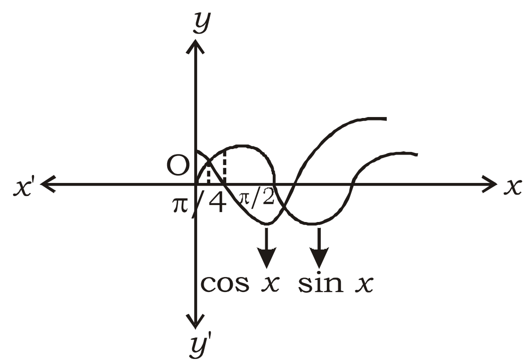
$\Rightarrow \sin^2 x - \cos^2 x = p$

$\Rightarrow -\cos 2x = p \Rightarrow \cos 2x = -p$

$[\because -1 \le \cos x \le 1 \Rightarrow |\cos 2x| \le 1$

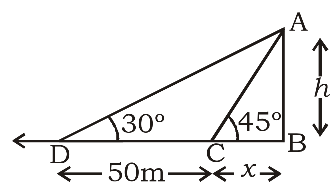
$|p| \le 1$

68. (B) We know that, for $\frac{\pi}{4} < \theta < \frac{\pi}{2}$ $\cos\theta < \sin\theta$



69. (C) In ΔABC,

$\tan 45^\circ = \frac{AB}{BC}$



$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow h = x$ ... (i)

In ΔABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x + 50}$

$\Rightarrow x + 50 = h\sqrt{3}$

$\Rightarrow h + 50 = h\sqrt{3}$ [From Eq. (i)]

$\Rightarrow h = \frac{50}{\sqrt{3} - 1}$

So, width of the river is $x = h = \frac{50}{\sqrt{3} - 1}$ m

70. (C) $\sin^2 x + \sin^2 y = 1$

$\Rightarrow \sin^2 x = 1 - \sin^2 y$

$\Rightarrow \sin^2 x = \cos^2 y$

$\Rightarrow \sin x = \cos y$ ... (i)

Similarly, we take

$\sin^2 y = 1 - \sin^2 x = \cos^2 x$

$\Rightarrow \cos x = \sin y$ ... (ii)

Now,

$\cot(x+y) = \frac{\cos(x+y)}{\sin(x+y)}$

$= \frac{\cos x \cos y - \sin x \sin y}{\sin(x+y)}$

$= \frac{\cos x \cos y - \cos x \cos y}{\sin(x+y)} = 0$

[from Eqs. (i) and (ii)]

71. (B) $\cos 10° + \cos 110° + \cos 130°$

$= (\cos 130° + \cos 10°) + \cos 110°$

$= 2\cos 60° \cos 70° + \cos 110°$

$= \cos 70° + \cos 110°$

$= \cos(180° - 110°) + \cos 110°$

$= -\cos 110° + \cos 110° = 0$

72. (A) $\frac{dy}{dx} = 1 + x + y + xy$ (given)

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y)$

$\Rightarrow \int \frac{1}{1+y} dy = \int (1+x) dx$

$\Rightarrow \log(1+y) = x + \frac{x^2}{2} + C$

At $x = -1$, $y = 0$

$\Rightarrow \log 1 = -1 + \frac{1}{2} + C$

$\Rightarrow C = \frac{1}{2}$ $(\because \log 1 = 0)$

$\therefore \log(1+y) = x + \frac{x^2}{2} + \frac{1}{2} = \frac{(1+x)^2}{2}$

$\Rightarrow y = e^{\frac{(1+x)^2}{2}} - 1$

73. (A) Let $I = \int_0^{\pi/2} \log(\tan x)\, dx$ ... (i)

and $I = \int_0^{\pi/2} \log\left\{\tan\left(\frac{\pi}{2} - x\right)\right\} dx$

$[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx]$

$\Rightarrow I = \int_0^{\pi/2} \log(\cot x)\, dx$ ... (ii)

From Eqs. (i) and (ii), we get

$2I = \int_0^{\pi/2} \log(\tan x \cot x)\, dx$

$= \int_0^{\pi/2} \log 1\, dx$

$= \int_0^{\pi/2} 0\, dx = 0$

$\Rightarrow I = 0$

74. (B) Let $I = \int \tan^2 x \sec^4 x\, dx$

$= \int \tan^2 x (1 + \tan^2 x) \sec^2 x\, dx$

Let $\tan x = t$, then $\sec^2 x\, dx = dt$

$\therefore I = \int t^2(1+t^2)dt = \int (t^2 + t^4)dt$

$= \frac{t^5}{5} + \frac{t^3}{3} + C = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

75. (A) $\lim_{x\to 0} \frac{\sin^2 ax}{bx}$

$= \lim_{x\to 0} \frac{\sin^2 ax}{a^2x^2} \times \frac{a^2x^2}{bx}$

$= \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)^2 \times \frac{a^2}{b} \lim_{x\to 0}(x)$

$= (1)^2 \times \frac{a^2}{b} \times 0$

$= 0$

76. (B) $\because f(x) = \tan x + e^{-2x} - 7x^3$

On differentating w.r.t, x, we get

$f'(x) = \sec^2 x - 2e^{-2x} - 21x^2$

$\Rightarrow f'(0) = \sec^2 0 - 2e^0 - 21 \times 0$

$= 1 - 2 = -1$

77. (D) $\because P(X \le 2) = 0.25$

$\Rightarrow P(X = 1) + P(X = 2) = 0.25$

$\Rightarrow k + p_1 = 0.25$

$\Rightarrow p_1 = 0.25 - k$ and $P(X \ge 4) = 0.35$

$\Rightarrow P(X = 4) + P(X = 5) = 0.35$

$\Rightarrow p_2 + 2k = 0.35$

$\Rightarrow p_2 = 0.35 - 2k$

$\Rightarrow p_1 \ne p_2$ and

$p_1 + p_2 = 0.25 - k + 0.35 - 2k$

$= 0.6 - 3k \ne P(X = 3)$

Hence, neither I nor II is correct.

78. (B) $\because$ A, B, C are in AP. ... (i)

$\therefore 2B = A + C$

But $A + B + C = 180°$

$\Rightarrow$ 3B = 180°

$\Rightarrow$ B = 60°

sin A + 2 sin B + sin C

= (sin A + sin C) + 2 sin B

$= 2\sin \frac{A+C}{2} \cos \frac{A-C}{2} + 2\sin B$

$= 2\sin \frac{(2B)}{2} \cos \left(\frac{A-C}{2}\right) + 2\sin B$

[from Eq. (i)]

$= 2\sin B\left[\cos \frac{A-C}{2} + 1\right]$

$= 2\sin B\left[2\cos^2\left(\frac{A-C}{4}\right)\right]$

$\left(\because 1 + \cos A = 2\cos^2 \frac{A}{2}\right)$

$= 4 \sin B \cos^2\left(\frac{A-C}{4}\right)$

79. (B) (I) cos(sin–1 x) = $\cos(\cos^{-1}\sqrt{1-x^2})$

$= \sqrt{1-x^2}$

when $x \in [-1, 1]$

(II) $\sin(\cos^{-1} x) = \sin(\sin^{-1}\sqrt{1-x^2})$

when $x \in [-1, 1]$

$= \sqrt{1-x^2}$

Hence, statement I is false and II is true.

80.(C) $\because$ $y = -\tan^{-1}(x^{-1}) + 1$

$\frac{dy}{dx} = -\frac{1}{1+x^{-2}}(-1)x^{-2}$

$= \frac{1}{1+x^2}$

Since, $\frac{dy}{dx}$ is positive for all values of x.

Therefore, y is an increasing function of x.

$\frac{dy}{dx}$ can be positive, negative or zero, for all values of x.

81. (D) The equation of the first circle is

$x^2 + y^2 - 2x - 2y = 0$

Radius of this circle

$= \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

and equation of the second circle is

$x^2 + y^2 = 1$

Radius of this circle = 1

From above it is clear that the radius of first circle is not twice that of the second circle. Also the first passes through the origin while the 2nd circle does not pass through the origin.

Hence, neither statements I or II is correct.

82. (C) Let $\frac{1}{ab}, \frac{1}{ca}, \frac{1}{bc}$ are in AP.

$\Rightarrow \frac{1}{ca} - \frac{1}{ab} = \frac{1}{bc} - \frac{1}{ca}$

$\Rightarrow \frac{b-c}{abc} = \frac{a-b}{abc}$

$\Rightarrow b - c = a - b \Rightarrow 2b = a + c$

$\Rightarrow a, b, c$ are in AP.

Now, $\frac{1}{\sqrt{b}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in AP.

$\therefore \frac{2}{\sqrt{c}+\sqrt{a}} = \frac{1}{\sqrt{b}+\sqrt{c}} + \frac{1}{\sqrt{a}+\sqrt{b}}$

$\Rightarrow 2(\sqrt{b}+\sqrt{c})(\sqrt{a}+\sqrt{b}) = (\sqrt{c}+\sqrt{a})$

$(\sqrt{a}+2\sqrt{b}+\sqrt{c})$

$\Rightarrow 2(\sqrt{ab}+b+\sqrt{ac}+\sqrt{bc}) =$

$\sqrt{ac}+2\sqrt{bc}+c+a+2\sqrt{ab}+\sqrt{ac}$

$\Rightarrow 2\sqrt{ab}+2b+2\sqrt{ac}+2\sqrt{bc}$

$= 2\sqrt{ac}+2\sqrt{bc}+2\sqrt{ab}+c+a$

$\Rightarrow 2b = a + c$

$\Rightarrow a, b, c$ are in A.P.

Hence, both the statements are correct.

83. (A) Let $u = \log_x x = 1$ $[\because \log_a a = 1]$

$\Rightarrow \frac{du}{dx} = 0$ and $v = \log x \Rightarrow \frac{dv}{dx} = \frac{1}{x}$

$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = 0$

84. (C) $\tan\left(7\frac{1}{2}\right)^\circ = \dfrac{2\sin^2\left(7\frac{1}{2}\right)^\circ}{2\sin\left(7\frac{1}{2}\right)^\circ\cos\left(7\frac{1}{2}\right)^\circ}$

$$\left(\because 2\sin^2\frac{\theta}{2} = 1-\cos\theta \text{ and } 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin\theta\right)$$

$$= \frac{1-\cos 15^\circ}{\sin 15^\circ} = \frac{1-\cos(45^\circ-30^\circ)}{\sin(45^\circ-30^\circ)}$$

$$= \frac{1-(\cos 45^\circ.\cos 30^\circ+\sin 45^\circ \sin 30^\circ)}{\sin 45^\circ.\cos 30^\circ-\cos 45^\circ.\sin 30^\circ}$$

$$= \frac{1-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}}$$

$$= \frac{(2\sqrt{2}-\sqrt{3}-1)}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{6}-3-\sqrt{3}+2\sqrt{2}-\sqrt{3}-1}{3-1}$$

$$= \sqrt{6}-\sqrt{3}+\sqrt{2}-2$$

85. (D) $\dfrac{\cos 15^\circ+\cos 45^\circ}{\cos^3 15^\circ+\cos^3 45^\circ}$

$$= \frac{\cos 15^\circ+\cos 45^\circ}{(\cos 15^\circ+\cos 45^\circ)(\cos^2 45^\circ+\cos^2 15^\circ-\cos 45^\circ \cos 15^\circ)}$$

$$\left[\because a^3+b^2 = (a+b)(a^2-ab+b^2)\right]$$

$$= \frac{1}{(\cos^2 45^\circ+\cos^2 15^\circ-\cos 45^\circ \cos 15^\circ)}$$

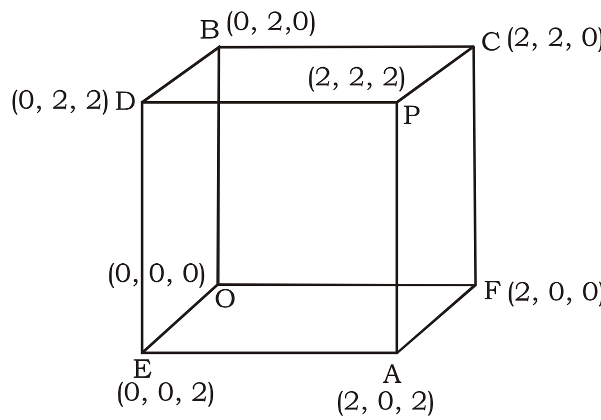
$$= \frac{1}{\frac{1}{2}+(\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ)^2-\frac{\cos 15^\circ}{\sqrt{2}}}$$

$$= \frac{1}{\frac{1}{2}+\left(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)^2-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}$$

$$= \frac{1}{\frac{1}{2}+\frac{3+1+2\sqrt{3}}{8}-\frac{\sqrt{3}+1}{4}}$$

$$= \frac{1}{\frac{4+4+2\sqrt{3}-2\sqrt{3}-2}{8}} = \frac{8}{6} = \frac{4}{3}$$

86. (B) Direction ratio of digonal OP
= 2 – 0, 2 – 0, 2 – 0 = < 2, 2, 2>



and direction cosines

$$\left\langle \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}} \right\rangle$$

$$\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

Direction ratio of digonal AB
= 2 – 0, 0 – 2, 2 – 0 = < 2, –2, 2> and direction cosines

$$\left\langle \frac{2}{2\sqrt{3}}, \frac{-2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}} \right\rangle$$

$$\left\langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

Let θ be the angle between OP and AB

$$\cos\theta = \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{3} - \frac{1}{3} + \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

87. (C) Direction ratio of side OB
= 0 – 0, 2 – 0, 0 – 0
= <0, 2, 0>
and direction cosines

$$\left\langle \frac{0}{2}, \frac{2}{2}, \frac{0}{2} \right\rangle$$

<0, 1, 0>

Let θ_1 be the angle between digaonal OP and side OB

$$\cos\theta_1 = (0)\left(\frac{1}{\sqrt{3}}\right) + (1)\left(\frac{1}{\sqrt{3}}\right) + (0)\left(\frac{1}{\sqrt{3}}\right)$$

$$= 0 + \frac{1}{\sqrt{3}} + 0$$

$$\Rightarrow \theta_1 = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

88. (C) Direction ratio of side OC = 2 – 0, 2 – 0, 0 – 0 = <2, 2, 0>

and direction cosines $\left\langle \frac{2}{2\sqrt{2}}, \frac{2}{2\sqrt{2}}, 0 \right\rangle$

$$\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

Let θ_2 be the angle between side OC and diagonal OP

$$\cos\theta_2 = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right) + (0)\times\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} + 0 = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow \theta_2 = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

89. (C) a.b |a| |b| cos α

$\Rightarrow \cos\alpha$ = a.b

($\because$ |a| = |b| = 1, a and b are unit vectors)

[$\because$ (a + b) is unit vector]

Now, |a + b| = 1

$\Rightarrow |a|^2 + |b|^2 + 2a.b = 1$

$\Rightarrow 1 + 1 + 2\cos\alpha = 1$

$\Rightarrow 2\cos\alpha = -1$

$$\Rightarrow \cos\alpha = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{2\pi}{3}$$

90. (A) $\cos\beta = \dfrac{\sin A}{2\sin C}$

$$\frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2} \times \frac{a}{c} \text{ (by sine Rule)}$$

$\Rightarrow a^2 + c^2 - b^2 = a^2$

$\Rightarrow b^2 = c^2$

$\Rightarrow b = c$

$\Rightarrow \Delta$ABC is isosceles

91. (C) Total literate people in a town= (100 –35.4)% = 64.6%

Education up to primary = 27%

Education up to middle = 18.6%

Education up to graduation= $\dfrac{660}{15000} \times 100$

= 4.4%

$\therefore$ Total = 50%

Let the number of pre-university students= x

Then, the number of high school students= $2x$

$\therefore$ According to the question,

$2x + x$ = (64.6 – 50)% of 15000

$$\Rightarrow 3x = \frac{14.6}{100} \times 15000$$

$\Rightarrow 3x = 2190$

$\Rightarrow x = 730$

$\therefore$ Total people up to high school= $2x$

= 2 × 730

= 1460

92. (B) $\therefore$ Required probability $\dfrac{^{25}C_3}{^{26}C_3}$

$$= \frac{25 \times 24 \times 23}{26 \times 25 \times 24} = \frac{23}{26}$$

93. (B)

Head	Tail
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

Hence, total number of points in the sample space is 11.

94. (D) Since, lines of regression passes through $(\overline{X}, \overline{Y})$.

$\therefore 3\overline{X} + \overline{Y} - 12 = 0$ ..... (i)

and $\overline{X} + 2\overline{Y} - 14 = 0$ ..... (ii)

On solving Eqs. (i) and (ii), we get

$\overline{X} = 2$ and $\overline{Y} = 6$

95. (D) Since, coefficient of variance = $\frac{SD}{Mean}$

Coefficient of variance $A = \frac{\sqrt{12}}{60} = \frac{3.46}{60}$

= 0.057

Coefficient of variance $B = \frac{\sqrt{25}}{90} = \frac{5}{90}$

=0.055

Coefficient of variance $C = \frac{\sqrt{36}}{80} = \frac{6}{80}$

=0.075

Coefficient of variance $D = \frac{\sqrt{16}}{120} = \frac{4}{120}$ =0.033

Hence, we see that minimum coefficient and variance is D, hence product is consistent.

96. (C) Let us consider any five integers be 3, 4, 5, 6 and 7.

Its mean $= \frac{25}{5} = 5$

$\therefore$ SD= $\sqrt{\frac{(5-3)^2+(5-4)^2+(5-5)^2+(5-6)^2+(5-7)^2}{5}}$

$= \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{2}$

97. (B) Total number of elementary events
= $^7C_1 \times {^6C_1} = 42$

98. (C) Since, $P(A \cup B) \leq 1$

$\Rightarrow P(A) + P(B) - (A \cap B) \leq 1$

$\Rightarrow 0.8 - 0.7 - P(A \cap B) \leq 1$

$\Rightarrow P(A \cap B) \geq 1.5 - 1$

$\Rightarrow P(A \cap B) \geq 0.5$

99. (B) $\because$ a = i – 2j + k and b = 4i – 4j + 7k
$\therefore$ Projection of a on b

$= \frac{a \bullet b}{|b|}$

$= \frac{4 \times 1 + (-2)(-4) + 1 \times 7}{\sqrt{16+16+49}}$

$= \frac{4+8+7}{\sqrt{81}}$

$= \frac{19}{9}$

100. (D) $\because \tan^2 30° = \frac{1}{3}$

$\tan^2 45° = 1$ and $\tan^2 60° = 3$
$\therefore \tan^2 30°, \tan^2 45°$ and $\tan^2 60°$ are in G.P. because its common ratio is same, *i.e.*, 3.

101. (D) Let the roots of the equation $x^2 + kx - b = 0$ be α and β.

$\Rightarrow \alpha + \beta = -k$, and $\alpha\beta = -b$

According to the question, $\alpha^2 + \beta^2 = 2b$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 2b$
$\Rightarrow k^2 + 2b = 2b$
$\Rightarrow k = 0$

102. (A) Let the roots of $ax^2 + bx + c = 0$, $a \neq o$ are α and $\frac{1}{\alpha}$.

$\therefore \alpha \cdot \frac{1}{\alpha} = \frac{c}{a} \Rightarrow c = a$

103. (D) $(2 - \omega + 2\omega^2)^{27} = [2(1+\omega^2) - \omega]^{27}$

$[\because 1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1]$

$= (-2\omega - \omega)^{27}$

$= (-3\omega)^{27}$

$= -3^{27}.\omega^{27}$

$= -3^{27}.(\omega^3)^9$
$= -3^{27} . 1$
$= -3^{27}$

104. (A) $\therefore \frac{1}{1+3i} - \frac{1}{1-3i}$

$= \frac{1-3i}{1+9} - \frac{1+3i}{1+9}$

$= \frac{1-3i-1-3i}{10}$

$= -\frac{6i}{10} = -\frac{3}{5}i$

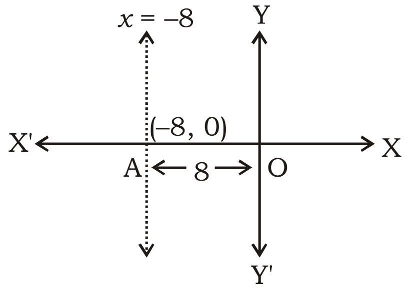
$\Rightarrow \left|-\frac{3}{5}i\right| = \frac{3}{5}$

105. (D) M = Set of men and R is a relation 'is son of' defined on M.
Reflexive $a \not R a$.
Since, a cannot be a son of a.
Symmetric relation
$aRb \Rightarrow bRa$
which is also not possible.
Transitive relation

aRb, $bRc \Rightarrow cRa$

which is not possible.

106. (C) Required locus is $x = -8$ which is at a distance of 8 units to be left of Y-axis.



107. (B) 10101111

$= 2^7 \times 1 + 2^6 \times 0 + 2^5 \times 1 + 2^4 \times 0 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 1$

$= 128 + 32 + 8 + 4 + 2 + 1$

$= 175$

108. (C) Required probability

$= \frac{1}{4!} = \frac{1}{4 \times 3 \times 2} = \frac{1}{24}$

109. (D) Let the three points A(3, 1), B(12, – 2) and C(0, 2) are collinear and the point P(h, k) are equidistant from these points A, B and C.

Now, $PA^2 = PB^2 = PC^2$

$\Rightarrow (h-3)^2 + (k-1)^2 = (h-12)^2 + (k+2)^2 = (h-0)^2 + (k-2)^2$

$\Rightarrow h^2 + k^2 - 6h - 2k + 10 = h^2 + k^2 - 24h + 4k + 148 = h^2 + k^2 - 4k + 4$

Taking first and third, we get

$3h - k = 3$ ... (i)

Taking second and third, we get

$3h - k = 18$ ... (ii)

Since, Eqs., (i) and (ii) are two parallel lines. Hence, the locus will be a null set.

110. (A) Let A(1, 0) and B(0, –2) are the two given points and let P(h, k) be any variable point, then

According to question,

PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (h-1)^2 + (k-0)^2 = (h-0)^2 + (k+2)^2$

$\Rightarrow h^2 + 1 - 2h + k^2 = h^2 + k^2 + 4 + 4k$

$\Rightarrow 4k + 2h + 3 = 0$

$\Rightarrow 2h + 4k + 3 = 0$

$\therefore$ locus of P(h, k) is $2x + 4y + 3 = 0$.

111. (A) Points (5, 1), (1, –1) and (11, 4) are collinear, if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Here, $x_1 = 5$, $y_1 = 1$, $x_2 = 1$, $y_2 = -1$, $x_3 = 11$, $y_3 = 4$

$\therefore$ From Eq. (i)

$5(-1-4) + 1(4-1) + 11(1+1) = 0$

$\Rightarrow 5(-5) + 1(3) + 22 = 0 \Rightarrow 0 = 0$

$\therefore$ (5, 1), (1, –1) and (11, 4) are collinear.

112. (D) we know that if $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ are two parallel lines, then distance between them is $\left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$.

$\therefore$ For lines $3x + 4y - 9 = 0$ ... (i)

and $9x + 12y + 28 = 0$

or $3x + 4y + \frac{28}{3} = 0$ ... (ii)

This distance them $= \left|\frac{\frac{28}{3} + 9}{\sqrt{9 + 16}}\right|$

$= \left|\frac{55}{3} \times \frac{1}{5}\right|$

$= \frac{11}{5}$ unit

113. (B) The distance between (2, 6) and (0, 0)

$p = \sqrt{36 + 4} = \sqrt{40}$

The distance between (3, 4) and (0, 0)

$q = \sqrt{9 + 16} = 5$

The distance between (4, 5) and (0, 0)

$r = \sqrt{16 + 25} = \sqrt{41}$

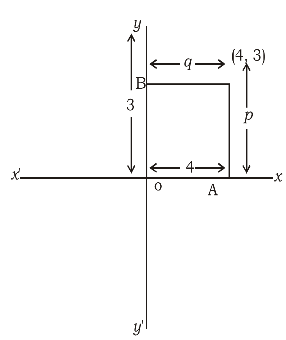
The distance between (–2, 5) and (0, 0)

$s = \sqrt{(-2)^2 + (5)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$

Clearly, the distance of q is a whole number.

114. (C)



From adjoining figure,

$p = 3$ and $q = 4$

$\therefore 4p = 3q$

115. (A) Given,

$(2x + 3y + 4) + \lambda(6x - y + 12) = 0$

$2x + 6\lambda x + 3y - \lambda y + 4 + 12\lambda = 0$

$2x(3\lambda + 1) + y(3 - \lambda) + 4 + 12\lambda = 0$

Since, line (i) is parallel to y-axis.

$\therefore$ The coefficient of y must be zero.

$3 - \lambda = 0 \Rightarrow \lambda = 3$

116. (C) Let the line $y = 0$ divides the line joining the points (3, – 5) and (–4, 7) in the ratio $n : m$, then

By section formula,

[For internally division]

$$y = \frac{m(-5) + n(7)}{m + n} = \frac{-5m + 7n}{m + n}$$

Given, $y = 0$

$$\Rightarrow \frac{-5m + 7n}{m + n} = 0$$

$\Rightarrow 5m = 7n$

$$\Rightarrow \frac{m}{n} = \frac{7}{5} \Rightarrow \frac{n}{m} = \frac{5}{7}$$

or $n : m = 5 : 7$

117. (B) Since, the sum of focal distances of a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to $2b$ when $b > a$.

$\because a^2 = 4, b^2 = 9$

$\Rightarrow a = 2, b = 3$

$\therefore$ Sum of the focal distances

$= 2 \times 3 = 6$ unit

118. (B) The eccentricity of ellipse lies between 0 and 1.

119. (D) We know that, if the line making an angle θ with the positive direction of x-axis with y intercept as C.

Then equation of the line is

$\because y = mx + c = \tan\theta\, x + c$

$\therefore \theta = 45°$ and $c = 101$ unit

$\Rightarrow y = 1 \times x + 101$

$\Rightarrow x - y + 101 = 0$

120. (B) Let A = (2, 4), B = (2, 6), C = $(2 + \sqrt{3}, k)$

$$AB = \sqrt{(2-2)^2 + (6-4)^2} = \sqrt{(2)^2} = 2$$

$$BC = \sqrt{(2 + \sqrt{3} - 2)^2 + (k-6)^2}$$

$$= \sqrt{3 + (k-6)^2}$$

$$= \sqrt{k^2 - 12k + 39}$$

Since, ABC is an equilateral triangle.

$\therefore$ AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow 4 = k^2 - 12k + 39$

$k^2 - 12k + 35 = 0$

$k^2 - 7k - 5k + 35 = 0$

$k(k - 7) - 5(k - 7) = 0$

$(k - 5)(k - 7) = 0$

$k = 5, 7$

NDA MATHS MOCK TEST- 55 (ANSWER KEY)

1. (D)	21. (D)	41. (A)	61. (B)	81. (D)	101. (D)
2. (A)	22. (B)	42. (B)	62. (D)	82. (C)	102. (A)
3. (A)	23. (A)	43. (B)	63. (A)	83. (A)	103. (D)
4. (D)	24. (D)	44. (A)	64. (D)	84. (C)	104. (A)
5. (C)	25. (C)	45. (A)	65. (A)	85. (D)	105. (D)
6. (B)	26. (D)	46. (D)	66. (B)	86. (B)	106. (C)
7. (B)	27. (C)	47. (C)	67. (D)	87. (C)	107. (B)
8. (A)	28. (B)	48. (A)	68. (B)	88. (C)	108. (C)
9. (A)	29. (C)	49. (B)	69. (C)	89. (C)	109. (D)
10. (B)	30. (A)	50. (C)	70. (C)	90. (A)	110. (A)
11. (C)	31. (D)	51. (A)	71. (B)	91. (C)	111. (A)
12. (B)	32. (D)	52. (B)	72. (A)	92. (B)	112. (D)
13. (D)	33. (B)	53. (D)	73. (A)	93. (B)	113. (B)
14. (A)	34. (C)	54. (C)	74. (B)	94. (D)	114. (C)
15. (A)	35. (C)	55. (A)	75. (A)	95. (D)	115. (A)
16. (D)	36. (A)	56. (C)	76. (B)	96. (C)	116. (C)
17. (D)	37. (B)	57. (B)	77. (D)	97. (B)	117. (B)
18. (B)	38. (C)	58. (D)	78. (B)	98. (C)	118. (B)
19. (A)	39. (C)	59. (D)	79. (B)	99. (B)	119. (D)
20. (D)	40. (B)	60. (B)	80. (C)	100. (D)	120. (B)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*