NDA (MATHS) MOCK TEST - 57 (SOLUTION)

1. (B) $\frac{\pi}{2} < 2^c < \frac{3\pi}{4}$ and $2\pi < 7^c < \frac{5\pi}{2}$

$\Rightarrow$ 2^c is in second quadrant and 7^c is in first quadrant

$\Rightarrow$ a = cos 2^c <0 and b = sin7^c>0

$\Rightarrow$ ab < 0

2. (C) $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\sin\theta}{|\cos\theta|}$

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\sin\theta}{-\cos\theta}$

$\Rightarrow \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \tan\theta - \sec\theta$

3. (A) Using sine rule, we have,

$\frac{a+c}{a-c}\tan\frac{B}{2} = \frac{\sin A + \sin C}{\sin A - \sin C}\tan\frac{B}{2}$

$= \frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{2\sin\frac{A-C}{2}\cos\frac{A+C}{2}} .tanB$

$= \frac{\cos\frac{B}{2}\cos\frac{A-C}{2}}{\sin\frac{A-C}{2}\sin\frac{B}{2}} . \tan\frac{B}{2}$

$= \cot\left(\frac{A-C}{2}\right)$

$= \cot\left(\frac{\pi-B-C-C}{2}\right)$

$= \tan\left(\frac{B}{2}+C\right)$

4. (D) $\frac{b-c}{a} = \frac{k(\sin B - \sin C)}{k\sin A}$

$= \frac{\sin B - \sin C}{\sin A}$

$= \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}$

$\Rightarrow \frac{b-c}{a} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos\frac{A}{2}} \Rightarrow (b-c)\cos\frac{A}{2}$

$= a\sin\left(\frac{B-C}{2}\right)$.

5. (B) $A = 45^\circ$ and $A = 75^\circ$

$C = 180^\circ - (A+B) =$

$C = 180^\circ - 120^\circ = 60^\circ$

Now, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin 45^\circ} = \frac{b}{\sin 75^\circ} = \frac{c}{\sin 60^\circ}$

$\Rightarrow \frac{a}{1/\sqrt{2}} = \frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{c}{\sqrt{3}/2}$

$\Rightarrow$ a = $\frac{2b}{\sqrt{3}+1}$ and c = $\frac{\sqrt{6}b}{\sqrt{3}+1}$

$\Rightarrow a + \sqrt{2}c = \frac{2b}{\sqrt{3}+1} + \frac{2\sqrt{3}b}{\sqrt{3}+1} = 2b$

6. (D) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k(say)$

$\therefore \frac{a}{b^2-c^2} + \frac{c}{b^2-a^2} = 0$

$\Rightarrow \frac{k\sin A}{k^2(\sin^2 B - \sin^2 C)} + \frac{k\sin C}{k^2(\sin^2 B - \sin^2 A)}$

= 0

$\Rightarrow \frac{\sin A}{\sin(B+C)\sin(B-C)} +$

$\frac{\sin C}{\sin(B+A)\sin(B-A)}$

$$\Rightarrow \frac{1}{\sin(B-C)} + \frac{1}{\sin(B-A)} = 0$$

$$\Rightarrow \sin(B-A) + \sin(B-C) = 0$$

$$\Rightarrow \sin(A-B) = \sin(B-C)$$

$$\Rightarrow A-B = B-C \Rightarrow A+C = 2B \Rightarrow B = 60^\circ$$

7. (B) $\tan^{-1}3 + \tan^{-1}x = \tan^{-1}8$

$$\Rightarrow \tan^{-1}x = \tan^{-1}8 - \tan^{-1}3$$

$$\Rightarrow \tan^{-1}x = \tan^{-1}\left(\frac{8-3}{1+24}\right) \Rightarrow x = \frac{1}{5}$$

8. (D) Let $x = \cos\theta$. Then,

$$0 < x < 1 \Rightarrow 0 < \cos\theta < 1 \Rightarrow 0 < \theta < \frac{\pi}{2}$$

Now, $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$

$$= \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

Thus, option (A) is true.

$$\cos^{-1}\sqrt{\frac{1+x}{2}} = \cos^{-1}\left(\cos\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

so, option (B) is true.

$$\sin^{-1}\sqrt{\frac{1-x}{2}} = \sin^{-1}\left(\sin\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\cos^{-1}x$$

so, option (C) is true.

9. (C) $2\sin^3 x + 2\cos^3 x - 3\sin 2x + 2 = 0$

$\sin^3 x + \cos^3 x + 1 - 3\sin x\cos x = 0$

$\sin x + \cos x + 1 = 0$

[If $a^3+b^3+c^3-3abc = 0$ then $a + b + c = 0$]

$$2\sin\frac{x}{2}\cos\frac{x}{2} + 2\cos^2\frac{x}{2} = 0$$

$$2\cos\frac{x}{2}\left\{\cos\frac{x}{2} + \sin\frac{x}{2}\right\} = 0$$

$$\cos\frac{x}{2} = 0 \text{ or, } \cos\frac{x}{2} + \sin\frac{x}{2} = 0$$

$$\Rightarrow \cos\frac{x}{2} = 0 \text{ or, } \tan\frac{x}{2} = -1$$

Now, $\cos\frac{x}{2} = 0 \Rightarrow x = \pi, 3\pi$ and, $\tan\frac{x}{2} = -1$

$$\Rightarrow x = \frac{3\pi}{2}, \frac{7\pi}{2}$$

10. (C) We have,

$\sin^4 x - (k+2)\sin^2 x - (k+3) = 0$

$$\sin^2 x = \frac{(k+2) \pm \sqrt{(k+2)^2 + 4(k+3)}}{2}$$

$$\Rightarrow \sin^2 x = \frac{(k+2) \pm (k+4)}{2}$$

$\Rightarrow \sin^2 x = k + 3, -1$

$\Rightarrow \sin^2 x = k + 3$

This equation will have a solution , if

$0 \le k + 3 \le 1 \Rightarrow -3 \le k \le -2$

11. (C) $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x\sin^2 x + (\cos^2 x + \sin^2 x)^3 - 3\cos^2 x\sin^2 x(\cos^2 x + \sin^2 x) = 2$

$$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x - \frac{5}{4}\sin^2 2x = 0$$

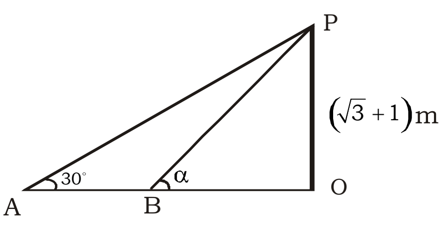
$$\Rightarrow \tan 2x = \pm 1 \quad x \in [0, 2\pi]$$

$$\Rightarrow 2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$$

$$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$$

clearly, these are 8 solutions.

12. (C) We, have OP = $(\sqrt{3}+1)$m and AB = 2 metres.



In Δ's AOP and BOP, we have

$$\tan 30^\circ = \frac{\sqrt{3}+1}{OA} \text{ and } \tan\alpha = \frac{\sqrt{3}+1}{OB}$$

$$\Rightarrow OA = (\sqrt{3}+1)\sqrt{3} \text{ and } OB = (\sqrt{3}+1)\cot\alpha$$

$$\Rightarrow OA - OB = (3+\sqrt{3}) - (\sqrt{3}+1)\cot\alpha$$

$\Rightarrow 2 = 3 + \sqrt{3} - (\sqrt{3}+1)\cot\alpha$

$\Rightarrow \cot\alpha = 1 \Rightarrow \alpha = 45^\circ$

13. (C) $f(x) = \frac{1}{2}\left\{\frac{\sin x}{|\cos x|} + \frac{|\sin x|}{\cos x}\right\}$

Since $\sin x$ and $|\cos x|$ are periodic with periods 2π and π respectively. Therefore, $\frac{\sin x}{|\cos x|}$ is periodic with period 2π. similarly, $\frac{|\sin x|}{\cos x}$ is periodic with period 2π.

Hence, $f(x)$ is periodic with period 2π.

14. (B) We know, that a polynomial function $f(x)$ of degree n satisfying.

$f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ for all $(x \neq 0) \in R$, is of the form

$f(x) = 1 \pm x^n$ for all $(\neq 0) \in R$.

We are given that $f(3) = -26$.

$\therefore f(x) = 1 - x^n$ ....(I)

$\Rightarrow f(3) = 1 - 3^n$

$\Rightarrow -26 = 1 - 3^n$ $[\because f(3) = -26]$

$\Rightarrow 3^n = 27 \Rightarrow 3^n = 3^3 \Rightarrow n=3$

substituting n = 3 in (I), we get

$f(x) = 1 - x^3 \Rightarrow f(4) = 1 - 4^3 = -63$

15. (C) We have, $f(x) = \sqrt{2-x} + \sqrt{1+x}$

clearly, $f(x)$ is defined for

$2 - x \geq 0$ and $1 + x \geq 0 \Rightarrow x \leq 2$ and $x \geq -1 \Rightarrow$ $x \in [-1, 2]$ so domain $(f) = [-1,2]$.

Let, $y = \sqrt{2-x} + \sqrt{1+x}$ (i)

$\Rightarrow y^2 = 3 + 2\sqrt{2 + x - x^2}$ (ii)

$\Rightarrow \left(\frac{y^2-3}{2}\right)^2 = 2 + x - x^2$

$\Rightarrow x^2 - x = 2 - \left(\frac{y^2-3}{2}\right)^2$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 = \frac{9}{4} - \left(\frac{y^2-3}{2}\right)^2$

$\Rightarrow x - \frac{1}{2} = \pm\sqrt{\frac{9}{4} - \left(\frac{y^2-3}{2}\right)^2}$

$\Rightarrow x = \frac{1}{2} \pm \sqrt{\frac{9}{4} - \left(\frac{y^2-2}{2}\right)^2}$

For x to be real, we must have

$\frac{9}{4} - \left(\frac{y^2-3}{2}\right)^2 \geq 0$

$\Rightarrow \lim_{x\to\sqrt{2}^-} f(x) = \lim_{x\to\sqrt{2}^+} f(x) = f(\sqrt{2}) \leq 0$

$\Rightarrow -\frac{3}{2} \leq \frac{y^2-3}{2} \leq \frac{3}{2}$

$\Rightarrow 0 \leq y^2 \leq 6 \Rightarrow -\sqrt{6} \leq y \leq \sqrt{6} \Rightarrow y \in [-\sqrt{6}, \sqrt{6}]$ (iii)

Also, from (i) and (ii), we have

$y^2 \geq 3$ and $y \geq 0 \Rightarrow y \geq \sqrt{3}$ (iv)

From (iii) and (iv), we have

$y \in [\sqrt{3}, \sqrt{6}]$

Hence, range $(f) = [\sqrt{3}, \sqrt{6}]$

16. (A) $\lim_{x\to\pi/4} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

$= \lim_{x\to\pi/4} \frac{\{(\cos x + \sin x)^2\}^{5/2} - (2)^{5/2}}{(1+\sin 2x) - 2}$

$\lim_{x\to\pi/4} \frac{(1+\sin 2x)^{5/2} - 2^{5/2}}{(1+\sin 2x) - 2}$

$\lim_{y\to 2} \frac{y^{5/2} - 2^{5/2}}{y - 2}$,where y = 1 + sin2x

$= \frac{5}{2} \times 2^{5/2 - 1} = 5\sqrt{2}$.

17. (C) $\lim_{x\to a}\left(2 - \frac{a}{x}\right)\tan\frac{\pi x}{2a} = \lim_{x\to a}\left\{1 + \left(1 - \frac{a}{x}\right)\right\}^{\tan\frac{\pi x}{2a}}$

$= e^{\lim_{x\to a}\left(1-\frac{a}{x}\right)\tan\frac{\pi x}{2a}} = e^{\lim_{x\to a}\left(\frac{x-a}{x}\right)\tan\frac{\pi x}{2a}}$

$= e^{\lim_{h\to0}\frac{h}{a+h}\tan\frac{\pi}{2a}(a+h)} = e^{\lim_{h\to0}\frac{h}{a+h}\tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right)}$

$= e^{\lim_{h\to0}-\frac{h}{a+h}\cot\frac{\pi h}{2a}} = e^{\lim_{h\to0}-\frac{h}{a\tan^{\pi h}/_{2a}}}$

$e^{\lim_{h\to0}-\frac{2}{\pi}\left\{\frac{\frac{\pi h}{2a}}{\tan^{\pi h}/_{2a}}\left(\frac{a}{a+h}\right)\right\}} = e^{-2/\pi}$

18. (D) $\lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0} f(-h)$

$\Rightarrow \lim_{x\to0^-} f(x) = \lim_{h\to0}\frac{\tan^{-1}(-h+[-h])}{[-h]+2h}$

$= \lim_{h\to0}\frac{\tan^{-1}(-1-h)}{-1+2h}$

$= \tan^{-1}1 = \frac{\pi}{4}$

and, $\lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h) = \lim_{h\to0} f(h)$

$\Rightarrow \lim_{x\to0^+} f(x) = \lim_{h\to0}\frac{\tan^{-1}(h+[h])}{[h]-2h}$

$\Rightarrow \lim_{x\to0^+} f(x) = \lim_{h\to0}\frac{\tan^{-1}h}{-2h} = \frac{1}{2} \quad [\because [h]=0]$

Hence, $\lim_{x\to0} f(x)$ does not exist.

19. (B) $\lim_{x\to0}\frac{x^n\sin^n x}{x^n-\sin^n x} = -\lim_{x\to0}\frac{x^n\left(\frac{\sin x}{x}\right)^n}{\left(\frac{\sin x}{x}\right)^n-1}$

$= -\lim_{x\to0}\frac{x^n\left(\frac{\sin x}{x}\right)^n}{\left\{\frac{\left(\frac{\sin x}{x}\right)^n-1^n}{\left(\frac{\sin x}{x}\right)-1}\right\}\times\left(\frac{\sin x}{x}-1\right)}$

$= -\frac{(1)}{n(1)^{n-1}}\times\lim_{x\to0}\frac{x^{n+1}}{\sin x - x}$

$= -\frac{1}{n}\lim_{x\to0}\frac{x^{n+1}}{\sin x - x}$

$= -\frac{1}{n}\lim_{x\to0}\frac{(n+1)x^n}{\cos x - 1}$

$= -\frac{(n+1)}{n}\lim_{x\to0}\frac{x^n}{\cos x-1} = -\left(\frac{n+1}{n}\right)\lim_{x\to0}\frac{n\,x^{n-1}}{-\sin x}$

$= (n+1)\lim_{x\to0}\frac{x^{n-1}}{\sin x} = (n+1)(n-1)\lim_{x\to0}\frac{x^{n-2}}{\cos x}$

$= (n^2-1)\times\lim_{x\to0} x^{n-2} = n^2-1, \text{if } n = 2$

20. (A) $f(x)$ will be continuous at $x = \pi/2$, if

$\lim_{x\to\pi/2} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \lim_{x\to\pi/2} f(x)\frac{1-\sin x}{(\pi-2x)^2} = \lambda$

$\Rightarrow \frac{1}{4}\lim_{x\to\pi/2}\frac{1-\cos(\pi/2-x)}{(\pi/2-x)^2} = \lambda$

$\Rightarrow \frac{1}{4}\times\frac{1}{2} = \lambda$

$\Rightarrow \lambda = \frac{1}{8}$

21. (C) It is given that $f(x)$ is continuous in $[0, \pi/2]$. so, it is continuous at $x = \pi/4$.

$\therefore f\left(\frac{\pi}{4}\right) = \lim_{x\to\pi/4} f(x) \Rightarrow f\left(\frac{\pi}{4}\right) = \lim_{x\to\pi/4}\frac{1-\tan x}{4x-\pi}$

$\Rightarrow f\left(\frac{\pi}{4}\right) = \lim_{x\to\pi/4}\frac{\cos x - \sin x}{4\cos x(x-\pi/4)}$

$\Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{4}\lim_{x\to\pi/4}\frac{\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)}{\left(x-\frac{\pi}{4}\right)\cos x}$

$\Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{1}{2\sqrt{2}}\lim_{x\to\pi/4}\frac{\sin\left(x-\frac{\pi}{4}\right)}{\left(x-\frac{\pi}{4}\right)}\times\frac{1}{\cos x}$

$\Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{1}{2\sqrt{2}}\times1\times\sqrt{2} = -\frac{1}{2}$

22. (A) we have,

$f(x)$ = Degree of $(u^{x^2} + u^2 + 2u + 3)$

$\Rightarrow f(x) = \begin{cases} x^2 & , x > \sqrt{2} \\ 2 & , x \le \sqrt{2} \end{cases}$

$\therefore \lim_{x\to\sqrt{2}^-} f(x) = 2 = f(\sqrt{2})$

and, $\lim_{x\to\sqrt{2}^+} f(x) = \lim_{x\to\sqrt{2}} x^2 = \left(\sqrt{2}\right)^2 = 2$

$\therefore \lim_{x\to\sqrt{2}^-} f(x) = \lim_{x\to\sqrt{2}^+} f(x) = f\left(\sqrt{2}\right)$

so, $f(x)$ is continuous at $x = \sqrt{2}$

Now, (LHD at $x = \sqrt{2}$) = $\left(\frac{d}{dx}(2)\right)_{x=\sqrt{2}} = 0$

and, (RHD at $x = \sqrt{2}$) = $\left(\frac{d}{dx}\left(x^2\right)\right)_{x=\sqrt{2}} = (2x)_{x=\sqrt{2}} =$

$2\sqrt{2}$

clearly, (LHD at $x = \sqrt{2}$) $\neq$ (RHD at $x = \sqrt{2}$)

so, $f(x)$ is not differentiate at $x = \sqrt{2}$.

23. (C) For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{x\to 0}\frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x} = f(0)$$

$$\Rightarrow f(0) = \lim_{x\to 0} e^x \frac{\left(e^{\tan x - x} - 1\right)}{\tan x - x}$$

$$+\lim_{x\to 0}\frac{\ln(\sec x + \tan x) - x}{\tan x - x}$$

$$\Rightarrow f(0) = \lim_{x\to 0} e^x\left(\frac{e^{\tan x - x} - 1}{\tan x - x}\right)$$

$$+\lim_{x\to 0}\frac{\log(\sec x + \tan x) - x}{x^3\left(\frac{\tan x - x}{x^3}\right)}$$

$$\Rightarrow f(0) = 1\times e^0 + 3\lim_{x\to 0}\frac{\ln(\sec x + \tan x) - x}{x^3}$$

$$\left[\because \lim_{x\to 0}\frac{\tan x - x}{x^3} = \frac{1}{3}\right]$$

$\Rightarrow f(0) = 1 + 3\lim_{x\to 0}\frac{\sec x - 1}{3x^2}$ [By L' Hospital's rule]

$$\Rightarrow f(0) = 1 + 3\lim_{x\to 0}\frac{1 - \cos x}{3\cos x . x^2}$$

$$\Rightarrow f(0) = 1 + 3\times\frac{1}{3}\times\frac{1}{2} = \frac{3}{2}$$

24. (B) Let $y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ and $z = \sqrt{1 - x^2}$ .Then,

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right) = \cos^{-1}\left(2x^2 - 1\right)$$

$$\Rightarrow y = \begin{cases} 2\cos^{-1} x & , if\ 0 \le x \le 1 \\ 2\pi - 2\cos^{-1} x & , if\ -1 \le x \le 0 \end{cases}$$

$$\therefore \frac{dy}{dx} = \begin{cases} \frac{-2}{\sqrt{1 - x^2}} & ,\text{if } 0 < x < 1 \\ \frac{2}{\sqrt{1 - x^2}} & ,\text{if} -1 < x < 0 \end{cases}$$

$z = \sqrt{1 - x^2} \Rightarrow \frac{dz}{dx} = \frac{-x}{\sqrt{1 - x^2}}$ for all $x \in (-1, 1)$

$$\therefore \frac{dy}{dz} = \begin{cases} \frac{2}{x}, 0 < x < 1 \\ \frac{-2}{x}, -1 < x < 0 \end{cases}$$

Hence, $\left(\frac{dy}{dz}\right)_{x=1/2} = 4$

25. (C) $y = x + e^x$

$$\Rightarrow \frac{dy}{dx} = 1 + e^x$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{1 + e^x}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{1 + e^x}\right)$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\frac{1}{\left(1 + e^x\right)^2}\frac{d}{dy}\left(1 + e^x\right)$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\frac{1}{\left(1 + e^x\right)^2}\frac{dx}{dy} = \frac{-e^x}{\left(1 + e^x\right)^3}$$

26. (B) $f(x) = \log|x| = \begin{cases} \log x & , x > 0 \\ \log(-x) & , x < 0 \end{cases}$

$$\therefore f'(x)\begin{cases} \frac{1}{x} & , x > 0 \\ -\frac{1}{x}\times(-1) = \frac{1}{x} & , x < 0 \end{cases}$$

$\Rightarrow f'(x) = \frac{1}{x}$ for all $x \neq 0$.

27. (D) $\frac{dx}{dy} = \left(\frac{dy}{dx}\right)^{-1}$

$\Rightarrow \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}$

$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^{-1}\right\}\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-2}\frac{d}{dx}\left(\frac{dy}{dx}\right).\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$

28. (C)The equations of the two curves are

$C_1 : x^3 - 3xy^2 = a$

$C_2 : 3x^2y - y^3 = b$

Differentiating (i) and (ii) w.r.t, we get

$\left(\frac{dy}{dx}\right)_{C_1} = \frac{x^2 - y^2}{2xy}$ and $\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2xy}{x^2 - y^2}$

Clearly, $\left(\frac{dy}{dx}\right)_{C_1} \times \left(\frac{dy}{dx}\right)_{C_2} = -1$

so, the two curves intersect at right angle.

29. (B) $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos\theta)$, $\frac{dy}{d\theta} = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{\theta}{2}$

Length of the normal = $y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$= a(1-\cos\theta)\sqrt{1+\tan^2\frac{\theta}{2}}$

$= 2a\sin^2\frac{\theta}{2}\times\sec\frac{\theta}{2} = 2a\tan\frac{\theta}{2}\sin\frac{\theta}{2}$

Length of normal at $\theta = \frac{\pi}{2}$ is

2a $\tan\frac{\pi}{4}\sin\frac{\pi}{4} = \sqrt{2}a$.

30. (C)

We have, $y = x^3 - ax^2 + x + 1$ (i)

$\Rightarrow \frac{dy}{dx} = 3x^2 - 2ax + 1$

It is given that at each point on the curve (i), the tangent is inclined at an acute angle with the positive direction of x-axis.

$\therefore \frac{dy}{dx} \geq 0$ for all (x,y) lying on the curve (i).

$\Rightarrow 3x^2 - 2ax + 1 \geq 0$ for all x

$\Rightarrow 4a^2 - 12 \leq 0 \Rightarrow a^2 - 3 \leq 0 \Rightarrow -\sqrt{3} \leq a \leq \sqrt{3}$

$\Rightarrow |a| \leq \sqrt{3}$

31. (B) $y = x^x = e^{x\log_e x}$, $x > 0$

$\Rightarrow \frac{dy}{dx} = x^{-x}(-1 - \log_e x)$

$\Rightarrow \frac{dy}{dx} = -x^{-x}(1 + \log_e x)$

For the point of local maximum, we must have $\frac{dy}{dx} = 0 \Rightarrow 1 + \log_e x = 0 \Rightarrow x = \frac{1}{e}$.

Clearly, $1 + \log_e x < 0$ for $0 < x < \frac{1}{e}$ and $1 + \log_e x > 0$ for $x > \frac{1}{e}$

Thus, $\frac{dy}{dx} > 0$ *for* $0 < x < \frac{1}{e}$ *and* $\frac{dy}{dx} < 0$ *for* $x > \frac{1}{e}$.

$\Rightarrow x = \frac{1}{e}$ is the point of local maximum.

Clearly, $\frac{dy}{dx} = 0$ at $x = \frac{1}{e}$. So, the normal at $x = \frac{1}{e}$ is parallel to y-axis and its equation is given by $x = \frac{1}{e}$

32. (D) Let r be the radius and V be the volume of the sphere. Then,

$V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$ $\left[\because \frac{dV}{dt} = \frac{dr}{dt}(given)\right]$

$\Rightarrow 4\pi r^2 = 1 \Rightarrow r = \frac{1}{2\sqrt{\pi}}$

33. (A) Let r, l and h denote respectively the radius, slant height and height of the cone at any time t. Then,

$l^2 = r^2 + h^2$

$\Rightarrow 2l\frac{dl}{dt} = 2r\frac{dr}{dt} + 2h\frac{dh}{dt}$

$\Rightarrow l\frac{dl}{dt} = r\frac{dr}{dt} + h\frac{dh}{dt}$

$\Rightarrow l\frac{dl}{dt} = 7\times 3 + 24\times -4$

$\Rightarrow l\frac{dl}{dt} = -75$

When r = 7 and h = 24, we have

$l^2 = 7^2 + 24^2$

$\Rightarrow l = 25$

$\therefore l\frac{dl}{dt} = -75 \Rightarrow \frac{dl}{dt} = -3$

Let S denote the lateral surface area. Then,

$S = \pi rl$

$\Rightarrow \frac{dS}{dt} = \pi\left\{\frac{dr}{dt}l + r\frac{dl}{dt}\right\} =$

$\pi\{3\times 25 + 7\times -3\} = 54\pi$.

34. (A) We are given that the side c and angle C remain constant.

$\therefore \frac{c}{\sin C} = k$ (constant)

$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = k$

$\Rightarrow$ a = k sin$A$ and b = k sin$B$

$\Rightarrow \frac{da}{dA} = k\cos A$ and $\frac{db}{dB} = k\cos B$

Now, da = $\frac{da}{dA}.dA \Rightarrow$ da = k cosA . dA $\Rightarrow$

$\frac{da}{\cos A} = k\,dA$

and, $db = \frac{db}{dB}$.dB $\Rightarrow$ db = k.cosB.dB

$\Rightarrow \frac{db}{\cos B} = kdB$

$\therefore \frac{da}{\cos A} + \frac{db}{\cos B}$ = kdA+ kdB = kd(A+B)= $kd(\pi - C)$

$\frac{da}{\cos A} + \frac{db}{\cos B} = k(0) = 0$ $[\because \pi - C = constant]$

$[\therefore d(\pi - C) = 0]$

35. (C) Let V be the volume of the cylinder of base radius r and height h. Then,

$V = \pi r^2 h$

$\Rightarrow dV = d(\pi r^2 h)$

$\Rightarrow dV = \pi(h\,dr^2 + r^2 dh)$

$\Rightarrow dV = \pi(2rhdr + r^2 dh)$

$\Rightarrow \frac{dV}{V} = \frac{\pi(2rh\,dr + r^2 dh)}{\pi r^2 h}$

$\Rightarrow \frac{dV}{V} = \frac{2}{r}dr + \frac{dh}{h}$

$\Rightarrow \frac{dV}{V}\times 100 = 2\frac{dr}{r}\times 100 + \frac{dh}{h}\times 100$

$\Rightarrow \frac{\Delta V}{V}\times 100 = 2\left(\frac{\Delta r}{r}\times 100\right) + \left(\frac{\Delta h}{h}\times 100\right)$

= 2 × 1+1=3

36. (C) Let r be the radius, C be the circumference and A be the area of the circle. Then,

$C = 2\pi r$ and $A = \pi r^2$

$\therefore \Delta C = \frac{dC}{dr}\Delta r$ and $\Delta A = \frac{dA}{dr}\Delta r$

$\Rightarrow \Delta C = 2\pi\Delta r$ and $\Delta A = 2\pi r\Delta r$

we have,C = 56 and $\Delta C = 0.02$

$\therefore \frac{\Delta C}{C} = \frac{0.02}{56} = \frac{1}{2800}$

$\Rightarrow \frac{2\pi\Delta r}{2\pi r} = \frac{1}{2800} \Rightarrow \frac{\Delta r}{r} = \frac{1}{2800}$

$\therefore \frac{\Delta A}{A}\times 100 = \frac{2\pi r\Delta r}{\pi r^2}\times 100 = 2\left(\frac{\Delta r}{r}\times 100\right)$

$= 2\times\frac{1}{28} = \frac{1}{14}$

37. (D) It is given that $f(x)$ satisfies all the conditions for Rolle's theorem. Therefore,

$f(3) = f(5) = 0$

$\Rightarrow x = 3$ and $x = 5$ are roots of $f(x)$.

$\Rightarrow f(x) = (x-3)(x-5) = x^2 - 8x + 15$

$\therefore \int_3^5 f(x)d(x) = \int_3^5 (x^2 - 8x + 15)dx$

$= \left[\frac{x^3}{3} - 4x^2 + 15x\right]_3^5$

$\Rightarrow \int_3^5 f(x)dx$

$= \frac{1}{3}(125-27) - 4(25-9) + 15(5-3) = -\frac{4}{3}$

38. (C) Consider the polynomial $f(x)$ given by

$$f(x) = ax^4 + bx^3 + cx^2 + dx$$

$$\Rightarrow f'(x) = 4ax^3 + 3bx^2 + 2cx + d$$

We have,
$f(0) = 0$ and,
$f(3) = 81a + 27b + 9c + 3d$
$= 3(27a + 9b + 3c + d) = 0$ [Given]
Therefore, 0 and 3 are roots of $f(x)=0$.
Consequently, by Rolle's theorem $f'(x)=0$ i.e. $4ax^3 + 3bx^2 + 2cx + d = 0$ has at least one root between 0 and 3.

39. (D) since $f(x)$ satisfies conditions of Rolle's theorem.

$$\therefore f(1) = f(3)$$

$\Rightarrow 1 - 6 + a + b = 27 - 54 + 3a + b \Rightarrow a=11$
Now,
$f'(x) = 3x^2 - 12x + a = 3x^2 - 12x + 11$.

Clearly, $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$. Hence, $a=11$.

40.(B) We have,

$$f'(x) = -x^3 + ax^2 + bx + \frac{5}{2}\sin 2x$$

$$\Rightarrow f'(x) = -3x^2 + 2ax + b + 5\cos 2x$$

For $f'(x)$ to be decreasing on R, we must have $f'(x) < 0$ for all $x \in R$
$\Rightarrow -3x^2 + 2ax + b + 5\cos 2x < 0$ for all $x \in R$
$\Rightarrow -3x^2 + 2ax + b + 5 < 0$ for all $x \in R$
$\Rightarrow 3x^2 - 2ax - b - 5 > 0$ for all $x \in R$
$\Rightarrow 4a^2 - 4\times3(-b-5) < 0$ $[\because Disc < 0]$
$\Rightarrow a^2 + 3b + 5 < 0$

41. (A)

$$f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$$

$$f'(x) = \begin{vmatrix} 1 & 0 & 0 \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix} +$$

$$\begin{vmatrix} x+a^2 & ab & ac \\ 0 & 1 & 0 \\ ac & bc & x+c^2 \end{vmatrix} + \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow f'(x) = 3x^2 + 2x(a^2 + b^2 + c^2)$$

For $f(x)$ to be decreasing, we must have $f'(x) < 0$

$$\Rightarrow 3x^2 + 2x(a^2 + b^2 + c^2) < 0$$

$$\Rightarrow x \in \left(-\frac{2}{3}(a^2 + b^2 + c^2), 0\right)$$

42. (D)

43. (*) We have,

$$f'(x) = 2\tan^{-1}\frac{1-x}{1+x} = 2(\tan^{-1}1 - \tan^{-1}x)$$

$$= \frac{\pi}{2} - 2\tan^{-1}x$$

$$\Rightarrow f'(x) = -\frac{2}{1+x^2} < 0 \text{ for all } x \in [0,1]$$

$\Rightarrow f(x)$ decreases on $[0,1]$

$\Rightarrow$ Range of $f = [f(1), f(0)] = [0, \pi/2]$

Hence, both the statements are true and statement -2 is a correct explanation of statement -1.

44. (B) We have,

$$f(x) = \sin x(1 + \cos x)$$

$$\Rightarrow f(x) = \sin x + \frac{1}{2}\sin 2x$$

$$\Rightarrow f'(x) = \cos x + \cos 2x \text{ and}$$

$$f''(x) = -\sin x - 2\sin 2x$$

For local maximum or minimum, we must have

$$f'(x) = 0$$

$$\Rightarrow \cos x + \cos 2x = 0$$

$$\Rightarrow 2\cos^2 x + \cos x - 1 = 0$$

$$\Rightarrow (2\cos x - 1)(\cos x + 1) = 0$$

$$\Rightarrow \cos x = \frac{1}{2}, -1 \Rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

Now,

$$f''\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} - 2\sin\frac{2\pi}{3} < 0, \; f''(\pi) = 0$$

$$and\; f''\left(\frac{5\pi}{3}\right) = -\sin\frac{5\pi}{3} - 2\sin\frac{10\pi}{3} > 0$$

Thus, $f(x)$ attains a local maximum at $x = \frac{\pi}{3}$

$\therefore$ Maximum ordinate $= f\left(\frac{\pi}{3}\right) =$

$$\sin\frac{\pi}{3}\left(1 + \cos\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \times \frac{3}{2} = \frac{3\sqrt{3}}{4}$$

45. (C) We have,

$f'(x) = 2\cos 2x - 1$

$\therefore f'(x) = 0$

$\Rightarrow 2\cos 2x - 1 = 0 \Rightarrow \cos 2x = \frac{1}{2}$

$\Rightarrow 2x = -\pi/3, \pi/3 \Rightarrow x = -\pi/6, \pi/6$

Now,

$f(-\pi/2) = \pi/2, f(\pi/2) = -\pi/2$

$f\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$ and $f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$.

Clearly, $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ is the greatest value of $f(x)$ and its least value is $-\pi/2$.

Hence, the greatest difference is

$\frac{\sqrt{3}}{2} - \frac{\pi}{6} - \left(-\frac{\pi}{2}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$

46. (B) We have,

$f(x) = (a^2 - 3a + 2)\left\{\cos^2\frac{x}{4} - \sin^2\frac{x}{4}\right\}$
$+ (a-1)x + \sin 1$

$\Rightarrow f(x) = (a-1)(a-2)\cos\frac{x}{2} + (a-1)x + \sin 1$

$\Rightarrow f'(x) = \frac{-1}{2}(a-1)(a-2)\sin\frac{x}{2} + (a-1)$

$\Rightarrow f'(x) = (a-1)\left\{1 - \frac{(a-2)}{2}\sin\frac{x}{2}\right\}$

If $f(x)$ does not possess critical points, then

$\Rightarrow f'(x) \neq 0$ for any $x \in R$

$\Rightarrow (a-1)\left\{1 - \frac{(a-2)}{2}\sin\frac{x}{2}\right\} \neq 0$ for any $x \in R$

$\Rightarrow a \neq 1$ and $1 - \left(\frac{a-2}{2}\right)\sin\frac{x}{2} = 0$ must not have any solution in R

$\Rightarrow a \neq 1$ and $\sin\frac{x}{2} = \frac{2}{a-2}$ is not solvable inR.

$\Rightarrow a \neq 1$ and $\left|\frac{2}{a-2}\right| > 1$

$\Rightarrow a \neq 1$ and $|a-2| < 2$

$\Rightarrow a \neq 1$ and $-2 < a-2 < 2$

$\Rightarrow a \neq 1$ and $0 < a < 4$

$\Rightarrow a \in (0,1) \cup (1,4)$.

47. (A) We have, $f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$

$\Rightarrow f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 - 3\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$
$-4\left\{\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 - 2\right\}$

$\Rightarrow f(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 - 4\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$
$-3\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) + 8$

Clearly, $f(x)$ is defined for all $x > 0$.

Let, $\sqrt{x} + \frac{1}{\sqrt{x}} = t$. Then, $t \geq 2$ for all $x > 0$.

Also, let $g(t) = t^3 - 4t^2 - 3t + 8$. Then,
$g'(t) = 3t^2 - 8t - 3$ and $g''(t) = 6t - 8$

$\Rightarrow g'(t) = (3t+1)(t-3)$ and $g''(t) = 6t - 8$

Clearly, $g'(t) = 0$ for $t = 3$ and $g''(t) = 10 > 0$

Thus, g is minimum when t=3 and the minimum value of g is g(3) = -10.

48. (*) We have, $g(x) = \tan^{-1}x$

$\Rightarrow g'(x) = \frac{1}{1+x^2} > 0$ for all $x \Rightarrow g(x)$ is increasing on $[0, \infty]$

so, statement -2 is true.

Now, $f(x) = \tan^{-1}\frac{1-x}{1+x}$

$\Rightarrow f(x) = \tan^{-1}1 - \tan^{-1}x = \pi/4 - g(x)$

$\because$ g(x) is increasing on $[0, \infty]$

$\Rightarrow$ -g(x) is decreasing on $[0, \infty]$

$\Rightarrow \frac{\pi}{4} - g(x)$ is decreasing on $[0, \infty]$

$\Rightarrow f(x)$ is decreasing on [0,1]

$\Rightarrow f(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$ is the greatest value

and, $f(1) = \frac{\pi}{4} - g(1) = \frac{\pi}{4} - \frac{\pi}{4} = 0$ is the least value of $f(x)$.

Hence, required difference $= \frac{\pi}{4} - 0 = \frac{\pi}{4}$

49. (B) $\int \frac{\cos^4 x}{\sin^2 x} dx$

$\Rightarrow I = \int \frac{(1-\sin^2 x)^2}{\sin^2 x} dx$

$\Rightarrow I = \int (\operatorname{cosec}^2 x + \sin^2 x - 2) dx$

$\Rightarrow I = -\cot x + \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) - 2x + D$

$\Rightarrow I = -\cot x - \frac{1}{4}\sin 2x - \frac{3}{2}x + D$

Hence, A=-1, B $= -\frac{1}{4}$, C =-3

50. (B) $I = \int \frac{4x+1}{x^2+3x+2} dx$

$\Rightarrow I = \int \frac{2(2x+3)-5}{x^2+3x+2} dx$

[Using $4x+1 = \lambda(2x+3)+\mu$]

$\Rightarrow I = 2\int \frac{2x+3}{x^2+3x+2} dx - 5\int \frac{1}{x^2+3x+2} dx$

$\Rightarrow I = 2\int \frac{1}{x^2+3x+2} d(x^2+3x+2)\, dx$

$-5\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$

$\Rightarrow I = 2\log|x^2+3x+2| - 5\log\left|\frac{x+1}{x+2}\right| + C$

$\Rightarrow I = 2\log|x+1| + 2\log|x+2| - 5\log|x+1|$

$+5\log|x+2| + C$

$\Rightarrow I = -3\log|x+1| + 7\log|x+2| + C$

$\therefore$ a = -3 and b = 7 $\Rightarrow a+b=4$

51. (A) We have, $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\therefore \tan^{-1}\sqrt{\frac{1-x}{1+x}} = \frac{1}{2}\cos^{-1}\left(\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}\right) = \frac{1}{2}\cos^{-1}x$

Thus, we have

$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}} dx$

$\Rightarrow I = \frac{1}{2}\int \cos^{-1} dx$

$\Rightarrow I = \frac{1}{2}\left\{x\cos^{-1}x - \int \frac{-1}{\sqrt{1-x^2}} \times x \times dx\right\}$

$\Rightarrow I = \frac{1}{2}\left\{x\cos^{-1}x - \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} dx\right\}$

$\Rightarrow I = \frac{1}{2}\left\{x\cos^{-1}x - \sqrt{1-x^2}\right\} + C$

52. (C) We have,

$\Rightarrow I = \int \frac{e^{\tan^{-1}x}(1+x+x^2)}{1+x^2} dx$

$\Rightarrow I = \int e^{\theta}(1+\tan\theta+\tan^2\theta)\, d\theta$,

where $x = \tan\theta$

$\Rightarrow I = \int e^{\theta}(\sec^2\theta + \tan\theta)\, d\theta$

$\Rightarrow I = e^{\theta}\tan\theta + C$

$\left[\because \int e^x\{f(x)+f'(x)\}dx = e^x f(x)\right]$

$I = xe^{\tan^{-1}x} + C$

53. (C) Let

$I = \int \frac{1}{\sin^6 x + \cos^6 x} dx = \int \frac{\sec^6 x}{1+\tan^6 x} dx$

$I = \int \frac{(1+\tan^2 x)^2}{1+\tan^6 x}\sec^2 x\, dx = \int \frac{(1+t^2)^2}{1+t^6} dt$,

Where t = tan$x$

$I = \int \frac{t^2+1}{t^4-t^2+1} dt = \int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-1} dt$

$= \int \frac{d\left(t-\frac{1}{t}\right)}{\left(t-\frac{1}{t}\right)^2 + 1^2}$ dt

$\Rightarrow$ I = $\tan^{-1}\left(t-\frac{1}{t}\right)$ + C = $\tan^{-1}(\tan x - \cot x)$+C

54. (D)

$I = \int \frac{1}{\sin\left(x-\frac{\pi}{3}\right)\cos x} dx$

$$I = \frac{1}{\cos\frac{\pi}{3}}\int \frac{\cos\left\{x-\left(x-\frac{\pi}{3}\right)\right\}}{\sin\left(x-\frac{\pi}{3}\right)\cos x}dx$$

$$\Rightarrow I = 2\int \frac{\cos x\cos\left(x-\frac{\pi}{3}\right)+\sin x\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x-\frac{\pi}{3}\right)\cos x}dx$$

$$\Rightarrow I = 2\int\left\{\cot\left(x-\frac{\pi}{3}\right)+\tan x\right\}dx$$

$$\Rightarrow I = 2\left\{\log\left|\sin\left(x-\frac{\pi}{3}\right)\right| - \log|\cos x|\right\}+C$$

$$\Rightarrow I = 2\log\left|\sin\left(x-\frac{\pi}{3}\right)\sec x\right|+C$$

55. (B) Let I = $\int \frac{1}{\tan x+\cot x+\sec x+\cos ecx}dx$

Then,

$$I = \int \frac{\sin x\cos x}{1+\sin x+\cos x}dx$$

$$\Rightarrow I = \frac{1}{2}\frac{(1+2\sin x\cos x-1)}{1+\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{(\sin x+\cos x)^2-1}{1+\sin x+\cos x}dx$$

$$\Rightarrow I = \frac{1}{2}\int \frac{(\sin x+\cos x+1)(\sin x+\cos x-1)}{1+\sin x+\cos x}dx$$

$$\Rightarrow I = \frac{1}{2}\int(\sin x+\cos x-1)dx$$

$$\Rightarrow I = \frac{1}{2}\left[(-\cos x+\sin x-x)\right]+c$$

56. (D) We have,

$$I = \int_e^{e^4}\sqrt{\log_e x}\,dx = \int_1^2 \sqrt{t^2}e^{t^2}2tdt$$,where $\log_e x = t^2$

$$\Rightarrow 2\int_1^2 t^2e^{t^2}dt \Rightarrow 2\int_1^2 \underset{I}{t}\,\underset{II}{e^{t^2}2t}\,dt$$

$$=\left[te^{t^2}\right]_1^2 - \int_1^2 e^{t^2}dt = 2e^4-e-a.$$

57. (B) We have, {x} = x- [x]

Let $k \le x < k+1$,where $k \in N$ .Then,

$$I = \int_0^X\left(\{x\}-\frac{1}{2}\right)dx = \int_0^X\left(x-[x]-\frac{1}{2}\right)dx$$

$$\Rightarrow I = \int_0^X(x-[x])dx - \int_0^x\frac{1}{2}dx$$

$$\Rightarrow I = \int_0^k(x-[x])dx - \int_k^x(x-[x])dx - \frac{x}{2}$$

$$\Rightarrow I = \frac{k}{2}+\int_k^x(x-k)dx - \frac{x}{2}$$

$$\Rightarrow I = \frac{k}{2}+\left(\frac{x^2}{2}-kx\right)-\left(\frac{k^2}{2}-k^2\right)-\frac{x}{2}$$

$$\Rightarrow I = \frac{x^2}{2}-kx+\frac{k(k+1)}{2}-\frac{x}{2}$$

$$\Rightarrow I = \frac{1}{2}(x^2-2kx+k^2)+\frac{1}{2}(k-x)$$

$$\Rightarrow I = \frac{1}{2}(x-k)^2-\frac{1}{2}(x-k)$$

$$\Rightarrow I = \frac{1}{2}(x-[x])^2-\frac{1}{2}(x-[x])$$

$$\Rightarrow I = \frac{1}{2}\{x\}^2-\frac{1}{2}\{x\} = \frac{1}{2}\{x\}(\{x\}-1)$$

58. (B) Consider the integral limit from

$$\int_{1/e}^{\cot x}\frac{dt}{(1+t^2)}$$

Putting, $t = \frac{1}{u}$,we get

$$\int_{1/e}^{\cot x}\frac{dt}{t(1+t)^2} = \int_e^{\tan x}\frac{-1/u^2du}{1/u(1+1/u^2)} = -\int_e^{\tan x}\frac{udu}{1+u^2}$$

$$\int_{1/e}^{\cot x}\frac{tdt}{t(1+t^2)} = -\int_e^{\tan x}\frac{tdt}{1+t^2} =$$

$$-\left\{\int_e^{1/e}\frac{tdt}{1+t^2}+\int_{1/e}^{\tan x}\frac{tdt}{1+t^2}\right\}$$

$$\Rightarrow \int_{1/e}^{\cot x}\frac{dt}{t(1+t^2)} = -\int_e^{1/e}\frac{t}{1+t^2}dt - \int_{1/e}^{\tan x}\frac{t\,dt}{1+t^2}$$

$$\therefore \int_{1/e}^{\tan x}\frac{t}{1+t^2}dt + \int_{1/e}^{\cot x}t(1+t^2)dt$$

$$= -\int_e^{1/e}\frac{t}{1+t^2}dt = -\frac{1}{2}\left[\log(1+t^2)\right]^{1/e}$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{e^2}\right) - \log\left(1+e^2\right)\right]$$

$$= -\frac{1}{2}\left[\log\left(e^2+1\right) - 2\log e - \log\left(1+e^2\right)\right]$$

$$= \log e = 1$$

59. (D) Let

$$f(x) = \sum_{r=1}^{10} \tan rx = \tan x + \tan 2x + ... + \tan 10x$$

Clearly, $f(x)$ is a periodic function with period π.

$$\therefore I = \int_0^{100\pi} \left(\sum_{r=1}^{10} \tan rx\right) dx$$

$$\Rightarrow I = 100\int_0^{\pi} \left(\sum_{r=1}^{10} \tan rx\right) dx$$

$$\Rightarrow I = 100\left[\sum_{r=1}^{10} \int_0^{\pi} \tan rx\, dx\right]$$

$$\Rightarrow I = 100 \times \sum_{r=1}^{10} 0 = 0$$

$$\left[\begin{array}{l} \because \tan r(\pi - x) = -\tan rx \text{ for } r = 1, 2, ...10 \\ \therefore \int_0^{\pi} \tan rx\, dx = 0 \end{array}\right]$$

60. (C) Let $I = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x) dx$

$$I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin\left(\frac{9x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx$$

$$\Rightarrow I = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}} dx$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 9\theta}{\sin\theta} d\theta$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2} \frac{(\sin 9\theta - \sin 7\theta) + (\sin 7\theta - \sin 5\theta) + (\sin 5\theta - \sin 3\theta) + (\sin 3\theta - \sin\theta) + \sin\theta}{\sin\theta} d\theta$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/2} (\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1) d\theta$$

$$\Rightarrow I = \frac{8}{\pi} \times \frac{\pi}{2} = 4$$

61. (B) Let $I = \int_0^{\pi} \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}\, dx$. Then,

$$I = \int_0^{\pi} \sqrt{\left(1 - 2\sin\frac{x}{2}\right)^2}\, dx$$

$$\Rightarrow I = \int_0^{\pi} \left|1 - 2\sin\frac{x}{2}\right| dx \quad \left[\because \sqrt{x^2} = |x|\right]$$

$$\Rightarrow I = \int_0^{\pi/3} \left|1 - 2\sin\frac{x}{2}\right| dx + \int_{\pi/3}^{\pi} \left(1 - 2\sin\frac{x}{2}\right) dx$$

$$\Rightarrow I = \int_0^{\pi/3} \left|1 - 2\sin\frac{x}{2}\right| dx + \int_{\pi/3}^{\pi} \left(2\sin\frac{x}{2} - 1\right) dx$$

$$\Rightarrow I = \left[x + 4\cos\frac{x}{2}\right]_0^{\pi/3} + \left[-4\cos\frac{x}{2} - x\right]_{\pi/3}^{\pi}$$

$$\Rightarrow I = \left(\frac{\pi}{3} + 4\cos\frac{\pi}{6} - 4\right) + \left(0 - \pi + 4\cos\frac{\pi}{6} + \frac{\pi}{3}\right)$$

$$\Rightarrow I = -\frac{\pi}{3} + 4\sqrt{3} - 4$$

62. (C) Since $\sin^{-1}(\cos x) + \cos^{-1}(\cos x)$ is a period function with period 2π.

$$\therefore I = \int_{t+2\pi}^{t+5\pi/2} \left\{\sin^{-1}(\cos x) + \cos^{-1}(\cos x)\right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \left\{\sin^{-1}(\cos x) + \cos^{-1}(\cos x)\right\} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi}{2} dx = \frac{\pi^2}{4}$$

63. (C) It is given that $\int_0^a f(x) dx = 1 + \frac{a^2}{2}\sin a$

Differentiating this w.r.t. to a, we get

$$f(a) = a\sin a + \frac{a^2}{2}\cos a$$

64. (A) We have, $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$

$\Rightarrow 2xy\,dy = (x^2 + y^2 + 1)dx$

$\Rightarrow 2xy\,dy - y^2dx = (x^2 + 1)dx$

$\Rightarrow xd(y^2) - y^2dx = (x^2 + 1)dx$

$\Rightarrow \frac{xd(y^2) - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx$

$\Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$

on integrating, we get

$\frac{y^2}{x} = x - \frac{1}{x} + C$

$y^2 = x^2 - 1 + Cx \Rightarrow y^2 = \left(x + \frac{C}{2}\right)^2 - 1 - \frac{C^2}{5}$

Clearly, it represents a hyperbola

65. (A) We have, $\frac{dy}{dx} = (x - y)^2$

Let $x - y = v$. Then, $1 - \frac{dy}{dx} = \frac{dv}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = (x - y)^2$

$\Rightarrow 1 - \frac{dv}{dx} = v^2$

$\Rightarrow 1 - v^2 = \frac{dv}{dx}$

$\Rightarrow dx = \frac{1}{1 - v^2}dv$

$\Rightarrow 2\int dx = 2\int \frac{1}{1 - v^2}dv$

$\Rightarrow 2x = \log\left(\frac{1 + v}{1 - v}\right) + \log C$

$\Rightarrow C\left(\frac{1 + v}{1 - v}\right) = e^{2x}$

$\Rightarrow C\left(\frac{x - y + 1}{y - x + 1}\right) = e^{2x}$

$\Rightarrow C(x - y + 1) = e^{2x}(y - x + 1)$.

66. (D) We have,

$y^2dx + (x^2 - xy + y^2)dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y^2}{x^2 - xy + y^2}$

putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} = -\frac{v^2}{1 - v + v^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{-v - v^3}{v^2 - v + 1}$

$\Rightarrow \frac{v^2 - v + 1}{v(v^2 + 1)}dv = -\frac{dx}{x}$

$\Rightarrow \left(\frac{1}{v} - \frac{1}{v^2 + 1}\right)dv = -\frac{dx}{x}$

on integrating, we get

$\log v - \tan^{-1}v = -\log x + C$

$\Rightarrow \log\left(\frac{y}{x}\right) - \tan^{-1}\frac{y}{x} = -\log x + C$

$\Rightarrow \log y = \tan^{-1}\frac{y}{x} + C$

67. (D) $(x + y)(dx - dy) = dx + dy$

$\Rightarrow dx - dy = \frac{dx + dy}{x + y}$

$\Rightarrow d(x - y) = \frac{d(x + y)}{x + y}$

$\Rightarrow x - y = \log(x + y) + \log C$ [on integrating]

$\Rightarrow c(x + y) = e^{x-y}$

$\Rightarrow x + y = k\,e^{x-y}$, where $k = \frac{1}{C}$

68. (C) We have, $\frac{d}{dt}(p(t)) = \frac{1}{2}p(t) - 200$

$\Rightarrow \frac{d}{dt}(p(t)) + \left(-\frac{1}{2}\right)(p(t)) = -200$

This is a linear differentital equation with

I.f.= $e^{\int -\frac{1}{2}dt} = e^{-\frac{t}{2}}$ .....(i)

Multiplying both sides of (i) by I.F.= $e^{\int -\frac{1}{2}dt} = e^{-\frac{t}{2}}$, we obtain

$e^{-t/2}\frac{d}{dt}(p(t)) + \left(-\frac{1}{2}\right)p(t)e^{-t/2} = -200e^{-t/2}$

Integrating both sides with respect to t, we get $p(t)\,e^{-t/2} = 400\,e^{-t/2} + C$ ....(ii)

Putting t = 0 and p(0)=100, we get

100 = 400 + C $\Rightarrow C = -300$

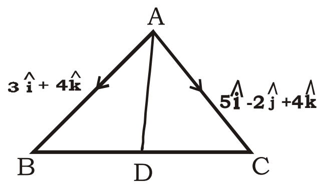
Putting C=-300, we get

$p(t)e^{-t/2} = 400e^{-t/2} - 300$

$\Rightarrow p(t) = 400 - 300e^{t/2}$

69. (C)Let D be the mid-point of BC. Then,

$\overrightarrow{AD} = \dfrac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$



$\overrightarrow{AD} = \dfrac{(3\hat{i} + 4\hat{k}) + (5\hat{i} - 2\hat{j} + 4\hat{k})}{2}$

$= 4\hat{i} + \hat{j} + 4\hat{k}$

$\Rightarrow \left|\overrightarrow{AD}\right| = \sqrt{16+1+16} = \sqrt{33}$

70. (B) Let $\vec{u} = \vec{a} + 2\vec{b}$ and $\vec{v} = 5\vec{a} - 4\vec{b}$ and let θ be the angle between $\vec{a}$ and $\vec{b}$. It is given that $\vec{u}$ and $\vec{v}$ are perpendicular to each other. Therefore, $\vec{u} \cdot \vec{v} = 0$

$\Rightarrow (\vec{a} + 2\vec{b}).(5\vec{a} - 4\vec{b}) = 0$

$\Rightarrow 5\left|\vec{a}\right|^2 - 8\left|\vec{b}\right|^2 + 10\left(\vec{a}.\vec{b}\right) - 4\left(\vec{a}.\vec{b}\right) = 0$

$\Rightarrow -3 + 6(\vec{a}.\vec{b}) = 0$

$\Rightarrow -3 + 6\cos\theta = 0$

$[\because \vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = \cos\theta$

$\cos\theta = \dfrac{1}{2} \Rightarrow \theta = 60^\circ$

71. (B) $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} \times \vec{b})$

$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) =$

$(\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} + (\vec{b}.\vec{b})\vec{a} - (\vec{b}.\vec{a})\vec{b}$

$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) =$

$(\vec{a}.\vec{b})\vec{a} - \left|\vec{a}\right|^2 \vec{b} + + \left|\vec{b}\right|^2 \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$

$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = (\vec{a}.\vec{b})\vec{a} - \vec{b} + \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$

$\left[\because \left|\vec{a}\right| = \left|\vec{b}\right| = 1\right]$

$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = (\vec{a}.\vec{b} + 1)\vec{a} - (\vec{a}.\vec{b} + 1)\vec{b}$

$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = (\vec{a}.\vec{b} + 1)\ (\vec{a} - \vec{b})$

Hence, $(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})$ is parallel to $(\vec{a} - \vec{b})$

72. (C) We know that z-coordinate of every point on xy-plance is zero. So, let $(x,y,0)$ be a point on xy-plane such that PA = PB = PC.

Now, PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y-0)^2 + (0-3)^2 =$

$(x-0)^2 + (y-3)^2 + (0-2)^2$

$\Rightarrow 4x - 6y = 0 \Rightarrow 2x - 3y = 0$

and, PB = PC

$\Rightarrow PB^2 = PC^2$

$\Rightarrow (x-0)^2 + (y-3)^2 + (0-2)^2$

$= (x-0)^2 + (y-0)^2 + (0-1)^2$

$\Rightarrow -6y + 12 = 0$

$\Rightarrow y = 2$

putting y = 2 in (i), we obtain $x = 3$

Hence, the required point is (3,2,0).

73. (C) We have, $l + m + n = 0$ .....(i)

and, $l^2 = m^2 + n^2$ ....(ii)

$\therefore (-m-n)^2 = m^2 + n^2$ [on eliminating l]

$\Rightarrow 2mn = 0 \Rightarrow m = 0$ *or* $n = 0$

Now,

$m = 0 \Rightarrow l + n = 0$ and $l^2 = n^2$ [putting m=0 in (i) and (ii)]

Thus, the direction ratios of one of the two lines are proportional, to -n,0,n or -1,0,1.

when n=0

$l + m + n = 0$ and $l^2 = m^2 + n^2$

$l + m = 0$ and $l^2 = m^2 \Rightarrow l = -m$

Thus, the direction ratios of one of two lines are proportional to -m,m,0 or -1,1,0.

Let θ be the angle between the given lines.Then,

$\cos\theta = \dfrac{-1 \times -1 + 0 \times 1 + 1 \times 0}{\sqrt{(-1)^2 + 0^2 + 1^2}\sqrt{(-1)^2 + 1^2 + 0}}$

$= \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

74. (C) The given equations are not in the standard form.The equations of the given lines in standard form can be written as

$$\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0} \quad \text{....(i)}$$

$$\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2} \quad \text{....(ii)}$$

It θ is the angle between the given lines, then

$$\cos\theta = \frac{(3)(1)+(-2)(3/2)+(0)(2)}{\sqrt{3^2+(-2)^2+0^2}\sqrt{1^2+(3/2)^2+2^2}} = 0$$

$\Rightarrow \theta = \pi/2$

75. (C) We have,
N = 100, F = 45 ,l= 20 and h=10 and, Median= 25 Let f be the frequency of the median class.

Then, Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$$\Rightarrow 25 = 20 + \frac{50-45}{f} \times 10$$

$$\Rightarrow 5 = \frac{50}{f} \Rightarrow f = 10$$

76. (C) Let $\overline{X}$ denote the mean of the given numbers. Then,

$$\overline{X} = \frac{1+(1+d)+(1+2d)+....+(1+100d)}{101}$$

$$\Rightarrow \overline{X} = \frac{\frac{101}{2}\{1+(1+100d)\}}{101} = 1 + 50d$$

$\therefore$ Mean deviation =

$$\frac{1}{101}\left\{\sum_{r=0}^{100} |(1+rd)-(1+50d)|\right\}$$

Mean deviation = $\frac{1}{101}\sum_{r=0}^{100}|r-50|$ d

Mean deviation= $\frac{d}{101} \times 2\sum_{r=1}^{50} r$

Mean deviation= $\frac{2d}{101} \times \frac{50\times 51}{2} = \frac{50\times 51}{101} d$

It is given that the mean deviation is 255.

$$\therefore 255 = \frac{50 \times 51}{101} d \Rightarrow d = 10.1$$

77. (B) $4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$$\Rightarrow 2 \times 2^{2x-1} + 2^{2x-1} = 3^{x-\frac{1}{2}} \times 3 + 3^{x-\frac{1}{2}}$$

$$\Rightarrow 2^{2x-1}(2+1) = 3^{x-\frac{1}{2}}(3+1)$$

$$\Rightarrow 2^{2x-1} \times 3 = 3^{x-\frac{1}{2}} \times 4$$

$$\Rightarrow 2^{2x-3} = 3^{x-\frac{3}{2}}$$

$$\Rightarrow (2^2)^{x-\frac{3}{2}} = 3^{x-\frac{3}{2}}$$

$$\Rightarrow 4^{x-\frac{3}{2}} = 3^{x-\frac{3}{2}}$$

$$\Rightarrow x - \frac{3}{2} = 0 \Rightarrow x = \frac{3}{2}$$

78. (D) We have,
$3^x = 4^{x-1}$
$\Rightarrow x \log_{10} 3 = (x-1)\log_{10} 4$
$\Rightarrow x = (x-1)\log_3 4$
$\Rightarrow x = 2(x-1)\log_3 2$
$\Rightarrow x(2\log_3 2 - 1) = 2\log_3 2$

$$\Rightarrow x = \frac{2\log_3 2}{2\log_3 2 - 1}$$

Now, $x = \frac{2\log_3 2}{2\log_3 2 - 1}$

$$\Rightarrow x = \frac{2}{2 - \frac{1}{\log_3 2}} = \frac{2}{2 - \log_2 3}$$

$$\Rightarrow x = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{1}{1 - \log_{2^2} 3} = \frac{1}{1 - \log_4 3}$$

Hence, option(A),(B) and (C) are correct and option (D) is not correct.

79. (C) $2^{x+2}.3^{\frac{3x}{x-1}} = 3^2$

$$\Rightarrow (x+2)\log 2 + \frac{3x}{x-1}\log 3 = 2\log 3$$

$$\Rightarrow (x+2)\log 2 + \left(\frac{3x}{x-1} - 2\right)\log 3 = 0$$

$$\Rightarrow (x+2)\log 2 + \left(\frac{x+2}{x-1}\right)\log 3 = 0$$

$$\Rightarrow (x+2)\left\{\log 2 + \frac{\log 3}{x-1}\right\} = 0$$

$\Rightarrow x + 2 = 0$ or, $\log 2 + \frac{\log 3}{x-1} = 0$

$\Rightarrow x = -2$ or, $x = 1 - \frac{\log 3}{\log 2}$

80. (A) We have, $x + y + z = 1$.
$\therefore xy(x+y)^2 + yz(y+z)^2 + zx(z+x)^2$
$= xy(1-z)^2 + yz(1-x)^2 + zx(1-y)^2$
$= xy + yz + zx - 6xyz + xyz(x+y+z)$
$= xy + yz + zx - 5xyz$ ..(i)
Using AM $\geq$ HM, we obtain

$$\frac{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}{3} \geq \frac{3}{x+y+z}$$

$\Rightarrow xy + yz + zx \geq 9xyz$ $[x + y + z = 1]$ ..(ii)
From (i) and (ii), we obtain
$xy(x+y)^2 + yz(y+z)^2 + zx(z+x)^2 \geq 9xyz - 5xyz$
$\Rightarrow xy(x+y)^2 + yz(y+z)^2 + zx(z+x)^2 \geq 4xyz$

81. (D) $f(f(x)) = x$ for all $x \neq -1$

$\Rightarrow f\left(\frac{\alpha x}{x+1}\right) = x$ for all $x \neq -1$

$\Rightarrow \frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\frac{\alpha x}{x+1}+1} = x$ for all $x \neq -1$

$\Rightarrow \frac{\alpha^2 x}{\alpha x + x + 1} = x$ for all $x \neq -1$

$\Rightarrow \alpha^2 x = (\alpha+1)x^2 + x$ for all $x \neq -1$

$\Rightarrow (\alpha+1)x^2 + (1-\alpha^2)x = 0$ for all $x \neq -1$

$\Rightarrow (\alpha+1) = 0$ and $1 - \alpha^2 = 0$

$\Rightarrow \alpha = -1$

82. (A) $|z_1| = |z_2| + |z_1 - z_2|$

$\Rightarrow |z_1 - z_2| = |z_1| - |z_2|$

$\Rightarrow |z_1 - z_2|^2 = (|z_1| - |z_2|)^2$

$\Rightarrow |z_1|^2 + |z_2|^2 - 2|z_1||z_2| \cos(\theta_1 - \theta_2)$

$= |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$,

where $\theta_1 = \arg(z)$ and $\theta_2 = \arg(z_2)$.

$\Rightarrow \cos(\theta_1 - \theta_2) = 1 \Rightarrow \cos(\theta_1 - \theta_2) = \cos 0^o$

$\Rightarrow \theta_1 - \theta_2 = 0$

$\Rightarrow \arg(z_1) - \arg(z_2) = 0$

83. (A) We have,

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$$

$\Rightarrow S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 \ldots$ upto 4n terms

$\Rightarrow S_n = (3^2 - 1^2) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2)$ upto 2n terms

$\Rightarrow S_n = 2(4+6+12+14+20+22+\ldots$ upto 2n terms

$\Rightarrow S_n = 2\left[\frac{n}{2}\{8 + (n-1)\times 8\} + \frac{n}{2}\{12 + (n-1)\times 8\right]$

$\Rightarrow S_n = 8n^2 + 8n^2 + 4n$
$\Rightarrow S_n = 4n(4n+1)$
$\Rightarrow S_n$ = Product of a multiple of 4 and its successor.

Clearly, 1056= 32×33=(4×8)((4×8)+1) and 1332=36×37=(4×9)((4×9)+1) are products of multiple of 4 and its successor.
Hence, s_n can take values 1056 and 1332.

84. (C) we have, $(x-a)(x-b) + c = 0$

$\Rightarrow x^2 - x(a+b) + ab + c = 0$

Since α, β are roots of this equation.

$\therefore \alpha + \beta = a + b$ and

$\alpha\beta = ab + c$

Now, $(x - c - \alpha)(x - c - \beta) = c$

$\Rightarrow (x-c)^2 - (x-c)(\alpha+\beta) + \alpha\beta = 0$

$\Rightarrow (x-c)^2 - (x-c)(a+b) + ab + c - c = 0$

$\Rightarrow (x-c)^2 - (x-c)(a+b) + ab = 0$

$\Rightarrow \{(x-c-a)(x-c) - b\} = 0$

$\Rightarrow x = c + a$ and $x = c + b$

Thus, the given equation has c + a and c + b as its roots.

85. (D) The total number of unordered pairs of disjoint subsets of S, except ordered pair, (θ, ϕ) is

$\left({}^4c_0 \times 2^4 + {}^4c_1 \times 2^3 + {}^4c_2 \times 2^2 + {}^4c_3 \times 2^1 + {}^4c_4 \times 2^2\right) - 1$

$= (1+2)^4 - 1 = 80$

Total number of ordered pairs of disjoint subsets of S is equal to $\frac{80}{2} + 1 = 41$.

86. (A) $(1-x)^5(1+x+x^2+x^3)^4$

$=(1-x)^5(1+x)^4(1+x^2)^4$

$=(1-x)(1-x^2)^4(1+x^2)^4$

$=(1-x)(1-x^4)^4$

$=(1-x)({}^4C_0-{}^4C_1x^4+{}^4C_2x^8-{}^4C_3x^{12}+{}^4C_4x^{16})$

Coefficient of $x^{13}={}^4C_3=4$.

87. (A) $\frac{2}{x}+\frac{2}{3x^3}+\frac{2}{5x^5}+\ldots..to\infty$

$=2\left\{\frac{1}{x}+\frac{1}{3}\left(\frac{1}{x}\right)^3+\frac{1}{5}\left(\frac{1}{x}\right)^5+\ldots\ldots\infty\right\}$

$=\log\left(\frac{1+\frac{1}{x}}{1-\frac{1}{x}}\right)$

$=\log\left(\frac{1+y^3-1}{1-y^3+1}\right)\left[\because x(y^3-1)=1\Rightarrow y^3-1=\frac{1}{x}\right]$

$=\log\left(\frac{y^3}{2-y^3}\right)$

88. (A) $\left(A(A+B)^{-1}B\right)^{-1}$

$=B^{-1}(A+B)A^{-1}$

$\left[\because (ABC)^{-1}=C^{-1}B^{-1}A^{-1}\right]$

$=B^{-1}AA^{-1}+B^{-1}BA^{-1}$

$=B^{-1}I+IA^{-1}=B^{-1}+A^{-1}$

$\therefore \left(A(A+B)^{-1}B\right)^{-1}(AB)=(B^{-1}+A^{-1})AB$

$=B^{-1}(AB)+A^{-1}(AB)$

$=B^{-1}(BA)+A^{-1}(AB)$

$[\because AB=BA]$

$=(B^{-1}B)A+(A^{-1}A)B$

$=IA+IB=A+B.$

89. (C) We have,

$\begin{vmatrix}a&b&c\\b&c&a\\c&a&b\end{vmatrix}=\begin{vmatrix}a+b+c&b&c\\a+b+c&c&a\\a+b+c&a&b\end{vmatrix}$

[Applying $C_1\to C_1+C_2+C_3$]

$=(a+b+c)\begin{vmatrix}1&b&c\\1&c&a\\1&a&b\end{vmatrix}$

$=(a+b+c)\begin{vmatrix}1&b&c\\0&c-b&a-c\\0&a-b&b-c\end{vmatrix}$

$=(a+b+c)\begin{vmatrix}c-b&a-c\\a-b&b-c\end{vmatrix}$

$=(a+b+c)(-b^2-c^2-a^2+ac+bc-2bc)$

$=-(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=-(a+b+c)\left(a+b\omega+c\omega^2\right)\left(a+b\omega^2+c\omega\right)$

$\Rightarrow k=w$

90. (B) We have, $P(A\cap B)=\frac{1}{6}$ and $P(\overline{A}\cap\overline{B})=\frac{1}{3}$

$\Rightarrow P(A)\,P(B)=\frac{1}{6}\Rightarrow P(\overline{A})P(\overline{B})=\frac{1}{3}$

$\Rightarrow xy=\frac{1}{6}$ and $(1-x)(1-y)=\frac{1}{3}$, where

P(A)= x, P(B)= y

$\Rightarrow xy=\frac{1}{6}$ and $x+y=\frac{5}{6}$

$\Rightarrow x=\frac{1}{2}$ and $y=\frac{1}{3}$ or $x=\frac{1}{3}$ and $y=\frac{1}{2}$

91. (C) The number of tosses required to get head is even mens that the head is obtain in either 2nd toss or in 4th toss or in 6th toss etc.

$\therefore (1-p)p+(1-p)^3p+(1-p)^5p+\ldots=\frac{2}{5}$

$\Rightarrow \frac{(1-p)p}{1-(1-p)^2}=\frac{2}{5}\Rightarrow 5p(1-p)=2(2p-p^2)$

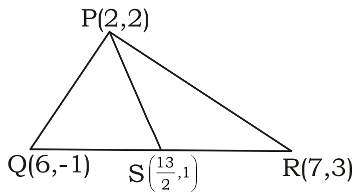
$\Rightarrow 3p^2-p=0\Rightarrow p=\frac{1}{3}$

92. (D) The slope of PS is

$m=\frac{1-2}{\frac{13}{2}-2}=\frac{-2}{9}$

so, the equation of the line passing through (1,-1) and parallel to PS is

$y+1=-\frac{2}{9}(x-1)$ or, $2x+9y+7=0$



93. (A)Let (h,k) be the coordinates of the centre of the circle. Since it touches *x*-axis. So, radius of the circle is $|k|$.
This circle also touches a circle of radius 2 having centre at (0,3). Therefore, distance between their centre is equal to sum or difference of their radii.
i.e. $\sqrt{(h-0)^2+(k-3)^2} = |k| \pm 2$
$\Rightarrow h^2+(k-3)^2 = k^2+4\pm 4k$
$\Rightarrow h^2-10k+5=0$ or, $h^2-2k+5=0$
Hence, the locus of (h,k) is
$x^2-10y+5=0$ or $x^2-2y+5=0$
which are equations of a parabola.

94. (D) Let P $(at_1^2, 2at_1)$ and Q$(at_2^2, 2at_2)$ be the end-ponts of a focal chord of the parabola $y^2 = 4ax$. Then,
PQ = $a(t_2-t_1)^2$
The equation of PQ is
$(t_1+t_2)y = 2x - 2a$
It is given that PQ = c and it is at a distance b from the vertex.
$\therefore a(t_2-t_1)^2 = c$ and, $\left|\dfrac{-2a}{\sqrt{(t_1+t_2)^2+4}}\right| = b$
$\Rightarrow (t_2-t_1)^2 = \dfrac{c}{a}$ and, $\dfrac{2a}{(t_2-t_1)} = b$ $[t_1t_2=-1]$
$\Rightarrow \left(\dfrac{2a}{b}\right)^2 = \dfrac{c}{a} \Rightarrow 4a^3 = b^2c$

95. (A) The equation of the parabola is
$(y-1)^2 = 4(x+1)$
The equation of any normal to this parabola is
$y-1 = m(x+1)-2m-m^3$
If it passess through {-2,1}. Then,
$0 = -m - 2m - m^3 \Rightarrow m^3 + 3m = 0 \Rightarrow m = 0$
$[\therefore m^2 + 3 \neq 0]$
so, there is only one normal passing through (-2,1).

96. (B) Each element of A can image the every element of B.
$\therefore$ Total number of functions = $n \times n \times n \times \ldots m$ times = n^m

97. (C) The composition of two bijection is a bijection.

98. (C) Domain of $\text{cosec}^{-1}(x) = (-\infty, -1) \cup [1, \infty]$

99. (B) Let the diagonals be
$a = 3i + 6j - 2k$
and $b = 4i - j - 3k$
Now, $a \cdot b = (3i + 6j - 2k) \cdot (4i - j + 3k)$
$= 12 - 6 - 6 = 0$
Now, $|a| = \sqrt{9+36+4} = 7$
and $|b| = \sqrt{16+1+9} = \sqrt{26}$
$\therefore |a| \neq |b|$
So, diagonals are perpendicular but they are not equal, therefore it is rhombus.

100. (C) $P(A) = \dfrac{1}{3}$, $P(A \cup B) = \dfrac{11}{12}$, $P(B) = \dfrac{3}{4}$
$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$\Rightarrow \dfrac{11}{12} = \dfrac{1}{3} + \dfrac{3}{4} - P(A \cap B)$
$\Rightarrow P(A \cap B) = \dfrac{1}{3} + \dfrac{3}{4} - \dfrac{11}{12}$
$P(A \cap B) = \dfrac{1}{6}$
Now, $P\left(\dfrac{B}{A}\right) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1/6}{1/3} = \dfrac{1}{2}$

101. (B) Let *r* be the radius of balloon.
$\therefore$ Its volume,
$$V = \frac{4}{3}\pi r^3$$
$$\Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi . 3r^2 \frac{dr}{dt}$$
$$\Rightarrow 4 = \frac{4}{3}\pi . 3(4)^2 \frac{dr}{dt}$$
$$\left(\because \frac{dV}{dt} = 4\,\text{cm}^3/\text{s} \text{ and } r = 4\,\text{cm}\right)$$
$$\Rightarrow \frac{dr}{dt} = \frac{1}{16\pi} \quad \ldots \text{(i)}$$
Now, surface area
$$S = 4\pi r^2$$
$$\frac{dS}{dt} = 4\pi . 2r \frac{dr}{dt}$$
$$= 4\pi \times 2 \times 4 \frac{1}{16\pi} \text{ [from Eq. (i)]}$$

$= 2\ cm^2/s$

102. (D) $\frac{1+\tan 15^\circ}{1-\tan 15^\circ} = \frac{\tan 45^\circ + \tan 15^\circ}{1-\tan 45 \tan 15^\circ}$ $[\because \tan 45^\circ = 1]$

$= \tan(45^\circ + 15^\circ)$

$= \tan 60^\circ = \sqrt{3}$

103. (C) $\because f(x) = kx^3 - 9x^2 + 9x + 3$

On differentiating w.r.t. x, we get

$\therefore\ f'(x) = 3kx^2 - 18x + 9$

For a function to be montonically increasing

$f'(x) > 0 \Rightarrow 3kx^2 - 18x + 9 > 0$

$\Delta = b^2 - 4ac < 0$

$\Rightarrow 36 - 12k < 0$

$\Rightarrow k > 3$

104. (C) $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{5}{x} + \cos^{-1}\frac{\sqrt{x^2-144}}{x} = \frac{\pi}{2}$

$[\because \sin^{-1}x = \cos^{-1}\sqrt{1-x^2}\,]$

But $\sin^{-1}y + \cos^{-1}y = \frac{\pi}{2}$

$\therefore\ \frac{5}{x} = \frac{\sqrt{x^2-144}}{x}$

$\Rightarrow 25 = x^2 - 144$

$\Rightarrow x^2 = 169$

$\Rightarrow x = 13$

105. (C) $\because \alpha$ and β be the roots of the equation $x^2 - x + 1 = 0$

$\therefore\ \alpha + \beta = 1$ and $\alpha\beta = 1$

Now, $\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{3}i$

$\Rightarrow \alpha = \frac{1+i\sqrt{3}}{2}$ and

$\beta = \frac{1-i\sqrt{3}}{2}$

Now, $\alpha = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\beta = \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

(a) $\alpha^4 - \beta^4 =$

$\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} - \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= 2i\sin\frac{4\pi}{3}$

$\Rightarrow \alpha^4 - \beta^4$ is not real.

(b) $2(\alpha^5 + \beta^5) =$

$2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} + \cos\frac{5\pi}{3} - i\sin\frac{5\pi}{3}\right)$

$= 2.2\cos\frac{5\pi}{3} = 4.\frac{1}{2} = 2$

Now, $(\alpha\beta)^5 = 1$

$\Rightarrow 2(\alpha^5 - \beta^5) \neq \alpha\beta$

(c) $\alpha^6 - \beta^6 = \cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3} -$

$\cos\frac{6\pi}{3} + i\sin\frac{6\pi}{3}$

$= 2i\sin 2\pi = 0$

(d) $\alpha^8 + \beta^8 = \cos\frac{8\pi}{3} + \sin\frac{8\pi}{3} +$

$\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}$

$= 2\cos\frac{8\pi}{3} = 2\left(-\frac{1}{2}\right) = -1$

Now, $(\alpha\beta)^8 = (1)^8 = 1$

$\Rightarrow (\alpha^8 + \beta^8) \neq \alpha\beta^8$

106. (A) We have,

$\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$

$\Rightarrow \tan^{-1}\left\{\frac{(x+3)-(x-3)}{1+(x^2-9)}\right\} = \tan^{-1}\frac{3}{4}$

$\Rightarrow \frac{6}{x^2-8} = \frac{3}{4} \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

107. (D) $\lim_{x\to\pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

$= \lim_{x\to\pi/2} \frac{\tan\left(\frac{\pi}{2}-x\right) - \sin\left(\frac{\pi}{2}-x\right)}{8\left(\frac{\pi}{2}-x\right)^3} = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16}$

108. (D) We have,

$$\lim_{x\to 0}\left\{1+x\ln\left(1+b^2\right)\right\}^{1/x}=2b\sin^2\theta$$

$$\Rightarrow e^{\lim_{x\to 0} x\ln\left(1+b^2\right)\times\frac{1}{x}}=2b\sin^2\theta$$

$$\Rightarrow e^{\ln\left(1+b^2\right)}=2b\sin^2\theta$$

$$\Rightarrow 1+b^2=2b\sin^2\theta$$

$$\Rightarrow \sin^2\theta=\frac{1+b^2}{2b}$$

$$\Rightarrow \sin^2\theta=\frac{1}{2}\left(b+\frac{1}{b}\right)$$

$$\left[b+\frac{1}{b}\ge 2\ \therefore \frac{1}{2}\left(b+\frac{1}{b}\right)\ge 1\right]$$

$$\Rightarrow \sin^2\theta=1$$

$$\Rightarrow \theta=\pm\frac{\pi}{2}$$

109. (D) We have, $\cos(\alpha+\beta)=\frac{4}{5}$ and

$$\sin(\alpha-\beta)=\frac{5}{13}$$

$$\tan(\alpha+\beta)=\frac{3}{5}\ \text{ and } \tan(\alpha-\beta)=\frac{5}{12}$$

$$\therefore \tan 2\alpha=\tan(\alpha+\beta+\alpha-\beta)$$

$$=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$\Rightarrow \tan 2\alpha=\frac{\frac{3}{5}+\frac{5}{12}}{1-\frac{3}{5}\times\frac{5}{12}}=\frac{56}{33}$$

110. (B) $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$

$$=\frac{\sin^2 A}{\cos A(\sin A-\cos A)}+\frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$=\frac{\sin^3 A-\cos^3 A}{\sin A\cos A(\sin A-\cos A)}$$

$$=\frac{1+\sin A\cos A}{\sin A\cos A}=\sec A\ \text{cosec} A+1$$

111. (A) we have,
sinA:sinC = sin(A-B):sin(B-C)
$\Rightarrow$ sin(B+C):sin(A+B) = sin(A-B):sin(B-C)
$\Rightarrow$ sin(B+C)sin(B-C) = sin(A+B)sin(A-B)
$\Rightarrow$ $\sin^2B-\sin^2C=\sin^2A-\sin^2B$
$\Rightarrow$ $2\sin^2B=\sin^2A+\sin^2C$
$\Rightarrow$ $2b^2=a^2+c^2$ [Using Sine rule]
$\Rightarrow$ a^2,b^2,c^2 are in A.P.

112. (D) Let at any time t the length of the diagonal be x cm.

Then, each side $=\frac{x}{\sqrt{2}}$ cm

we have, $\frac{dx}{dt}=0.2$ cm/sec

Let A be the area of the square. Then,

$$A=\left(\frac{x}{\sqrt{2}}\right)^2=\frac{1}{2}x^2$$

$$\Rightarrow \frac{dA}{dt}=x\frac{dx}{dt}$$

$$\Rightarrow \left(\frac{dA}{dt}\right)_{\frac{x}{\sqrt{2}}=\frac{30}{\sqrt{2}}}=30\times 0.2=6cm^2/\sec$$

113. (C) Let at any time t, h cm be the thickness of ice. Then,

V = volume of ice $=\frac{4}{3}\pi(10+h)^3-\frac{4}{3}\pi\times 10^3$

$$\Rightarrow \frac{dV}{dt}=4\pi(10+h)^2\frac{dh}{dt}$$

$$\Rightarrow -50=4\pi(10+h)^2\times\frac{dh}{dt}$$

$$\left[\because \frac{dv}{dt}=-50\,cm^3/\min \text{ and h = 5}\right]$$

$$\Rightarrow \frac{dh}{dt}=-\frac{1}{18\pi}cm/\min$$

114. (C) We have,

$$\Rightarrow I=\int\frac{1+\cos 8x}{\tan 2x-\cot 2x}dx$$

$$\Rightarrow I=\int\frac{2\cos^2 4x}{-2\cot 4x}dx$$

$$[\because \cot\theta-\tan\theta=2\cot 2\theta]$$

$$\Rightarrow I=-\int\cos 4x\sin 4x\,dx$$

$$=-\frac{1}{2}\int\sin 8x\,dx=\frac{1}{16}\cos 8x+C$$

$$\therefore a\cos 8x+C=\frac{1}{16}\cos 8x+C$$

$$\Rightarrow a=\frac{1}{16}$$

115. (*) We have,

$$I = \int \frac{\cos 2x - \cos 2\alpha}{\sin x - \sin \alpha} dx$$

$$\Rightarrow I = \int \frac{(1 - 2\sin^2 x) - (1 - 2\sin^2 \alpha)}{\sin x - \sin \alpha} dx$$

$$\Rightarrow I = -2\int (\sin x + \sin \alpha) dx$$

$$\Rightarrow I = 2(\cos x - x \sin \alpha) + C$$

116. (B)

$$I = \int \frac{1}{x^2 (x^4 + 1)^{3/4}} dx$$

$$\Rightarrow I = \int \frac{1}{x^5 \left(1 + \frac{1}{x^4}\right)^{3/4}} dx$$

$$\Rightarrow I = -\frac{1}{4} \int \left(1 + \frac{1}{x^4}\right)^{-3/4} \left(\frac{-4}{x^5}\right) dx$$

$$\Rightarrow I = -\frac{1}{4} \int \left(1 + \frac{1}{x^4}\right)^{-3/4} d\left(1 + \frac{1}{x^4}\right)$$

$$\Rightarrow I = -\frac{1}{4} \left\{ \frac{\left(1 + \frac{1}{x^4}\right)^{1/4}}{1/4} \right\} + C = -\left(1 + \frac{1}{x^4}\right)^{1/4} + C$$

117. (A) we have,

$$I = \int \frac{1 + \log x}{\sqrt{x^{2x} - 1}} dx$$

$$\int \frac{1}{x^x \sqrt{(x^x)^2 - 1^2}} d(x^x) = \sec^{-1}(x^x) + C$$

118. (B) Let

$$I = \int f(x) d(x^2) = \int \sqrt{1 + x^2}\, d(x^2)$$

$$\Rightarrow I = \int \sqrt{1 + x^2}\, d(1 + x^2) \qquad [\because d(x^2) = d(1 + x^2)$$

$$\Rightarrow I = \frac{2}{3}(1 + x^2)^{3/2} + C$$

119. (A) We have,

$$I = \int \frac{1}{x\sqrt{1 - x^3}} dx$$

$$\Rightarrow I = -\frac{1}{3} \int \frac{1}{x^3 \sqrt{1 - x^3}} (-3x^2) dx$$

$$\Rightarrow I = -\frac{1}{3} \int \frac{1}{x^3 \sqrt{1 - x^3}} d(1 - x^3)$$

$$\Rightarrow I = -\frac{1}{3} \int \frac{1}{(1 - t^2)\sqrt{t^2}} 2t\,dt \text{, where } t^2 = 1 - x^2$$

$$\Rightarrow I = -\frac{2}{3} \int \frac{1}{1 - t^2} dt$$

$$\Rightarrow I = \frac{2}{3} \int \frac{1}{t^2 - 1^2} dt$$

$$\Rightarrow I = \frac{1}{3} \log\left|\frac{t - 1}{t + 1}\right| + C$$

$$\Rightarrow I = \frac{1}{3} \log\left|\frac{\sqrt{1 - x^3} - 1}{\sqrt{1 - x^3} + 1}\right| + C$$

120.(A) Let

$$I = \int \frac{x^{5/2}}{\sqrt{1 + x^2}} dx$$

$$\Rightarrow I = \frac{2}{7} \int \frac{1}{\sqrt{1^2 + (x^{7/2})^2}} \frac{7}{2} x^{\frac{5}{2}} dx$$

$$\Rightarrow I = \frac{2}{7} \int \frac{1}{\sqrt{1^2 + (x^{7/2})^2}} d(x^{7/2})$$

$$= \frac{2}{7} \log\left|x^{7/2} + \sqrt{1 + x^7}\right| + C.$$

NDA (MATHS) MOCK TEST - 57 (Answer Key)

1.	(B)	21.	(C)	41.	(A)	61.	(B)	81.	(D)	101.	(B)
2.	(C)	22.	(A)	42.	(D)	62.	(C)	82.	(A)	102.	(D)
3.	(A)	23.	(C)	43.	(*)	63.	(C)	83.	(A)	103.	(C)
4.	(D)	24.	(B)	44.	(B)	64.	(A)	84.	(C)	104.	(C)
5.	(B)	25.	(C)	45.	(C)	65.	(A)	85.	(D)	105.	(C)
6.	(D)	26.	(B)	46.	(B)	66.	(D)	86.	(A)	106.	(A)
7.	(B)	27.	(D)	47.	(A)	67.	(D)	87.	(A)	107.	(D)
8.	(D)	28.	(C)	48.	(*)	68.	(C)	88.	(A)	108.	(D)
9.	(C)	29.	(B)	49.	(B)	69.	(C)	89.	(C)	109.	(D)
10.	(C)	30.	(C)	50.	(B)	70.	(B)	90.	(B)	110.	(B)
11.	(C)	31.	(B)	51.	(A)	71.	(B)	91.	(C)	111.	(A)
12.	(C)	32.	(D)	52.	(C)	72.	(C)	92.	(D)	112.	(D)
13.	(C)	33.	(A)	53.	(C)	73.	(C)	93.	(A)	113.	(C)
14.	(B)	34.	(A)	54.	(D)	74.	(C)	94.	(D)	114.	(C)
15.	(C)	35.	(C)	55.	(B)	75.	(C)	95.	(A)	115.	(*)
16.	(A)	36.	(C)	56.	(D)	76.	(C)	96.	(B)	116.	(B)
17.	(C)	37.	(D)	57.	(B)	77.	(B)	97.	(C)	117.	(A)
18.	(D)	38.	(C)	58.	(B)	78.	(D)	98.	(C)	118.	(B)
19.	(B)	39.	(D)	59.	(D)	79.	(C)	99.	(B)	119.	(A)
20.	(A)	40.	(B)	60.	(C)	80.	(A)	100.	(C)	120.	(A)

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003