NDA MATHS MOCK TEST - 82 (SOLUTION)

1. (C) Given that $a = 15$

and H.M. = 24

then H.M. = $\frac{2ab}{a+b}$

$24 = \frac{2\times 15\times b}{15+b}$

$4 = \frac{5b}{15+b}$

$b = 60$

2. (C) $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}$, $\vec{b} = 2\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{b} + \vec{a} = 4\hat{i} - 2\hat{j} - \hat{k}$

$3\vec{a} - 2\vec{b} = 3(2\hat{i} + 3\hat{j} - 5\hat{k}) - 2(2\hat{i} - 5\hat{j} + 4\hat{k})$

$= 6\hat{i} + 9\hat{j} - 15\hat{k} - 4\hat{i} + 10\hat{j} - 8\hat{k}$

$= 2\hat{i} + 19\hat{j} - 23\hat{k}$

$(\vec{b} + \vec{a}) . (3\vec{a} - 2\vec{b})$

$\Rightarrow (4\hat{i} - 2\hat{j} - \hat{k}).\ (2\hat{i} + 19\hat{j} - 23\hat{k})$

$\Rightarrow 8 - 38 + 23$

$\Rightarrow -7$

3. (A)

4. (D) We know that

$\int \text{cosec}\,x\,dx = \log|\text{cosec}\,x - \cot x| + c$

5. (A) $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots}}}$

$y = \sqrt{\tan x + y}$

On squaring both side

$y^2 = \tan x + y$

On differentiating both side w.r.t 'x'

$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$

$(2y-1)\frac{dy}{dx} = \sec^2 x$

$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$

6. (B) Given word 'COPPER'

(OE) CPPR

as a one letter

total arrangement = $\frac{5!}{2!} \times 2!$ (OE)

= 120

7. (A) $\begin{vmatrix} bc-c^2 & c-a & ab-ca \\ b^2-bc & b-c & bc-c^2 \\ ab-ca & a-b & b^2-bc \end{vmatrix}$

$\Rightarrow \begin{vmatrix} c(b-c) & c-a & a(b-c) \\ b(b-c) & b-c & c(b-c) \\ a(b-c) & a-b & b(b-c) \end{vmatrix}$

$\Rightarrow (b-c)^2 \begin{vmatrix} c & c-a & a \\ b & b-c & c \\ a & a-b & b \end{vmatrix}$

$C_2 \to C_2 + C_3$

$\Rightarrow (b-c)^2 \begin{vmatrix} c & c & a \\ b & b & c \\ a & a & b \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two columns are identical.]

8. (B)

9. (B) Equation

$bx^2 + cx + a = 0$

roots are m and n.

then $m + n = -\frac{c}{b}$

$mn = \frac{a}{b}$

and given that

roots of new equation are $\frac{m^2-1}{2m}$ and $\frac{n^2-1}{2n}$.

then sum of roots = $\frac{m^2-1}{2m} + \frac{n^2-1}{2n}$

$= \frac{(m+n)(mn-1)}{2mn}$

$= \frac{-\frac{c}{b}\left(\frac{a}{b}-1\right)}{\frac{2a}{b}}$

$= -\left[\frac{ac-bc}{2ab}\right]$

product of the roots = $\frac{m^2-1}{2m} \times \frac{n^2-1}{2n}$

$= \frac{m^2n^2 - n^2 - m^2 + 1}{4mn}$

$= \frac{m^2n^2 + 1 - (m+n)^2 + 2mn}{4mn}$

$= \frac{\frac{a^2}{b^2} + 1 - \frac{c^2}{b^2} + \frac{2a}{b}}{\frac{4a}{b}}$

$= \frac{(a+b)^2 - c^2}{4ab}$

Then equation

$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$

$x^2 + \frac{(ac-bc)}{2ab}x + \frac{(a+b)^2 - c^2}{4ab} = 0$

$4abx^2 + 2(ac - bc)x + (a + b)^2 - c^2 = 0$

10. (C) We know that

35 < 45

sin 35 < sin 45 and cos 35 > cos 45

sin 35 < cos 45

then

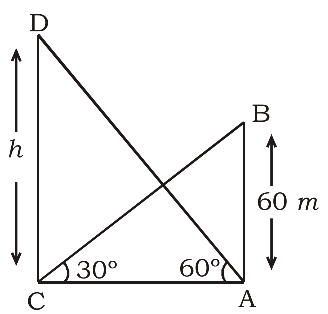
sin 35 < cos 45 < cos 35

sin 35 < cos 35

cos 35 – sin 35 > 0

positive put less than 1

11. (B)



Let height of second tower (bigger)

= $CD = h$ m

then

distance between the two towers $AC = n \times CD$

$AC = n \times h$

In ΔACD

$\tan 60^\circ = \frac{CD}{AC}$

$\sqrt{3} = \frac{h}{n \times h}$

$n = \frac{1}{\sqrt{3}}$

$n^2 = \frac{1}{3}$

12. (C) $\sin\theta + 2\cos\theta = \sqrt{2}$

On squaring both side

$\sin^2\theta + 4\cos^2\theta + 4\sin\theta.\cos\theta = 2$

$1 - \cos^2\theta + 4 - 4\sin^2\theta + 4\sin\theta.\cos\theta = 2$

$5 - 2 = \cos^2\theta + 4\sin^2\theta - 4\sin\theta.\cos\theta$

$3 = (\cos\theta - 2\sin\theta)^2$

$\cos\theta - 2\sin\theta = \sqrt{3}$

13. (C) Let

$S = 8^3 + 9^3 + ... + 15^3$

$= (1^3 + 2^3 + ... + 7^3 + 8^3 + 9^3 ... + 15^3) - (1^3 + 2^3 + ... + 7^3)$

$= \left[\frac{1}{2} \times 15(15+1)\right]^2 - \left[\frac{1}{2} \times 7(7+1)\right]^2$

$\left[\because 1^3 + 2^3 + ... + n^3 = \left[\frac{1}{2}n(n+1)^2\right]^2\right]$

$= \left(\frac{1}{2} \times 15 \times 16\right)^2 - \left(\frac{1}{2} \times 7 \times 8\right)^2$

$= 14400 - 784$

$= 13416$

14. (A) Side of polygon (n) = 15

then number of diagonals = $\frac{n(n-3)}{2}$

$= \frac{15(15-3)}{2}$

$= \frac{15 \times 12}{2}$

$= 90$

15. (C) Let $z = (\sin\theta + i\cos\theta)^3$

$z = \left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]^3$

by De-movire's theorem

$$z = \left[\cos 3\left(\frac{\pi}{2}-\theta\right)+i\sin 3\left(\frac{\pi}{2}-\theta\right)\right]$$

$$z = \left[\cos\left(\frac{3\pi}{2}-3\theta\right)+i\sin\left(\frac{3\pi}{2}-3\theta\right)\right]$$

$z = [-\sin 3\theta - i\cos 3\theta]$

Imaginary part of $z = -\cos 3\theta$

16. (A) $I = \int \frac{dx}{x^2+8x+28}$

$$I = \int \frac{dx}{(x+4)^2-16+28}$$

$$I = \int \frac{dx}{(x+4)^2+\left(2\sqrt{3}\right)^2}$$

$$I = \frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{x+4}{2\sqrt{3}}\right) + c$$

17. (B)

class	x_i	f_i	$f_i \times x_i$	$\lvert x_i - A\rvert$	$f_i\lvert x_i - A\rvert$
10 – 20	15	6	90	24	144
20 – 30	25	10	250	14	140
30 – 40	35	8	280	4	32
40 – 50	45	10	450	6	60
50 – 60	55	16	880	16	256
		50	1950		632

Mean A $= \frac{\Sigma f_i \times x_i}{\Sigma f_i}$

$= \frac{1950}{50}$

$= 39$

Mean Deviation $= \frac{\Sigma f_i\lvert x_i - A\rvert}{\Sigma f_i}$

$= \frac{632}{50}$

$= 12.64$

18. (D) $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

On squaring both side

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b} = |\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$

$4\vec{a}\cdot\vec{b} = 0$

$|\vec{a}||\vec{b}|\cos\theta = 0$

$\cos\theta = \cos\frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

$\vec{a}$ is perpendicular to $\vec{b}$.

19. (B) Probability of drawing three kings when card are drawn successively without replacement

$P(E) = \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50}$

$= \frac{1}{13} \times \frac{1}{17} \times \frac{1}{25}$

$= \frac{1}{5525}$

20. (A) The mean of 6 numbers = 28

sum of six number = 6 × 28 = 168

one number excluded, their mean become 31,

then $168 - x = 31 \times 5$

$168 - x = 155$

$x = 13$

The excluded number = 13

21. (C) Series 2, $\sqrt{2}$, 1, ... ∞

$a = 2,\ r = \frac{1}{\sqrt{2}}$

then

$S_\infty = \frac{a}{1-r}$ [$r < 1$]

$= \frac{2}{1-\frac{1}{\sqrt{2}}}$

$= \frac{2\sqrt{2}}{\sqrt{2}-1}$

$= 2(2+\sqrt{2})$

22. (A) $\frac{\cos 15+\cos 45}{\cos^3 15+\cos^3 45}$

$$= \frac{\cos 15+\cos 45}{(\cos 15+\cos 45)(\cos^2 15+\cos^2 45-\cos 15.\cos 45)}$$

$$= \frac{1}{\cos^2 15+\cos^2 45-\cos 15.\cos 45}$$

$$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2-\frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{1}{\sqrt{2}}}$$

On solving

$\Rightarrow \frac{4}{3}$

23. (D) We know that

$$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$$

$$\cos 18^\circ = \sqrt{1-\sin^2 18}$$

$$\cos 18^\circ = \sqrt{1-\left(\frac{\sqrt{5}-1}{4}\right)^2}$$

$$\cos 18^\circ = \sqrt{\frac{10+2\sqrt{5}}{4}}$$

24. (C) In point (x, y), x and y can positive or negative but distance never negative, then perpendicular distance from x-axis = $|y|$

25. (B) $\Sigma x_i = 18$, $\Sigma y_i = 36$, $\Sigma x_i y_i = 144$, $n = 6$

then $\text{cov}(x, y) = \frac{\sum x_i y_i}{n} - \frac{\sum x_i \sum y_i}{n^2}$

$$= \frac{144}{6} - \frac{18\times 36}{(6)^2}$$

$= 24 - 18$

$= 6$

26. (C) Time is 9 : 25

angle $(\theta) = \left|\frac{11M-60H}{2}\right|$ where H $\rightarrow$ Hour

M $\rightarrow$ minute

$$= \left|\frac{11\times 25-60\times 9}{2}\right|$$

$$= \left|\frac{275-540}{2}\right|$$

$$= \frac{265}{2}$$

$$= \left(132\frac{1}{2}\right)^\circ$$

27. (B) $y = \log\left(x-\sqrt{1+x^2}\right)$

On differentiating w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{x-\sqrt{1+x^2}}\left(1-\frac{1\times(2x)}{2\sqrt{1+x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\left(x-\sqrt{1+x^2}\right)}\left(\frac{\sqrt{1+x^2}-x}{2\sqrt{1+x^2}}\right)$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1+x^2}}$$

28. (B) Let $z = \frac{1-2i}{1+(1+i)^2}$

$$z = \frac{1-2i}{1+1+i^2+2i}$$

$$z = \frac{1-2i}{1+2i}\times\frac{1-2i}{1-2i}$$

$$z = \frac{-3-4i}{5}$$

$$|z| = \sqrt{\left(\frac{-3}{5}\right)^2+\left(\frac{-4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25}+\frac{16}{25}}$$

$|z| = 1$

29. (A) Angles of a triangle are in the ratio 1 : 6 : 5,

Let angles x, $6x$, $5x$

$x + 6x + 5x = 180$

$12x = 180$

$x = 15$

angles 15, 90, 75

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 15} = \frac{b}{\sin 90} = \frac{c}{\sin 75}$$

$$\frac{a}{\frac{(\sqrt{3}-1)}{2\sqrt{2}}} = \frac{b}{1} = \frac{c}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}}$$

$$\frac{a}{\sqrt{3}-1} = \frac{b}{2\sqrt{2}} = \frac{c}{\sqrt{3}+1}$$

$a : b : c = (\sqrt{3}-1) : 2\sqrt{2} : (\sqrt{3}+1)$

30. (A) Given that

$C(n, 8) = C(n, 10)$

${}^nC_8 = {}^nC_{10}$

$n = 8 + 10$

$n = 18$

then $C(20, n) = C(20, 18)$

$$= \frac{20!}{18!2!}$$

$$= \frac{20\times 19\times 18!}{18!\times 2\times 1} = 190$$

31. (C) $\lim_{x\to 0} \dfrac{a^{\sin x} - \cos x}{x + \sin x} \quad \left[\dfrac{0}{0}\right]$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \dfrac{a^{\sin x}\log a + \sin x}{1 + \cos x}$

$\Rightarrow \dfrac{a^{\sin 0}\log a + \sin 0}{1 + \cos 0}$

$\Rightarrow \dfrac{\log a}{2}$

32. (D) $xdy + ydx = x^2 dx$

$d(xy) = x^2 dx$

On integrating

$xy = \dfrac{x^3}{3} + \dfrac{c}{3}$

$3xy = x^3 + c$

33. (D) $(y-4)^2 = 4(x+3)$

$Y^2 = 4X$ where $Y = y - 4$

$4a = 4 \qquad X = x + 3$

$a = 1$

vertex of parabola $(X, Y) = (0, 0)$

$X = 0 \quad , \quad Y = 0$

$x + 3 = 0 \quad , \quad y - 4 = 0$

$x = -3 \quad , \quad y = 4$

vertex $(-3, 4)$

34. (C) S = {(HHH), (HHT), (HTH), (HTT), (THT), (THH), (TTH), (TTT)}

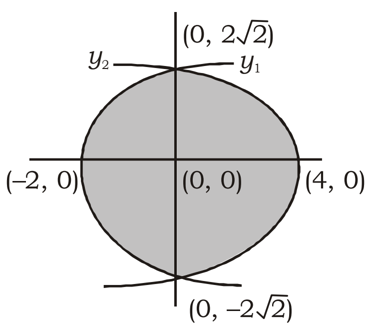
$n(E) = 8$

E = {(HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)}

$n(E) = 7$

$P(E) = \dfrac{n(E)}{n(S)} = \dfrac{7}{8}$

35. (B)



parabolas

$y^2 = 4(x + 2)$

$y_1 = y = 2\sqrt{(x+2)} \qquad \ldots(i)$

and $y^2 = -2(x - 4)$

$y_2 \Rightarrow y = \sqrt{8 - 2x} \qquad \ldots(ii)$

On solving equation (i) and equation (ii)

$x = 0,\ y = \pm 2\sqrt{2}$

$\text{Area} = \int_{-2}^{0} y_1\, dx + \int_{0}^{4} y_2 dx$

$= \int_{-2}^{0} 2\sqrt{x+2}\, dx + \int_{0}^{4} \sqrt{8-2x}\, dx$

$= \left[2 \times \dfrac{2(x+2)^{\frac{3}{2}}}{3}\right]_{-2}^{0} + \left[\dfrac{2(8-2x)^{\frac{3}{2}}}{3(-2)}\right]_{0}^{4}$

$= \dfrac{4}{3}\left[(x+2)^{\frac{3}{2}}\right]_{-2}^{0} - \dfrac{1}{3}\left[(8-2x)^{\frac{3}{2}}\right]_{0}^{4}$

$= \dfrac{4}{3}\left[2^{\frac{3}{2}} - 0\right] - \dfrac{1}{3}\left[0 - 8^{\frac{3}{2}}\right]$

$= \dfrac{4}{3} \times 2\sqrt{2} + \dfrac{1}{3} \times 16\sqrt{2}$

$= \dfrac{24\sqrt{2}}{3}$

$= 8\sqrt{2}$

36. (A) 1 1 0 1 1

$1 \times 2^0 = 1$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$1 \times 2^3 = 8$

$1 \times 2^4 = 16$

27

$(11011)_2 = (27)_{10}$

and

0. 0 0 1

$0 = 2^{-1} \times 0$

$0 = 2^{-2} \times 0$

$\dfrac{1}{8} = 2^{-3} \times 0$

0.125

$(0.001)_2 = (0.125)_{10}$

then $(11011.001)_2 = (27.125)_{10}$

37.(B) $\log_{10}\dfrac{3}{5} - \log_{10}\dfrac{9}{25} + \log_{10}\dfrac{27}{5}$

$\Rightarrow \log_{10}\left(\dfrac{\frac{3}{5}}{\frac{9}{25}}\right) + \log_{10}\dfrac{27}{5}$

$\left[\because \log_a m - \log_a n = \log_a \dfrac{m}{b}\right]$

$\Rightarrow \log_{10}\dfrac{5}{3} + \log_{10}\dfrac{27}{5}$

$\Rightarrow \log_{10}\dfrac{5}{3} + \log_{10}\dfrac{27}{5}$

$\Rightarrow \log_{10}\left(\dfrac{5}{3}\times\dfrac{27}{5}\right) = \log_{10}3^2 = 2\log_{10}3$

$= 2 \times 0.4771 = 0.9542$

38. (A) $\cos\left[\sin^{-1}\dfrac{4}{5} - 2\tan^{-1}\dfrac{1}{5}\right]$

$\Rightarrow \cos\left[\tan^{-1}\dfrac{4}{3} - \tan^{-1}\dfrac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right]$

$\left[\because \sin^{-1}x = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}}\right]$

$\Rightarrow \cos\left[\tan^{-1}\dfrac{4}{3} - \tan^{-1}\dfrac{5}{12}\right]$

$\Rightarrow \cos\left[\tan^{-1}\dfrac{\frac{4}{3}-\frac{5}{12}}{1+\frac{4}{3}\times\frac{5}{12}}\right]$

$\Rightarrow \cos\left[\tan^{-1}\left(\dfrac{33}{56}\right)\right]$

$= \cos\left[\cos^{-1}\dfrac{56}{65}\right]$

$\left[\because \tan^{-1}x = \cos^{-1}\dfrac{1}{\sqrt{1+x^2}}\right]$

$= \dfrac{56}{65}$

39. (A) $(1 + e^x)ydy = dx$

$ydy = \dfrac{dx}{1+e^x}$

$-ydy = \dfrac{-e^{-x}}{(e^{-x}+1)}\,dx$

On integrating

$-\int y\,dy = \int\dfrac{-e^{-x}}{(e^{-x}+1)}\,dx$

$-\dfrac{y^2}{2} = \log(e^{-x}+1) + \log c$

$-\dfrac{y^2}{2} = \log c(e^{-x}+1)$

$y^2 = -2\log c\left(\dfrac{1+e^x}{e^x}\right)$

$y^2 = -2\log c(1+e^x) + 2\log e^x$

$y^2 = -2\log c(1+e^x) + 2x$

$y^2 = 2x - 2\log c(1+e^x)$

40. (B) $I = \int e^{2\ln x}\sin x\,dx$

$I = \int e^{\ln x^2}\sin x\,dx$

$I = \int x^2\sin x\,dx$

$I = x^2\int\sin x\,dx - \int\left\{\dfrac{d}{dx}(x^2).\int\sin x\,dx\right\}dx$

$I = x^2(-\cos x) - \int 2x.(-\cos x)dx$

$I = -x^2\cos x + 2\int x.\cos x\,dx$

$I = -x^2\cos x + 2$

$\left[x.\int\cos x\,dx - \int\left\{\dfrac{d}{dx}(x).\int\cos x\,dx\right\}dx\right]$

$I = -x^2\cos x + 2\left[x.\sin x - \int\sin x\,dx\right]$

$I = -x^2\cos x + 2[x\sin x + \cos x] + c$

$I = -x^2\cos x + 2x\sin x + 2\cos x + c$

41. (C)

42. (A) $z = \left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{201} + \left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{201}$

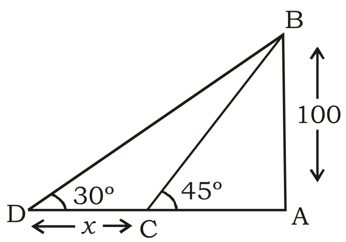
$z = (\cos 60 + i\sin 60)^{201} + (\cos 60 - i\sin 60)^{201}$

$z = \cos(201\times 60) + i\sin(201\times 60) + \cos(201\times 60) - i\sin(201\times 60)$

$z = 2\cos(201\times 60)$

$z = 2\cos(360 \times 33 + 180)$
$z = 2\cos 180$
$z = -2$
Real part of $z = -2$

43. (C)



Given that time taken by boat from C to D = 5 min
Let CD = x m

In ΔABC

$\tan 45° = \frac{AB}{AC}$

$1 = \frac{100}{AC}$

AC = 100 m

In ΔABD

$\tan 30° = \frac{AB}{AD}$

$\frac{1}{\sqrt{3}} = \frac{100}{100 + x}$

$x + 100 = 100\sqrt{3}$

$x = 100(\sqrt{3} - 1)\ m$

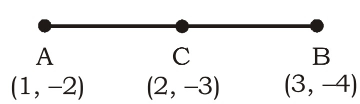
speed of boat = $\frac{\text{distance}}{\text{time}}$

$= \frac{100(\sqrt{3}-1)}{5}$ metre/min

$= \frac{100(\sqrt{3}-1)}{5} \times 60$

$= 1200(\sqrt{3} - 1)$ metre/hr.

44. (B)



Slope of the line AB

$m = \frac{-4+2}{3-1}$

$= \frac{-2}{2} = -1$

mid-point of AB = $\left(\frac{1+3}{2}, \frac{-2-4}{2}\right)$

$= (2, -3)$

slope of line which is perpendicular to the line AB

$m_1 = -\frac{1}{m} = \frac{-1}{-1} = 1$

equation of the line which is perpendicular to line AB and passes through the point $C(2, -3)$

$y - y_1 = m_1(x - x_1)$
$y + 3 = 1(x - 2)$
$y + 3 = x - 2$
$x - y = 5$

45. (C) $\cos 36° = \frac{\sqrt{5}+1}{4}$

then $\cos 54° = \cos(90° - 36°)$
$= \sin 36°$
$= \sqrt{1 - \cos^2 36°}$

$= \sqrt{1 - \left(\frac{\sqrt{5}+1}{4}\right)^2}$

$= \sqrt{1 - \frac{5+1+2\sqrt{5}}{16}}$

$\cos 54° = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

46. (B) Latus-rectum $\frac{2b^2}{a} = 16$

$b^2 = 8a$ ...(i)

and eccentricity $e = \frac{2}{\sqrt{3}}$

then $e^2 = 1 + \frac{b^2}{a^2}$

$\left(\frac{2}{\sqrt{3}}\right)^2 = 1 + \frac{8a}{a^2}$

$\frac{4}{3} = 1 + \frac{8}{a}$

$\frac{8}{a} = \frac{1}{3}$

$a = 24$

from equation (i)

$b^2 = 8a$

$b^2 = 8 \times 24$

equation of hyperbola

$$\frac{x^2}{(24)^2} - \frac{y^2}{8\times 24} = 1$$

$$x^2 - 3y^2 = 576$$

47. (A) Straight line

$x - 2y = -3$ ...(i)

and $3x + y = 5$ ...(ii)

On solving equation (i) and (ii)

$y = 2$ and $x = 1$

intersection point of line (i) and line (ii) is (1, 2)

equation of the line which is parallel to the line $4x - 3y = 7$

$4x - 3y + c = 0$ ...(ii)

it passes through the point (1, 2)

$4 - 6 + c = 0$

$c = 2$

equation of the line

$4x - 3y + 2 = 0$

48. (A) $I = \int e^{\cos x} \dfrac{x\sin^3 x + \cos x}{\sin^2 x}\, dx$

$$I = \int e^{\cos x}(x\sin x + \cot x.\operatorname{cosec} x)\, dx$$

$$I = \int e^{\cos x}(x\sin x + 1 - 1 + \operatorname{cosec} x.\cot x) dx$$

$$I = \int e^{\cos x}(x\sin x + 1) dx - \int e^{\cos x}(1 - \operatorname{cosec} x.\cot x) dx$$

$$I = \int e^{\cos x}(-\sin x)(-x - \operatorname{cosec} x)\, dx - \int e^{\cos x}(1 - \operatorname{cosec} x\cot x) dx$$

$$I = (-x - \operatorname{cosec} x)\int e^{\cos x}(-\sin x) dx - \int\left\{\frac{d}{dx}(-x - \operatorname{cosec} x)\int e^{\cos x}(-\sin x)\, dx\right\} dx - \int e^{\cos x}(1 - \operatorname{cosec} x\cot x) dx$$

$$I = (-x - \operatorname{cosec} x)\, e^{\cos x} - \int(-1 + \operatorname{cosec} x.\cot x)\, e^{\cos x} dx - \int e^{\cos x}(1 - \operatorname{cosec} x.\cot x)\, dx + c$$

$$I = -(x + \operatorname{cosec} x)\, e^{\cos x} + c$$

$$I = c - (x + \operatorname{cosec} x) e^{\cos x}$$

49. (B) $\dfrac{dy}{dx} + y\left(\cot x - \dfrac{1}{x}\right) = \dfrac{1}{x}\operatorname{cosec} x$

On comparing with general equation

$$\frac{dy}{dx} + Py = Q$$

$$P = \cot x - \frac{1}{x} \text{ and } Q = \frac{1}{x}\operatorname{cosec} x$$

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{\int\left(\cot x - \frac{1}{x}\right)dx}$$

$$= e^{(\log\sin x - \log x)}$$

$$= e^{\log\left(\frac{\sin x}{x}\right)}$$

$$= \frac{\sin x}{x}$$

Solution of differential equation

$$y \times I.F. = \int Q \times \text{I.F.}\, dx$$

$$y \times \frac{\sin x}{x} = \int \frac{1}{x} \times \operatorname{cosec} x \times \frac{\sin x}{x}\, dx$$

$$\frac{y}{x} \times \sin x = \int \frac{1}{x^2} dx$$

$$\frac{y}{x} \times \sin x = \frac{-1}{x} + c$$

$$y\sin x = -1 + c.x$$

$$y \times \sin x = cx - 1$$

50. (C)

51. (C)

52. (A) $I = \int \dfrac{e^{e^{\sqrt{x}}} e^{\sqrt{x}}}{\sqrt{x}}\, dx$

Let $e^{\sqrt{x}} = t$

$$e^{\sqrt{x}}\frac{1}{2\sqrt{x}}\, dx = dt$$

$$\frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2dt$$

$$I = \int e^t \times 2dt$$

$$I = 2e^t + c$$

$$I = 2e^{e^{\sqrt{x}}} + c$$

53. (D) Let $y = \cos(\sin x)$

On differentiating w.r.t.'x'

$$\frac{dy}{dx} = -\cos x.\sin(\sin x)$$

54. (C) Three digit numbers formed from the digit 2, 3 and 4 is

234
243
342
324
423
432
1998

55. (A) 1 a 1 1 1
  1 b 0 0
10 1 0 c 1
$c = 1$, $b = 1$, $a = 1$

56. (C) Given that

$$b_{xy} = \frac{-1}{2} \text{ and } b_{yx} = \frac{-32}{25}$$

then

$$\text{Correlation coefficient } (r) = \pm\sqrt{b_{xy} \times b_{yx}}$$

$$= \pm\sqrt{\left(\frac{-1}{2}\right)\times\left(\frac{-32}{25}\right)}$$

$$= \pm\sqrt{\frac{16}{25}}$$

$$= \pm\frac{4}{5}$$

$$\text{Correlation coefficient } (r) = -\frac{4}{5}$$

57. (B) $y = \log_{10}(3x^2 - 5x)$

$$y = \frac{\log_e(3x^2-5x)}{\log_e 10}$$

$y = \log_{10}e \log_e(3x^2 - 5x)$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = \log_{10}e \; \frac{1}{(3x^2-5x)} \; (6x-5)$$

$$\frac{dy}{dx} = \frac{(6x-5)\log_{10}e}{(3x^2-5x)}$$

58. (A) $I = \int \frac{1}{x(x^6-1)}dx$

$$= \int \frac{x^5}{x^6(x^6-1)}dx$$

Let $x^6 = t$

$6x^5dx = dt$

$$x^5dx = \frac{1}{6}dt$$

$$I = \frac{1}{6}\int \frac{dt}{t(t-1)}$$

$$I = \frac{1}{6}\int\left[\frac{1}{t-1} - \frac{1}{t}\right]dt$$

$$I = \frac{1}{6}[\log(t-1) - \log t] + c$$

$$I = \frac{1}{6}\log\left(\frac{t-1}{t}\right) + c$$

$$I = \frac{1}{6}\log\left(\frac{x^6-1}{x^6}\right) + c$$

59. (C) $I = \int_0^{\frac{\pi}{4}} \frac{d\theta}{1+\sin\theta}$

$$I = \int_0^{\frac{\pi}{4}} \frac{(1-\sin\theta)}{1-\sin^2\theta}d\theta$$

$$I = \int_0^{\frac{\pi}{4}} \frac{1-\sin\theta}{\cos^2\theta}d\theta$$

$$I = \int_0^{\frac{\pi}{4}} (\sec^2\theta - \sec\theta.\tan\theta)\,d\theta$$

$$I = [\tan\theta - \sec\theta]_0^{\frac{\pi}{4}}$$

$$I = \left(\tan\frac{\pi}{4} - \sec\frac{\pi}{4}\right) - (\tan 0 - \sec 0)$$

$$I = 1 - \sqrt{2} - (0-1) \Rightarrow 2 - \sqrt{2}$$

60. (D) $\vec{a} = 3\hat{i} + 4\hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ are perpendicular,

then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$3 \times 2 + 4 \times (-3) + \lambda \times 1 = 0$

$6 - 12 + \lambda = 0$

$\lambda = 6$

61. (C) $\lim_{x\to 0} \frac{e^x - \cos x}{\sin x}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$$\lim_{x\to 0}\frac{e^x + \sin x}{\cos x}$$

$$\Rightarrow \frac{e^0 + \sin 0}{\cos 0}$$

$$\Rightarrow \frac{1+0}{1}$$

$\Rightarrow 1$

62. (A) Let $f(x) = \begin{cases} x^2-4, \text{if } -2 \geq x \geq 2 \\ 4-x^2, \text{if } -2 \leq x \leq 2 \end{cases}$

at $x = -1$

$f(x) = 4 - x^2$

$f'(x) = -2x$

$f'(-1) = -2(-1)$

$f'(-1) = 2$

63. (B)

64. (A) limes $\frac{x+2}{-1} = \frac{y-2}{3} = \frac{z+3}{-2}$

and $\frac{x-2}{-3} = \frac{y-0}{2} = \frac{z+4}{1}$

angle between two lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{(-1)(-3)+3\times 2+(-2)\times 1}{\sqrt{(-1)^2+(3)^2+(-2)^2}\sqrt{(-3)^2+(2)^2+(1)^2}}$$

$$= \frac{3+6-2}{\sqrt{14}\sqrt{14}}$$

$$= \frac{7}{14}$$

$$\cos q = \frac{1}{2}$$

$$\cos q = \cos\frac{\pi}{3}$$

$$q = \frac{\pi}{3}$$

65. (D) Planes $x = 3y - 5$ and $y = 4z + 2$

$\frac{x+5}{3} = y$ $\qquad y = 4\left(z+\frac{1}{2}\right)$

$$y = \frac{z+\frac{1}{2}}{\frac{1}{4}}$$

then $\frac{x+5}{3} = \frac{y-0}{1} = \frac{z+\frac{1}{2}}{\frac{1}{4}}$

D.R. = < 3, 1, $\frac{1}{4}$ >

66. (A) $x^n - y^n = 1$

On differentiating w.r.t 'x'

$$nx^{n-1} - ny^{n-1}\frac{dy}{dx} = 0$$

$$x^{n-1} = y^{n-1}\frac{dy}{dx}$$

$$x^{n-1}\,dx = y^{n-1}\,dy$$

$$\frac{dx}{dy} = \left(\frac{y}{x}\right)^{n-1}$$

and given that $\frac{dx}{dy} = \left(\frac{y}{x}\right)^2$

then $n-1 = 2$

$n = 3$

67. (C) **Statement I :**

$y = ln(\cot x - \text{cosec}\,x)$

$$\frac{dy}{dx} = \frac{-\text{cosec}^2 x + \text{cosec}\,x.\cot x}{\cot x - \text{cosec}\,x}$$

$$\frac{dy}{dx} = \frac{\text{cosec}\,x(\cot x - \text{cosec}\,x)}{(\cot x - \text{cosec}\,x)}$$

$$\frac{dy}{dx} = \text{cosec}\,x$$

Statement I is correct.

Statement II

$y = ln(x + \sqrt{x^2+1})$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1+\frac{1\times 2x}{2\sqrt{x^2+1}}\right]$$

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}} \times \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1}}$$

Statement II is correct.

68. (A) $\lim_{x\to 3}\frac{x^2-9}{x-3}$ $\quad \left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 3}\frac{2x-0}{1-0}$

$\Rightarrow 2 \times 3$

$\Rightarrow 6$

69. (A) $I = \int_0^{\frac{\pi}{2}} \ln(\tan x).\text{cosec}\,2x\,dx$ ...(i)

$I = \int_0^{\frac{\pi}{2}} \ln\left\{\tan\left(\frac{\pi}{2} - x\right)\right\}.\ \text{cosec}2\left(\frac{\pi}{2} - x\right)dx$

[Property IV]

$I = \int_0^{\frac{\pi}{2}} \ln(\cot x).\text{cosec}2x\,dx$ ...(ii)

On adding equation (i) and equation (ii)

$2I = \int_0^{\frac{\pi}{2}} \text{cosec}\,2x.[ln(\tan x) + ln(\cot x)]dx$

$2I = \int_0^{\frac{\pi}{2}} \ln 1.\text{cosec}\,2x\,dx$

$2I = 0$

$I = 0$

70. (A) $I = \int_0^{\frac{\pi}{2}} \ln 1.\text{cosec}\,2x\,dx$

$I = \int a^x e^{-x}\,dx$

$I = a^x \int e^{-x}dx - \int\left\{\frac{d}{dx}(a^x)\int e^{-x}dx\right\}dx$

$I = a^x \times \frac{e^{-x}}{-1} - \int a^x\ ln\ a.\ \frac{e^{-x}}{-1}\ dx + e$

$I = -a^x e^{-x} + lna\int a^x\ .e^{-x}dx + c$

$I = -a^x e^{-x} + lna(I) + c$

$[1 - ln\ a]\ I = -a^x e^{-x} + c$

$(ln\ a - 1)\ I = a^x e^{-x} - c$

$(ln\ a - ln\ e)\ I = \frac{a^x}{e^x} - c$

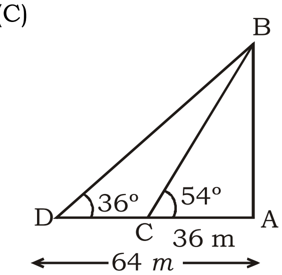
$ln\left(\frac{a}{e}\right)I = \frac{a^x}{e^x} - c$

$I = \frac{a^x}{e^x \ln\left(\frac{a}{e}\right)} - \frac{c}{\ln\left(\frac{a}{e}\right)}$

$I = \frac{a^x}{e^x \ln\left(\frac{a}{e}\right)} + k$

71. (B)

72. (C)



In ΔABC

$\tan 54 = \frac{AB}{AC}$

$\tan 54 = \frac{AB}{36}$ ...(i)

In ΔABD

$\tan 36° = \frac{AB}{AD}$

$\cot 54° = \frac{AB}{64}$

$\tan 54° = \frac{64}{AB}$ ...(ii)

from equation (i) and equation (ii)

$\frac{AB}{36} = \frac{64}{AB}$

$(AB)^2 = 36 \times 64$

$AB = 6 \times 8$

$AB = 48\ m$

height of the tower = 48 m

73. (A) An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

it passes through the point $(2\sqrt{3}, 1)$

$\frac{12}{a^2} + \frac{1}{b^2} = 1$ ...(i)

latus rectum $\frac{2b^2}{a} = 2$

$b^2 = a$ ...(ii)

from equation (i)

$\frac{12}{a^2} + \frac{1}{a} = 1$

$\frac{12 + a}{a^2} = 1$

$a^2 - a - 12 = 0$

$(a - 4)(a + 3) = 0$

$a = 4, -3$

length of major axis = $2a$

$= 2 \times 4$

$= 8$

74. (A)

75. (B) S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

$n(S) = 8$

E = {(HTT), (THT), (TTH)}

$n(E) = 3$

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{3}{8}$$

76. (D)

77. (C) Given data

3, 5, 7, 9, 11, 13, 15

$$\text{Mean } \bar{x} = \frac{3+5+7+9+11+13+15}{7}$$

$$= \frac{63}{7}$$

$= 9$

$$\text{standard deviation} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$= \sqrt{\frac{(3-9)^2+(5-9)^2+(7-9)^2+(9-9)^2+(11-9)^2+(13-9)^2+(15-9)^2}{7}}$$

$$= \sqrt{\frac{36+16+4+0+4+16+36}{7}}$$

$$= \sqrt{\frac{112}{7}}$$

$= \sqrt{16}$

$= 4$

78. (C) Two dice thrown.

$n(S) = 6 \times 6 = 36$

Sum = 2 → {(1, 1)}

= 3 → {(1, 2), (2, 1)}

= 4 → {(1, 3), (3, 1), (2, 2)}

= 5 → {(1, 4), (4, 1), (2, 3), (3, 2)}

$n(E) = 10$, then $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{10}{36} = \frac{5}{18}$$

79. (A) The straight line

$6x - 8y + 5 = 0$ ...(i)

and $12x - 16y + 7 = 0$

$6x - 8y + \frac{7}{2} = 0$ ...(ii)

perpendicular distance between lines

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2}}$$

$$= \frac{\left|5 - \frac{7}{2}\right|}{\sqrt{(6)^2+(8)^2}}$$

$$= \frac{\frac{3}{2}}{10}$$

$$= \frac{3}{20} \text{ unit}$$

80. (C) Let $a - ib = \sqrt{7-24i}$

On squaring both side

$(a^2 - b^2) - 2abi = 7 - 74i$

$a^2 - b^2 = 7$ and $2ab = 24$

$(a^2 + b^2)^2 - 4a^2b^2 = (a^2 - b^2)$ ...(i)

$(a^2 + b^2)^2 - 576 = 49$

$(a^2 + b^2)^2 = 625$

$a^2 + b^2 = 25$ ...(ii)

from equation (i) and equation (ii)

$a^2 = 16$ $b^2 = 9$

$a = \pm 4$ $b = \pm 3$

square root of $(7 - 24i) = \pm(4 - 3i)$

81. (B) $I = \int \cot^2 x \ \text{cosec}^4 x \, dx$

$I = \int \cot^2 x \, \text{cosec}^2 x . \text{cosec}^2 x \, dx$

$I = \int \cot^2 x \, (1 + \cot^2 x) \, \text{cosec}^2 x dx$

$I = \int \cot^2 x . \text{cosec}^2 x \, dx + \int \cot^4 x \, \text{cosec}^2 x \, dx$

Let $\cot x = t$

$-\text{cosec}^2 x \, dx = dt$

$\text{cosec}^2 x \, dx = -dt$

$I = \int -t^2 \, dt + \int -t^4 \, dt$

$$I = -\frac{t^3}{3} - \frac{t^5}{5} + c$$

$$I = -\frac{\cot^3 x}{3} - \frac{\cot^5 x}{5} + c$$

82. (A) $f(x) = \begin{cases} 4x-5, 0 \le x \le 3 \\ x-\lambda, 3 < x \le 5 \end{cases}$ is continuous at $x = 3$

then $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x) = f(3)$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^-} f(x)$

$\lim_{h\to 0} f(3+h) = \lim_{h\to 0} f(3-h)$

$\lim_{h\to 0} 3+h-\lambda = \lim_{h\to 0} 4(3-h)-5$

$3-\lambda = 12-5$

$3-\lambda = 7$

$\lambda = -4$

83. (C) $f(x) = ln(\tan x^2)$

On differentiating w.r.t.'x'

$f'(x) = \frac{1}{\tan x^2} \times \sec^2 x^2 \times (2x)$

$f'(x) = \frac{2x \times \cos x^2}{\sin x^2} \times \frac{1}{\cos^2 x^2}$

$f'(x) = \frac{2\times 2x}{2\sin x^2.\cos x^2}$

$f'(x) = \frac{4x}{\sin 2x^2}$

$f'(x) = 4x \operatorname{cosec} 2x^2$

84. (A) Given that

$S = 18t - 9t^2$

On differentiating w.r.t.'t'

$\frac{dS}{dt} = 18 - 18t$

again, differentiating

$\frac{d^2S}{dt^2} = -18$ ...(i)

for maxima or minima

$\frac{dS}{dt} = 0$

$18 - 18t = 0$

$t = 1$

from equation (i)

$\frac{d^2S}{dt^2}_{(at\, t=1)} = -18$ (maxima)

The stone will gain maximum height at ***t* = 1**.

85.(A) $I = \int \frac{(1-x)}{e^x \sin^2\left(\frac{x}{e^x}\right)} dx$

Let $\frac{x}{e^x} = t$

$xe^{-x} = t$

$e^{-x}(1-x)\,dx = dt$

$\frac{(1-x)}{e^x} dx = dt$

$I = \int \frac{dt}{\sin^2 t}$

$I = \int \operatorname{cosec}^2 t\, dt$

$I = -\cot t + c$

$I = -\cot\left(\frac{x}{e^x}\right) + c$

$I = -\cot(xe^{-x}) + c$

86. (B) $\log_2 8 - 3\log_9 3$

$\Rightarrow \log_2 2^3 - \frac{3}{\log_3 9}$

$\Rightarrow 3\log_2 2 - \frac{3}{2\log_3 3}$

$\Rightarrow 3 - \frac{3}{2}$

$\Rightarrow \frac{3}{2}$

87. (B) plane $4x - 6y + 2z = 24$

$\frac{4x}{24} - \frac{6y}{24} + \frac{2z}{24} = 1$

$\frac{x}{6} + \frac{y}{(-4)} + \frac{z}{12} = 0$

intercepts (6, – 4, 17)

88. (C) circle $(x+1)^2 + (y-2)^2 = 18$

centre (–1, 2)

and circle

$x^2 + y^2 + 4x - 10y + 8 = 0$

$g = 2, f = -5, c = 8$

centre $(-g, -f) = (-2, 5)$

distance between the centres (–1, 2) and (–2, 5)

$d = \sqrt{(-1+2)^2 + (2-5)^2}$

$= \sqrt{1+9} = \sqrt{10}$

89. (A) Given that major axis $2a = 12$

$a = 6$

and $e = \frac{2}{3}$

then $e^2 = 1 - \frac{b^2}{a^2}$

$\frac{4}{9} = 1 - \frac{b^2}{36}$

$\frac{b^2}{36} = \frac{5}{9}$

$b^2 = 20$

equation of an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{36} + \frac{y^2}{20} = 1$

$\frac{5x^2 + 9y^2}{180} = 1$

$5x^2 + 9y^2 = 180$

90. (A) We know that

The sum of focal radii of any point on an ellipse = $(a + x) + (a - x)$

$= 2a$

= length of major axis

91. (C) Point (a, b) on the y-axis

So $a = 0$

and equidistance from the points (–1, 2) and (4, 5)

then

$\sqrt{(a+1)^2 + (b-2)^2} = \sqrt{(a-4)^2 + (b-5)^2}$

$(0 + 1)^2 + (b - 2)^2 = (0 - 4)^2 + (b - 5)^2$

$1 + b^2 + 4 - 4b = 16 + b^2 + 25 - 10b$

$5 - 4b = 41 - 10b$

$b = 6$

then point $(a, b) = (0, 6)$

92. (A) **From option (A)** : line 10x + 5y = 9

$10x + 5y - 9 = 0$

at point (2, 6)

$20 + 30 - 9 > 0$

$41 > 0$

and at point (3, –5)

$30 - 25 - 9 < 0$

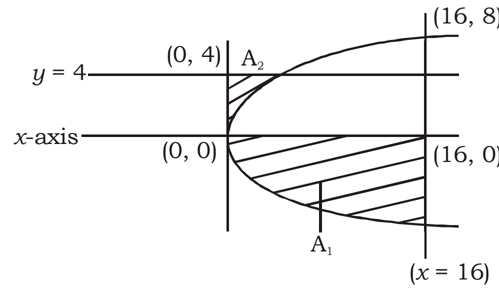
$-4 < 0$

(2, 6) and (3, – 5) are opposite side of line $10x + 5y = 9$.

93. (B) $|z_1| = |z_2| = 1$

$|z_1 z_2| = |z_1||z_2| = 1 \times 1 = 1$

(94-96)



curve $y^2 = 4x$

line $y = 4$

and $x = 16$

94. (B) $y_1 \Rightarrow y = 2\sqrt{x}$

$A_1 = \int_0^{16} y_1 dx$

$A_1 = \int_0^{16} 2\sqrt{x}\, dx$

$= 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{16}$

$= \frac{4}{3}\left[(16)^{\frac{3}{2}} - 0\right]$

$= \frac{4}{3} \times 64$

$= \frac{256}{3}$ sq. unit

95. (C) $x_1 \Rightarrow x = \frac{y^2}{4}$

$A_2 = \int_0^4 x_1\, dy$

$= \int_0^4 \frac{y^2}{4} dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^4$

$= \frac{1}{4}\left[\frac{4^3}{3} - 0\right]$

$= \frac{16}{3}$ sq. unit

96. (A) $A_1 = \frac{256}{3}$ and $A_2 = \frac{16}{3}$

$$A_1 : A_2 = \frac{256}{3} : \frac{16}{3}$$

$= 16 : 1$

97. (B) Let $y = \sin\theta + \cos 2\theta$ ...(i)

On differentiating w.r.t. 'θ'

$$\frac{dy}{d\theta} = \cos\theta - 2\sin 2\theta$$

again, differentiating

$$\frac{d^2y}{d\theta^2} = -\sin\theta - 4\cos 2\theta \quad \text{...(ii)}$$

For maxima or minima

$$\frac{dy}{d\theta} = 0$$

$\cos\theta - 2\sin 2\theta = 0$

$\cos\theta - 2 \times 2\sin\theta.\cos\theta = 0$

$\cos\theta (1 - 4\sin\theta) = 0$

$\cos\theta = 0 \quad , \quad 1 - 4\sin\theta = 0$

$\theta = 90^\circ \quad , \quad \sin\theta = \frac{1}{4}$

from equation (ii)

$$\left(\frac{d^2y}{d\theta^2}\right)_{\text{at }\theta=90} = -\sin 90 - 4\cos 180$$

$= -1 - 4(-1)$

$= 3$ (minima)

The minimum value (at $\theta = 90^\circ$)

$= \sin 90 + \cos 180$

$= 1 - 1$

$= 0$

98. (B) In the expansion of $\left(\sqrt{3}x^2 + \frac{1}{2x^2}\right)^8$

$$T_{r+1} = {}^8C_r (\sqrt{3}\,x^2)^{8-r} \left(\frac{1}{2x^2}\right)^r$$

$$\text{5th term} = T_5 = T_{4+1} = {}^8C_4 (\sqrt{3}\,x^2)^{8-4} \left(\frac{1}{2x^2}\right)^4$$

$$= \frac{8!}{4!4!} (\sqrt{3})^4\, x^8\, \frac{1}{2^4}\, \frac{1}{x^8}$$

$$= \frac{70 \times 3^2}{2 \times 2 \times 2 \times 2}$$

$$= \frac{35 \times 9}{8}$$

$$= \frac{315}{8}$$

99. (B) $(3x + 8y)^{12} - (3x - 8y)^{12}$

$\Rightarrow [{}^{12}C_0 (3x)^{12} (8y)^0 + {}^{12}C_1 (3x)^{11} (8y)^1 + ... + {}^{12}C_{12} (3x)^0 (8y)^{12}]$
$- [{}^{12}C_0 (3x)^{12} (-8y)^0 + {}^{12}C_1 (3x)^{11} (-8y)^1 + {}^{12}C_2 (3x)^{10} (-8y)^2 + ... + {}^{12}C_{12} (3x)^o (-8y)^{12}]$

$\Rightarrow [{}^{12}C_0 (3x)^{12} (8y)^0 + {}^{12}C_1 (3x)^{11} (8y)^1 + ... + {}^{12}C_{12} (3x)^0 (8y)^{12}$
$- {}^{12}C_0 (3x)^{12} (8y)^0 + {}^{12}C_1 (3x)^{11} (8y)^1 - {}^{12}C_2 (3x)^{10} (8y)^2 + {}^{12}C_3 (3x)^9 (8y)^3 ... - {}^{12}C_{12} (3x)^0 (8y)^{12}]$

$\Rightarrow [{}^{12}C_1 (3x)^{11} (8y)^1 + {}^{12}c_3 (3x)9 (8y)^3 + ... + {}^{12}C_{11} (3x)^1 (8y)^3]$

total term = 6

100. (A) Given equation

$x - y + 3z = 7, \; x - y + 2z = 3, \; 2x - y - z = 2$

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 3 \\ 1 & -1 & 2 \\ 2 & -1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}$$

then $AX = B$

$X = A^{-1}B$

$X = A^{-1}B$ ...(i)

$|A| = 1(1 + 2) + 1(-1 - 4) + 3(-1 + 2)$

$= 1$

Co-factors of A

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 2 \\ -1 & -1 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$$

$= 3 \qquad = 5 \qquad = 1$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 3 \\ -1 & -1 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$$

$= -4 \qquad = -7 \qquad = -1$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 3 \\ -1 & 2 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}$$

$= 1 \qquad = 1 \qquad = 0$

$$C = \begin{bmatrix} 3 & 5 & 1 \\ -4 & -7 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} 3 & -4 & 1 \\ 5 & -7 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{AdjA}{|A|}$$

$$= \begin{bmatrix} 3 & -4 & 1 \\ 5 & -7 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

from equation (i)

$X = A^{-1} B$

$$= \begin{bmatrix} 3 & -4 & 1 \\ 5 & -7 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 11 \\ 16 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 16 \\ 4 \end{bmatrix}$$

then $x = 11$, $y = 16$, $z = 4$

101. (B) Given that $AB = A$ ...(i)

and $BA = B$ ...(ii)

then $A^2 = A.A$

$= (AB).(AB)$ [from equation (i)]

$= A.(BA).B$

$= A.B.B$ [from equation (ii)]

$= (AB).B$

$= AB = A$ [from equation (i)]

102. (B) $\begin{vmatrix} b-c & c-a & a-b \\ p-q & q-r & r-p \\ z-x & x-y & y-z \end{vmatrix}$

$C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 0 & c-a & a-b \\ 0 & q-r & r-p \\ 0 & x-y & y-z \end{vmatrix}$$

$\Rightarrow 0$

103. (C) $\begin{vmatrix} \omega^{2n} & \omega^n & 1 \\ \omega^n & 1 & \omega^{2n} \\ 1 & \omega^{2n} & \omega^n \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} \omega^{2n} + \omega^n + 1 & \omega^n + 1 + \omega^{2n} & 1 + \omega^{2n} + \omega^n \\ \omega^n & 1 & \omega^{2n} \\ 1 & \omega^{2n} & \omega^n \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 + \omega^n + \omega^{2n} & 1 + \omega^n + \omega^{2n} & 1 + \omega^n + \omega^{2n} \\ \omega^n & 1 & \omega^{2n} \\ 1 & \omega^{2n} & \omega^n \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ \omega^n & 1 & \omega^{2n} \\ 1 & \omega^{2n} & \omega^n \end{vmatrix} \quad [\because n \neq 3k, \text{ then } 1 + \omega^n + \omega^{2n} = 0]$$

$\Rightarrow 0$

104. (C) $\begin{vmatrix} 2 & 3 & 1 \\ x & -1 & 0 \\ 1 & -2 & 3 \end{vmatrix} = 6$

$2(-3 - 0) - 3(3x - 0) + 1(-2x + 1) = 6$

$-6 - 9x - 2x + 1 = 6$

$x = -1$

(105-112) :

Year	Male			Female			Total
	Urban	Rural	Total	Urban	Rural	Total	
1991	180	**170**	350	150	**300**	450	**800**
1992	250	150	**400**	**230**	320	**550**	950
1993	**260**	120	380	**290**	180	**470**	**850**
1994	230	180	**410**	310	**170**	**480**	**890**
Total	**920**	**620**	**1540**	980	970	**1950**	**3490**

105. (B) total population for the year 1993 = 850

106. (A) Male rural population for the year 1991 =170

107. (C) Total female population in the year 1994 = 480

108. (C) The number of male in the year 1992 = 400

The number of female in the year 1992 = 550

difference = 550 – 400

= 150

109. (D) Female population minimum in 1991.

110. (A) Male population maximum in 1994.

111. (C) Urban female population in the year 1991 = 150

Total population in the year 1991 = 800

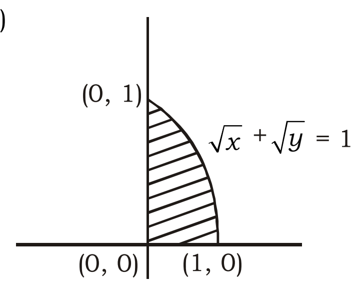
$$\text{Percentage} = \frac{150}{800} \times 100$$

$= 18.75\%$

112. (B) Total population in the four year = 3490

113. (D) $\phi = \{ \}$

114. (A)



curve $\sqrt{x} + \sqrt{y} = 1$ ...(i)

it meets x-axis *i.e* $y = 0$

$x = 1$

and it meets y-axis i.e $x = 0$

$y = 1$

$$\text{Area} = \int_0^1 y\, dx$$

$= \int_0^1 \left(1-\sqrt{x}\right)^2 dx$

$= \int_0^1 \left(1+x-2\sqrt{x}\right) dx$

$= \left[x + \frac{x^2}{2} - \frac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= 1 + \frac{1}{2} - \frac{4}{3} - 0$

$= \frac{1}{6}$ square unit

115. (C) $z = (1 - \cos\theta) + i\sin\theta$

argument $\tan\phi = \frac{b}{a}$

$\tan\phi = \frac{\sin\theta}{1-\cos\theta}$

$= \frac{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$

$\tan\phi = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$\tan\phi = \cot\frac{\theta}{2}$

$\tan\phi = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$

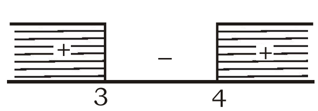
argument $\phi = \frac{\pi}{2} - \frac{\theta}{2}$

116. (B) $f(x) = \frac{\sqrt{\log_e(x^2-7x+3)}}{x-2}$

$x^2 - 7x + 13 \geq 1$ or $x - 2 \neq 0$

$x^2 - 7x + 12 \geq 0$ $\quad x \neq 2$

$(x-4)(x-3) \geq 0$



$x = (-\infty, 3] \cup [4, \infty) - \{2\}$

117. (D) $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\}$

Reflexive :

$1R1, 2R2, 3R3$

So R is reflexive.

Symmetric :

$(1R2 \Rightarrow 2R1), (1R3 \Leftrightarrow 3R1)$

So R is symmetric.

Transitive :

1R2, 2R3 then 1R3

So R is transitive.

Hence R is an equivalence Relation.

118. (D) **Reflexive :**

$1R1 \Leftrightarrow 1 - 1 = 0$ is divisible by 7

So R is reflexive.

Symmetric :

$7R14 \Leftrightarrow 7 - 14 = -7$ is divisible by 7

$14R7 \Leftrightarrow 14 - 7 = 7$ is divisible by 7

So R is symmetric.

Transitive :

$7R14 \Leftrightarrow 7 - 14 = -7$ is divisible by 7

$14R7 \Leftrightarrow 14 - 7 = 7$ is divisible by 7

then

$7R21 \Leftrightarrow 7 - 21 = -14$ is divisible by 7

So R is transitive.

Hence R is an equivalence relation.

119. (C) $y = (1 + x^{\frac{1}{8}})(1 + x^{\frac{1}{4}})(1 + x^{\frac{1}{2}})(1 - x^{\frac{1}{8}})$

$y = (1 + x^{\frac{1}{8}})(1 - x^{\frac{1}{8}})(1 + x^{\frac{1}{4}})(1 + x^{\frac{1}{2}})$

$y = (1 - x^{\frac{1}{4}})(1 + x^{\frac{1}{4}})(1 + x^{\frac{1}{2}})$

$y = (1 - x^{\frac{1}{2}})(1 + x^{\frac{1}{2}})$

$y = 1 - x$

On differentiation both side w.r.t. 'x'

$\frac{dy}{dx} = 0 - 1$

$\frac{dy}{dx} = -1$

120. (C) 8^{th} term of a G.P.

$ar^7 = 18$ ...(i)

2^{nd} term of a G.P

$ar = 72$ ...(ii)

from equation (i) and equation (ii)

$(ar^7)(ar) = 18 \times 72$

$a^2r^8 = 18 \times 72$

$(ar^4)^2 = 18 \times 18 \times 4$

$ar^4 = 18 \times 2$

$ar^4 = 36$

5^{th} term = 36

NDA (MATHS) MOCK TEST - 82 (Answer Key)

1.	(C)	21.	(C)	41.	(C)	61.	(C)	81.	(B)	101.	(B)
2.	(C)	22.	(A)	42.	(A)	62.	(A)	82.	(A)	102.	(B)
3.	(A)	23.	(D)	43.	(C)	63.	(B)	83.	(C)	103.	(C)
4.	(D)	24.	(C)	44.	(B)	64.	(A)	84.	(A)	104.	(C)
5.	(A)	25.	(B)	45.	(C)	65.	(D)	85.	(A)	105.	(B)
6.	(B)	26.	(C)	46.	(B)	66.	(A)	86.	(B)	106.	(A)
7.	(A)	27.	(B)	47.	(A)	67.	(C)	87.	(B)	107.	(C)
8.	(B)	28.	(B)	48.	(A)	68.	(A)	88.	(C)	108.	(C)
9.	(B)	29.	(A)	49.	(B)	69.	(A)	89.	(A)	109.	(D)
10.	(C)	30.	(A)	50.	(C)	70.	(A)	90.	(A)	110.	(A)
11.	(B)	31.	(C)	51.	(C)	71.	(B)	91.	(C)	111.	(C)
12.	(C)	32.	(D)	52.	(A)	72.	(C)	92.	(A)	112.	(B)
13.	(C)	33.	(D)	53.	(D)	73.	(A)	93.	(B)	113.	(D)
14.	(A)	34.	(C)	54.	(C)	74.	(A)	94.	(B)	114.	(A)
15.	(C)	35.	(B)	55.	(A)	75.	(B)	95.	(C)	115.	(C)
16.	(A)	36.	(A)	56.	(C)	76.	(D)	96.	(A)	116.	(B)
17.	(B)	37.	(B)	57.	(B)	77.	(C)	97.	(B)	117.	(D)
18.	(D)	38.	(A)	58.	(A)	78.	(C)	98.	(B)	118.	(D)
19.	(B)	39.	(A)	59.	(C)	79.	(A)	99.	(B)	119.	(C)
20.	(A)	40.	(B)	60.	(D)	80.	(C)	100.	(A)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*