NDA MATHS MOCK TEST - 84 (SOLUTION)

1. (C) $(\log_2 x)(\log_x 3x)(\log_{3x} y) = \log_x x^2$

$(\log_2 3x)(\log_{3x} y) = \log_x x^2$

$\log_2 y = \log_x x^2$

On comparing

$x = 2$

and $y = x^2$

$y = (2)^2 = 4$

2. (A)

2	191	
2	95	1
2	47	1
2	23	1
2	11	1
2	5	1
2	2	1
2	1	0
	0	1

$(191)_{10} = (10111111)_2$

3. (B) $n(S) = 6 \times 6 = 36$

same number appears each of them.

then $E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$n(E) = 6$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

$$= \frac{1}{6}$$

4. (A) Latus rectum of ellipse = $\frac{1}{3}$ × major axis

$$\frac{2b^2}{a} = \frac{1}{3} \times 2a$$

$$b^2 = \frac{a^2}{3}$$

$$\text{then } e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{a^2}{3a^2}$$

$$e^2 = \frac{2}{3}$$

$$e = \frac{\sqrt{2}}{\sqrt{3}}$$

5. (C) Given line

$\sqrt{3}\,x - y = -7$

$y = \sqrt{3}\,x + 7$

$m = \tan\theta = \sqrt{3}$

angle with x-axis $\theta = 60°$

slope of the line which making the angle with y-axis is equal to the inclination of the given line with x-axis.

the slope of the line $m_1 = \tan(90 - 60)$

$m_1 = \tan 30$

$$m_1 = \frac{1}{\sqrt{3}}$$

equation of the line which is passing through the point (– 2, 3)

$y - 3 = m_1 (x + 2)$

$$y - 3 = \frac{1}{\sqrt{3}}(x + 2)$$

$\sqrt{3}\,y - 3\sqrt{3} = x + 2$

$x - \sqrt{3}\,y + 2 + 3\sqrt{3} = 0$

6. (A) **In ΔABC**

angles A, B, C are in A.P.

Let $A = x - y$, $B = x$, $C = x + y$

then $x - y + x + x - y = 180$

$3x = 180$

$x = 60$

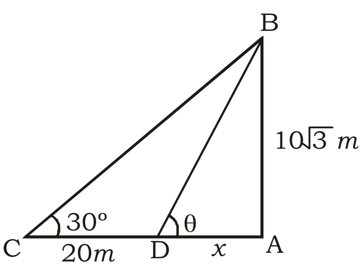
$B = 60$

$$\cos B = \cos 60 = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\frac{1}{2} = \frac{b^2 + c^2 - a^2}{2bc}$$

$a^2 = b^2 + c^2 - bc$

7. (C)



Let AD = x m

In ΔABC

$$\tan 30° = \frac{AB}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{10\sqrt{3}}{20 + x}$$

$20 + x = 30$

$x = 10$

In ΔABD

$\tan\theta = \dfrac{AB}{AD}$

$\tan\theta = \dfrac{10\sqrt{3}}{10}$

$\tan\theta = \sqrt{3}$

$\theta = 60^\circ$

8. (A) $z = -\bar{z}$

Let $z = x + iy$ then $\bar{z} = x - iy$

$x + iy = -(x - iy)$

$x + iy = -x + iy$

$2x = 0$

$x = 0$

The real part of z is 0.

9. (C)

10. (B) $9y^2 - 4x^2 = 36$

$4x^2 - 9y^2 = -36$

$\dfrac{x^2}{9} - \dfrac{y^2}{4} = -1$

$a^2 = 9,\ b^2 = 4$

then $e^2 = 1 + \dfrac{a^2}{b^2}$

$e^2 = 1 + \dfrac{9}{4}$

$e = \dfrac{\sqrt{13}}{2}$

11. (C) We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)

then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$\Rightarrow 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$\Rightarrow 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

from equation (i)

$\Rightarrow 2 \times 1 - 3$

$\Rightarrow -1$

12. (B) Equations

$3x - 4y = 17$

and $6x - 8y = 34$

$\dfrac{3}{6} = \dfrac{-4}{-8} = \dfrac{17}{34}$

Hence equations have infinitely many solutions.

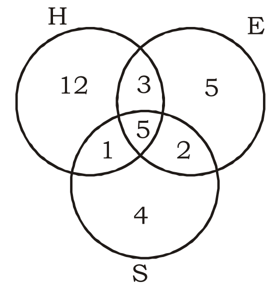
13. (C) The equation

$x^2 - 4x + 29 = 0$

one root is $(2 - 5i)$

then other root is $(2 + 5i)$

(14-17) :



given that

total students = 48

$n(H) = 21,\ n(E) = 15,\ n(S) = 12$

$n(H \cap E) = 8,\ n(H \cap S) = 6,\ n(E \cap S) = 7$

and $n(H \cap E \cap S) = 5$

14. (B) The number of students who had taken only Sanskrit = 4

15. (C) The number of students who had taken only two Subjects = 1 + 3 + 2 = 6

16. (A) The number of students who had not taken any subjects = 48 – (12 + 3 + 5 + 5 + 1 + 2 +4)
= 48 – 32
= 16

17. (C) The number of students who had taken at least two subjects = 1 + 2 + 3 + 5
= 11

18. (C) $\cos^{-1}\left[\cos\left(\dfrac{17\pi}{5}\right)\right]$

$\Rightarrow \cos^{-1}\left[\cos\left(4\pi - \dfrac{3\pi}{5}\right)\right]$

$\Rightarrow \cos^{-1}\left(\cos\dfrac{3\pi}{5}\right)$

$\Rightarrow \dfrac{3\pi}{5}$

19. (C) Let $z = \dfrac{1}{1 + \cos\theta - i\sin\theta}$

$z = \dfrac{(1 + \cos\theta + i\sin\theta)}{(1 + \cos\theta - i\sin\theta)(1 + \cos\theta + i\sin\theta)}$

$z = \dfrac{1 + \cos\theta + i\sin\theta}{1 + \cos^2\theta + 2\cos\theta - i^2\sin^2\theta}$

$z = \dfrac{(1 + \cos\theta) + i\sin\theta}{2(1 + \cos\theta)}$

$z = \dfrac{1}{2} + \dfrac{i\sin\theta}{2(1 + \cos\theta)}$

$z = \dfrac{1}{2} + \dfrac{i \times \tan\dfrac{\theta}{2}}{2}$

imaginary part of $z = \dfrac{\tan\dfrac{\theta}{2}}{2}$

20. (A) $f(x) = \sqrt{49+x^2}$

$$f'(x) = \frac{x}{\sqrt{49+x^2}}$$

then $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to 1} \frac{f'(x)-0}{1}$$

$$\Rightarrow \lim_{x\to 1} \frac{x}{\sqrt{49+x^2}}$$

$$\Rightarrow \frac{1}{\sqrt{49+1}}$$

$$\Rightarrow \frac{1}{5\sqrt{2}}$$

21. (A) $\lim_{x\to 0} x \operatorname{cosec} x$

$$\Rightarrow \lim_{x\to 0} \frac{x}{\sin x} \quad \left[\because \lim_{x\to 0}\frac{x}{\sin x} = \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$\Rightarrow 1$

22. (B) $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$$I = \int e^x \left[\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right] dx$$

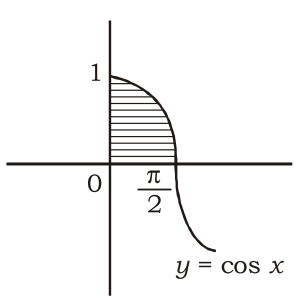
$$I = \int e^x \left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] dx$$

$$I = \int e^x \left[\tan\frac{x}{2} + \frac{1}{2}\sec^2\frac{x}{2}\right] dx$$

$$I = e^x .\tan\frac{x}{2} + c$$

$[\because e^x [f(x) + f'(x)]dx = e^x.f(x) + c]$

23. (B)



curve $y_1 \Rightarrow y = \cos x$

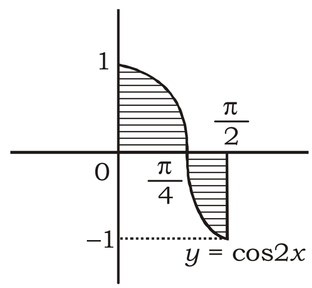
Area $A_1 = \int_0^{\frac{\pi}{2}} y_1\, dx$

$$A_1 = \int_0^{\frac{\pi}{2}} \cos x\, dx$$

$$A_1 = [\sin x]_0^{\frac{\pi}{2}}$$

$$A_1 = \sin\frac{\pi}{2} - \sin 0$$

$$A_1 = 1$$



curve $y_2 \Rightarrow y = \cos 2x$

$$A_2 = 2\int_0^{\frac{\pi}{4}} y_2\, dx$$

$$A_2 = 2\int_0^{\frac{\pi}{4}} \cos 2x\, dx$$

$$A_2 = 2\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$A_2 = \sin\frac{\pi}{2} - \sin 0$$

$A_2 = 1$

ratio $A_1 : A_2 = 1 : 1$

24. (B) $\sin 54^\circ = \sin(90^\circ - 36^\circ)$

$= \cos 36^\circ$

$$= \frac{\sqrt{5}+1}{4}$$

25. (A) The equation of line which is perpendicular to the line $8x + y = 17$

$x - 8y + c = 0$ ...(i)

equation (i) passing through the point (–2, 4)

$-2 - 8 \times 4 + c = 0$

$c = 34$

from equation (i)

$x - 8y + 34 = 0$

26. (C) $\frac{\cos 5x + 2\sin 3x - \cos x}{\sin 5x + \sin x}$

$$\Rightarrow \frac{(\cos 5x - \cos x) + 2\sin 3x}{\sin 5x + \sin x}$$

$$\Rightarrow \frac{2\sin\frac{5x+x}{2}\cdot\sin\frac{x-5x}{2}+2\sin 3x}{2\sin\frac{5x+x}{2}\cdot\cos\frac{5x-x}{2}}$$

$$\Rightarrow \frac{2\sin 3x(-\sin 2x)+2\sin 3x}{2\sin 3x.\cos 2x}$$

$$\Rightarrow \frac{2\sin 3x(-\sin 2x+1)}{2\sin 3x.\cos 2x}$$

$$\Rightarrow \frac{(1-\sin 2x)}{\cos 2x}$$

$$\Rightarrow \frac{\left[1-\cos\left(\frac{\pi}{2}-2x\right)\right]}{\sin\left(\frac{\pi}{2}-2x\right)}$$

$$\Rightarrow \frac{2\times\sin^2\left(\frac{\pi}{4}-x\right)}{2\sin\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-x\right)}$$

$$\Rightarrow \tan\left(\frac{\pi}{4}-x\right)$$

27. (A) We know that

$\tan A.\tan(60-A).\tan(60+A) = \tan 3A$

then $\cot A.\cot(60-A).\cot(60+A) = \cot 3A$

28. (C) We know that minimum value of

$\left(ax^2+\frac{b}{x^2}\right) = 2\sqrt{ab}$

So minimum value of ($4\sin^2\theta + 16\,\text{cosec}^2\theta$)

$= 2\sqrt{4\times 16}$

$= 2\times 8$

$= 16$

29. (B) Let $a - ib = \sqrt{24-70i}$

On squaring both side

$(a^2-b^2)-(2ab)i = 24-70i$

On comparing

$a^2-b^2 = 24$ and $2ab = 70$ …(i)

$(a^2+b^2)^2 = (a^2-b^2)^2 + 4a^2b^2$

$(a^2+b^2)^2 = 576 + 4900$

$(a^2+b^2)^2 = 5476$

$a^2+b^2 = 74$ …(ii)

from equation (i) and equation (ii)

$a^2 = 49$ and $b^2 = 25$

$a = \pm 7$ $b = \pm 5$

then square root of $(24-70i) = \pm(7-5i)$

30. (A) Hyperbola $6x^2 - 7y^2 = 21$

$$\frac{x^2}{\frac{7}{2}} - \frac{y^2}{3} = 1$$

$a^2 = \frac{7}{2}$, $b^2 = 3$

then $e^2 = 1 + \frac{b^2}{a^2}$

$$e^2 = 1 + \frac{3}{\frac{7}{2}}$$

$$e = \frac{\sqrt{13}}{\sqrt{7}}$$

foci $(\pm ae, 0) = \left(\pm\frac{\sqrt{7}}{\sqrt{2}}\times\frac{\sqrt{13}}{\sqrt{7}}, 0\right)$

$$= \left(\pm\sqrt{\frac{13}{2}}, 0\right)$$

31. (A) Matrix $\begin{bmatrix}0 & -1 & 1\\ 2 & 2 & 2\\ 1 & x & -3\end{bmatrix}$ is a singular matrix.

then $\begin{vmatrix}0 & -1 & 1\\ 2 & 2 & 2\\ 1 & x & -3\end{vmatrix} = 0$

$0 + 1(-6-2)+1(2x-2) = 0$

$2x = 10$

$x = 5$

32. (C) Sides of polygon (n) = 27

then number of diagonals $= \frac{n(n-3)}{2}$

$$= \frac{27\times(27-3)}{2}$$

$$= \frac{27\times 24}{2}$$

$= 324$

33. (B) $A = \begin{bmatrix}-1 & 1 & 0\\ -2 & 3 & 1\\ 3 & -1 & 4\end{bmatrix}$

$|A| = -1(12+1) - 1(-8-3) + 0$

$= -13 + 11$

$= -2$

Co-factors of A –

$C_{11} = (-1)^{1+1}\begin{vmatrix}3 & 1\\ -1 & 4\end{vmatrix} = 13$, $C_{12} = (-1)^{1+2}\begin{vmatrix}-2 & 1\\ 3 & 4\end{vmatrix} = 11$, $C_{13} = (-1)^{1+3}\begin{vmatrix}-2 & 3\\ 3 & -1\end{vmatrix} = -7$

$C_{21} = (-1)^{2+1}\begin{vmatrix}1 & 0\\ -1 & 4\end{vmatrix} = -4$, $C_{22} = (-1)^{2+2}\begin{vmatrix}-1 & 0\\ 3 & 4\end{vmatrix} = -4$, $C_{23} = (-1)^{2+3}\begin{vmatrix}-1 & 1\\ 3 & -1\end{vmatrix} = 2$

$C_{31} = (-1)^{3+1}\begin{vmatrix}1 & 0\\ 3 & 1\end{vmatrix} = 1$, $C_{32} = (-1)^{3+2}\begin{vmatrix}-1 & 0\\ -2 & 1\end{vmatrix} = 1$, $C_{33} = (-1)^{3+3}\begin{vmatrix}-1 & 1\\ -2 & 3\end{vmatrix} = -1$

$$C = \begin{bmatrix} 13 & 11 & -7 \\ -4 & -4 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

Adj A = Transpose of C

$$= \begin{bmatrix} 13 & -4 & 1 \\ 11 & -4 & 1 \\ -7 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{AdjA}{|A|}$$

$$= \begin{bmatrix} -\frac{13}{2} & 2 & -\frac{1}{2} \\ -\frac{11}{2} & 2 & -\frac{1}{2} \\ \frac{7}{2} & -1 & \frac{1}{2} \end{bmatrix}$$

34. (A) Three digits are chosen from 1, 2, 3, 4, 5, 6, 7, 8

$n(S) = {}^8C_3 = 56$

E = sum is even

= two digits are chosen from (1, 3, 5, 7) and one digit is chosen from (2, 4, 6, 8) + Three digits are chosen from (2, 4, 6, 8)

$n(E) = {}^4C_2 \times {}^4C_1 + {}^4C_3 = 24 + 4$

$= 28$

Required probability $P(E) = \frac{n(E)}{n(S)}$

$= \frac{28}{56} = \frac{1}{2}$

35. (C) $\sin^2 10 + \sin^2 20 + \ldots \sin^2 80 + \sin^2 90$

$\Rightarrow (\sin^2 10 + \sin^2 80) + (\sin^2 20 + \sin^2 70) + (\sin^2 30 + \sin^2 60) + (\sin^2 40 + \sin^2 50) + 1$

$\Rightarrow (\sin^2 10 + \cos^2 10) + (\sin^2 20 + \cos^2 20) + (\sin^2 30 + \cos^2 30) + (\sin^2 40 + \cos^2 40) + 1$

$\Rightarrow$ 5

36. (C) Series

$\sqrt{3} + \sqrt{12} + \sqrt{27} + \sqrt{48} + \ldots$

$\Rightarrow \sqrt{3} + 2\sqrt{3} + 3\sqrt{3} + 4\sqrt{3} + \ldots$

$\Rightarrow \sqrt{3}\ (1 + 2 + 3 + 4 + \ldots)$

$n = 8$

$\Rightarrow \sqrt{3} \times \frac{8}{2}\ (2 \times 1 + (8 - 1) \times 1)$

$= \sqrt{3} \times 4 \times 9$

$= 36\sqrt{3}$

37. (A) Let $y = x - \tan x$ and $z = \log(x - \sin x)$

$\frac{dy}{dx} = 1 - \sec^2 x$, $\frac{dz}{dx} = \frac{1}{x - \sin x} \times (1 - \cos x)$

$\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$= (1 - \sec^2 x) \times \frac{(x - \sin x)}{(1 - \cos x)}$

$= \frac{(\cos^2 x - 1)}{\cos^2 x} \times \left(\frac{x - \sin x}{1 - \cos x}\right)$

$= -\frac{(1 + \cos x)(x - \sin x)}{\cos^2 x}$

$= \frac{(1 + \cos x)(\sin x - x)}{\cos^2 x}$

38. (B) $f : N \to N$

where N = 1, 2, 3, 4, 5, 6, 7,...

$f(x) = 2x + 8$

$f(1) = 10$

$f(2) = 12$

$f(3) = 14$

$\vdots$

Hence function is injective but not surjective.

39. (D) $x = a(\sin\theta - \theta\cos\theta)$

$\frac{dx}{d\theta} = a(\cos\theta - \theta(-\sin\theta) - \cos\theta)$

$\frac{dx}{d\theta} = a\theta \sin\theta$

and $y = a(\cos\theta + \theta\sin\theta)$

$\frac{dy}{d\theta} = a(-\sin\theta + \theta.\cos\theta + \sin\theta.1)$

$\frac{dy}{d\theta} = a\theta\cdot\cos\theta$

then $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$= a\theta.\cos\theta \times \frac{1}{a\theta.\sin\theta}$

$= \cot\theta$

40. (B) U = {1, 2, 3, 4, 5, 6, 7}, A = {1, 2, 3}, B = {3, 4, 5}, C = {5, 6, 7}

$(A \cap B) = \{3\}$ and $(B \cap C) = \{5\}$

$(A \cap B) \cup (B \cap C) = \{3, 5\}$

$C - \{(A \cap B) \cup (B \cap C)\} = \{5, 6, 7\} - \{3, 5\}$

$= \{6, 7\}$

41. (B) $\log_2 (\log_3 (\log_2 512))$

$\Rightarrow \log_2(\log_3(\log_2 2^9))$

$\Rightarrow \log_2(\log_3 9)$

$\Rightarrow \log_2(\log_3 3^2)$

$\Rightarrow \log_2 2$

$\Rightarrow 1$

42. (C) Equation

$b(c-a)x^2 + c(a-b)x + a(b-c) = 0$

one root = 1 and let other root = α

product of roots $\Rightarrow 1.\alpha = \dfrac{a(b-c)}{b(c-a)}$

$$\alpha = \frac{a(b-c)}{b(c-a)}$$

43. (A) $[1\ 2\ 3x]\begin{bmatrix}1 & 2 & 3\\ 4 & -1 & 2\\ 2 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ -2\\ 1\end{bmatrix} = [0]$

$$[1, 2, 3x]\begin{bmatrix}0\\ 8\\ 3\end{bmatrix} = [0]$$

$[1 \times 0 + 2 \times 8 + 3x \times 3] = [0]$

$16 + 9x = 0$

$$x = -\frac{16}{9}$$

44. (A) $X = 2, 3, 2, 2, 3, 4, 3, 4, 3, 3, 5, 5, 6$

Mode = 3

45. (A) $\dfrac{\sin x}{1-\cos x} - \dfrac{1-\cos x}{\sin x}$

$$\Rightarrow \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} - \frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}}$$

$$\Rightarrow \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$\Rightarrow \frac{2\cos x}{\sin x}$$

$\Rightarrow 2\cot x$

46. (C)

47. (B) The required probability = $\dfrac{5}{8} \times \dfrac{4}{7}$

$$= \frac{5}{14}$$

48. (C) **Statement I :** $(\omega^7 + 1)^9 + 1 = 0$

L.H.S. $= (\omega^7 + 1)^9 + 1$

$= (\omega + 1)^9 + 1$

$= (-\omega^2)^9 + 1$

$= -\omega^{18} + 1$

$= -1 + 1 = 0 =$ R.H.S

Statement (I) is correct.

Statement II : $(\omega^{213} + 1)^{13} = 2^{13}$

L.H.S. $= (\omega^{213} + 1)^{13}$

$= ((\omega^3)^{71} + 1)^{13}$ $[\because \omega^3 = 1]$

$= (1 + 1)^{13} = 2^{13} =$ R.H.S.

Statement (II) is correct.

49. (A) $I = \int e^x(\sin x - \cos x)dx$

$I = -\int e^x(\cos x - \sin x)\,dx$

$I = -\int e^x[\cos x + (-\sin x)]dx$

$I = -e^x\cos x + c\ [\because \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c]$

50. (C) Equation

$x^2 - 9x - 22 = 0$

$(x - 11)(x + 2) = 0$

$x = -2, 11$

root are distinct and real.

51. (C) Let $f(x) = y = \dfrac{x^2}{1-x^2}$

$y - yx^2 = x^2$

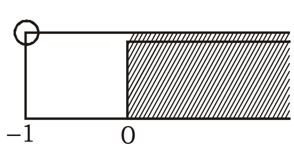
$y = x^2(1 + y)$

$$x^2 = \frac{y}{1+y}$$

$$x = \sqrt{\frac{y}{1+y}}$$

Case-I $y \geq 0$ and $1 + y > 0$

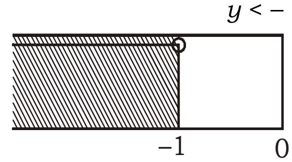
$y > -1$



$\boldsymbol{y \in [0, \infty)}$

Case-II $y \leq 0$ and $1 + y < 0$

$y < -1$



$\boldsymbol{y \in (-\infty, -1)}$

Range of function $f(x) = (-\infty, -1) \cup [0, \infty)$

52. (A) Let $y = 2^{91}$

taking log

$\log_{10} y = 91 \log_{10} 2$

$\log_{10} y = 91 \times 0.3010$

$\log_{10} y = 27.391$

number of digits are in 2^{91} = 28

53. (B) $\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

$R_3 \to R_3 - R_1 + 2R_2$

$\Rightarrow \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$\Rightarrow a^2(b-c) - b^2(a-c) + c^2(a-b)$

$\Rightarrow (a-b)(b-c)(a-c)$

54. (A) $\dfrac{\sin(x-y)}{\sin(x+y)} = \dfrac{a-b}{a+b}$

by componendo & dividendo Rule

$\dfrac{\sin(x-y)+\sin(x+y)}{\sin(x-y)-\sin(x+y)} = \dfrac{a-b+a+b}{a-b-a-b}$

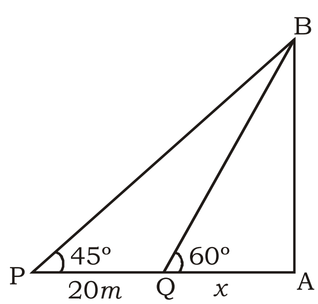
$\dfrac{2\sin x.\cos y}{2\cos x.(-\sin y)} = \dfrac{a}{-b}$

$\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$

$\dfrac{\tan y}{\tan x} = \dfrac{b}{a}$

55. (B) Let the distance between the base of the tower and the point $Q = x\,m$

Let AB = $h\,m$



In ΔABQ

$\tan 60° = \dfrac{AB}{AQ}$

$\sqrt{3} = \dfrac{h}{x}$

$h = x\sqrt{3}$ ...(i)

In ΔABP

$\tan 45° = \dfrac{AB}{AP}$

$1 = \dfrac{h}{x+20}$

$x + 20 = x\sqrt{3}$

$x = \dfrac{20}{\sqrt{3}-1}$

$x = 10(\sqrt{3}+1)$

distance between the base of the tower and the point $P(AP) = x + 20$

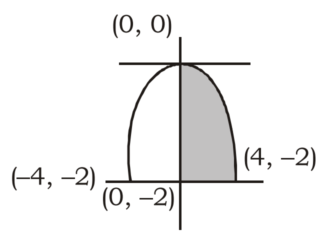
$= 10\sqrt{3} + 30$

$= 10(3+\sqrt{3})$

56. (B) parabola

$x^2 = -8y$

$a = 2$



$y_1 \Rightarrow y = -\dfrac{x^2}{8}$

Area $= \left|2\int_0^4 y_1 dx\right|$

$= \left|2\int_0^4 \left(-\dfrac{x^2}{8}\right) dx\right|$

$= \left|-\dfrac{1}{4}\left[\dfrac{x^3}{3}\right]_0^4\right|$

$= \left|-\dfrac{1}{12}(64-0)\right|$

$= \dfrac{16}{3}$ sq. unit

57. (C) The required probability $P(E) = \dfrac{1}{52}$

58. (C) $y = x\ln x + xe^x$

On differentiating both side w.r.t 'x'

$$\frac{dy}{dx} = x \times \frac{1}{x} + (\ln x) \times 1 + x.e^x + e^x \times 1$$

$$\frac{dy}{dx} = 1 + \ln x + x.e^x + e^x$$

$$\left(\frac{dy}{dx}\right)_{at\,x=1} = 1 + \ln 1 + 1.e^1 + e$$

$$= 1 + 2e$$

59. (C)

60. (C)

61. (B) $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

On integrating

$\sin^{-1}y = -\sin^{-1}x + c$

$\sin^{-1}x + \sin^{-1}y = c$

62. (B) $4x^2 + 9y^2 + 16x + 54y + 61 = 0$

$4(x^2 + 4x) + 9(y^2 + 6y) + 61 = 0$

$4(x + 2)^2 - 16 + 9(y + 3)^2 - 81 + 61 = 0$

$4(x + 2)^2 + 9(y + 3)^2 = 36$

$$\frac{(x+2)}{9} + \frac{(y+3)^2}{4} = 1$$

$a^2 = 9,\ b^2 = 4$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{4}{9}$$

$$e = \frac{\sqrt{5}}{3}$$

$$\text{foci } (\pm ae, 0) = \left(\pm 3 \times \frac{\sqrt{5}}{3}, 0\right)$$

$$= (\pm\sqrt{5}, 0)$$

63. (C)

64. (B) $I = \int e^x \left(\frac{(x-1)^2}{(1+x^2)^2}\right) dx$

$$I = \int e^x \left[\frac{1+x^2-2x}{(1+x^2)^2}\right] dx$$

$$I = \int e^x \left[\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right] dx$$

$$I = \int e^x \frac{1}{1+x^2} + c$$

$$\left[\because \int e^x \left[f(x) + f'(x)\right] dx = e^x f(x) + c\right]$$

65. (C) $\frac{1-\cos\theta}{1+\cos\theta} = 3$

$1 - \cos\theta = 3 + 3\cos\theta$

$4\cos\theta = -2$

$$\cos\theta = -\frac{1}{2}$$

$$\cos\theta = \cos\frac{2\pi}{3}$$

$$\theta = 2n\pi \pm \frac{2\pi}{3}$$

66. (C) Given that

$\int_{-2}^{4} f(x) = 5$ ...(i)

and $\int_{0}^{4} (2 + f(x))\,dx = 9$ ...(ii)

$$\int_0^4 2dx + \int_0^4 f(x)\,dx = 9$$

$$2[x]_0^4 + \int_0^4 f(x)\,dx = 9$$

$$2(4 - 0) + \int_0^4 f(x)\,dx = 9$$

$\int_0^4 f(x)\,dx = 1$ ...(iii)

from equation (i)

$$\int_{-2}^4 f(x)\,dx = 5$$

$$\int_{-2}^0 f(x)\,dx + \int_0^4 f(x)\,dx = 5$$

$\int_{-2}^0 f(x)\,dx + 1 = 5$ [From equation (iii)]

$$\int_{-2}^0 f(x)\,dx = 4$$

67. (C) Given that $A = 34°$, B = 96°

then $C = 50°$

$$\text{L.H.S.} = \tan\left(\frac{B}{2} + C\right)$$

= tan(48 + 50)

= tan(98°)

R.H.S. = tanB

= tan96

tan(98°) > tan96°

then $\tan\left(\frac{B}{2} + C\right) >$ tanB

Statement (I) is incorrect.

Statement (II)

$$\tan\left(\frac{A+C}{2}\right).\sin\frac{B}{2} > \cos\frac{B}{2}$$

$$\tan\left(90-\frac{B}{2}\right).\sin\frac{B}{2} > \cos\frac{B}{2}$$

$$\cot\frac{B}{2}.\sin\frac{B}{2} > \cos\frac{B}{2}$$

$$\cos\frac{B}{2} > \cos\frac{B}{2}$$

Statement (I) is incorrect.

Statement (III)

ΔABC is Δ equilateral triangle

A = 60º, B = 60º, C = 60º

L.H.S. = tan (A + C) + tan B

= tan (60 + 60) + tan 60

= $-\sqrt{3}+\sqrt{3}$ = 0 = R.H.S

Statement (III) is correct.

68. (B) $I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$ ...(i)

From Property IV

$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$ ...(ii)

from equation (i) and equation (ii)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\tan x}+\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}\,dx$$

$$2I=\int_0^{\frac{\pi}{2}}1.\,dx$$

$$2I=[x]_0^{\frac{\pi}{2}}$$

$$I=\frac{\pi}{4}$$

(69-71) :

69. (C) Given that

$f(x) = 3x^2 + 5x - 8$ ...(i)

$f'(x) = 6x + 5$ ...(ii)

$f'(3) = 6 \times 3 + 5$

$f'(3) = 23$

70. (B) From equation (ii)

$f'(x) = 6x + 5$

$f''(x) = 6$

$f''(1.5) = 6$

71. (A) from equation (i)

$f(x) = 3x^2 + 5x - 8$

$f(-1) = 3 \times (-1)^2 + 5(-1) - 8$

$= -10$

(72-74) :

72. (C) $f(x) = |x|$ and $g(x) = [x]$

$$fog\left(-\frac{7}{4}\right)+gof\left(-\frac{7}{4}\right)$$

$$\Rightarrow f\left[g\left(\frac{-7}{4}\right)\right]+g\left[f\left(\frac{-7}{4}\right)\right]$$

$\Rightarrow f[g(-1.75)] + g[f(-1.75)]$

$\Rightarrow f(-2) + g(1.75)$

$\Rightarrow 2 + 1 = 3$

73. (A) $gog\left(\frac{-9}{2}\right)+fof(-3)$

$\Rightarrow g[g(-4.5)] + f[f(-3)]$

$\Rightarrow g(-5) + f(3)$

$\Rightarrow -5 + 3 = -2$

74. (C) $\dfrac{f(-4)-g(-4)}{f(-2)+g\left(-\frac{7}{2}\right)}$

$$\Rightarrow \frac{4+4}{2+g(-3.5)}$$

$$\Rightarrow \frac{8}{2-4}$$

$\Rightarrow -4$

75. (A) **Statement I**

If $xy = 1$

$$y=\frac{1}{x}$$

L.H.S = $\tan^{-1}x + \tan^{-1}y$

$= \tan^{-1}x + \tan^{-1}\frac{1}{x}$

$= \tan^{-1}x + \cot^{-1}x \quad \left[\because \tan^{-1}x=\cot^{-1}\frac{1}{x}\right]$

$= \frac{\pi}{2}$ = R.H.S

So statement (I) is correct.

Statement II

we know that

$$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$$

So statement II is incorrect.

76. (B) Differential equation

$y = c(x-c)^2$ ...(i)

$\frac{dy}{dx} = 2c(x-c)$

$y_1 = 2c(x-c)$ ...(ii)

from equation (i) and equation (ii)

$\frac{y}{y_1} = \frac{x-c}{2}$

$c = x - \frac{2y}{y_1}$

from equation (i)

$y = \left(x - \frac{2y}{y_1}\right)\left(\frac{2y}{y_1}\right)^2$

$y = \left(x - \frac{2y}{y_1}\right)\left(\frac{4y^2}{y_1^2}\right)$

$y = \frac{xy_1 - 2y}{y_1} \times \frac{4y^2}{y_1^2}$

$y = \frac{(xy_1 - 2y)}{y_1^3} \times 4y^2$

$y_1^3 = (xy_1 - 2y) \times 4y$

order = 1 and degree = 3

77. (D) **Statement I :**

The equation of y-axis is $x = 0$

So statement I is incorrect.

Statement II :

The equation to a straight line parallel to x-axis is $y = d$ where d is a correct.

So statement II is incorrect.

78. (C) Planes

$3x - 4y - 5z = 8$ and $-3x + 4y - 5z = 7$

angle between the planes

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$\cos\theta = \frac{-9-16+25}{\sqrt{50}\sqrt{50}}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

79. (A) $[(A \cup B) \cap (C \cap D)]'$

$\Rightarrow (A \cup B)' \cup (C \cap D)'$ $[\because (x \cap y)' = x' \cup y']$

$\Rightarrow (A' \cap B') \cup (C' \cup D')$

80. (D) $\begin{vmatrix} x & z+x & z \\ y & y+z & x \\ z & x+y & y \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} x+y+z & 2(x+y+z) & x+y+z \\ y & y+z & x \\ z & x+y & y \end{vmatrix}$

$\Rightarrow (x+y+z)\begin{vmatrix} 1 & 2 & 1 \\ y & y+z & x \\ z & x+y & y \end{vmatrix}$

$\Rightarrow (x+y+z)\,[1(y^2 + yz - x^2 - xy) - 2(y^2 - xz) + 1(xy + y^2 - yz - z^2)]$

$\Rightarrow -(x+y+z)(x^2 - 2xz + z^2)$

$\Rightarrow -(x+y+z)(x-z)^2$

81. (A) $1^c = \left(1 \times \frac{180}{\pi}\right)^\circ$

$= \left(1 \times \frac{180}{22} \times 7\right)^\circ$

$= \left(\frac{630}{11}\right)^\circ$

$= 57^\circ\ 16'\ 21''$

$\simeq 57^\circ$

82. (B) $f(x) = \sin x + e^{-x} - 8x^2$

On differentiating both side w.r.t 'x'

$f'(x) = \cos x - e^{-x} - 16x$

$f'(0) = \cos 0 - e^0 - 16 \times 0$

$f'(0) = 1 - 1 - 0$

$f'(0) = 0$

83. (C) $f(x) = \begin{cases} 4x-1, & 0 \le x \le 3 \\ 3x - \frac{\lambda}{2}, & 3 < x0 \le 5 \end{cases}$ is continuous at $x = 3$,

then $\lim_{x\to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

$\lim_{x\to 3}\left(3x - \frac{\lambda}{2}\right) = \lim_{x\to 3}(4x - 1)$

$3 \times 3 - \frac{\lambda}{2} = 4 \times 3 - 1$

$\lambda = -4$

84. (D)

85. (A)

86. (B) $2^x - 2^y = 2^{x+y}$ ...(i)

On differentiating both side w.r.t. 'x'

$$2^x \log_e 2 - 2^y \log_e 2 \frac{dy}{dx} = 2^{(x+y)} \log_e 2 \left(1 + \frac{dy}{dx}\right)$$

$$2^x - 2^y \frac{dy}{dx} = 2^{(x+y)} \left(1 + \frac{dy}{dx}\right)$$

$$2^x - 2^{(x+y)} = (2^y + 2^{(x+y)}) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2^x - 2^{x+y}}{2^y + 2^{x+y}}$$

$$\frac{dy}{dx} = \frac{2^y}{2^x} \text{ [from equation (i)]}$$

$$\frac{dy}{dx} = \frac{1}{2^{x-y}}$$

87. (A) $I = \int_0^{\frac{\pi}{2}} \ln(\tan x)\, dx$ ...(i)

$$I = \int_0^{\frac{\pi}{2}} \ln\left[\tan\left(\frac{\pi}{2} - x\right)\right] dx \text{ (Property IV)}$$

$$I = \int_0^{\frac{\pi}{2}} \ln \cot x\, dx \quad \text{...(ii)}$$

from equation (i) and equation (ii)

$$2I = \int_0^{\frac{\pi}{2}} [\ln(\tan x) + \ln(\cot x)]\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} \ln(\tan x \times \cot x)\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} 0\, dx$$

$I = 0$

88. (C) Given that

$\cos^2 x + \cos^2 y = 1$

$\cos^2 x = \sin^2 y$

$$\cos^2 x = \cos^2\left(\frac{\pi}{2} - y\right)$$

$$x = \frac{\pi}{2} - y$$

$$x + y = \frac{\pi}{2}$$

then $\tan(x + y) = \tan\frac{\pi}{2}$

$= \infty$

89. (B) $I = \int \tan^2 x . \sec^4 x\, dx$

$I = \int \tan^2 x\ (\sec^2 x) \sec^2 x\, dx$

$I = \int \tan^2 x (1 + \tan^2 x) \sec^2 x\, dx$

$I = \int \tan^2 x . \sec^2 x\, dx + \int \tan^4 x . \sec^2 x\, dx$

Let $\tan x = t$

$\sec^2 x dx = dt$

$I = \int t^2\, dt + \int t^4\, dt$

$$I = \frac{t^3}{3} + \frac{t^5}{5} + c$$

$$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$$

90. (A) cos10.cos50.cos70

$\Rightarrow$ sin80.sin40.sin20

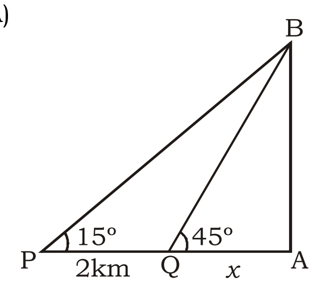
$$\Rightarrow \frac{1}{4}\sin(3\times 20)$$

$$\left[\because \sin A.\sin(60 - A).\sin(60 + A) = \frac{1}{4}\sin 3A\right]$$

$$\Rightarrow \frac{1}{4} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\sqrt{3}}{8}$$

91. (A)



Let the height of balloon (AB) = h km

and AQ = x km

In ΔABQ

$$\tan 45^\circ = \frac{AB}{AQ}$$

$$1 = \frac{h}{x}$$

$x = h$

In ΔABP

$\tan 15^\circ = \frac{AB}{AP}$

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{x+20}$

$\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h}{h+20}$

$20(\sqrt{3} - 1) = h \times 2$

$h = 10(\sqrt{3} - 1)$

$h = 10 \times 0.732$

$h = 7.32$

height of balloon (AB) = 7.32 km

92. (B) $\cos\theta > \sin\theta$ when $0 < \theta < \frac{\pi}{4}$

and $\cos\theta < \sin\theta$ when $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

93. (C) $\sin^{-1}\left(\frac{2x}{1+x^2}\right) - \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2}\right) - \tan^{-1}\left(\frac{2y}{1-y^2}\right)$

$\left[\because \sin^{-1}\left(\frac{2a}{1+a^2}\right) = \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$

$\Rightarrow 2\tan^{-1}x - 2\tan^{-1}y$

$\left[\because 2\tan^{-1}a = \tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$

$\Rightarrow 2(\tan^{-1}x - \tan^{-1}y)$

$\Rightarrow 2\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

94. (C) Let A = $\begin{bmatrix} 1+\omega^2 & \omega^2 & \omega \\ 1+\omega & \omega & \omega^2 \\ \omega+\omega^2 & \omega & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 1+\omega^2 & \omega^2 & \omega \\ 1+\omega & \omega & \omega^2 \\ \omega+\omega^2 & \omega & 1 \end{vmatrix}$

$C_1 \rightarrow C_1 + C_3$

$= \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega^2 & \omega \\ 0 & \omega & \omega^2 \\ 0 & \omega & 1 \end{vmatrix} = 0$

95. (C) In the expansion of $\left[\sqrt{\frac{x}{2}} + \frac{2}{3x}\right]^9$

general term

$T_{r+1} = {}^9C_r \left\{\left(\frac{x}{2}\right)^{\frac{1}{2}}\right\}^{9-r} \left(\frac{2}{3x}\right)^r$

$= {}^9C_r \, x^{\frac{9-r}{2}-r} \left(\frac{1}{2}\right)^{\frac{9-r}{2}} \left(\frac{2}{3}\right)^r$

then

$\frac{9-r}{2} - r = 0$

$3r = 9$

$r = 3$

$T_{3+1} = T_4 = {}^9C_3 \left(\frac{1}{2}\right)^{\frac{6}{2}} \left(\frac{2}{3}\right)^3$

$= \frac{28}{9}$

96. (C) Let $y = \log_{10}(3x^3 + 5x)$

$\frac{dy}{dx} = \frac{1}{3x^3+5x} \times (9x^2 + 5)\log_{10}e$

$\frac{dy}{dx} = \frac{(9x^2+5)}{3x^3+5x} \log_{10}e$

97. (A) $f(x) = \begin{cases} 3x-1, & -3 < x < 0 \\ x-1, & 0 \le x < 2 \\ 2x+1, & 2 \le x < 4 \end{cases}$

$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} (0 + h - 1)$

$= -1$

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= 3(0 - h) - 1$

$= -1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

function is continuous at $x = 0$.

$\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} 2(2+h) + 1$

$= 5$

$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} (2-h) - 1$

$= 1$

$\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

function is discontinuous at $x = 2$.

98. (B) $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

On putting $x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$y = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$y = \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$y = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$y = \frac{\theta}{2}$$

$$\frac{dy}{d\theta} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \frac{1}{2} \times \frac{1}{\sec^2\theta}$$

$$= \frac{1}{2(1+\tan^2\theta)}$$

$$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$$

99. (B) $I = \int x^4 \log(2x) dx$

$$I = \log(2x) \int x^4\, dx - \int\left\{\frac{d}{dx}\log(2x)\int x^4\, dx\right\}$$

$$I = \log(2x) \times \frac{x^5}{5} - \int \frac{1}{2x} \times 2 \times \frac{x^5}{5}\, dx$$

$$I = \frac{x^5}{5}\log 2x - \frac{1}{5} \times \frac{x^5}{5} + c$$

$$I = \frac{x^5}{5}\log 2x - \frac{x^5}{25} + c$$

100. (A) **Statement (I) :**

$$I = \int e^{[f(x)]^2} f(x).\, f'(x)\, dx$$

Let $[f(x)]^2 = t$

$2f(x).\, f'(x)\, dx = dt$

$$f(x).\, f'(x) dx = \frac{1}{2}\, dt$$

$$I = \int e^t \times \frac{1}{2}\, dt$$

$$I = \frac{1}{2} e^t + c$$

$$I = \frac{1}{2}\, e^{[f(x)]^2} + c$$

$$\int e^{[f(x)]^2} f(x).f'(x)\, dx = \frac{1}{2}\, e^{[f(x)]^2} + c$$

Statement (I) is correct.

Statement (II)

$$I = \int \frac{f'(x)}{f(x)}\, dx$$

Let $f(x) = t$

$f'(x)\, dx = dt$

$$I = \int \frac{1}{t}\, dt$$

$I = \log t + c$

$I = \log f(x) + c$

Statement (II) is incorrect.

Statement (III)

$$\int \log f(x)\, dx \neq \frac{1}{f(x)} + c$$

Statement (III) is incorrect.

101. (C) $I = \int_1^e \frac{(\ln x)^2}{x}\, dx$

Let $\ln x = t$ when $x \to 1,\ t = 0$

$\frac{1}{x}\, dx = dt$ $x \to e,\ t = 1$

$$I = \int_0^1 t^2\, dt = \left[\frac{t^3}{3}\right]_0^1 = \frac{1}{3}$$

102. (D) $I = \int_a^b \frac{(x)^{\frac{3}{2}}}{x^{\frac{3}{2}} + (a+b-x)^{\frac{3}{2}}} dx$ ...(i)

Property $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

$I = \int_a^b \frac{(a+b-x)^{\frac{3}{2}}}{(a+b-x)^{\frac{3}{2}} + x^{\frac{3}{2}}} \quad dx$ ...(ii)

from equation (i) and equation (ii)

$2I = \int_a^b \frac{x^{\frac{3}{2}} + (a+b-x)^{\frac{3}{2}}}{x^{\frac{3}{2}} + (a+b-x)^{\frac{3}{2}}} dx$

$2I = \int_a^b 1 .dx$

$2I = [x]_a^b$

$2I = b - a$

$I = \frac{b-a}{2}$

103. (B) $\frac{dx}{dy}$ + (tan$y$)x = 2cos$y$

On comparing with $\frac{dx}{dy} + Px = Q$

where P and Q are function of y.

$P = \tan y$ and $Q = 2\cos y$

I.F. = $e^{\int P dy}$

= $e^{\int \tan y\, dy}$

= $e^{\log \sec y}$

= sec$y$

solution of differential equation

$x \times$ I.F. = $\int Q \times \text{I.F.}\ dy$

$x \times \sec y = \int 2\cos y \times \sec y\ dy$

$x \times \sec y = 2y + c$

$x = (2y + c).\cos y$

104. (D) $y^2 = 5 + c.e^{-x} + x$ ...(i)

On differentiating w.r.t. 'x'

$2y \frac{dy}{dx} = c.e^{-x}(-1) + 1$

$c.e^{-x} = 1 - 2y \frac{dy}{dx}$

from equation (i)

$y^2 = 6 - 2y\frac{dy}{dx} + x$

$2y\frac{dy}{dx} = x + 6 - y^2$

105. (D) Points (x, y), $(-2, 3)$, $(0, 4)$ are collinear then

$$\begin{vmatrix} x & y & 1 \\ -2 & 3 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$$

$x(3-4) - y(-2-0) + 1(-8-0) = 0$

$-x + 2y - 8 = 0$

$x - 2y + 8 = 0$

106. (D)

107. (D) $x^2 + y^2 - x - \frac{1}{2}y + 5 = 0$

On comparing with general equation

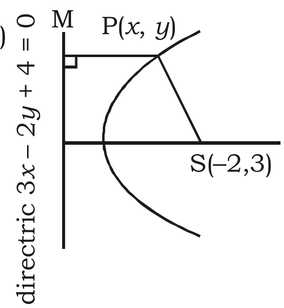
$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = -1$, $2f = -\frac{1}{2}$

$g = -\frac{1}{2}$, $f = -\frac{1}{4}$

centre $(-g, -f) = \left(\frac{1}{2}, \frac{1}{4}\right)$

108. (A)



Then SP = PM

$\sqrt{(x+2)^2 + (y-3)^2} = \frac{3x - 2y + 4}{\sqrt{(3)^2 + (2)^2}}$

on squaring

$(x+2)^2 + (y+3)^2 = \frac{(3x-2y+4)^2}{13}$

On solving

$4x^2 + 9y^2 + 12xy + 28x - 62y + 153 = 0$

109. (B) A (2, 3) —— m —— C $\left(\frac{-1}{2}, \frac{-3}{4}\right)$ —— n —— B (–4, –6)

$-\frac{1}{2} = \frac{m(-4) + n(2)}{m+n}$

$-m - n = -8m + 4n$

$7m = 5m$

$\frac{m}{n} = \frac{5}{7}$

then the point C divides the line joining the point A and B in ratio 5 : 7.

110. (D) Vertices of a triangle
$(-3, x)$, $(-5, -3)$ and $(y, 8)$
and centroid $(-6, -2)$

then $-6 = \frac{-3-5+y}{3}$ and $-2 = \frac{x-3+8}{3}$

$-18 = -8 + y$ $\quad$ $-6 = x + 5$

$y = -10$ $\quad$ $x = -11$

So value of x and y is -11 and -10.

111. (D) $\vec{a} = 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} - \lambda\hat{j} + 3\hat{k}$
are perpendicular,
then $\vec{a}\cdot\vec{b} = 0$

$2(-1) + 4(-\lambda) + (-2) \times 3 = 0$

$\lambda = -2$

112. (A)

(113 -116) :

113. (C) Given that

$\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{c} = -\vec{b}$

On squaring both side

$|\vec{a}|^2 + |\vec{c}|^2 + 2|\vec{a}|.|\vec{c}|\cos\theta = |-\vec{b}|^2$

$25 + 9 + 2 \times 5 \times 3 \cos\theta = 49$

$30\cos\theta = 15$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

114. (B) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

On squaring both side

$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}|.|\vec{b}|\cos\theta = |-\vec{c}|^2$

$(5)^2 + (7)^2 + 2 \times 5 \times 7 \cos\theta = (3)^2$

$\cos\theta = -\frac{65}{70}$

$\cos\theta = -\frac{13}{14}$

115. (A) $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

On squaring both side

$(\vec{a})^2 + (\vec{b})^2 + 2\vec{a}\cdot\vec{b} = |-\vec{c}|^2$

$(5)^2 + (7)^2 + 2\vec{a}\cdot\vec{b} = (3)^2$

$2\vec{a}\cdot\vec{b} = -65$

$\vec{a}\cdot\vec{b} = -\frac{65}{2}$

such that $\vec{b}\cdot\vec{c} = -\frac{33}{2}$ and $\vec{c}\cdot\vec{a} = \frac{15}{2}$

then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{65}{2} - \frac{33}{2} + \frac{15}{2}$

$= -\frac{83}{2}$

116. (C) $|\vec{a} + \vec{c}| = \sqrt{|\vec{a}|^2 + |\vec{c}|^2 + 2|\vec{a}||\vec{c}|}$

$= \sqrt{(5)^2 + (3)^2 + 2 \times 5 \times 3}$

$= \sqrt{64} = 8$

(117-118) :

Let the equation of sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ...(i)

it passes through the point (0, 0, 0)

$d = 0$

equation (i) passes through the point (2, 0, 0)

$u = -1$

equation (i) passes through the point (0, 3, 0)

$v = \frac{-3}{2}$

equation (i) passes through the point (0, 0, –2)

$w = 1$

from equation (i)

$x^2 + y^2 + z^2 - 2x - 3y + 2z = 0$

117. (B) Radius $r = \sqrt{u^2 + v^2 + w^2 - d}$

$r = \sqrt{1 + \frac{9}{4} + 1 - 0}$

radius $r = \frac{\sqrt{17}}{2}$

118. (B) Centre $(-u, -v, -w) = \left(1, \frac{3}{2}, -1\right)$

119. (B)

120. (A) lines

$\frac{x-4}{-2} = \frac{y+4}{3} = \frac{z-2}{5}$ and

$\frac{x+3}{-5} = \frac{y+1}{2} = \frac{z-0}{-3}$

angle between the lines

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1)^2 + (b_1)^2 + (c_1)^2}\sqrt{(a_2)^2 + (b_2)^2 + (c_2)^2}}$

$\cos\theta = \frac{1}{38}$

$\theta = \cos^{-1}\left(\frac{1}{38}\right)$

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NDA (MATHS) MOCK TEST - 84 (Answer Key)

1.	(C)	21.	(A)	41.	(B)	61.	(B)	81.	(A)	101.	(C)
2.	(A)	22.	(B)	42.	(C)	62.	(B)	82.	(B)	102.	(D)
3.	(B)	23.	(B)	43.	(A)	63.	(C)	83.	(C)	103.	(B)
4.	(A)	24.	(B)	44.	(A)	64.	(B)	84.	(D)	104.	(D)
5.	(C)	25.	(A)	45.	(A)	65.	(C)	85.	(A)	105.	(D)
6.	(A)	26.	(C)	46.	(C)	66.	(C)	86.	(B)	106.	(D)
7.	(C)	27.	(A)	47.	(B)	67.	(C)	87.	(A)	107.	(D)
8.	(A)	28.	(C)	48.	(C)	68.	(B)	88.	(C)	108.	(A)
9.	(C)	29.	(B)	49.	(A)	69.	(C)	89.	(B)	109.	(B)
10.	(B)	30.	(A)	50.	(C)	70.	(B)	90.	(A)	110.	(D)
11.	(C)	31.	(A)	51.	(C)	71.	(A)	91.	(A)	111.	(D)
12.	(B)	32.	(C)	52.	(A)	72.	(C)	92.	(B)	112.	(A)
13.	(C)	33.	(B)	53.	(B)	73.	(A)	93.	(C)	113.	(C)
14.	(B)	34.	(A)	54.	(A)	74.	(C)	94.	(C)	114.	(B)
15.	(C)	35.	(C)	55.	(B)	75.	(A)	95.	(C)	115.	(A)
16.	(A)	36.	(C)	56.	(B)	76.	(B)	96.	(C)	116.	(C)
17.	(C)	37.	(A)	57.	(C)	77.	(D)	97.	(A)	117.	(B)
18.	(C)	38	(B)	58.	(C)	78.	(C)	98.	(B)	118.	(B)
19.	(C)	39.	(D)	59.	(C)	79.	(A)	99.	(B)	119.	(B)
20.	(A)	40.	(B)	60.	(C)	80.	(D)	100.	(A)	120.	(A)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*