NDA MATHS MOCK TEST - 86 (SOLUTION)

1. (B) Let $f(x) = \frac{(x-2)^2}{|x-2|}$

L.H.L = $\lim_{x\to2^-} f(x) = \lim_{h\to0} f(2-h)$

$= \lim_{h\to0} \frac{(2-h-2)^2}{|2-h-2|}$

$= \lim_{h\to0} \frac{h^2}{h}$

$= \lim_{h\to0} h$

$= 0$

R.H.L. = $\lim_{x\to2^+} f(x) = \lim_{h\to0} f(2+h)$

$= \lim_{h\to0} \frac{(2+h-2)^2}{|2+h-2|}$

$= \lim_{h\to0} \frac{h^2}{h}$

$= \lim_{h\to0} h$

$= 0$

then $\lim_{x\to2} \frac{(x-2)^2}{|x-2|} = 0$

2. (C) $\sec^{-1}(-2) = \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$

$= \sec^{-1}\left[\sec\left(\pi-\frac{\pi}{3}\right)\right]$

$= \sec^{-1}\left[\sec\left(\frac{2\pi}{3}\right)\right]$

$= \frac{2\pi}{3}$

3. (A) Given that $f(x) = x-4$

and $gof(x) = (x-4)^3$

$g[f(x)] = [f(x)]^3$

Let $f(x) = y$

$g(y) = y^3$

$g(-2) = (-2)^3$

$= -8$

4. (B) Given that

$$\int x^3\, lnx\, dx = \frac{x^4}{a}\, lnx + \frac{x^4}{b} + c \qquad \ldots(i)$$

Let $I = \int x^3\, lnx\, dx$

$= lnx \int x^3dx - \int\left\{\frac{d}{dx}\ln x.\int x^3dx\right\}dx$

$= (lnx)\,\frac{x^4}{4} - \int \frac{1}{x} \times \frac{x^4}{4}\, dx$

$= \frac{x^4}{4}\, lnx - \frac{1}{4} \times \frac{x^4}{4} + c$

$I = \frac{x^4}{4}\, lnx - \frac{1}{16}x^4 + c$

On comparing with equation (i)

$a = 4, \quad b = -16$

5. (A) $I = \int_0^1 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{1-x}}\, dx \qquad \ldots(i)$

$I = \int_0^1 \frac{\sqrt{1-x}}{\sqrt{x-1}+\sqrt{x}}\, dx \qquad \ldots(ii)$ [Property IV]

from equation (i) and equation (ii)

$2I = \int_0^1 \frac{\sqrt{x}+\sqrt{1-x}}{\sqrt{x}+\sqrt{1-x}}\, dx$

$2I = [x]_0^1$

$2I = 1-0$

$I = \frac{1}{2}$

6. (A) $y^2 = 2a(x+a) \qquad \ldots(i)$

On differentiating both side w.r.t.'x'

$2yy_1 = 2a \qquad \ldots(ii)$

from equation (i) and equation (ii)

$\frac{y}{2y_1} = x+a$

$a = \frac{y}{2y_1} - x$

On putting equation (i)

$y^2 = 2\left(\frac{y}{2y_1}-x\right)\left(\frac{y}{2y_1}\right)$

$$y^2 = 2\left(\frac{y-2xy_1}{2y_1}\right) \times \frac{y}{2y_1}$$

$$2y^2 y_1^2 = y(y-2xy_1)$$

$$2y y_1^2 = y - 2xy_1$$

$$2y y_1^2 + 2xy_1 - y = 0$$

$$2y_1(yy_1 + x) - y = 0$$

7. (C) Differential equation

$$\frac{d^3y}{dx^3} - \sqrt{1+\left(\frac{d^2y}{dx^2}\right)^4} = 0$$

$$\left(\frac{d^3y}{dx^3}\right) = \sqrt{1+\left(\frac{d^2y}{dx^2}\right)^4}$$

$$\left(\frac{d^3y}{dx^3}\right)^2 = 1 + \left(\frac{d^2y}{dx^2}\right)^4$$

degree = 2

8. (C) Differential equation

$2\cot y\, dx + (1 + e^x)\,\mathrm{cosec}^2 y\, dy = 0$

$(1 + e^x)\,\mathrm{cosec}^2 y\, dy = -2\cot y\, dx$

$$-\frac{\mathrm{cosec}^2 y}{\cot y}\, dy = 2\,\frac{dx}{1+e^x}$$

$$\frac{-\mathrm{cosec}^2 y}{\cot y}\, dx = -2\left(\frac{-e^{-x}dx}{(e^{-x}+1)}\right)$$

On integrating both side

$\log(\cot y) = -2\log(e^{-x} + 1) + \log c$

$\log(\cot y) + \log(1 + e^{-x})^2 = \log c$

$\log[\cot y(1 + e^{-x})^2] = \log c$

$\cot y(1 + e^{-x})^2 = c$

$$\cot y\frac{(e^x+1)^2}{e^{2x}} = c$$

$(1 + e^x)^2 = ce^{2x}\tan y$

9. (C) We know that

$\det(\lambda A) = \lambda^m \det(A)$ if matrix $m \times m$

then $r = m$

10. (B) $$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 45^\circ & \cos 45^\circ \end{vmatrix} \times \begin{vmatrix} \sin 15^\circ & \cos 45^\circ \\ \cos 15^\circ & \sin 45^\circ \end{vmatrix}$$

$\Rightarrow (\cos 15.\cos 45 - \sin 45.\sin 15) \times (\sin 15.\sin 45 - \cos 15.\cos 45)$

$\Rightarrow \cos(45 + 15) \times [-\cos(45 + 15)]$

$\Rightarrow -\cos 60 \times \cos 60$

$$\Rightarrow -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$$

11. (A) Vectors $2\hat{i} - \hat{j} + 3\hat{k}$, $2\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 3\hat{k}$ are coplanar,

then $$\begin{vmatrix} 2 & -1 & 3 \\ 2\lambda & -1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 0$$

$2(-3 + 2) + 1(6\lambda - 1) + 3(-4\lambda + 1) = 0$

On solving

$\lambda = 0$

12. (B) $$y = \cos^{-1}\left(\frac{1-9x^2}{1+9x^2}\right)$$

$$y = \tan^{-1}\left[\frac{2\times 3x}{1-9x^2}\right] \quad \left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}\right]$$

$$y = \tan^{-1}\left[\frac{2\times 3x}{1-(3x)^2}\right]$$

$$y = 2\tan^{-1}(3x) \quad \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = 2 \times \frac{1}{1+(3x)^2} \times 3$$

$$\frac{dy}{dx} = \frac{6}{1+9x^2}$$

13. (B)

2	17	
2	8	1
2	4	0
2	2	0
2	1	0
	0	1

$(17)_{10} = (10001)_2$

$$\begin{array}{r} 0.125 \\ \times 2 \\ \hline \boxed{0}.250 \\ \times 2 \\ \hline \boxed{0}.500 \\ \times 2 \\ \hline \boxed{1}.000 \end{array}$$

$(0.125)_{10} = (0.001)_2$

then $(17.125)_{10} = (10001.001)_2$

14. (A) TELESCOPE

$$\text{total arrangement} = \frac{9!}{3!}$$

arrangement when E appear together = 7!

$$\text{The required arrangement} = \frac{9!}{3!} - 7!$$

$$= 12 \times 7! - 7!$$

$$= 11 \times 7!$$

15. (C) Sample space $n(S) = {}^{10}C_3 = 120$

at least one ball red.

$$n(E) = {}^3C_1 \times {}^2C_1 \times {}^5C_1 + {}^3C_1 \times {}^2C_2 \times {}^5C_0 + {}^3C_1 \times {}^2C_0 \times {}^5C_2 + {}^3C_2 \times {}^2C_1 \times {}^5C_0 + {}^3C_2 \times {}^2C_0 \times {}^5C_1 + {}^3C_3 \times {}^2C_0 \times {}^5C_0$$

$= 30 + 3 + 30 + 6 + 15 + 1$

$= 85$

$$\text{The required probability } P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{85}{120} = \frac{17}{24}$$

16. (A) There coin tossed

$S = \{(\text{HHH}), (\text{HTT}), (\text{HTH}), (\text{HHT}), (\text{THH}), (\text{THT}), (\text{TTH}), (\text{TTT})\}$

$n(S) = 8$

at most two tails

$$E = \left\{\begin{matrix} (\text{HHH}) \text{ for '0' tail} \\ (\text{HTH}), (\text{HHT}), (\text{THH}) \text{ for '1' tail} \\ (\text{HTT}), (\text{THT}), (\text{TTH}) \text{ for '2' tail} \end{matrix}\right\}$$

$n(E) = 7$

$$\text{The required probability } P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

17. (B) $$\lim_{n\to\infty} \frac{n(1+2+3+4+\ldots+n)}{(1^2+2^2+3^2+4^2+\ldots+n^2)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{n \times \frac{1}{2} n(n+1)}{\frac{n}{6}(n+1)(2n+1)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{3n}{n\left(2+\frac{1}{n}\right)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{3}{\left(2+\frac{1}{n}\right)} = \frac{3}{2}$$

18. (A) $y = x \ln x + xe^{-x}$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x \times 1 - xe^{-x} + e^{-x} \times 1$$

$$\frac{dy}{dx} = 1 + \ln x - xe^{-x} + e^{-x}$$

$$\left(\frac{dy}{dx}\right)_{at x=1} = 1 + \ln 1 - 1.e^{-1} + e^{-1} = 1$$

19. (C)

20. (B) Equation $px^2 - 7x + 8 = 0$

its roots are real and unequal,

then

$b^2 - 4ac > 0$

$(-7)^2 - 4p \times 8 > 0$

$49 - 32p > 0$

$$p < \frac{49}{32}$$

(21-23) :- $\frac{15}{2}(2a + 14d) = 180$

$a + 7d = 12$ ...(i)

and $\frac{25}{2}(2a + 24d) = 800$

$a + 12d = 32$ ...(ii)

from equation (i) and equation (ii)

$5d = 20$

$d = 4$

21. (B) Common difference $d = 4$

22. (A) From equation (i)

$a + 7 \times 4 = 12$

first term $a = -16$

23. (C) Sum of first 10 terms

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{10} = \frac{10}{2}(2 \times (-16) + 9 \times 4)$$

$= 5(-32 + 36)$

$= 5 \times 4$

$= 20$

24. (A) $A = \{2, 3\}$, $B = \{1, 2, 3, 4\}$ and $C = \{1, 2, 3\}$,

then $(A \cup C) = \{1, 2, 3\}$ and $(A \cup B) = \{1, 2, 3, 4\}$

no. of element in $(A \cup C) = 3$

no. of element in $(A \cup B) = 4$

no. of element in $(A \cup C) \times (A \cup B) = 3 \times 4 = 12$

25. (C) $\begin{bmatrix} y \\ y \\ z \end{bmatrix} + \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} z \\ z \\ x \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 8 \end{bmatrix}$

$x + y + z = 16 \quad \ldots(i)$

$y + z = 4 \quad \ldots(ii)$

$x + z = 8 \quad \ldots(iii)$

from equation (iii) and equation (ii)

$x - y = 4$

26. (B)

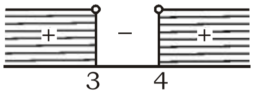
27. (A) $f(x) = \dfrac{1}{\sqrt{\log(x^2 - 7x + 13)}}$

$\log(x^2 - 7x + 13) > 0$

$x^2 - 7x + 13 > 1$

$x^2 - 7x + 12 > 0$

$(x - 3)(x - 4) > 0$



$x \in (-\infty, 3) \cup (4, \infty)$

28. (B) Variance of 30 observations = 6

we know that

$\text{var}(\lambda x) = \lambda^2 \,\text{var}(x)$

If each observation is multiplied by 3, then variance of new observations

$\text{var}(3x) = (3)^2 \times 6$

$= 54$

29. (B) Given that $r = |z| = 2$

and $\arg|z| = \theta = \dfrac{3\pi}{4}$

then $z = r(\cos\theta + i\sin\theta)$

$$= 2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 2\left(-\frac{1}{\sqrt{2}} + i \times \frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}}(i - 1)$$

$$= \sqrt{2}(i - 1)$$

30. (C)

31. (A)

32. (A) Given that $\begin{vmatrix} a & l & p \\ b & m & q \\ c & n & r \end{vmatrix} = 3$

then

$$\begin{array}{cc} & \begin{matrix} 2 & \;5 & \;2 \\ \uparrow & \;\uparrow & \;\uparrow \end{matrix} \\ 2\leftarrow & \begin{vmatrix} 4a & 10b & 4c \\ 2l & 5m & 2n \\ 2p & 5q & 2r \end{vmatrix} \end{array}$$

$\Rightarrow 2 \times 2 \times 5 \times 2 \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$

$\Rightarrow 2 \times 2 \times 5 \times 2 \begin{vmatrix} a & l & p \\ b & m & q \\ c & n & r \end{vmatrix}$

$\Rightarrow 40 \times 3 = 120$

33. (C) A = (1, 2, 5, 6, 7, 8)

no. of element in A = 6

then

no. of proper subsets of A $= 2^6 - 1$

$= 64 - 1$

$= 63$

34. (C) digits (1, 2, 3, 4, 5, 6, 7, 8)

for four digit even numbers

$\boxed{7}\boxed{6}\boxed{5}\boxed{4} = 7 \times 6 \times 5 \times 4 = 840$

only (2, 4, 6, 8)

number of four digit even numbers formed by using the given digits = 840

35. (C) Foci $(0 \pm be) = (0, \pm 3)$

$be = 3 \Rightarrow e = \dfrac{3}{b}$

and semi-minor axis $a = 2$

then $e^2 = 1 - \dfrac{a^2}{b^2}$

$\dfrac{9}{b^2} = 1 - \dfrac{4}{b^2}$

$\dfrac{13}{b^2} = 1$

$b^2 = 13$

equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{13} = 1$$

ellipse passes through the point **(–2, 0).**

36. (A) Let $y = x\sqrt{x^2+a^2} + a^2\log|x + \sqrt{x^2+a^2}\,|$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = x \times \frac{1}{2\sqrt{x^2+a^2}}(2x) + \sqrt{x^2+a^2}$$

$$+ a^2\frac{1}{x+\sqrt{x^2+a^2}} \times \left[1+\frac{1}{2\sqrt{x^2+a^2}}\times 2x\right]$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+a^2}} + \sqrt{x^2+a^2}$$

$$+ \frac{a^2}{x+\sqrt{x^2+a^2}}\left[\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right]$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{x^2+a^2}} + \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

$$\frac{dy}{dx} = \frac{(x^2+a^2)}{\sqrt{x^2+a^2}} + \sqrt{x^2+a^2}$$

$$\frac{dy}{dx} = \sqrt{x^2+a^2} + \sqrt{x^2+a^2}$$

$$\frac{dy}{dx} = 2\sqrt{x^2+a^2}$$

37. (A) $I = \int\frac{1}{1-e^{-x}}\,dx$

$$I = \int\frac{e^x}{e^x-1}\,dx$$

Let $e^x - 1 = t$

$e^x\,dx = dt$

$$I = \int\frac{1}{t}\,dt$$

$I = \log t + c$

$I = \log(e^x - 1) + c$

(38-40) :

Class	x	frequency	$f\times x$
0-10	5	8	40
10-20	15	9	135
20-30	25	f_1	$25f_1$
30-40	35	f_2	$35f_2$
40-50	45	3	135

$\Sigma f = 20 + f_1 + f_2$, $\Sigma f \times x = 310 + 25f_1 + 35f_2$

total frequency $20 + f_1 + f_2 = 32$

$f_1 + f_2 = 12$ ...(i)

$$\text{A.M.} = \frac{\Sigma f\times x}{\Sigma f}$$

$$20 = \frac{310+25f_1+35f_2}{20+f_1+f_2}$$

$5f_1 + 15f_2 = 90$

$f_1 + 3f_2 = 18$ ...(ii)

from equation (i) and equation (ii)

$f_1 = 9$ and $f_2 = 3$ ...(ii)

38. (B) $f_1 = 9$

39. (C)

Class	x_i	f_i	$\|x_i–A\|$	$f_i\times\|x_i–A\|$
0-10	5	8	15	120
10-20	15	9	5	45
20-30	25	9	5	45
30-40	35	3	15	45
40-50	45	3	25	75

$\Sigma f_i = 32$ $\Sigma f_i \times |x_i – A| = 330$

$$\text{Mean deviation} = \frac{\Sigma f\times|x_i - A|}{\Sigma f}$$

$$= \frac{330}{32}$$

$$= \frac{165}{16}$$

40. (C) $f_2 = 3$

(41-43) :

Lines $L_1 \Rightarrow \frac{x+2}{3} = \frac{y-3}{-4} = \frac{z-2}{1}$

and $L_2 \Rightarrow \frac{x-1}{4} = \frac{y+1}{-1} = \frac{z-0}{-3}$

41. (C) Angle between L_1 and L_2

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{13}{\sqrt{26}\sqrt{26}}$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

42. (B) $L_2 \Rightarrow \frac{x-1}{4} = \frac{y+1}{-1} = \frac{z-1}{-3}$

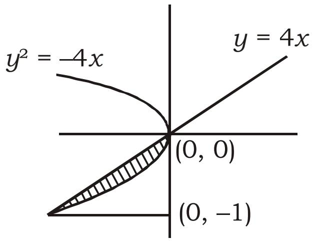
DR of L_2 = < 4, –1, – 3 >

43. (A) $L_1 \Rightarrow \frac{x+2}{3} = \frac{y-3}{-4} = \frac{z-2}{1}$

DR of L_2 = <3, –4, 1>

Direction cosine of $L_1 = <\frac{3}{\sqrt{26}}, \frac{-4}{\sqrt{26}}, \frac{1}{\sqrt{26}}>$

44. (A)



Parabola

$x_1 \Rightarrow x = -\frac{y^2}{4}$ ...(i)

and line $x_2 \Rightarrow x = \frac{y}{4}$ ...(ii)

from equation (i) and equation (ii)

$x = 0, \qquad x = -\frac{1}{4}$

$y = 0, \qquad y = -1$

$$\text{Area} = \int_{-1}^{0} (x_1 - x_2)\, dy$$

$$= \int_{-1}^{0} \left(\frac{-y^2}{4} - \frac{y}{4}\right) dy$$

$$= \left(\frac{-y^3}{12} - \frac{y^2}{8}\right)_{-1}^{0}$$

$$\text{Area} = \left[(0-0) - \left(-\frac{(-1)^3}{12} - \frac{(-1)^2}{8}\right)\right]$$

$$= \frac{-1}{12} + \frac{1}{8} = \frac{1}{24} \text{ sq. unit}$$

45. (A) $I = \int e^x \left[\frac{x}{(x+2)^3}\right] dx$

$$I = \int e^x \left[\frac{x+2-2}{(x+2)^3}\right] dx$$

$$I = \int e^x \left[\frac{1}{(x+2)^2} - \frac{2}{(x+2)^3}\right] dx$$

$$I = \frac{e^x}{(x+2)^2} + c$$

46. (C) $\lim\limits_{x\to 0} \left(\frac{x+a+b}{a+b}\right)^{\frac{1}{x}}$

$$\Rightarrow \lim_{x\to 0} \left[1 + \frac{x}{a+b}\right]^{\frac{a+b}{x} \times \frac{1}{a+b}}$$

$$\Rightarrow \lim_{x\to 0} \left[1 + \frac{x}{a+b}\right]^{\frac{a+b}{x} \times \frac{1}{a+b}}$$

$$\Rightarrow \lim_{x\to 0} \left[\left(1 + \frac{x}{a+b}\right)^{\frac{a+b}{x}}\right]^{\frac{1}{a+b}}$$

$$\Rightarrow e^{\frac{1}{a+b}}$$

47. (C) A cricket team of 11 players be chosen out of a batch of 15 players

The number of ways = ${}^{15-1}C_{11-1}$

$= {}^{14}C_{10} = 1001$

48. (B) $f(x) = x^3 + 2x^2 + x + 6$ ...(i)

$f'(x) = 3x^2 + 4x + 1$

$f''(x) = 6x + 4$ ...(ii)

for maxima or minima

$f'(x) = 0$

$3x^2 + 4x + 1 = 0$

$(3x + 1)(x + 1) = 0$

$$x = \frac{-1}{3}, -1$$

from equation (ii)

$$f''\left(-\frac{1}{3}\right) = 6\left(-\frac{1}{3}\right) + 4 = 2 \text{ (minima)}$$

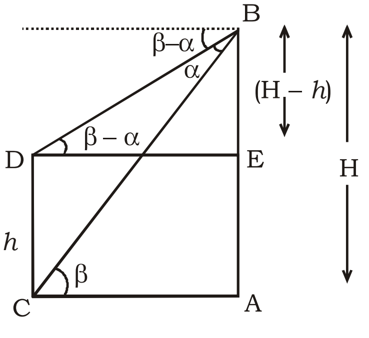
$$f''(-1) = 6(-1) + 4 = -2 \text{(maxima)}$$

from equation (i)

maximum value = $(-1)^3 + 2(-1)^2 + (-1) + 6$

= 6

49. (A)



Let height of tower (AB) = H

In ΔABC

$$\tan\beta = \frac{AB}{AC}$$

$$\tan\beta = \frac{H}{AC} \quad \text{...(i)}$$

In ΔDEB

$$\tan(\beta - \alpha) = \frac{BE}{DE}$$

$$\tan(\beta - \alpha) = \frac{H - h}{AC} \quad \ldots(ii)$$

from equation (i) and equation (ii)

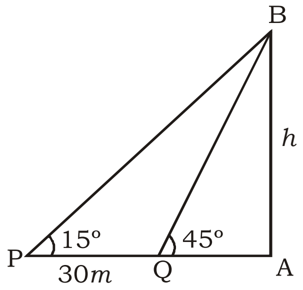
$$\frac{\tan\beta}{\tan(\beta - \alpha)} = \frac{H}{H - h}$$

$$H \tan\beta - h\tan\beta = H\tan(\beta - \alpha)$$

$$H = \frac{h \tan\beta}{\tan\beta - \tan(\beta - \alpha)}$$

$$H = \frac{h\cot(\beta - \alpha)}{\cot(\beta - \alpha) - \cot\beta}$$

50. (A)



Let AB = h m

In ΔABQ

$$\tan 45^\circ = \frac{AB}{AQ}$$

$$1 = \frac{h}{AQ} \quad \ldots(i)$$

In ΔABP

$$\tan 15^\circ = \frac{AB}{30 + AQ}$$

$$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{h}{30 + h}$$

height of the tower $h = 15(\sqrt{3} - 1)$ m

51. (C) $\sin^{-1}\frac{8}{x} + \sin^{-1}\frac{15}{x} = \frac{\pi}{2}$

$$\sin^{-1}\frac{8}{x} = \frac{\pi}{2} - \sin^{-1}\frac{15}{x}$$

$$\sin^{-1}\frac{8}{x} = \cos^{-1}\frac{15}{x} \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$\sin^{-1}\frac{8}{x} = \sin^{-1}\frac{\sqrt{x^2 - 225}}{x}$$

$$\left[\because \cos^{-1}x = \sin\sqrt{1 - x^2}\right]$$

$$\frac{8}{x} = \frac{\sqrt{x^2 - 225}}{x}$$

$$64 = x^2 - 225$$

$$x^2 = 289$$

$$x = 17$$

52. (D) $\log_9 a + \log_9\frac{4}{5} = \frac{3}{2}$

$$\log_9\frac{4a}{5} = \frac{3}{2}$$

$$\frac{4a}{5} = (9)^{\frac{3}{2}}$$

$$\frac{4a}{5} = 27 \Rightarrow a = \frac{135}{4}$$

53. (C) In the expansion of $\left(\sqrt{x} - \frac{1}{2x}\right)^9$

general term

$$T_{r+1} = {}^9C_r\,(\sqrt{x})^{9-r}\left(\frac{-1}{2x}\right)^r$$

$$= {}^9C_r\, x^{\frac{9-r}{2} - r}\left(\frac{-1}{2}\right)^r$$

then $\frac{9 - r}{2} - r = 0$

$$9 - 3r = 0$$

$$r = 3$$

Constant term = $T_4 = {}^9C_3\left(\frac{-1}{2}\right)^3$

$$= -\frac{9 \times 8 \times 7}{3 \times 2} \times \frac{1}{8} = -\frac{21}{2}$$

54. (A) Ellipse $25x^2 + 100x + 9y^2 - 125 = 0$

$$25x^2 + 100x + 9y^2 - 125 = 0$$

$$25(x^2 + 4x + 4 - 4) + 9y^2 - 125 = 0$$

$$25(x + 2)^2 + 9y^2 = 225$$

$$\frac{(x + 2)^2}{9} + \frac{y^2}{25} = 1$$

$\frac{X^2}{9} + \frac{Y^2}{25} = 1$ where $X = x + 2$

$a^2 = 9,\ b^2 = 25$ $\qquad$ $Y = y$

$$e^2 = 1 - \frac{a^2}{b^2}$$

$$e^2 = 1 - \frac{9}{25}$$

$$e = \frac{4}{5}$$

foci of the ellipse $(X, Y) = (0, \pm be)$

$X = 0 \qquad Y = \pm be$

$x + 2 = 0 \qquad y = \pm 5 \times \frac{4}{5}$

$x = -2 \qquad y = \pm 4$

foci $(-2, \pm 4)$

55. (B) $y = \sqrt{\operatorname{cosec} x + \sqrt{\operatorname{cosec} x + \sqrt{\operatorname{cosec} x + \ldots}}}$

$$y = \sqrt{\operatorname{cosec} x + y}$$

$$y^2 = \operatorname{cosec} x + y$$

On differentiating both side w.r.t. 'x'

$$2y\frac{dy}{dx} = -\operatorname{cosec} x . \cot x + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{\operatorname{cosec} x . \cot x}{1 - 2y}$$

56. (C) Probability of drawing two ace when cards are drawn successively without replacement.

$$P(E) = \frac{4}{52} \times \frac{3}{51}$$

$$= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

57. (B) When $\theta = 180$

then $M = \frac{60}{11}$ (H ± 6)

between 3 and 4

$$M = \frac{60}{11}(3 + 6)$$

$$= \frac{540}{11}$$

$= 49\frac{1}{11}$, time $= 3 : 49\frac{1}{11}$

58. (A) $C(2n, 4) = C(2n, n)$

$${}^{2n}C_4 = {}^{2n}C_n$$

$$2n = 4 + n$$

$$n = 4$$

then $C(10, n) = {}^{10}C_n = {}^{10}C_4$

$$= \frac{10!}{4!6!}$$

$$= 210$$

59. (A) $y = \log(x + \sqrt{1 + x^2})$

On differentiating w.r.t. 'x'

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}}\left(1 + \frac{1}{2\sqrt{1 + x^2}} \times 2x\right)$$

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}}\left(\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}$$

60. (B) $\left(\frac{1 - i}{1 + i}\right)^n = 1$

$$\left[\frac{(1 - i)(1 - i)}{(1 + i)(1 - i)}\right]^n = 1$$

$$\left[\frac{1 + i^2 - 2i}{1 - i^2}\right]^n = 1$$

$$\left(\frac{-2i}{2}\right)^n = 1$$

$$(-i)^n = (-i)^4$$

$$n = 4$$

61. (C) Given that $\vec{a} = (1, 2, -3)$

and $\vec{a} \cdot \vec{b} = 7$

from option C

$$\vec{b} = \frac{1}{2}, 1, -\frac{3}{2}$$

$$\vec{a} = 2\vec{b}$$

and $\vec{a} \cdot \vec{b} = \frac{1}{2} + 2 + \frac{9}{2} = 7$

vector $\left(\frac{1}{2}, 1, -\frac{3}{2}\right)$ is collinear with the vector $\vec{a} = (1, 2, -3)$ and satisfies the condition $\vec{a} \cdot \vec{b} = 7$.

62. (A) $I = \int \frac{x^3 + 1}{x^2 + 1}\, dx$

$$I = \int\left(x + \frac{1 - x}{x^2 + 1}\right) dx$$

$$I = \int x dx + \int \frac{1}{1 + x^2}\, dx - \frac{1}{2}\int \frac{2x}{x^2 + 1}\, dx$$

$$I = \frac{x^2}{2} + \tan^{-1} x - \frac{1}{2} \log(x^2 + 1) + c$$

$$I = \frac{x^2}{2} + \tan^{-1} x - \log\sqrt{1 + x^2} + c$$

63. (A) $2x = 4 + 3i$

$2x - 4 = 3i$ ...(i)

$(2x - 4)^3 = (3i)^3$

$8x^3 - 64 - 24x(2x - 4) = - 27i$

$8x^3 - 64 - 48x^2 + 96x = - 9(3i)$

$8x^3 - 48x^2 + 96x - 64 = - 9(2x - 4)$ from(i)

$8x^3 - 48x^2 + 96x - 64 = - 18x + 36$

$8x^3 - 48x^2 + 114x - 100 = 0$

$4x^3 - 24x^2 + 57x - 50 = 0$

$4x^3 - 24x^2 + 57x - 41 - 9 = 0$

$4x^3 - 24x^2 + 57x - 41 = 9$

64. (C) Let $a + ib = \sqrt{20 + 21i}$

On squaring both side

$(a^2 - b^2) + (2ab)i = 20 + 21i$

On comparing

$a^2 - b^2 = 20$ and $2ab = 21$ ...(i)

then

$(a^2 + b^2) = (a^2 - b^2)^2 + (2ab)^2$

$= (20)^2 + (21)^2$

$(a^2 + b^2)^2 = 400 + 441$

$(a^2 + b^2)^2 = 841$

$a^2 + b^2 = 29$ ...(ii)

from equation (i) and equation (ii)

$a = \pm \frac{7}{\sqrt{2}}$ $\qquad b = \pm \frac{3}{\sqrt{2}}$

Hence square root of $(20 + 21i) = \pm \left(\frac{7 + 3i}{\sqrt{2}}\right)$

65. (D) $1430 = 13 \times 11 \times 10$

$13! = 13 \times 12 \times 11 \times 10 \times 9!$

13! will be divisible by 1430

then $n = 13$

66. (B) First term = a, common ratio = r

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$S_7 = \frac{a\left(r^7 - 1\right)}{r - 1}$$

A.M. of first seven terms of G.P.

$$\Rightarrow \frac{a\left(r^7 - 1\right)}{7\left(r - 1\right)}$$

67. (A) We know that

$$\omega = \frac{-1 + \sqrt{3}i}{2} \text{ and } \omega^2 = \frac{-1 - \sqrt{3}i}{2}$$

then $\left(\frac{-1 + i\sqrt{3}}{2}\right)^{305} + \left(\frac{-1 - i\sqrt{3}}{2}\right)^{600}$

$\Rightarrow (\omega)^{305} + (\omega^2)^{600}$

$\Rightarrow (\omega)^{101\times3+2} + (\omega^3)^{400}$

$\Rightarrow \omega^2 + 1$

$\Rightarrow - \omega$ $\qquad [\because 1 + \omega + \omega^2 = 0]$

$$\Rightarrow - \left(\frac{-1 + \sqrt{3}i}{2}\right) = \frac{1 - \sqrt{3}i}{2}$$

68. (A) A $\Rightarrow x - 3 = 0$

B $\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)\ (x + 1) = 0$

$C \Rightarrow x^3 - x^2 - 5x - 3 = 0$

$x^2 - 3x^2 + 2x^2 - 6x + x - 3 = 0$

$(x - 3)\ (x^2 + 2x + 1) = 0$

$(x - 3)\ (x + 1)^2 = 0$

if $A \neq B = C$

then $x = - 1$

69. (C) $k\text{P}(30, 5) = \text{C}(30, 5)$

$$k \times \frac{30!}{25!} = \frac{30!}{5!25!}$$

$$k = \frac{1}{5!} = \frac{1}{120}$$

70. (D)

71. (C) $\cot^2 \theta = 2\cot^2 \phi + 1$

$1 + \cot^2 \theta = 2\cot^2\phi + 2$

$\text{cosec}^2\theta = 2(\text{cosec}^2 \phi)$

$$\frac{1}{\sin^2\theta} = \frac{2}{\sin^2\phi}$$

$2\sin^2\phi = 2 \times 2\sin^2\theta$

$1 - \cos2\phi = 2(1 - \cos2\theta)$

$1 - \cos2\phi = 2 - 2\cos2\theta$

$$\cos2\theta = \frac{(\cos 2\phi + 1)}{2}$$

72. (D) $\mathbf{\sin 15 = \frac{\sqrt{3} - 1}{2\sqrt{2}}}$

$\cos105 = - \sin15$

$$= - \left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)$$

$\mathbf{\cos105 = \frac{1 - \sqrt{3}}{2\sqrt{2}}}$

$\tan165 = - \tan15$

$= - (2 - \sqrt{3})$

$\mathbf{\tan165 = \sqrt{3} - 2}$

73. (B) **Statement (I)**

$$I = \int \ln 10\, dx$$

$I = \ln 10 \int 1.dx$

$I = x\, ln10 + c$

Statement (I) is incorrect.

Statement (II)

$I = \int 10^x\, dx$

$I = \frac{10^x}{\ln 10} + c$

Statement II is correct.

Statement(III)

$I = \int 1.\ln x\, \mathrm{dx}$

$I = lnx\int 1.\ln x \; - \int\left\{\frac{d}{dx}\ln x.\int 1.dx\right\}dx$

$I = xlnx - \int \frac{1}{x} \times xdx$

$I = xlnx - \int 1.dx$

$I = xlnx - x + c$

Statement III is incorrect.

74. (B) $[a\ b\ c]_{1\times 3}\begin{bmatrix}a & h & f\\ h & b & g\\ f & g & c\end{bmatrix}_{3\times3}\begin{bmatrix}x\\ y\\ z\end{bmatrix}_{3\times1}$

order = 1× 1

75. (A) $\lim_{x\to 4}\frac{x-4}{x^3-64}\left[\frac{0}{0}\right]$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 4}\frac{1-0}{3x^2-0}$

$\Rightarrow \frac{1}{3(4)^2} = \frac{1}{48}$

76. (B) Curve $x = y^2 - 6y + 7$

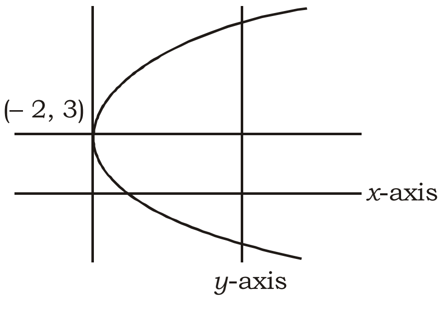
$x = (y-3)^2 - 9 + 7$

$x + 2 = (y-3)^2$

$(y-3)^2 = x + 2$

$Y^2 = X$ where $Y = y - 3$

$a = \frac{1}{4}$ $X = x + 2$



centre of parabola $(X, Y) = (0, 0)$

$X = 0, \quad Y = 0$

$x + 2 = 0, \; y - 3 = 0$

$x = -2, \quad y = 3$

One line is parallel to the y-axis at (– 2, 3).

77. (B) 6 + 6 + 6 = 18 < 20

The sum of the numbers appearing on them can not be 20.

So $P(E) = 0$

78. (B) Given that

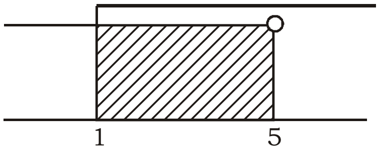
$A^2 = I$

$A^{-1}(A.A) = A^{-1}.I$

$(A^{-1}A)A = A^{-1}$

$A^{-1} = A$

79. (C) $A \Rightarrow x + y < 5$and $B \Rightarrow x + y \geq 1$



then $(A \cap B) \Rightarrow 1 \leq x + y < 5$

$(A \cap B) = \{(x, y) / 1 \leq x + y < 5\}$

80. (B) $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right)\right]$

$\Rightarrow \tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{4}\right]$

$\left[\because \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x} \text{ and } 2\tan^{-1}x = \frac{2x}{1-x^2}\right]$

$\Rightarrow \tan\left[2\tan^{-1}\frac{3}{4}\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{24}{7}\right)\right] = \frac{24}{7}$

81. (A) $\frac{d^2y}{dx^2} + \sec^2 x = 0$

$\frac{d^2y}{dx^2} = -\sec^2 x$

On integration

$\frac{dy}{dx} = -\tan x + c$

On integrating

$y = -\log\sec x + cx + d$

$y = \log\cos x + cx + d$

82. (C) Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad a < b$

$$e^2 = 1 - \frac{a^2}{b^2} \qquad \ldots(i)$$

and $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$$e^2 = 1 - \frac{9}{16} \qquad \ldots(ii)$$

from equation (i) and equation (ii)

$$1 - \frac{a^2}{b^2} = 1 - \frac{9}{16}$$

$$\frac{a^2}{b^2} = \frac{9}{16}$$

$$\frac{a}{b} = \frac{3}{4}$$

$$4a = 3b$$

83. (A) $f(x) = \begin{cases} \frac{3x-1}{\sqrt{3+x}-\sqrt{3}}, -3 < x < \infty, x \neq 0 \\ k \qquad , x = 0 \end{cases}$

is continuous at $x = 0$,

then $\lim_{x \to 0^+} \frac{3^x - 1}{\sqrt{3+x}-\sqrt{3}} = k$

by L-Hospital's Rule

$$\lim_{x \to 0} \frac{3^x \log 3}{\frac{1}{2\sqrt{3+x}}} = k$$

$$\lim_{x \to 0} 2 \times 3^x \sqrt{3+x} \log 3 = k$$

$$2\sqrt{3} \log 3 = k$$

84. (D) Given that

$S_n = n^2 - 3n + 2 \qquad \ldots(i)$

$S_{n-1} = (n-1)^2 - 3(n-1) + 2$

$S_{n-1} = n^2 + 1 - 2n - 3n + 3 + 2$

$S_{n-1} = n^2 - 5n + 6 \qquad \ldots(ii)$

n^{th} term of A.P.

$T_n = S_n - S_{n-1}$

$= n^2 - 3n + 2 - n^2 + 5n - 6$

$T_n = 2n - 4$

$T_9 = 2 \times 9 - 4$

$T_9 = 14$

85. (A) Assertion (A)

$$C(21, 5) + \sum_{r=1}^{5} C(26 - r, 4)$$

$\Rightarrow {}^{21}C_5 + {}^{25}C_4 + {}^{24}C_4 + {}^{23}C_4 + {}^{22}C_4 + {}^{21}C_4$

$\Rightarrow {}^{21}C_5 + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4 + {}^{24}C_4 + {}^{25}C_4$

$\Rightarrow {}^{22}C_5 + {}^{22}C_4 + {}^{23}C_4 + {}^{24}C_4 + {}^{25}C_4$

$[\because {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{n+1}]$

$\Rightarrow {}^{23}C_5 + {}^{23}C_4 + {}^{24}C_4 + {}^{25}C_4$

$\Rightarrow {}^{24}C_5 + {}^{24}C_4 + {}^{25}C_4$

$\Rightarrow {}^{25}C_5 + {}^{25}C_4$

$\Rightarrow {}^{26}C_5$

Assertion (A) is correct.

Reason (R) is also correct.

Hence option (A) is correct.

86. (C) In the expansion of $\left(2x - \frac{1}{4x^2}\right)^7$

general term

$$T_{r+1} = {}^7C_r (2x)^{7-r} \left(-\frac{1}{4x^2}\right)^r$$

$$= {}^7C_r\, 2^{7-r}\, x^{7-3r} \left(\frac{-1}{4}\right)^r$$

then $7 - 3r = -2$

$9 = 3r$

$r = 3$

coefficient of $x^{-2} = {}^7C_3\, 2^4 \left(\frac{-1}{4}\right)^3$

$$= -35 \times \frac{16}{64} = -\frac{35}{4}$$

87. (A) $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & -4 \\ -3 & 0 & -1 \end{bmatrix}$

$|A| = 2(-3 - 0) - 0 + 1(0 + 9)$

$|A| = 3$

Co-factors of A –

$C_{11} = (-1)^{1+1}\begin{vmatrix} 3 & -4 \\ 0 & -1 \end{vmatrix}, C_{12} = (-1)^{1+2}\begin{vmatrix} -2 & -4 \\ -3 & -1 \end{vmatrix}, C_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 3 \\ -3 & 0 \end{vmatrix}$

$= -3 \qquad = 10 \qquad = 9$

$C_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}, C_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 1 \\ -3 & -1 \end{vmatrix}, C_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix}$

$= 0 \qquad = 1 \qquad = 0$

$C_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 1 \\ 3 & -4 \end{vmatrix}, C_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 1 \\ -2 & -4 \end{vmatrix}, C_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 0 \\ -2 & 3 \end{vmatrix}$

$= -3 \qquad = 6 \qquad = 6$

$$C = \begin{bmatrix} -3 & 10 & 9 \\ 0 & 1 & 0 \\ -3 & 6 & 6 \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} -3 & 0 & -3 \\ 10 & 1 & 6 \\ 9 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj A}}{|A|}$$

$$= \begin{bmatrix} -1 & 0 & -1 \\ \frac{10}{3} & \frac{1}{3} & 2 \\ 3 & 0 & 2 \end{bmatrix}$$

88.(A)

89. (B) Parabola

$$y^2 - 4y + 8x = 0$$
$$(y-2)^2 - 4 + 8x = 0$$
$$(y-2)^2 = -8x + 4$$
$$(y-2)^2 = -8\left(x - \frac{1}{2}\right)$$
$$Y^2 = -8X \text{ where } Y = y - 2$$
$$X = x - \frac{1}{2}$$

The axis of parabola is x-axis

$i.e$ $Y = 0$

$y - 2 = 0$

$y = 2$

90. (A) Let $y = \sin^2\sqrt{x}$

$$\frac{dy}{dx} = 2\sin\sqrt{x}\,.\cos\sqrt{x}\,.\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\sin 2\sqrt{x}}{2\sqrt{x}}$$

91. (A) $\Rightarrow \dfrac{\log_{\sqrt{abc}} P}{\log_{\sqrt{ab}} P}$

$$\Rightarrow \frac{\log_P \sqrt{ab}}{\log_P \sqrt{abc}} \quad \left[\because \log_a b = \frac{1}{\log_b a}\right]$$

$$\Rightarrow \frac{\frac{1}{2}\log_p ab}{\frac{1}{2}\log_p abc} \quad \left[\because \log_a b^m = m\log_a b\right]$$

$$\Rightarrow \log_{abc} ab \left[\because \log_a b = \frac{\log_c b}{\log_c a}\right]$$

92. (B) X = (multiples of 2)

= (2, 4, 6, 8, 10, 12 ...)

Y = (multiples of 3)

= (3, 6, 9, 12, 15, 18...)

Z = (multiples of 6)

= (6, 12, 18, 24...)

$X \cap (Y \cap Z)$ = (6, 12, 18, 24...)

= multiples of 6

93. (A) Given that $x^2 + y^2 = 1$

$$\frac{1 + x - iy}{1 + x + iy}$$

$$\Rightarrow \frac{(1+x-iy)(1+x-iy)}{(1+x+iy)(1+x-iy)}$$

$$\Rightarrow \frac{1 + x^2 - y^2 + 2x - 2xyi - 2iy}{(1+x)^2 + y^2}$$

$$\Rightarrow \frac{(1-y^2) + x^2 + 2x - 2iy(x+1)}{1 + x^2 + 2x + y^2} \quad [\therefore x^2 + y^2 = 1]$$

$$\Rightarrow \frac{x^2 + x^2 + 2x - 2iy(x+1)}{1 + (x^2 + y^2) + 2x}$$

$$\Rightarrow \frac{2x(x+1) - 2iy(x+1)}{2(x+1)}$$

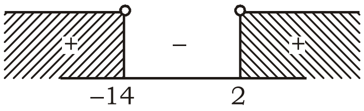
$$\Rightarrow \frac{2(x+1)(x-iy)}{2(x+1)}$$

$$\Rightarrow x - iy$$

94. (C) $f(x) = \dfrac{1}{\sqrt{x^2 + 12x - 28}}$

$x^2 + 12x - 28 > 0$

$(x + 14)(x - 2) > 0$



domain of $f(x) = (-\infty, -14) \cup (2, \infty)$

95. (C) $\dfrac{\tan 38}{\cot 128} + \dfrac{\cos 42}{\sin 132}$

$$\Rightarrow \frac{\tan 38}{\cot(90+38)} + \frac{\cos 42}{\sin(90+42)}$$

$$\Rightarrow \frac{\tan 38}{-\tan 38} + \frac{\cos 42}{\cos 42}$$

$$\Rightarrow -1 + 1$$

$$\Rightarrow 0$$

(96-98) : Given that

$$\cos(A + B) = \frac{1}{2} \text{ and } \cos(A - B) = \frac{\sqrt{3}}{2}$$

$$A + B = \frac{\pi}{3} \quad \ldots\text{(i)}$$

$$A - B = \frac{\pi}{6} \quad \ldots\text{(ii)}$$

from equation (i) and equation (ii)

$$A = \frac{\pi}{4} \text{ and } B = \frac{\pi}{12}$$

96. (B) $B = \frac{\pi}{12}$

97. (A) $\cot(A + 2B).\cot(4A + B)$

$$\Rightarrow \cot\left(\frac{\pi}{4} + \frac{\pi}{6}\right).\cot\left(\pi + \frac{\pi}{12}\right)$$

$$\Rightarrow \cot\frac{5\pi}{12}.\cot\frac{\pi}{12}$$

$$\Rightarrow \tan\frac{\pi}{12} \times \cot\frac{\pi}{12} = 1$$

98. (C) $\cos^2 A + \cos^2 2B$

$$\Rightarrow \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{6}$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

99. (A)

100. (C) Let $y = \sqrt{x^2 + 8}$ and $z = (x^3 + 4)$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x^2+8}} \times 2x, \quad \frac{dz}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2 + 8}}$$

Rate of change of y with respect to z.

$$\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$$

$$\frac{dy}{dz} = \frac{x}{\sqrt{x^2+8}} \times \frac{1}{3x^2} = \frac{1}{3x\sqrt{x^2+8}}$$

$$\left(\frac{dy}{dz}\right)_{\text{at}x=-1} = \frac{1}{3(-1)\sqrt{(-1)^2+8}}$$

$$= \frac{-1}{3 \times 3} = \frac{-1}{9}$$

101. (A) $v = -x\, lnx$

On differentiating both side w.r.t. 'x'

$$\frac{dv}{dx} = -x \times \frac{1}{x} - lnx \times 1$$

$$\frac{dv}{dx} = -1 - lnx$$

again, differentiating

$$\frac{d^2v}{dx^2} = -\frac{1}{x} \quad \ldots\text{(i)}$$

for maxima or minima

$$\frac{dv}{dx} = 0$$

$$-1 - lnx = 0$$

$$ln\, x = -1$$

$$x = e^{-1}$$

from equation (i)

$$\left(\frac{d^2v}{dx^2}\right)_{\text{at}x=e^{-1}} = \frac{-1}{e^{-1}}$$

$$= -e \qquad \text{(maxima)}$$

The velocity of telegraphic communication is maximum at $\boldsymbol{x = e^{-1}}$.

102. (C) **Statement I :**

$f(x) = |x - 4|$

$$Lf'(1) = \text{L.H.D.} = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$

$$= \lim_{h \to 0} \frac{|1 - h - 4| - 3}{-h}$$

$$\text{L.H.D.} = \lim_{h \to 0} \frac{3 + h - 3}{-h} = -1$$

$$Rf'(1) = \text{R.H.D.} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{|1 + h - 4| - 3}{h}$$

$$= \lim_{h \to 0} \frac{3 - h - 3}{h} = -1$$

L.H.D. = R.H.D.

So $f(x)$ is differentiable at $x = 1$

Statement I is correct.

Statement II :

$f(x) = |x - 4|$

$$Lf'(5) = \text{L.H.D.} = \lim_{h \to 0} \frac{f(5-h) - f(5)}{-h}$$

$$= \lim_{h \to 0} \frac{|5 - h - 4| - 1}{-h}$$

$$= \lim_{h \to 0} \frac{1 - h - 1}{-h} = 1$$

$$Rf'(5) = \text{R.H.D.} = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h \to 0} \frac{|5 + h - 4| - 1}{h}$$

$$= \lim_{h \to 0} \frac{1 + h - 1}{h}$$

$$= 1$$

L.H.D. = R.H.D.

So $f(x)$ is differentiable at $x = 5$

Statement II is correct.

103. (C) $xy^2 = c_1e^x - c_2e^{-x}$ ...(i)

On differentiating both side w.r.t. x

$$2xy\frac{dy}{dx} + y^2 = c_1e^x + c_2e^{-x}$$

again, differentiating

$$2xy\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 + 2y\frac{dy}{dx} + 2y\frac{dy}{dx} = c_1e^x - c_2e^{-x}$$

$$2xy\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 + 4y\frac{dy}{dx} = xy^2$$

second order and first degree.

104. (B) $f'(x) = 3\cos 2x + 4\sin 2x$

On integrating

$$f(x) = 3\frac{\sin 2x}{2} - \frac{4\cos 2x}{2} + c$$

$$f(x) = \frac{3}{2}\sin 2x - 2\cos 2x + c$$

Given $f(0) = -2$

then

$$-2 = \frac{3}{2}\sin 0 - 2\cos 0 + c$$

$$c = 0$$

Hence $f(x) = \frac{3}{2}\sin 2x - 2\cos 2x$

105. (B) $2x\hat{i} + 4y\hat{j} - \hat{k}$ and $6x\hat{i} + 2y\hat{j} + \hat{k}$ are orthogonal to each other.

then $2x \times 6x + 4y \times 2y + (-1)(1) = 0$

$$12x^2 + 8y^2 - 1 = 0$$

$$12x^2 + 8y^2 = 1$$

$$\frac{x^2}{\frac{1}{12}} + \frac{y^2}{\frac{1}{8}} = 1$$

locus of the point (x, y) is an ellipse.

106. (D) Given that

$\overrightarrow{OA} = -2\hat{i} + 4\hat{j} + 3k$ and $\overrightarrow{OB} = \hat{i} - 2\hat{j} + \hat{k}$

$$\overrightarrow{AB} = (\hat{i} - 2\hat{j} + \hat{k}) - (-2\hat{i} + 4\hat{j} + 3\hat{k})$$

$$\overrightarrow{AB} = 3\hat{i} - 6\hat{j} - 2\hat{k}$$

length of (AB) = $|AB| = \sqrt{(3)^2 + (-6)^2 + (-2)^2}$

$$= \sqrt{9 + 36 + 4}$$

$$= \sqrt{49}$$

length of (AB) = $|AB| = 7$

107. (C) $I = \int_0^1 x^2 e^{x^3} dx$

Let $x^3 = t$ when $x \to 0$, $t \to 0$

$3x^2\,dx = dt$ $x \to 1$, $t \to 1$

$$x^2 dx = \frac{1}{3}dt$$

$$I = \int_0^1 e^t \frac{1}{3}dt$$

$$= \frac{1}{3}\left[e^t\right]_0^1$$

$$I = \frac{1}{3}[e^1 - e^0] = \frac{(e-1)}{3}$$

108. (D) $\frac{dy}{dx} + \frac{1}{x}y = \frac{1}{x^2}$

On comparing with the equation

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{x}, \qquad Q = \frac{1}{x^2}$$

$$I.F. = e^{\int Pdx}$$

$$= e^{\int \frac{1}{x}dx}$$

$$= e^{\log x} = x$$

Solution of differential equation

$$y \times I.F. = \int Q \times I.F.\,dx$$

$$y \times x = \int \frac{1}{x^2} \times xdx$$

$$xy = \int \frac{1}{x}dx$$

$$xy = \log x + \log c$$

$$xy = \log xc$$

$$xc = e^{xy}$$

$$e^{xy} = xc$$

109. (A) $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1, (\lambda \geq 0)$

$$\text{eccentricity } e = \sqrt{1-\frac{b^2+\lambda}{a^2+\lambda}}$$

$$e = \sqrt{\frac{a^2+\lambda-b^2-\lambda}{a^2+\lambda}}$$

$$e = \sqrt{\frac{a^2-b^2}{a^2+\lambda}}$$

eccentricity will decrease with increase in λ.

110. (D) Two planes $3x + 4y + 5z + 2 = 0$ and $3x + 4y + 5z - 1 = 0$ are parallel to each other. There are no points in common.

111. (B) Length of the tangent from (3, – 2) to the circle $x^2 + y^2 + 2x - 6y + 5 = 0$

$$= \sqrt{(3)^2+(-2)^2+2(3)-6(-2)+5}$$

$$= \sqrt{9+4+6+12+5}$$

$$= \sqrt{36} = 6$$

112. (B) Given that $b_{yx} = \frac{16}{9}$ and $b_{xy} = \frac{1}{4}$

$$\text{then } r = \pm\sqrt{b_{yx} \times b_{xy}}$$

$$r = \pm\sqrt{\frac{16}{9}\times\frac{1}{4}}$$

$$r = \frac{2}{3}$$

correlation coefficient $(r) = \frac{2}{3}$

113. (A) $I = \int \frac{dx}{(x^2+9)(x^2+16)}$

$$I = \frac{1}{7}\int\left(\frac{1}{x^2+9}-\frac{1}{x^2+16}\right)dx$$

$$I = \frac{1}{7}\left[\frac{1}{3}\tan^{-1}\frac{x}{3}-\frac{1}{4}\tan^{-1}\frac{x}{4}\right]+c$$

$$I = \frac{1}{7}\left[\frac{4\tan^{-1}\frac{x}{3}-3\tan^{-1}\frac{x}{4}}{12}\right]+c$$

$$I = \frac{1}{84}\left[4\tan^{-1}\frac{x}{3}-3\tan^{-1}\frac{x}{4}\right]+c$$

114. (C) line $\frac{2x-1}{4} = \frac{y+1}{-3} = \frac{z-3}{5}$

$$\frac{x-\frac{1}{2}}{2} = \frac{y+1}{-3} = \frac{z-3}{5}$$

and plane $3x - 2y + 5z - 6 = 0$

angle between the line and plane

$$\sin\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{2\times3+(-3)(-2)+5\times5}{\sqrt{(2)^2+(-3)^2+(5)^2}\sqrt{(3)^2+(-2)^2+(5)^2}}$$

$$= \frac{6+6+25}{\sqrt{38}\sqrt{38}}$$

$$\sin\theta = \frac{37}{38}$$

$$\theta = \sin^{-1}\left(\frac{37}{38}\right)$$

115. (A)

116. (C) $\int_0^p (3x^2-2x-5)\,dx = p^3 + 6$

$$\left[3\frac{x^3}{3}-2\frac{x^2}{2}-5x\right]_0^p = p^3 + 6$$

$$p^3 - p^2 - 5p - 0 = p^3 + 6$$

$$p^2 + 5p + 6 = 0$$

$$p = -2, -3$$

117. (A) $y = a\cos(bx + c)$ ...(i)

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = -ab\sin(bx+c)$$

again, differentiating

$$\frac{d^2y}{dx^2} = -ab^2\cos(bx+c)$$

$$\frac{d^2y}{dx^2} = -b^2y \quad \text{[from equation (i)]}$$

$$\frac{d^2y}{dx^2} + b^2y = 0$$

18. (C) Equation of hyperbola

$$x^2 - 4y^2 = k^2$$

$$\frac{x^2}{k^2} - \frac{y^2}{\frac{k^2}{4}} = 1$$

$a = k$, $\quad b = \dfrac{k}{2}$

then

$$e^2 = 1 + \frac{b^2}{a^2}$$

$$e^2 = 1 + \frac{\frac{k^2}{4}}{k^2}$$

$$e^2 = 1 + \frac{1}{4}$$

$$e = \frac{\sqrt{5}}{2}$$

then foci $(\pm\, ae, 0) = (\pm\, 5, 0)$

$$ae = 5$$

$$k \times \frac{\sqrt{5}}{2} = 5$$

$$k = 2\sqrt{5}$$

119. (C)

120. (A) Given that $P(A) = 0.6$, $P(B) = 0.5$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

for the minimum value of $P(A \cap B)$, $P(A \cup B) = 1$, then

$$P(A \cap B) = 0.6 + 0.5 - 1$$

$$= 1.1 - 1$$

$$= 0.1$$

NDA (MATHS) MOCK TEST - 86 (Answer Key)

1.	(B)	21.	(B)	41.	(C)	61.	(C)	81.	(A)	101.	(A)
2.	(C)	22.	(A)	42.	(B)	62.	(A)	82.	(C)	102.	(C)
3.	(A)	23.	(C)	43.	(A)	63.	(A)	83.	(A)	103.	(C)
4.	(B)	24.	(A)	44.	(A)	64.	(C)	84.	(D)	104.	(B)
5.	(A)	25.	(C)	45.	(A)	65.	(D)	85.	(A)	105.	(B)
6.	(A)	26.	(B)	46.	(C)	66.	(B)	86.	(C)	106.	(D)
7.	(C)	27.	(A)	47.	(C)	67.	(A)	87.	(A)	107.	(C)
8.	(C)	28.	(B)	48.	(B)	68.	(A)	88.	(A)	108.	(D)
9.	(C)	29.	(B)	49.	(A)	69.	(C)	89.	(B)	109.	(A)
10.	(B)	30.	(C)	50.	(A)	70.	(D)	90.	(A)	110.	(D)
11.	(A)	31.	(A)	51.	(C)	71.	(C)	91.	(A)	111.	(B)
12.	(B)	32.	(A)	52.	(D)	72.	(D)	92.	(B)	112.	(B)
13.	(B)	33.	(C)	53.	(C)	73.	(B)	93.	(A)	113.	(A)
14.	(A)	34.	(C)	54.	(A)	74.	(B)	94.	(C)	114.	(C)
15.	(C)	35.	(C)	55.	(B)	75.	(A)	95.	(C)	115.	(A)
16.	(A)	36.	(A)	56.	(C)	76.	(B)	96.	(B)	116.	(C)
17.	(B)	37.	(A)	57.	(B)	77.	(B)	97.	(A)	117.	(A)
18.	(A)	38.	(B)	58.	(A)	78.	(B)	98.	(C)	118.	(C)
19.	(C)	39.	(C)	59.	(A)	79.	(C)	99.	(A)	119.	(C)
20.	(B)	40.	(C)	60.	(B)	80.	(B)	100.	(C)	120.	(A)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*