NDA MATHS MOCK TEST - 88 (SOLUTION)

1. (B) $\cos^4 x - \sin^4 x = A$

$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = A$

$\cos 2x = A$

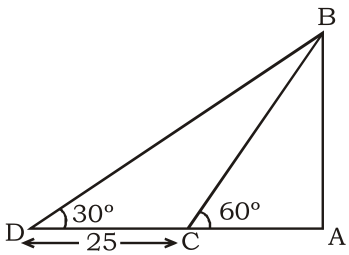
We know that

$-1 \le \cos 2x \le 1$

$-1 \le A \le 1$

$|A| \le 1$

2. (C)



Let height of the tower $(AB) = hm$

In ΔABC

$\tan 60° = \frac{AB}{AC}$

$\sqrt{3} = \frac{h}{AC}$

$AC = \frac{h}{\sqrt{3}}$ ... (i)

In ΔABD

$\tan 30° = \frac{AB}{AD}$

$\frac{1}{\sqrt{3}} = \frac{h}{AC + 25}$

$\frac{1}{\sqrt{3}} = \frac{h}{\frac{h}{\sqrt{3}} + 25}$

$\frac{25}{\sqrt{3}} = \frac{2h}{3}$

$h = \frac{25 \times 3}{2\sqrt{3}}$

$h = \frac{25\sqrt{3}}{2}$

3. (C) $f(x) = x^2 - 3x + 2$

$f'(x) = 2x - 3$

for $f(x)$ to be increasing

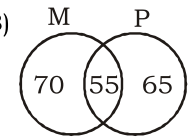
$f'(x) > 0$

$2x - 3 > 0$

$x > \frac{3}{2}$

4. (A)

5. (B)



$n(M) = 125$, $n(P) = 120$

$n(M \cap P) = 55$

The number of students passed in exactly one of the two subjects = 70 + 65

= 135

6. (A) 1 1 0

$0 \times 2^0 = 0$

$1 \times 2^1 = 2$

$1 \times 2^2 = 4$

6

$(110)_2 = (6)_{10}$

0.0 1 1

$0 = 0 \times 2^{-1}$

$\frac{1}{4} = 1 \times 2^{-2}$

$\frac{1}{8} = 1 \times 2^{-3}$

$\frac{3}{8} = 0.375$

$(0.011)_2 = (0.375)_{10}$

then $(110.011)_2 = (6.375)_{10}$

7. (B) $A = \begin{bmatrix} -3 & -1 \\ 2 & 4 \end{bmatrix}$

Co-factors of A

$C_{11} = (-1)^{1+1}(4)$, $= 4$ $\quad C_{12} = (-1)^{1+2}(2)$ $= -2$

$C_{21} = (-1)^{2+1}(-1)$, $= 1$ $\quad C_{22} = (-1)^{2+2}(-3)$ $= -3$

$C = \begin{bmatrix} 4 & -2 \\ 1 & -3 \end{bmatrix}$

$Adj\,A = C^T = \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}$

$A(Adj\,A) = \begin{bmatrix} -3 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} -12+2 & -3+3 \\ 8-8 & 2-12 \end{bmatrix}$

$= \begin{bmatrix} -10 & 0 \\ 0 & -10 \end{bmatrix}$

$= -10 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -10I$

8. (A) Given that $a = 12$ cm, $b = 20$ cm and $c = 28$ cm

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{144 + 400 - 784}{2 \times 12 \times 20}$$

$$\cos C = \frac{-240}{2 \times 240}$$

$$\cos C = \frac{-1}{2}$$

$$C = 120^\circ$$

9. (C) $I = \int \sec x \, dx$

$I = \log|\sec x + \tan x| + c$

10. (B) Word 'MOUSE'

(OUE) MS
↓
as one letter

(OUE)
↓
Total arrangement = 3! × 3!
= 6 × 6 = 36

11. (A) $|z_1| = |z_2| = 1$

$|z_1 z_2| = |z_1|\,|z_2| = 1 \times 1 = 1$

12. (C)
$$\begin{vmatrix} x & y & x+y+2z \\ 2x+y+z & y & z \\ x & x+2y+z & z \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 2(x+y+z) & y & x+y+2z \\ 2(x+y+z) & y & z \\ 2(x+y+z) & x+2y+z & z \end{vmatrix}$$

$$\Rightarrow 2(x+y+z) \begin{vmatrix} 1 & y & x+y+2z \\ 1 & y & z \\ 1 & x+2y+z & z \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow 2(x+y+z) \begin{vmatrix} 1 & y & x+y+2z \\ 0 & 0 & -x-y-z \\ 0 & x+y+z & -x-y-z \end{vmatrix}$$

$$\Rightarrow 2(x+y+z)\,[1(x+y+z)^2 - y \times 0 + (x+y+z) \times 0]$$

$$\Rightarrow 2(x+y+z)^3$$

13. (A) $45^\circ < 65^\circ$

$\sin 45^\circ < \sin 65^\circ$ and $\cos 45^\circ > \cos 65^\circ$

$\cos 45^\circ < \sin 65^\circ$

then $\sin 65^\circ > \cos 45^\circ > \cos 65^\circ$

$\sin 65^\circ > \cos 65^\circ \Rightarrow \sin 65^\circ - \cos 65^\circ > 0$

$\sin 65^\circ - \cos 65^\circ$ positive but less than 1.

14. (B) $I = \int \frac{dx}{16 + 6x - x^2}$

$$I = \int \frac{dx}{25 - 9 + 6x - x^2}$$

$$I = \int \frac{dx}{(5)^2 - (x-3)^2}$$

$$I = \frac{1}{2 \times 5} \log\left|\frac{5 + x - 3}{5 - x + 3}\right| + c$$

$$I = \frac{1}{10} \log\left|\frac{2 + x}{8 - x}\right| + c$$

15. (B) We know that

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$A = 22\frac{1}{2}$$

$$\tan 45 = \frac{2\tan 22\frac{1}{2}}{1 - \tan^2 22\frac{1}{2}}$$

$$1 = \frac{2\tan 22\frac{1}{2}}{1 - \tan^2 22\frac{1}{2}}$$

$$\tan^2 22\frac{1}{2} + 2\tan 22\frac{1}{2} - 1 = 0$$

$$\tan 22\frac{1}{2} = \frac{-2 \pm \sqrt{(-2)^2 - 4 \times (-1)}}{2 \times 1}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

then $\tan 22\frac{1}{2} = \sqrt{2} - 1$

16. (B) Let angles $2x$, $5x$, $5x$

$2x + 5x + 5x = 180$

$x = 15$

angles of a triangle = 30°, 75°, 75°

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 30} = \frac{b}{\sin 75} = \frac{c}{\sin 75}$$

$$\frac{a}{\frac{1 \times \sqrt{2}}{2 \times \sqrt{2}}} = \frac{b}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \frac{c}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}}$$

$$\frac{a}{\sqrt{2}} = \frac{b}{(\sqrt{3}+1)} = \frac{c}{\sqrt{3}+1}$$

$$a : b : c = \sqrt{2} : (\sqrt{3} + 1) : (\sqrt{3} + 1)$$

17. (C) Let $w_1 = \omega$ and $w_2 = \omega^2$

then $(w_1^2 + w_2^2 + 3w_1w_2)^2 = (\omega^2 + (\omega^2)^2 + 3 \times \omega \times \omega^2)^2$

$= (\omega^2 + \omega^4 + 3\omega^3)^2$

$= (\omega^2 + \omega + 3)^2$

$= (-1 + 3)^2$

$= 4$

18. (C) $\log_{ab} b = y$

$$\frac{1}{\log_b ab} = y$$

$$\frac{1}{\log_b a + \log_b b} = y$$

$$\frac{1}{\log_b a + 1} = y$$

$$\frac{1}{y} = \log_b a + 1$$

$$\log_b a = \frac{1-y}{y}$$

then $\log_a(ab) = \log_a a + \log_a b$

$$= 1 + \frac{1}{\log_b a}$$

$$= 1 + \frac{y}{1-y}$$

$$\log_a(ab) = \frac{1}{1-y}$$

19. (B) Three-digit number formed from (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

| 9 | 9 | 8 | = 9×9×8 = 648

'0' can not put here.

then the number of three-digit decimal number = 648

20. (A) Quadratic equation

$(a-b)x^2 - ax + 1 = 0$

one root = α

other root = 3α

then $\alpha + 3\alpha = \frac{-(-a)}{a-b}$

$$4\alpha = \frac{a}{a-b} \quad \ldots(i)$$

$$\alpha \times 3\alpha = \frac{1}{a-b}$$

$$\alpha^2 = \frac{1}{3(a-b)}$$

$$\frac{a^2}{16(a-b)^2} = \frac{1}{3(a-b)} \quad \text{from (i)}$$

$(a-b)\,[3a^2 - 16(a-b)] = 0$

$a - b \neq 0 \qquad 3a^2 - 16a + 16b = 0$

$$a = \frac{+16 \pm \sqrt{(16)^2 - 4\times3\times16b}}{2\times1}$$

$$a = \frac{16 \pm \sqrt{256 - 192b}}{2}$$

a is real

then $256 - 192b \geq 0$

$4 \geq 3b$

$$\frac{4}{3} \geq b$$

gratest value of $b = \frac{4}{3}$

21. (B) A coin tossed four times.

$n(S) = 2^4 = 16$

E = {(HTHT), (THTH)}

$n(E) = 2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{16} = \frac{1}{8}$$

22. (C) One year = 366 days

= 52 week and 2 days

$$S = \left\{\begin{matrix}(\text{Monday,Tuesday}), (\text{Tuesday, Wednesday}),\\ (\text{Wednesday, Thursday})(\text{Thursday, Friday}),\\ (\text{Friday, Saturday}), (\text{Saturday,Sunday})\\ (\text{Sunday, Monday})\end{matrix}\right\}$$

$n(S) = 7$

E = {(Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

$n(E) = 3$

The required Probability $P(E) = \frac{n(E)}{n(S)} = \frac{3}{7}$

23. (B) $I = \int_1^4 e^x \left(\frac{2x-1}{2(x)^{\frac{3}{2}}}\right) dx$

$$I = \int_1^2 e^x \left(\frac{1}{x^{\frac{1}{2}}} - \frac{1}{2x^{\frac{3}{2}}}\right) dx$$

$$I = \left[\frac{e^x}{x^{\frac{1}{2}}}\right]_1^4 \quad [\because e^x[f(x) + f'(x)]\,dx = e^x f(x) + c]$$

$$I = \frac{e^4}{(4)^{\frac{1}{2}}} - \frac{e^1}{(1)^{\frac{1}{2}}}$$

$$I = \frac{e^4}{2} - e = e\left(\frac{e^3}{2} - 1\right)$$

24. (C) $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ ...(i)

Property (iv)

$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ ...(ii)

On adding equation (i) and (ii)

$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$

$2I = \int_0^{\frac{\pi}{2}} 1.dx$

$2I = [x]_0^{\frac{\pi}{2}}$

$I = \frac{\pi}{4}$

25. (D) $x = \sin 2t$

On differentiating both side w.r.t 'x'

$\frac{dx}{dt} = 2\cos 2t$

and $y = \cos^2 t$

$\frac{dy}{dt} = 2\cos t(-\sin t)$

$\frac{dy}{dt} = -\sin 2t$

then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = -\sin 2t \times \frac{1}{2\cos 2t}$

$\frac{dy}{dx} = -\frac{1}{2}\tan 2t$

26. (D) $\frac{\text{A.M.}}{\text{G.M.}} = \frac{13}{5}$

$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{13}{5}$

$\frac{a+b}{2\sqrt{ab}} = \frac{13}{5}$

by componendo and dividendo Rule

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{18}{8}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{9}{4}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3}{2}$

by componendo and dividendo Rule

$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{3+2}{3-2}$

$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{5}{1}$

$\frac{a}{b} = \frac{25}{1}$

$a : b = 25 : 1$

27. (D) Given line $\frac{x}{5} + \frac{y}{3} = 1$

$3x + 5y = 15$

Equation of line which is perpendicular to the given line

$5x - 3y + c = 0$

It is passes through the point (– 3, 2)

$-15 - 6 + c = 0$

$c = 21$

from equation (i)

$5x - 3y + 21 = 0$

28. (D) Let $z = \frac{1+2i}{1+(1+i)^2}$

$z = \frac{1+2i}{1+(1+i^2+2i)}$

$z = \frac{1+2i}{1+2i}$

$z = 1$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{0}{1}$

$\theta = 0$

29. (A) $y = \log(x + \sqrt{1+x^2})$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\left[1 + \frac{1}{2\sqrt{1+x^2}} \times 2x\right]$

$\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\left[\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right]$

$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

again, differentiating

$\frac{d^2y}{dx} = -\frac{1}{2} \times (1+x^2)^{-3/2}(2x)$

$\frac{d^2y}{dx^2} = -\frac{x}{(1+x^2)^{\frac{3}{2}}}$

30. (D) In the expansion of $\left(x^2 + \frac{1}{2x}\right)^8$

$n = 8$ (even)

then middle term = $\left(\frac{8}{2}+1\right)^{th}$ = 5^{th}

$$T_5 = T_{4+1} = {}^8C_4\,(x^2)^{8-4}\left(\frac{1}{2x}\right)^4$$

$$T_5 = {}^8C_4\,x^8\left(\frac{1}{2x}\right)^4$$

$$T_5 = \frac{70}{16} \times x^4 = \frac{35}{8}x^4$$

The coefficient of the middle term in the expansion of $\left(x^2 + \frac{1}{2x}\right)^8$ is $\mathbf{\frac{35}{8}}$.

31. (A) $I = \int \frac{x+3}{x^2+4x-5}\,dx$

by partial fraction

$$I = \frac{1}{3}\int\left(\frac{1}{x+5} + \frac{2}{x-1}\right)dx$$

$$I = \frac{1}{3}[\log(x+5) + 2\log(x-1)] + c$$

$$I = \frac{1}{3}\log\{(x+5)(x-1)^2\} + c$$

$$I = \frac{1}{3}\log\{(x^2+4x-5)(x-1)\} + c$$

32. (B) $I = \int \tan^4\theta\, d\theta$

$$I = \int \tan^2\theta\,.\tan^2\theta\, d\theta$$

$$I = \int \tan^2\theta\,(\sec^2\theta - 1)\,d\theta$$

$$I = \int \tan^2\theta.\sec^2\theta.d\theta - \int \tan^2\theta\, d\theta$$

$$I = \int (\tan\theta)^2.\sec^2\theta d\theta - \int(\sec^2\theta - 1)\,d\theta$$

$$I = \frac{(\tan\theta)^3}{3} - \tan\theta + \theta + c$$

$$I = \frac{\tan^3\theta}{3} - \tan\theta + \theta + c$$

33. (A) $\int_1^2 \{k^2 + (3k-3)x - 3x^2\}\,dx \le 16$

$$\left[k^2x + (3k-3)\frac{x^2}{2} - \frac{3x^3}{3}\right]_1^2 \le 16$$

$$\{2k^2+2(3k-3)-8\} - \left\{k^2 + (3k-3)\times\frac{1}{2} - 1\right\} \le -16$$

$$k^2 + \frac{9k}{2} - \frac{23}{2} \le -16$$

$2k^2 + 9k - 23 \le -32$

$2k^2 + 9k + 9 \le 0$

$(2k+3)(k+3) \le 0$

+ | − | +

$-3 \quad\quad \frac{-3}{2}$

$$-3 \le k \le \frac{-3}{2}$$

34. (B) We know that planes never intersect at points, they intersect in a line.

35. (B)

2	101	
2	50	1
2	25	0
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

$(101)_{10} = (1100101)_2$

36. (D) $I = \int \frac{e^{2\ln x}}{x^2}\,dx$

$$I = \int \frac{e^{\ln x^2}}{x^2}\,dx$$

$$I = \int \frac{x^2}{x^2}\,dx$$

$$I = \int 1.dx$$

$I = x + c$

37. (D) $(1 - e^y)x\,dx = dy$

$$x\,dx = \frac{dy}{1-e^y}$$

$$-x\,dx = -\frac{e^{-y}}{e^{-y}-1}\,dy$$

On integrating

$$\frac{-x^2}{2} = \log(e^{-y} - 1) - \log c$$

$$-\frac{x^2}{2} = \log\left(\frac{1-e^y}{c.e^y}\right)$$

$$x^2 = 2\log\left(\frac{c.e^y}{1-e^y}\right)$$

38. (A) $i^3 + i^6 + i^9 + e^{12} \ldots + i^{99}$

We know that $i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

then $(i^3 + i^6 + i^9 + e^{12}) + (i^{15} + i^{18} + i^{21} + i^{24})$ $+\ldots+ (i^{87} + i^{40} + i^{93} + i^{96}) + i^{99}$

$\Rightarrow 0 + 0 + \ldots + 0 + i^{24 \times 4 + 3}$

$\Rightarrow 0 + i^3 = -i$

39. (D) Data 2, 3, 3, 4, 5, 6, 7, 4, 9, 8

$n = 10$

Mean $\overline{X} = \dfrac{2+3+3+4+5+6+7+4+9+8}{10}$

$\overline{X} = \dfrac{51}{10}$

$\overline{X} = 5.1$

Mean deviation $= \dfrac{\Sigma\left|X - \overline{X}\right|}{n}$

$= \dfrac{3.1+2.1+2.1+1.1+0.1+0.9+1.9+1.1+3.9+2.9}{10}$

$= \dfrac{19.2}{10} = 1.92$

40. (D) (I) Range of cosec$\theta \Rightarrow (-\infty, -1) \cup (1, \infty)$

Statement I is incorrect.

(II) Range of sec$\theta \Rightarrow (-\infty, -1) \cup (1, \infty)$

Statement II is incorrect.

(41-43) $S = \{8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23\ldots\}$

$B = \{9, 15\}$

$C = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22\ldots\}$

$D = \{1, 2, 3, 4, 5\ldots16, 17, 18\}$

41. (C) $(A \cap B \cap C) = \phi$

A, B and C are mutually exclusive events.

and $(A \cup B \cup C) \supseteq S$

A, B and C are exhaustive events.

42. (C) $(A \cap C \cap D) = \{2\} \neq \phi$

A, C and D are not mutually exclusive events.

and $(A \cup C \cup D) \supseteq S$

A, C and D are exhaustive events.

43. (A) $(A \cap B) = \phi$

A and B are mutually exclusive events.

and $(A \cup B) \not\supseteq S$

A and B are not exhaustive events.

44. (C)

45. (B) $(a + b - c) \tan\dfrac{C}{2}$

$\Rightarrow (2s - c - c)\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

$\Rightarrow 2(s - c)\sqrt{\dfrac{s(s-a)(s-b)(s-c)}{s^2(s-c)^2}}$

$\Rightarrow \dfrac{2(s-c)}{s(s-c)}\Delta = \dfrac{2\Delta}{s}$

46. (B) In ΔABC

AB = 5 cm, BC = 6 cm and CA = 7 cm

$c = 5$ cm, $a = 6$ cm, $b = 7$ cm

$s = \dfrac{a+b+c}{2}$

$= \dfrac{5+6+7}{2} = 9$

We know that

$\tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\tan\dfrac{A}{2} = \sqrt{\dfrac{(9-7)\times(9-5)}{9(9-6)}}$

$\tan\dfrac{A}{2} = \sqrt{\dfrac{2\times4}{9\times3}}$

$\tan^2\dfrac{A}{2} = \dfrac{8}{27}$

47. (C) **Statement I**

$r_2 = r_1 + r_3 + r$

$r_2 - r = r_1 + r_3$

$\dfrac{\Delta}{s-b} - \dfrac{\Delta}{s} = \dfrac{\Delta}{s-a} + \dfrac{\Delta}{s-c}$

$\dfrac{s-(s-b)}{s(s-b)} = \dfrac{s-c+s-a}{(s-a)(s-c)}$

$\dfrac{b}{s^2 - sb} = \dfrac{2s-a-c}{s^2 - sa - sc + ac}$

$\dfrac{b}{s^2 - sb} = \dfrac{b}{s^2 - s(a+c) + ac}$

$s^2 - s(a + c) + ac = s^2 - sb$

$2s(-b + a + c) = 2ac$

$(a + b + c)(a + c - b) = 2ac$

$(a + c)^2 - b^2 = 2ac$

$a^2 + c^2 + 2ac - b^2 = 2ac$

$a^2 + c^2 = b^2$

ΔABC is a right-angled triangle.

Statement I is correct.

Statement II

L.H. S $= \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_2}$

$= \dfrac{s-a}{\Delta} + \dfrac{s-b}{\Delta} + \dfrac{s-c}{\Delta}$

$= \dfrac{3s-a-b-c}{\Delta}$ $[\because a + b + c = 2s]$

$= \dfrac{3s-2s}{\Delta}$

$= \dfrac{s}{\Delta} = \dfrac{1}{r} =$ R.H.S. $\left[\because r = \dfrac{\Delta}{5}\right]$

Statement II is correct.

48. (C) **Statement I**

$a^2 + b^2 + c^2 = 8R^2$

$\Rightarrow (2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2 = 8R^2$

$\Rightarrow 2R^2 \times 2\sin^2 A + 2R^2 \times 2\sin^2 B + 2R^2 \times 2\sin^2 C = 2R^2 \times 4$

$\Rightarrow 2\sin^2 A + 2\sin^2 B + 2\sin^2 C = 4$

$\Rightarrow 1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C = 4$

$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$

$\Rightarrow 2\cos(A+B).\cos(A-B) + 1 + \cos 2C = 0$

$\Rightarrow 2\cos(\pi - C)\cos(A-B) + 2\cos^2 C = 0$

$\Rightarrow -2\cos C.\cos(A-B) + 2\cos^2 C = 0$

$\Rightarrow 2\cos C[\cos C - \cos(A-B)] = 0$

$\Rightarrow \cos C[\cos\{\pi - (A+B)\} - \cos(A-B)] = 0$

$\Rightarrow \cos C[-\cos(A+B) - \cos(A-B)] = 0$

$\Rightarrow \cos C[\cos(A+B) + \cos(A-B)] = 0$

$\Rightarrow \cos C \times 2\cos A.\cos B = 0$

$\Rightarrow \cos A.\cos B.\cos C = 0$

if $\cos A = 0 \Rightarrow A = 90^\circ$, if $\cos B = 0 \Rightarrow B = 90^\circ$ if $\cos C = 0 \Rightarrow C = 90^\circ$, thus ΔABC is right angled triangle.

Statement I is correct.

Statement-II

In ΔABC

$a = \sqrt{3} + 1$, $\angle B = 30^\circ$, $b = \dfrac{\sqrt{3}+1}{\sqrt{3}}$

Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{\sqrt{3}+1} = \frac{\sqrt{3}\times\sin 30}{\left(\sqrt{3}+1\right)}$$

$$\sin A = \frac{\sqrt{3}}{2}$$

$A = 60^\circ$

Statement II is incorrect.

Statement III

In ΔABC

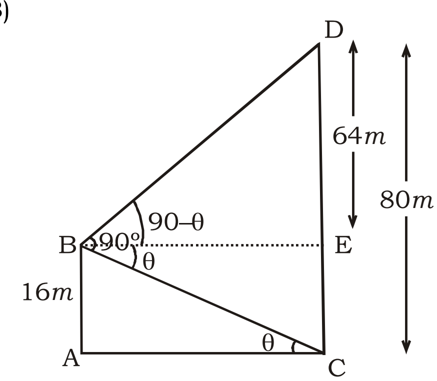
$a = 2$, $b = 4$ and $c = 7$

then $\cos B = \dfrac{(2)^2 + (7)^2 - (4)^2}{2\times 2\times 7}$

$\cos B = \dfrac{4 + 49 - 16}{28} = \dfrac{37}{28}$

Statement III is correct.

49. (B)



Let $\angle ACB = \theta$

In ΔABC

$\tan\theta = \dfrac{AB}{AC}$

$\tan\theta = \dfrac{16}{AC}$ ...(i)

In ΔEBD

$\tan(90 - \theta) = \dfrac{DE}{BE}$

$\cot\theta = \dfrac{64}{AC}$

$\tan\theta = \dfrac{AC}{64}$ ...(ii)

from equation (i) and (ii)

$(AC)^2 = 16 \times 64$

$AC = 4 \times 8$

$AC = 32\ m$

Thus distance between two houses = 32 m

50. (A) $\tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{4} + \tan^{-1}\dfrac{1}{5}$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{4}}{1-\frac{1}{3}\times\frac{1}{4}}\right] + \tan^{-1}\frac{1}{5}$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \tan^{-1}\frac{7}{11} + \tan^{-1}\frac{1}{5}$$

$$\Rightarrow \tan^{-1}\frac{7}{11} + \tan^{-1}\frac{1}{5}$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{7}{11}+\frac{1}{5}}{1-\frac{7}{11}\times\frac{1}{5}}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{46}{48}\right) = \tan^{-1}\left(\frac{23}{24}\right)$$

51. (C) Differential equation

$$xdy - ydx = \sqrt{x^2 - y^2}\, dx$$

$$x\frac{dy}{dx} - y = \sqrt{x^2 - y^2}$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2 - y^2} + y}{x}$$

Let $y = tx$

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + x\frac{dt}{dx} = \frac{\sqrt{x^2 - t^2x^2} + tx}{x}$$

$$t + x\frac{dt}{dx} = \sqrt{1 - t^2} + t$$

$$\frac{dt}{\sqrt{1 - t^2}} = \frac{dx}{x}$$

$$\sin^{-1} t = \log x + \log c$$

$$\sin^{-1}\left(\frac{y}{x}\right) = \log cx$$

52. (A) Ellipse $3x^2 + 4y^2 = 5$

$$\frac{x^2}{\frac{5}{3}} + \frac{y^2}{\frac{5}{4}} = 1$$

$$a^2 = \frac{5}{3}, \qquad b^2 = \frac{5}{4}$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$e^2 = 1 - \frac{\frac{5}{4}}{\frac{5}{3}}$$

$$e^2 = 1 - \frac{3}{4}$$

$$e^2 = \frac{1}{4} \Rightarrow e = \frac{1}{2}$$

and hyperbola $5x^2 - 4y^2 = 3$

$$\frac{y^3}{\frac{3}{5}} - \frac{y^3}{\frac{3}{4}} = 1$$

$$a^2 = \frac{3}{5}, \; b^2 = \frac{3}{4}$$

$$(e')^2 = 1 + \frac{b^2}{a^2}$$

$$(e')^2 = 1 + \frac{\frac{3}{4}}{\frac{3}{5}}$$

$$(e')^2 = 1 + \frac{5}{4}$$

$$(e')^2 = \frac{9}{4}$$

$$(e') = \frac{3}{2}$$

$$\text{then } \frac{e}{e^1} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

53. (B) $I = \int x^4 (\log x)^2\, dx$

Let $\log x = t$

$$x = e^t$$

$$dx = e^t\, dt$$

$$I = \int \left(e^t\right)^4 (t^2) e^t\, dt$$

$$I = \int e^{5t}\, t^2\, dt$$

$$I = t^2 \int e^{5t}\, dt - \int\left\{\frac{d}{dt}(t^2).\int e^{5t} dt\right\} dt$$

$$I = t^2\, \frac{e^{5t}}{5} - \int 2t\,.\frac{e^{5t}}{5}\, dt$$

$$I = \frac{1}{5} e^{5t}\, t^2 - \frac{2}{5}\int t.e^{5t}\, dt$$

$$I = \frac{1}{5} e^{5t} t^2 - \frac{2}{5}\left[t.\int e^{5t} dt - \int\left\{\frac{d}{dt}(t).\int e^{5t} dt\right\} dt\right]$$

$$I = \frac{1}{5} e^{5t} t^2 - \frac{2}{5}\left[t.\frac{e^{5t}}{5} - \int 1.\frac{e^{5t}}{5}\, dt\right]$$

$$I = \frac{1}{5} e^{5t} t^2 - \frac{2}{25}\, t.e^{5t} + \frac{2}{125}\, e^{5t} + c$$

$$I = \frac{1}{5} x^5 (\log x)^2 - \frac{2}{25} x^5 (\log x) + \frac{2}{125} x^5 + c$$

54. (A) $\frac{dy}{dx} = 5y + e^{-2x} y^{-2}$

$$\frac{dy}{dx} = 5y + \frac{e^{-2x}}{y^2}$$

$$y^2\frac{dy}{dx} = 5y^3 + e^{-2x}$$

$$y^2\frac{dy}{dx} - 5y^3 = e^{-2x} \quad \ldots(i)$$

Let $y^3 = t$

$$3y^2\frac{dy}{dx} = \frac{dt}{dx}$$

$$y^2\frac{dy}{dx} = \frac{1}{3}\frac{dt}{dx}$$

$$\frac{1}{3}\frac{dt}{dx} - 5t = e^{-2x}$$

$$\frac{dt}{dx} - 15t = 3e^{-2x}$$

On comparing general equation $\frac{dt}{dx} + Pt = Q$

$P = -15,\ Q = 3e^{-2x}$

$$I.F. = e^{\int P dx}$$

$$= e^{\int -15 dx}$$

$$= e^{-15x}$$

Solution of differential equation

$$t \times I.F. = \int Q \times \text{I.F. } dx$$

$$t \times e^{-15x} = \int 3e^{-2x} \times e^{-15x}\, dx$$

$$t \times e^{-15x} = 3 \times \frac{e^{-17x}}{-17} + c$$

$$y^3 = \frac{-3}{17}e^{-2x} + c.e^{15x}$$

when $y(0) = 0$

$$c = \frac{3}{17}$$

$$y^3 = \frac{-3}{17}e^{-2x} + \frac{3}{17}e^{15x}$$

$$17y^3 = 3\left(e^{15x} - \frac{1}{e^{2x}}\right)$$

$$17y^3 = 3\left(\frac{e^{17x} - 1}{e^{2x}}\right)$$

55.(C) Differential equation

$$x\frac{dy}{dx} + 2y = x^2 + x + 2$$

$$\frac{dy}{dx} + \frac{2}{x}y = x + 1 + \frac{2}{x}$$

On comparing with the general equation

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2}{x}, \qquad Q = x + 1 + \frac{2}{x}$$

$$I.F. = e^{\int P dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$$= e^{2\log x} = x^2$$

Solution of differential equation

$$y \times I.F. = \int Q \times I.F. dx$$

$$y \times x^2 = \int \left(x + 1 + \frac{2}{x}\right) x^2\, dx$$

$$x^2 y = \int (x^3 + x^2 + 2x)\, dx$$

$$x^2 y = \frac{x^4}{4} + \frac{x^3}{3} + x^2 + c$$

$y(0) = 0$

$0 = 0 + 0 + 0 + c$

$c = 0$

$$x^2 y = \frac{x^4}{4} + \frac{x^3}{3} + x^2$$

$$y = \frac{x^2}{4} + \frac{x}{3} + 1$$

56. (D) 1 year = 365 days

= 52 week and 1 day

$n(S)$ = 7(Mon, Tue, Wed, Thu, Fri, Sat, Sun)

$n(E)$ = 1(Wednesday)

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$$

57. (B) $n(S) = {}^5C_3 = 10$

$n(E) = {}^3C_3 = 1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{10}$$

58. (B) Bowlers takes 3 wickets in 1 over

number of ways = ${}^6C_3$

$$= \frac{6!}{3!3!}$$

$$= \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1} = 20$$

59. (C) $\lim_{x\to 0} \frac{a^{\sin x}-1}{b^{\sin x}-1}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{a^{\sin x}\log_e a - 0}{b^{\sin x}\log_e b - 0}$

$\Rightarrow \frac{a^{\sin 0}\log_e a}{b^{\sin 0}\log_e b}$

$\Rightarrow \frac{\log_e a}{\log_e b}$

60. (A) sin 1140° = sin(360 × 3 + 60)

$= \sin 60 = \frac{\sqrt{3}}{2}$

61. (C) In the expansion of $(1 + x)^{16}$

$T_{2r+6} = T_{(2r+5)+1} = {}^{16}C_{2r+5}$

and $T_{r-3} = T_{(r-4)+1} = {}^{16}C_{r-4}$

then ${}^{16}C_{2r+5} = {}^{16}C_{r-4}$

$2r + 5 + r - 4 = 16$

$3r = 15$

$r = 5$

62. (B) $(1 + x + x^2 + x^3)^{10}$

$\Rightarrow [(1 + x)(1 + x^2)]^{10}$

$\Rightarrow (1 + x)^{10}(1 + x^2)^{10}$

$\Rightarrow ({}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + {}^{10}C_4 x^4 + {}^{10}C_5 x^5 + {}^{10}C_6 x^6 \ldots)({}^{10}C_0 + {}^{10}C_1 x^2 + {}^{10}C_2 x^4 + {}^{10}C_3 x^6 + \ldots)$

Coefficient of x^6

$\Rightarrow {}^{10}C_0 \times {}^{10}C_3 + {}^{10}C_2 \times {}^{10}C_2 + {}^{10}C_4 \times {}^{10}C_1 + {}^{10}C_6 \times {}^{10}C_0$

$\Rightarrow 1 \times 120 + 45 \times 45 + 210 \times 10 + 210 \times 1$

$\Rightarrow 120 + 2025 + 2100 + 210$

$\Rightarrow 4455$

63. (C) $n(S) = 7 + 8 + 9 = 24$

E = event that the ball drawn is neither red nor blue.

= event that the ball drawn is green

$n(E) = 9$

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{9}{24} = \frac{3}{8}$

64. (C) Two dice are thrown.

$n(S) = 6 \times 6 = 36$

E = Product of odd.

= {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

$n(E) = 9$

$P(E) = \frac{n(E)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

The Probability of getting two numbers whose product is even.

$P(\overline{E}) = 1 - P(E)$

$= 1 - \frac{1}{4} = \frac{3}{4}$

65. (A) Two dice are tossed

$n(S) = 6 \times 6 = 36$

Event (E) = the sum is a prime number

= {(1, 1) (1, 2), (2, 1), (1, 4), (4, 1), (2, 3), (3, 2), (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3), (5, 6), (6, 5)}

$n(E) = 15$

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{15}{36} = \frac{5}{12}$

66. (C) In a group of 8 boys and 5 girls, four children are to be selected.

no. of ways $= {}^8C_2 \times {}^5C_2 + {}^8C_3 \times {}^5C_1 + {}^8C_4 \times {}^5C_0$

$= 28 \times 10 + 56 \times 5 + 70 \times 1$

$= 280 + 280 + 70$

$= 630$

67. (B) $\log_{10}2 + \log_{10}(2x + 1) = \log_{10}(x + 2) + 1$

$\log_{10}2(2x + 1) = \log_{10}(x + 2) + \log_{10}10$

$\log_{10}2(2x + 1) = \log_{10}10(x + 2)$

$2(2x + 1) = 10(x + 2)$

$2x + 1 = 5x + 10$

$3x = -9$

$x = -3$

68. (C) $\log_{10}(998 + \sqrt{x^2 - 5x + 10}) = 3$

$998 + \sqrt{x^2 - 5x + 10} = 1000$

$x^2 - 5x + 10 = 4$

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x = 2, 3$

69. (B) $\frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2a_1 + (n-1)d_1]} = \frac{5n-1}{2n+26}$

$\frac{a + \frac{(n-1)}{2}d}{a_1 + \frac{(n-1)}{2}d_1} = \frac{5n-1}{2n+26}$

on putting $n = 17$

$\frac{a + 8d}{a_1 + 8d_1} = \frac{5 \times 17 - 1}{2 \times 17 + 26}$

ratio of 9th terms $= \frac{84}{60} = \frac{7}{5}$

ratio of 9th terms of two AP's = 7 : 5

70. (B) $\begin{vmatrix} 1-i & i & 1+i \\ i & 1-i & 1+i \\ 1+i & i & 1-i \end{vmatrix}$

$C_1 \to C_1 + C_2,\ C_3 \to C_3 - C_2$

$\Rightarrow \begin{vmatrix} 1 & i & 1 \\ 1 & 1-i & 2i \\ 1+2i & i & 1-2i \end{vmatrix}$

$\Rightarrow 1(1 - i - 2i + 2i^2 - 2i^2) - i(1 - 2i - 2i - 4i^2)$
$+1(i - 1 - 2i + i + 2i^2)$

$\Rightarrow 1 - 3i - i(5 - 4i) + (-3)$

$\Rightarrow 1 - 3i - 5i + 4i^2 - 3$

$\Rightarrow -6 - 8i$

71. (D) $I = \int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$

$I = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\frac{a^2}{b^2}\tan^2 x + 1}\ dx$

Let $\frac{a}{b}\tan x = t$ $\qquad x \to 0,\ t \to 0$

$\frac{a}{b}\sec^2 x dx = dt$ $\qquad x \to \frac{\pi}{2},\ t \to \infty$

$\sec^2 x dx = \frac{b}{a} dt$

$I = \frac{1}{b^2} \times \frac{b}{a} \int_0^{\infty} \frac{dt}{1+t^2}$

$I = \frac{1}{ab}\left[\tan^{-1} t\right]_0^{\infty}$

$I = \frac{1}{ab}[\tan^{-1}\infty - \tan^{-1}0]$

$I = \frac{1}{ab}\left[\frac{\pi}{2} - 0\right] = \frac{\pi}{2ab}$

72. (A) $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} - \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{4}} = 0$

$\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{4}}$

$\left[\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}}\right]^{12} = \left[\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{4}}\right]^{12}$

$\left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^3$

degree = 2, order = 3

73. (B) $I = \int \sin h^3 x\ dx$

$I = \int \sin h^2 x \times \sin hx\ dx$

$I = \int (\cos h^2 x - 1) \sin hx\ dx$

Let $\cos hx = t$

$\sin hx\ dx = dt$

$I = \int (t^2 - 1)\ dt$

$I = \frac{t^3}{3} - t + c$

$I = \frac{\cos h^3 x}{3} - \cos hx + c$

74. (D) $f(x) = \ln\left(4e^{3\sqrt{x}}\right)$

$f(x) = \ln(4) + \ln e^{3\sqrt{x}}$

$f(x) = \ln 4 + 3\sqrt{x}$

On differentiating both side w.r.t 'x'

$f'(x) = 0 + 3 \times \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{3}{2\sqrt{x}}$

75. (D) $\lim_{x\to\frac{\pi}{2}} \frac{\log[1 + \cos a \cos x]}{\cos x}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\cos a(-\sin x)}{(1 + \cos a \cos x)(-\sin x)}$

$\Rightarrow \frac{\cos a}{1 + \cos a \cos \frac{\pi}{2}}$

$\Rightarrow \cos a$

76. (C) $f(x) = [x^2] = \begin{cases} 0 & 0 \le x < 1 \\ 1 & 1 \le x < \sqrt{2} \\ 2 & \sqrt{2} \le x < \sqrt{3} \\ 3 & \sqrt{3} \le x < \sqrt{2} \end{cases}$

then $I = \int_0^2 [x^2]\,dx$

$= \int_0^1 [x^2]\,dx + \int_1^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\,dx + \int_{\sqrt{3}}^{2} [x^2]\,dx$

$= \int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^{2} 3.dx$

$= 0 + \{x\}_1^{\sqrt{2}} + 2\{x\}_{\sqrt{2}}^{\sqrt{3}} + 3\{x\}_{\sqrt{3}}^{2}$

$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3[2 - \sqrt{3}]$

$= 5 - \sqrt{2} - \sqrt{3}$

77. (B) **Statement I.**

Let $I = \int [\log f(x)] f'(x) dx$

Let $f(x) = t$

$f'(x) dx = dt$

$I = \int \log t \, dt$

$I = \log t \int 1.dt - \int \left\{ \frac{d}{dt}(\log t). \int 1.dt \right\} dt$

$I = t.\log t - \int \frac{1}{t} \times t dt$

$I = t.\log t - t + c$

$I = f(x) [\log f(x) - 1] + c$

$\int \log f(x). f'(x) dx = f(x)[\log f(x) - 1] + c$

Statement I is correct.

Statement II

$I = \int e^{f(x)} f'(x) \, dx$

Let $f(x) = t$

$f'(x) dx = dt$

$I = \int e^t \, dt$

$I = e^t + c$

$\int e^{f(x)} f'(x) dx = e^{f(x)} + c$

Statement II is correct.

Statement III

$I = \int \frac{f'(x)}{f(x)} dx$

Let $f(x) = t$

$f'(x) dx = dt$

$I = \int \frac{1}{t} dt$

$I = \log t + c$

$\int \frac{f'(x)}{f(x)} \, dx = \log \{f(x)\} + c$

Statement III is incorrect.

78. (B)

79. (C) General equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$\Delta \neq 0$, $h = 0$, $a = b$ for circle

80. (A) $\frac{1}{x-4} = \frac{1}{x+3} + \frac{1}{2}$

$\frac{1}{x-4} = \frac{2+x+3}{2(x+3)}$

$2(x + 3) = (x - 4)(x + 5)$

$x^2 - x - 26 = 0$

degree = 2

81. (B)

82. (D) Five coins are tossed simultaneously

$n(S) = 2^5 = 32$

getting exactly 3 tails

$n(E) = {}^5C_3 = 10$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$= \frac{10}{32} = \frac{5}{16}$

83. (B) Harmonic mean (H) = 6

$\frac{2ab}{a+b} = 6$

$a + b = \frac{ab}{3}$ ...(i)

Now $G^2 - 4A = 16$

$ab - 4\left(\frac{a+b}{2}\right) = 16$

$ab - 2(a + b) = 16$

from equation (i)

$ab - 2\left(\frac{ab}{3}\right) = 16$

$\frac{ab}{3} = 16$

$ab = 48$

from equation (i)

$a + b = 16$ ...(ii)

then $(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^2 = (16)^2 - 4 \times 48$

$(a - b)^2 = 256 - 192$

$(a - b)^2 = 64$

$a - b = 8$ ...(ii)

from equation (ii) and equation (iii)

$a = 12$, $b = 4$

84. (A) A dice are thrown.

A = {odd number} = {1, 3, 5}

B = {prime number} = {2, 3, 5}

C = {even number} = {2, 4, 6}

$A \cap C = \phi$

A and C are mutually exclusive.

85. (C)

86. (D) $I = \int \frac{dx}{x.\ln x.\ln(\ln x)}$

Let $\ln(\ln x) = t$

$\frac{1}{\ln x} \times \frac{1}{x} dx = dt$

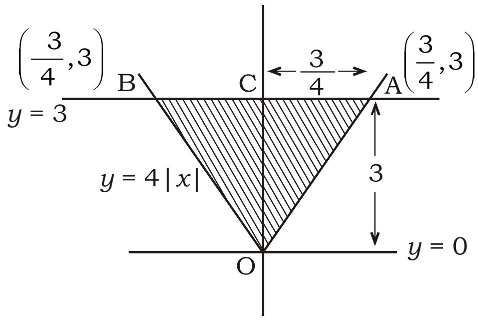
$\frac{dx}{x.\ln x} = dt$

$I = \int \frac{dt}{t}$

$I = \ln t + c$

$I = \ln\{\ln(\ln x)\} + c$

87. (C)



Total Area = 2 × Area of ΔOAC

$= 2 \times \frac{1}{2} \times OC \times AC$

$= 1 \times 3 \times \frac{3}{4} = \frac{9}{4}$ sq. unit

88. (B) $z = -1 + i$

$\tan\theta = \frac{1}{-1}$

$\tan\theta = -1$

$\tan\theta = \tan\frac{3\pi}{4}$

$\theta = \frac{3\pi}{4}$

89. (A)

2	57	
2	28	1
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

$(57)_{10} = (111001)_2$

90. (C) Equations $4x + y = 7$ and $12x - ky = 8$ have no solution.

then

$\frac{4}{12} = \frac{1}{-k}$

$k = -3$

91. (B)

92. (D) $2A^3 + 5A^2 - 3A + I = O$

$A^{-1}(2A^3 + 5A^2 - 3A + I) = A^{-1}O$

$2A^2 + 5A - 3I + A^{-1} = O$

$A^{-1} = -2A^2 - 5A + 3I$

93. (C) $f(x) = \begin{cases} \frac{1-\cos x}{\sin x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at $x = 0$

then

$\lim_{x\to 0^+} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos x}{\sin x} = k$

by L-Hospital's Rule

$\lim_{x\to 0} \frac{0+\sin x}{\cos x} = k$

$\frac{\sin 0}{\cos 0} = k$

$0 = k$

94.(A) $f(x) = \frac{\sin\left(e^{x-3}-1\right)}{\log(x-2)}$

$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{\sin\left(e^{x-3}-1\right)}{\log(x-2)}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$= \lim_{x\to 3} \frac{\cos\left(e^{x-3}-1\right)\left(e^{x-3}-0\right)}{\frac{1}{x-2}}$

$= \lim_{x\to 3} \cos(e^{x-3} - 1)e^{x-3}.(x-2)$

$= \cos(e^o - 1)\, e^o(1)$

$= 1$

95. (D) $\lim_{x\to\infty} \frac{3x^3 \sin\left(\frac{1}{x}\right) + x^2}{2 - x^2}$

$= \lim_{x\to\infty} \frac{3x\sin\left(\frac{1}{x}\right) + 1}{\frac{2}{x^2} - 1}$

$= \lim_{x\to\infty} \frac{3\frac{\sin\frac{1}{x}}{\frac{1}{x}} + 1}{\frac{2}{x^2} - 1}$

$= \frac{3\times 1 + 1}{0 - 1}$ $\left[\because \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1\right]$

$= \frac{4}{-1} = -4$

96. (D) $f(x) = 1 - 2x^5 - 3x^3$

On differentiating w.r.t. 'x'

$f'(x) = -10x^4 - 9x^2$

$= -(10x^4 + 9x^2)$

$f'(x) \leq 0$ for all value of x.

97. (C) $f(x) = \begin{cases} \frac{e^{ax} - e^{2x} + 3x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ is continuous at $x = 0$,

then

$$\lim_{x\to 0^+} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{ax} - e^{2x} + 3x}{x} = 2$$

by L-Hospital's Rule

$$\lim_{x\to 0} \frac{e^{ax} \times a - e^{2x} \times 2 + 3}{1} = 2$$

$$a - 2 + 3 = 2$$

$$a = 1$$

98. (D) $f(x) = |x[x]|$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0 - h)$

$= \lim_{h\to 0} |(0 - h)[0 - h]|$

$= \lim_{h\to 0} |(-h)(-1)|$

$= \lim_{h\to 0} h = 0$

R.H.L. $= \lim_{h\to 0^+} f(x) = \lim_{h\to 0} f(0 + h)$

$= \lim_{h\to 0} |(0 + h)[0 + h]|$

$= \lim_{h\to 0} |h \times 0| = 0$

L.H.L. = R.H.L.

$f(x)$ is continuous at $x = 0$.

L.H.D. $= \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$

$= \lim_{h\to 0} \frac{|(0-h)[0-h]| - 0}{-h}$

$= \lim_{h\to 0} \frac{|(-h)(-1)|}{-h} = -1$

R.H.D. $= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{|(0+h)[0+h]| - 0}{h}$

$= \lim_{h\to 0} \frac{h \times 0 - 0}{h} = 0$

L.H.D. $\neq$ R.H.D

$f(x)$ is not differentiable at $x = 0$

L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1 - h)$

$= \lim_{h\to 0} |(1 - h)[1 - h]|$

$= \lim_{h\to 0} (1 - h) \times 0 = 0$

R.H.L. $= \lim_{x+1^+} f(x) = \lim_{h\to 0} f(1 + h)$

$= \lim_{h\to 0} |(1 + h)[1 + h]|$

$= \lim_{h\to 0} (1 + h) \times 1 = 1$

L.H.R. $\neq$ R.H.L.

$f(x)$ is discontinuous at $x = 1$

99. (A) Let the point $A(x, x)$

then

$$\sqrt{(x+2)^2 + (x-0)^2} = \sqrt{(x-0)^2 + (x+4)^2}$$

$$x^2 + 4 + 4x + x^2 = x^2 + x^2 + 8x + 16$$

$$4x = -12$$

$$x = -3$$

Hence point is $(-3, -3)$.

100. (A) Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Difference of focal distances of any point on a hyperbola = **$2a$ (transverse axis)**

101. (A) Line

$$x - \sqrt{3}\, y + 1 = 0$$

$$x + 1 = \sqrt{3}\, y$$

$$y = \frac{1}{\sqrt{3}} x + \frac{1}{\sqrt{3}}$$

$$m = \tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

102. (A) Planes $-2x + 2y - z + 4 = 0$
and $5x + 3y - 4z + 6 = 0$
angle between planes

$$\cos\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$\cos\theta = \left| \frac{-10 + 6 + 4}{3 \times 5\sqrt{2}} \right|$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

103. (C) Equation of the line passes through the points $(3 - 2)$ and $(-2, 1)$

$$y + 2 = \frac{1+2}{-2-3}(x - 3)$$

$$y + 2 = \frac{3}{-5}(x - 3)$$

$$3x + 5y + 1 = 0 \quad \text{...(i)}$$

The length of perpendicular from the point (2,4) on the equation (i)

$$(L) = \frac{3 \times 2 + 5 \times 4 + 1}{\sqrt{(3)^2 + (5)^2}} = \frac{27}{\sqrt{34}}$$

104. (D) We know that
if a line makes α, β and γ with positive axis,
then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\alpha = \alpha, \beta = 30^\circ, \gamma = 45^\circ$
$\cos^2\alpha + \cos^2 30 + \cos^2 45 = 1$

$$\cos^2\alpha + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\cos^2\alpha + \frac{3}{4} + \frac{1}{2} = 1$$

$$\frac{5}{4} = 1 - \cos^2\alpha$$

$$\sin^2\alpha = \frac{5}{4}$$

105. (A) Line $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-4}$
and plane $2x - \lambda y + z - 4 = 0$ is parallel to each other.
then $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$2 \times 2 + 3(-\lambda) + (-4) \times 1 = 0$
$4 - 3\lambda - 4 = 0$
$\lambda = 0$

106. (B) Data 11, 12, 10, 9, 8, 7, 6, 13, 14, 15, 14, 7
On arranging in ascending order
6, 7, 7, 8, 9, 10, 11, 12, 13, 14, 14, 15
$n = 12$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}}\text{term} + \left(\frac{n}{2}+1\right)^{\text{th}}\text{term}}{2}$$

$$= \frac{6^{\text{th}}\text{ term} + 7^{\text{th}}\text{ term}}{2}$$

$$= \frac{10+11}{2} = \frac{21}{2} = 10.5$$

107. (B) $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \sqrt{2}$
$|\vec{a} + \vec{b}| = \sqrt{6}$
On squaring
$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = 6$
$3 + 2 + 2 \times \sqrt{3} \times \sqrt{2}\cos\theta = 6$

$$\cos\theta = \frac{1}{2\sqrt{6}}$$

$|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$
$|\vec{a} - \vec{b}|^2 = 3 + 2 - 2 \times \sqrt{3}\sqrt{2}\cos\theta$
$|\vec{a} - \vec{b}|^2 = 5 - 1$
$|\vec{a} - \vec{b}|^2 = 4$
$|\vec{a} - \vec{b}| = 2$

108. (D)

Class	x_i	f_i	$f_i \times x_i$	$\|x_i - A\|$	$f_i \times \|x_i - A\|$
0-10	5	4	20	22	88
10-20	15	16	240	12	192
20-30	25	14	350	2	28
30-40	35	16	560	8	128
40-50	45	10	450	18	180

$\Sigma f_i = 60, \Sigma f_i \times x_i = 1620, \Sigma f_i \times |x_i - A| = 616$

$$A = \frac{\Sigma f_i \times x_i}{\Sigma f_i}$$

$$= \frac{1620}{60} = 27$$

$$\text{Mean deviation} = \frac{\Sigma f_i \times (x_i - A)}{\Sigma f_i}$$

$$= \frac{616}{60} = 10\frac{4}{15}$$

109. (D) $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$,
$\vec{c} = \hat{i} - \hat{j} - 5\hat{k}$ and $\vec{d} = \hat{i} + \hat{j} + 2\hat{k}$
$\vec{a}\cdot\vec{b} = (\hat{i} + 2\hat{j} + \hat{k}).(\hat{i} - 2\hat{j} - 3\hat{k})$
$= 1 - 4 - 3 = -6$
$\vec{c}\cdot\vec{a} = (\hat{i} - \hat{j} - 5\hat{k}).(\hat{i} + 2\hat{j} + \hat{k})$
$= 1 - 2 - 5 = -6$
$\boldsymbol{\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a}}$

$$\vec{a} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$\vec{a} \times \vec{d} = \hat{i}(4-1) - \hat{j}(2-1) + \hat{k}(1-2)$
$\boldsymbol{\vec{a} \times \vec{d} = 3\hat{i} - \hat{j} - \hat{k}}$

$$[a\ b\ c] = \begin{vmatrix} 1 & 2 & 1 \\ 1 & -2 & -3 \\ 1 & -1 & -5 \end{vmatrix}$$

$[a\ b\ c] = 1(10-3) - 2(-5+3) + 1(-1+2)$
$[a\ b\ c] = 7 + 4 + 1$
$\boldsymbol{[a\ b\ c] = 12}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 1 & -1 & -5 \end{vmatrix}$$

$\vec{b} \times \vec{c} = \hat{i}(10-3) - \hat{j}(-5+3) + \hat{k}(-1+2)$
$\boldsymbol{\vec{b} \times \vec{c} = 7\hat{i} + 2\hat{j} + \hat{k}}$

110. (B) eccentricity of the ellipse $e = \frac{3}{5}$
and distance between foci $2ae = 6$

$$2a \times \frac{3}{5} = 6$$
$$a = 5$$

then $e^2 = 1 - \frac{b^2}{a^2}$

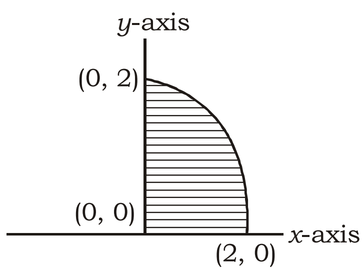
$$\left(\frac{3}{5}\right)^2 = 1 - \frac{b^2}{(5)^2} \Rightarrow \frac{b^2}{25} = \frac{16}{25} \Rightarrow b = 4$$

equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$16x^2 + 25y^2 = 400$$

111. (C)



curve $\sqrt{x} + \sqrt{y} = \sqrt{2}$

curve cut the x – axis *i.e.* $y = 0$

$x = 2$

and curve cut the y-axis *i.e.* $x = 0$

$y = 2$

$$\text{Area} = \int_0^2 y\,dx$$

$$= \int_0^2 \left(2 + x - 2\sqrt{2}\sqrt{x}\right) dx$$

$$\text{Area} = \left(2x + \frac{x^2}{2} - 2\sqrt{2} \times 2 \times \frac{x^{\frac{3}{2}}}{3}\right)_0^2$$

$$\text{Area} = 4 + 2 - \frac{16}{3} = \frac{2}{3} \text{ sq. unit}$$

112. (A) Data 3, 5, 4, 7, 8, 9, 6, 2, 1

$n = 9$

$$\sum_{i=1}^{n} x_i^2 = (3)^2 + (5)^2 + (4)^2 + (7)^2 + (8)^2 + (9)^2 + (6)^2 + (2)^2 + (1)^2$$

$= 285$

$$\sum_{i=1}^{n} x_i = 3 + 5 + 4 + 7 + 8 + 9 + 6 + 2 + 1$$

$= 45$

$$\text{S.D. } (\sigma) = \sqrt{\frac{\sum_{i=0}^{n} x_i^2}{n} - \left(\frac{\sum_{i=0}^{n} x_i}{n}\right)^2}$$

$$\text{S.D. } (\sigma) = \sqrt{\frac{285}{9} - \left(\frac{45}{9}\right)^2}$$

$$\text{S.D. } (\sigma) = \sqrt{\frac{540}{81}} = \sqrt{\frac{20}{3}}$$

$$\text{Variance } (\sigma^2) = (\text{S.D})^2 = \frac{20}{3}$$

113. (C)

114. (C)

115. (B) Parabola

$y^2 - 16y - 4x + 32 = 0$

$(y - 8)^2 - 64 = 4x - 32$

$(y - 8)^2 = 4(x + 8)$

$Y^2 = 4X$ Where $Y = y - 8$

$4a = 4$ $X = x + 8$

$a = 1$

focus of parabola $(X, Y) = (a, 0)$

$X = a, \quad Y = 0$

$x + 8 = 1, \quad y - 8 = 0$

$x = -7, \quad y = 8$

focus = $(-7, 8)$

116. (D) Lines $3x + 4y + 2 = 0$, $x - 5y + 6 = 0$ and $2x + 3y - \lambda$ are concurrent,

then
$$\begin{vmatrix} 3 & 4 & 2 \\ 1 & -5 & 6 \\ 2 & 3 & -\lambda \end{vmatrix} = 0$$

$3(5\lambda - 18) - 4(-\lambda - 12) + 2(3 + 10) = 0$

$15\lambda - 54 + 4\lambda + 48 + 26 = 0$

$19\lambda = -20$

$$\lambda = -\frac{20}{19}$$

117. (D) $\sin(1548) = \sin(360 \times 4 + 108)$

$= \sin 108$

$= \sin(90 + 18)$

$$= \cos 18 = \frac{\sqrt{10 + 2\sqrt{5}}}{4}$$

118. (C) $$\lim_{n\to\infty} \frac{(2n+1)(1^2 + 2^2 + 3^2 + 4^2 + .. + n^2)}{(1^3 + 2^3 + 3^3 + \ldots + n^3)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{(2n+1)\frac{n}{6}(n+1)(2n+1)}{\frac{1}{4}n^2(n+1)^2}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{1}{6} \times n^4 \left(2 + \frac{1}{n}\right)\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{\frac{1}{4}n^4\left(1 + \frac{1}{n}\right)^2}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2}{3} \frac{\left(2 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{\left(1 + \frac{1}{n}\right)}$$

$$\Rightarrow \frac{2}{3} \times \frac{2 \times 2}{1} = \frac{8}{3}$$

119. (D)

120. (C) $f(x) = \sin x^2$

On differentiating both side w.r.t 'x'

$f'(x) = \cos x^2 \times 2x$

$f'(x) = 2x.\cos x^2$

NDA (MATHS) MOCK TEST - 88 (Answer Key)

1.	(B)	21.	(B)	41.	(C)	61.	(C)	81.	(B)	101.	(A)
2.	(C)	22.	(C)	42.	(C)	62.	(B)	82.	(D)	102.	(A)
3.	(C)	23.	(B)	43.	(A)	63.	(C)	83.	(B)	103.	(C)
4.	(A)	24.	(C)	44.	(C)	64.	(C)	84.	(A)	104.	(D)
5.	(B)	25.	(D)	45.	(B)	65.	(A)	85.	(C)	105.	(A)
6.	(A)	26.	(D)	46.	(B)	66.	(C)	86.	(D)	106.	(B)
7.	(B)	27.	(D)	47.	(C)	67.	(B)	87.	(C)	107.	(B)
8.	(A)	28.	(D)	48.	(C)	68.	(C)	88.	(B)	108.	(D)
9.	(C)	29.	(A)	49.	(B)	69.	(B)	89.	(A)	109.	(D)
10.	(B)	30.	(D)	50.	(A)	70.	(B)	90.	(C)	110.	(B)
11.	(A)	31.	(A)	51.	(C)	71.	(D)	91.	(B)	111.	(C)
12.	(C)	32.	(B)	52.	(A)	72.	(A)	92.	(D)	112.	(A)
13.	(A)	33.	(A)	53.	(B)	73.	(B)	93.	(C)	113.	(C)
14.	(B)	34.	(B)	54.	(A)	74.	(D)	94.	(A)	114.	(C)
15.	(B)	35.	(B)	55.	(C)	75.	(D)	95.	(D)	115.	(B)
16.	(B)	36.	(D)	56.	(D)	76.	(C)	96.	(D)	116.	(D)
17.	(C)	37.	(D)	57.	(B)	77.	(B)	97.	(C)	117.	(D)
18.	(C)	38.	(A)	58.	(B)	78.	(B)	98.	(D)	118.	(C)
19.	(B)	39.	(D)	59.	(C)	79.	(C)	99.	(A)	119.	(D)
20.	(A)	40.	(D)	60.	(A)	80.	(A)	100.	(A)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*