NDA MATHS MOCK TEST - 90 (SOLUTION)

1\. (B) $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$

$\Rightarrow i^{2n}(1 + i + i^2 + i^3)$

$\Rightarrow i^{2n}(1 + i - 1 - i) = 0$

2\. (A) Let $y = x + \sqrt{1+x^2}$

on differentiating w.r.t. 'x'

$$\frac{dy}{dx} = 1 + \frac{1}{2}\left(1+x^2\right)^{\frac{-1}{2}}(2x)$$

$$\frac{dy}{dx} = \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}$$

3\. (B) $I = \int x^2 \sin x\, dx$

$$I = x^2.\int \sin x\, dx - \int\left\{\frac{d}{dx}(x^2).\int \sin x\, dx\right\} dx$$

$$I = x^2(-\cos x) - \int 2x.(-\cos x)\, dx$$

$$I = -x^2\cos x + 2\left[x\int \cos x dx - \int\left\{\frac{d}{dx}(x).\int \cos x\, dx\right\}dx\right]$$

$$I = -x^2\cos x + 2\left[x\sin x - \int 1.\sin x\, dx\right]$$

$$I = -x^2\cos x + 2x\sin x + 2\cos x + c$$

4\. (A) sum of p terms $\Rightarrow q = \frac{p}{2}[2a + (p-1)d]$

$$2a + (p-1)d = \frac{2q}{p} \quad \ldots\text{(i)}$$

sum of q terms $\Rightarrow p = \frac{q}{2}[2a + (q-1)d]$

$$2a + (q-1)d = \frac{2p}{q} \quad \ldots\text{(ii)}$$

from eq.(i) and eq.(ii)

$$d = \frac{-2(p+q)}{pq} \text{ and } a = \frac{q^2+p^2-p+pq-q}{pq}$$

$$\text{sum of } (p+q) \text{ terms} = \frac{p+q}{2}[2a+(p+q-1)d]$$

$$= (p+q)\left(-\frac{pq}{pq}\right)$$

$$= -(p+q)$$

5\. (C) $(1011001)_2 = (89)_{10}$ and $(101)_2 = (5)_{10}$

then $(89 \div 5)$

remainder $= (4)_{10}$ and quatient $= (17)_{10}$

$= (100)_2$ and $= (10001)_2$

6\. (B) Word 'COMBINATION'

$$\text{Total Permutation} = \frac{11!}{2!2!2!}$$

$$= 4989600$$

7\. (D) $A = \begin{bmatrix} 2i-3 & 5i \\ 7i & 3+2i \end{bmatrix}$

$$\lambda A = \frac{1}{i}\begin{bmatrix} 2i-3 & 5i \\ 7i & 3+2i \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2i-3}{i} & 5 \\ 7 & \frac{3+2i}{i} \end{bmatrix}$$

$$= \begin{bmatrix} 2+3i & 5 \\ 7 & 2-3i \end{bmatrix}$$

8\. (C) equation

$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

roots $\alpha = \frac{3}{2}$ and $\beta = -2$

then $\alpha + \beta^{-1} = \frac{3}{2} + (-2)^{-1}$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

$$\beta - \alpha^{-1} = -2 - \left(\frac{3}{2}\right)^{-1}$$

$$= -2 - \frac{2}{3} = \frac{-8}{3}$$

equation whose roots 1 and $\frac{-8}{3}$

$x^2 -$ (sum of roots)x + product of roots = 0

$$x^2 - \left(\frac{-5}{3}\right)x + \left(\frac{-8}{3}\right) = 0$$

$$3x^2 + 5x - 8 = 0$$

9. (B) $(1 + \omega)(1 + \omega^2)(1 + \omega^3)(1 + \omega^4)(1 + \omega + \omega^2) = 0$ $[\because 1 + \omega + \omega^2 = 0]$

10. (A) $\sin 1020 = \sin(360 \times 3 - 60)$

$$= -\sin 60 = -\frac{\sqrt{3}}{2}$$

11. (C) differential equation

$$\frac{dy}{dx} - \frac{1}{x}y = x \text{ compare with } \frac{dy}{dx} + py = Q$$

$$p = \frac{-1}{x} \text{ and } Q = x$$

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{\int -\frac{1}{x}dx}$$

$$= e^{-\log x} = \frac{1}{x}$$

Solution of differential equation

$$y \times I.F = \int Q \times I.F.dx$$

$$y \times \frac{1}{x} = \int x \times \frac{1}{x}\ dx$$

$$\frac{y}{x} = \int 1.dx$$

$$\frac{y}{x} = x + c$$

$$y = x^2 + cx$$

12.(D) $f(x) = \lim_{x\to 0} \frac{\sin x - x\cos x}{x^2}$ Form $\left[\frac{0}{0}\right]$

by L-Hospital's Rule

$$= \lim_{x\to 0} \frac{\cos x - x(-\sin x) - \cos x}{2x}$$

$$= \lim_{x\to 0} \frac{\sin x}{2} = 0$$

13. (A) $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ ...(i)

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan\left(\frac{\pi}{2} - x\right)}}{\sqrt{\tan\left(\frac{\pi}{2} - x\right)} + \sqrt{\cot\left(\frac{\pi}{2} - x\right)}} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} \quad \text{...(ii)}$$

From equation (i) and equation (ii)

$$2I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$2I = [x]_0^{\frac{\pi}{2}} \Rightarrow I = \frac{\pi}{4}$$

14. (C) $\sin^{-1}\left(\frac{5}{13}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

$$\left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{56}{33}\right)$$

15. (B) $\frac{1}{\log_2 e} + \frac{1}{\log_2 e^2} + \frac{1}{\log_2 e^4} + ...\infty$

$$\Rightarrow \frac{1}{\log_2 e} + \frac{1}{2\log_2 e} + \frac{1}{4\log_2 e} + ...\infty$$

$$\Rightarrow \log_e 2 + \frac{1}{2}\log_e 2 + \frac{1}{4}\log_e 2 + ...\infty$$

$$\Rightarrow \log_e 2\left[1 + \frac{1}{2} + \frac{1}{4} + ...\infty\right]$$

$$\Rightarrow \log_e 2\left[\frac{1}{1 - \frac{1}{2}}\right] = 2\log_e 2$$

16. (D) $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{3}{7}\right)$

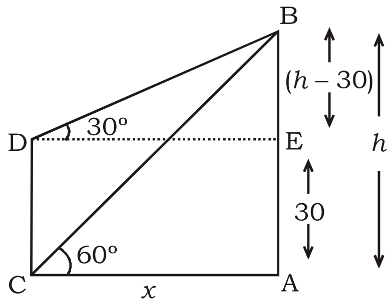
$$\Rightarrow \tan^{-1}\left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}}\right) + \tan^{-1}\left(\frac{3}{7}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{7}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{4}{3} + \frac{3}{7}}{1 - \frac{4}{3} \times \frac{3}{7}}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{37}{9}\right)$$

17. (A)



Let height of the tower $(AB) = h\ m$

$DE = CA = x\ m$

In ΔDEB,

$$\tan 30° = \frac{h-30}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{h-30}{x} \qquad \ldots(i)$$

In ΔABC,

$$\tan 60° = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x} \qquad \ldots(ii)$$

From equation (i) and equation (ii)

$$\frac{1}{3} = \frac{h-30}{h}$$

$h = 45\ m$

18. (B) $y = (\sin x)^{(\sin x)^{(\sin x)^{(\sin x)\ldots\infty}}}$

$y = (\sin x)^y$

taking log both side

$\log y = y \log \sin x$

On differentiating both side w.r.t. 'x'

$$\frac{1}{y}\frac{dy}{dx} = y \times \frac{1}{\sin x} \times \cos x + (\log \sin x)\frac{dy}{dx}$$

$$\left(\frac{1}{y} - \log \sin x\right)\frac{dy}{dx} = y \cot x$$

$$\frac{1 - y\log \sin x}{y}\frac{dy}{dx} = y \cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{1 - y \log \sin x}$$

19. (D) $I = \int \sqrt{1+\sin x}\ dx$

$$I = \int \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}\ dx$$

$$I = \int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$$

$$I = -\frac{\cos\frac{x}{2}}{\frac{1}{2}} + \frac{\sin\frac{x}{2}}{\frac{1}{2}} + C$$

$$I = -2\cos\frac{x}{2} + 2\sin\frac{x}{2} + C$$

$$I = 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$$

20. (B) $\left[1 + \left(\frac{dy}{dx}\right)^{\frac{4}{3}}\right]^2 = \left(\frac{d^2y}{dx^2}\right)^4$

$$1 + \left(\frac{dy}{dx}\right)^{\frac{4}{3}} = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\left(\frac{dy}{dx}\right)^{\frac{4}{3}} = \left(\frac{d^2y}{dx^2}\right)^2 - 1$$

$$\left(\frac{dy}{dx}\right)^4 = \left(\frac{d^2y}{dx^2}\right)^6 - 1 - 3\left(\frac{d^2y}{dx^2}\right)^2$$

$$\left[\left(\frac{d^2y}{dx^2}\right)^2 - 1\right]$$

order = 2 and degree = 6

21. (C) $A = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\downarrow$$

$$n^2 = \begin{bmatrix} 9+9 & -9-9 \\ -9-9 & 9+9 \end{bmatrix}$$

$$A^2 = -6\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}$$

$A^2 = -6A$

22. (C) $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ 1 & 0 & 6 \end{bmatrix}$

Co-factors of A –

$$C_{11}=(-1)^{1+1}\begin{vmatrix} -1 & 4 \\ 0 & 6 \end{vmatrix}, C_{12}=(-1)^{1+2}\begin{vmatrix} 3 & 4 \\ 1 & 6 \end{vmatrix}, C_{13}=(-1)^{1+3}\begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix}$$

$= -6 \qquad = -14 \qquad = 1$

$$C_{21}=(-1)^{2+1}\begin{vmatrix} 2 & 0 \\ 0 & 6 \end{vmatrix}, C_{22}=(-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 1 & 6 \end{vmatrix}, C_{23}=(-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix}$$

$= -12 \qquad = 6 \qquad = 2$

$$C_{31}=(-1)^{3+1}\begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}, C_{32}=(-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix}, C_{33}=(-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$$

$= 8 \qquad = -4 \qquad = -7$

$$C = \begin{bmatrix} -6 & -14 & 1 \\ -12 & 6 & 2 \\ 8 & -4 & -7 \end{bmatrix}$$

Adj A = Transpose of C

$$= \begin{bmatrix} -6 & -12 & 8 \\ -14 & 6 & -4 \\ 1 & 2 & -7 \end{bmatrix}$$

23. (B) $\begin{vmatrix} 1-b & b-b^2 & b^2 \\ 1-a & a-a^2 & a^2 \\ 1-c & c-c^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 + C_1 + C_3$

$$\Rightarrow \begin{vmatrix} 1-b & 1 & b^2 \\ 1-a & 1 & a^2 \\ 1-c & 1 & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1-b & 1 & b^2 \\ b-a & 0 & a^2-b^2 \\ b-c & 0 & c^2-b^2 \end{vmatrix}$$

$$\Rightarrow (a-b)(c-b)\begin{vmatrix} 1-b & 1 & b^2 \\ -1 & 0 & a+b \\ -1 & 0 & c+b \end{vmatrix}$$

$\Rightarrow (a-b)(c-b)[0-1(-c-b+a+b)+0]$

$\Rightarrow (a-b)(c-b)(c-a)$

24. (A) The required probability

$$= \frac{{}^2C_1 \times {}^3C_2 + {}^2C_2 \times {}^3C_1}{{}^5C_3}$$

$$= \frac{2\times3+1\times3}{\frac{5\times4}{2}} = \frac{9}{10}$$

25. (A) Coefficient of correlation $= \sqrt{r_1 \times r_2}$

$= \sqrt{0.4 \times 0.9}$

$= 0.6$

26. (C) $\sum_{r=0}^{4} C(25 + r, 4) + C(25, 5)$

$\Rightarrow {}^{25}C_4 + {}^{26}C_4 + {}^{27}C_4 + {}^{28}C_4 + {}^{29}C_4 + {}^{25}C_5$

$\Rightarrow {}^{25}C_5 + {}^{25}C_4 + {}^{26}C_4 + {}^{27}C_4 + {}^{28}C_4 + {}^{29}C_4$

$\Rightarrow {}^{26}C_5 + {}^{26}C_4 + {}^{27}C_4 + {}^{28}C_4 + {}^{29}C_4$

$\Rightarrow {}^{27}C_5 + {}^{27}C_4 + {}^{28}C_4 + {}^{29}C_4$

$\Rightarrow {}^{28}C_5 + {}^{28}C_4 + {}^{29}C_4$

$\Rightarrow {}^{29}C_5 + {}^{29}C_4$

$\Rightarrow {}^{30}C_5 = 142506$

27. (D) cot 10°.cot 50°.cot 70°

$\Rightarrow$ tan 80°.tan40°.tan20°

$\Rightarrow$ tan (3 × 20)°

[$\because$ tan θ.tan(60 – θ).tan(60+ θ) = tan 3θ]

$= \sqrt{3}$

28. (B) $E - (E - (E - (E - A)))$

$\Rightarrow E - (E - (E - A))$

$\Rightarrow E - (E - A)$

$\Rightarrow E - A'$

$\Rightarrow A = B \cup C$

29.(C) $\left(1-\cos\frac{\pi}{8}\right)\left(1-\cos\frac{2\pi}{8}\right)\left(1-\cos\frac{5\pi}{8}\right)\left(1-\cos\frac{7\pi}{8}\right)$

$$\Rightarrow 2\sin^2\frac{\pi}{16}\times2\sin^2\frac{3\pi}{16}\times2\sin^2\frac{5\pi}{16}\times2\sin^2\frac{7\pi}{16}$$

$$\Rightarrow 2\sin^2\frac{\pi}{16}\times2\sin^2\frac{3\pi}{16}\times2\cos^2\frac{3\pi}{16}\times2\cos^2\frac{\pi}{16}$$

$$\Rightarrow 2\sin^2\frac{\pi}{16}\times2\cos^2\frac{\pi}{16}\times2\sin^2\frac{3\pi}{16}\times2\cos^2\frac{3\pi}{16}$$

$$\Rightarrow \left(2\sin\frac{\pi}{16}\cos\frac{\pi}{16}\right)^2\times\left(2\sin\frac{3\pi}{16}\times\cos\frac{3\pi}{16}\right)^2$$

$$\Rightarrow \sin^2\frac{\pi}{8}\times\sin^2\frac{3\pi}{16}$$

$$\Rightarrow \frac{1}{4}\left(2\sin\frac{\pi}{8}\times\cos\frac{\pi}{8}\right)^2$$

$$\Rightarrow \frac{1}{4}\sin^2\frac{\pi}{4} = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

30. (C) $\frac{1-\tan 18.\cot 48}{\tan 42-\tan 162}$

$\Rightarrow \frac{1-\tan 18.\tan 42}{\tan 42-\tan(180-18)}$

$\Rightarrow \frac{1-\tan 18.\tan 42}{\tan 42+\tan 18}$

$\Rightarrow \frac{1}{\frac{\tan 42+\tan 18}{1-\tan 18.\tan 42}}$

$\Rightarrow \frac{1}{\tan 60} = \frac{1}{\sqrt{3}}$

31. (D) $(a-b)x^2 + (b-c)x + (c-a) = 0$

sum of the roots

$\alpha + \beta = -\frac{(b-c)}{a-b}$ ...(i)

product of the roots

$\alpha.\beta = \frac{c-a}{a-b}$ ...(ii)

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\,\alpha\beta$

$(\alpha-\beta)^2 = \frac{b^2+c^2-2bc}{(a-b)^2} - \frac{4(c-a)}{(a-b)}$

$(\alpha-\beta)^2 = \frac{(2a-b-c)^2}{(a-b)^2}$

$\alpha-\beta = \frac{2a-b-c}{a-b}$ ...(iii)

from eq. (i) and eq. (ii)

$\alpha = 1,\ \beta = \frac{c-a}{a-b}$

32. (B) $x\sin\theta - y\cos\theta = z$

on squaring both side

$x^2\sin^2\theta + y^2\cos^2\theta - 2xy\sin\theta.\cos\theta = z^2$

$x^2(1-\cos^2\theta) + y^2(1-\sin^2\theta) - 2xy\sin\theta.\cos\theta = z^2$

$x^2 + y^2 - z^2 = x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta.\cos\theta$

$x^2 + y^2 - z^2 = (x\cos\theta + y\sin\theta)^2$

$(x\cos\theta + y\sin\theta)^2 = x^2 + y^2 - z^2$

33.(C) $m = \begin{vmatrix} 1 & -1 \\ -1 & -1 \end{vmatrix}$ and $n = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}$

$= -1-1 = -2 \qquad = -1-1 = -2$

then $m\cos^2\theta + n\sin^2\theta = -2\cos^2\theta + (-2)\sin^2\theta$

$= -2(\cos^2\theta + \sin^2\theta)$

$= -2$

34. (A) In the expansion of $\left(4x + \frac{1}{2\sqrt{x}}\right)^8$

Total term = 9

middle term = 5th

$T_5 = T_{4+1} = {}^8C_4\,(4x)^{8-4}\left(\frac{1}{2\sqrt{x}}\right)^4$

$= \frac{70\times 256x^4}{16x^2} = 1120x^2$

coefficient of middle term = 1120

35. (D) lines $x + 3y = 5$ ...(i)

and $2x + y = 6$ ...(ii)

from eq. (i) and eq. (ii)

$x = \frac{13}{5}$ and $y = \frac{4}{5}$

intersecting point $\left(\frac{13}{5}, \frac{4}{5}\right)$

equation of the line which is perpendicular to the line $3x - 5y = 10$

$5x + 3y = c$ ...(ii)

it passes through the point $\left(\frac{13}{5}, \frac{4}{5}\right)$

then $c = \frac{77}{5}$

The equation of required line

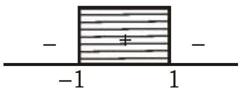
$5x + 3y = \frac{77}{5}$

$25x + 15y = 77$

36. (A) $f(x) = \frac{1}{\sqrt{1-x^2}}$

$1 - x^2 > 0$

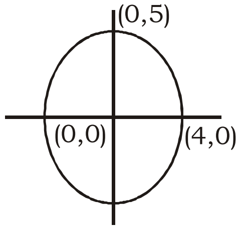
$1 > x^2$



domain $= -1 < x < 1$

$= (-1, 1)$

37. (C)



Ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Area of ellipse $= \pi ab$

$= \pi \times 4 \times 5 = 20\pi$

38. (D) $ydx - xdy = y^2dy$

$$\frac{ydx - xdy}{y^2} = 1.dy$$

$$d\left(\frac{x}{y}\right) = 1.dy$$

On Integrating

$$\frac{x}{y} = y + c$$

$$\frac{x}{y} = y + c \qquad \ldots(i)$$

$y(1) = -1$

$$\frac{1}{-1} = -1 + c$$

$c = 0$

from eq. (i)

$$\frac{x}{y} = y$$

$y^2 = x$

$y = \pm\sqrt{x}$

$y(4) = \pm\sqrt{4} = \pm 2$

39. (B) $I = \int e^x \left(\frac{2x-1}{x\sqrt{x}}\right)$

$$I = \int e^x \left(\frac{2}{\sqrt{x}} - \frac{1}{x^{\frac{3}{2}}}\right)$$

$$I = \int e^x \times \frac{2}{\sqrt{x}} + c$$

$$\left[\because \int e^x \left[f(x) + f'(x)\right]dx = e^x f(x) + c\right]$$

$$I = \frac{2e^x}{\sqrt{x}} + c$$

40. (D) $I = \int \tan^3 x\, dx$

$$I = \int \tan x .\tan^2 x\, dx$$

$$I = \int \tan x (\sec^2 x - 1)dx$$

$$I = \int \tan x .\sec^2 x\, dx - \int \tan x.\, dx$$

$$I = \frac{(\tan x)^2}{2} - \log\sec x + c$$

$$I = \frac{1}{2}\tan^2 x - \log\sec x + c$$

41. (A) Given that $e = \frac{2}{3}$ ...(i)

and $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ...(ii)

then $e^2 = 1 - \frac{b^2}{a^2}$

$$\frac{4}{9} = 1 - \frac{b^2}{a^2} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$$

from eq. (ii)

$$\frac{2a}{a^2} = \frac{5}{9} \Rightarrow a = \frac{18}{5}$$

from eq. (ii)

$$b^2 = \frac{36}{5}$$

equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{\left(\frac{18}{5}\right)^2} + \frac{y^2}{\frac{36}{5}} = 1$$

$25x^2 + 45y^2 = 324$

42. (C) Days in July = 31(28 + 3)

The required probability = $\frac{3}{7}$

43. (A) $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $f(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(\theta)\times f(\phi) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta.\cos\theta - \sin\theta.\sin\phi & -\cos\theta.\sin\phi - \sin\theta.\cos\phi & 0 \\ \sin\theta.\cos\phi + \cos\theta.\sin\phi & -\sin\theta.\sin\phi + \cos\theta.\cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(\theta)\times f(\theta) = \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$f(\theta)\times f(\theta) = f(\theta + \phi)$

44. (D) $n(S) = 6 \times 6 = 36$

$$E = \begin{bmatrix} (1,1)\text{ for sum} = 2 \\ (1,2),(2,1)\text{ for sum} = 3 \\ (2,2),(1,3),(3,1)\text{ for sum} = 4 \\ (1,4),(4,1),(2,3),(3,2)\text{ for sum} = 5 \end{bmatrix}$$

$n(E) = 10$

The required Probability = $\dfrac{n(E)}{n(S)} = \dfrac{10}{36} = \dfrac{5}{18}$

45. (B) 1. $\begin{vmatrix} 2 & 0 & -2 \\ 3 & 4 & 1 \\ 3 & 1 & -2 \end{vmatrix}$

$\Rightarrow 2(-8-1) + 0 - 2(3-12)$

$\Rightarrow -18 + 18 = 0$

2. $\begin{vmatrix} a+b & c & 1 \\ b+c & a & 1 \\ c+a & b & 1 \end{vmatrix}$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} a+b+c & c & 1 \\ a+b+c & a & 1 \\ a+b+c & b & 1 \end{vmatrix}$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & c & 1 \\ 1 & a & 1 \\ 1 & b & 1 \end{vmatrix}$$

$\Rightarrow 0$ [$\because$ Two columns are identical.]

3. $\begin{vmatrix} a & -a & 1 \\ b & b & 1 \\ c & -c & 1 \end{vmatrix}$

$\Rightarrow a(b+c) + a(b-c) + 1(-bc - bc)$

$\Rightarrow ab + ac + ab - ac - 2bc$

$\Rightarrow 2(ab - bc) \neq 0$

46. (C) lines $x - \sqrt{3}y + 5 = 0$

$$y = \frac{1}{\sqrt{3}}x + \frac{5}{\sqrt{3}}$$

$m_1(\tan\theta) = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$

and $\sqrt{3}x - y + 7 = 0 \Rightarrow y = \sqrt{3}x + 7$

$m_2(\tan\phi) = \sqrt{3} \Rightarrow \phi = 60^\circ$

angle between two lines $= |\phi - \theta|$

$= 60^\circ - 30^\circ$

$= 30^\circ$

47. (A) Given that $f(x) = [x]$

$$I = \int_1^3 f(x)\,dx$$

$$= \int_1^3 [x]\,dx$$

$$= \int_1^2 [x]\,dx + \int_2^3 [x]\,dx$$

$$= \int_1^2 1.dx + \int_2^3 2.dx$$

$$= \{x\}_1^2 + 2\{x\}_2^3$$

$$= (2-1) + 2\{x\}_2^3$$

$= (2-1) + 2(3-2)$

$= 1 + 2 = 3$

48. (C) Given that $f(x) = [x]$ and $g(x) = xf(x)$

$= x[x]$

$$I = \int_0^3 g(x)dx$$

$$I = \int_0^3 x[x]dx$$

$$I = \int_0^1 x \times 0.dx + \int_1^2 x \times 1.dx + \int_2^3 x \times 2.dx$$

$$I = 0 + \left[\frac{x^2}{2}\right]_1^2 + \left[2 \times \frac{x^2}{2}\right]_2^3$$

$$I = \left(2 - \frac{1}{2}\right) + (9-4) = \frac{13}{2}$$

49. (D) $x = \cos 30.\cos 80$ and $y = \sin 70.\sin 50$

$xy = \cos 30.\cos 80.\sin 70.\sin 50$

$$xy = \frac{\sqrt{3}}{2}\cos 80.\cos 20.\cos 40$$

$$xy = \frac{\sqrt{3}}{2} \times \frac{1}{4}\cos(3 \times 20)$$

$[\because \cos\theta.\cos(60+\theta).\cos(60-\theta) = \dfrac{1}{4}\cos 3\theta]$

$$xy = \frac{\sqrt{3}}{8} \times \frac{1}{2} = \frac{\sqrt{3}}{16}$$

50. (B) We know that

$\cos 2A = 1 - 2\sin^2 A$

$$\sin^2 A = \frac{1-\cos{}^2 A}{2}$$

$$A = \left(22\frac{1}{2}\right)^\circ$$

$$\sin^2\left(22\frac{1}{2}\right)^\circ = \frac{1-\cos 45}{2}$$

$$\sin^2\left(22\frac{1}{2}\right)^\circ = \frac{1-\frac{1}{\sqrt{2}}}{2}$$

$$\sin^2\left(22\frac{1}{2}\right)^\circ = \frac{\sqrt{2}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\sin^2\left(22\frac{1}{2}\right)^\circ = \frac{2-\sqrt{2}}{4}$$

51. (C) Data 7, 2, 3, 2, 6, 3, 2, 7, 3, 3, 6

on arranging in ascending order

2, 2, 2, 3, 3, 3, 3, 6, 6, 7, 7

mean (x)

$$= \frac{2+2+2+3+3+3+3+6+6+7+7}{11}$$

$$= \frac{44}{11} = 4$$

mode (y) = 3

median (z) = 6th term = 3

$x \neq y = z$

52. (B) $f(x) = \sin(\log\sqrt{x})$

On differenting both side w.r.t.'x'

$$f'(x) = \cos(\log\sqrt{x}) \times \frac{1}{\sqrt{x}} \times \frac{1}{2} \times \frac{1}{\sqrt{x}}$$

$$f'(x) = \frac{1}{2x}\cos(\log\sqrt{x})$$

53.(A) The required Probability = $\frac{1}{52}$

54. (A) **Statement I**

In equilateral ΔABC, $A = B = C = 60°$

L.H.S. $= \tan(A + C).\tan B$

$= \tan 120.\tan 60$

$= -\sqrt{3} \times \sqrt{3}$

$= -3$ = R.H.S.

Statement I is correct.

Statement II

In ΔABC, $A = 76°$, $C = 52°$ and $B = 52°$

$$\tan\left(\frac{A}{2}+C\right) < \tan B.$$

$$\tan\left(\frac{76}{2}+52\right) < \tan 52$$

tan 90 < tan 52

it is wrong.

statement II is incorrect.

55. (B)

$$\begin{array}{r|l|l} 2 & 3 & \\ \hline 2 & 1 & 1 \\ \hline & 0 & 1 \end{array}\uparrow \qquad \downarrow\begin{array}{r} 0.0625 \\ \times 2 \\ \hline \mathbf{0}.1250 \\ \times 2 \\ \hline \mathbf{0}.2500 \\ \times 2 \\ \hline \mathbf{0}.5000 \\ \times 2 \\ \hline \mathbf{1}.0000 \end{array}$$

$(3.0625)_{10} = (11.0001)_2$

56. (C) $I = \int_0^{2\pi} |\sin x|\ dx$

$$I = \int_0^{\pi} \sin x\, dx + \left|\int_{\pi}^{2\pi} \sin x\, dx\right|$$

$$I = [-\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$$

$I = [-\cos\pi + \cos 0] + |-\cos 2\pi + \cos\pi|$

$I = 1 + 1 + |-1-1|$

$I = 2 + 2 = 4$

57. (D) $I = \int_1^4 |1 - x^3|\, dx$

$$I = \left|\int_1^4 (1-x^3)\, dx\right|$$

$$I = \left|\left[x - \frac{x^4}{4}\right]_1^4\right|$$

$$I = \left|(4-64) - \left(1-\frac{1}{4}\right)\right|$$

$$I = \left|\frac{-243}{4}\right| = \frac{243}{4}$$

58. (A) $I = \int \frac{x^4+1}{x^2\sqrt{x^4+2x^2-1}}\,dx$

$$I = \int \frac{x^2+\frac{1}{x^2}}{\sqrt{x^2\left(x^2+2-\frac{1}{x^2}\right)}}\,dx$$

$$I = \int \frac{x+\frac{1}{x^3}}{\sqrt{x^2-\frac{1}{x^2}+2}}\,dx$$

Let $x^2 - \frac{1}{x^2} + 2 = t$

$$\left(2x+\frac{2}{x^3}\right)dx = dt$$

$$\left(x+\frac{1}{x^3}\right)dx = \frac{1}{2}\,dt$$

$$I = \int \frac{1}{2}\frac{1}{\sqrt{t}}\,dt$$

$$I = \sqrt{t} + c$$

$$I = \sqrt{x^2-\frac{1}{x^2}+2} + c$$

$$I = \frac{\sqrt{x^4-1+2x^2}}{x} + c$$

(59-60) : Equation
$bx^2 + cx + a = 0$
roots cotα and cotβ

$\cot\alpha + \cot\beta = -\frac{c}{b}$

$\frac{\tan\alpha+\tan\beta}{\tan\alpha.\tan\beta} = -\frac{c}{b}$ ...(i)

and $\cot\alpha.\cot\beta = \frac{a}{b}$

$\tan\alpha.\tan\beta = \frac{b}{a}$ ...(ii)

from eq. (i)

$\frac{\tan\alpha+\tan\beta}{\frac{b}{a}} = -\frac{c}{b}$

$\tan\alpha + \tan\beta = -\frac{c}{a}$ ...(iii)

59. (B) $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha.\tan\beta}$

$$\tan(\alpha+\beta) = \frac{-\frac{c}{a}}{1-\frac{b}{a}}$$

$$\tan(\alpha+\beta) = \frac{-c}{a-b}$$

60. (C) sin(α + β).cosecα.cosecβ

$\Rightarrow \frac{\sin\alpha.\cos\beta+\cos\alpha.\sin\beta}{\sin\alpha.\sin\beta}$

$\Rightarrow \cot\alpha + \cot\beta$

$\Rightarrow -\frac{c}{b}$ [from eq.(i)]

61. (C) **Statement 1**

$$\lim_{x\to 0} x\tan\frac{1}{x} = \lim_{x\to 0}\frac{\tan\frac{1}{x}}{\frac{1}{x}}$$

$$= 1 \qquad \left[\because \lim_{x\to\infty}\frac{\tan x}{x} = 1\right]$$

$\lim_{x\to 0} x\tan\frac{1}{x}$ exists.

Statement 1 is correct.

Statement 2

$\lim_{x\to 0}\tan\frac{1}{x}$ does not exists.

Statement 2 is correct.

62. (D) $f(x) = \begin{cases} x^2, & x>1 \\ 5x-k, & x\le 1\end{cases}$ is continuous at $x = 1$, then

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$\lim_{x\to 1} 5x - k = \lim_{x\to 1} x^2$$

$5 - k = 1 \Rightarrow k = 4$

63. (C) Planes $x + 2y - 2z + 7 = 0$ ...(i)

$2x + 4y - 4z + 9 = 0$

$x + 2y - 2z + \frac{9}{2} = 0$ ...(ii)

$$\text{distance (D)} = \left|\frac{7-\frac{9}{2}}{\sqrt{(1)^2+(2)^2+(-2)^2}}\right| = \frac{5}{6}$$

64. (A) Planes $2x + y + 2z + 1 = 0$
and $5x - 3y + 4z + 10 = 0$
angle between planes

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{2\times 5 + 1\times(-3) + 2\times 4}{\sqrt{(2)^2 + (1)^2 + (2)^2}\sqrt{(5)^2 + (-3)^2 + (4)^2}}$$

$$\cos\theta = \frac{15}{3\times 5\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

65. (B) $y = x\ln x - x\sin x$
On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = x\times\frac{1}{x} + \ln x \times 1 - x\cos x - \sin x \times 1$$

$$\frac{dy}{dx} = 1 + \ln x - x\cos x - \sin x$$

(66-68). In the expansion of $\left(2x - \frac{1}{4x^2}\right)^{12}$

general term

$$T_{r+1} = {}^{12}C_r\,(2x)^{12-r}\left(-\frac{1}{4x^2}\right)^r$$

$$T_{r+1} = {}^{12}C_r(2)^{12-r}\left(\frac{-1}{4}\right)^r x^{12-3r} \quad \ldots\text{(i)}$$

66. (B) $12 - 3r = 0$
$r = 4$

The term independent of $x = {}^{12}C_4(2)^8\left(\frac{-1}{4}\right)^4$

$= 495$

67. (C) From eq. (i)
$12 - 3r = 6$
$r = 2$

$$T_3 = T_{2+1} = {}^{12}C_2(2)^{10}\left(\frac{-1}{4}\right)^2$$

$$= 66 \times 2^6$$

The required Ratio $= \frac{66\times 2^6}{495}$

$= \frac{128}{15} = 128 : 15$

68. (A) Statement 1 is correct.

Statement 2

From eq. (i)

$$T_{r+1} = {}^{12}C_r(2)^{12-r}\left(\frac{-1}{4}\right)^r x^{12-3r} \quad \ldots\text{(ii)}$$

$12 - 3r = 9$
$r = 1$

Coefficient of $x^9 = {}^{12}C_1(2)^{11}\left(\frac{-1}{4}\right)^1 = -12\times 2^9$

from eq. (ii)
$12 - 3r = 3$
$r = 3$

Coefficient of $x^3 = {}^{12}C_3(2)^9\left(\frac{-1}{4}\right)^3 = -55 \times 2^5$

The coefficient of x^9 is not equal to that of x^3.
So statement 2 is incorrect.

69. (D) In ΔABC,
$\angle A = 60°$, $\angle C = 75°$, then $\angle B = 45°$
Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 45} = \frac{3}{\sin 75}$$

$$\frac{b\times\sqrt{2}}{1} = \frac{3\times 2\sqrt{2}}{\left(\sqrt{3}+1\right)}$$

$$b = \frac{6}{\left(\sqrt{3}+1\right)} \Rightarrow b = 3(\sqrt{3} - 1)$$

70. (C) $\dfrac{\cos 8x - \cos 2x}{\sin 8x - 2\sin 5x + \sin 2x}$

$$\Rightarrow \frac{2\sin 5x.\sin(-3x)}{\sin 8x + \sin 2x - 2\sin 5x}$$

$$\Rightarrow \frac{-2\sin 5x.\sin 3x}{2\sin 5x.\cos 3x - 2\sin 5x}$$

$$\Rightarrow \frac{\sin 3x}{1 - \cos 3x}$$

$$\Rightarrow \frac{2\sin\frac{3x}{2}.\cos\frac{3x}{2}}{2\sin^2\frac{3x}{2}} = \cot\frac{3x}{2}$$

71. (A) $f(x) = \lim_{x\to 0} \frac{6^x - 1}{x}$ Form $\left[\frac{0}{0}\right]$

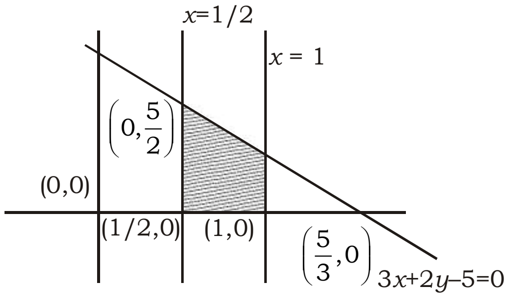
by L-Hospital's Rule

$= \lim_{x\to 0} \frac{6^x \log 6 - 0}{1} = \log 6$

72. (B) $\sin^2(2\pi) + \tan^2(3\pi) - \cos^2(5\pi)$

$\Rightarrow 0 + 0 - 1 = -1$

73. (C)



line $3x + 2y - 5 = 0$

$$y = -\frac{3}{2}x + \frac{5}{2}$$

Area $= \int_{\frac{1}{2}}^{1} y\,dx$

Area $= \int_{\frac{1}{2}}^{1} \left(\frac{-3}{2}x + \frac{5}{2}\right) dx$

Area $= \left[-\frac{3}{2} \times \frac{x^2}{2} + \frac{5}{2}x\right]_{\frac{1}{2}}^{1}$

Area $= \left(-\frac{3}{4} + \frac{5}{2}\right) - \left(\frac{-3}{4} \times \frac{1}{4} + \frac{5}{2} \times \frac{1}{2}\right)$

Area $= \frac{7}{4} - \frac{17}{16} = \frac{11}{16}$

74. (C) line $\frac{x-5}{2} = \frac{y+3}{4} = \frac{z-1}{5}$

from option C

$5x - 5y + 2z + 7 = 0$

$2 \times 5 + 4 \times (-5) + 5 \times 2 = 0$

Hence given line is parallel to the plane

$5x - 5y + 2z + 7 = 0$

75. (D) $9x^2 + 16y^2 + 72x - 64y + 64 = 0$

$9(x^2 + 8x) + 16(y^2 - 4y) + 64 = 0$

$9(x + 4)^2 - 144 + 16(y - 2)^2 - 64 + 64 = 0$

$$\frac{(x+4)^2}{16} + \frac{(y-2)^2}{9} = 1$$

$a = 4, \quad b = 3$

eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$$e = \sqrt{1 - \frac{9}{16}} \Rightarrow e = \frac{\sqrt{7}}{4}$$

76. (A) $\frac{1}{2}\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 4\theta}}}$

$\Rightarrow \frac{1}{2}\sqrt{2 + \sqrt{2 + \sqrt{2 \times 2\cos^2 2\theta}}}$

$\Rightarrow \frac{1}{2}\sqrt{2 + \sqrt{2 + 2\cos 2\theta}}$

$\Rightarrow \frac{1}{2}\sqrt{2 + \sqrt{2 \times 2\cos^2\theta}}$

$\Rightarrow \frac{1}{2}\sqrt{2 + 2\cos\theta}$

$\Rightarrow \frac{1}{2}\sqrt{2 \times 2\cos^2\frac{\theta}{2}}$

$\Rightarrow \frac{1}{2} \times 2\cos\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2}$

77. (D) $\cos^{-1}\frac{24}{x} + \cos^{-1}\frac{7}{x} = \frac{\pi}{2}$

$\cos^{-1}\frac{24}{x} = \frac{\pi}{2} - \cos^{-1}\frac{7}{x}$

$\cos^{-1}\frac{24}{x} = \sin^{-1}\frac{7}{x}$

$\cos^{-1}\frac{24}{x} = \cos^{-1}\sqrt{1 - \frac{49}{x^2}}$

$\frac{24}{x} = \sqrt{1 - \frac{49}{x^2}}$

$x^2 = 625$

$x = 25$

78. (B) ${}^{17}C_{3r} = {}^{17}C_{r+5}$

$3r + r + 5 = 17$

$4r = 12$

$r = 3$

79. (D) $\boxed{1}\boxed{5}\boxed{4}\boxed{3} = 60$

Only 1 can put here.

80. (C) $3^{2 - 2\log_3 4 + 3\log_3 2}$

$\Rightarrow 3^{2 + \log_3 2^3 - \log_3 4^2}$

$\Rightarrow 3^{2 + \log_3 \frac{2^3}{4^2}}$

$\Rightarrow 3^2 \times 3^{\log_3 \frac{1}{2}}$

$\Rightarrow 9 \times \frac{1}{2} = \frac{9}{2}$

81. (A) $\begin{vmatrix} \log_z x & \log_z y & 1 \\ \log_x y & 1 & \log_x z \\ 1 & \log_y x & \log_y z \end{vmatrix}$

$\Rightarrow \log_z x(\log_y z - \log_y x \times \log_x z)$
$-\log_z y(\log_x y \times \log_y z - \log_x z) + 1(\log_x y \times \log_y x - 1)$

$\Rightarrow \log_z x(\log_y z - \log_y z) - \log_z y(\log_x z - \log_x z) + 1(1-1)$

$\Rightarrow 0$

82. (A) Centre of circle $(-g, -f) = (2, -1)$

$g = -2$ and $f = 1$

and area $\pi r^2 = 64\pi$

$r = 8$

then $g^2 + f^2 - c = r^2$

$4 + 1 - c = 64$

$c = -59$

The equation of circle

$x^2 + y^2 + 2(-2)x + 2(1)y + (-59) = 0$

$x^2 + y^2 - 4x + 2y - 59 = 0$

83. (B) $\tan 780° = \tan(2 + 360 + 60)°$

$= \tan 60° = \sqrt{3}$

84. (A) $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2$

On comparing with the general equation

$P = \dfrac{1}{x}$ and $Q = x^2$

$I.F. = e^{\int P.dx}$

$= e^{\int \frac{1}{x}dx}$

$= e^{\log x} = x$

Solution of differential equation

$y \times \text{I.F.} = \int Q \times \text{I.F.}\ dx$

$y \times x = \int x^2 \times x\, dx$

$xy = \dfrac{x^4}{4} + \dfrac{c}{4}$

$4xy = x^4 + c$

85. (C) $\left(-\sqrt{-1}\right)^{12n+2} + \left(-\sqrt{-1}\right)^{4n-5}$

$\Rightarrow (-i)^{12n}.(-i)^2 + (-i)^{4n}.(-i)^{-5}$

$\Rightarrow 1 \times i^2 + 1 \times \left(\dfrac{-1}{i}\right)^5$

$\Rightarrow -1 + \dfrac{-1}{i}$

$\Rightarrow -1 + i = i - 1$

86. (D) $\tan\left[\cos^{-1}\dfrac{4}{5} + \tan^{-1}\dfrac{7}{24}\right]$

$\Rightarrow \tan\left[\tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{7}{24}\right]$

$\left[\because \cos^{-1} x = \tan^{-1}\dfrac{\sqrt{1-x^2}}{x}\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{7}{24}}{1-\frac{3}{4}\times\frac{7}{24}}\right)\right]$

$\Rightarrow \dfrac{100}{75} = \dfrac{4}{3}$

87. (B) $\begin{vmatrix} a & b & x+c \\ x+a & b & c \\ a & x+b & c \end{vmatrix} = 0$

$C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} x+a+b+c & b & x+c \\ x+a+b+c & b & c \\ x+a+b+c & x+b & c \end{vmatrix} = 0$

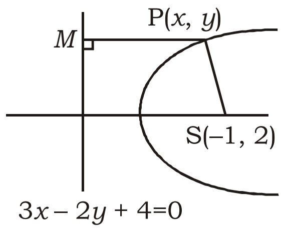
$(x + a + b + c)\begin{vmatrix} 1 & b & x+c \\ 1 & b & c \\ 1 & x+b & c \end{vmatrix} = 0$

$x + a + b + c = 0$

$x = -(a + b + c)$

One roots is $-(a + b + c)$.

88. (C)



Let $P(x, y)$

defination of parabola

$PM^2 = PS^2$

$\Rightarrow \left[\dfrac{3x-2y+4}{\sqrt{(3)^2+(-2)^2}}\right]^2 = \left(\sqrt{(x+1)^2+(y-2)^2}\right)^2$

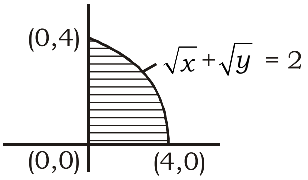
$\Rightarrow \dfrac{9x^2+4y^2+16-12xy-16y+24x}{13}$

$= x^2 + 1 + 2x + y^2 + 4 - 4y$

On solving

$\Rightarrow 4x^2 + 9y^2 + 12xy + 2x - 36y + 49 = 0$

89. (B)



Curve $\sqrt{x} + \sqrt{y} = 2$

$\Rightarrow y = 4 + x - 4\sqrt{x}$

Area = $\int_0^4 y.dx$

Area = $\int_0^4 \left(4 + x - 4\sqrt{x}\right) dx$

Area = $\left[4x + \frac{x^2}{2} - 4 \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= \left(16 + 8 - \frac{8}{3} \times (4)^{\frac{3}{2}}\right) - 0 = \frac{8}{3}$ sq. unit

90. (A) $3f(x-1) + f\left(\frac{1}{x-1}\right) = x^2$ ...(i)

On putting $x = 3$

$3f(2) + f\left(\frac{1}{2}\right) = 9$ ...(ii)

On putting $x = \frac{3}{2}$ in eq. (i)

$3f\left(\frac{1}{2}\right) + f(2) = \frac{9}{4}$ ...(iii)

3 × eq.(ii) – eq. (iii)

$9f(2) - f(2) = 27 - \frac{9}{4}$

$8f(2) = \frac{99}{4} \Rightarrow f(2) = \frac{99}{32}$

91. (B) Lines $3x + 4y - 5 = 0$ and $12x + 16y + 21 = 0$

Slope $m_1 = -\frac{3}{4}$ and $m_2 = -\frac{3}{4}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{-\frac{3}{4} + \frac{3}{4}}{1 - \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right)}\right| = 0 \Rightarrow \theta = 0°$

92. (D) lines $3x + 4y - 5 = 0$
and $12x + 16y + 21 = 0$

$3x + 4y + \frac{21}{4} = 0$

distance (d) = $\left|\frac{-5 - \frac{21}{4}}{\sqrt{(3)^2 + (4)^2}}\right|$

distance (d) = $\left|\frac{-\frac{41}{4}}{5}\right| = \frac{41}{20}$

93. (A) Equation of line which is parallel to the given line $3x + 4y - 5 = 0$ is

$3x + 4y = c$ ...(i)

it passes through the point (–4, 7)

$3(-4) + 4(7) = c$

$c = 16$

from eq.(i)

equation of the line $3x + 4y = 16$

94. (C) Equation of the line which is perpendicular to the given line $12x + 16y + 21 = 0$ is

$16x - 12y = c$ ...(i)

it passes through the point (3, –2)

$16 \times 3 - 12(-2) = c$

$c = 72$

from eq. (i)

equation of line

$16x - 12y = 72$

$4x - 3y = 18$

95. (D) Number of diagonals = 44

$\frac{n(n-3)}{2} = 44$

$n^2 - 3n - 88 = 0$

$(n - 11)(n + 8) = 0$

$n = 11, -8$

Number of sides = 11

96. (C) Vectors $4\hat{i} + 3\hat{j} + 5\hat{k}$ and $2\hat{i} + 2\hat{k}$

angle between the vectors

$\cos\theta = \frac{4 \times 2 + 3 \times 0 + 5 \times 2}{\sqrt{(4)^2 + (3)^2 + (5)^2}\sqrt{(2)^2 + (0)^2 + (2)^2}}$

$\cos\theta = \frac{18}{\sqrt{50} \times 2\sqrt{2}}$

$\cos\theta = \frac{9}{10} \Rightarrow \theta = \cos^{-1}\left(\frac{9}{10}\right)$

(97-99)

Class	x	f	C	$f \times x$
0-10	5	6	6	30
10-20	15	7	13	105
20-30	25	11	24	275
30-40	35	17	41	595
40-50	45	15	56	675
50-60	55	19	75	1045
60-70	65	20	95	1300
70-80	75	5	100	375

Σf=100=N $\Sigma f \times x$ =4400

97. (B) $\frac{N}{2} = \frac{100}{2} = 50$

Median class is (40 – 50).

$L_1 = 40,\ L_2 = 50,\ C = 41,\ f = 15$

$$\text{Median} = L_1 + \frac{\frac{N}{2} - C}{f} \times (L_2 - L_1)$$

$$= 40 + \frac{50-41}{15} \times (50 - 40)$$

$$= 40 + \frac{9}{15} \times 10 = 46$$

98. (C) Modal class is (60 – 70).

$L_1 = 60,\ L_2 = 70,\ f = 20,\ f_0 = 19,\ f_1 = 5$

$$\text{Mode} = L_1 + \frac{f - f_0}{2f - f_0 - f_1} \times (L_2 - L_1)$$

$$\text{Mode} = 60 + \frac{20-19}{2\times20-19-5} \times (70 - 60)$$

$$\text{Mode} = 60 + \frac{1}{16} \times 10 = 60.625$$

99. (A) $\text{Mean} = \frac{\sum f \times x}{\sum f}$

$$= \frac{4400}{100} = 44$$

100. (D) Let $f(x) = \frac{[x]}{x}$

$$\text{L.H.L.} = \lim_{x\to1^-} f(x) = \lim_{h\to0} f(1-h)$$

$$= \lim_{h\to0} \frac{[1-h]}{(1-h)} = \lim_{h\to0} \frac{0}{(1-h)} = 0$$

$$\text{R.H.L.} = \lim_{x\to1^+} f(x) = \lim_{h\to0} f(1+h)$$

$$= \lim_{h\to0} \frac{[1+h]}{(1+h)}$$

$$= \lim_{h\to0} \frac{1}{(1+h)} = 1$$

L.H.L. ≠ R.H.L.

limit does not exist.

101. (A) $\lim_{x\to0} \left[\frac{1+\tan x}{1+\sin x}\right]^{\cot x}$ [1^∞ Form]

$$\Rightarrow \lim_{x\to0} \frac{(1+\tan x)^{\frac{1}{\tan x}}}{\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\cos x}}$$

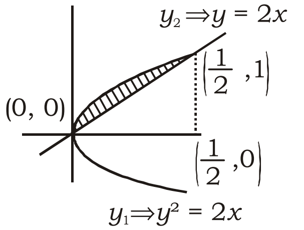
$$\Rightarrow \frac{e^1}{e^{1.\lim\limits_{x\to0}\cos x}} \qquad \left[\because \lim_{x\to0}(1+\lambda x)^{\frac{1}{x}} = e^\lambda\right]$$

$$\Rightarrow \frac{e}{e} = 1$$

102. (C) Both statements are correct.

103. (D) one-one but not onto.

104. (C)



curve

$y_1 \Rightarrow y = \sqrt{2}\ \sqrt{x}$

and line

$y_2 \Rightarrow y = 2x$

$$\text{The required Area} = \int_0^{\frac{1}{2}} (y_1 - y_2)\, dx$$

$$= \int_0^{\frac{1}{2}} \left(\sqrt{2}\sqrt{x} - 2x\right) dx$$

$$= \left[\sqrt{2} \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \times \frac{x^2}{2}\right]_0^{\frac{1}{2}}$$

$$= \left(\sqrt{2} \times \frac{2}{3}\left(\frac{1}{2}\right)^{\frac{3}{2}} - \left(\frac{1}{2}\right)^2\right)$$

$$= \frac{1}{12} \text{ sq. unit}$$

105. (A) Line $\frac{x-3}{4} = \frac{y+2}{7} = \frac{z-1}{5}$

direction cosine

$$= < \frac{4}{\sqrt{(4)^2+(7)^2+(5)^2}}, \frac{7}{\sqrt{(4)^2+(7)^2+(5)^2}}, \frac{5}{\sqrt{(4)^2+(7)^2+(5)^2}} >$$

$$= < \frac{4}{3\sqrt{10}}, \frac{7}{3\sqrt{10}}, \frac{5}{3\sqrt{10}} >$$

106. (B) U = {1, 2, 3, 4, 5, 6, 7}
A = {2, 5, 7}, B = {1, 2, 7} and C = {4, 5, 7}
then $(B \cap A) = \{2, 7\}$
$C - (B \cap A) = \{4, 5, 7\} - \{2, 7\}$
$= \{4, 5\}$

107. (A) $z = \dfrac{(1+3i)(1-i)}{(2-i)}$

$$z = \frac{(4+2i)(2+i)}{(2-i)(2+i)}$$

$$z = \frac{8i+6}{5} \Rightarrow z = \frac{6}{5} + \frac{8}{5}i$$

argument $\theta = \tan^{-1}\left(\dfrac{b}{a}\right) \Rightarrow \theta = \tan^{-1}\left(\dfrac{\frac{8}{5}}{\frac{6}{5}}\right)$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

108. (C) Let $a + ib = \sqrt{(8+4\sqrt{5}i)}$

On squaring both side

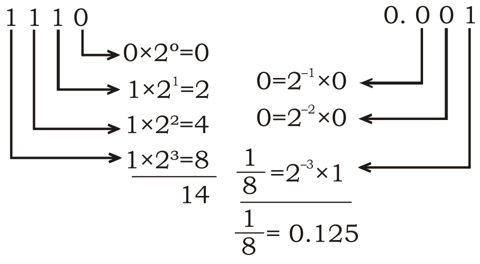
$(a^2 - b^2) + 2abi = 1 + 4\sqrt{5}\,i$

On comparing

$a^2 - b^2 = 1$ and $2ab = 4\sqrt{5}$ …(i)
$(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$
$(a^2 + b^2)^2 = 1 + 80$
$a^2 + b^2 = 9$ …(ii)
from eq.(i) and eq. (ii)
$a^2 = 5$ and $b^2 = 4$
$a = \pm\sqrt{5}$ $b = \pm 2$

square root of $(1 + 4\sqrt{5}\,i)$ is $\pm(\sqrt{5} + 2i)$.

109. (A)



then $(1110.001)_2 = (14.125)_{10}$

110. (B) $f(x) = 2x^3 + 5x^2 - 4x - 5$ …(i)
On differentiating both side w.r.t 'x'
$f'(x) = 6x^2 + 10x - 4$ …(ii)
$f''(x) = 12x + 10$ …(iii)
For minima and maxima
$f'(x) = 0$
$6x^2 + 10x - 4 = 0$
$2(3x^2 + 5x - 2) = 0$
$(x + 2)(3x - 1) = 0$

$$x = -2, \frac{1}{3}$$

From eq. (iii)

$f''(-2) = 12(-2) + 10$ and $f''\left(\dfrac{1}{3}\right) = 12 \times \dfrac{1}{3} + 10$
$= -14$(max.) $= 14$ (min.)

Minimum value $\left(at\ x = \dfrac{1}{3}\right)$

$$= 2\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^2 - 4\left(\frac{1}{3}\right) - 5$$

$$= \frac{2}{27} + \frac{5}{9} - \frac{4}{3} - 5 = -\frac{154}{27}$$

111. (C) NATION

$\boxed{\text{AIO}}$ NNT
as one letter

Total arrangement $= \dfrac{4!}{2!} \times \underset{\uparrow\ \text{AIO}}{3!}$
$= 72$

112. (A) $e = \dfrac{\sqrt{13}}{2}$ and $\dfrac{2b^2}{a} = \dfrac{9}{2} \Rightarrow b^2 = \dfrac{9}{4}a$

$$\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{13}{2}}$$

$$1 + \frac{9a}{4a^2} = \frac{13}{4}$$

$$\frac{9}{4a} = \frac{9}{4}$$

$a = 1 \Rightarrow a^2 = 1$

and $b^2 = \dfrac{9}{4}$

The equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$x^2 - \frac{4y^2}{9} = 1 \Rightarrow 9x^2 - 4y^2 = 9$$

113. (B) $\frac{1^2}{1} + \frac{1^2+2^2}{1+2} + \frac{1^2+2^2+3^2}{1+2+3} \ldots\infty$

$$n^{\text{th}} \text{ term } T_n = \frac{1^2+2^2+3^2+\ldots+n^2}{1+2+3+\ldots+n}$$

$$T_n = \frac{\frac{n}{6}(n+1)(2n+1)}{\frac{n(n+1)}{2}}$$

$$T_n = \frac{2n+1}{3}$$

114. (D) Let $y = 7^{37}$

$\log y = 37 \log_{10} 7$

$\log y = 37 \times 0.8451$

$\log y = 31.3687$

Number of digit in $7^{37} = 32$

115. (C) $f(x) = \begin{cases} 2x-\lambda, & x<1 \\ 3x^2+1, & x \geq 1 \end{cases}$ is continuous at $x = 1$

then $\lim_{x\to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$$\lim_{x\to 1}(2x-\lambda) = \lim_{x\to 1}(3x^2+1)$$

$$2-\lambda = 3+1$$

$$\lambda = -2$$

(116-117) $\vec{a} = -4\hat{i} + 7\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$

116. (B) projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{(-4)\times 2 + 7\times 2 + 4(-1)}{\sqrt{(2)^2+(2)^2+(-1)^2}} = \frac{2}{3}$$

117. (D) projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{(-4)\times 2 + 7\times 2 + 4\times(-1)}{\sqrt{(-4)^2+(7)^2+(4)^2}} = \frac{2}{9}$$

118. (B)

119. (C) $\frac{a^2-b^2}{c^2}$

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow \frac{k^2\sin^2 A - k^2 \sin^2 B}{k^2 \sin^2 C}$$

$$\Rightarrow \frac{\sin(A+B).\sin(A-B)}{\sin^2[\pi-(A+B)]}$$

$$\Rightarrow \frac{\sin(A+B).\sin(A-B)}{\sin^2(A+B)}$$

$$\Rightarrow \frac{\sin(A-B)}{\sin(A+B)} \Rightarrow \frac{\sin(A-B)}{\sin C}$$

120. (C) Time is 9 : 25.

angle = $\frac{11M-60H}{2}$ where M→ minute

H→ Hour

$$= \frac{11\times 25 - 60\times 9}{2}$$

$$= \frac{265}{2} = 132.5°$$

NDA (MATHS) MOCK TEST - 90 (Answer Key)

1.	(B)	21.	(C)	41.	(A)	61.	(C)	81.	(A)	101.	(A)
2.	(A)	22.	(C)	42.	(C)	62.	(D)	82.	(A)	102.	(C)
3.	(B)	23.	(B)	43.	(A)	63.	(C)	83.	(B)	103.	(D)
4.	(A)	24.	(A)	44.	(D)	64.	(A)	84.	(A)	104.	(C)
5.	(C)	25.	(A)	45.	(B)	65.	(B)	85.	(C)	105.	(A)
6.	(B)	26.	(C)	46.	(C)	66.	(B)	86.	(D)	106.	(B)
7.	(D)	27.	(D)	47.	(A)	67.	(C)	87.	(B)	107.	(A)
8.	(C)	28.	(B)	48.	(C)	68.	(A)	88.	(C)	108.	(C)
9.	(B)	29.	(C)	49.	(D)	69.	(D)	89.	(B)	109.	(A)
10.	(A)	30.	(C)	50.	(B)	70.	(C)	90.	(A)	110.	(B)
11.	(C)	31.	(D)	51.	(C)	71.	(A)	91.	(B)	111.	(C)
12.	(D)	32.	(B)	52.	(B)	72.	(B)	92.	(D)	112.	(A)
13.	(A)	33.	(C)	53.	(A)	73.	(C)	93.	(A)	113.	(B)
14.	(C)	34.	(A)	54.	(A)	74.	(C)	94.	(C)	114.	(D)
15.	(B)	35.	(D)	55.	(B)	75.	(D)	95.	(D)	115.	(C)
16.	(D)	36.	(A)	56.	(C)	76.	(A)	96.	(C)	116.	(B)
17.	(A)	37.	(C)	57.	(D)	77.	(D)	97.	(B)	117.	(D)
18.	(B)	38.	(D)	58.	(A)	78.	(B)	98.	(C)	118.	(B)
19.	(D)	39.	(B)	59.	(B)	79.	(D)	99.	(A)	119.	(C)
20.	(B)	40.	(D)	60.	(C)	80.	(C)	100.	(D)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*