NDA MATHS MOCK TEST - 96 (SOLUTION)

1. (B) $S = 6 + 66 + 666 + 6666 + \ldots$

$S = \frac{6}{9}[9 + 99 + 999 + 9999 + \ldots)$

$S = \frac{2}{3}[(10-1) + (100-1) + (1000-1)+\ldots]$

$S = \frac{2}{3}[(10+100 + 1000+\ldots 9\text{times}) - (1+1+1+\ldots 9\text{times})]$

$S = \frac{2}{3}\left[\frac{10(10^9-1)}{10-1} - 9\right]$

$S = \frac{2}{3}\left[\frac{10^{10}-10-81}{9}\right] = \frac{2}{27}[10^{10} - 91]$

2. (A) $T_n = 4n + 5$

$S_n = 4\Sigma n + 5\Sigma 1$

$S_n = 4 \times \frac{n(n+1)}{2} + 5n$

$S_n = 2n^2 + 7n$

$S_{45} = 2(45)^2 + 7\times 45 = 4365$

3. (B) $\begin{vmatrix} 1 & 6 & \pi \\ \log_e e & 6 & \sqrt{7} \\ \log_5 5 & \log_2 64 & e \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 6 & \pi \\ 1 & 6 & \sqrt{7} \\ 1 & 6 & e \end{vmatrix}$

$\Rightarrow 6\begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{7} \\ 1 & 1 & e \end{vmatrix} = 0$ [∵two columns are idential.]

4. (C) $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 4 & 3 \\ 8 & 6 & 5 \end{vmatrix}$

$R_1 \to R_1 + 2R_2$ and $R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} 5 & 6 & 7 \\ 2 & 4 & 3 \\ 6 & 2 & 2 \end{vmatrix} \Rightarrow \Delta = 2\begin{vmatrix} 5 & 3 & 7 \\ 2 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix} = 2\Delta'$

5. (A) $\sin\theta$, $(3\sin\theta + 1)$ and $(2 + 5\sin\theta)$ are in G.P., then $(3\sin\theta + 1)^2 = \sin\theta(2 + 5\sin\theta)$

$9\sin^2\theta + 1 + 6\sin\theta = 2\sin\theta + 5\sin^2\theta$

$4\sin^2\theta + 1 + 4\sin\theta = 0$

$(2\sin\theta+1)^2 = 0$

$\sin\theta = -\frac{1}{2} \Rightarrow \theta = 210$

then

$\frac{1-\tan\theta}{\tan\theta} = \frac{1-\tan 210}{\tan 210}$

$= \frac{1-\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = \sqrt{3} - 1$

6. (B) $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^5 x \; dx$

Let $f(x) = \tan^5 x$

$f(-x) = -\tan^5 x = -f(x)$

function is odd.

Then $I = 0$

7. (C) Let $y = \log_x x = 1$ and $z = x^5$

$\frac{dy}{dx} = 0$, $\frac{dz}{dx} = 5x^4$

then $\frac{dy}{dz} = 0$

8. (B) $e^y + xy = x^2$

On differentiating w.r.t 'x'

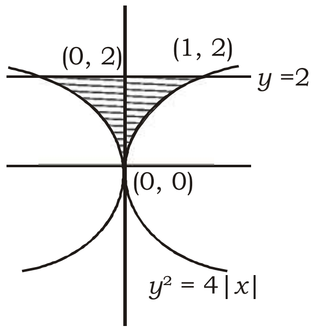
$e^y \frac{dy}{dx} + x.\frac{dy}{dx} + y.1 = 2x$

$(e^y + x)\frac{dy}{dx} = (2x - y)$

$\frac{dy}{dx} = \frac{2x-y}{e^y+x}$

$\frac{dx}{dy} = \frac{e^y+x}{2x-y}$

9. (A)



Curve

$x_1 \Rightarrow x = \frac{y^2}{4}$

and line $y = 2$

$\text{Area} = 2\int_0^2 x_1 \, dy$

$= 2\int \frac{y^2}{4} dy$

$= 2 \times \left[\frac{y^3}{4 \times 3}\right]_0^2$

$= \frac{2}{12} [8 - 0] = \frac{4}{3}$ sq. unit

10.(B) $I = \int \sin^{-1}\left(\frac{1-x}{1+x}\right) dx$

$x = \tan^2\theta \Rightarrow dx = 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}(\cos 2\theta) \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \times 2\tan\theta.\sec^2\theta.d\theta$

$I = \int 2\left(\frac{\pi}{2} - 2\theta\right)\sec^2\theta.\tan\theta d\theta$

$I = \pi \int \sec^2\theta \tan\theta \, d\theta - 4\int \theta.\sec^2\theta \tan\theta \, d\theta$

$I = \pi \frac{\tan^2\theta}{2} -$

$4\left[\theta.\int \tan\theta.\sec^2\theta.d\theta - \int\left\{\frac{d}{d\theta}(\theta).\int \tan\theta.\sec^2\theta\right\}dx\right]$

$I = \frac{\pi}{2}\tan^2\theta - 4\left[\theta.\frac{\tan^2\theta}{2} - \int 1.\frac{\tan^2\theta}{2} d\theta\right]$

$I = \frac{\pi}{2}\tan^2\theta - 2\theta.\tan^2\theta + 2\int(\sec^2\theta - 1)\,d\theta$

$I = \frac{\pi}{2}\tan^2\theta - 2\theta.\tan^2\theta + 2[\tan\theta - \theta] + c$

$I = \frac{\pi}{2}x - 2x.\tan^{-1}(\sqrt{x}) + 2[\sqrt{x} - \tan^{-1}\sqrt{x}] + c$

$I = \frac{\pi}{2}x - 2(x + 1)\tan^{-1}\sqrt{x} + 2\sqrt{x} + c$

11. (A)

2	76	
2	38	0
2	19	0
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

↑

$(76)_{10} = (1001100)_2$

12. (B) $\Delta \neq 0$, $h^2 < ab$

13. (D) Three-digit numbers

| 9 | 10 | 10 | $= 9 \times 10 \times 10 = 900$

'0'can't put here

14. (B) $\frac{dy}{dx} = \text{cosec}(x + y)$

Let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \text{cosec}t$

$\frac{dt}{dx} - 1 = \text{cosec}t + 1$

$\frac{dt}{\text{cosec}t + 1} = dx$

$\frac{dt(\text{cosec}t - 1)}{\text{cosec}^2 t - 1} = dx$

$dt(\sec t - \tan t) = dx$

On integrating

$\log|\sec t + \tan t| - \log\sec t = x + c$

$\log\left|\frac{\sec t + \tan t}{\sec t}\right| = x + c$

$\log|1 + \sin t| = x + c$

$\log|1 + \sin(x + y)| = x + c$

15. (C) $f(x) = (x^2 + 2x + 1)\tan^{-1}(e^{-3x})$

On differentiating w.r.t. 'x'

$$f'(x) = \tan^{-1}(e^{-3x}).(2x + 2) + (x^2 + 2x + 1)\times\frac{(-3).e^{-3x}}{1+\left(e^{-3x}\right)^2}$$

$$f'(x) = (2x + 2)\tan^{-1}(e^{-3x}) - \frac{3\left(x^2+2x+1\right).e^{-3x}}{1+e^{-6x}}$$

$$f'(0) = (0 + 2)\tan^{-1}(e^{0}) - \frac{3(0+1)e^0}{1+e^0}$$

$$f'(0) = 2.\tan^{-1}(1) - \frac{3}{2}$$

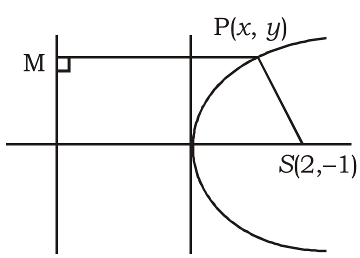
$$f'(0) = 2\times\frac{\pi}{4} - \frac{3}{2} = \frac{\pi-3}{2}$$

16. (A) $\left(\frac{d^3y}{dx^3}+5\frac{dy}{dx}\right)^{\frac{1}{3}} = \left(\frac{dy}{dx}-\frac{d^2y}{dx^2}\right)^{\frac{1}{4}}$

$$\left(\frac{d^3y}{dx^3}+\frac{5dy}{dx}\right)^4 = \left(\frac{dy}{dx}-\frac{d^2y}{dx^2}\right)^3$$

Order = 3 and degree = 4

17. (B)



$2x+3y+5 = 0$

Directrix $2x + 3y + 5 = 0$ and foci = (2, –1)

PM = PS

$$\Rightarrow \frac{2x+3y+5}{\sqrt{(2)^2+(3)^2}} = \sqrt{(x-2)^2+(y+1)^2}$$

On squaring both side

$$\Rightarrow \frac{4x^2+9y^2+25+12xy+30y+20x}{13} = x^2 + 4 - 4x + y^2 + 1 + 2y$$

On solving

$\Rightarrow 9x^2 + 4y^2 - 12xy - 72x - 4y + 40 = 0$

18. (A) $(1 - x + x^2 - x^3 + \ldots\infty)^{-n} = \left(\frac{1}{1+x}\right)^{-n}$

$(1 - x - x^2 - x^3 + \ldots\infty)^{-n} = (1 + x)^n$

Coefficient of x^n in $(1+ x)^n = {}^nC_n = 1$

19. (A) In the expansion of $\left(x^2-\frac{3}{x}\right)^{13}$

$$T_{r+1} = {}^{13}C_r\,(x^2)^{13-r}\left(\frac{-3}{x}\right)^r$$

$T_{r+1} = {}^{13}C_r\,x^{26-3r}(-3)^r$ ... (i)

then $26 - 3r = 8 \Rightarrow r = 6$

coefficient of $x^8 = {}^{13}C_6\,(-3)^6$ ... (ii)

from eq. (i)

$26 - 3r = 2 \Rightarrow r = 8$

coefficient of $x^2 = {}^{13}C_8(-3)^8$

$$\text{Ratio} = \frac{{}^{13}C_6\,(-3)^6}{{}^{13}C_8\,(-3)^8} = \frac{4}{27} = 4 : 27$$

20. (D) $\cot A + \cot B - \cot C$

$$\Rightarrow \frac{\cos A}{\sin A}+\frac{\cos B}{\sin B}-\frac{\cos C}{\sin C}$$

$$\Rightarrow \frac{\cos A\sin B+\sin A.\cos B}{\sin A.\sin B}-\frac{\cos C}{\sin C}$$

$$\Rightarrow \frac{\sin(A+B)}{\sin A.\sin B}-\frac{\cos C}{\sin C}$$

$$\Rightarrow \frac{\sin C}{\sin A.\sin B}-\frac{\cos C}{\sin C} \quad [\because A + B + C = \pi]$$

$$\Rightarrow \frac{\sin^2 C-\sin A.\sin B.\cos C}{\sin A.\sin B.\sin C}$$

$$\Rightarrow \frac{c^2-ab\left(\frac{a^2+b^2-c^2}{2ab}\right)}{abc\times k} \quad \text{[by Sine Rule]}$$

given that $a^2 + b^2 = 3c^2$

$$\Rightarrow \frac{c^2-c^2}{abc\times k} = 0$$

21. (B) $\frac{n(n-3)}{2} = 44$

$n^2 - 3n - 88 = 0$

$(n - 11)(n + 8) = 0$

$n = 11, -8$

No. of sides = 11

22. (C) $\left(\frac{1+\sqrt{2}i}{1-\sqrt{2}i}\right)^3 = \left[\frac{1+\sqrt{2}i}{1-\sqrt{2}i}\times\frac{1+\sqrt{2}i}{1+\sqrt{2}i}\right]^3$

$$= \left[\frac{-1+2\sqrt{2}i}{3}\right]^3 = \frac{23-10\sqrt{2}i}{27}$$

23. (C) Vectors $3\hat{i} + \hat{j} + \lambda\hat{k}$, $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + \hat{j} - 4\hat{k}$ are coplanar, then

$$\begin{vmatrix} 3 & 1 & \lambda \\ 3 & -1 & 2 \\ 1 & 1 & -4 \end{vmatrix} = 0$$

$3(4 - 2) - 1(-12 - 2) + \lambda(3 + 1) = 0$

$6 + 14 + 4\lambda = 0 \Rightarrow \lambda = -5$

24. (B) $AB = \begin{bmatrix} \cos 75^\circ & \sin 75^\circ \\ \cos 45^\circ & -\sin 45^\circ \end{bmatrix} \times \begin{bmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 15^\circ & -\cos 15^\circ \end{bmatrix}$

$$AB = \begin{bmatrix} \cos(75-15) & -\sin(75-15) \\ \cos(45+15) & \sin(45+15) \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos 60 & -\sin 60 \\ \cos 60 & \sin 60 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

then $|AB| = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

25. (B) $\tan^2\theta = 2\tan^2\phi + 1$

$1 + \tan^2\theta = 2\tan^2\phi + 2$

$\sec^2\theta = 2(\sec^2\phi)$

$$\frac{1}{\cos^2\theta} = \frac{2}{\cos^2\phi}$$

$2\cos^2\phi = 4\cos^2\theta$

$1 + \cos 2\phi = 2(1 + \cos 2\theta)$

$\cos 2\phi = 2\cos 2\theta + 1$

26.(A) $[x\ y\ z]_{1\times3} \begin{bmatrix} a & d & e \\ b & b & f \\ c & f & c \end{bmatrix}_{3\times3} \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{3\times1}$

order = 1×1

27. (C) $\lim_{n\to\infty} \frac{(1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2)}{n(1 + 2 + 3 + \ldots + n)}$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n}{6}(n+1)(2n+1)}{n \times \frac{n}{2}(n+1)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{\frac{n^3}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{h}\right)}{\frac{n^3}{2}\left(1+\frac{1}{n}\right)}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{3}\left(2+\frac{1}{n}\right) = \frac{2}{3}$$

28. (A) $y^2 = 2b(x - b)$ ...(i)

On differentiating both side w.r.t. 'x'

$2yy_1 = 2b$ ...(ii)

from eq. (i) and eq. (ii)

$$\frac{y}{2y_1} = x - b$$

$$b = x - \frac{y}{2y_1}$$

On putting equation (i)

$$y^2 = 2\left(x - \frac{y}{2y_1}\right)\left(\frac{y}{2y_1}\right)$$

$2yy_1^2 + y = 2xy_1$

29. (C) $nRm \Leftrightarrow n$ is divisible by m

Reflexive :

$nRn \Leftrightarrow n$ is divisible by n

So R is Reflexive.

Symmetric :

$nRm \Leftrightarrow n$ is divisible by m

but $mRn \Leftrightarrow m$ will not be divisible by n

So R is not symmetric.

Transitive:

$nRm \Leftrightarrow n$ is divisible by m

$nRl \Leftrightarrow m$ is divisible by l

then $nRl \Leftrightarrow m$ is also divisible by l

So R is transitive.

30. (B) Given that

$f(x) = 4x - 8$

x	$f(x)$
1	-4
2	0
3	4
⋮	⋮

so on

So function injective but not surjective.

31. (C) $\begin{bmatrix} 1 & 0 \\ -5 & 3 \end{bmatrix} A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$

Let $B = \begin{bmatrix} 1 & 0 \\ -5 & 3 \end{bmatrix} \Rightarrow |B| = 3 - 0 = 3$

$$BA = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

$$B^{-1}(BA) = B^{-1}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

$$A = B^{-1}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \quad \ldots(i)$$

Co-factors of B -

$C_{11} = (-1)^{1+1}(3) = 3, \quad C_{12} = (-1)^{1+2}(-5) = 5$

$C_{21} = (-1)^{2+1}(0) = 0, \quad C_{22} = (-1)^{2+2}(1) = 1$

$$C = \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}$$

$$\text{Adj } B = C^T = \begin{bmatrix} 3 & 0 \\ 5 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{Adj\,B}{|B|} = \begin{bmatrix} 1 & 0 \\ \frac{5}{3} & \frac{1}{3} \end{bmatrix}$$

from equation (i)

$$A = \begin{bmatrix} 1 & 0 \\ \frac{5}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & -2 \\ \frac{8}{3} & -2 \end{bmatrix}$$

32. (C) $I = \int_0^{\frac{\pi}{2}} \frac{\delta(x)}{\delta(x) + \left(\frac{\pi}{2} - x\right)} dx \quad \ldots(i)$

$$I = \int_0^{\frac{\pi}{2}} \frac{\delta\left(\frac{\pi}{2} - x\right)}{\delta\left(\frac{\pi}{2} - x\right) + \delta(x)} dx \quad \ldots(ii)$$

from eq. (i) and eq. (ii)

$$I + I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

33. (A) $x = a(\theta.\cos\theta - \sin\theta)$

$$\frac{dx}{d\theta} = a[\theta(-\sin\theta) + \cos\theta.1 - \cos\theta]$$

$$\frac{dx}{d\theta} = -a\theta.\sin\theta$$

and $y = a(\cos\theta + \theta.\sin\theta)$

$$\frac{dx}{d\theta} = a(-\sin\theta + \theta.\cos\theta + \sin\theta.1)$$

$$\frac{dy}{d\theta} = a\theta.\cos\theta$$

then $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$$\frac{dy}{dx} = a\theta.\cos\theta \times \left(\frac{-1}{a\theta.\sin\theta}\right)$$

$$\frac{dy}{dx} = -\cot\theta$$

$$\frac{d^2y}{dx^2} = -(-\text{cosec}^2\theta).\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \text{cosec}^2\theta.\left(\frac{-1}{a\theta.\sin\theta}\right) = \frac{-\text{cosec}^3\theta}{a\theta}$$

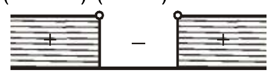
34. (B) $f(x) = \frac{1}{\sqrt{\log_e(2x^2 + 5x - 2)}}$

$\log_e(2x^2 + 5x - 20) > 0$

$2x^2 + 5x - 2 > 1$

$2x^2 + 5x - 3 > 0$

$(2x - 1)(x + 3) > 0$



$-3 \quad \frac{1}{2}$

$x \in (-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$

35. (A)

```
     1101
   × 110
    0000
   1101×
  1101 ×
  1001110
```

36. (C) $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \text{cosec}^{-1}\left(-\text{cosec}\frac{\pi}{3}\right)$

$$= \text{cosec}^{-1}\left[\text{cosec}\left(-\frac{\pi}{3}\right)\right] = \frac{-\pi}{3}$$

37. (A)

38. (D) Given that $f(x) = x + 6$

$gof(x) = x^2 + 12x + 38$

$g[f(x)] = (x + 6)^2 + 2$

$g[(f(x))] = [f(x)]^2 + 2$

$g(x) = x^2 + 2$

$g(-3) = (-3)^2 + 2 = 11$

39. (B) Word 'ARRANGE'

Total arrangement $= \frac{7!}{2!2!} = 1260$

when A appear together

Arrangement $= \frac{6!}{2!} = 360$

The required arrangement $= 1260 - 360$
$= 900$

40. (B) $\tan y\, dx - (1 - e^x)\sec^2 y\, dy = 0$

$\tan y\, dx = (1 - e^x)\sec^2 y\, dy$

$$\frac{dx}{1 - e^x} = \frac{\sec^2 y}{\tan y} dy$$

$$\frac{e^{-x}}{e^{-x} - 1} dx = \frac{\sec^2 y}{\tan y} dy$$

On integrating

$-\log(e^{-x} - 1) = \log\tan y + \log c$

$$-\log\left(\frac{1 - e^x}{e^x}\right) = \log(c.\tan y)$$

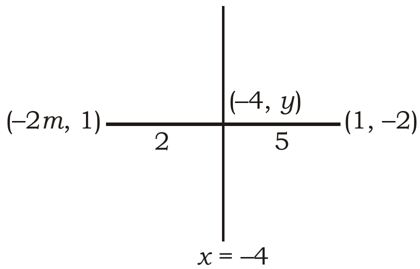
$$\log\left(\frac{e^x}{1 - e^x}\right) = \log(c.\tan y)$$

$$\frac{e^x}{1 - e^x} = c.\tan y$$

$$(1 - e^x)\tan y = \frac{1}{c} e^x \Rightarrow (1 - e^x)\tan y = C.e^x$$

41. (B) We know that
det(λA) = λ^n det(A), if matrix $n \times n$
Then $\lambda = n$

42. (B)



$$\frac{2\times 1+5\times(-2m)}{2+5} = -4$$

$2 - 10m = -28$
$10m = 30 \Rightarrow m = 3$

43. (A) $A = \{1, 2, 3, 4, 6, 7, 9\}$
no. of elements = 7
then
No. of subsets of A = $2^7 = 128$

44. (D) $I = \int \frac{e^{-x}}{1+e^{-x}}\,dx$
$(1 + e^{-x}) = t$
$-e^{-x}dx = dt \Rightarrow e^{-x}dx = -dt$
$I = \int -\frac{dt}{t}$
$I = -\log t + c$
$I = -\log(1+e^{-x}) + c$
$I = \log\left(\frac{e^x}{1+e^x}\right) + c$

45. (C) given that
$$\int x.\ln x\,dx = \frac{x^2}{a} + \frac{x^2.\ln x}{b} + c \qquad \ldots(i)$$
$$\int x.\ln x\,dx = \ln x\int x.dx - \int\left\{\frac{d}{dx}(\ln x).\int x.dx\right\}dx$$
$$\int x.\ln x dx = (\ln x).\frac{x^2}{2} - \int\frac{1}{x}.\frac{x^2}{2}\,dx$$
$$\int x.\ln x dx = \frac{x^2}{2}\ln x - \frac{1}{2}\times\frac{x^2}{2} + c$$
$$\int x.\ln x\,dx = -\frac{x^2}{4} + \frac{x^2}{2}\ln x + c$$
On comparing with equation (i)
$a = -4$ and $b = 2$

46. (A) $\begin{bmatrix} x \\ x \\ y \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} y \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 5 \end{bmatrix}$
$x + y + z = 20$ ...(i)
$x + y = 10$ ...(ii)
$y + z = 5$ ...(iii)
eq. (ii) + eq.(iii)
$(x + y + z) + y = 15$
from eq. (i)
$20 + y = 15$
$y = -5$
from eq. (ii) and eq. (iii)
$x = 15$ and $z = 10$
then $x - z = 15 - 10 = 5$

47. (B) Variance of 25 observations var $(x) = 4$
We know that
Var $(\lambda x) = \lambda^2$ var(x)
If each observation multiplied by 4
then variance of new observations
var $(4x) = 4^2 \times$ var(x)
var $(4x) = 4^2 \times 4 = 64$

48. (A) $n(S) = {}^{12}C_4 = 495$
$n(E) = {}^3C_2\times{}^5C_1\times{}^4C_1 + {}^3C_2\times{}^5C_2\times{}^4C_0 + {}^3C_2\times{}^5C_0\times{}^4C_2 + {}^3C_3\times{}^5C_1\times{}^4C_0 + {}^3C_3\times{}^5C_0\times{}^4C_1$
$n(E) = 3\times5\times4 + 3\times10\times1 + 3\times1\times6 + 1\times5\times1 + 1\times1\times4$
$n(E) = 117$
The required probability = $\frac{n(E)}{n(S)} = \frac{117}{495}$
$= \frac{13}{55}$

49. (A) a, b, c are in A.P.
$2b = a + c$ ...(i)
l, m, n in A.P
$2m = l + n$ ...(ii)
from eq. (i) and eq. (ii)
$2(b + m) = (a + l) + (c + n)$
then $(a + l), (b + m), (c + n)$ also are in A.P.

50. (B) Digits 0, 1, 2, 3, 4, 7, 9
| 5 | 5 | 4 | = 5 × 5 × 4 = 100
'0' can not put here
only (1,3,7,9) for odd number

51. (C) Sphere $x^2+y^2+z^2-6x-7y+2z-8=0$
On comparing with general equation
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$
$u = -3,\ v = \frac{-7}{2},\ w = 1,\ d = -8$
radius of sphere $(r) = \sqrt{u^2+v^2+w^2-d}$
$(r) = \sqrt{9+\frac{49}{4}+1+8}$
$r = \sqrt{\frac{121}{4}} = \frac{11}{2}$ unit

52. (B) Roots of the quadratic equation $ax^2 + 7x + 11 = 0$ are imaginary, then

$B^2 - 4AC < 0$

$(7)^2 - 4 \times a \times 11 < 0$

$49 - 44a < 0$

$49 < 44a \Rightarrow a > \frac{49}{44}$

53. (D) $y = x\, lnx + \frac{e^x}{x}$

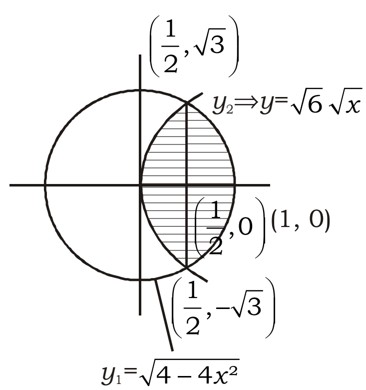
On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = x.\frac{1}{x} + lnx.1 + e^x\left(\frac{-1}{x^2}\right) + \frac{1}{x}.e^x$

$\frac{dy}{dx} = 1 + lnx - \frac{e^x}{x^2} + \frac{e^x}{x}$

$\left(\frac{dy}{dx}\right)_{at x=1} = 1 + ln1 - \frac{e^1}{1} + \frac{e^1}{1} = 1$

54. (B)



Parabola $y_2 \Rightarrow y = \sqrt{6}\ \sqrt{x}$ ...(i)

and

ellipse $y_1 \Rightarrow \sqrt{4 - 4x^2}$ ...(ii)

from equation (i) and equation (ii)

$x = \frac{1}{2}, \quad y = \pm\sqrt{3}$

Area $= \int_0^{\frac{1}{2}} y_2\, dx + \int_{\frac{1}{2}}^{1} y_1.dx$

$= \int_0^{\frac{1}{2}} \sqrt{6}\ \sqrt{x}\, dx + \int_{\frac{1}{2}}^{1} \sqrt{4 - 4x^2}$

$= \sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^{\frac{1}{2}} + 2\int_{\frac{1}{2}}^{1} \sqrt{1 - x^2}\, dx$

Let $x = \sin\theta$ when $x \to \frac{1}{2}, \theta \to \frac{\pi}{6}$

$dx = \cos\theta\, d\theta$ $x \to 1, \theta \to \frac{\pi}{2}$

$= \sqrt{6} \times \frac{2}{3} \times \frac{1}{2\sqrt{2}} + 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos\theta.\cos\theta\, d\theta$

$= \frac{1}{\sqrt{3}} + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos 2\theta).d\theta$

$= \frac{1}{\sqrt{3}} + \left[\theta + \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= \frac{1}{\sqrt{3}} + \left[\frac{\pi}{2} + \frac{\sin\pi}{2} - \frac{\pi}{6} - \frac{\sin\frac{\pi}{3}}{2}\right]$

$= \frac{1}{\sqrt{3}} + \left[\frac{\pi}{3} + 0 - \frac{\sqrt{3}}{4}\right] = \frac{\sqrt{3} + 4\pi}{12}$

55. (A) $z = 1 + \cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$z = 2\cos^2\frac{\pi}{6} - i \times 2\sin\frac{\pi}{6}.\cos\frac{\pi}{6}$

$z = 2\cos\frac{\pi}{6}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$|z| = 2\cos\frac{\pi}{6}$

$|z| = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

56. (C) given that $P(A) = 0.8$ and $P(B) = 0.4$

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

for minimum value of $P(A \cap B)$, $P(A \cup B) = 1$

$1 = 0.8 + 0.4 - P(A \cap B)$

$P(A \cap B) = 0.2$

57. (A) line $\frac{2x + 3}{4} = \frac{y + 2}{-3} = \frac{z - 1}{6}$

$\frac{x + \frac{3}{2}}{2} = \frac{y + 2}{-3} = \frac{z - 1}{6}$

and plane $3x - 6y + 2z = 6$

angle between line and plane

$\sin\theta = \frac{2 \times 3 + (-3)(-6) + 6 \times 2}{\sqrt{(2)^2 + (-3)^2 + (6)^2}\sqrt{(3)^2 + (-6)^2 + (2)^2}}$

$\sin\theta = \frac{36}{7 \times 7} \Rightarrow \theta = \sin^{-1}\left(\frac{36}{49}\right)$

58. (B) In the expansion of $\left(2\sqrt{x}+\frac{1}{4\sqrt{x}}\right)^9$

Total term = 9 + 1 = 10
middle term = 5^{th} and 6^{th}

$T_5 = T_{4+1} = {}^9C_4(2\sqrt{x})^5\left(\frac{1}{4\sqrt{x}}\right)^4$

$T_5 = {}^9C_4 \times \frac{1}{2^3}\, x^{\frac{1}{2}}$

$T_6 = T_{5+1} = {}^9C_5(2\sqrt{x})^4\left(\frac{1}{4\sqrt{x}}\right)^5$

$T_6 = {}^9C_5 \times \frac{1}{2^6}\, x^{-\frac{1}{2}}$

Sum of coefficients of middle terms

$= {}^9C_4 \times \frac{1}{2^3} + {}^9C_4 \times \frac{1}{2^6}$

$= {}^9C_4\left(\frac{1}{8}+\frac{1}{64}\right)$

$= 21 \times 6 \times \frac{72}{64} = \frac{567}{4}$

59. (C) Let $a + ib = \sqrt{(9+4\sqrt{13}i)}$

On squaring both side

$(a^2 - b^2) + i(2ab) = 9 + 4\sqrt{13}\, i$

On comparing

$a^2 - b^2 = 9$ and $2ab = 4\sqrt{13}$ ...(i)

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = 81 + 208$

$a^2 + b^2 = 17$ ...(ii)

from eq. (i) and eq. (ii)

$a = \pm\sqrt{13}$ and $b = \pm 2$

then $\sqrt{9+4\sqrt{13}i} = \pm(\sqrt{13} + 2i)$

60. (A) Let $y = x^2\cos x$ and $z = \tan x$

$\frac{dy}{dx} = -x^2\sin x + 2x\cos x$, $\frac{dz}{dx} = \sec^2 x$

then

$\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\frac{dy}{dz} = (-x^2\sin x + 2x\cos x) \times \frac{1}{\sec^2 x}$

$\frac{dy}{dz} = (-x^2\sin x + 2x\cos x)\cos^2 x$

61. (C) two point (–1, 3) and (–3, 4)

midpoint $\left(-2, \frac{7}{2}\right)$

equation of the line which is perpendicular to the line $12x + 7y = 8$

$7x - 12y = c$ ...(i)

its passes through the point $\left(-2, \frac{7}{2}\right)$

$7 \times (-2) - 12 \times \frac{7}{2} = c$

$c = -86$

from eq. (i)

$7x - 12y = -86$

$7x - 12y + 86 = 0$

(62-64)

Class	x	frequency (f)	$f \times x$
0-10	5	f_1	$5f_1$
10-20	15	16	240
20-30	25	15	375
30-40	35	11	385
40-50	45	f_2	$45f_2$
		$\Sigma f = 42+f_1+f_2$	$\Sigma f \times x = 1000+5f_1+45f_2$

given that $\Sigma f = 50$

$42 + f_1 + f_2 = 50$

$f_1 + f_2 = 8$ ...(i)

and A.M = $\frac{\Sigma f \times x}{\Sigma f}$

$24 = \frac{1000 + 5f_1 + 45f_2}{50}$

$5f_1 + 45f_2 + 1000 = 1200$

$f_1 + 9f_2 = 40$ ...(ii)

from eq. (i) and eq. (ii)

62. (A) $f_1 = 4$

63. (B) $f_2 = 4$

64. (A) given that $A = 24$

Class	x	f	$d = \lvert x - A \rvert$	$f \times d$
0-10	5	4	19	76
10-20	15	16	9	144
20-30	25	15	1	15
30-40	35	11	11	121
40-50	45	4	21	84
		$\Sigma f = 50$		$\Sigma f \times d = 440$

Mean deviation = $\frac{\Sigma f \times d}{\Sigma f} = \frac{440}{50} = 8.8$

65. (C) $L_1 \Rightarrow \frac{1-\sqrt{2}x}{4} = \frac{y+4}{7} = \frac{z-1}{8}$

$$\Rightarrow \frac{x-\frac{1}{\sqrt{2}}}{-2\sqrt{2}} = \frac{y+4}{7} = \frac{z-1}{8}$$

and $L_2 \Rightarrow \frac{x-0}{\sqrt{2}} = \frac{y-0}{1} = \frac{z-0}{1}$

angle between lines

$$\cos\theta = \frac{-2\sqrt{2}\times\sqrt{2}+7\times1+8\times1}{\sqrt{(-2\sqrt{2})^2+(7)^2+(8)^2}\sqrt{(\sqrt{2})^2+(1)^2+(1)^2}}$$

$$\cos\theta = \frac{11}{11\times 2} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

66. (B) Direction ratio of L_1 = $(-2\sqrt{2}, 7, 8)$

67. (D) Direction cosine of L_2 = $< \frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2} >$

68. (A) $I = \int e^x \frac{2x-1}{(2x+1)^2} dx$

$$I = \int e^x \left(\frac{1}{2x+1} - \frac{2}{(2x+1)^2}\right) dx$$

$$I = e^x \times \frac{1}{2x+1} + c$$

$$I = \frac{e^x}{(2x+1)} + c$$

69. (B) $\lim_{x\to\infty}\left(\frac{x+5}{x}\right)^x = \lim_{x\to\infty}\left(1+\frac{5}{x}\right)^x$ $[1^\infty]$ Form

$$= \lim_{x\to\infty}\left\{\left(1+\frac{5}{x}\right)^{\frac{x}{5}}\right\}^5$$

$$= e^{5.\lim_{x\to\infty} 5} = e^{5\times5} = e^{25}$$

70. (A) $\sin^{-1}\frac{36}{x} + \sin^{-1}\frac{77}{x} = \frac{\pi}{2}$

$$\sin^{-1}\frac{36}{x} = \frac{\pi}{2} - \sin^{-1}\frac{77}{x}$$

$$\sin^{-1}\frac{36}{x} = \cos^{-1}\frac{77}{x}$$

$$\sin^{-1}\frac{36}{x} = \sin^{-1}\frac{\sqrt{x^2-5929}}{x}$$

$$\frac{36}{x} = \frac{\sqrt{x^2-5929}}{x}$$

$$1296 = x^2 - 5929$$

$$x^2 = 7225 \Rightarrow x = 85$$

71. (B) We know that

$C_0 + C_1x + C_2x^2 + \ldots C_{n-1}x^{n-1} + C_nx^n = (1+x)^n$ …(i)

Multiply by x

$\Rightarrow C_0x + C_1x^2 + \ldots + C_{n-1}x^n + C_nx^{n+1} = x(1+x)^n$

On differentiate both side w.r.t. 'x'

$\Rightarrow C_0 + 2C_1x + 3C_2x^2 + \ldots + nC_{n-1}x^{n-1} + (n+1)C_nx^n$

$= nx.(1+x)^{n-1} + (1+x)^n$ …(ii)

$x \to \frac{1}{x}$ in eq. (i)

$\Rightarrow C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots + \frac{C_{n-1}}{x^{n-1}} + \frac{C_n}{x^n} = \left(1+\frac{1}{x}\right)^n$

…(iii)

from eq. (ii) and eq. (iii)

$\Rightarrow$ coeff. of x^0 in $\left(1+\frac{1}{x}\right)^n [nx(x+1)^{n-1} + (1+x)^n]$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow$ coeff. of x^{n-1} in $n(1+x)^{2n-1}$ + coeff. of x^n in$(1+x)^{2n} = C_0^2 + 2C_1^2 + \ldots + nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow n^{2n-1}C_{n-1} + {}^{2n}C_n = C_0^2 + 2C_1^2 + \ldots + nC_{n-1}^2 + (n+1)C_2^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{(2n)!}{n!n!}$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots + nC_{n-1}^2 + (n+1)C_n^2$

$\Rightarrow \frac{n\times(2n-1)!}{(n-1)!n!} + \frac{2n(2n-1)!}{n(n-1)!n!}$

$= C_0^2 + 2C_1^2 + 3C_2^2 + \ldots + nC_{n-1}^2 + (n+1)c_n^2$

$\Rightarrow \frac{(2n-1)!}{(n-1)!n!}(n+2) = C_0^2 + 2C_1^2 + \ldots + (n+1)C_n^2$

$\Rightarrow (n+2)\ {}^{2n-1}C_{n-1} = C_0^2 + 2C_1^2 \ldots + (n+1)C_n^2$

72. (C) Total students = 6

the table is round. One students is fixed.

No. of ways = (6 –1)! = 5! = 120

73. (A) Let $y = 5^{61}$

taking log both side

$\log_{10}y = 61 \log_{10}5$

$\log_{10}y = 61 \times 0.699$

$\log_{10}y = 42.639$

No. of digits = 42 + 1 = 43

74. (B) $C(3n, 6) = C(3n, n)$

$3n = 6 + n$

$2n = 6$

then $C(9, 2n) = C(9, 6)$

$$= \frac{9!}{6!3!} = 84$$

75. (C) When $\theta = 180°$

$M = \frac{60}{11}(H \pm 6)$ when $- \to H > 6$

$+ \to H < 6$

$H = 7$ (between 7 and 8 O'clock)

$M = \frac{60}{11}(7-6)$

$M = \frac{60}{11} = 5\frac{5}{11}$ minute

time $= 7 : 5\frac{5}{11}$

76. (B) $(\log_2 x)(\log_x 4x)(\log_{4x} y) = \log_y y^3$

$\frac{\log x}{\log 2} \times \frac{\log 4x}{\log x} \times \frac{\log y}{\log 4x} = 3\log_y y$

$\frac{\log y}{\log 2} = 3$

$\log_2 y = 3 \Rightarrow y = 2^3 = 8$

77. (D) Let $z = x + iy$

$z = \bar{z}$

$x + iy = x - iy$

$2iy = 0 \Rightarrow y = 0$

imaginary part of z is zero.

78. (C) curve $5x^2 + 4y^2 = 20$

$\frac{x^2}{4} + \frac{y^2}{5} = 1$

$a^2 = 4,\ b^2 = 5$

eccentricity $e = \sqrt{1 - \frac{a^2}{b^2}}$

$e = \sqrt{1 - \frac{4}{5}} = \frac{1}{\sqrt{5}}$

79. (B)

80. (C) We know that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \cos^2\gamma = 2$

81. (A) $\lim_{x\to 0} x.\cot x = \lim_{x\to 0} \frac{x}{\tan x}$

$= 1$

82. (B) We know that

minimum value of $\left(ax^2 + \frac{b}{x^2}\right) = 2\sqrt{ab}$

then

minimum value of $(9\tan^2\theta + 4\cot^2\theta)$

$= 2\sqrt{9 \times 4} = 12$

83. (C) $f(x) = \sqrt{24 + x^2}$

$f'(x) = \frac{1}{2} \times \frac{1}{\sqrt{24+x^2}} \times 2x = \frac{x}{\sqrt{24+x^2}}$

then

$\lim_{x\to 1} \frac{f(x) - f(1)}{x - 1}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 1} \frac{f'(x) - 0}{1 - 0}$

$\Rightarrow \lim_{x\to 1} \frac{x}{\sqrt{24+x^2}} = \frac{1}{\sqrt{24+1}} = \frac{1}{5}$

84. (D) $I = \int e^x \left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

$I = \int e^x \left(\frac{1 - 2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$

$I = \int e^x \left(\frac{1}{2}\text{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$

$I = -\int e^x \left(\cot\frac{x}{2} - \frac{1}{2}\text{cosec}^2\frac{x}{2}\right) dx$

$I = -e^x \cot\frac{x}{2} + c$

85. (A) $\frac{\cos 5x + 2\cos 3x + \cos x}{\sin 5x - \sin x}$

$\Rightarrow \frac{\cos 5x + \cos x + 2\cos 3x}{\sin 5x - \sin x}$

$\Rightarrow \frac{2\cos 3x.\cos 2x + 2\cos 3x}{2\cos 3x.\sin 2x}$

$\Rightarrow \frac{2\cos 3x(\cos 2x + 1)}{2\cos 3x.\sin 2x}$

$\Rightarrow \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}} = \cot\frac{x}{2}$

86. (C) Planes $6x + 3y - 5z + 4 = 0$

and $6x + 3y + 5z - 1 = 0$

angle between planes

$\cos\theta = \frac{6\times 6 + 3\times 3 + (-5)(5)}{\sqrt{6^2 + 3^2 + (-5)^2}\sqrt{6^2 + 3^2 + 5^2}}$

$\cos\theta = \frac{36 + 9 - 25}{\sqrt{70}\sqrt{70}}$

$\cos\theta = \frac{20}{70} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{7}\right)$

87. (D) $2ae = 2\sqrt{3}$ and $e = \sqrt{3}$

$2a \times \sqrt{3} = 2\sqrt{3}$, $\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{3}$

$a = 1$, $\frac{b^2}{a^2} = 2 \Rightarrow b^2 = 2$

equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{2} = 1 \Rightarrow 2x^2 - y^2 = 1$

88. (B) sin20°. sin40°. sin100

$\Rightarrow$ sin40°.sin(60°– 40°).sin(60° + 40°)

$\Rightarrow \frac{1}{4}\sin(3 \times 40)$

$\left[\because \sin A.\sin(60 - A).\sin(60 + A) = \frac{1}{4}\sin 3A\right]$

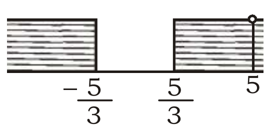
$\Rightarrow \frac{1}{4}\sin 120 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

89. (C) $f(x) = \frac{\sqrt{9x^2 - 25}}{x - 5}$

$9x^2 - 25 \geq 0$, $x \neq 5$

$x^2 \geq \frac{25}{9} \Rightarrow x \geq \pm\frac{5}{3}$

$x \geq \frac{5}{3}$ and $x \leq \frac{-5}{3}$



$x \in \left[\left(-\infty, \frac{-5}{3}\right] \cup \left[\frac{5}{3}, \infty\right)\right] - \{5\}$

90. (A) $I = \int_0^1 \sum_{n=1}^{6} \left(x^n - x^{n-1}\right)$

$I = \int_0^1 \left[\left(x - x^0\right) + \left(x^2 - x\right) + \left(x^3 - x^2\right) + \ldots + \left(x^6 - x^5\right)\right]$

$I = \int_0^1 \left(-1 + x^6\right) dx$

$I = \left[-x + \frac{x^7}{7}\right]_0^1$

$I = -1 + \frac{1}{7} - 0 \Rightarrow I = \frac{-6}{7}$

91. (C) $\sum_{i=0}^{8} \left(i^{n+1} - i^n\right) = (i - i^0) + (i^2 - i) + \ldots + (i^9 - i^8)$

$= -i^0 + i^9$

$= -1 + i = i - 1$

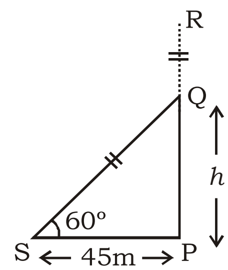
92. (B) $(n-1)!$, $4 \times (n-1)!$ and $n!$ are in G.P.

then $[4 \times (n-1)!]^2 = (n-1)! \times n!$

$16(n-1)!\,(n-1)! = (n-1)! \times n(n-1)!$

$16 = n \Rightarrow n = 16$

93. (A)



We know that QR = QS

Let PQ = h m

In ΔPSQ :

$\tan 60° = \frac{PQ}{PS}$

$\sqrt{3} = \frac{h}{45} \Rightarrow h = 45\sqrt{3}$ ...(i)

$\sin 60° = \frac{PQ}{QS}$

$\frac{\sqrt{3}}{2} = \frac{h}{QS} \Rightarrow \frac{\sqrt{3}}{2} = \frac{45\sqrt{3}}{QS}$

QR = QS = 90

Length of a tree = PQ + QR

$= 45\sqrt{3} + 90 = 45(\sqrt{3} + 2)$m

94. (B) Given that $f(x) = bx + c$, $g(x) = ax + d$

$fog(x) = gof(x)$

$f[g(x)] = g[f(x)]$

$f[ax + d] = g[bx + c]$

$b(ax + d) + c = a(bx + c) + d$

$abx + bd + c = abx + ac + d$

$bd + c = ac + d$

$f(d) = g(c)$

95. (C) $y = e^{\cos\sqrt{x^2+1}}$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = e^{\cos\left(\sqrt{x^2+1}\right)}\left(-\sin\sqrt{x^2+1}\right) \times \frac{1}{2\sqrt{x^2+1}} \times 2x$

$\frac{dy}{dx} = -\frac{x}{\sqrt{x^2+1}}.\sin\sqrt{x^2+1} \cdot e^{\cos\sqrt{x^2+1}}$

96. (B) Given that $\vec{a} = \hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

projection of $\vec{a}$ on $\vec{b} = \dfrac{1\times 2 - 3(-2) + 4\times 1}{\sqrt{(2)^2 + (-2)^2 + (1)^2}}$

projection of $\vec{a}$ on $\vec{b} = \dfrac{12}{3} = 4$

97. (A) $\lim\limits_{x\to 0} \dfrac{\sqrt{a+x^2} - \sqrt{a-x^2}}{x^2}$ $\left[\dfrac{0}{0}\right]$ Form

$\Rightarrow \lim\limits_{x\to 0} \left[\dfrac{\sqrt{a+x^2} - \sqrt{a-x^2}}{x^2} \times \dfrac{\sqrt{a+x^2} + \sqrt{a-x^2}}{\sqrt{a+x^2} + \sqrt{a-x^2}}\right]$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{a + x^2 - a + x^2}{x^2\left(\sqrt{a+x^2} + \sqrt{a-x^2}\right)}$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{2x^2}{x^2\left(\sqrt{a+x^2} + \sqrt{a-x^2}\right)}$

$\Rightarrow \dfrac{2}{\sqrt{a} + \sqrt{a}} = \dfrac{1}{\sqrt{a}}$

98. (C) $f(x) = \begin{cases} \dfrac{x^2 - 36}{x-6}, & x \neq 6 \\ kx + 7, & x = 6 \end{cases}$ is continuous

at $x = 6$, then

$\lim\limits_{x\to 6^-} f(x) = f(6)$

$\lim\limits_{x\to 6} \dfrac{x^2 - 36}{x - 6} = k \times 6 + 7$

$\lim\limits_{x\to 6} \dfrac{(x-6)(x+6)}{(x-6)} = k \times 6 + 7$

$12 = 6k + 7 \Rightarrow k = \dfrac{5}{6}$

99. (A) $(-\sqrt{-1})^{4n+3} + (-\sqrt{-1})^{8n-7} = (-i)^{4n+3} + (-i)^{8n-7}$

$= (-i)^3 + (-i)^{-7}$

$= -i^3 + \left(\dfrac{-1}{i}\right)^7$

$= i + (i)^7$

$= i + (-i) = 0$

100. (C) $\log_{10}[99 + \sqrt{(x-6)^2}\,] = 2$

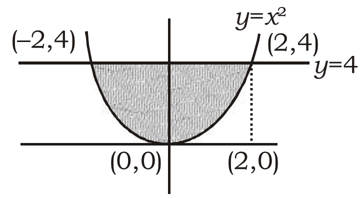
$99 + \sqrt{x^2 - 12x + 36} = 10^2$

$\sqrt{x^2 - 12x + 36} = 1$

$x^2 - 12x + 35 = 0$

$(x-5)(x-7) = 0 \Rightarrow x = 5, 7$

101. (C)



curve $\Rightarrow y_1 = y = x^2$

and $y_2 = y = 4$

Area $= 2\int_0^2 (y_2 - y_1)\,dx$

$= 2\int_0^2 (4 - x^2)\,dx$

$= 2\left[4x - \dfrac{x^3}{3}\right]_0^2$

$= 2\left[8 - \dfrac{8}{3}\right] = \dfrac{32}{3}$ sq. unit

$= \left(\dfrac{6^3}{3} - 4\times 6^2 + 12\times 6\right) - \left(\dfrac{2^3}{8} - 4\times 2^2 + 12\times 2\right)$

$= (72 - 144 + 72) - (1 - 16 + 24)$

$= 9$ sq. unit

102. (B) $I = \int_0^{\pi} |\cos x|\,dx$

$I = 2\int_0^{\frac{\pi}{2}} \cos x\,dx$

$= 2[\sin x]_0^{\frac{\pi}{2}}$

$= 2\left(\sin\dfrac{\pi}{2} - \sin 0\right) = 2$

103. (C) Word 'CONCLUSION'

Total Arrangement $= \dfrac{10!}{2!2!2!} = 453600$

(OUIO) CNCLSN

as one letter

Arrangement when vowels always come together $= \dfrac{7!}{2!\,2!} \times \dfrac{4!}{2!} = 15120$

Arrangement when vowels never come together = 453600 – 15120 = 438480

104. (C) Slope $m = \dfrac{x^2}{y-1}$ and point (–1, 2)

equation of the curve

$y - 2 = \dfrac{x^2}{y-1}(x + 1)$

$y^2 - 3y + 2 = x^3 + x^2$

$x^3 + x^2 - y^2 + 3y - 2 = 0$

105. (B) $\dfrac{\log_{81} 9 \times \log_{64} 8}{\log_{36} 6} = \dfrac{\dfrac{1}{\log_9 81} \times \dfrac{1}{\log_8 64}}{\dfrac{1}{\log_6 36}}$

$= \dfrac{\dfrac{1}{\log_9 9^2} \times \dfrac{1}{\log_8 8^2}}{\dfrac{1}{\log_6 6^2}}$

$= \dfrac{\dfrac{1}{2} \times \dfrac{1}{2}}{\dfrac{1}{2}} = \dfrac{1}{2}$

106. (C) 73, 74, 64, 65, 32, 48, 69, 53
On arranging in ascending order
32, 48, 53, 64, 65, 69, 73, 74
middle terms = 64 and 65
median = $\dfrac{64+65}{2}$ = 64.5

107. (B) $\cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right) = \cos^{-1}\left(-\cos\dfrac{\pi}{4}\right)$

$= \cos^{-1}\left[\cos\left(\pi - \dfrac{\pi}{4}\right)\right]$

$= \cos^{-1}\left[\cos\left(\dfrac{3\pi}{4}\right)\right] = \dfrac{3\pi}{4}$

108.(B) curve $2x^2 + 2y^2 = 1$

$x^2 + y^2 = \dfrac{1}{2}$

It is circle, then $r = \dfrac{1}{\sqrt{2}}$

Area of circle $= \pi r^2$

$= \pi \times \dfrac{1}{2} = \dfrac{\pi}{2}$ sq. unit

109. (B) Differential equation

$\left(\dfrac{dy}{dx}\right)^4 = (xy)^3$

$\dfrac{dy}{dx} = x^{3/4}.y^{3/4}$

$\dfrac{dy}{y^{\frac{3}{4}}} = x^{3/4}dx$

On integrating

$4y^{1/4} = \dfrac{4}{7}x^{\frac{7}{4}} + \dfrac{4}{7}c$

$7y^{1/4} = x^{\frac{7}{4}} + C$

110. (C) $x = 5 + 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$

$\Rightarrow (x-5)^3 = \left[5^{\frac{1}{3}} + 5^{\frac{2}{3}}\right]^3$

$\Rightarrow x^3 - 125 - 15x(x-5) = 5 + 5^2 + 15(5^{\frac{1}{3}} + 5^{\frac{2}{3}})$

$\Rightarrow x^3 - 125 - 15x^2 + 75x = 30 + 15(x-5)$

$\Rightarrow x^3 - 15x^2 + 75x - 125 - 30 - 15x + 75 = 0$

$\Rightarrow x^3 - 15x^2 + 60x = 80$

111. (A) $\begin{vmatrix} 7! & 6! & 5! \\ 6! & 5! & 4! \\ 5! & 4! & 3! \end{vmatrix}$

$C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$

$\Rightarrow \begin{vmatrix} 7!-5! & 6!-5! & 5! \\ 6!-4! & 5!-4! & 4! \\ 5!-3! & 4!-3! & 3! \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 41\times5! & 5\times5! & 5! \\ 29\times4! & 4\times4! & 4! \\ 19\times3! & 3\times3! & 3! \end{vmatrix}$

$\Rightarrow (5!)(4!)(3!)\begin{vmatrix} 41 & 5 & 1 \\ 29 & 4 & 1 \\ 19 & 3 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_3$ and $R_1 \to R_1 - R_3$

$\Rightarrow (5!)(4!)(3!)\begin{vmatrix} 22 & 2 & 0 \\ 10 & 1 & 0 \\ 19 & 3 & 1 \end{vmatrix}$

$\Rightarrow (5!)(4!)(3!)[22(1-0) - 2(10-0) + 0]$

$\Rightarrow (5!)(4!)(3!)(2) = 2 \times 5! \times 4! \times 3!$

112. (B) $a = 3$ cm

then $R = \dfrac{a}{\sqrt{3}}$

$R = \dfrac{3}{\sqrt{3}} = \sqrt{3}$ cm

Area of circumcircle $= \pi R^2 = 3\pi$ cm²

113. (A) cotA, cotB and cotC are in A.P.
then 2cotB = cotA + cotC

$\Rightarrow \dfrac{2\cos B}{\sin B} = \dfrac{\cos A}{\sin A} + \dfrac{\cos C}{\sin C}$

$\Rightarrow \dfrac{2\cos B}{\sin B} = \dfrac{\sin C.\cos A + \cos C.\sin A}{\sin A.\sin C}$

$\Rightarrow 2\cos B.\sin A.\sin C = \cos A.\sin B.\sin C + \sin A.\sin B.\cos C$

$\Rightarrow \cos B\sin A.\sin C - \cos A.\sin B.\sin C = \sin A.\sin B.\cos C - \cos B.\sin A.\sin C$

$\Rightarrow \sin C.\sin(A-B) = \sin A.\sin(B-C)$

$\Rightarrow \sin(A+B).\sin(A-B) = \sin(B+C).\sin(B-C)$

$\Rightarrow \sin^2A - \sin^2B = \sin^2B - \sin^2C$

$\Rightarrow 2\sin^2B = \sin^2A + \sin^2C$

$\Rightarrow 2b^2k^2 = a^2k^2 + c^2k^2$ [by Sine Rule]

$\Rightarrow 2b^2 = a^2 + c^2$

a^2, b^2 and c^2 are in A.P.

114. (B) $S = 3 + 7 + 13 + 21 + ... + a_n$

$S = \quad 3 + 7 + 13 + ... + a_{n-1} + a_n$

$– \quad – \quad – \quad – \quad – \quad – \quad –$

$0 = 3 + (4 + 6 + 8 + ... + a_{n-1}) - a_n$

$a_n = 3 + \frac{n-1}{2}\left(2\times4+(n-2)\times2\right)$

$a_n = 3 + (n-1)(n+2)$

$a_{24} = 3 + (24 - 1)(24 + 2)$

$a_{24} = 3 + 23 \times 26 = 601$

115. (C) line $\frac{x}{5} + \frac{y}{7} = 2$

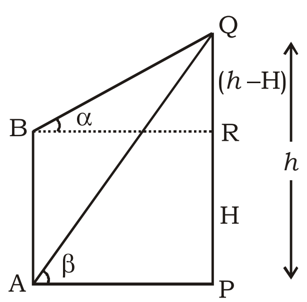
Slope $m_1 = -\frac{7}{5}$

Slope of the line which is perpendicular to the given line

$m_2 = \frac{-1}{m_1}$

$m_2 = \frac{-1\times5}{-7} = \frac{5}{7}$

116. (A)



Let height of a building (AB) = H m

In ΔRBQ :

$\tan\alpha = \frac{RQ}{BR} = \frac{h - H}{BR}$

$AP = BR = \frac{h - H}{\tan\alpha}$ ...(i)

In ΔAPQ :

$\tan\beta = \frac{PQ}{AP}$

$\tan\beta = \frac{h\times\tan\alpha}{h - H}$ [from eq. (i)]

$h\tan\beta - H\tan\beta = h\tan\alpha$

$H\tan\beta = h(\tan\beta - \tan\alpha)$

$$H = \frac{h(\tan\beta - \tan\alpha)}{\tan\beta}$$

117. (C) $y(x) = \tan^{-1}\left[\frac{\sqrt{x}(1+x)}{1-x^2}\right]$

$y(x) = \tan^{-1}\left[\frac{\sqrt{x}+x^{\frac{3}{2}}}{1-\sqrt{x}.x^{\frac{3}{2}}}\right]$

$y(x) = \tan^{-1}(x^{\frac{1}{2}}) + \tan^{-1}(x^{\frac{3}{2}})$

On differentiating both side w.r.t. 'x'

$y'(x) = \frac{1}{1+x} \times \frac{1}{2} \times \frac{1}{\sqrt{x}} + \frac{1}{1+x^3} \times \frac{3}{2} x^{\frac{1}{2}}$

$y'(x) = \frac{1}{2\sqrt{x}} \times \frac{1}{1+x} + \frac{3\sqrt{x}}{2(1+x^3)}$

$y'(1) = \frac{1}{2} \times \frac{1}{2} + \frac{3}{2\times2} = 1$

118. (B) $I = \int_0^1 e^{x^3} . \frac{x^2(x-1)^2}{(1+x^2)^2} dx$

$I = \frac{1}{3}\int_0^1 \left(3x^2.e^{x^3}\right)\left[\frac{x^2+1-2x}{(1+x^2)^2}\right]dx$

$I = \frac{1}{3}\int_0^1 \left(3x^2.e^{x^3}\right)\left[\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right]dx$

$I = \frac{1}{3}\left[e^{x^3}\left(\frac{1}{1+x^2}\right)\right]_0^1$

$I = \frac{1}{3}\left[\frac{e}{2} - \frac{e^0}{1}\right] = \frac{e-2}{6}$

119. (B) $\begin{vmatrix} x & -1 & 0 \\ 1 & 0 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 7$

$x(0 - 2) + 1(3 - 8) + 0 = 7$

$-2x - 5 = 7$

$2x = -12 \Rightarrow x = -6$

120. (C) $I = \int \frac{dx}{\sqrt{4x^2 - 20x + 29}}$

$I = \frac{1}{2}\int \frac{dx}{\sqrt{x^2 - 5x + \frac{29}{4}}}$

$I = \frac{1}{2}\int \frac{dx}{\sqrt{\left(x - \frac{5}{2}\right)^2 + 1^2}}$

$I = \frac{1}{2}\log\left|\left(x - \frac{5}{2}\right) + \sqrt{\left(x - \frac{5}{2}\right)^2 + 1^2}\right| + c$

$I = \frac{1}{2}\log|2x - 5 + \sqrt{4x^2 - 20x + 29}| + c$

NDA (MATHS) MOCK TEST - 96 (Answer Key)

1.	(B)	21.	(B)	41.	(B)	61.	(C)	81.	(A)	101.	(C)
2.	(A)	22.	(C)	42.	(B)	62.	(A)	82.	(B)	102.	(B)
3.	(B)	23.	(C)	43.	(A)	63.	(B)	83.	(C)	103.	(C)
4.	(C)	24.	(B)	44.	(D)	64.	(A)	84.	(D)	104.	(C)
5.	(A)	25.	(B)	45.	(C)	65.	(C)	85.	(A)	105.	(B)
6.	(B)	26.	(A)	46.	(A)	66.	(B)	86.	(C)	106.	(C)
7.	(C)	27.	(C)	47.	(B)	67.	(D)	87.	(D)	107.	(B)
8.	(B)	28.	(A)	48.	(A)	68.	(A)	88.	(B)	108.	(B)
9.	(A)	29.	(C)	49.	(A)	69.	(B)	89.	(C)	109.	(B)
10.	(B)	30.	(B)	50.	(B)	70.	(A)	90.	(A)	110.	(C)
11.	(A)	31.	(C)	51.	(C)	71.	(B)	91.	(C)	111.	(A)
12.	(B)	32.	(C)	52.	(B)	72.	(C)	92.	(B)	112.	(B)
13.	(D)	33.	(A)	53.	(D)	73.	(A)	93.	(A)	113.	(A)
14.	(B)	34.	(B)	54.	(B)	74.	(B)	94.	(B)	114.	(B)
15.	(C)	35.	(A)	55.	(A)	75.	(C)	95.	(C)	115.	(C)
16.	(A)	36.	(C)	56.	(C)	76.	(B)	96.	(B)	116.	(A)
17.	(B)	37.	(A)	57.	(A)	77.	(D)	97.	(A)	117.	(C)
18.	(A)	38.	(D)	58.	(B)	78.	(C)	98.	(C)	118.	(B)
19.	(A)	39.	(B)	59.	(C)	79.	(B)	99.	(A)	119.	(B)
20.	(D)	40.	(B)	60.	(A)	80.	(C)	100.	(C)	120.	(C)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*