NDA MATHS MOCK TEST - 98 (SOLUTION)

1. (B) $\Rightarrow \frac{dy}{dx} = \sec(x - y)$

Let $x - y = t$

$1 - \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$\Rightarrow 1 - \frac{dt}{dx} = \sec t$

$\Rightarrow \frac{dt}{dx} = 1 - \sec t$

$\Rightarrow \frac{dt}{\sec t - 1} = -dx$

$\Rightarrow \frac{(\sec t + 1)}{(\sec^2 t - 1)} dt = -dx$

$\Rightarrow \frac{\sec t + 1}{\tan^2 t} dt = -dx$

$\Rightarrow (\text{cosec}t.\cot t + \cot^2 t)\, dt = -dx$

$\Rightarrow \int (\text{cosec}.\cot t + \text{cosec}^2 t - 1)\, dt = -\int dx$

$\Rightarrow -\text{cosec}t - \cot t - t = -x - c$

$\Rightarrow \text{cosec}(x - y) + \cot(x - y) + x - y = x + c$

$\Rightarrow \text{cosec}(x - y) + \cot(x - y) = y + c$

2. (A)

2	93	
2	46	1
2	23	0
2	11	1
2	5	1
2	2	1
2	1	0
	0	1

$(93)_{10} = (1011101)_2$

3. (B) $y = \cos(n\sin^{-1}x)$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = -\sin(n\sin^{-1}x) \times \frac{n}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{-n\sin(n\sin^{-1}x)}{\sqrt{1-x^2}}$

again, differentiating

$$\frac{d^2y}{dx^2} = -n\left[\frac{\sqrt{1-x^2} \times \frac{n\cos(n\sin^{-1}x)}{\sqrt{1-x^2}} - \sin(n\sin^{-1}x) \times \left(\frac{-2x}{2\sqrt{1-x^2}}\right)}{\left(\sqrt{1-x^2}\right)^2}\right]$$

$$\frac{d^2y}{dx^2} = -n\left[\frac{n\cos(n\sin^{-1}x) + \frac{x\sin(n\sin^{-1}x)}{\sqrt{1-x^2}}}{1-x^2}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{at\,x=0} = -n\left[\frac{n\cos(n\sin^{-1}0) + 0}{1-0}\right] = -n^2$$

4. (A) We know that

$C_0 + C_1x + C_2x^2 + \ldots C_{n-1}x^{n-1} + C_nx^n = (1+x)^n$ ...(i)

Multiply by x

$\Rightarrow C_0x + C_1x^2 + \ldots C_{n-1}x^n + C_nx^{n+1} = x(1+x)^n$

On differentiating both side w.r.t. 'x'

$\Rightarrow C_0 + 2C_1x + 3C_2x^2 + \ldots + nC_{n-1}x^{n-1} + (n+1)C_nx^n$

$= nx(1+x)^{n-1} + (1+x)^n.1$

On putting $x = 1$

$\Rightarrow C_0 + 2C_1 + 3C_2 + \ldots + nC_{n-1} + (n+1)C_n = n.2^{n-1} + 2^n$

$\Rightarrow C_0 + 2C_1 + 3C_2 + \ldots + nC_{n-1} + (n+1)C_n = 2^{n-1}[n+2]$

5. (B) $\text{cosec}\theta + \cot\theta = A$ ...(i)

$\text{cosec}\theta - \cot\theta = \frac{1}{A}$ ...(ii)

from eq. (i) and eq. (ii)

$2\text{cosec}\theta = A + \frac{1}{A}$

$\text{cosec}\theta = \frac{A^2+1}{2A} \Rightarrow \sin\theta = \frac{2A}{1+A^2}$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\cos\theta = \sqrt{1 - \left(\frac{2A}{1+A^2}\right)^2} = \frac{A^2-1}{A^2+1}$

6. (D) Given that $f(x) = \frac{x}{x+1}$

then $f(f(x)) = f\left(\frac{x}{x+1}\right)$

$= \frac{\frac{x}{x+1}}{\frac{x}{x+1}+1} = \frac{x}{2x+1}$

7. (B) Given that $g(x) = \frac{1}{x}$, $f(x) = \frac{1}{g(x)} = x$

From option (B)

L.H.S = $f(g(f(g(g(x)))))$

$= f\left(g\left(f\left(g\left(\frac{1}{x}\right)\right)\right)\right)$

$= f(g(f(x)))$

$= f(g(x))$

$= f\left(\frac{1}{x}\right) = \frac{1}{x}$

R.H.S. = $g(f(g(g(f(x)))))$

$= g(f(g(g(x))))$

$= g\left(f\left(g\left(\frac{1}{x}\right)\right)\right)$

$= g(f(x))$

$= g(x) = \frac{1}{x}$

L.H.S. = R.H.S.

Hence option B is correct.

8. (C) **From option (C)**

L.H.S. = $\sin^{-1}\left[\sin\left(\frac{5\pi}{6}\right)\right]$

$= \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{5}\right)\right]$

$= \sin^{-1}\left[\sin\frac{\pi}{6}\right] = \frac{\pi}{6}$ = R.H.S.

Hence option (C) is correct.

9. (D) $x^y = e^{x+y}$

taking log both side

$y \log x = x + y$

On differentiating both side w.r.t. 'x'

$y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$

$(\log x - 1)\frac{dy}{dx} = 1 - \frac{y}{x}$

$\frac{dy}{dx} = \frac{x-y}{x(\log x - 1)}$

10. (B) $\frac{i-\sqrt{3}}{i+\sqrt{3}} = \frac{i-\sqrt{3}}{i+\sqrt{3}} \times \frac{i-\sqrt{3}}{i-\sqrt{3}}$

$= \frac{-1+\sqrt{3}i}{2} = \omega$

and $\frac{i+\sqrt{3}}{i-\sqrt{3}} = \frac{i+\sqrt{3}}{i-\sqrt{3}} \times \frac{i+\sqrt{3}}{i+\sqrt{3}}$

$= \frac{-1-\sqrt{3}i}{2} = \omega^2$

then $\left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{197} + \left(\frac{i+\sqrt{3}}{i-\sqrt{3}}\right)^{197} + 1$

$\Rightarrow \omega^{197} + (\omega^2)^{197} + 1 \Rightarrow \omega^2 + \omega + 1 = 0$

11. (A) $I = \int_{-1}^{1}\left(px^3 + qx^2 + rx\right)dx$

$= \int_{-1}^{1} px^3\,dx + \int_{-1}^{1} qx^2\,dx + \int_{-1}^{1} rx\,dx$

Prop $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is even} \\ 0 \quad , & \text{if } f(x) \text{ is odd} \end{cases}$

$= 0 + 2\int_0^1 qx^2\,dx + 0$

$= 2q\left[\frac{x^3}{3}\right]_0^1 = \frac{2q}{3}$

Hence value of $\int_{-1}^{1}\left(px^3 + qx^2 + rx\right)dx$ depends on the value of q only.

12. (B) Let $y = 10^{x\cot x}$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = 10^{x\cot x}\ln 10[x(-\text{cosec}^2 x) + \cot x.1]$

$\frac{dy}{dx} = \ln 10[\cot x - x\,\text{cosec}^2 x]10^{x\cot x}$

13. (D) Let $f(x) = e^{-\text{cosec}x} = e^{\frac{-1}{\sin x}}$

L.H.L. $= \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} e^{\frac{-1}{\sin(-h)}}$

$= \lim_{h\to 0} e^{\frac{1}{\sinh}} = e^{\infty} = \infty$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} e^{\frac{-1}{\sinh}} = e^{-\infty} = 0$

R.H.L. ≠ R.H.S.

limit does not exist.

14. (B) $\frac{dy}{dx} + 4y = \frac{dx}{dy}$

$\frac{dy}{dx} + 4y = \frac{1}{\frac{dy}{dx}}$

$\left(\frac{dy}{dx}\right)^2 + 4y\frac{dy}{dx} = 1$

order = 1 and degree = 2

15. (A) $S = \frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \ldots$ upto ∞

$S = \frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\ldots\text{upto}\,\infty\right]$

$S = \frac{1}{2}[1+0] = \frac{1}{2}$

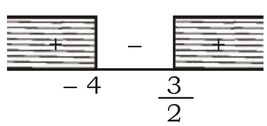
16. (B) $f(x) = \sqrt{\log_e(2x^2+5x-11)}$

$\log_e(2x^2 + 5x - 11) \geq 0$

$2x^2 + 5x - 11 \geq 1$

$2x^2 + 5x - 12 \geq 0$

$(x + 4)(2x - 3) \geq 0$



domain = $(-\infty, -4] \cup \left[\frac{3}{2}, \infty\right)$

17. (A) Sphere $x^2 + y^2 + z^2 + 2x - 6y + 4z + \lambda = 0$

$u = 1, v = -3, w = 2, d = -\lambda$

then radius of sphere $r = 1$

$\sqrt{u^2+v^2+w^2-d} = 1$

$1 + 9 + 4 + \lambda = 1 \Rightarrow \lambda = -13$

18. (C) $I = \int \tan x.\sec^4 x\, dx$

$I = \int \tan x.\sec^2 x.\sec^2 x\, dx$

$I = \int \tan x.\sec^2 x(1+\tan^2 x)dx$

$I = \int \tan x.\sec^2 x\, dx + \int \tan^3 x.\sec^2 x\, dx$

$I = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$

19. (A) Given that $f(x) = \log x$, $g(x) = \sin x$

$y = f\text{o}g(x)$

$y = f[g(x)]$

$y = f[\sin x]$

$y = \log(\sin x)$

On differentiating both side w.r. t.'x'

$\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$

$\frac{dy}{dx} = \cot x$

$\left(\frac{dy}{dx}\right)_{at\,x=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$

20. (C) $2^{x+2} + 3.2^{y-1} = 4$

$4.2^x + \frac{3}{2}.2^y = 4$ ...(i)

and $2^{x+2} + 2^{y+2} = 9$

$4.2^x + 4.2^y = 9$ ...(ii)

Let $2^x = a$ and $2^y = b$

from eq. (i) and eq. (ii)

$4a + \frac{3}{2}b = 4$ ...(iii)

$4a + 4b = 9$ ...(iv)

from eq. (iii) and eq. (ii)

$a = \frac{1}{4}$ and $b = 2$

$2^x = 2^{-2}$, $2^y = 2^1$

$x = -2$, $y = 1$

21. (D) $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

and $\vec{c} = -\hat{i} + \hat{j} - \hat{k}$

then $\vec{a}\times(\vec{b}+\vec{c}) + \vec{b}\times(\vec{c}+\vec{a}) + \vec{c}\times(\vec{a}+\vec{b})$

$\Rightarrow \vec{a}\times\vec{b} + \vec{a}\times\vec{c} + \vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a} + \vec{c}\times\vec{b}$

$\Rightarrow \vec{a}\times\vec{b} - \vec{c}\times\vec{a} + \vec{b}\times\vec{c} - \vec{a}\times\vec{b} + \vec{c}\times\vec{a} - \vec{b}\times\vec{c} = 0$

22. (C) $\begin{vmatrix} x-1 & x-2 & x-a \\ x-3 & x-4 & x-b \\ x-5 & x-6 & x-c \end{vmatrix}$

$C_1 \to C_1 - C_2$ and $C_3 \to C_3 - C_2$

$\Rightarrow \begin{vmatrix} 1 & x-2 & 2-a \\ 1 & x-4 & 4-b \\ 1 & x-6 & 6-c \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & x-2 & 2-a \\ 0 & -2 & 2+a-b \\ 0 & -4 & 4+a-c \end{vmatrix}$

$\Rightarrow 1(-8 - 2a + 2c + 8 + 4a - 4b) - (x - 2) \times 0 + (2 - a) \times 0$

$\Rightarrow 2a + 2c - 4b$

$\Rightarrow 2(a + c - 2b)$

a, b, c are in A.P. *i.e.* $2b = a + c$

$\Rightarrow 2(a + c - a - c) = 0$

23. (A) $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{4}$ and $\left(\frac{A}{B}\right) = \frac{1}{6}$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{6} = \frac{P(A \cap B)}{\frac{1}{4}} \Rightarrow P(A \cap B) = \frac{1}{24}$$

$$\text{then } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{24}}{\frac{2}{3}} = \frac{1}{16}$$

24. (C) $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ -1 & -4 & 2+x \end{vmatrix} = 15$

$\Rightarrow$ $1(2 + x) - 2(0) + 3(1) = 15$

$\Rightarrow$ $x + 5 = 15 \Rightarrow x = 10$

25. (A) $I = \int_0^1 x\,(1-x)^5\, dx$

$I = \int_0^1 (1-x)\,x^5\, dx$ [from prop. (iv)]

$I = \int_0^1 \left[x^5 - x^6\right] dx$

$I = \left[\frac{x^6}{6} - \frac{x^7}{7}\right]_0^1$

$I = \frac{1}{6} - \frac{1}{7} = \frac{1}{42}$

26. (A) Differential equation

$(\sin x - \cos x)dy + (\sin x + \cos x)dx = 0$

$$dy = -\left[\frac{\sin x + \cos x}{\sin x - \cos x}\right] dx$$

$$dy = \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}\, dx$$

$$dy = \frac{1 + \tan x}{1 - \tan x}\, dx$$

$$dy = \tan\left(\frac{\pi}{4} + x\right) dx$$

On integrating

$$y = \log\left|\sec\left(\frac{\pi}{4} + x\right)\right| + c$$

27. (B) $n(S) = {}^{11}C_3 = 165$

$n(E) = {}^4C_2 \times {}^2C_1 \times {}^5C_0 + {}^4C_2 \times {}^2C_0 \times {}^5C_1 + {}^4C_3 \times {}^2C_0 \times {}^5C_0$

$n(E) = 6 \times 2 \times 1 + 6 \times 1 \times 5 + 4 \times 1 \times 1 = 46$

The required probability $= \frac{n(E)}{n(S)} = \frac{46}{165}$

28. (C) Roots of the quadratic equation

$ax^2 + 6x + 13 = 0$ are real and equal

then $B^2 - 4AC = 0$

$36 - 4 \times a \times 13 = 0 \Rightarrow a = \frac{9}{13}$

29. (B) Let $(a - ib) = \sqrt{45 - 28i}$

On squaring both side

$(a^2 - b^2) - 2abi = 45 - 28i$

On comparing both side

$a^2 - b^2 = 45$ and $2ab = 28$ ...(i)

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = (45)^2 + (28)^2$

$(a^2 + b^2)^2 = (53)^2$

$a^2 + b^2 = 53$ ...(ii)

from eq. (i) and eq. (ii)

$a = \pm 7$, $b = \pm 2$

Square root of $(45 - 28i) = \pm(7 - 2i)$

30. (A) 4-digits numbers formed from the digits (0, 1, 2, 3,4, 5, 6)

6	6	5	4

$= 6 \times 6 \times 5 \times 4 = 720$

'0'can't put here

31. (C) $y = x \ln x + \sin^{-1} x$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = x \times \frac{1}{x} + \ln x.1 + \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{dy}{dx} = 1 + \ln x + \frac{1}{\sqrt{1 - x^2}}$$

$$\left(\frac{dy}{dx}\right)_{at\, x = \frac{1}{2}} = 1 + ln\left(\frac{1}{2}\right) + \frac{2}{\sqrt{3}}$$

$$= 1 - \ln 2 + \frac{2\sqrt{3}}{3} = \frac{3 + 2\sqrt{3} - 3\ln 2}{3}$$

32. (B) lines $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-0}{1}$ and

$\frac{x+6}{1} = \frac{y-4}{-2} = \frac{z+3}{2}$

angle between lines

$$\cos\theta = \frac{2 \times 1 + 2 \times (-2) + 1 \times 2}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{1^2 + (-2)^2 + 2^2}}$$

$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

33\. (C) Area = $\int_0^1 \left(xe^{2x} - xe^{-2x}\right) dx$

$$= \left[\left(x.\frac{e^{2x}}{2} + e^{2x}.1\right) - \left(x.\frac{e^{-2x}}{-2} + e^{-2x}.1\right)\right]_0^1$$

$$= \left[\frac{x}{2}e^{2x} + e^{2x} + \frac{x}{2}e^{-2x} - e^{-2x}\right]_0^1$$

$$= \left[\left(\frac{1}{2}e^2 + e^2 + \frac{1}{2}e^{-2} - e^{-2}\right) - (0 + 1 + 0 - 1)\right]$$

$$= \frac{3}{2}e^2 - \frac{1}{2}e^{-2}$$

34\. (A) Let $y = 6^{73}$
taking log both side
$\log_{10}y = 73 \log_{10}6$
$\log_{10}y = 73 \times 0.778$
$\log_{10}y = 56.794$
No. of digits = 56 +1 = 57

35\. (D) $\{x : x + 6 = 6\} = \{0\}$

36\. (C) $\dfrac{1 + \cos(B - C)\cos A}{1 + \cos(B - A)\cos C}$

$$= \frac{1 - \cos(B - C)\cos(B + C)}{1 - \cos(B - A)\cos(B + A)}$$

$$= \frac{1 - \cos^2 B + \sin^2 C}{1 - \cos^2 B + \sin^2 A}$$

$$= \frac{\sin^2 B + \sin^2 C}{\sin^2 B + \sin^2 A} = \frac{b^2 + c^2}{b^2 + a^2}$$

37\. (C) $A = \dfrac{\sin 60 - \sin 30}{\cos 60 + \cos 30}$

$$A = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}}$$

$$A = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3}$$

and $B = \dfrac{\sec 60 - \tan 45}{\operatorname{cosec} 30 + \cot 30}$

$$B = \frac{2 - 1}{2 + \sqrt{3}} = 2 - \sqrt{3}$$

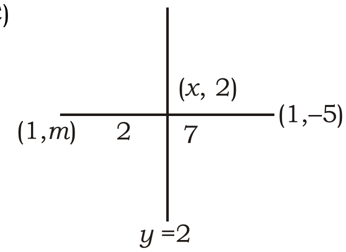
Hence $A = B$

38\. (A) $\lim_{x \to 0} \dfrac{\ln(1 + x) + e^x - 1}{x}$ $\left[\dfrac{0}{0}\right]$ Form
by L-Hospital's Rule

$$\Rightarrow \lim_{x \to 0} \frac{\frac{1}{1 + x} + e^x}{1}$$

$\Rightarrow 1 + e^0 = 2$

39\. (C)



$$\frac{2 \times (-5) + 7 \times m}{2 + 7} = 2$$

$-10 + 7m = 18 \Rightarrow m = 4$

40\. (C) $\sin^{-1} x = 2\cos^{-1} x$

for option (C) $-x = \dfrac{\sqrt{3}}{2}$

L.H.S = $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

R.H.S = $2\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = 2 \times \dfrac{\pi}{6} = \dfrac{\pi}{3}$

L.H.S = R.H.S
Hence option (C) is correct.

41\. (A) $A = \{1, 3, 5, 6, 7, 8, 9\}$
$n = 7$
Number of proper subsets $P(A) = 2^n - 1$
$= 2^7 - 1$
$= 128 - 1$
$= 127$

42\. (D) $x = 1 + \left(\dfrac{y}{3}\right) + \left(\dfrac{y}{3}\right)^2 + \left(\dfrac{y}{3}\right)^3 + \ldots$
where $|y| < 3$

$$\Rightarrow x = \frac{1}{1 - \frac{y}{3}}$$

$$\Rightarrow x = \frac{3}{3 - y}$$

$\Rightarrow 3x - xy = 3$

$\Rightarrow xy = 3x - 3 \Rightarrow y = \dfrac{3x - 3}{x}$

43. (B) Equation $x + y + 2z = 3$, $x - y + 4z = 4$, $2x + y - kz = 3$ have a unique solution, then

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 4 \\ 2 & 1 & -k \end{vmatrix} \neq 0$$

$1(k - 4) - 1(-k - 8) + 2(1 + 2) \neq 0$

$k \neq -5$

44. (C) Given that $f(x) = \dfrac{1}{\sqrt{32 - x^2}}$

$$f'(x) = \frac{(-2x)}{2(32 - x^2)^{\frac{3}{2}}} = \frac{x}{(32 - x^2)^{\frac{3}{2}}}$$

then $\lim\limits_{x \to 4} \dfrac{f(4) - f(x)}{x^2 - 16}$ $\left[\dfrac{0}{0}\right]$ Form

by L-Hospitals' Rule

$$\Rightarrow \lim_{x \to 4} \frac{-f'(x)}{2x}$$

$$\Rightarrow \lim_{x \to 4} \frac{-\left(\dfrac{x}{(32 - x^2)^{\frac{3}{2}}}\right)}{2x}$$

$$\Rightarrow -\frac{1}{2(32 - 16)^{\frac{3}{2}}} = -\frac{1}{128}$$

45. (B) Mean $= \dfrac{2 + 3 + 6 + 7 + 10 + 12 + 15 + 20}{8}$

$$= \frac{75}{8} = 9.375$$

46. (A) $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sec x}{\operatorname{cosec} x + \sec x}\, dx$ ...(i)

$$I = \int_0^{\frac{\pi}{2}} \frac{\sec\left(\frac{\pi}{2} - x\right)}{\operatorname{cosec}\left(\frac{\pi}{2} - x\right) + \sec\left(\frac{\pi}{2} - x\right)}\, dx$$

$I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\operatorname{cosec} x}{\sec x + \operatorname{cosec} x}\, dx$ ...(ii)

from eq. (i) and eq. (ii)

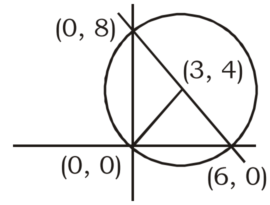
$$I + I = \int_0^{\frac{\pi}{2}} \frac{\sec x + \operatorname{cosec} x}{\sec x + \operatorname{cosec} x}\, dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1.dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

47. (B) Centre of circle $\Rightarrow$ (3, 4)



and radius $(r) = \sqrt{(3)^2 + (4)^2} = 5$

equation of circle

$(x - 3)^2 + (y - 4)^2 = (5)^2$

$x^2 + y^2 = 6x + 8y$

48. (D) $I = \displaystyle\int_0^{2\pi} |\sin x|\, dx$

$$I = 2\int_0^{\pi} \sin x\, dx$$

$$I = 2[-\cos x]_0^{\pi}$$

$$I = 2[-\cos \pi + \cos 0]$$

$$I = 2[+1 + 1] = 4$$

49. (B) Let $y = \sin^{-1} x$ and $z = 2\sqrt{1 - x^2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}, \quad \frac{dz}{dx} = 2 \times \frac{1}{2}\frac{(-2x)}{\sqrt{1 - x^2}}$$

$$= \frac{-2x}{\sqrt{1 - x^2}}$$

then $\dfrac{dy}{dz} = \dfrac{dy}{dx} \times \dfrac{dx}{dz}$

$$\frac{dy}{dz} = \frac{1}{\sqrt{1 - x^2}} \times \left(\frac{\sqrt{1 - x^2}}{-2x}\right) = \frac{-1}{2x}$$

50. (A) In the expansion of $\left(3\sqrt{x} - \dfrac{1}{6x}\right)^8$

$$T_{r+1} = {}^8C_r\, (3\sqrt{x})^{8-r} \left(\frac{-1}{6x}\right)^r$$

$$T_{r+1} = {}^8C_r\, 3^{8-2r}\, (-1)^r \left(\frac{1}{2}\right)^r x^{\frac{8-3r}{2}}$$

then $\dfrac{8 - 3r}{2} = -2 \Rightarrow r = 4$

coefficient of $x^{-2} = {}^8C_4\, 3^0\, (-1)^4 \left(\dfrac{1}{2}\right)^4 = \dfrac{35}{8}$

51. (B) Let $z = \frac{(1+3i)(3+i)}{1+i}$

$z = \frac{10i}{1+i} \times \frac{1-i}{1-i}$

$z = \frac{10i+10}{2} \Rightarrow z = 5 + 5i$

$\arg(z) = \tan^{-1}\left(\frac{5}{5}\right)$

$\arg(z) = \tan^{-1}(1) = \frac{\pi}{4}$

52. (C) $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

53. (A) $x = t - \frac{1}{t}$ and $y = t^2 + \frac{1}{t^2}$

$\frac{dx}{dt} = 1 + \frac{1}{t^2}, \quad \frac{dy}{dt} = 2t - \frac{2}{t^3}$

$\frac{dx}{dt} = \frac{t^2+1}{t^2} \quad \frac{dy}{dt} = 2\,\frac{t^4-1}{t^3}$

then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{2(t^4-1)}{t^3} \times \frac{t^2}{t^2+1}$

$\frac{dy}{dx} = \frac{2(t^2-1)}{t}$

$\frac{dy}{dx} = 2\left(t - \frac{1}{t}\right) = 2x$

54. (C) Given that

$x + y = 25$ ...(i)

A.T.Q.

$A = x^3y^2$

$\Rightarrow A = x^3(25 - x)^2$

$\Rightarrow A = 625x^3 + x^5 - 50x^4$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dA}{dx} = 1875x^2 + 5x^4 - 200x^3$ ...(i)

again, differentiating

$\Rightarrow \frac{d^2A}{dx^2} = 3750x + 20x^3 - 600x^2$ ...(ii)

for maxima and minima

$\frac{dA}{dx} = 0$

$\Rightarrow 1875x^2 + 5x^4 - 200x^3 = 0$

$\Rightarrow 5x^2(x^2 - 40x + 375) = 0$

$\Rightarrow x^2(x - 25)(x - 15) = 0$

$\Rightarrow x = 0, 15, 25$

from eq. (ii)

$\left(\frac{d^2A}{dx^2}\right)_{dtx=15} = 3750\times15+20\times15^3-600\times(15)^2$

$= -11250$ (maxima)

$\left(\frac{d^2A}{dx^2}\right)_{dtx=25} = 3750 \times 25 + 20(25)^3-600\times(25)^2$

$= 31250$ (minima)

for maximum value, $x = 15$ and $y = 10$

55. (A) $\sin 660^\circ = \sin(2 \times 360 - 60)$

$= -\sin 60 = \frac{-\sqrt{3}}{2}$

56. (B) $\begin{vmatrix} 1 & \omega & \omega^5 \\ \omega^2 & \omega & \omega^6 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & \omega & 1 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$\Rightarrow C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega & 1 \\ 1+\omega+\omega^2 & \omega^2 & \omega \end{vmatrix}$

$= \begin{vmatrix} 0 & \omega & \omega^5 \\ 0 & \omega & 1 \\ 0 & \omega^2 & \omega \end{vmatrix} = 0 \; [\because \omega^2 + \omega + 1 = 0]$

57. (D)

```
   1101
  +110
 101011
```

58. (C) $\log_3[\log_3(\sqrt{3\sqrt{3}})]$

$\Rightarrow \log_3[\log_3 3^{\frac{3}{4}}]$

$\Rightarrow \log_3\left[\frac{3}{4}\log_3 3\right]$

$\Rightarrow \log_3\left(\frac{3}{4}\right)$

$\Rightarrow \log_3 3 - \log_3 4 = 1 - 2\log_3 2$

59. (A) $\sin^{-1}\frac{20}{29} + \sin^{-1}\frac{x}{29} = \frac{\pi}{2}$

$\sin^{-1}\frac{x}{29} = \frac{\pi}{2} - \sin^{-1}\frac{20}{29}$

$\sin^{-1}\frac{x}{29} = \cos^{-1}\frac{20}{29}$

$\sin^{-1}\frac{x}{29} = \sin^{-1}\frac{21}{29} \Rightarrow x = 21$

60. (B) Equation of line which makes equal intercept on coordinate axes

$x + y = c$ ...(i)

Its passes through the point (– 1, 6)

$-1 + 6 = c \Rightarrow c = 5$

from eq. (i)

$x + y = 5$

61. (A) $I = \int \frac{1+\ln x}{\sin^2(x\ln x)}dx$

Let $x\, lnx = t$

$(1 + lnx)dx = dt$

$I = \int \frac{dt}{\sin^2 t}$

$I = \int \text{cosec}^2\, t\, dt$

$I = -\cot t + c$

$I = -\cot(xlnx) + c$

62. (C) $\Rightarrow \int_0^1 \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx = \frac{1}{2}\left[e^{\pi}-1\right]$

Let $m\sin^{-1}x = t$ when $x \to 0,\ t \to 0$

$\frac{m}{\sqrt{1-x^2}}dx = dt$ $x \to 1,\ t \to m\frac{\pi}{2}$

$\frac{1}{\sqrt{1-x^2}}dx = \frac{1}{m}dt$

$\Rightarrow \frac{1}{m}\int_0^{m\frac{\pi}{2}} e^t\, dt = \frac{1}{2}[e^{\pi}-1]$

$\Rightarrow \frac{1}{m}\left[e^t\right]_0^{m\frac{\pi}{2}} = \frac{1}{2}[e^{\pi}-1]$

$\Rightarrow \frac{1}{m}\left[e^{\frac{m\pi}{2}}-1\right] = \frac{1}{2}[e^{\pi}-1]$

On comparing

$\Rightarrow m = 2$

63. (D) Let $y = \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$

$\Rightarrow y = \tan^{-1}\sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$

$\Rightarrow y = \tan^{-1}(\tan x)$

$\Rightarrow y = x$

On differentiating both side w.r.t 'x'

$\Rightarrow \frac{dy}{dx} = 1$

64. (B) $y = e^x$

$\Rightarrow \frac{dy}{dx} = e^x$

Slope = $\left(\frac{dy}{dx}\right)_{at(0,1)} = e^0 = 1$

Equation of tangent

$y - 1 = 1(x - 0)$

$x - y + 1 = 0$

65. (C) Equation of tangent

$x - y + 1 = 0$

Its meets the x-axis i.e $y = 0$

$x - 0 + 1 \Rightarrow 0 = x = -1$

Co-ordinate (– 1, 0)

66. (D) $y = e^x$

$\Rightarrow \frac{dy}{dx} = e^x$

$m_1 = \left(\frac{dy}{dx}\right)_{at(1,e)} = e^1 = e$

Slope of normal $m_2 = \frac{-1}{m_1} \Rightarrow m_2 = \frac{-1}{e}$

67. (A) When $\theta = 180$

$M = \frac{60}{11}(H \pm 6)$ $+ \to H < 6$

$- \to H < 6$

H = 3(between 3 and 4 O'clock)

$M = \frac{60}{11}(3 + 6)$

$M = \frac{540}{11} = 49\frac{1}{11}$

time = $3 : 49\frac{1}{11}$

68. (A)

69. (B) $f(x) = \sqrt{x^2 + \sqrt{x^2 + \sqrt{x^2 + \ldots\infty}}}$

$\Rightarrow f(x) = \sqrt{x^2 + f(x)}$

$\Rightarrow [f(x)]^2 = x^2 + f(x)$

On differentiating both side w.r.t. 'x'

$\Rightarrow 2f(x).f'(x) = 2x + f'(x)$

$\Rightarrow f'(x)[2f(x) - 1] = 2x$

$\Rightarrow f'(x) = \dfrac{2x}{2f(x) - 1}$

70. (C) $\lim_{x\to 0}\left(\dfrac{x+7}{7}\right)^{\frac{1}{x}}$ $[1^\infty]$ Form

$\Rightarrow \lim_{x\to 0}\left\{\left(1+\dfrac{x}{7}\right)^{\frac{7}{x}}\right\}^{\times\frac{1}{7}}$

$\Rightarrow e^{\frac{1}{7}\lim_{x\to 0}\frac{1}{7}}$

$\Rightarrow e^{\frac{1}{7}\times\frac{1}{7}} = e^{\frac{1}{49}}$

71. (D) $\dfrac{dy}{dx} = 2xy - 2x + y - 1$

$\Rightarrow \dfrac{dy}{dx} = (y-1)(2x+1)$

$\Rightarrow \dfrac{dy}{y-1} = (2x+1)dx$

On differentiating

$\Rightarrow \log(y-1) = x^2 + x + \log c$

$\Rightarrow \log\left(\dfrac{y-1}{c}\right) = x^2 + x$

$\Rightarrow y - 1 = c.e^{x(x+1)}$

72. (D) $y = 3^{\frac{1}{\log_x 9}}$

$\Rightarrow y = 3^{\log_9 x}$

$\Rightarrow y = 3^{\log_3 \sqrt{x}}$

$\Rightarrow y = \sqrt{x} \Rightarrow x = y^2$

73. (C) $\left(\sin 7\dfrac{1}{2} + \cos 7\dfrac{1}{2}\right)^4$

$\Rightarrow \left[\left(\sin 7\dfrac{1}{2} + \cos 7\dfrac{1}{2}\right)^2\right]^2$

$\Rightarrow \left(1 + 2\sin 7\dfrac{1}{2}.\cos 7\dfrac{1}{2}\right)^2$

$\Rightarrow (1 + \sin 15)^2$

$\Rightarrow 1 + \sin^2 15 + 2\sin 15$

$\Rightarrow 1 + \dfrac{2-\sqrt{3}}{4} + 2\times\dfrac{\sqrt{3}-1}{2\sqrt{2}} = \dfrac{6+2\sqrt{6}-2\sqrt{2}-\sqrt{3}}{4}$

74. (B) $\lim_{x\to 0}\dfrac{3^x - 1}{\sqrt{3+x}-\sqrt{3}}$ $\left[\dfrac{0}{0}\right]$ Form

by L - Hospital's Rule

$\Rightarrow \lim_{x\to 0}\dfrac{3^x - 1}{\sqrt{3+x}-\sqrt{3}}$

$\Rightarrow \lim_{x\to 0}\dfrac{3^x \log 3 - 0}{\dfrac{1}{2\sqrt{3+x}} - 0}$

$\Rightarrow \lim_{x\to 0} 2\log 3\sqrt{3+x}.3^x$

$\Rightarrow 2(\log 3)\sqrt{3}.1 = 2\sqrt{3}\log 3$

75. (A) 5 points out of 11 are in the straight line, then

No. of triangle = ${}^{11}C_3 - {}^{5}C_3$

= 165 – 10 = 155

76. (B) 2 radian = $\left(2\times\dfrac{180}{\pi}\right)^\circ$

$= \left(\dfrac{2\times 180\times 7}{22}\right)^\circ = 114°32'44''$

77. (C) We know that

$y = a\sin\theta + b\cos\theta$

$-\sqrt{a^2+b^2} \le y \le \sqrt{a^2+b^2}$

then maximum value of $(24\sin\theta + 7\cos\theta)$

$= \sqrt{(24)^2 + (7)^2} = 25$

78. (A) $I = \int_0^2 |1-x|\,dx$

$I = \int_0^1 (1-x)\,dx + \int_1^2 |1-x|\,dx$

$I = \left[x - \dfrac{x^2}{2}\right]_0^1 + \left|\left[x - \dfrac{x^2}{2}\right]_1^2\right|$

$I = \left|1 - \dfrac{1}{2}\right| + \left|(2-2) - \left(1 - \dfrac{1}{2}\right)\right|$

$I = \dfrac{1}{2} + \left|0 - \dfrac{1}{2}\right|$

$I = \dfrac{1}{2} + \dfrac{1}{2} = 1$

79. (C) We know that

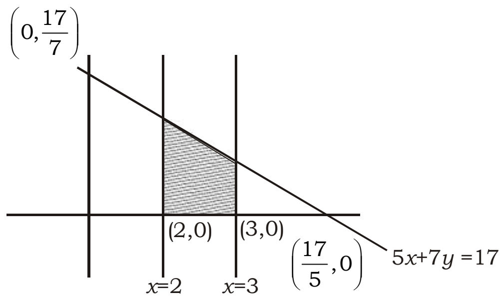
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \dfrac{1+\cos 2\alpha}{2} + \dfrac{1+\cos 2\beta}{2} + \dfrac{1+\cos 2\gamma}{2} = 1$

$\Rightarrow 3 + \cos 2\alpha + \cos 2\beta + \cos 2\beta + \cos 2\gamma = 2$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

80. (C)



line $y_1 \Rightarrow \frac{17-5x}{7}$

Area = $\int_2^3 y_1 dx$

$= \int_2^3 \frac{17-5x}{7} dx$

$= \frac{1}{7}\left[17x - \frac{5x^2}{2}\right]_2^3$

$= \frac{1}{7}$

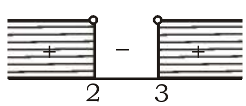
$\left[\left(17\times 3 - \frac{5}{2}\times(3)^2\right) - \left(17\times 2 - \frac{5}{2}\times 2^2\right)\right]$

$= \frac{1}{7}\left[51 - \frac{45}{2} - 34 + 10\right]$

$= \frac{1}{7}\left[\frac{9}{2}\right) = \frac{9}{14}$

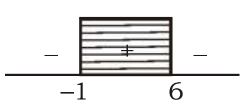
81. (C) $x^2 - 5x + 6 > 0$

$(x-2)(x-3) > 0$

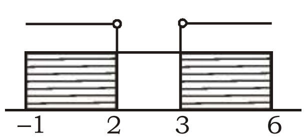


and $x^2 - 5x - 6 \leq 0$

$(x-6)(x+1) \leq 0$



then



$x \in [-1, 2) \cup (3, 6]$

82. (D) six-digit no. formed from 0, 1, 3, 5, 7, 9

5	5	4	3	2	1

= 5×5×4×3×2×1=600

'0'can't put here

83.(C) (a, c), (b, d) and $(a+b, c+d)$ are collinear, then

$$\begin{vmatrix} a & c & 1 \\ b & d & 1 \\ a+b & c+d & 1 \end{vmatrix} = 0$$

$\Rightarrow a(d-c-d) - c(b-a-b) + 1(bc+bd-ad-bd) = 0$

$\Rightarrow \quad -ac + ca + bc - ad = 0$

$\Rightarrow \quad bc = ad$

84. (D) $\cos\frac{\pi}{24} > \tan\frac{\pi}{24} > \sin\frac{\pi}{24}$

85. (A) **In A.P.**

$T_{n+1} = a + nd$

$T_n = a + (n-1)d$

difference $= T_{n+1} - T_n$

$= (a + nd) - [a + (n-1)d]$

$= d$ = independent of n

86. (B) $\frac{P(9, n+2)}{P(8, n+2)} = \frac{3}{2}$

$\Rightarrow \frac{\frac{9!}{(7-n)!}}{\frac{8!}{(6-n)!}} = \frac{3}{2}$

$\Rightarrow \frac{9!\times(6-n)!}{8!\times(7-n)!} = \frac{3}{2}$

$\Rightarrow \frac{9\times 8!\times(6-n)!}{8!\times(7-n)(6-n)!} = \frac{3}{2}$

$\Rightarrow \frac{9}{7-n} = \frac{3}{2}$

$\Rightarrow 18 = 21 - 3n \Rightarrow n = 1$

87. (D) $\begin{vmatrix} x+2 & x+3 & x \\ x+4 & x+5 & x+2 \\ x+7 & x+8 & x+5 \end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow \begin{vmatrix} x+2 & 1 & -2 \\ x+4 & 1 & -2 \\ x+7 & 1 & -2 \end{vmatrix}$

$\Rightarrow -2\begin{vmatrix} x+2 & 1 & 1 \\ x+4 & 1 & 1 \\ x+7 & 1 & 1 \end{vmatrix}$

$= 0$ [$\because$ two columns are identical.]

88. (C) Vectors $(\hat{i} - 2\hat{j} + 3\hat{k})$ and $(2\hat{i} - 3\hat{j} + \hat{k})$

angle between vectors

$$\cos\theta = \frac{1\times 2 - 2\times(-3) + 3\times 1}{\sqrt{14}\sqrt{14}}$$

$$\cos\theta = \frac{11}{44}$$

then $\sin\theta = \frac{5\sqrt{3}}{14}$

89. (A) In ΔABC,

A.T.Q.,

Let $\angle A = B - x$, $\angle B = B$, $\angle C = B + x$

$B - x + B + B + x = 180 \Rightarrow B = 60°$

then

$$\cos B = \cos 60 = \frac{1}{2}$$

90. (A) We know that

$$\int_0^{\pi} f(x)\,dx = \begin{cases} 2\int_0^{\frac{\pi}{2}} f(x)\,dx, & \text{if } f(x) \text{ is even} \\ 0 \quad , & \text{if } f(x) \text{ is an odd} \end{cases}$$

Hence $\int_0^{\pi} f(\tan x)\,dx = 0$ because $f(x)$ is odd.

91. (C) $(3 - 2\sqrt{2})x^2 - (7 + 5\sqrt{2})x + (3 + 2\sqrt{2}) = 0$

$$\text{Sum of roots} = \frac{-\left[-\left(7+5\sqrt{2}\right)\right]}{3-2\sqrt{2}}$$

$$= \frac{7+5\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$$

$$= \frac{21+15\sqrt{2}+14\sqrt{2}+20}{9-8}$$

$$= 41 + 29\sqrt{2}$$

92. (B) In the expansion of $\left(1-\frac{x}{2}\right)^{10}$

total term = 10 + 1 = 11

middle term = $\left(\frac{10}{2}+1\right)^{\text{th}} = 6^{\text{th}}$

$$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{-x}{2}\right)^5$$

$$= -\frac{10!}{5!5!} \times \frac{x^5}{32} = -\frac{63}{8}x^5$$

93. (C) $x^2 + y^2 = 5$ ...(i)

and $x^2 + y^2 - 6x - 11 = 0$ ...(ii)

from eq. (i) and eq. (ii)

$x = -1$ and $y = \pm 2$

two intersecting points (–1, 2) and (–1, –2).

general equation of circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(iii)

it passes through the point (– 1, 2)

$\Rightarrow 1 + 4 - 2g + 4f + c = 0$

$\Rightarrow 2g - 4g - c = 5$ ...(iv)

eq. (i) passes through the point (– 1, – 2)

$\Rightarrow 1 + 4 - 2g - 4f + c = 0$

$\Rightarrow 2g + 4f - c = 5$ ...(v)

eq. (i) passes through the point (3, – 2)

$\Rightarrow 9 + 4 + 6g - 4f + c = 0$

$\Rightarrow 6g - 4f + c = -13$ ...(vi)

On Solving eq. (iv), (v) and (vi)

$\Rightarrow g = -1, f = 0, c = -7$

from eq. (iii)

$\Rightarrow x^2 + y^2 - 2x = 7$

94. (A) $\sin x \frac{dy}{dx} + y\cos x = e^x$

$\Rightarrow \frac{dy}{dx} + y\cot x = e^x.\text{cosec}x$

On comparing with general equation

$P = \cot x$, $Q = e^x.\text{cosec}x$

I.F. $= e^{\int P dx}$

$= e^{\int \cot x\, dx}$

$= e^{\log \sin x} = \sin x$

Solution of differential equation

$y \times I.F. = \int Q \times \text{I.F.}\,dx$

$\Rightarrow y \times \sin x = \int e^x .\text{cosec}x.\sin x\, dx$

$\Rightarrow y\sin x = \int e^x\, dx$

$\Rightarrow y\sin x = e^x + c$

95. (B) Let $y = \sin^{-1}\sqrt{1-x^2}$ and $z = \sqrt{1-x^2}$

$\Rightarrow y = \sin^{-1} z$

On differentiating w.r.t. 'z'

$$\Rightarrow \frac{dy}{dz} = \frac{1}{\sqrt{1-z^2}}$$

$$\Rightarrow \frac{dy}{dz} = \frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}$$

$$\Rightarrow \frac{dy}{dz} = \frac{1}{\sqrt{1-1+x^2}}$$

$$\Rightarrow \frac{dy}{dz} = \frac{1}{\sqrt{x^2}} = \frac{1}{x}$$

96. (C) $S = 3^2 + 6^2 + 9^2 + \ldots + 45^2$

$S = 3^2(1^2 + 2^2 + 3^2 + \ldots + 15^2)$

$S = 3^2 \times \frac{15}{6}(15 + 1)(2 \times 15 + 1)$

$S = 9 \times \frac{5}{2} \times 16 \times 31 = 11160$

97. (B) $I = \int_0^{\frac{\pi}{4}} \frac{\tan 2x}{\tan 2x + \cot 2x}$ ...(i)

$$I = \int_0^{\frac{\pi}{4}} \frac{\tan 2\left(\frac{\pi}{4} - x\right)}{\tan 2\left(\frac{\pi}{4} - x\right) + \cot 2\left(\frac{\pi}{4} - x\right)} dx$$

$$I = \int_0^{\frac{\pi}{4}} \frac{\tan\left(\frac{\pi}{2} - 2x\right)}{\tan\left(\frac{\pi}{2} - 2x\right) + \cot\left(\frac{\pi}{2} - 2x\right)} dx$$

$I = \int_0^{\frac{\pi}{4}} \frac{\cot 2x}{\tan 2x + \cot 2x}$ ...(ii)

from eq. (i) and eq (ii)

$$I + I = \int_0^{\frac{\pi}{4}} \frac{\tan 2x + \cot 2x}{\tan 2x + \cot 2x} dx$$

$$2I = \int_0^{\frac{\pi}{4}} 1.dx$$

$$2I = [x]_0^{\frac{\pi}{4}}$$

$$2I = \frac{\pi}{4} \Rightarrow I = \frac{\pi}{8}$$

98. (C) $\lim_{x\to 0} \frac{\log_7(1-x)}{x}$ $\left[\frac{0}{0}\right]$ Form

by L-Hospital's Rule

$$\Rightarrow \lim_{x\to 0} \frac{\frac{-1}{1-x}\log_7 e}{1} = -\log_7 e$$

99. (A) **Statement I**

$$f(x) = \begin{cases} x^2 - 3, & x \le 4 \\ 2x + 5, & x > 4 \end{cases}$$

L.H.L. $= \lim_{x\to 4^-} f(x) = \lim_{x\to 0} f(4 - h)$

$= \lim_{h\to 0} (4 - h)^2 - 3$

$= 16 - 3 = 13$

R.H.L. $= \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4 + h)$

$= \lim_{h\to 0} 2(4 + h) + 5$

$= 2(4) + 5 = 13$

L.H.L. = R.H.L.

$f(x)$ is continuous at $x = 4$

Statement I is correct.

Statement II

$L'f(4) = \text{L.H.D.} = \lim_{h\to 0} \frac{f(4-h) - f(4)}{-h}$

$= \lim_{h\to 0} \frac{(4-h)^2 - 3 - 13}{-h}$

$= \lim_{h\to 0} \frac{16 + h^2 - 8h - 16}{-h}$

$= \lim_{h\to 0} -h + 8 = 8$

$R'f(4) = \text{R.H.D.} = \lim_{h\to 0} \frac{f(4+h) - f(4)}{h}$

$= \lim_{h\to 0} \frac{2(4+h) + 5 - 13}{h}$

$= \lim_{h\to 0} \frac{2h}{h} = 2$

L.H.D. $\neq$ R.H.D.

Hence $f(x)$ is not differentiable at $x = 4$.

Statement II is incorrect.

100. (C) $f(x) = x^2 - 3,\ x < 4$

On differentiating both side w.r.t. 'x'

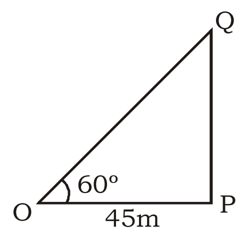
$\Rightarrow f'(x) = 2x$

$\Rightarrow f'(-2) = 2 \times (-2) = -4$

101. (A) **from sol-99**

$\lim_{x\to 4} f(x) = 13$

102. (B)



In ΔPOQ

$\tan 60° = \frac{PQ}{OP}$

$\sqrt{3} = \frac{PQ}{45} \Rightarrow PQ = 45\sqrt{3}$

Hence height of the tower = $45\sqrt{3}$ m.

103. (A) Given that $v = 2s^2 - 3s + 7$

$\frac{dv}{ds} = 4s - 3$

acceleration $\left(\frac{dv}{ds}\right)_{at\, s=7} = 4 \times 7 - 3 = 25$

104. (B) $I = \int \cos^4 x \,.\sin x\, dx$

Let $\cos x = t$

$-\sin x\, dx = dt \Rightarrow \sin x\, dx = -dt$

$\Rightarrow I = -\int t^4 dt$

$\Rightarrow I = -\frac{t^5}{5} + c$

$\Rightarrow I = -\frac{\cos^5 x}{5} + c$

105. (C)

2	45	
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0
	0	1

0.125
×2
0.250
×2
0.500
×2
1.000

$(45)_{10} = (101101)_2$, $(0.125)_{10} = (0.001)_2$

Hence $(45.125)_{10} = (101101.001)_2$

106. (A) Let $\vec{a} = 3\hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

$= \frac{3\times 2 + 2(-1) - 4\times(-1)}{\sqrt{(2)^2 + (-1)^2 + (-1)^2}}$

$= \frac{8}{\sqrt{6}}$

107. (B) data 2, 3, 4, 8, 9, 16, 27, 32, 81, 243

G.M. = $(2.3.4.8.9.16.27.32.81.243)^{\frac{1}{10}}$

$= (2^{15} \times 3^{15})^{\frac{1}{10}}$

$= 6^{15\times\frac{1}{10}}$

$= 6^{\frac{3}{2}} = 6\sqrt{6}$

108. (D) Plane $2x - 5y + 6z = 16$

Point (4, 2, 3) satisfy the plane.

hence point (4, 2, 3) lies on the plane.

109. (C) $2x - 5y = -1$ ...(i)

$5x - y = 9$ ...(ii)

Intersecting point of both equations is (2, 1).

Equation of line which passes through the points (2, 1) and (0, 0)

$y - 0 = \frac{1-0}{2-0}(x - 0) \Rightarrow 2y = x$

110. (A) $f(x) = 3^x e^x$

On differentiating both side w.r.t. 'x'

$\Rightarrow f'(x) = 3^x.e^x + e^x.3^x \log 3$

again, differentiating

$\Rightarrow f''(x) = 3^x.e^x + e^x.3^x \log 3$
$+\log 3[e^x.3^x \log 3 + 3^x e^x]$

$\Rightarrow f''(x) = 3^x.e^x + (\log 3)e^x.3^x + (\log 3)^2 e^x.3^x$
$+(\log 3)3^x.e^x$

$\Rightarrow f''(x) = (\log 3)^2 e^x.3^x + 2(\log 3)e^x.3^x + 3^x.e^x$

$\Rightarrow f''(x) = e^x.3^x[\log 3 + 1]^2$

111. (C) $\lim_{x\to 0} \frac{2x}{\sqrt{4x^2+3x+1} - \sqrt{4x^2+1}}$ $\left[\frac{0}{0}\right]$ Form

$\Rightarrow \lim_{x\to 0} \frac{2x\left(\sqrt{4x^2+3x+1} + \sqrt{4x^2+1}\right)}{4x^2+3x+1-4x^2-1}$

$\Rightarrow \lim_{x\to 0} \frac{2x\left(\sqrt{4x^2+3x+1} + \sqrt{4x^2+1}\right)}{3x}$

$\Rightarrow \frac{2}{3}(\sqrt{1} + \sqrt{1}) = \frac{4}{3}$

112. (B) tan 1740 = tan (360 × 4 + 300)

= tan 300

= tan (360 – 60)

= – tan 60 = $-\sqrt{3}$

113. (A) Conic $3x^2 + 6y^2 = 24$

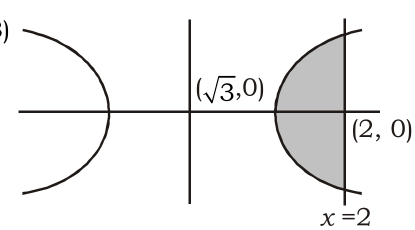
$\frac{x^2}{8} + \frac{y^2}{4} = 1$

$a^2 = 8$, $b^2 = 4$

eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{4}{8}} = \frac{1}{\sqrt{2}}$

114. (B)



Curve

$y_1 \Rightarrow \sqrt{4x^2 - 12}$

and $x = 2$

$$\text{Area} = 2\int_{\sqrt{3}}^{2} y_1 dx$$

$$= 2\int_{\sqrt{3}}^{2} \sqrt{4x^2 - 12}\, dx$$

$$= 2 \times 2\int_{\sqrt{3}}^{2} \sqrt{x^2 - \left(\sqrt{3}\right)^2}\, dx$$

$$= 4\left[\frac{1}{2}x\sqrt{x^2-3} - \frac{\left(\sqrt{3}\right)^2}{2}\log\left|x + \sqrt{x^2-3}\right|\right]_{\sqrt{3}}^{2}$$

$$= 4\left[1 - \frac{3}{2}\log 3 + \frac{3}{2}\log 3^{\frac{1}{2}}\right]$$

$$= 4\left[1 - \frac{3}{2}\log 3 + \frac{3}{4}\log 3\right]$$

$$= 4\left[1 - \frac{3}{4}\log 3\right]$$

= (4 – 3log3) sq. unit.

115. (D) $\left(\frac{dy}{dx}\right)^{\frac{1}{4}} + 3y = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$

$$\left(\frac{dy}{dx}\right)^{\frac{1}{4}} = \left[\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} - 3y\right]$$

$$\frac{dy}{dx} = \left[\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} - 3y\right]^4$$

order = 2 and degree = 6

116. (D) tan100.tan140.tan160

⇒ tan(90 + 10).tan(90 +50).tan(90 + 70)

⇒ (– cot10).(– cot50).(– cot70)

⇒ – cot10.cot50.cot70

⇒ –cot(3 ×10)

[∵ cotθ.cot(60 – θ).cot(60 + θ) = cot3θ]

⇒ $-\sqrt{3}$

117. (C) $\frac{\sqrt{\log_e\left(3x+5-x^2\right)}}{2x-3}$

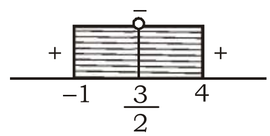
$\log_e(3x + 5 - x^2) \geq 0$, $2x - 3 \neq 0$

$3x + 5 - x^2 \geq 1$, $x \neq \frac{3}{2}$

$3x + 4 - x^2 \geq 0$

$x^2 - 3x - 4 \leq 0$

$(x - 4)(x + 1) \leq 0$



domain $x \in [-1, 4] - \left\{\frac{3}{2}\right\}$

118. (B) $\tan^{-1}\left(\frac{1}{3}\right) + 2\tan^{-1}\left(\frac{1}{7}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\frac{2\times\frac{1}{7}}{1-\frac{1}{49}}$$

$$\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{7}{24}$$

$$\Rightarrow \tan^{-1}\frac{\frac{1}{3}+\frac{7}{24}}{1-\frac{1}{3}\times\frac{7}{24}}$$

$$\Rightarrow \tan^{-1}\frac{45}{65} = \tan^{-1}\frac{9}{13}$$

119. (C) 27

120. (D) $(-\sqrt{-1})^{8n+4} + (-\sqrt{-1})^{4n+9}$

⇒ $(-i)^{8n+4} + (-i)^{4n+9}$

⇒ $(-i)^{8n}.(-i)^4 + (-i)^{4n}.(-i)^9$

⇒ $1 \times 1 + 1.(-i)$

⇒ $1 - i$

NDA (MATHS) MOCK TEST - 98 (Answer Key)

1.	(B)	21.	(D)	41.	(A)	61.	(A)	81.	(C)	101.	(A)
2.	(A)	22.	(C)	42.	(D)	62.	(C)	82.	(D)	102.	(B)
3.	(B)	23.	(A)	43.	(B)	63.	(D)	83.	(C)	103.	(A)
4.	(A)	24.	(C)	44.	(C)	64.	(B)	84.	(D)	104.	(B)
5.	(B)	25.	(A)	45.	(B)	65.	(C)	85.	(A)	105.	(C)
6.	(D)	26.	(A)	46.	(A)	66.	(D)	86.	(B)	106.	(A)
7.	(B)	27.	(B)	47.	(B)	67.	(A)	87.	(D)	107.	(B)
8.	(C)	28.	(C)	48.	(D)	68.	(A)	88.	(C)	108.	(D)
9.	(D)	29.	(B)	49.	(B)	69.	(B)	89.	(A)	109.	(C)
10.	(B)	30.	(A)	50.	(A)	70.	(C)	90.	(A)	110.	(A)
11.	(A)	31.	(C)	51.	(B)	71.	(D)	91.	(C)	111.	(C)
12.	(B)	32.	(B)	52.	(C)	72.	(D)	92.	(B)	112.	(B)
13.	(D)	33.	(C)	53.	(A)	73.	(C)	93.	(C)	113.	(A)
14.	(B)	34.	(A)	54.	(C)	74.	(B)	94.	(A)	114.	(B)
15.	(A)	35.	(D)	55.	(A)	75.	(A)	95.	(B)	115.	(D)
16.	(B)	36.	(C)	56.	(B)	76.	(B)	96.	(C)	116.	(D)
17.	(A)	37.	(C)	57.	(D)	77.	(C)	97.	(B)	117.	(C)
18.	(C)	38.	(A)	58.	(C)	78.	(A)	98.	(C)	118.	(B)
19.	(A)	39.	(C)	59.	(A)	79.	(C)	99.	(A)	119.	(C)
20.	(C)	40.	(C)	60.	(B)	80.	(C)	100.	(C)	120.	(D)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*