NDA MATHS MOCK TEST - 100 (SOLUTION)

1. (B) We have, $f(x) = \begin{cases} x^2 - 5, x \le 3 \\ \sqrt{x+13}, x > 3 \end{cases}$

To find $\lim_{x\to 3} f(x)$ -

L.H.L. = $\lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} [(3-h)^2 - 5]$

$= \lim_{h\to 0} (9-5) = 4$

R.H.L. = $\lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} (\sqrt{3+h+13})$

$= \lim_{h\to 0} (\sqrt{16+h}) = 4$

$\because \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 4$

$\therefore \lim_{x\to 3} f(x) = 4$

2. (D) 1. For continuous,

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$

$\therefore \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 4$

Hence, $f(x)$ is continuous at $x = 4$.
Statement 1 is incorrect.
2. We have, $f(x) = x^2 - 5$, $x \le 3$

$\Rightarrow f'(x) = 2x \Rightarrow f'(0) = 0$.

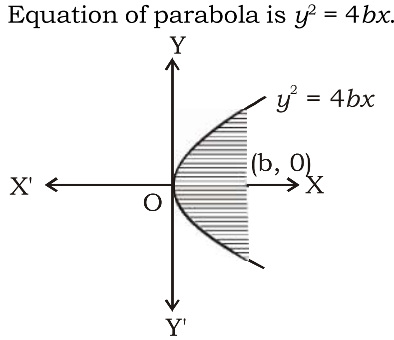
Hence, $f(x)$ is differentiable at $x = 0$.
Statement 2 is incorrect.
So, neither statement 1 nor 2 is correct.

3. (D) We have, $f(x) = \sqrt{x+13}$, $x > 3$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x+13}}$

$\therefore f'(12) = \frac{1}{2\sqrt{12+13}} = \frac{1}{2\times 5} = \frac{1}{10}$

4. (D) Equation of parabola is $y^2 = 4bx$.



Area $= 2\int_0^b \sqrt{4bx}\, dx$

$= 4\sqrt{b} \times \frac{2}{3}\left[x^{3/2}\right]_0^b$

$= \frac{8\sqrt{b}}{3}[b^{3/2} - 0] = \frac{8b^2}{3}$ sq units

5. (B) $\lim_{x\to 0} \frac{\log_5(1+x)}{x}$ $\left(\frac{0}{0}\right)$ Form

$= \lim_{x\to 0} \frac{\log_e(1+x)}{x\log_e 5}$ $\left(\because \log_a b = \frac{\log_e b}{\log_e a}\right)$

$= \frac{1}{\log_e 5} \lim_{x\to 0} \frac{\log_e(1+x)}{x}$

$= \log_5 e$

$\left(\because \lim_{x\to 0} \frac{\log_e(1+x)}{x} = 1 \text{ and } \log_a b = \frac{1}{\log_b a}\right)$

6. (A) Equation of circles are

$x^2 + y^2 + 2ax + c = 0$

and $x^2 + y^2 + 2by + c = 0$

Since, the centres of two circles are $(-a, 0)$ and $(0, -b)$

$\therefore$ Distance between two centres $= \sqrt{a^2 + b^2}$

7. (B) Two circles touch each other, if

$\Rightarrow$ Distance between two centres
= Sum of radius of two circles

$\Rightarrow \sqrt{a^2+b^2} = \sqrt{a^2-c} + \sqrt{b^2-c}$

On squaring both sides, we get

$\Rightarrow a^2 + b^2 = a^2 - c + b^2 - c + 2\sqrt{(a^2-c)(b^2-c)}$

$\Rightarrow c = \sqrt{(a^2-c)(b^2-c)}$

Again, squaring both sides, we get

$\Rightarrow c^2 = a^2b^2 - a^2c - b^2c + c^2$

$\Rightarrow a^2b^2 = (a^2 + b^2)\, c \Rightarrow \frac{1}{c} = \frac{1}{a^2} + \frac{1}{b^2}$

8. (C) We have, $A(3, 4)$ and $B(5, -2)$

Let $P(x, y)$

Given that, $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16$

$= x^2 - 10x + 25 + y^2 + 4y + 4$

$\Rightarrow 4x - 12y = 4$

$\Rightarrow x - 3y = 1$ ...(i)

$\because$ Area of $\Delta PAB = 10$

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 4 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \pm 10$$

$\Rightarrow x(4+2) - y(3-5) + 1(-6-20) = \pm 20$

$\Rightarrow 6x + 2y - 26 = \pm 20$

$\Rightarrow 6x + 2y - 26 = 20$

or $6x + 2y - 26 = -20$

$\Rightarrow \quad 6x + 2y = 46 \quad \ldots$(ii)

or $\quad 6x + 2y = 6 \quad \ldots$(iii)

On solving Eqs. (i) and (ii) , we get

$x = 7, y = 2$

Similarly, solving Eqs. (i) and (iii), we get

$x = 1, y = 0$

Hence, coordinates of P are (7, 2) or (1,0).

9. (B) We have equation of line is

$bx\cos\alpha + ay\sin\alpha = ab$

Perpendicular distance from point

$(\sqrt{a^2 - b^2}, 0)$ is

$$d_1 = \left|\frac{b\cos\alpha\sqrt{a^2-b^2} + 0 - ab}{\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}}\right|$$

Similarly, perpendicular distance from point $(-\sqrt{a^2 - b^2}, 0)$ is

$$d_2 = \left|\frac{-b\cos\alpha\sqrt{a^2-b^2} + 0 - ab}{\sqrt{b^2\cos^2 a + a^2\sin^2 a}}\right|$$

Now, $d_1 \times d_2$

$$= \frac{\left(b\cos\alpha\sqrt{a^2-b^2} - ab\right)\left(b\cos\alpha\sqrt{a^2-b^2} + ab\right)}{\left(\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}\right)\left(\sqrt{b^2\cos^2\alpha + a^2\sin^2\alpha}\right)}$$

$$= \frac{b^2\cos^2\alpha\left(a^2-b^2\right) - a^2b^2}{b^2\cos^2 a + a^2\sin^2 a}$$

$$= \frac{a^2b^2\cos^2\alpha - b^4\cos^2\alpha - a^2b^2}{b^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$= \frac{a^2b^2\left(\cos^2\alpha - 1\right) - b^4\cos^2\alpha}{b^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$= \frac{-b^2\left[a^2\sin^2\alpha + b^2\cos^2\alpha\right]}{b^2\cos^2\alpha + a^2\sin^2\alpha}$$

$= -b^2 = b^2$ (Since, distance is positive).

10. (D) Equation of line passing through the points (2, 1, 3) and (4, –2, 5) is

$$\frac{x-2}{4-2} = \frac{y-1}{-2-1} = \frac{z-3}{5-3} = \lambda$$

$$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-3} = \frac{z-3}{2} = \lambda$$

$\Rightarrow x = 2\lambda + 2,\ y = -3\lambda + 1$ and $z = 2\lambda + 3$

Since, this line cuts the plane $2x + y - z = 3$.

So, $(2\lambda + 2, -3\lambda + 1, 2\lambda + 3)$ satisfies the equation of plane.

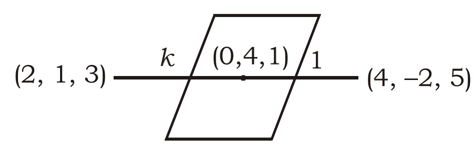
$\therefore \quad 2(2\lambda + 2) - 3\lambda + 1 - 2\lambda - 3 = 3$

$\Rightarrow \quad \lambda = -1$

Hence, points are

$[2(-1) + 2, -3(-1) + 1, 2(-1) + 3]$ *i.e.* **(0, 4,1)**.

11. (D) Let the ratio plane divides the line is $k : 1$



Then, $0 = \dfrac{4k+2}{k+1}$

$\Rightarrow 4k + 2 = 0 \Rightarrow k = -\dfrac{1}{2}$

and $4 = \dfrac{-2k+1}{k+1}$

$\Rightarrow 4k + 4 = -2k + 1 \Rightarrow k = -\dfrac{1}{2}$

Hence, plane divides the line in ratio **1 : 2** externally.

12. (A) $P(A/B^C) = \dfrac{P\left(A \cap B^C\right)}{P\left(B^C\right)} = \dfrac{P(A) - P(B)}{1 - P(B)}$

13. (D)Angle between the regression lines will be

$$\tan\theta = \left\{\left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\right)\right\}$$

$$\Rightarrow \tan\frac{\pi}{2} = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$

$$\Rightarrow \quad \frac{1}{0} = \left(\frac{1-r^2}{r}\right)\left(\frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$$

$\Rightarrow r(\sigma_x^2 + \sigma_y^2) = 0$

$\therefore \quad r = 0$

14. (A) $\therefore$ Total number of arrangements = $\frac{10!}{2!}$

Total number of arrangements when I's comes together = 9!

and favourable arrangements = $\frac{10!}{2!} - 9!$

$\therefore$ Required probability = $\dfrac{\frac{10!}{2!} - 9!}{\frac{10!}{2!}}$

$= \frac{(10-2)\times 9!}{10\times 9!} = \frac{4}{5}$

15. (D) Let A and B be the events that X and Y qualify the examination respectively, We have, $P(A) = 0.05$, $P(B) = 0.10$ and $P(A\cap B) = 0.02$,

then

P (only one of A and B will qualify the examination) = $P(A\cap \overline{B}) + P(B\cap \overline{A})$

$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$

$= P(A) + P(B) - 2P(A\cap B)$

$= 0.05 + 0.1 - 2(0.02)$

$= 0.15 - 0.04 = 0.11$

16. (B) Let S be the sample space of the experiment and E be the event that at most three tails occur.

Clearly, $n(S) = 2^4 = 16$

and $n(E) = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3$

$= 1 + 4 + 6 + 4 = 15$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{15}{16}$

17. (B) $\sin^2 66\frac{1}{2}^\circ - \sin^2 23\frac{1}{2}^\circ$

$\Rightarrow \left[\sin\left(90^\circ - 23\frac{1}{2}^\circ\right)\right]^2 - \sin^2 23\frac{1}{2}^\circ$

$\Rightarrow \cos^2 23\frac{1}{2}^\circ - \sin^2 23\frac{1}{2}^\circ$

$\Rightarrow \cos 2\left(23\frac{1}{2}^\circ\right)$

$\Rightarrow \cos\left[2\times\left(\frac{47}{2}\right)^\circ\right] = \cos 47^\circ$

(18-20) : We know that

De Moivre's Theorem

$(\cos n\theta + i\sin n\theta) = (\cos\theta + i\sin\theta)^n \quad \ldots(i)$

$\Rightarrow \cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$

$\Rightarrow \cos 5\theta + i\sin 5\theta = {}^5C_0(\cos\theta)^5(i\sin\theta)^0 + {}^5C_1(\cos\theta)^4(i\sin\theta)^1 + {}^5C_2(\cos\theta)^3(i\sin\theta)^2 + {}^5C_3(\cos\theta)^2(i\sin\theta)^3 + {}^5C_4(\cos\theta)(i\sin\theta)^4 + {}^5C_5(\cos\theta)^0(i\sin\theta)^5$

On comparing

$\sin 5\theta = {}^5C_1\cos^4\theta.\sin\theta - {}^5C_3\cos^2\theta.\sin^3\theta + {}^5C_5\sin^5\theta$

$\Rightarrow \sin 5\theta = 5\cos^4\theta.\sin\theta - 10\cos^2\theta.\sin^3\theta + \sin^5\theta$

$\Rightarrow \sin 5\theta = 5(1-\sin^2\theta)^2.\sin\theta - 10(1-\sin^2\theta).\sin^3\theta + \sin^5\theta$

$\Rightarrow \sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \quad \ldots(ii)$

Similarly

$\cos 3\theta + i\sin 3\theta = {}^3C_0(\cos\theta)^3(i\sin\theta)^0 + {}^3C_1(\cos\theta)^2(i\sin\theta)^1 + {}^3C_2(\cos\theta)^1(i\sin\theta)^2 + {}^3C_3(\cos\theta)^0(i\sin\theta)^3$

On comparing

$\sin 3\theta = {}^3C_1\cos^2\theta.\sin\theta - {}^3C_3\sin^3\theta$

$\Rightarrow \sin 3\theta = 3\cos^2\theta.\sin\theta - \sin^3\theta$

$\Rightarrow \sin 3\theta = 3(1-\sin^2\theta).\sin\theta - \sin^3\theta$

$\Rightarrow \sin 3\theta = 3\sin\theta - 4\sin^3\theta \quad \ldots(iii)$

Given that

$16\sin^5\theta = p\sin 5\theta + q\sin 3\theta + r\sin\theta$

$\Rightarrow 16\sin^5\theta = p(16\sin^5\theta - 20\sin^3\theta + 5\sin\theta)$ $q(3\sin\theta - 4\sin^3\theta) + r\sin\theta$

$\Rightarrow 16\sin^5\theta = 16p\sin^5\theta + (-20p - 4q)\sin^3\theta + (5p + 3q + r)\sin\theta$

On comparing

$16 = 16p \Rightarrow p = 1$

or $-20p - 4q = 0$

$\Rightarrow -20 - 4q = 0 \Rightarrow q = -5$

or $5p + 3q + r = 0$

$\Rightarrow 5 + 3(-5) + r = 0 \Rightarrow r = 10$

18. (A) $p = 1$

19. (D) $q = -5$

20. (C) $r = 10$

21. (A) Let $\vec{a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$

and $\vec{b} = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + \hat{k}$

$\therefore \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$\cos\theta = \frac{\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right).\left(\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}+\hat{k}\right)}{\sqrt{\frac{1}{2}+\frac{1}{2}+1}\sqrt{\frac{1}{2}+\frac{1}{2}+1}}$$

$$\cos\theta = \frac{\frac{1}{2}-\frac{1}{2}+1}{\sqrt{2}.\sqrt{2}}$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

22. (A) Let $\vec{a} = \lambda\hat{i} + (1+\lambda)\hat{j} + (1+2\lambda)\hat{k}$

and $\vec{b} = (1-\lambda)\hat{i} + \lambda\hat{j} + 2\hat{k}$

$\vec{a}$ and $\vec{b}$ are perpendicular to each other

$\vec{a}.\vec{b} = 0$

$\Rightarrow [\lambda\hat{i} + (1+\lambda)\hat{j} + (1+2\lambda)\hat{k}].[(1-\lambda)]\hat{i} + \lambda\hat{j} + 2\hat{k}] = 0$

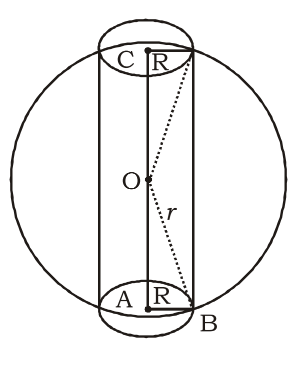
$\Rightarrow \lambda(1-\lambda) + (1+\lambda)\lambda + (1+2\lambda)\times 2 = 0$

$\Rightarrow 6\lambda + 2 = 0$

$\Rightarrow \lambda = -\frac{2}{6} = -\frac{1}{3}$

23. (A) Let h be the height, R be the radius and V be the volume of cylinder.

In ΔOAB : OA = OC = $h/2$



$r^2 = R^2 + \left(\frac{h}{2}\right)^2$ ...(i)

Clearly, $V = \pi R^2 h$

$\Rightarrow V(h) = \pi\left(r^2 - \frac{h^2}{4}\right)h$ [using Eq. (i)]

$\Rightarrow V(h) = \pi\left(r^2 h - \frac{h^3}{4}\right)$

$\Rightarrow V'(h) = \pi\left(r^2 - \frac{3h^2}{4}\right)$ ...(ii)

For maximum, put $V'(h) = 0$

$\Rightarrow r^2 = \frac{3h^2}{4} \Rightarrow h^2 = \frac{4r^2}{3}$

$\Rightarrow h = \frac{2r}{\sqrt{3}}$ $(\because h > 0)$

Again, differentiating Eq. (ii) w.r.t.'h'

$V''(h) = \left(\frac{-6h}{4}\right)$

$\Rightarrow V''\left(\frac{2r}{\sqrt{3}}\right) = \pi\left(\frac{-6}{4}\times\frac{2r}{\sqrt{3}}\right) < 0$ (maxima)

Thus, the volume is maximum when $h = \frac{2r}{\sqrt{3}}$.

24. (B) Clearly, volume of cylinder is maximum when $h = \frac{2r}{\sqrt{3}}$.

By using the relation $r^2 = R^2 + \left(\frac{h}{2}\right)^2$,

we have

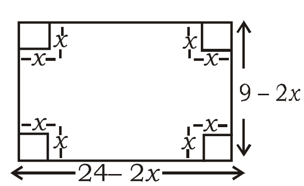
$R^2 = r^2 - \frac{h^2}{4}$

$R^2 = r^2 - \frac{r^2}{3}$

$R^2 = \frac{2r^2}{3} \Rightarrow R = \frac{\sqrt{2}r}{\sqrt{3}}$ $(\because R > 0)$

(25-26)

Given, a rectangular box is to be made from a sheet of size 24'' × 9'' by cutting out identical square of side length x from the four corners.



Clearly, the length of rectangular box = 24 – 2x, the height of rectangular box = x and the width of rectangular box = 9 – 2x.

25. (C) Let V be the volume of the box.

$\therefore$ $V(x) = (24 - 2x).(9 - 2x).x$

$V(x) = (216 - 48x - 18x + 4x^2).x$

$V(x) = 4x^3 - 66x^2 + 216x$

$\Rightarrow$ $V'(x) = 12x^2 - 132x + 216$

For maximum, put $V'(x) = 0$

$\Rightarrow 12x^2 - 132x + 216 = 0$

$\Rightarrow x^2 - 11x + 18 = 0$

$\Rightarrow (x - 9)(x - 2) = 0$

$\Rightarrow x = 9$ or $x = 2$

Now, $V''(x) = 24x - 132$

$\therefore$ $V''(9) = 216 - 132 = 84 > 0$ (minima)

and $V''(2) = 48 - 132 = -84 < 0$ (maxima)

Thus, volume is maximum when $x = 2$ inch.

26. (A) Maximum volume of box = $(24 - 4).(9 - 4).2$

$= 20 \times 5 \times 2 = 200$ cu inch

27. (B) Consider the given expression is

$$y = \frac{2}{3C}(Cx - 1)^{3/2} + B$$

On differentiating both sides w.r.t.'x'

$$\frac{dy}{dx} = \frac{2}{3C}.\frac{3}{2}(Cx - 1)^{1/2}.C + 0 = (Cx - 1)^{1/2}$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^2 = Cx - 1$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = Cx \quad \text{...(i)}$$

Now, on differentiating w.r.t.'x', we get

$$2\left(\frac{dy}{dx}\right).\frac{d^2y}{dx^2} = C$$

From Eq. (i)

$$\left(\frac{dy}{dx}\right)^2 + 1 = 2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$$

28. (A) $\because$ $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Also, $\alpha + h + \beta + h = -\frac{q}{p}$

$$\Rightarrow \alpha + \beta + 2h = -\frac{q}{p}$$

$$\Rightarrow 2h = -\frac{q}{p} + \frac{b}{a} \qquad \left(\because \alpha + \beta = -\frac{b}{a}\right)$$

$$\Rightarrow h = \frac{1}{2}\left[\frac{b}{a} - \frac{q}{p}\right]$$

29. (B) $\begin{vmatrix} a & b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$

$\Rightarrow a[a^2 - 0] - b[-b^2] + 0$

$\Rightarrow a^3 + b^3 = 0$

$\Rightarrow a^3 = -b^3 \Rightarrow \left(\frac{a}{b}\right)^3 = -1$

Hence, $\frac{a}{b}$ is one of the cube roots of -1.

30. (B) Given that, X = Collection of all people living in a city

Let R is related to x where $x < y$ if y is atleast 5 years older than x.

It is clear that $x \nless x$, Hence R is not reflexive.

Now, let xRy such that $x < y$, *i.e.* y is at least 5 year older than x.

Then, x must be younger than y.

It is clear that $y \nless x$, hence R is not symmetric.

Now, let xRy and yRz.

Then, $x < y$ and $y < z$.

It is clear that $x < z$.

Hence, R is transitive.

31. (B) $\begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an elementary matrix

because its value = 1.

32. (C) Given that, $(x^2 + 2)^2 + 8x^2 = 6x(x^2 + 2)$

$\Rightarrow x^4 + 4 + 4x^2 + 8x^2 = 6x^3 + 12x$

$\Rightarrow x^4 - 6x^3 + 12x^2 - 12x + 4 = 0$

let $P(x) = x^4 - 6x^3 + 12x^2 - 12x + 4$

and $P(-x) = x^4 + 6x^3 + 12x^2 + 12x + 4$

Thus we can say that any negative real roots is not possible but four positive roots are possible. Instead of these, this is clear that sum of all the roots is 6. Hence, both the statements given are true.

33. (D) A square matrix A is called skew Hermitian; if $(\overline{A})' = -A$

Now, $A = \begin{bmatrix} 0 & -4 + i \\ 4 + i & 0 \end{bmatrix}$

$\Rightarrow \overline{A} = \begin{bmatrix} 0 & -4 - i \\ 4 - i & 0 \end{bmatrix}$

$\Rightarrow (\overline{A})' = \begin{bmatrix} 0 & 4 - i \\ -4 - i & 0 \end{bmatrix}$

$\Rightarrow (\overline{A})' = -\begin{bmatrix} 0 & -4 + i \\ 4 + i & 0 \end{bmatrix}$

$\Rightarrow (\overline{A})' = -A$

Hence, it is clear that given matrix is a skew Hermitian.

34. (D) 0.5 + 0.55 + 0.555 + … + n terms

$$\Rightarrow \frac{5}{10} + \frac{55}{100} + \frac{555}{1000} + \ldots n \text{ terms}$$

$$\Rightarrow 5\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots n \text{ terms}\right]$$

$$\Rightarrow \frac{5}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots n \text{ terms}\right]$$

$$\Rightarrow \frac{5}{9}\left[\frac{(10-1)}{10} + \frac{(10^2-1)}{10^2} + \ldots n \text{ terms}\right]$$

$$\Rightarrow \frac{5}{9}(1 + 1 + 1 \ldots n \text{ terms}) -$$

$$- \frac{5}{9}\left[\frac{1}{10} + \frac{1}{10^2} + \ldots n \text{ terms}\right]$$

$$\Rightarrow \frac{5}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$$

$$\Rightarrow \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

35. (C) The no. of subsets of A = ${}^{10}C_2$

$$= \frac{10 \times 9}{2} = 45$$

36. (A) $\because AX = B$

$$\therefore \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3p+q & -4p-q \\ 3r+s & -4r-s \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & 1 \end{bmatrix}$$

$\Rightarrow 3p + q = 5$ and $-4p - q = 2$

For solving $p = -7$ and $q = 26$

Now, $3r + s = -2$ and $-4r - s = 1$

For solving , $r = 1$ and $s = -5$

$$\therefore A = \begin{bmatrix} -7 & 26 \\ 1 & -5 \end{bmatrix}$$

37. (C) Required no. of words = $\frac{6!}{2!} - \frac{4! \times 3!}{2!}$

= 360 – 72 = 288

38. (B) $(x^3 - 1) = (x-1)(x^2 + 1 + x)$

$= (x-1)(x^2 + x - \omega - \omega^2)$ ($\because 1 + \omega + \omega^2 = 0$)

$= (x-1)(x^2 - \omega^2 + x - \omega)$

$= (x-1)[(x-\omega)(x+\omega) + (x-\omega)]$

$= (x-1)(x-\omega)(x+\omega+1)$

$= (x-1)(x-\omega)(x-\omega^2)$

39. (C) let $Z = \left[\dfrac{\sin\frac{\pi}{6} + i\left(1 - \cos\frac{\pi}{6}\right)}{\sin\frac{\pi}{6} - i\left(1 - \cos\frac{\pi}{6}\right)}\right]^3$

$$= \left[\frac{2\sin\frac{\pi}{12}.\cos\frac{\pi}{12} + i.2\sin^2\frac{\pi}{12}}{2\sin\frac{\pi}{12}.\cos\frac{\pi}{12} - i.2\sin^2\frac{\pi}{12}}\right]^3$$

$$= \left[\frac{\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}}{\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}}\right]^3$$

$$= \left[\frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}\right]^3$$

$$= \left[\frac{\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^2}{\cos^2\frac{\pi}{12} + \sin^2\frac{\pi}{12}}\right]^3$$

$$= \left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)^6$$

$$= \left[\cos\left(6 \times \frac{\pi}{12}\right) + i\sin\left(6 \times \frac{\pi}{12}\right)\right]$$

$$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i = i$$

40. (C) $\dfrac{\theta^o}{\theta^C} = \dfrac{180^o}{\pi} \Rightarrow \theta^C = \dfrac{\pi \times \theta^o}{180^o}$

and $\theta^o \times \theta^C = \dfrac{125\pi}{9}$

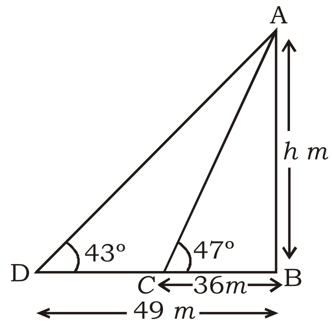
$$\therefore \theta^o \times \frac{\pi \times \theta^o}{180^o} = \frac{125\pi}{9}$$

$$\Rightarrow (\theta^o)^2 = \frac{125}{9} \times 180^o$$

$$\Rightarrow (\theta^o)^2 = 125 \times 20^o$$

$$\Rightarrow (\theta^o)^2 = (5 \times 10^o)^2 \Rightarrow \theta = 50^o$$

41. (B) Let AB is tower and its height = h m



In ΔABC :

$\tan 47° = \frac{h}{36}$ ...(i)

In ΔABD :

$\tan 43° = \frac{h}{49}$ ...(ii)

from eq. (i) and (ii)

$\Rightarrow \tan 47°.\tan 43° = \frac{h^2}{36 \times 49}$

$\Rightarrow \tan 47°.\cot 47° = \frac{h^2}{36 \times 49}$

$\Rightarrow 1 = \frac{h^2}{36 \times 49}$

$\Rightarrow h^2 = 49 \times 36$

$\Rightarrow h = 7 \times 6 = 42$

Hence, height of tower = 42 metre

42. (B) $(1 - \sin A + \cos A)^2$

$\Rightarrow 1 + \sin^2 A + \cos^2 A - 2\sin A - 2\sin A.\cos A + 2\cos A$

$\Rightarrow 1 + 1 - 2\sin A - 2\sin A.\cos A + 2\cos A$

$\Rightarrow 2 + 2\cos A - 2\sin A - 2\sin A.\cos A$

$\Rightarrow 2(1 + \cos A) - 2\sin A(1 + \cos A)$

$\Rightarrow 2(1 + \cos A)(1 - \sin A)$

$\Rightarrow 2(1 - \sin A)(1 + \cos A)$

43. (A) let $P(x, y)$ is circumcentre of ΔABC.

$\therefore AP^2 = PB^2$

$\Rightarrow (x+2)^2 + (y-3)^2 = (x-2)^2 + (y-1)^2$

$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y$
$= x^2 + 4 - 4x + y^2 + 1 - 2y$

$\Rightarrow 4x + 9 - 6y = -4x + 1 - 2y$

$\Rightarrow 8x - 4y + 8 = 0$

$\Rightarrow 2x - y + 2 = 0$ ...(i)

and $AP^2 = PC^2$

$\Rightarrow (x+2)^2 + (y-3)^2 = (x-1)^2 + (y-2)^2$

$\Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y$
$= x^2 + 1 - 2x + y^2 + 4 - 4y$

$\Rightarrow 4x - 6y + 9 = -2x - 4y + 1$

$\Rightarrow 6x - 2y + 8 = 0$

$\Rightarrow 3x - y + 4 = 0$ ...(ii)

On solving the equation (i) *and* (ii)

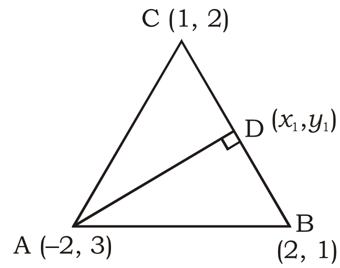
$x = y = -2$

$\therefore$ circumcentre of circle = (–2, –2)

44. (B) cenroid of $\Delta ABC = \left(\frac{-2+2+1}{3}, \frac{3+1+2}{3}\right)$

$= \left(\frac{1}{3}, 2\right)$

45. (D) In ΔABC, let D is the foot of altitude drawn from point A.



Slope of $BC(m_1) = \frac{1-2}{2-1} = -1$

and slope of $AD(m_2) = \frac{y_1 - 3}{x_1 + 2}$

We know that

$m_1.m_2 = -1$

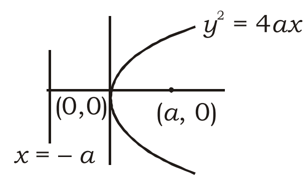
$\Rightarrow -1 \times \frac{y_1 - 3}{x_1 + 2} = -1$

$\Rightarrow y_1 - 3 = x_1 + 2$

$\Rightarrow y_1 - x_1 = 5$

Only (–1, 4) satisfies the above equation from given point.

46. (A) Hence point on the parabola $y^2 = 4ax$ nearest to the focus has its abscissa $x = 0$



47. (B) Equation of given line is $y = 3 - 3x$

Slope of given line $m = -3$

$\therefore$ slope of the perpendicular line

$m' = \frac{-1}{m} = \frac{1}{3}$

$\therefore$ Required equation of line

$y - 2 = \frac{1}{3}(x - 2)$

$\Rightarrow 3y - 6 = x - 2$

$\Rightarrow 3y - x = 4$

$\Rightarrow \frac{x}{-4} + \frac{y}{4/3} = 1$

$\therefore$ y-intercepts $= \frac{4}{3}$

48. (A) Given that,

$\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

and $\frac{x}{b} + \frac{y}{a} = 1$ ...(ii)

$\Rightarrow bx + ay = ab$ and $ax + by = ab$

$\therefore\ bx + ay = ax + by$

$\Rightarrow x(b - a) = y(b - a)$

$\Rightarrow\ x = y$

From equation (i)

$\frac{y}{a} + \frac{y}{b} = 1 \Rightarrow y = \frac{ab}{a+b}$ and $x = \frac{ab}{a+b}$

$\therefore$ The line joining the Points (0, 0) and $\left(\frac{ab}{a+b}, \frac{ab}{a+b}\right)$ is

$\Rightarrow y - 0 = \frac{\frac{ab}{a+b} - 0}{\frac{ab}{a+b} - 0}(x - 0)$

$\Rightarrow y = x \Rightarrow x - y = 0$

(49-50)

49. (D) length of the line segment

$= \sqrt{(12)^2 + (4)^2 + (3)^2}$

$= \sqrt{144 + 16 + 9}$

$= \sqrt{169} = 13$ units

50. (A) Direction Cosines of the line segment

$= \left(\pm\frac{12}{13}, \pm\frac{4}{13}, \pm\frac{3}{13}\right)$

51. (A) Given that centroid = (1, 2, 3)

Let equation of plane is $\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$

Hence, 3, 6, 9 are respectively intercepts cut on x-axis, y-axis, and z-axis.

52. (D) $\therefore$ Equation of plane ABC is $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$

$6x + 3y + 2z = 18$

53. (C) **Statement 1**

$y = f(x) = \frac{e^x + e^{-x}}{2}$

$\Rightarrow f'(x) = \frac{e^x - e^{-x}}{2}$

$\Rightarrow f'(x) = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right)$

$\Rightarrow f'(x) = \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right)$ ...(i)

Now, for $x \geq 0,\ 2x \geq 0$

$\Rightarrow e^{2x} \geq e^0$ ($\because e^x$ is a increasing function)

$\therefore$ for $x \geq 0,\ e^x \geq 1$

From equation (i)

$f'(x) = \frac{1}{2}\left(\frac{e^{2x} - 1}{e^x}\right) \geq 0$

Hence, $y = f(x) = \frac{e^x + e^{-x}}{2}$, increasing function in $[0, \infty]$

Statement 1 is correct.

Statement 2

$y = g(x) = \frac{e^x - e^{-x}}{2}$

$\Rightarrow g'(x) = \frac{e^x + e^{-x}}{2}$

[$\because$ both e^x and e^{-x} is more than 0 in $(-\infty, \infty)$]

Hence, $y = g(x) = \frac{e^x - e^{-x}}{2}$ increasing function in intervel $(-\infty, \infty)$.

Statement 2 is correct.

54. (B) let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ and $v = \tan^{-1}x$

On putting $x = \tan\theta$

$\Rightarrow u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right)$

$\Rightarrow u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$\Rightarrow u = \tan^{-1}\left(\frac{1 - \cos\theta}{\sin\theta}\right)$

$\Rightarrow u = \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right)$

$\Rightarrow u = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2}$

$\Rightarrow u = \frac{1}{2}\tan^{-1}x$

$\Rightarrow u = \frac{1}{2}v$

On differentiating both side w.r.t. 'v'

$\Rightarrow \frac{du}{dv} = \frac{1}{2}$

55. (B) $f(x) = \log_e\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^2}{1+3x^2}$

Now, $gof\left(\frac{e-1}{e+1}\right) = g\left[f\left(\frac{e-1}{e+1}\right)\right]$ ...(i)

$\Rightarrow \quad gof\left(\frac{e-1}{e+1}\right) = g\left[\log_e\left(\frac{1+\frac{e-1}{e+1}}{1-\frac{e-1}{e+1}}\right)\right]$

$\Rightarrow \quad gof\left(\frac{e-1}{e+1}\right) = g\left[\log_e\left(\frac{e+1+e-1}{e+1-e+1}\right)\right]$

$\Rightarrow \quad gof\left(\frac{e-1}{e+1}\right) = g\left[\log_e\left(\frac{2e}{2}\right)\right]$

$\Rightarrow \quad gof\left(\frac{e-1}{e+1}\right) = g[\log_e e]$

$\Rightarrow \quad gof\left[\frac{e-1}{e+1}\right] = g\,(1)$

$\Rightarrow \quad gof\left[\frac{e-1}{e+1}\right] = \frac{3(1)+(1)^2}{1+3(1)}$

$\Rightarrow \quad gof\left[\frac{e-1}{e+1}\right] = \frac{3+1}{1+3} = 1$

56. (C) 1. Given that $f(x) = x^3,\ x \in R$

$\Rightarrow f'(x) = 3x^2 \geq 0$

$\Rightarrow f$ is increasing function.

$\Rightarrow f$ is unique.

Hence f is inverse on its range.

2. Given that, $f(x) = \sin x,\ 0 < x < 2\pi$

it is clear that, $f\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

and $f\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad f\left(\frac{2\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\Rightarrow f$ is not unique.

Hence, f is not inverse on its range.

3. given that, $f(x) = e^x,\ x \in R$

$\Rightarrow f'(x) = e^x,\ x > 0$

Hence, f is a increasing function.

$\Rightarrow f$ is unique.

Hence, f is inverse on its range.

(57 -58) let $I = \int \frac{dx}{a\cos x + b\sin x}$

now, put $a = r\sin\alpha$ and $b = r\cos\alpha$ where,

$r = \sqrt{a^2+b^2}$ and $\alpha = \tan^{-1}\left(\frac{a}{b}\right)$

$\therefore I = \frac{1}{r}\int \frac{dx}{\sin\alpha.\cos x + \cos\alpha.\sin x}$

$= \frac{1}{r}\int \frac{dx}{\sin(x+\alpha)} = \frac{1}{r}\int \text{cosec}(x+\alpha)\,dx$

$= \frac{1}{r}\ \ln[\text{cosec}(x+\alpha) - \cot(x+a)] + C$

$= \frac{1}{r}\ \ln\left[\frac{1}{\sin(x+\alpha)} - \frac{\cos(x+\alpha)}{\sin(x+\alpha)}\right] + C$

$= \frac{1}{r}\ \ln\left[\frac{1-\cos(x+\alpha)}{\sin(x+\alpha)}\right] + C$

$= \frac{1}{r}\ \ln\left[\frac{2\sin^2\left(\frac{x+\alpha}{2}\right)}{2\sin\left(\frac{x+\alpha}{2}\right).\left(\frac{x+\alpha}{2}\right)}\right] + C$

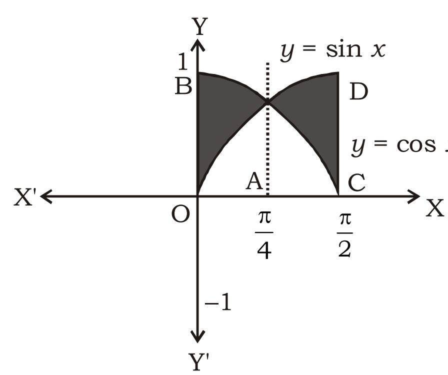
$= \frac{1}{r}\ \ln\left[\tan\frac{x+\alpha}{2}\right] + C$

57. (B)

58. (A)

(59 – 60)

curves $y = \sin x$ and $y = \cos x$



59. (A)Required Area = Area of curve OABO.

$= \int_0^{\frac{\pi}{4}}(\cos x - \sin x)\,dx$

$= [\sin x + \cos x]_0^{\pi/2}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1 = (\sqrt{2} - 1)$

60. (A) Required Area = Area of curve ACDA

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)\, dx$$

$$= [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= -[0 + 1 - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= (\sqrt{2} - 1)$$

(61-63)

61. (B) $x = \frac{a(1-t^2)}{(1+t^2)}$, $y = \frac{2at}{1+t^2}$

On squaring and adding

$$\Rightarrow x^2 + y^2 = \frac{a^2(1-t^2)^2}{(1+t^2)^2} + \frac{4a^2t^2}{(1+t^2)^2}$$

$$\Rightarrow x^2 + y^2 = \frac{a^2}{(1+t^2)^2}[(1-t^2)^2 + 4t^2]$$

$$\Rightarrow x^2 + y^2 = \frac{a^2}{(1+t^2)^2}[1 + t^4 - 2t^2 + 4t^2]$$

$$\Rightarrow x^2 + y^2 = \frac{a^2}{(1+t^2)^2} \times (1+t^2)^2 = a^2$$

$\Rightarrow x^2 + y^2 = a^2$ ...(i)

Equation (i) represents a circle whose radius is a.

62. (D) $x = \frac{a(1-t^2)}{(1+t^2)}$

$$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = -2at\left[\frac{1+t^2+1-t^2}{(1+t^2)^2}\right] = \frac{-2at \times 2}{(1+t^2)^2}$$

$$\Rightarrow \frac{dx}{dt} = \frac{-4at}{(1+t^2)^2}$$

and $y = \frac{2at}{1+t^2}$

$$\Rightarrow \frac{dy}{dt} = 2a\left[\frac{(1+t^2).1 - t.(2t)}{(1+t^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{2a(1-t^2)}{(1+t^2)^2}$$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = \frac{2a(1-t^2)}{(1+t^2)^2} \times \frac{(1+t^2)^2}{-4at}$$

$$\frac{dy}{dx} = \frac{-(1-t^2)}{2t} = -\frac{x}{y}$$ ...(ii)

63. (D) From equation (ii)

$$y\frac{dy}{dx} = -x$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -1$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \frac{x^2}{y^2} = -1$$ [from equation (ii)]

$$\Rightarrow y\frac{d^2y}{dx^2} = -1 - \frac{x^2}{y^2}$$

$$\Rightarrow y\frac{d^2y}{dx^2} = -\frac{(y^2+x^2)}{y^2}$$

$$\therefore \frac{d^2y}{dx^2} = -\frac{a^2}{y^3}$$ [from equation (i)]

(64 - 65)

$$\Rightarrow \frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = Ax + B$$

$$\Rightarrow \frac{d}{dx}\left[\frac{(x^2+x+1)(x^2-x-1)}{x^2+x+1}\right] = Ax + B$$

$$\Rightarrow \frac{d}{dx}(x^2 - x + 1) = \text{A}x + \text{B}$$

$\Rightarrow 2x - 1 = \text{A}x + \text{B}$

Now, equating both the sides ;

$A = 2$, $B = -1$

64. (C)

65. (A)

(66-67) Given that, $\lim_{x\to\infty}\left(\frac{2+x^2}{1+x}-Ax-B\right)=3$

$\Rightarrow \lim_{x\to\infty}\left(\frac{2+x^2-Ax-Ax^2-B-Bx}{1+x}\right)=3$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to\infty}\left(\frac{2x-A-2Ax-B}{1}\right)=3$

$\Rightarrow \lim_{x\to\infty}[x(2-2A)-(A+B)]=3$

Now, equating both the sides,

$\Rightarrow 2-2A=0$ and $(A+B)=-3$

$\Rightarrow A=1$ and $(A+B)=-3$

$\Rightarrow A=1$ and $B=-4$

66. (B)

67. (C)

68. (A) Differential equation

$\sin\left(\frac{dy}{dx}\right)-a=0$

$\Rightarrow \sin\frac{dy}{dx}=a$

$\Rightarrow \frac{dy}{dx}=\sin^{-1}a$

$\Rightarrow dy=(\sin^{-1}a)\,dx$

Integrating both side

$\Rightarrow \int dy=\int(\sin^{-1}a)\,dx+C$

$\Rightarrow y=x(\sin^{-1}a)+C$

69. (D) In ΔABC, $\overrightarrow{AB}=-2\hat{i}+3\hat{j}+2\hat{k}$

and $\overrightarrow{AC}=-4\hat{i}+5\hat{j}+2\hat{k}$

$\therefore \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&3&2\\-4&5&2\end{vmatrix}$

$\overrightarrow{AB}\times\overrightarrow{AC}=-4\hat{i}-4\hat{j}+2\hat{k}$

Area of $\Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

Area of $\Delta ABC=\frac{1}{2}\sqrt{(-4)^2+(-4)^2+(2)^2}$

Area of $\Delta ABC=\frac{1}{2}\sqrt{36}$

Area of $\Delta ABC=\frac{1}{2}\times 6=3$ square units

70. (C) Median is used for the measure of central tendency.

71. (A) Let X and Y are two persons and they hit a target with the probability A and B respectively.

$\therefore P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$

Now, P (Probability of hitting the target by any one X *or* Y).

$=P(A\cap\overline{B})+P(\overline{A}\cap B)$

$=P(A).P(\overline{B})+P(\overline{A}).P(B)$

$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{1}{3}=\frac{3}{6}=\frac{1}{2}$

72. (D) let probability of success $(p)=\frac{1}{2}$

and probability of unsuccess $(q)=\frac{1}{2}$

let x is random variable which show for solving 5 questions. It is clear that

x ~bionomial distribution $\left(5,\frac{1}{2}\right)$

$\therefore P(X=x)={}^5C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{5-x}$

(where, $x=0, 1, \ldots\ldots, 5$)

$\therefore$ Required Probability

$=P(X\geq 2)=1-[P(X=0)+P(X=1)]$

$=1-\left[{}^5C_0\left(\frac{1}{2}\right)^5+{}^5C_1\left(\frac{1}{2}\right)^5\right]$

$=1-\frac{6}{32}=\frac{26}{32}=\frac{13}{16}$

73. (C) If $A\subseteq B$ then $A\cup B=B$ and $A\cap B=A$ then

it is clear that $P(A\cap\overline{B})=0$

$P(A/B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A)}{P(B)}$

$P(A/(A\cup B))=P(A/B)=\frac{P(A)}{P(B)}$

$\therefore P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{P(A)}{P(A)}=1$

74. (C) Sample space in throwing of two dice.

(1, 1) (1, 2), (1, 3), (1, 4) (1, 5) (1, 6)

(2, 1).................................. (2, 6)

⋮

(5, 1).................................. (5, 6)

(6, 1).................................. (6, 6)

∵ 5 will never come on any dice.

∴ Number of exhaustive events

= 36 – 6 – 6 + 1 = 25

75. (B) $\dfrac{\left[1+\left(i^5\right)^{4n-1}\right]^{4n+1}}{\left[1+\left(i^5\right)^{4n+1}\right]^{4n-1}}$

$\Rightarrow \dfrac{\left[1+(i)^{4n-1}\right]^{4n+1}}{\left[1+(i)^{4n+1}\right]^{4n-1}} \Rightarrow \dfrac{\left[1+i^{-1}\right]^{4n+1}}{\left[1+i\right]^{4n-1}}$

$\Rightarrow \dfrac{\left[1+\frac{1}{i}\right]^{4n+1}}{\left[1+i\right]^{4n-1}} \Rightarrow \dfrac{\left[1+i\right]^{4n+1}}{\left[1+i\right]^{4n-1}.i^{4n+1}}$

$\Rightarrow \dfrac{(1+i)^2}{i} \Rightarrow \dfrac{1+i^2+2i}{i} \Rightarrow \dfrac{2i}{i} = 2$

76. (A) $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)\ldots$

...to $2n$ factors

$\Rightarrow (1-\omega+\omega^2)(1-\omega^2+\omega)(1-\omega+\omega^2)\ldots$

... to $2n$ factor

$\Rightarrow (-2\omega)(-2\omega^2)(-2\omega)(-2\omega^2)\ldots$ to $2n$ factors

$\Rightarrow (2^2\omega^3)(2^2\omega^3)\ldots$ to n factors

$\Rightarrow (2^2)^n = 2^{2n}$

77. (A) We have, $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$

$\Rightarrow \dfrac{\omega^2\left(a+b\omega+c\omega^2\right)}{\omega^2\left(c+a\omega+b\omega^2\right)} + \dfrac{\omega\left(a+b\omega+c\omega^2\right)}{\omega\left(b+c\omega+a\omega^2\right)}$

$\Rightarrow \dfrac{\omega^2\left(a+b\omega+c\omega^2\right)}{\left(c\omega^2+a\omega^3+b\omega^4\right)} + \dfrac{\omega\left(a+b\omega+c\omega^2\right)}{\left(b\omega+c\omega^2+a\omega^3\right)}$

$\Rightarrow \dfrac{\omega^2\left(a+b\omega+c\omega^2\right)}{\left(a+b\omega+a\omega^2\right)} + \dfrac{\omega\left(a+b\omega+c\omega^2\right)}{a+b\omega+a\omega^2}$

$\Rightarrow \omega^2+\omega = -1$

78. (A) Given, $(x-1)^3 + 8 = 0$

$\Rightarrow (x-1)^3 = -8$

$\Rightarrow x-1 = (-8)^{1/3}$

$\Rightarrow x-1 = 2(-1)^{1/3}$

$\Rightarrow x-1 = 2(-1)$ or $x-1 = 2(-\omega)$

or $x-1 = 2(-\omega^2)$

$\Rightarrow x-1 = -2$ or $x-1 = -2\omega$ or $x-1 = -2\omega^2$

$\Rightarrow x = -1$ or $x = 1-2\omega$ or $x = 1-2\omega^2$

79.(B) Given equations are $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}\,x + q = 0$.

They have real roots.

∴ $4q^2 - 4pr \geq 0 \Rightarrow q^2 \geq pr$ ...(i)

and from second $4(pr) - 4q^2 \geq 0$

$\Rightarrow pr \geq q^2$ ...(ii)

From Eqs. (i) and (ii), we get $q^2 = pr$

80. (C) We have, $\dfrac{1^3}{1} + \dfrac{1^3+2^3}{1+3} + \dfrac{1^3+2^3+3^3}{1+3+5} + \ldots$

$\therefore T_n = \dfrac{1^3+2^3+3^3+\ldots+n^3}{1+3+5+\ldots\text{upto } n \text{ terms}}$

$T_n = \dfrac{\sum n^3}{\frac{n}{2}\left[2\times1+(n-1)\times2\right]}$

$T_n = \dfrac{1}{4}\dfrac{n^2(n+1)^2}{n^2} = \dfrac{1}{4}(n^2+2n+1)$

81. (B) $I = \int_1^2 e^x\left(\dfrac{x-3}{x^4}\right)dx$

$I = \int_1^2 e^x\left(\dfrac{1}{x^3}-\dfrac{3}{x^4}\right)dx$

$I = \left[\dfrac{e^x}{x^3}\right]_1^2$

$I = \dfrac{e^2}{8} - \dfrac{e^1}{1} = e\left(\dfrac{e}{8}-1\right)$

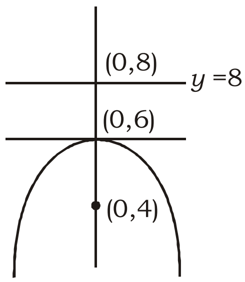
82. (C) $\begin{vmatrix} 3 & \log_7 7 & 2\pi \\ \log_5 125 & \log_3 3 & \sqrt{17} \\ 3 & 1 & 5 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 3 & 1 & 2\pi \\ 3\log_5 5 & 1 & \sqrt{17} \\ 3 & 1 & 5 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 3 & 1 & 2\pi \\ 3 & 1 & \sqrt{17} \\ 3 & 1 & 5 \end{vmatrix}$

$\Rightarrow 3\begin{vmatrix} 1 & 1 & 2\pi \\ 1 & 1 & \sqrt{17} \\ 1 & 1 & 5 \end{vmatrix} = 0$

83. (A)



Equation of directrixe

$$y = 8$$

84. (D) Required probability = $^5C_2 \left(\frac{1}{7}\right)^2 \left(\frac{6}{7}\right)^3$

$$= \frac{10 \times 6^3}{7^5}$$

85. (D) I. If $\cot\theta = x$,

then $x + \frac{1}{x} = \cot\theta + \frac{1}{\cot\theta}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{1}{\sin\theta.\cos\theta} = \text{cosece}\theta.\sec\theta$

$\therefore$ Statement I is correct.

II. If $x + \frac{1}{x} = \sin\theta$,

then $\left(x + \frac{1}{x}\right)^2 = \sin^2\theta$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = \sin^2\theta$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = \sin^2\theta - 2$

$\therefore$ Statement II is correct.

III. If $x = p\sec\theta$ and $y = q\tan\theta$, then

$x^2q^2 - y^2p^2 = p^2q^2\sec^2\theta - p^2q^2\tan^2\theta$

$x^2q^2 - y^2p^2 = p^2q^2(\sec^2\theta - \tan^2\theta)$

$x^2q^2 - y^2p^2 = p^2q^2$

$\therefore$ Statement III is correct.

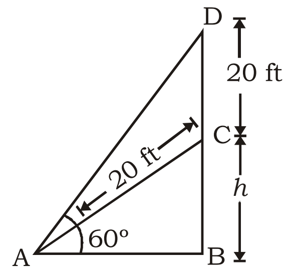
IV. Maximum value of $(\cos\theta - \sqrt{3}\sin\theta)$

$$= \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$$

Statement IV is incorrect.

$\therefore$ Only I, II and III are correct.

86. (B) Let BD is flag and $BD = (h + 20)$ ft



In ΔABD :

$\tan 60° = \frac{BD}{AB}$

$\Rightarrow \sqrt{3} = \frac{h+20}{AB}$

$\Rightarrow AB = \frac{(h+20)}{3}\sqrt{3}$ ...(i)

In ΔABC :

$AC^2 = AB^2 + BC^2$

$\Rightarrow 20^2 = \frac{3(h+20)^2}{9} + h^2$ [from Eq. (i)]

$\Rightarrow 400 = \frac{(h+20)^2 + 3h^2}{3}$

$\Rightarrow 1200 = h^2 + 40h + 400 + 3h^2$

$\Rightarrow 4h^2 + 40h - 800 = 0$

$\Rightarrow (h + 20)(h - 10) = 0$

$\Rightarrow h = 10$ $(\because h \neq -20)$

$\therefore$ Height of flag = $BD = (h + 20)$ ft

$= (10 + 20)$ ft

$= 30$ ft

87. (B) From equation (i)

$\Rightarrow AB = \frac{(h+20)}{3}\sqrt{3}$

On putting $h = 10$

$\Rightarrow AB = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$ ft

88. (A) $\because f(x) = \frac{\sec^4 x + \text{cosec}^4 x}{x^3 + x^4 \cot x}$

$\therefore \quad f(-x) = \frac{\sec^4(-x) + \text{cosec}^4(-x)}{(-x)^3 + (-x)^4\cot(-x)}$

$f(-x) = -\frac{\sec^4 x + \text{cosec}^4 x}{x^3 + x^4\cot x} = -f(x)$

$f(x)$ is an odd function.

Thus, both A and R are true and R is the correct explanation of A.

89. (D) **Assertion (A)** $f(x) = x$ and $F(x) = \frac{x^2}{x}$

At $x = 0$, $F(x) \neq f(x)$

$\therefore$ It is incorrect statement.

Reason (R) It is true that $F(x)$ is not defined at $x = 0$.

$\therefore$ Option (D) is correct.

90. (C) Let $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1}x$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$\Rightarrow y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

On differentiating *w.r.t.* '*x*', we get

$$\frac{dy}{dx} = -\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{1}{2\sqrt{1-x^2}}$$

91. (C) Let $y = \frac{d}{dx}[\sin^{-1}(2x\sqrt{1-x^2})]$

Put $x = \sin\alpha \Rightarrow \alpha = \sin^{-1}x$

$$\Rightarrow y = \frac{d}{dx}[\sin^{-1}(2\sin\alpha\sqrt{1-\sin^2\alpha})]$$

$$\Rightarrow y = \frac{d}{dx}[\sin^{-1}(2\sin\alpha.\cos\alpha)]$$

$$\Rightarrow y = \frac{d}{dx}[\sin^{-1}(\sin 2\alpha)]$$

$$\Rightarrow y = \frac{d}{dx}(2\alpha)$$

$$\Rightarrow y = \frac{d}{dx}(2\sin^{-1}x)$$

$$\Rightarrow y = \frac{2}{\sqrt{1-x^2}}$$

92.(C) From mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Given, $f(x) = x(x-1)(x-2)$

$a = 0 \Rightarrow f(a) = 0$

and $b = \frac{1}{2} \Rightarrow f(\text{b}) = \frac{3}{8}$

$f'(x) = (x-1)(x-2) + x(x-2) + x(x-1)$

$\therefore$ $f'(c) = (c-1)(c-2) + c(c-2) + c(c-1)$

$\Rightarrow f'(\text{c}) = 3c^2 - 6c + 2$

By definition of mean value theorem,

$$f'(\text{c}) = \frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow 3c^2 - 6c + 2 = \frac{(3/8)-0}{(1/2)-0} = \frac{3}{4}$$

$$\Rightarrow 3c^2 - 6c + \frac{5}{4} = 0 \Rightarrow 12c^2 - 24c + 5 = 0$$

This is a quadratic equation in c,

$$\Rightarrow c = \frac{24 \pm\sqrt{(24)^2 - 4\times 12\times 5}}{2\times 12}$$

$$\Rightarrow c = \frac{6 \pm\sqrt{21}}{6} = 1 \pm \frac{\sqrt{21}}{6}$$

Since, 'c' lies between [0, 1/2],

$\therefore$ $c = 1 - \frac{\sqrt{21}}{6}$ (neglecting $c = 1 + \frac{\sqrt{21}}{6}$)

93. (C) Equation of parabola

$\Rightarrow y^2 = 4ax$

On differentiating *w.r.t.* '*x*'

$\Rightarrow 2yy' = 4a$

Again, differentiating, we get

$\Rightarrow 2\,yy'' + 2(y')^2 = 0 \Rightarrow yy'' + (y')^2 = 0$

94. (D) Equation of of ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$$\therefore e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = \sqrt{1-\frac{5}{9}} \Rightarrow e = \frac{2}{3}$$

and equation of hyperbola $\frac{x^2}{9} - \frac{y^2}{\frac{45}{4}} = 1$

$$\therefore e' = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{45/4}{9}} = \frac{3}{2}$$

$$\therefore ee' = \frac{2}{3}\times\frac{3}{2} = 1$$

95. (C) Let $\vec{v} = \lambda\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}} + \frac{\hat{j}+\hat{k}}{\sqrt{2}} + \frac{\hat{k}+\hat{i}}{\sqrt{2}}\right)$

$\Rightarrow \vec{v} = \frac{\lambda}{\sqrt{2}}[2\hat{i} + 2\hat{j} + 2\hat{k}]$ ...(i)

$\Rightarrow |\vec{v}|^2 = \frac{\lambda^2}{2}(4 + 4 + 4)$

$\Rightarrow 16 = \frac{\lambda^2}{2} \times 12$ $[\because |\vec{v}| = 4]$

$\Rightarrow \lambda^2 = \frac{8}{3} \Rightarrow \lambda = \frac{2\sqrt{2}}{\sqrt{3}}$

From eq. (i)

$\vec{v} = \frac{2\sqrt{2}}{\sqrt{3}\sqrt{2}}(2\hat{i} + 2\hat{j} + 2\hat{k})$

$\Rightarrow \vec{v} = \frac{4}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

96. (A) Given, $\bar{x} = 65$, $\bar{y} = 67$

$\sigma_x = 5.0$, $\sigma_y = 2.5$

$r = 0.8$

The line of regression of y on x is

$y - \bar{y} = r.\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$\Rightarrow y - 67 = \frac{0.8 \times 2.5}{5}(x - 65)$

$\Rightarrow y - 67 = \frac{2}{5}(x - 65)$

97. (C) The line of regression of x on y is

$x - \bar{x} = r.\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\Rightarrow x - 65 = \frac{0.8 \times 5}{2.5}(y - 67)$

$\Rightarrow x - 65 = \frac{8}{5}(y - 67)$

98. (D) $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$

Let $3x - 4y = X$

$3 - 4\frac{dy}{dx} = \frac{dX}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{4}\left(3 - \frac{dX}{dx}\right)$

$\Rightarrow \frac{1}{4}\left(3 - \frac{dX}{dx}\right) = \frac{X-2}{X-3}$

$\Rightarrow \frac{3}{4} - \frac{1}{4}\frac{dX}{dx} = \frac{X-2}{X-3}$

$\Rightarrow -\frac{1}{4}\frac{dX}{dx} = \frac{X-2}{X-3} - \frac{3}{4}$

$\Rightarrow -\frac{1}{4}\frac{dX}{dx} = \frac{X+1}{4(X-3)}$

$\Rightarrow -\frac{(X-3)}{(X+1)}dX = dx$

$\Rightarrow -\left(1 - \frac{4}{X+1}\right)dX = dx$

$\Rightarrow \left(-1 + \frac{4}{X+1}\right)dX = dx$

On integration

$\Rightarrow -X + 4\log(X + 1) = x + 4C$

$\Rightarrow 4\log(X + 1) = X + x + 4C$

$\Rightarrow 4\log(3x - 4y + 1) = 3x - 4y + x + 4C$

$\Rightarrow 4\log(3x - 4y + 1) = 4x - 4y + 4C$

$\Rightarrow \log(3x - 4y + 1) = x - y + C$

99. (B) $x = \sin^{-1}(t)$, $y = \log(1 - t^2)$

$\frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}, \frac{dy}{dt} = \frac{1}{1-t^2}(-2t)$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2t}{1-t^2}}{\frac{1}{\sqrt{1-t^2}}}$

$\frac{dy}{dx} = -\frac{2t}{\sqrt{1-t^2}}$

$\frac{d^2y}{dx^2} = \frac{d}{dt}\left[-\frac{2t}{\sqrt{1-t^2}}\right]\frac{dt}{dx}$

$\frac{d^2y}{dx^2} = \frac{-2\sqrt{1-t^2} - (-2t).\frac{-2t}{2\sqrt{1-t^2}}}{1-t^2} \times \sqrt{1-t^2}$

$\frac{d^2y}{dx^2} = -\frac{2}{(1-t^2)^{\frac{3}{2}}} \times \sqrt{1-t^2} = -\frac{2}{1-t^2}$

100. (D) $\begin{vmatrix} -a^2 & -ab & ac \\ ba & -b^2 & bc \\ ac & -bc & c^2 \end{vmatrix}$

On taking common a, b, c from R_1, R_2, R_3

$\Rightarrow abc\begin{vmatrix} -a & -b & c \\ a & -b & c \\ a & -b & c \end{vmatrix}$

On taking b, c from C_2, C_3

$\Rightarrow -ab^2c^2\begin{vmatrix} -a & 1 & 1 \\ a & 1 & 1 \\ a & 1 & 1 \end{vmatrix} = 0$

101. (D) $\frac{df}{dx} = 0$

102. (C)

2	1753	1
2	876	0
2	438	0
2	219	1
2	109	1
2	54	0
2	27	1
2	13	1
2	6	0
2	3	1
	1	

$(1753)_{10} = (11011011001)_2$

103.(A) Class-size = Difference between two consecutive class marks = 10 – 6 = 4

104. (B) Committee of 3 is to be chosen from 4 men and 5 women,

So required probability = $\frac{^4C_2 \times {^5C_1}}{^9C_3}$

$= \frac{6 \times 5}{12 \times 7} = \frac{5}{14}$

105. (B) $\int_{-2}^{2} |1 - x^2|\, dx = \int_{-2}^{-1} |1 - x^2|\, dx + \int_{-1}^{1} |1 - x^2|\, dx + \int_{1}^{2} |1 - x^2|$

$= \int_{-2}^{-1} (x^2 - 1)\, dx + \int_{-1}^{1} (1 - x^2)\, dx + \int_{1}^{2} (x^2 - 1)\, dx$

$= \left(\frac{x^3}{3} - x\right)_{-2}^{-1} + \left(x - \frac{x^3}{3}\right)_{-1}^{1} + \left(\frac{x^3}{3} - x\right)_{1}^{2}$

$= \left[\left(-\frac{1}{3} + 1\right) - \left(-\frac{8}{3} + 2\right)\right] + \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] + \left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)\right]$

$= \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

$= 6 \times \frac{2}{3} = 4$

106. (A) $[a\ b\ c] = a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$

Using the given property,

$[a\ \ b + c\ \ a + b + c]$

$\Rightarrow (a + b + c) \cdot (a \times (b + c))$

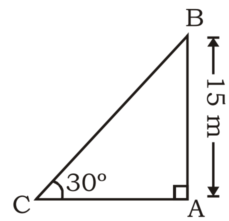
$\Rightarrow (a + b + c) \cdot (a \times b + a \times c)$

$\Rightarrow a \cdot (a \times b) + a \cdot (a \times c) + b \cdot (a \times b) + b \cdot (a \times c) + c \cdot (a \times b) + c \cdot (a \times c)$

$\Rightarrow [a\ a\ b] + [a\ a\ c] + [b\ a\ b] + [b\ a\ c] + [c\ a\ b] + [c\ a\ c]$

$\Rightarrow 0 + 0 + 0 - [a\ b\ c] + [a\ b\ c] + 0 = 0$

107. (A) Let AB be a tower whose height is 15 m.



In ΔBAC

$\tan 30^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{15}{AC} \Rightarrow AC = 15\sqrt{3}$

The distance of the point from the foot of the tower = $15\sqrt{3}$ m

108. (A)

2	55	1
2	27	1
2	13	1
2	6	0
2	3	1
2	1	1
	0	

0.625
× 2
1.250
× 2
0.500
× 2
1.000

$(55)_{10} = (110111)_2$, $(0.625)_{10} = (0.101)_2$

Hence $(55.625)_{10} = (110111.101)_2$

109. (B) Required probability = $\frac{^4C_1 \times {^4C_1}}{^{52}C_2}$

$= \frac{4 \times 4}{26 \times 51} = \frac{8}{663}$

110. (B) $I = \int \sin^3 x . \cos x\, dx$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$I = \int t^3\, dt$

$I = \frac{t^4}{4} + C \Rightarrow I = \frac{1}{4} \sin^4 x + C$

111. (B) $v = 2s^2 + 4s + 5$

On differentiating both side w.r.t.'s'

$\frac{dv}{ds} = 2(2s) + 4$

$\frac{dv}{ds} = 4s + 4$

At s = 5, Acceleration = 4 × 5 + 4 = 24

112. (B) sin2A = λ sin2B $\Rightarrow \frac{\sin 2A}{\sin 2B} = \frac{\lambda}{1}$

Using Componendo and Dividendo Rule

$$\frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \frac{\lambda+1}{\lambda-1}$$

$$\frac{2\sin(A+B)\cdot\cos(A-B)}{2\cos(A+B)\cdot\sin(A-B)} = \frac{\lambda+1}{\lambda-1}$$

$$\frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

113. (D) Shaded Region is $(P \cap Q) \cup (P \cap R)$

114. (D) $(9)^{200} = (1 + 8)^{200}$

$(9)^{200} = 1 + {}^{200}C_1(8) + {}^{200}C_2(8)^2 + \ldots$

$(9)^{200} = 1 + 1600 + 1273600 + \ldots$

So, last two digit are 01.

115. (C) Equation of plane be $ax + by + cz + d = 0$

So, no.of arbitrary constants = 4(a,b,c,d)

116. (B) $0.\overline{2} + 0.\overline{23} = \frac{2}{9} + \frac{23}{99}$

$$= \frac{45}{99} = 0.\overline{45}$$

117. (B) $(a + b + c)^n = [a + (b + c)]^n$

$\Rightarrow a^n + {}^nC_1\, a^{n-1}(b + c) + {}^nC_2 a^{n-2}(b + c)^2 + \ldots$

Number of terms = 1 + 2 + 3 + …+ (n + 1)

$$= \frac{1}{2}(n + 1)(n + 2)$$

118. (A) It the word 'DELHI'

When, we fix L in mid place, the first two letters arranged in ${}^4p_2$ ways, then last two letters are arranged in ${}^2P_2$ ways.

Total number of ways = ${}^4P_2 \times {}^2P_2 = 24$

119. (B) $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

Projection of $\vec{a}$ on $\vec{b}$ = $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{1\times4 + (-2)\times(-4) + 1\times7}{\sqrt{(4)^2 + (-4)^2 + (7)^2}}$$

$$= \frac{19}{\sqrt{81}} = \frac{19}{9}$$

120. (D) Given, $\left|z - \frac{2}{z}\right| = 6$

$\Rightarrow z - \frac{2}{z} = \pm 6 \Rightarrow z^2 - 2 = \pm 6z$

$\Rightarrow z^2 - 6z - 2 = 0$ or $z^2 + 6z - 2 = 0$

$\Rightarrow z = \frac{6 \pm \sqrt{36+8}}{2}$ or $z = \frac{-6 \pm \sqrt{36+8}}{2}$

$\Rightarrow z = \frac{6 \pm \sqrt{44}}{2}$ or $z = \frac{-6 \pm \sqrt{44}}{2}$

$\Rightarrow z = \frac{6 \pm 2\sqrt{11}}{2}$ or $z = \frac{-6 \pm 2\sqrt{11}}{2}$

$\Rightarrow z = 3 \pm \sqrt{11}$ or $z = -3 \pm \sqrt{11}$

$\Rightarrow z = 3+\sqrt{11}, 3-\sqrt{11}, -3+\sqrt{11}, -3-\sqrt{11}$

$\Rightarrow |z| = |3 + \sqrt{11}|, |3 - \sqrt{11}|,$
$|-3 + \sqrt{11}|, |-3-\sqrt{11}|$

$\Rightarrow |z| = (3+\sqrt{11}), (\sqrt{11}-3), (\sqrt{11}-3), (3+\sqrt{11})$

So, maximum value of $|z| = 3 + \sqrt{11}$

and minimum value of $|z| = \sqrt{11} - 3$

NDA (MATHS) MOCK TEST - 100 (Answer Key)

1.	(B)	21.	(A)	41.	(B)	61.	(B)	81.	(B)	101.	(D)
2.	(D)	22.	(A)	42.	(B)	62.	(D)	82.	(C)	102.	(C)
3.	(D)	23.	(A)	43.	(A)	63.	(D)	83.	(A)	103.	(A)
4.	(D)	24.	(B)	44.	(B)	64.	(C)	84.	(D)	104.	(B)
5.	(B)	25.	(C)	45.	(D)	65.	(A)	85.	(D)	105.	(B)
6.	(A)	26.	(A)	46.	(A)	66.	(B)	86.	(B)	106.	(A)
7.	(B)	27.	(B)	47.	(B)	67.	(C)	87.	(B)	107.	(A)
8.	(C)	28.	(A)	48.	(A)	68.	(A)	88.	(A)	108.	(A)
9.	(B)	29.	(B)	49.	(D)	69.	(D)	89.	(D)	109.	(B)
10.	(D)	30.	(B)	50.	(A)	70.	(C)	90.	(C)	110.	(B)
11.	(D)	31.	(B)	51.	(A)	71.	(A)	91.	(C)	111.	(B)
12.	(A)	32.	(C)	52.	(D)	72.	(D)	92.	(C)	112.	(B)
13.	(D)	33.	(D)	53.	(C)	73.	(C)	93.	(C)	113.	(D)
14.	(A)	34.	(D)	54.	(B)	74.	(C)	94.	(D)	114.	(D)
15.	(D)	35.	(C)	55.	(B)	75.	(B)	95.	(C)	115.	(C)
16.	(B)	36.	(A)	56.	(C)	76.	(A)	96.	(A)	116.	(B)
17.	(B)	37.	(C)	57.	(B)	77.	(A)	97.	(C)	117.	(B)
18.	(A)	38.	(B)	58.	(A)	78.	(A)	98.	(D)	118.	(A)
19.	(D)	39.	(C)	59.	(A)	79.	(B)	99.	(B)	119.	(B)
20.	(C)	40.	(C)	60.	(A)	80.	(C)	100.	(D)	120.	(D)

Note : *If your opinion differ regarding any answer, please message the mock test and Question number to 8860330003*

Note : *If you face any problem regarding result or marks scored, please contact : 9313111777*

Note : *Whatsapp with Mock Test No. and Question No. at 705360571 for any of the doubts. Join the group and you may also share your sugesstions and experience of Sunday Mock Test.*