NDA MATHS MOCK TEST - 104 (SOLUTION)

1. (A) Total number of arrangements

$= \frac{10!}{2!2!2!2!} = \frac{10!}{16}$

$\Rightarrow$ The total number of arrangements when I's come together $= \frac{9!}{2!\,2!\,2!} = \frac{9!}{8}$

$\Rightarrow$ The total number of arrangements when I's do not come together

$= \frac{10!}{16} - \frac{9!}{8} = \frac{9!}{2}$

$\therefore$ The required probabitlity

$= \frac{\frac{9!}{2}}{\frac{10!}{16}} = \frac{4}{5}$

2. (C) $\left[\frac{\sin\frac{\pi}{2} - i\left(1-\cos\frac{\pi}{2}\right)}{\sin\frac{\pi}{2} + i\left(1-\cos\frac{\pi}{2}\right)}\right]^3$

$\Rightarrow \left[\frac{1-i(1-0)}{1+i(1-0)}\right]^3$

$\Rightarrow \left[\frac{1-i}{1+i}\right]^3$

$\Rightarrow \left[\frac{(1-i)(1-i)}{(1+i)(1-i)}\right]^3$

$\Rightarrow \left(\frac{-2i}{2}\right)^3$

$\Rightarrow (-i)^3 = -i^3 = -(-i) = i$

3. (C) Let $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ and $z = \tan^{-1}x$

$x = \tan z$

$\Rightarrow y = \sin^{-1}\left(\frac{\tan z}{\sqrt{1+\tan^2 z}}\right)$

$\Rightarrow y = \sin^{-1}\left(\frac{\tan z}{\sec z}\right)$

$\Rightarrow y = \sin^{-1}(\sin z)$

$\Rightarrow y = z$

On diffrentiating both side w. r. t. 'z'

$\Rightarrow \frac{dy}{dz} = 1$

4. (D) A.T.Q,

$2b = \frac{2}{3} \times 2a \Rightarrow b = \frac{2a}{3}$

Now, eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{1 - \frac{4a^2}{9a^2}}$

$\Rightarrow e = \sqrt{\frac{5}{9}} \Rightarrow e = \frac{\sqrt{5}}{3}$

5. (D) $\cos^2 43\frac{1}{2}^\circ + \cos^2 46\frac{1}{2}^\circ$

$\Rightarrow \cos^2 43\frac{1}{2}^\circ + \cos^2\left(90 - 43\frac{1}{2}^\circ\right)$

$\Rightarrow \cos^2 43\frac{1}{2}^\circ + \sin^2 43\frac{1}{2}^\circ = 1$

6. (B) $\sin\left[\cos^{-1}\left(\cos\left(\frac{13\pi}{4}\right)\right)\right]$

$\Rightarrow \sin\left[\cos^{-1}\left(\cos\left(2\pi + \frac{5\pi}{4}\right)\right)\right]$

$\Rightarrow \sin\left[\cos^{-1}\left(\cos\frac{5\pi}{4}\right)\right]$

$\Rightarrow \sin\left[\cos^{-1}\left(\cos\left(\pi + \frac{\pi}{4}\right)\right)\right]$

$\Rightarrow \sin\left[\left\{\cos^{-1}\left(-\cos\frac{\pi}{4}\right)\right\}\right]$

$\Rightarrow \sin\left[\cos^{-1}\left(\cos\frac{3\pi}{4}\right)\right]$

$\Rightarrow \sin\frac{3\pi}{4}$

$\Rightarrow \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

7. (A) I = $\int x \cdot e^{x^2} \log x \, dx + \int \frac{e^{x^2}}{2x} dx$

I = $\frac{1}{2}\int 2x \cdot e^{x^2} \log x \, dx + \int \frac{e^{x^2}}{2x} dx$

I = $\frac{1}{2}\left[\log x \cdot \int 2x e^{x^2} dx - \int \left\{\frac{d}{dx}(\log x) \int 2x \cdot e^{x^2} dx\right\} dx\right]$

$+ \int \frac{e^{x^2}}{2x} dx$

I = $\frac{1}{2}\left[(\log x) e^{x^2} - \int \frac{1}{x} \cdot e^{x^2} dx\right] + \int \frac{e^{x^2}}{2x} dx$

I = $\frac{1}{2} e^{x^2} \cdot \log x - \int \frac{e^{x^2}}{2x} dx + \int \frac{e^{x^2}}{2x} dx + c$

I = $\frac{1}{2} e^{x^2} \cdot \log x + c$

8. (C) A = $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 32 & 32 \\ 32 & 32 \end{bmatrix} = \begin{bmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{bmatrix}$

Thus, $A^n = \begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$

9. (B) 1 0 0 1 1

$1 \times 2^0 = 1$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

$0 \times 2^3 = 0$

$1 \times 2^4 = 16$

19

$(10011)_2 = (17)_{10}$

and

0.101

$\frac{1}{2} = 1 \times 2^{-1}$

$0 = 0 \times 2^{-2}$

$\frac{1}{8} = 1 \times 2^{-3}$

$\frac{1}{2} + \frac{1}{8} = \frac{5}{8} = 0.625$

$(0.101)_2 = (0.625)_{10}$

Hence, $(10011.101)_2 = (19.625)_{10}$

10. (A) We Know that,

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3$ ....+ $C_n x^n$

On differeticating both side w.r. t. 'x'

$\Rightarrow x(1 + x)^{n-1} = 0 + C_1 + 2C_2 x + 3C_3 x^2 + \ldots\ldots + nC_n x^{n-1}$

On putting $x = 1$

$\Rightarrow n(1 + 1)^{n-1} = C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n$

$\Rightarrow C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n = n.2^{n-1}$

11. (B) Given that

$\int x^2 . \ln x \, dx = \frac{x^3}{a} \ln x + \frac{x^3}{b} + c$ ....(i)

Let $I = \int x^2 . \ln x \, dx$

I = $\ln x \int x^2 dx - \int \left\{\frac{d}{dx}(\ln x) . \int x^2 dx\right\} dx$

I = $\ln x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$

I = $\frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$

I = $\frac{x^3}{3} \ln x - \frac{1}{9} \cdot x^3 + C$

On Comparing with equation (i)

$a = 3$ and $b = -9$

12. (C) 4 digit numbers formed from the digits 1, 2, 3, 4 , 5, 6 , 7

7	7	7	7

= 7 × 7 × 7 × 7 = 2401

13. (D) P(29, 25) = k.C(29, 4)

$\Rightarrow \frac{29!}{4!} = k \cdot \frac{29!}{4!\,25!} \Rightarrow k = 25!$

14. (A) Let $a - ib = \sqrt{47 - 14\sqrt{2}i}$

On squaring both side

$(a^2 - b^2) - 2abi = 47 - 14\sqrt{2}i$

On Comparing

$a^2 - b^2 = 47$ and $2ab = 14\sqrt{2}$ ...(i)

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = (47)^2 + \left(14\sqrt{2}\right)^2$

$(a^2 + b^2)^2 = 2209 + 392$

$(a^2 + b^2)^2 = 2601$

$a^2 + b^2 = 51$ ...(ii)

from equation (i) and (ii)

$2a^2 = 98$ and $2b^2 = 4$

$a = \pm 7$, $b = \pm \sqrt{2}$

Hence, $\sqrt{47 - 14\sqrt{2}i} = \pm\left(7 - \sqrt{2}i\right)$

15. (B) $\cos^4\theta - \sin^4\theta = (\cos^2\theta)^2 - (\sin^2\theta)^2$

$\Rightarrow \cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow \cos^4\theta - \sin^4\theta = \cos 2\theta \cdot 1$

$\Rightarrow \cos^4\theta - \sin^4\theta = 2\cos^2\theta - 1$

16. (A) angle describe in 12 hr = 360°

angle describe in 1 hr (60 min) = $\frac{360}{12}$

angle describe in 1 min = $\frac{360}{12\times 60}$

angle describe in 12 min

$= \frac{360}{12\times 60} \times 12 = 6°$

17. (C) Given that

$\sin x \cdot \cos x = \frac{1}{2}$

$\Rightarrow 2\sin x \cdot \cos x = 1$

$\Rightarrow \sin 2x = \sin 90$

$\Rightarrow 2x = 90 \Rightarrow x = 45$

Now,

$\sec^n x + \text{cosec}^n x = (\sec 45)^n + (\text{cosec} 45)^n$

$= \left(\sqrt{2}\right)^n + \left(\sqrt{2}\right)^n = 2^{\frac{n+2}{2}}$

18. (D) Let $y = 3^{49}$

taking log both side

$\log_{10} y = 49 \log_{10} 3$

$\log_{10} y = 49 \times 0.4771 = 23.3779$

Hence the number of digits = 23 + 1 = 24

19. (C) $\tan(-1020) = -\tan(1020)$

$= -\tan(360 \times 3 - 60)$

$= -(-\tan 60) = \sqrt{3}$

20. (A) **Statement 1**

L.H.S = $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

$= \tan^2\theta \cdot \cot^2\theta = 1$ = R.H.S

Statement 1 is correct.

Statment 2

L.H.S = $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} + \frac{2\sin^2\frac{\theta}{2}\cdot 2\cos^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$

$= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} + \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$= \frac{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} = \frac{1\times 2}{2\sin\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}$

$= \frac{2}{\sin\theta} = 2\text{cosec}\theta \neq$ R.H.S

statement 2 is incorrect.

Hence only statement 1 is correct.

21. (B) $ax^2 - x + c = 0$

Let roots = α and $\frac{1}{\alpha}$

$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} \quad \Rightarrow c = a$

22. (B) A' = cofactor of A

$|A'| = |\text{cofactor of } A|$

$|A'| = (A)^{4-1}$ [$\because$ order = 4]

$|A'| = A^3$

23. (A) Given that = A = $\begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix}$

(A) = –3 × 2 – 5 + 1 = –11

We know that,

A (Adj A) = $|A|\ I_n$

A (Adj A) = $-11\ I_2$

A (Adj A) = $-11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A (Adj A) = $\begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$

24. (B) $\left(\frac{d^2y}{dx^2}\right)^{2/3} + 3\frac{d^3y}{dx^3} = \frac{dy}{dx}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{2/3} = \left[\frac{dy}{dx} - 3\frac{d^3y}{dx^3}\right]^3$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^3 - 27\left(\frac{d^3y}{dx^3}\right)^3$

$- 9\frac{dy}{dx}\left(\frac{d^3y}{dx^3}\right)\left[\frac{dy}{dx} - 3\frac{d^3y}{dx^3}\right]$

Order = 3 and degree = 3

25. (C) $f(x) = \begin{cases} x-3, & x \le 4 \\ 7x+\lambda, & x > 4 \end{cases}$ is continuous at $x = 4$, then

$\Rightarrow \lim_{x\to 4^+} f(x) = f(4)$

$\Rightarrow \lim_{x\to 4} (7x+\lambda) = 4 - 3$

$\Rightarrow 7 \times 4 + \lambda = 1 \Rightarrow \lambda = -27$

26. (B) $y = \left(1 - x^{\frac{1}{8}}\right)\left(1 + x^{\frac{1}{4}}\right)\left(1+x^{\frac{1}{2}}\right)\left(1+x^{\frac{1}{8}}\right)$

$y = \left(1+x^{\frac{1}{2}}\right)\left(1+x^{\frac{1}{4}}\right)\left(1-x^{\frac{1}{8}}\right)\left(1+x^{\frac{1}{8}}\right)$

$y = \left(1+x^{\frac{1}{2}}\right)\left(1+x^{\frac{1}{4}}\right)\left(1^2-\left(x^{\frac{1}{8}}\right)^2\right)$

$y = (1 + x^{1/2})(1 + x^{1/4})(1 - x^{1/4})$

$y = \left(1+x^{\frac{1}{2}}\right)\left(1-x^{\frac{1}{2}}\right)$

$y = 1 - x$

On differentiting both side w.r.t. 'x'

$\frac{dy}{dx} = -1$

27. (C) Let $I = \int_{-\pi/2}^{\pi/2} \frac{sin^3 x}{cos\, x} dx$

$= 0$ [$\because$ function is an odd.]

28. (C) Planes

$2x - y + z = 6$ and $x - 2y - z = 11$

angle between planes

$\cos\theta = \frac{2\times1+(-1)\times(-2)+1\times(-1)}{\sqrt{(2)^2+(-1)^2+(1)^2}\sqrt{(1)^2+(-2)^2+(-1)^2}}$

$\cos\theta = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6}$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

29. (C) $\lim_{x\to 0} \frac{\sin x - \tan x}{\sin x + \tan x}$

by L - Hospital's rule

$\Rightarrow \lim_{x\to 0} \frac{\cos x - \sec^2 x}{\cos x + \sec^2 x}$

$\Rightarrow \lim_{x\to 0} \frac{\cos x - \frac{1}{\cos^2 x}}{\cos x + \frac{1}{\cos^2 x}}$

$\Rightarrow \lim_{x\to 0} \frac{\cos^3 x - 1}{\cos^3 x + 1}$

$\Rightarrow \frac{(\cos 0)^3 - 1}{(\cos 0)^3 + 1} = \frac{1-1}{1+1} = 0$

30. (D) 71, 70, 72, 75,76, 69, 68, 71, 70, 66

Mean

$= \frac{71+70+72+75+76+69+68+71+70+66}{10}$

$= \frac{708}{10} = 70.8$

The required number of students = 5

31. (A) Let $z = \frac{(1-3i)(2+i)}{1-i}$

$z = \frac{5-5i}{1-i}$

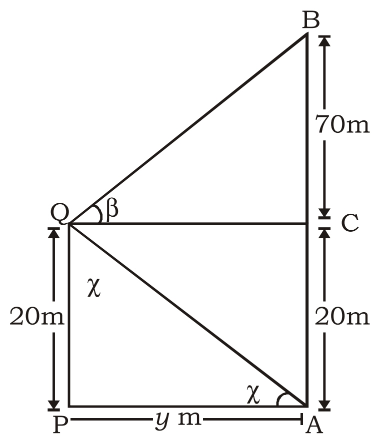
$z = \frac{5(1-i)}{(1-i)} = 5$

$\arg(z) = \tan^{-1}\left(\frac{0}{1}\right) = 0$

32. (B) $\sin x \frac{dy}{dx} - y = x$

$\frac{dy}{dx} - y \operatorname{cosec} x = x \operatorname{cosec} x$

33. (C) **Case I:-**



In ΔPQA

$\tan\beta = \frac{PQ}{PA}$

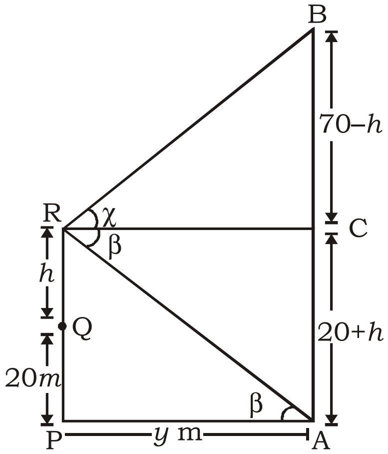
$\tan\beta = \frac{20}{PA}$ .... (i)

In ΔBCQ:-

$\tan\alpha = \frac{BC}{QC}$

$\tan\alpha = \frac{70}{PA}$ .... (ii)

Case II:-



Let he climps h m.

In ΔPAR:-

$\tan\alpha = \dfrac{PR}{PA}$

$\tan\alpha = \dfrac{20+h}{PA}$ ....(iii)

In ΔBCR:-

$\tan\beta = \dfrac{BC}{RC}$

$\tan\beta = \dfrac{70-h}{PA}$ .... (iv)

from equation (i) and (iv) or equation (ii) and equation (iii)

$\dfrac{20}{\text{PA}} = \dfrac{70-h}{\text{PA}}$ or $\dfrac{70}{\text{PA}} = \dfrac{20+h}{\text{PA}}$

$\boxed{h = 50\,m}$ $\boxed{h = 50\,m}$

34. (C) Equation

$3x^2 + 4x + 2 = 0$

$\alpha + \beta = \dfrac{-4}{3}$

$\alpha.\beta = \dfrac{2}{3}$

Now,

$\alpha + \alpha^{-1} + \beta + \beta^{-1} = \alpha + \beta + \dfrac{1}{\alpha} + \dfrac{1}{\beta}$

sum of roots $= \alpha + \beta + \dfrac{\alpha+\beta}{\alpha\beta}$

sum of roots $= \dfrac{-4}{3} + \dfrac{\frac{-4}{3}}{\frac{2}{3}} = \dfrac{-10}{3}$

$(\alpha + \beta^{-1})(\beta + \alpha^{-1}) = \alpha\beta + 1 + 1 + \alpha^{-1}\beta^{-1}$

$= 2\beta + 2 + \dfrac{1}{\beta\beta}$

product of roots $= \dfrac{2}{3} + 2 + \dfrac{3}{2} = \dfrac{25}{6}$

The required equation

$x^2 -$ (sum of the roots)x + product of roots = 0

$x^2 - \left(-\dfrac{10}{3}\right)x + \dfrac{25}{6} = 0$

$\Rightarrow 6x^2 + 20x + 25 = 0$

35. (B) $\begin{vmatrix} 1-a & a^2 & a-a^2 \\ 1-c & c^2 & c-c^2 \\ 1-b & b^2 & b-b^2 \end{vmatrix}$

$C_3 \to C_3 + C_2$

$\Rightarrow \begin{vmatrix} 1-a & a^2 & a \\ 1-c & c^2 & c \\ 1-b & b^2 & b \end{vmatrix}$

$C_1 \to C_1 + C_3$

$\Rightarrow \begin{vmatrix} 1 & a^2 & a \\ 1 & c^2 & c \\ 1 & b^2 & b \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & a^2 & a \\ 0 & c^2-a^2 & c-a \\ 0 & b^2-a^2 & b-a \end{vmatrix}$

$\Rightarrow (c-a)\,(b-a) \begin{vmatrix} 1 & a^2 & a \\ 0 & c+a & 1 \\ 0 & b+a & 1 \end{vmatrix}$

$R_2 \to R_2 - R_3$

$\Rightarrow (c-a)\,(b-a) \begin{vmatrix} 1 & a^2 & a \\ 0 & c-b & 0 \\ 0 & b+a & 1 \end{vmatrix}$

$\Rightarrow (c-a)\,(b-a)\,[1\{(c-b)-0\} + a^2 .0 + a .0]$

$\Rightarrow (c-a)(b-a)\,(c-b)$

$\Rightarrow (a-b)(b-c)\,(c-a)$

36. (A) We know that,

minimum value of $\left(ax^2 + \dfrac{b}{x^2}\right) = 2\sqrt{ab}$

So, minimum value of $(98\sin^2\theta + 50\,\text{cosec}^2\theta)$

$= 2\sqrt{98 \times 50} = 140$

37. (C) Let $I = \int_0^{\pi} \frac{\phi\left(\frac{x}{2}\right)}{\phi\left(\frac{x}{2}\right)+\phi\left(\frac{\pi-x}{2}\right)} dx$ ...(*i*)

$I = \int_0^{\pi} \frac{\phi\left(\frac{\pi-x}{2}\right)}{\phi\left(\frac{\pi-x}{2}\right)+\phi\left(\frac{x}{2}\right)} dx$

from equation (i) and equation (ii)

$I + I = \int_0^{\pi} \frac{\phi\left(\frac{x}{2}\right)+\phi\left(\frac{\pi-x}{2}\right)}{\phi\left(\frac{x}{2}\right)+\phi\left(\frac{\pi-x}{2}\right)} dx$

$2I = \int_0^{\pi} 1.dx$

$2I = [x]_0^{\pi}$

$2I = \pi - 0 \Rightarrow I = \frac{\pi}{2}$

38. (B) Given that,

$\Rightarrow \int_{-1}^{0}[-4+f(x)]\,dx = 9$

$\Rightarrow -\int_{-1}^{0} 4\,dx + \int_{-1}^{0} f(x)dx = 9$

$\Rightarrow -4[x]_{-1}^{0} + \int_{-1}^{0} f(x)dx = 9$

$\Rightarrow -4 + \int_{-1}^{0} f(x)dx = 9$

$\Rightarrow \int_{-1}^{0} f(x)dx = 13$

Now,

$\int_{-4}^{0} f(x)dx = \int_{-4}^{-1} f(x)dx + \int_{-1}^{0} f(x)dx$

$7 = \int_{-4}^{-1} f(x)dx + 13$

$\int_{-4}^{-1} f(x)dx = -6$

39. (C) We know that

No. of diagonals $= \frac{n(n-3)}{2}$

$\Rightarrow \quad 65 = \frac{n(n-3)}{2}$

$\Rightarrow n^2 - 3n - 130 = 0$

$\Rightarrow (n-13)(n+10) = 0$

$n = 13, -10$

Number of sides = 13

40. (C) Equation

$x^2 - 3x + 21 = 0$

Now,

$b^2 - 4ac = (-3)^2 - 4 \times 1 \times 21$

$b^2 - 4ac = 9 - 84 = -75 < 0$

Hence, roots are imaginary.

41. (B) $(\overline{A} \cap B \cap C)$

42. (C) One leap year = 366 days

= 52 weeks and 2 days

S = {(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon)}

$n(S) = 7$

E = (Thu, Fri)

$n(E) = 1$

The required probability $= \frac{n(E)}{n(S)} = \frac{1}{7}$

43. (C) Data 2, 3, 9 , 8, 7, 5, 8, 13, 18, 19

$n = 10$

$\sum_{i=0}^{n} x_i = 2+3+9+8+7+5+8+13+18+19$

$= 92$

$\sum_{i=0}^{n} x_i^2 = 2^2 + 3^2 + 9^2 + 8^2 + 7^2 + 5^2 + 8^2 + 13^2 + 18^2 + 19^2 = 1150$

S.D $(\sigma) = \sqrt{\frac{\sum_{i=0}^{n} x_i^2}{n} - \left(\frac{\sum_{i=0}^{n} x_i}{n}\right)^2}$

S.D $(\sigma) = \sqrt{\frac{1150}{10} - \left(\frac{92}{10}\right)^2}$

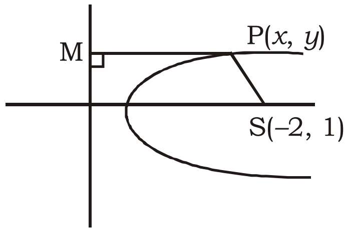
S.D $(\sigma) = \sqrt{\frac{1150}{10} - \frac{8464}{100}}$

S.D $(\sigma) = \sqrt{\frac{3036}{100}}$

Variance = $(\text{S.D})^2$

$= \left(\sqrt{\frac{3036}{100}}\right)^2 = \frac{3036}{100} = 30.36$

44. (A)



$PS^2 = PM^2$

$\Rightarrow \left[\sqrt{(x+2)^2+(y-1)^2}\right]^2 = \left[\dfrac{2x-5y-7}{\sqrt{(2)^2+(-5)^2}}\right]^2$

$\Rightarrow x^2 + 4 + 4x + y^2 + 1 - 2y$

$= \dfrac{4x^2+25y^2+49-20xy+70y-28x}{4+25}$

$\Rightarrow 29x^2 + 116 + 116x + 29y^2 + 29 - 58y$

$= 4x^2 + 25y^2 + 49 - 20xy + 70y - 28x$

On solving

$\Rightarrow 25x^2 + 4y^2 + 20xy + 144x - 128y + 96 = 0$

45. (A) $[(A \cup B) \cup (C \cap D)]' = (A \cup B)' \cap (C \cap D)'$

$[(A \cup B) \cup (C \cap D)]' = [(A' \cap B') \cap (C' \cup D')]$

46. (B) $I = \int \dfrac{dx}{\cos x\sqrt{\sin 2x}}$

$I = \int \dfrac{dx}{\cos x\sqrt{2\sin x.\cos x}}$

$I = \dfrac{1}{\sqrt{2}}\int \dfrac{dx}{(\cos x)^{3/2}\cdot(\sin x)^{1/2}}$

$I = \dfrac{1}{\sqrt{2}}\int \dfrac{dx}{\cos^2 x(\tan x)^{1/2}}$

$I = \dfrac{1}{\sqrt{2}}\int \dfrac{\sec^2 x\,dx}{(\tan x)^{1/2}}$

Let $\tan x = t$

$\sec^2 x\,dx = dx$

$I = \dfrac{1}{\sqrt{2}}\int \dfrac{dt}{t^{1/2}}$

$I = \dfrac{1}{\sqrt{2}}\ \dfrac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C$

$I = \sqrt{2}\ \sqrt{t} + C$

$I = \sqrt{2\tan x} + C$

47. (A) $I = \int_0^1 \sqrt{\dfrac{1+x}{1-x}}\,dx$

$I = \int_0^1 \sqrt{\dfrac{1+x}{1-x}\times\dfrac{1+x}{1+x}}\ dx$

$I = \int_0^1 \dfrac{1+x}{\sqrt{1-x^2}}\ dx$

$I = \int_0^1 \dfrac{1}{\sqrt{1-x^2}}\,dx + \int_0^1 \dfrac{1}{\sqrt{1-x^2}}\,dx$

Let $1 - x^2 = t$ When $x\to 0,\ t\to 1$

$-2x\,dx = dt$ $x\to 1,\ t\to 0$

$xdx = \dfrac{-1}{2}\ dt$

$I = [\sin^{-1}x]_0^1 - \dfrac{1}{2}\int_1^0 \dfrac{dt}{\sqrt{t}}$

$I - \sin^{-1}{}_1 - \sin^{-10} - \dfrac{-1}{2}\left[\dfrac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}\right]_1^0$

$I = \dfrac{\pi}{2} - 0 - [0 - (1)^{1/2}]$

$I = \dfrac{\pi}{2} + 1$

48. (C) $y = cot^{-1}\left[\dfrac{x-1}{x^{1/3}\left(x^{1/3}+1\right)}\right]$

$\Rightarrow y = cot^{-1}\left[\dfrac{x^{2/3}.x^{1/3}-1}{x^{2/3}+x^{1/3}}\right]$

$\Rightarrow y = cot^{-1}\left[cot\left(x^{2/3}+x^{1/3}\right)\right]$

$\Rightarrow y = x^{2/3} + x^{1/3}$

On differentiating both side w.r.t. 'x'

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2}{3}x^{\frac{2}{3}-1} + \dfrac{1}{3}x^{\frac{2}{3}-1}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2}{3}x^{-1/3} + \dfrac{1}{3}x^{-2/3}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}\left[\dfrac{2}{x^{1/3}} + \dfrac{1}{x^{2/3}}\right]$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3}\left[\dfrac{2x^{1/3}+1}{x^{2/3}}\right]$

49. (B) $a + 46d = 434$ ....(i)

$a + 433d = 47$ ....(ii)

from equation (i) and equation (ii)

$d = -1$ and $a = 480$

let n^{th} term is 0.

then $0 = a + (n-1)d$

$\Rightarrow 0 = 480 + (n-1)(-1)$

$\Rightarrow n-1 = 480 \quad \Rightarrow n = 481$

50. (D) matrix A $\rightarrow y \times (y-7)$

matrix B $\rightarrow x \times (9-x)$

Both AB and BA exist,

then $y - 7 = x \quad \Rightarrow x - y = -7$ ...(i)

and $y = 9 - x \quad \Rightarrow x + y = 9$ ...(ii)

from equation (i) and (ii)

$x = 1$ and $y = 8$

51. (A) equation

$ax^2 + cx - b = 0$

roots are cot (B/2) and cot (C/2).

then $cot\frac{B}{2} + cot\frac{C}{2} = \frac{-c}{a}$

and $cot\frac{B}{2}.cot\frac{C}{2} = \frac{-b}{a}$

Now,

$$cot\left(\frac{B}{2}+\frac{C}{2}\right) = \frac{cot\frac{B}{2}.cot\frac{C}{2} - 1}{cot\frac{B}{2} + cot\frac{C}{2}}$$

$$cot\left(\frac{180-A}{2}\right) = \frac{-\frac{b}{a} - 1}{\frac{-c}{a}}$$

$$tan\frac{A}{2} = \frac{-b-a}{-c}$$

We know that A =90°

$$tan 45° = \frac{b+a}{c}$$

c = a+b

52. (B) We know that

AM $\geq$ G.M $\geq$ H.M

Hence $\frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2ab}{a+b}$

$\frac{2ab}{a+b} \leq \sqrt{ab} \leq \frac{a+b}{2}$

53. (C) $y = 9 - 9^{1/3} + 9^{2/3}$

$\Rightarrow y - 9 = 9^{2/3} - 9^{1/3}$ ...(i)

$\Rightarrow (y-9)^3 = (9^{2/3} - 9^{1/3})^3$

$\Rightarrow y^3 - 729 - 3 \times y \times 9(y-9)$

$= 9^2 - 9 - 3 \times 9^{2/3} \times 9^{1/3}(9^{2/3} - 9^{1/3})$

$\Rightarrow y^3 - 729 - 27y^2 + 243y = 81 - 9 - 27(y-9)$

from equation (i)

$\Rightarrow y^3 - 27y^2 + 270y - 1044 = 0$

$\Rightarrow y^3 - 27y^2 + 270y - 44 = 1044 - 44$

$\Rightarrow y^3 - 27y^2 + 280y - 44 = 1000$

54. (D) direction ratio (3,–1,–2) and (2,y,–3) angle between lines

$$cos\theta = \frac{3\times2+(-1)\times y+(-2)\times(-3)}{\sqrt{9+1+4}\sqrt{4+y^2+9}}$$

$$cos\frac{\pi}{2} = \frac{6-y+6}{\sqrt{14}\sqrt{y^2+13}}$$

$$0 = \frac{12-y}{\sqrt{14}\sqrt{y^2+13}}$$

$12 - y = 0 \Rightarrow y = 12$

55. (A) According to question;

$$\frac{a+b}{2} = 4\sqrt{ab}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{4}{1}$$

by Componendo and Dividendo Rule

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{4+1}{4-1}$$

$$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{5}{3}$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+-\sqrt{b}} = \frac{\sqrt{5}}{\sqrt{3}}$$

by Componendo and Dividendo Rule

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

On squaring both side

$$\Rightarrow \frac{a}{b} = \frac{4+\sqrt{15}}{4-\sqrt{15}}$$

by Componendo and Dividendo Rule

$$\Rightarrow \frac{a-b}{a+b} = \frac{2\sqrt{15}}{8} \quad \Rightarrow \frac{a-b}{a+b} = \frac{\sqrt{15}}{4}$$

56. (B) The required remainder = 1

57. (A) word " STILL"

No. of words start with 'I' = $\frac{4!}{2!} = 12$

No. of words start with 'L' = 4! = 24

No. of words start with 'SI' = $\frac{3!}{2!} = 3$

No. of words start with 'SL' $\rightarrow$ 3! = 6

word STILL $\rightarrow$ 1

Position of word 'STILL = 12 + 24 + 3 + 6 + 1 = 46th

58. (B) $\lim_{x\to5}\frac{\sqrt{2x-6}-2}{\sqrt{5x}-5}$ $\left[\frac{0}{0}\right]$ from

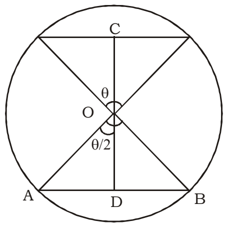
by L- Hospital's Rule

$$\Rightarrow \lim_{x\to5}\frac{\frac{1}{2}(2x-6)^{-\frac{1}{2}}(2)-0}{\frac{1}{2}(5x)^{-\frac{1}{2}}(5)-0}$$

$$\Rightarrow \lim_{x\to5}\frac{2(5x)^{\frac{1}{2}}}{5(2x-6)^{\frac{1}{2}}}$$

$$\Rightarrow \frac{2.(5\times5)^{\frac{1}{2}}}{5.(2\times5-6)^{\frac{1}{2}}} \Rightarrow \frac{2\times5}{5\times2}=1$$

59. (A)



given, $\angle$ AOB = θ = $sin^{-1}\left(\frac{4}{5}\right)$

$sin\theta = \frac{4}{5}$

Now, $cos\theta = \frac{4}{5}$

$2cos^2\frac{\theta}{2}-1=\frac{3}{5}$

$2cos^2\frac{\theta}{2}=\frac{8}{5}$

Now, $cos^2\frac{\theta}{2}=\frac{4}{5}$

$cos\frac{\theta}{2}=\frac{2}{\sqrt{5}}$

OD = $\frac{2}{\sqrt{5}}$ × AO

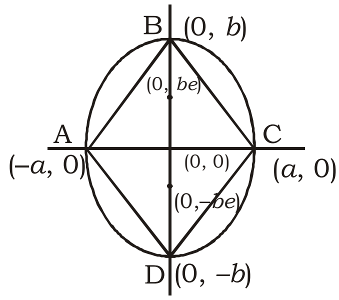
OD = $\frac{2}{\sqrt{5}}$ ×10

Now, CD = 2 × OD

CD = 2 × $\frac{2}{\sqrt{5}}$ × 10

CD = $\frac{40}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$ ⇒ CD = $8\sqrt{5}$ unit

60. (C)



Given that $e = \frac{12}{13}$

and $2be = 24$

$2 \times b \times \frac{12}{13} = 24 \Rightarrow b = 13$

Now,

$e = \sqrt{1-\frac{a^2}{b^2}}$

$\frac{12}{13} = \sqrt{1-\frac{a^2}{169}}$

$\frac{144}{169} = 1-\frac{a^2}{169}$

$\frac{25}{169} = \frac{a^2}{169} \Rightarrow a = 5$

In ΔAOB:

Area of ΔAOB = $\frac{1}{2}\times a\times b = \frac{1}{2}\times5\times13=\frac{65}{2}$

Area of ▱ ABCD = 4 × area of ΔAOB

= 4 × $\frac{65}{2}$ = 130 sq. unit

61. (B) G. M. of a and b = $\frac{a+b}{2} = \frac{a^{n-5}+b^{n-5}}{a^{n-6}+a^{n-6}}$

On comparing

$n = 6$

62. (C) I = $\int_{\pi/6}^{\pi/2} \frac{16\cos x}{\left[\tan\frac{x}{2}+\cot\frac{x}{2}\right]^4}\,dx$

$\Rightarrow$ I = $\int_{\pi/6}^{\pi/2} \frac{16\cos x}{\left[\frac{1}{\sin\frac{x}{2}.\cos\frac{x}{2}}\right]^4}\,dx$

$\Rightarrow$ I = $\int_{\pi/6}^{\pi/2} 16\cos x\left(\sin\frac{x}{2}.\cos\frac{x}{2}\right)^4 dx$

$\Rightarrow$ I = $\int_{\pi/6}^{\pi/2} \cos x\left(2\sin\frac{x}{2}.\cos\frac{x}{2}\right)^4 dx$

$\Rightarrow$ I = $\int_{\pi/6}^{\pi/2} \cos x(\sin x)^4\,dx$

let sin x = t *when,* $x \to \frac{\pi}{6}$, $t \to \frac{1}{2}$

cosx dx = dt $x \to \frac{\pi}{2}$, $t \to 1$

$\Rightarrow$ I = $\int_{1/2}^{1} t^4\,dx$

$\Rightarrow$ I = $\left[\frac{t^5}{5}\right]_{1/2}^{1}$

$\Rightarrow$ I = $\frac{1}{5}\left(1-\left(\frac{1}{2}\right)^5\right)$

$\Rightarrow$ I = $\frac{1}{5}\times\frac{31}{32} = \frac{31}{160}$

63. (B) $\frac{\log 5\times\log_7 9}{2\log_7 4\times\log_2 5}$

$\Rightarrow \frac{\log_7 3^2\times\log_3 5}{2\log_7 2^2\times\log_2 5}$

$\Rightarrow \frac{2\log_7 3\times\log_3 5}{4\log_7 2\ \times\log_2 5}$

$\Rightarrow \frac{2\log_7 5}{4\log_7 5} = \frac{1}{2}$

64. (A) Given that $(x-iy)^2 = 20-21i$ ...(i)

$(x^2-y^2) - 2xyi = 20 - 21i$

$x^2 - y^2 = 20$ and $2xy = 21$ ...(ii)

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$(x^2+y^2)^2 = 400 + 441$

$x^2 + y^2 = 841$

$x^2 + y^2 = 29$ ...(iii)

from equation (i) and (ii)

$2x^2 = 49$ and $2y^2 = 8$

$x = \pm\frac{7}{\sqrt{2}}$, $y = \pm 2$

Now ,

$\Rightarrow \left(20+\sqrt{-441}\right)^{1/2}+\left(20-\sqrt{-441}\right)^{1/2}$

$\Rightarrow (20+21i)^{1/2} + (20-21i)^{1/2}$

$\Rightarrow x+iy + x-iy$

$\Rightarrow 2x = 2\times\frac{7}{\sqrt{2}} = 7\sqrt{2}$

65. (C) $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^4\\ 1 & \omega^2 & \omega^7\end{vmatrix} = 2k$

$\Rightarrow \begin{vmatrix}1 & 1 & 1\\ 1 & \omega & \omega\\ 1 & \omega^2 & \omega\end{vmatrix} = 2k$

$\Rightarrow 1(\omega^2-\omega^3) - 1(\omega-\omega) + 1(\omega^2-\omega) = 2k$

$\Rightarrow \omega^2 - 1 - 0 + \omega^2 - \omega = 2k$

$\Rightarrow 2\omega^2 + \omega^2 = 2k$

$\Rightarrow 3\omega^2 = 2k$

$\Rightarrow z = 2k \Rightarrow k = \frac{z}{2}$

66. (B) $\lim_{x\to\pi/2} \frac{\cot\left(\frac{\pi}{4}+x\right)-\cos\left(\frac{\pi}{4}+x\right)}{\left(\frac{\pi}{2}-2x\right)^2}$ $\left[\frac{0}{0}\right]$from

by L–Hospital 's Rule

$\Rightarrow \lim_{x\to\pi/4} \frac{-\text{cosec}^2\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}+x\right)}{2\left(\frac{\pi}{2}-2x\right)(-2)}$

$\Rightarrow \lim_{x\to\pi/4} \frac{\sin\left(\frac{\pi}{4}+x\right)-\text{cosec}^2\left(\frac{\pi}{4}+x\right)}{8x-2\pi}$ $\left[\frac{0}{0}\right]$from

by L– Hospital's Rule

$\Rightarrow \frac{\cos\left(\frac{\pi}{4}+x\right)+2\text{cosec}\left(\frac{\pi}{4}+x\right).\text{cosec}\left(\frac{\pi}{4}+x\right).\cot\left(\frac{\pi}{4}+x\right)}{8}$

$\Rightarrow \frac{\cos\frac{\pi}{2}+2\text{cosec}^2\frac{\pi}{2}.\cot\frac{\pi}{2}}{8} = 0$

67. (A) $({}^{15}C_1-{}^{7}C_1)+({}^{15}C_2-{}^{7}C_2)+\ldots\ldots+({}^{15}C_7-{}^{7}C_7)$

$\Rightarrow (1+{}^{15}C_1+{}^{15}C_2+{}^{15}C_3\ldots\ldots\ldots{}^{15}C_7)$
$-\ (1+{}^{7}C_1+{}^{7}C_2+{}^{7}C_3\ldots\ldots{}^{7}C_7)$

$\Rightarrow ({}^{15}C_0+{}^{15}C_1+{}^{15}C_2+{}^{15}C_3\ldots\ldots\ldots{}^{15}C_7)$
$-({}^{7}C_0+{}^{7}C_1+{}^{7}C_2\ldots\ldots{}^{7}C_7)$

$\Rightarrow \frac{(1+1)^{15}}{2}-(1+1)^7$

$\therefore\ [(1+x)^n={}^{n}C_0+{}^{n}C_1+{}^{n}C_2\ldots\ldots{}^{n}C_n]$

$\Rightarrow \frac{2^{15}}{2}-2^7$

$\Rightarrow 2^{14}-2^7$

68. (A) cosA.sin (90°–A) – sin (180°–A).cos (270°– A)

$\Rightarrow$ cosA.cosA – sinA(–sinA)

$\Rightarrow \cos^2A+\sin^2A = 1$

69. (C) $A = \begin{bmatrix} 3 & -1 \\ 2 & -5 \end{bmatrix}$

Let $Z = 2A^2-3A$

$Z = 2\begin{bmatrix} 3 & -1 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -5 \end{bmatrix}-3\begin{bmatrix} 3 & -1 \\ 2 & -5 \end{bmatrix}$

$Z = 2\begin{bmatrix} 7 & 2 \\ -4 & 23 \end{bmatrix}-\begin{bmatrix} 9 & -3 \\ 6 & -15 \end{bmatrix}$

$Z = \begin{bmatrix} 5 & 7 \\ -14 & 61 \end{bmatrix}$

Co-factors of Z

$C_{11}=(-1)^{1+1}(61)= 61,\ \ C_{12}=(-1)^{1+2}(-14)= 14$
$C_{21}= (-1)^{2+1}(7)= -7,\ \ \ C_{22}= (-1)^{2+2}(5)= 5$

$C = \begin{bmatrix} 61 & 14 \\ -7 & 5 \end{bmatrix}$

$\text{Adj}Z = C^T = \begin{bmatrix} 61 & -7 \\ 14 & 5 \end{bmatrix}$

$\text{Adj}(2A^2-3A) = \begin{bmatrix} 61 & -7 \\ 14 & 5 \end{bmatrix}$

70. (B) ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

On differetiating both side w.r.t 'x'

$\frac{2x}{9}+\frac{2y\frac{dy}{dx}}{4}=0$

$\frac{dy}{dx}=\frac{-4x}{9y}$

slope of tangent

$\left(\frac{dy}{dx}\right)_{at\left(\frac{3}{\sqrt2},\sqrt2\right)} = \frac{-4}{9}\times\frac{\frac{3}{\sqrt2}}{\sqrt2}$

$m_1 = \frac{-2}{3}$

Slope of normal $m_2 = \frac{-1}{m_1} = \frac{3}{2}$

equation of normal at $\left(\frac{3}{\sqrt2},\sqrt2\right)$

$y-\sqrt2 = m_2\left(x-\frac{3}{\sqrt2}\right)$

$y-\sqrt2 = \frac{3}{2}\left(x-\frac{3}{\sqrt2}\right)$

$3x-2y = \frac{5}{\sqrt2}$ ...(i)

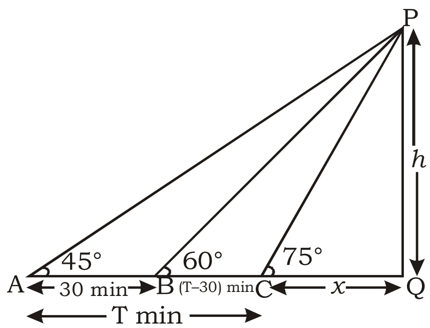
From option (B) $\left(\frac{1}{\sqrt2},-\frac{1}{\sqrt2}\right)$

$\frac{3}{\sqrt2}+\frac{2}{\sqrt2}=\frac{5}{\sqrt2}$

Option (B) satisfy the equation (ii)

Hence point $\left(\frac{1}{\sqrt2},-\frac{1}{\sqrt2}\right)$ lies on the ellipse.

71. (A)



Let PQ = h

CQ = x

In ΔPQC :-

$\tan 75° = \frac{PQ}{CQ}=\frac{h}{x}$

$\frac{\sqrt3+1}{\sqrt3-1}=\frac{h}{x} \Rightarrow h = (2+\sqrt3)x$ ....(i)

In ΔPQB :-

$\tan 60° = \frac{PQ}{QB}$

$\sqrt{3} = \frac{h}{T-30+x}$

$\sqrt{3}$ (T–30+x)= (2+ $\sqrt{3}$)x

$\sqrt{3}$ (T–30) = 2x ....(ii)

In ∆PQA :-

$tan\,45° = \frac{PQ}{AQ}$

$1 = \frac{h}{T+x}$

T+x = (2 + $\sqrt{3}$)x

T = $x + \sqrt{3}\,x$

T = (1 + $\sqrt{3}$)x .....(iii)

T = (1+ $\sqrt{3}$)× $\frac{\sqrt{3}(T-30)}{2}$ [from eq. (i)]

On solving this equation

T = 30 $\sqrt{3}$

72. (B) $\frac{1}{sin\,345°} - \frac{1}{\sqrt{3}\,cos\,195°}$

$\Rightarrow \frac{1}{sin(360°-15°)} - \frac{1}{\sqrt{3}\,cos(180°+15°)}$

$\Rightarrow -\frac{1}{sin\,15°} - \frac{1}{\sqrt{3}(-cos\,15°)}$

$\Rightarrow -\frac{2\sqrt{2}}{\sqrt{3}-1} + \frac{1\times 2\sqrt{2}}{\sqrt{3}(\sqrt{3}+1)}$

$\Rightarrow \frac{-2\sqrt{2}(\sqrt{3}+1)}{2} + \frac{2\sqrt{2}(\sqrt{3}-1)}{\sqrt{3}\times 2}$

$\Rightarrow -\sqrt{6}-\sqrt{2} + \frac{\sqrt{6}-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \frac{-\sqrt{18}-\sqrt{6}+\sqrt{6}-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \frac{-3\sqrt{2}-\sqrt{2}}{\sqrt{3}} = -\frac{4\sqrt{2}}{\sqrt{3}}$

73. (A) $I_n = \int tan^n x\ dx$

$I_n = \int tan^{n-2} x.tan^2 x\,dx$

$I_n = \int tan^{n-2} x.(sec^2 x - 1)\ dx$

$I_n = \int (tan\,x)^{n-2}.sec^2 x\,dx - \int tan^{n-2} x\ \ dx + c$

$I_n = \frac{(tan\,x)^{n-2+1}}{n-2+1} - I_{n-2} + C$

$I_n + I_{n-2} = \frac{tan^{n-1} x}{n-1} + C \Rightarrow n = 7$

$I_7 + I_5 = \frac{tan^6 x}{6} + C$ ...(i)

given that $I_5 + I_7 = \frac{tan^6 x}{a} + bx^6 + C$

On companing with equation (i)

$a = 6,\ b = 0$

Hence ordered pair (a, b) = (6, 0)

74. (D) S = $i^2 + i^5 + i^8 + ...+ (4n+1)^{th}$ terms

$S = \frac{i^2\left(1-(i^3)^{4n+1}\right)}{1-i^3}$

$S = \frac{-1\left(1-(i^3)^{4n}\,i^3\right)}{1+i}$

$S = \frac{-1[1-1.(-i)]}{1+i}$

$S = \frac{-1(1+i)}{1+i} = -1$

75. (A) $\frac{d^2y}{dx^2} = x.e^{-3x}$

On integrating both side w.r.t 'x'

$\Rightarrow \int \frac{d^2y}{dx^2}dx = \int x.e^{-3x}dx$

$\Rightarrow \frac{dy}{dx} = x\int e^{-3x}dx - \int 1.\frac{e^{-3x}}{-3}dx + c$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{3}x.e^{-3x} + \frac{1}{3}\frac{e^{-3x}}{-3} + c$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{3}x.e^{-3x} - \frac{1}{9}e^{-3x} + c$

Again, Integrating

$y = \frac{-1}{3}\left[\int x.e^{-3x}dx\right] - \frac{-1}{9}\int e^{-3x}dx + \int c.dx$

$y = \frac{-1}{3}\left[x.\frac{e^{-3x}}{-3} - \int 1.\frac{e^{-3x}}{-3}\ dx\right] - \frac{1}{9}\frac{e^{-3x}}{-3} + cx + d$

$y = \frac{-1}{3}\left[\frac{-x}{3}e^{-3x} + \frac{1}{3}\frac{e^{-3x}}{-3}dx\right] + \frac{1}{27}e^{-3x} + cx + d$

$y = \frac{x}{9}e^{-3x} + \frac{1}{27}e^{-3x} + \frac{1}{27}e^{-3x} + cx + d$

$y = \frac{x}{9}e^{-3x} + \frac{2}{27}e^{-3x} + cx + d$

76. (C) differential equation

$$\sqrt{1-y^2}+\left(2x-e^{\sin^{-1}y}\right)\frac{dy}{dx}=0$$

$$\sqrt{1-y^2}\frac{dx}{dy}+2x-e^{\sin^{-1}y}=0$$

$$\frac{dx}{dy}+\frac{2}{\sqrt{1-y^2}}x=\frac{e^{\sin^{-1}y}}{\sqrt{1-y^2}}$$

On comparing with $\frac{dx}{dy}+P(y)x=Q(y)$

$$P=\frac{2}{\sqrt{1-y^2}},\ Q=\frac{e^{\sin^{-1}y}}{\sqrt{1-y^2}}$$

$$I.F.=e^{\int P\,dy}$$

$$I.F.=e^{\int\frac{2}{\sqrt{1-y^2}}dy}$$

$$I.F.=e^{2\sin^{-1}y}$$

Solution of the differential equation

$$x\times I.F.=\int Q\times I.F.\,dy$$

$$x\times e^{2\sin^{-1}y}=\int\frac{e^{\sin^{-1}y}}{\sqrt{1-y^2}}\times e^{2\sin^{-1}y}\,dy$$

$$x\times e^{2\sin^{-1}y}=\int\frac{e^{3\sin^{-1}y}}{\sqrt{1-y^2}}\,dy$$

$$x\times e^{2\sin^{-1}y}=\frac{e^{3\sin^{-1}y}}{3}+x$$

$$x=\frac{e^{\sin^{-1}y}}{3}+c.e^{-2\sin^{-1}y}$$

77. (C) $\lim_{x\to\infty}\left[\frac{x^2+4x+5}{x^2+x+5}\right]^x\Rightarrow\lim_{x\to\infty}\left[\frac{1+\frac{4}{x}+\frac{5}{x^2}}{1+\frac{1}{x}+\frac{5}{x^2}}\right]^x$

We know that,

$$\lim_{x\to\infty}[f(x)]^{g(x)}=e^{\lim_{x\to\infty}[f(x)-1]g(x)}$$

$$\Rightarrow e^{\lim_{x\to\infty}}\left[\frac{x^2+4x+5}{x^2+x+5}-1\right]x$$

$$\Rightarrow e^{\lim_{x\to\infty}}\left[\frac{3x}{x^2+x+5}\right]x$$

$$\Rightarrow e^{\lim_{x\to\infty}}\left[\frac{3x^2}{x^2+x+5}\right]$$

$$\Rightarrow e^{\lim_{x\to\infty}}\left[\frac{3}{1+\frac{1}{x}+\frac{5}{x^2}}\right]=e^3$$

78. (D) Curve $y=x^2+3x-6$

$$\frac{dy}{dx}=2x+3$$

$$m_1=\left(\frac{dy}{dx}\right)_{at(-2,-8)}=2\times(-2)+3=-1$$

$$m_2=\left(\frac{dy}{dx}\right)_{at(-1,-8)}=2\times(-1)+3=1$$

angle between lines

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta=\left|\frac{-1-1}{1+(-1)\times1}\right|$$

$$\tan\theta=\left|\frac{-2}{0}\right|$$

$$\tan\theta=\infty\Rightarrow\theta=\frac{\pi}{2}$$

79. (A) The required sum = (2 + 4 +……….+ 150) $+(5+10+15+20+...150)-(10+20+\dots...+150)$

= 2(1 + 2 +…+ 75) + 5(1 + 2 +…..+ 30) – 10 (1 + 2 + 7…..+15)

$$=2\times\frac{75\times76}{2}+5\times\frac{30\times31}{2}-10\times\frac{15\times16}{2}$$

= 5700 + 2325 – 1200 = 6825

80. (B) ${}^{35}C_5+\sum_{r=0}^{5}{}^{40-r}C_4$

$\Rightarrow {}^{35}C_5+{}^{40}C_4+{}^{39}C_4+{}^{38}C_4+{}^{37}C_4+{}^{36}C_4+{}^{35}C_4$

$\Rightarrow {}^{35}C_5+{}^{35}C_4+{}^{36}C_4+{}^{37}C_4+{}^{38}C_4+{}^{39}C_4+{}^{40}C_4$

We know that ${}^{n}C_{n+1}+{}^{n}C_r={}^{n+1}C_{r+1}$

$\Rightarrow {}^{36}C_5+{}^{36}C_4+{}^{37}C_4+{}^{38}C_4+{}^{39}C_4+{}^{40}C_4$

$\Rightarrow {}^{37}C_5+{}^{37}C_4+{}^{38}C_4+{}^{39}C_4+{}^{40}C_4$

$\Rightarrow {}^{38}C_5+{}^{38}C_4+{}^{39}C_4+{}^{40}C_4$

$\Rightarrow {}^{39}C_5+{}^{39}C_4+{}^{40}C_4$

$\Rightarrow {}^{40}C_5+{}^{40}C_4={}^{41}C_5$

81. (A) $f(x)=\cos^{-1}\left[\log_4\left(\frac{x}{2}\right)\right]$ exist,

$$if\ -1\le\log_4\left(\frac{x}{2}\right)\le1$$

$$\Rightarrow 4^{-1}\le\frac{x}{2}\le4$$

$$\Rightarrow\frac{1}{4}\le\frac{x}{2}\le4\Rightarrow\frac{1}{2}\le x\le8$$

Hence $x\in\left[\frac{1}{2},8\right]$

82. (B) $(1+\omega)^5 = a + b\omega$

$(-\omega^2)^5 = a + b\omega$

$-\omega^{10} = a + b\omega$

$-\omega = a + b\omega$

On comparing

$a = 0, \quad b = -1$

Hence $(a, b) = (0,-1)$

83. (B) $T_3 = a + 2d$, $T_6 = a + 5d$, $T_8 = a + 7d$

According to question:

$a + 2d,\ a + 5d,\ a + 7d$ are in G.P,

Then $(a + 2d)^2 = (a + 2d)(a + 7d)$

$a^2 + 25d^2 + 10ad = a^2 + 2ad + 7ad + 14d^2$

$11d^2 = -ad \Rightarrow d = \frac{-a}{11}$

comon ratio $(r) = \frac{a+5d}{a+2d}$

$$r = \frac{a - 5\times\frac{a}{11}}{a - 2\times\frac{a}{11}}$$

$$r = \frac{11a-5a}{11a-2a} = \frac{6a}{9a} = \frac{2}{3}$$

84. (B) Let $\vec{a} = 2\hat{i} + (1-\lambda)\hat{j} + 2\lambda\hat{k}$

and $\vec{b} = (1-\lambda)\hat{i} + \hat{j} - \hat{k}$

$\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Then $\vec{a}.\vec{b} = 0$

$\Rightarrow 2 \times (1-\lambda) + (1-\lambda)\times 1 + 2\lambda(-1) = 0$

$\Rightarrow 2 - 2\lambda + 1 - \lambda + -2\lambda = 0 \Rightarrow \lambda = \frac{3}{5}$

85. (A) Let S be the sample space of the experiment and E be the event that at most four head occur.

clearly, $n(S) = 2^6 - 64$

and $n(E) = {}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4$

$= 1 + 6 + 15 + 20 + 15 = 57$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{57}{64}$

86. (B) $S_n = 0.6 + 0.66 + 0.666 + \ldots\ldots$ n terms

$$S_n = \frac{6}{10} + \frac{66}{100} + \frac{666}{1000} + \ldots\ldots \text{ n terms}$$

$$S_n = \frac{6}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots n \text{ terms}\right]$$

$$S_n = \frac{6}{9}\left[\frac{10-1}{10} + \frac{100-1}{100} + \frac{1000-1}{1000} \ldots . n \text{ terms}\right]$$

$$S_n = \frac{6}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{100}\right) + \ldots\ldots n \text{ terms}\right]$$

$$S_n = \frac{6}{9}\left[n - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^n\right)}{1-\frac{1}{10}}\right]$$

$$S_n = \frac{2}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

87. (C) $x = \frac{a(1+t^2)}{1-t^2}$

$$\frac{dx}{dt} = a\left[\frac{(1-t^2)(2t) - (1+t^2)(-2t)}{(1-t^2)^2}\right]$$

$$\frac{dx}{dt} = a\left[\frac{4t}{(1-t^2)^2}\right]$$

and $y = \left[\frac{4at}{1-t^2}\right]$

$$\frac{dy}{dt} = 4a\left[\frac{(1-t^2).1 - t(-2t)}{(1-t^2)^2}\right] = \frac{4a(1+t^2)}{(1-t^2)^2}$$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\frac{dy}{dx} = \frac{4a(1+t^2)}{(1-t^2)^2} \times \frac{(1-t^2)^2}{4at}$$

$$\frac{dy}{dx} = \frac{1+t^2}{t} \qquad \ldots\ldots(i)$$

given that $x = \frac{a(1+t^2)}{1-t^2}$, $y = \frac{4at}{1-t^2}$

Now, $\frac{x}{y} = \frac{1+t^2}{4t}$

from equation (i)

$$\frac{dy}{dx} = \frac{4x}{y}$$

88. (A) In the expansion of $\left(3x+\frac{6}{x^2}\right)^{12}$

$$T_{r+1} = {}^{12}C_r.(3x)^{12-r}\left(\frac{6}{x^2}\right)^r$$

$$T_{r+1} = {}^{12}C_r.3^{12}.2^r.x^{12-3r}$$

Now, $12 - 3r = 0 \Rightarrow r = 4$

term independent of x = 4 +1 = 5th term

89. (B) $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$$

$$\Rightarrow (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$$

$C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_1$

$$\Rightarrow (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x & x^2-x & 1-x \\ x^2 & 1-x^2 & x-x^2 \end{vmatrix}$$

$$\Rightarrow (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x & x(x-1) & 1-x \\ x^2 & (1-x)(1+x) & x(1-x) \end{vmatrix}$$

$$\Rightarrow (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1 & 0 & 0 \\ x & -x & 1 \\ x & 1+x & x \end{vmatrix}$$

$\Rightarrow (1+x+x^2)(1-x)(1-x)\ 1(-x^2-1-x)$

$\Rightarrow -(1+x+x^2)^2(1-x)^2$

$\Rightarrow -(1-x^3)^2$

90. (B) Area of triangle = $\begin{vmatrix} 3 & 4 & 1 \\ a & -4 & 1 \\ 5 & 3 & 1 \end{vmatrix}$

$15 = \frac{1}{2}\ [3(-4-3) - 4(a-5) + 1(3a+20)]$

$30 = -21 - 4a + 20 + 3a + 20$

$30 = 19 - a \Rightarrow a = -11$

91. (D) $f'(x) = 3x^2 + \frac{4}{x^3}$

On integrating both side w.r.t. 'x'

$$f(x) = 3.\frac{x^2}{3} + 4.\frac{x^{-2}}{-2} + C$$

$$f(x) = x^3 - \frac{2}{x^2} + C \qquad ......(i)$$

given that $f(-1) = 6$

$6 = -1 - 2 + C \Rightarrow C = 9$

from equation (i)

$$f(x) = x^3 - \frac{2}{x^2} + 9$$

92. (C) Median

93. (B) $\dfrac{\left[1+\left(i^9\right)^{8n-1}\right]^{8n+1}}{\left[1+\left(i^9\right)^{8n+1}\right]^{8n-1}}$

$$\Rightarrow \frac{\left[1+(i)^{8n-1}\right]^{8n+1}}{\left[1+(i)^{8n+1}\right]^{8n-1}} \Rightarrow \frac{\left[1+(i)^{8n}(i)^{-1}\right]^{8n+1}}{\left[1+(i)^{8n}(i)^{1}\right]^{8n-1}}$$

$$\Rightarrow \frac{\left[1+\frac{1}{-i}\right]^{8n+1}}{(1+i)^{8n-1}} \Rightarrow \frac{(1+i)^{8n+1}}{(1+i)^{8n-1}}$$

$\Rightarrow (1+i)^2 \Rightarrow 2i$

94. (C) $\dfrac{1^2}{1} + \dfrac{1^2+3^2}{1+3} + \dfrac{1^2+3^2+5^2}{1+3+5}$

$$T_n = \frac{1^2+3^2+5^2+......(2n-1)^3}{1+3+5+.......+(2n-1)}$$

$$T_n = \frac{\left[1^2+2^2+3^2+4^2+5^2+.....+(2n-1)^3+(2n)^2\right]-\left[\left(2^2+4^2+...+(2n)^2\right)\right]}{\left[(1+2+3+4+....+(2n-1)+2n)\right]-\left[(2+4+....+2n)\right]}$$

$$T_n = \frac{\frac{2n}{6}(2n+1)(2\times 2n+1) - 2^2\left(1^2+2^2+.....n^2\right)}{\frac{2n(2n+1)}{2} - 2(1+2+......+n)}$$

$$T_n = \frac{\frac{n}{3}(2n+1)(4n+1) - 4\times\frac{n}{6}(n+1)(2n+1)}{n(2n+1) - 2\times\frac{n(n+1)}{2}}$$

$$T_n = \frac{\frac{n}{3}(2n+1)\left[(4n+1)-2(n+1)\right]}{n(n)}$$

$$T_n = \frac{(2n+1)(2n-1)}{3n}$$

95. (A) $y = 2\sqrt{tan\, x^3}$ and $z = x^3$

$\Rightarrow y = 2\sqrt{tan\, z}$

On differentiating both side w.r.t. 'z'

$\Rightarrow \frac{dy}{dz} = 2\times\frac{1}{2}(\tan z)^{-1/2}\sec^2 z$

$\Rightarrow \frac{dy}{dz} = \frac{sec^2 z}{\sqrt{tan\, z}}$

$\Rightarrow \frac{dy}{dz} = \frac{1+tan^2 z}{\sqrt{tan\, z}}$

$\Rightarrow \frac{dy}{dz} = \frac{1+tan^2 x^3}{\sqrt{tan\, x^3}}$

96. (B) $x\frac{dy}{dz} + y = x\log x$

$\frac{dy}{dz} + \frac{1}{x}y = log\, x$

On comparing with $\frac{dy}{dz} + Py = Q$

$P = \frac{1}{x}$, $Q = log\, x$

$I.F. = e^{\int P.dx}$

$I.F. = e^{\int \frac{1}{x}.dx}$

I.F. $= e^{logx} = x$

Solution of the differential equation

$\Rightarrow y \times I.F. = \int Q\, .I.F.dx$

$\Rightarrow y \times x = \int x.log\, x\, dx$

$\Rightarrow xy = log\, x\, .\int x.dx - \int\left\{\frac{d}{dx}(log\, x)\int x.dx\right\}dx$

$\Rightarrow xy = (log\, x)\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx + C$

$\Rightarrow xy = (log\, x)\frac{x^2}{2} - \frac{1}{2}\times\frac{x^2}{2} + C$

$\Rightarrow xy = \frac{x^2}{2}(log\, x) - \frac{x^2}{4} + C$

97. (A) **Statement I :-**

Given that, $tan\theta = x$

$cot\theta = \frac{1}{x}$

Now,

$\Rightarrow x - \frac{1}{x} = tan\theta - cot\theta$

$\Rightarrow x - \frac{1}{x} = \frac{sin\,\theta}{cos\,\theta} - \frac{cos\,\theta}{sin\,\theta}$

$\Rightarrow x - \frac{1}{x} = \frac{sin^2\theta - cos^2\theta}{sin\,\theta.cos\,\theta}$

$\Rightarrow x - \frac{1}{x} = \frac{-2(cos^2\theta - sin^2\theta)}{2\,sin\,\theta.cos\,\theta}$

$\Rightarrow x - \frac{1}{x} = \frac{-2\,cos\,2\theta}{sin\,2\theta}$

$\Rightarrow x - \frac{1}{x} = -2\,cot\,2\theta$

Statement - I is incorrect.

Statement II:-

$\Rightarrow x - \frac{1}{x} = \sqrt{2}\,tan\,\theta$

$\Rightarrow x^2 + \frac{1}{x^2} = (\sqrt{2}\,tan\,\theta)^2 + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2tan^2\theta + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2(1 + tan^2\theta) = 2\,sec^2\theta$

Statement II is correct.

Statement III :-

given that $x = m\,cos\theta$ and $y = n\,sin\theta$

$(xn)^2 + (my)^2 = (mn\,cos\theta)^2 + (mn\,sin\theta)^2$

$(xn)^2 + (my)^2 = (mn)^2(\cos^2\theta + sin\theta)^2$

$(xn)^2 + (my)^2 = (mn)^2$

Statement III is correct.

Statement IV :-

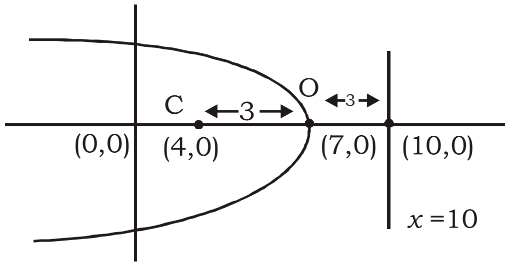
max. value of $7\sin\theta + 25\cos\theta = \sqrt{(7)^2 + (24)^2}$

max. value of $7\sin\theta + 25\cos\theta = \sqrt{49 + 576}$

max. value of $7\sin\theta + 25\cos\theta = \sqrt{625} = 25$

Statement IV is incorrect.

98. (C)



equation of direction $x = 10$

99. (D) $I=\int_0^{\pi/4} e^x\left(\frac{\sin 2x+2}{\cos^2 x}\right)dx$

$$I=\int_0^{\pi/4} e^x\left(\frac{2\sin x\cos x+2}{\cos^2 x}\right)dx$$

$$I=2\int_0^{\pi/4} e^x\left(\frac{\sin x\cos x}{\cos^2 x}+\frac{1}{\cos^2 x}\right)dx$$

$$I=2\int_0^{\pi/4} e^x\left(\tan x+\sec^2 x\right)dx$$

$$I=2\left[e^x \tan x\right]_0^{\pi/4}$$

$\therefore [e^x\{f(x)+f'(x)\}\,dx = e^x f(x)]$

$$I=2\left[e^{\pi/4}\tan\frac{\pi}{4}-e^{\circ}\tan 0\right]$$

$I = 2\,e^{\pi/4}\times 1 - 0 = 2e^{\pi/4}$

100.(B) Given that $f(x) = x^2 + 2x + 1$

$a = 1 \Rightarrow f(a) = 4$

$b = \frac{3}{2} \Rightarrow f(b) = \frac{25}{4}$

$f'(x) = 2x + 2$

$f'(c) = 2c + 2$

by definition of mean value theorem

$$\Rightarrow f'(x) = \frac{f(b)-f(a)}{b-a}$$

$$\Rightarrow 2c+2=\frac{\frac{25}{4}-4}{\frac{3}{2}-1}$$

$$\Rightarrow 2c+2=\frac{\frac{9}{4}}{\frac{1}{2}}$$

$$\Rightarrow 2c+2=\frac{9}{2}$$

$$\Rightarrow 2c=\frac{5}{2} \Rightarrow c=\frac{5}{4}$$

101. (A) curves

$y_1 \Rightarrow x = 3y^2$ and $y_2 \Rightarrow y = 3x^2$

Solving the equations,

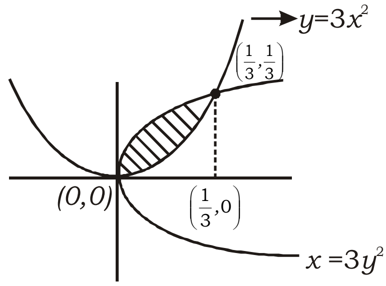
$x = 3\times 9x^4$

$27x^4 - x = 0$

$x(27x^3 - 1) = 0$

$x = 0,\quad x = \frac{1}{3}$

$y = 0,\quad y = \frac{1}{3}$



$$\text{Area} = \int_0^{1/3}(y_1 - y_2)\,dx$$

$$\text{Area} = \int_0^{1/3}\left(\sqrt{\frac{x}{3}}-3x^2\right)dx$$

$$\text{Area} = \left[\frac{1}{\sqrt{3}}.\frac{x^{3/2}}{\frac{3}{2}}-3.\frac{x^3}{3}\right]_0^{1/3}$$

$$\text{Area} = \frac{2}{3\sqrt{3}}.\left(\frac{1}{3}\right)^{3/2}-\left(\frac{1}{3}\right)^3-0+0$$

$$\text{Area} = \frac{2}{3\sqrt{3}}.\frac{1}{3\sqrt{3}}-\frac{1}{27}=\frac{1}{27}\text{ sq. units}$$

102. (B) $I=\int\left(1+x+\frac{1}{x}\right)e^{x-\frac{1}{x}}dx$

let $\quad xe^{x-\frac{1}{x}} = t$

$$\left[xe^{x-\frac{1}{x}}\left(1+\frac{1}{x^2}\right)+e^{x-\frac{1}{x}}.1\right]dx = dt$$

$$\left(1+x+\frac{1}{x^2}\right)e^{x-\frac{1}{x}}dx = dt$$

$$\Rightarrow I=\int dt$$

$$\Rightarrow I = t + c$$

$$\Rightarrow I = x.e^{x-\frac{1}{x}}+c$$

103. (D) given that $X = \{9(n-1) : n \in N\}$

$n = 1,2,3,4,\ldots\ldots$

$X = \{0,9,18,27,\ldots\ldots\}$

$Y = \{4^n - 3n - 1 : n \in N\}$

$n = 1, 2, 3, 4,\ldots\ldots$

$Y = \{0, 9, 54, 243,\ldots\ldots\}$

$(X \cap Y) = \{0, 9, 54, 243\} = Y$

104. (B) $(10)^6 + 6(10)^5(2)^1 + 15(10)^4(2)^2 + \ldots\ldots + (2)^6 = k \times 3^6 \times 2^{10}$

${}^6C_0(10)^6(2)^0 + {}^6C_1(10)^5(2)^1 + {}^6C_2(10)^4(2)^2 + \ldots\ldots + {}^6C_6(10)^0(2)^6 = k \times 3^6 \times 2^{10}$ ...(i)

we know that

$(x + a)^n = {}^nC_0 x^n (a)^0 + {}^nC_1 x^{n-1}(a)^1 + \ldots\ldots + {}^nC_n x^0 (a)^n$

On putting $x = 10$, $a = 2$ and $n = 6$

$(10 + 2)^6 = {}^6C_0(10)^6(2)^0 + {}^6C_1(10)^5(2)^1 + {}^6C_2(10)^4(2)^2 + \ldots\ldots {}^6C_6(10)^0(2)^6$

From equation (i)

$(10+2)^6 = k \times 3^6 \times 2^{10}$

$\Rightarrow (12)^6 = k \times 3^6 \times 2^{10}$

$\Rightarrow 3^6 \times 4^6 = k \times 3^6 \times 2^{10}$

$\Rightarrow 2^{12} = k \times 2^{10}$

$\Rightarrow k = 2^2 = 4$

105. (C) given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

we know that $A^{-1}A = I$

$$A^{-1}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & .0 & 1 \end{bmatrix}$$

by elementary operation

$R_3 \to R_3 - 3R_1$

$$A^{-1}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 + 5R_2$

$$A^{-1}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix}$$

$R_3 \to \frac{R_3}{2}$

$$A^{-1}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ \frac{-3}{2} & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$R_2 \to R_2 - 2R_3$ and $R_1 \to R_1 + R_3$

$$A^{-1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 3 & -4 & -1 \\ \frac{-3}{2} & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$$A^{-1} I = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 3 & -4 & -1 \\ \frac{-3}{2} & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 3 & -4 & -1 \\ \frac{-3}{2} & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

106. (B) $\lim_{x \to 0} \frac{(1 - \cos x)\sin 2x}{x \tan x}$ $\left[\frac{0}{0}\right]$ form

$$\Rightarrow \lim_{x \to 0} \frac{2(1 - \cos x)}{x} \frac{\frac{\sin 2x}{2x}}{\frac{\tan x}{x}}$$

We know that $\left[\because \lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$

$\Rightarrow \lim_{x \to 0} \frac{2(1 - \cos x)}{x}$ $\left[\frac{0}{0}\right]$ form

by L – Hospital's rule

$$\Rightarrow \lim_{x \to 0} \frac{2(\sin x)}{1} = 0$$

107. (D) given that $f(x) = |3x - 2|$, $g(x) = x - 3$

Now, $fog(x) = f[g(x)]$

$fog(x) = f[x-3]$

$fog(x) = |3(x-3)-2|$

$fog(x) = |3x-9-2|$

$fog(x) = |3x-11|$

$fog(2) = |3 \times 2 - 11| = 5$

108. (C) $f(x) = \dfrac{\sqrt{log_e\left(6+8x-4x^2\right)}}{3x-2}$

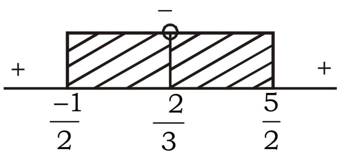
$\log_e(6 + 8x - 4x^2) \geq 0$ and $3x - 2 \neq 0$

$6 + 8x - 4x^2 \geq 1,$ $\quad x \neq \dfrac{2}{3}$

$4x^2 - 8x - 5 \geq 0$

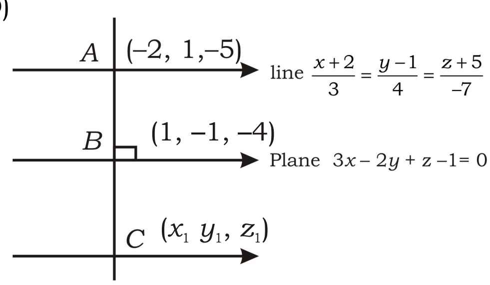
$(2x + 1)(2x - 5) \leq 0$

$x = -\dfrac{1}{2}, \dfrac{5}{2}$



$x \in \left[\dfrac{-1}{2}, \dfrac{5}{2}\right] - \left\{\dfrac{2}{3}\right\}$

109. (D)



equation of AB

$\dfrac{x+2}{3} = \dfrac{y-1}{-2} = \dfrac{z+5}{1} = \lambda$

co-ordinate of point B

$x = 3\lambda - 2,\ y = -2\lambda + 1,\ z = \lambda - 5$

point satisfy the equation of plane

$3(3\lambda - 2) - 2(-2\lambda + 1) + \lambda - 5 - 1 = 0$

$9\lambda - 6 + 4\lambda - 2 + \lambda - 6 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

Co-ordinate of B = (1,–1,–4)

Let Co-ordinate of C = (x_1, y_1, z_1)

Now, $\dfrac{x_1 - 2}{2} = 1 \Rightarrow x_1 = 4$

$\dfrac{y_1 + 1}{2} = -1 \quad \Rightarrow y_1 = -3$

$\dfrac{z_1 - 5}{2} = -4 \quad \Rightarrow z_1 = -3$

Co- ordinate of C = (4, –3, –3)

equation of line passing through the point C

$\dfrac{x-4}{3} = \dfrac{y+3}{4} = \dfrac{z+3}{-7}$

110. (A) A triangle with vertices $(k, 2k)$, $(5,k)$ and $(-k,0)$

Area of triangle = $\dfrac{1}{2}\begin{vmatrix} k & 2k & 1 \\ 5 & k & 1 \\ -k & 0 & 1 \end{vmatrix}$

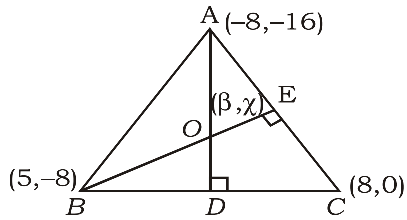
$40 = \dfrac{1}{2}[k(k-0) - 2k(5+k) + 1(0+k^2)$

$80 = k^2 - 10k - 2k^2 + k^2$

$80 = -10k \Rightarrow k = -8$

hence vertices are

(–8, –16), (5,–8) and (8,0)



Let Orthocentre O = (α, β)

Slope of AC $(m_1) = \dfrac{16}{16} = 1$

Slope of BE $(m_2) = \dfrac{\beta+8}{\alpha-5}$

Now, $\quad 1 \times \dfrac{\beta+8}{\alpha-5} = -1$

$\alpha + \beta = -3$ ......(i)

Similarly

$\dfrac{8}{3} \times \dfrac{\beta+16}{\alpha+8} = -1$

$3\alpha + 8\beta = -152$ ......(ii)

On solving eq. (i) and (ii)

$\beta = \dfrac{-143}{5}$ and $\alpha = \dfrac{128}{5}$

Orthocentre = $\left(\dfrac{128}{5}, \dfrac{-143}{5}\right)$

111. (C) Given plane $-3x + 4y - 12z + 8 = 0$ perpendicular distance from a point (2,3,–1) to the ellipse

$D = \left|\dfrac{-3\times2+4\times3+(-12)(-1)+8}{\sqrt{(3)^2+(-4)^2+(12)^2}}\right|$

$D = \left|\dfrac{-6+12+12+8}{\sqrt{169}}\right| = \dfrac{26}{13} = 2$ unit

112. (A) Normal vector = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 5 \\ -4 & 2 & 1 \end{vmatrix}$

Normal vector = $\hat{i}\,(-3-10) - \hat{j}\,(-1+20) + \hat{k}\,(-2-12)$

Normal vector $= -13\hat{i} - 19\hat{j} - 14\hat{k}$

So equation of plane

$-13(x-1) - 19(y+2) - 14(z+6) = 0$

$-13x + 13 - 19y - 38 - 14z - 84 = 0$

$13x + 19y + 14z + 109 = 0$

113. (B) $f(x) = \begin{cases} x^2 - k, & x > 2 \\ 3x - 2, & x \le 2 \end{cases}$ is continuous at $x = 2$, then

$\lim_{x \to 2^+} f(x) = f(2)$

$\lim_{x \to 2} (x^2 - k) = 3 \times 2 - 2$

$4 - k = 4 \Rightarrow k = 0$

114. (A) $\dfrac{b + c\omega + a\omega^2}{a + b\omega + c\omega^2} + \dfrac{b + c\omega + a\omega^2}{c + a\omega + b\omega^2}$

$\Rightarrow \dfrac{\omega^2(b + c\omega + a\omega^2)}{\omega^2(a\omega + b\omega + c\omega^2)} + \dfrac{\omega(b + c\omega + a\omega^2)}{\omega(c + a\omega + b\omega^2)}$

$\dfrac{\omega^2(b + c\omega + a\omega^2)}{a\omega^2 + b + c\omega} + \dfrac{\omega(b + c\omega + a\omega^2)}{(c\omega + a\omega^2 + b)}$

$\omega^2 + \omega = -1$ $[\because 1 + \omega + \omega^2 = 0]$

115. (B) $\sin^{-1}\dfrac{3}{5} + 2\tan^{-1}\dfrac{1}{3}$

$\Rightarrow \tan^{-1}\dfrac{3}{4} + \tan^{-1}\left(\dfrac{2 \times \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}\right)$

$\Rightarrow \tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{3}{4}$

$\Rightarrow 2\tan^{-1}\dfrac{3}{4} = \tan^{-1}\dfrac{24}{7} \Rightarrow \sin^{-1}\dfrac{24}{25}$

116. (A) Given that $e = \dfrac{1}{3}$ and

$\dfrac{a}{e} = 6 \Rightarrow \dfrac{a \times 3}{1} = 6 \Rightarrow a = 2$

Now, $e^2 = 1 - \dfrac{b^2}{a^2}$

$\dfrac{1}{9} = 1 - \dfrac{b^2}{4} \Rightarrow b^2 = \dfrac{32}{9}$

equation of ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{x^2}{4} + \dfrac{y^2 \times 9}{32} = 1$

$\Rightarrow 8x^2 + 9y^2 = 32$ ...(i)

On differentiating both side w.r.t 'x'

$\Rightarrow 16 + 18y\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-8x}{9y}$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{at\left(-\sqrt{2}, \frac{4}{3}\right)} = \dfrac{-8 \times (-\sqrt{2}) \times 3}{9 \times 4}$

$\Rightarrow m = \dfrac{2\sqrt{2}}{3}$

equation of tangent

$y - \dfrac{4}{3} = \dfrac{2\sqrt{2}}{3}(x + \sqrt{2})$

$2\sqrt{2}\,x - 3y + 8 = 0$

117. (C) $f(x) = x^3 + 2x^2 - 4x + 2$

$f'(x) = 3x^2 + 4x^2 - 4$

$f'(x) = 6x + 4$ ...(i)

for maxima and minima

$f'(x) = 0$

$3x^2 + 4x - 4 = 0$

$(x + 2)(3x - 2) = 0$

$x = -2 \quad x = \dfrac{2}{3}$

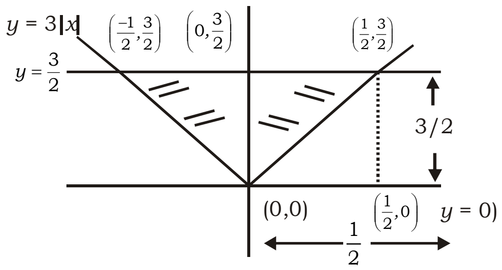
from equation (ii)

$f''(-2) = 6(-2) + 4 = -8$ (maxima)

$f''\left(\dfrac{2}{3}\right) = 6 + \left(\dfrac{2}{3}\right) + 4 = 8$ (minima)

The function $f(x)$ will attain minimum value at $x = \dfrac{2}{3}$

118. (A)



$y = 3|x|$ and $y = \frac{3}{2}$

The required Area = $2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{3}{2}$

$= \frac{3}{4}$ sq. unit

119. (A) transverse axis

120. (A) Only 1

NDA (MATHS) MOCK TEST - 104 (Answer Key)

1. (A)	21. (B)	41. (B)	61. (B)	81. (A)	101. (A)
2. (C)	22. (B)	42. (C)	62. (C)	82. (B)	102. (B)
3. (C)	23. (A)	43. (C)	63. (B)	83. (B)	103. (D)
4. (D)	24. (B)	44. (A)	64. (A)	84. (B)	104. (B)
5. (D)	25. (C)	45. (A)	65. (C)	85. (A)	105. (C)
6. (B)	26. (B)	46. (B)	66. (B)	86. (B)	106. (B)
7. (A)	27. (C)	47. (A)	67. (A)	87. (C)	107. (D)
8. (C)	28. (C)	48. (C)	68. (A)	88. (A)	108. (C)
9. (B)	29. (C)	49. (B)	69. (C)	89. (B)	109. (D)
10. (A)	30. (D)	50. (D)	70. (B)	90. (B)	110. (A)
11. (B)	31. (A)	51. (A)	71. (A)	91. (D)	111. (C)
12. (C)	32. (B)	52. (B)	72. (B)	92. (C)	112. (A)
13. (D)	33. (C)	53. (C)	73. (A)	93. (B)	113. (B)
14. (A)	34. (C)	54. (D)	74. (D)	94. (C)	114. (A)
15. (B)	35. (B)	55. (A)	75. (A)	95. (A)	115. (B)
16. (A)	36. (A)	56. (B)	76. (C)	96. (B)	116. (A)
17. (C)	37. (C)	57. (A)	77. (C)	97. (A)	117. (C)
18. (D)	38. (B)	58. (B)	78. (D)	98. (C)	118. (A)
19. (C)	39. (C)	59. (A)	79. (A)	99. (D)	119. (A)
20. (A)	40. (C)	60. (C)	80. (B)	100. (B)	120. (A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777