NDA MATHS MOCK TEST - 106 (SOLUTION)

1. (C) Let $y = \tan^{-1}\dfrac{x}{\sqrt{1-x^2}}$ and $z = \cos^{-1}x$

$x = \cos z$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos z}{\sqrt{1-\cos^2 z}}\right)$$

$$\Rightarrow y = \tan^{-1}\left(\frac{\cos z}{\sin z}\right)$$

$$\Rightarrow y = \tan^{-1}\left[\tan\left(\frac{\pi}{2} - z\right)\right]$$

$$\Rightarrow y = \frac{\pi}{2} - z$$

On diffrentiating both side w.r.t. 'z'

$$\Rightarrow \frac{dy}{dx} = -1$$

2. (A) angle describe in 12hr by hour-hand = 360º

angle describe in 1hr(60min) by hour hand = $\dfrac{360}{12}$

angle describe in 1 min by hour-hand $= \dfrac{360}{12 \times 60}$

angle describe in 18 min by hour-hand

$= \dfrac{360}{12 \times 60} \times 18 = 9^\circ$

3. (B) 1 0 0 1 0

$0 \times 2^0 = 0$
$1 \times 2^1 = 2$
$0 \times 2^2 = 0$
$0 \times 2^3 = 0$
$0 \times 2^4 = 16$
18

0.11

$\dfrac{1}{2} = 1 \times 2^{-1}$

$\dfrac{1}{4} = 1 \times 2^{-2}$

$\dfrac{1}{2} + \dfrac{1}{4} = \dfrac{3}{4} = 0.75$

$(10010)_2 = (18)_{10}$, $(0.11)_2 = (0.75)_{10}$

Hence $(10010.11)_2 = (18.75)_{10}$

4. (A) digits 0,2,4,6,8,9

5	5	4

$= 5 \times 5 \times 4 = 100$

'0' can not put here.

5. (D) Let $a - ib = \sqrt{1 - 2\sqrt{6}i}$

On squaring both side

$(a^2 - b^2) - 2abi = 1 - 2\sqrt{6}i$

On comparing

$a^2 - b^2 = 1$ and $2ab = 2\sqrt{6}$ (i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = 1 + 24$

$a^2 + b^2 = 5$

from eq (i) and eq (ii)

$2a^2 = 6$ and $2b^2 = 4$

$a = \pm\sqrt{3}$, $b = \pm\sqrt{2}$

Hence $\sqrt{1 - 2\sqrt{6}i} = \pm\left(\sqrt{3} - \sqrt{2}\,i\right)$

6. (A) $(1 - 3\cos x)\dfrac{dy}{dx} + 3(2+y)\sin x = 0$

$$\frac{d}{dx}\left[(1 - 3\cos x)(2+y)\right] = 0$$

On integrating

$(1-3\cos x)(2+y) = c$ .....(i)

on putting

$x = \dfrac{\pi}{2}, y = 1$

then c = 3

On puting c = 3 in eq. (i)

$(1-3\cos x)(2+y) = 3$

On putting $x = \dfrac{\pi}{3}$

$$\Rightarrow \left(1 - 3\cos\frac{\pi}{3}\right)(2+y) = 3$$

$$\Rightarrow \left(1 - 3 \times \frac{1}{2}\right)(2+y) = 3$$

$$\Rightarrow -\frac{1}{2} \times (2+y) = 3$$

$\Rightarrow 2 + y = -6 \Rightarrow y = -8$

7. (B) $I = \displaystyle\int_{\pi/3}^{2\pi/3} \frac{dx}{1 + \sin 2x}$ ....(i)

We know that

$$\int_a^b f(x) = \int_a^b f(a + b - x)\,dx$$

$$I = \int_{\pi/3}^{2\pi/3} \frac{dx}{1 - \sin 2\left(\frac{\pi}{3} + \frac{2\pi}{3} - x\right)} dx$$

$$I = \int_{\pi/3}^{2\pi/3} \frac{dx}{1 - \sin 2x} \qquad \text{(i)}$$

from eq (i) and eq (ii)

$$I+I= \int_{\pi/3}^{2\pi/3} \left[\frac{1}{1+\sin 2x} + \frac{1}{1-\sin 2x}\right] dx$$

$$2I= \int_{\pi/3}^{2\pi/3} \frac{1 - \sin 2x + 1 + \sin 2x}{1 - \sin^2 2x}$$

$$2I= \int_{\pi/3}^{2\pi/3} \frac{2}{\cos^2 2x} dx$$

$$2I= \int_{\pi/3}^{2\pi/3} 2\sec^2 2x \, dx$$

$$2I= 2\left[\frac{\tan 2x}{2}\right]_{\pi/3}^{2\pi/3}$$

$$2I= \tan\frac{4\pi}{3} - \tan\frac{2\pi}{3}$$

$$2I= \sqrt{3} + \sqrt{3} \Rightarrow I = \sqrt{3}$$

8. (D) $10! \times C(19,11) = k.P(19,8)$

$$10! \times \frac{19!}{11!\,8!} = k.\frac{19!}{11!}$$

$$\frac{10!}{8!} = k \Rightarrow k = 90$$

9. (C) $\text{cosec}\,y \dfrac{dx}{dy} - x = 2\cot y$

$$\frac{dx}{dy} - x \sin y = 2\cos y$$

10. (a) Word " STRONG"
No. of words start with "G" = 5! = 120
No. of words start with "N" = 5! = 120
No. of words start with "O" = 5! = 120
No. of words start with "R" = 5! = 120
No. of words start with "SG" = 4! = 24
No. of words start with "SN" = 4! = 24
No. of words start with "SO" = 4! = 24
No. of words start with "SR" =4! = 24
No. of words start with "STG" = 3! = 6
No. of words start with "STN" = 3! = 6
No. of words start with "STO" = 3! = 6
No. of words start with "STRG" = 2! = 2
No. of words start with "STRN" = 2! = 2
No. of words start with "STROG" = 1! = 1
Word "STRONG"=1
Position of word "STRONG"= 4×120 + 4×24 + 3×6 + 2×2 + 1 + 1 = 600

11. (B) sinA. sin(180+A) + cos(180+A). sin (270–A)
= sinA. (–sinA)+(– cosA).(– cosA)
= $-\sin^2 A + \cos^2 A = \cos 2A$

12. (A) $\dfrac{1}{\cos 255^o} + \dfrac{\sqrt{3}}{\sin 165^o}$

$$\Rightarrow \frac{1}{\cos(270-15)} + \frac{\sqrt{3}}{\sin(180-15)}$$

$$\Rightarrow \frac{1}{-\sin 15} + \frac{\sqrt{3}}{\sin 15}$$

$$= \frac{1}{\sin 15}(-1 + \sqrt{3})$$

$$\Rightarrow \frac{2\sqrt{2}}{\sqrt{3}-1}(\sqrt{3}-1) = 2\sqrt{2}$$

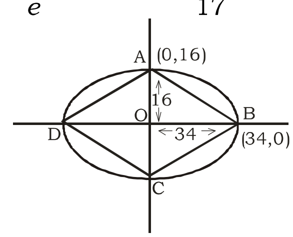
13. (C) $\lim\limits_{x \to 6} \dfrac{\sqrt{6x} - 6}{\sqrt{3x-2} - 4}$ $\qquad \left[\dfrac{0}{0}\right]$form
by L- Hospital's Rule

$$\Rightarrow \lim_{x \to 6} \frac{\frac{1}{2\sqrt{6x}} \times 6 - 0}{\frac{1}{2\sqrt{3x-2}} \times 3 - 0}$$

$$\Rightarrow \lim_{x \to 6} \frac{6\sqrt{3x-2}}{3\sqrt{6x}} \Rightarrow \frac{6 \times 4}{3 \times 6} = \frac{4}{3}$$

14. (B) given that $e = \dfrac{17}{30}$

and $\dfrac{2a}{e} = 120 \Rightarrow \dfrac{2a \times 30}{17} = 120 \Rightarrow a = 34$



Now, $e^2 = 1 - \dfrac{b^2}{(34)^2}$

$$\Rightarrow \frac{64}{289} = \frac{b^2}{(34)^2} \Rightarrow \frac{8}{17} = \frac{b}{34} \Rightarrow b = 16$$

Area of $\Delta AOB = \dfrac{1}{2} \times OA \times OB$

$$= \frac{1}{2} \times 16 \times 34 = 272$$

Area of ABCD= 4×Area of ΔAOB
= 4×272 =1088 sq. unit

15. (C) planes

$3x - 4y + 12z = 6$ and $-4x - 12y - 3z = 8$

Angle b/w planes

$$\cos\theta = \frac{3\times(-4)+(-4)(-12)+12(-3)}{\sqrt{(3)^2+(-4)^2+(12)^2}}$$

$$\cos\theta = \frac{-12+48-36}{13}$$

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

16. (C) $I = \int_{-\pi 2}^{\pi/2} \frac{\tan^2 x}{\sin x} dx$

$I = 0$ [$\because$Function is an odd.]

17. (B) equation $3x^2 - 4x + 8 = 0$

$$\alpha+\beta = \frac{4}{3}, \alpha\beta = \frac{8}{3}$$

Now, $\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\frac{16}{9}-\frac{16}{3}}{\frac{64}{9}} = \frac{-1}{2}$$

and $\frac{1}{\alpha^2}.\frac{1}{\beta^2} = \frac{1}{\left(\frac{8}{3}\right)^2} = \frac{9}{64}$

The required equation

$$x^2 - \left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2}.\frac{1}{\beta^2} = 0$$

$$x^2 - \left(\frac{-1}{2}\right)x + \frac{9}{64} = 0$$

$$64x^2 + 32x + 9 = 0$$

18. (B) $y = \text{cosec}(\tan^{-1}x)$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = -\text{cosec}(\tan^{-1}x).\cot(\tan^{-1}x).\frac{1}{1+x^2}$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\text{cosec}\left(\frac{\pi}{4}\right)\cot\left(\frac{\pi}{4}\right).\frac{1}{2}$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\frac{\sqrt{2}\times 1}{2} = \frac{-1}{\sqrt{2}}$$

19. (D) $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ -(i)

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} dx$$

$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ -(ii)

from eq (i) and eq (ii)

$$I + I = \int_0^{\pi/2}\left[\frac{\sin x}{\sin x+\cos x} + \frac{\cos x}{\cos x + \sin x}\right]dx$$

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1.dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

20. (C) $\begin{vmatrix} 8! & 9! & 10! \\ 9! & 10! & 11! \\ 10! & 11! & 12! \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 8! & 9! & 10! \\ 8\times 8! & 9\times 9! & 10\times 10! \\ 89\times 8! & 109\times 9! & 131\times 10! \end{vmatrix}$$

$$\Rightarrow 8!\times 9!\times 10! \begin{vmatrix} 1 & 0 & 0 \\ 8 & 1 & 2 \\ 89 & 20 & 42 \end{vmatrix}$$

$$\Rightarrow 8!\times 9!\times 70![1(42-40)-0-0]$$

$\Rightarrow$ $2\times 8!\times 9!\times 10!$

21. (A) $\vec{a} = 3\hat{i}+2\hat{j}-5\hat{k}$ and $\vec{b} = -\hat{i}+\hat{j}-4\hat{k}$

$$\vec{b} - 2\vec{a} = (-\hat{i}+\hat{j}-4\hat{k}) - 2(3\hat{i}+2\hat{j}-5\hat{k})$$

$$\vec{b} - 2\vec{a} = (-7\hat{i} - 3\hat{j} + 6\hat{k})$$

$$3\vec{a} - \vec{b} = 3(3\hat{i}+2\hat{j}-5\hat{k}) - (-\hat{i}+\hat{j}-4\hat{k})$$

$$= (10\hat{i} + 5\hat{j} - 11\hat{k})$$

Now $(\vec{b} - 2\vec{a}).(3\vec{a} - \vec{b})$

$$\Rightarrow (-7\hat{i} - 3\hat{j} + 6\hat{k}).(10\hat{i} + 5\hat{j} - 11\hat{k})$$

$$\Rightarrow -70 - 15 - 66 \Rightarrow -151$$

22. (C) $\sin\frac{\pi}{3}+\sin\frac{5\pi}{9}+\sin\frac{8\pi}{9}+\sin\frac{14\pi}{9}$

$\Rightarrow \sin\frac{\pi}{3}+\sin\frac{5\pi}{9}+\sin\frac{8\pi}{9}+\sin\frac{14\pi}{9}$

$\Rightarrow \frac{\sqrt{3}}{2}+\sin\frac{5\pi}{9}+2\sin\frac{22\pi}{9}.\cos\frac{6\pi}{9}$

$\Rightarrow \frac{\sqrt{3}}{2}+\sin\frac{5\pi}{9}+2\sin\left(2\pi+\frac{4\pi}{9}\right).\cos\frac{2\pi}{3}$

$\Rightarrow \frac{\sqrt{3}}{2}+\sin\frac{5\pi}{9}+2\sin\frac{4\pi}{9}.\cos\left(\pi-\frac{\pi}{3}\right)$

$\Rightarrow \frac{\sqrt{3}}{2}+\sin\frac{5\pi}{9}+2\sin\left(\pi-\frac{5\pi}{9}\right)\left(-\cos\frac{\pi}{3}\right)$

$\Rightarrow \frac{\sqrt{3}}{2}+\sin\frac{5\pi}{9}+2\sin\frac{5\pi}{9}\times\left(-\frac{1}{2}\right)$

$\Rightarrow \frac{\sqrt{3}}{2}$

23. (C) $y=\tan^{-1}\left[\frac{1-x}{1+x}\right]$

Let $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$

$\Rightarrow y=\tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}.\tan\theta}\right]$

$\Rightarrow y=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$

$\Rightarrow y=\frac{\pi}{4}-\theta$

$\Rightarrow y=\frac{\pi}{4}-\tan^{-1}x$

On differentiating both side w.r.t "x"

$\Rightarrow \frac{dy}{dx}=-\frac{1}{1+x^2}$

24. (D) A.T.Q –

$a+33d=235$ ……(i)

$a+234d=34$ ……(ii)

from eq. (i) and eq (ii)

$d=-1$ and $a=268$

Let $T_n=0$

$\Rightarrow a+(n-1)d=0$

$\Rightarrow 268+(n-1)(-1)=0 \Rightarrow n=269$

25. (C) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=\operatorname{cosec}^{-1}\left(-\operatorname{cosec}\frac{\pi}{4}\right)$

$\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=\operatorname{cosec}^{-1}\left[\operatorname{cosec}\left(-\frac{\pi}{4}\right)\right]=-\frac{\pi}{4}$

26. (B) $I=\int\frac{dx}{x\left(x^5+1\right)}$

$I=\int\frac{x^4dx}{x^5\left(x^5+1\right)}$

Let $x^5=t$

$5x^4dx=dt$

$x^4dx=\frac{1}{5}dt$

$I=\int\frac{1}{5}\frac{1}{t(t+1)}dt$

$I=\frac{1}{5}\int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt$

$I=\frac{1}{5}\left[\log t-\log(t+1)\right]+C$

$I=\frac{1}{5}\log\frac{t}{t+1}+c$

$I=\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+C$

27. (A) Let $y=\log_{10}(3x^2-5)$ and $z=x^2$

$y=\log_{10}(3z-5)$

$y=\log_{10}e\times\log_e(3z-5)$

On differentialting both side w.r.t. 'z'

$\frac{dy}{dz}=\log_{10}e\times\frac{1}{3z-5}\times 3$

$\frac{dy}{dz}=\frac{3\log_{10}e}{3z-5} \Rightarrow \frac{dy}{dz}=\frac{3\log_{10}e}{3x^2-5}$

28. (C) Probability of selecting Rohan $P(R)=\frac{2}{5}$

and $P(\overline{R})=1-\frac{2}{5}=\frac{3}{5}$

probability of selecting Sumit $P(S)=\frac{1}{4}$

$P(\overline{S})=1-\frac{1}{4}=\frac{3}{4}$

Probability of one of them is selected

$=\frac{2}{5}\times\frac{3}{4}+\frac{3}{5}\times\frac{1}{4} \Rightarrow \frac{6}{20}+\frac{3}{20}=\frac{9}{20}$

29. (B) $\begin{vmatrix} \log_5 5 & 7 & 5 \\ 1 & 2 & \log_2 8 \\ \log_2 2 & 8 & 3\log_3 \sqrt{9} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 7 & 3 \\ 1 & 2 & \log_2 2^3 \\ 1 & 8 & 3\log_3 \sqrt{3^2} \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 7 & 3 \\ 1 & 2 & 3\log_2 2 \\ 1 & 8 & 3\log_3 3 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 7 & 3 \\ 1 & 2 & 3 \\ 1 & 8 & 3 \end{vmatrix}$

$\Rightarrow 3\begin{vmatrix} 1 & 7 & 1 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix}$

$\Rightarrow 0$ [$\because$ Two columns are idential.]

30. (A) $f(x) = |2x - 26|$ and $g(x) = x^2$

Now $fog(x) = f[g(x)]$

$fog(x) = f[x^2]$

$fog(x) = |2x^2 - 26|$

$fog(3) = |2 \times (3)^2 - 26|$

$fog(3) = |18 - 26| = 8$

31. (B) degree = 2

32. (C) Let $y = \sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + ...}}}$

$y = \sqrt{3 + 2y}$

$y^2 = 3 + 2y$

$y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = -1, 3$

Hence $\sqrt{3 + 2\sqrt{3 + 2\sqrt{3 + ...}}} = 3$

33. (D) $\begin{vmatrix} 1+y & 1 & 1 \\ 1 & 1+z & 1 \\ 1 & 1 & 1+x \end{vmatrix} = k$

$\Rightarrow (1 + y)[(1 + z)(1 + x) - 1]$

$-1[1 + x - 1] + 1(1 - 1 - z) = k$

$\Rightarrow (1 + x)(1 + y)(1 + z) - 1 - y - x - z = k$

$\Rightarrow 1 + y + z + yz + x + xy + xz + xyz$

$-1 - y - x - z = k$

$\Rightarrow \frac{xy + yz + zx + xyz}{xyz} = \frac{k}{xyz}$

$\Rightarrow z^{-1} + x^{-1} + y^{-1} + 1 = \frac{k}{xyz}$

given that $x^{-1} + y^{-1} + z^{-1} = 0$

$\Rightarrow 0 + 1 = \frac{k}{xyz} \Rightarrow k = xyz$

34. (C) given that

$\frac{x^2}{2} + \frac{y^2}{18} = 1$

$a = \sqrt{2}$, $b = \sqrt{18}$

Area of an ellipse = πab

$= \pi\sqrt{2} \times \sqrt{18}$

$= 6\pi$ sq. unit

35. (B) $z = \frac{1 - 2i}{1 - i} - \frac{3 - i}{1 + 2i}$

$z = \frac{(1 - 2i)(1 + i)}{(1 - i)(1 + i)} - \frac{(3 - i)(1 - 2i)}{(1 + 2i)(1 - 2i)}$

$z = \frac{3 - i}{2} - \frac{1 - 7i}{3}$

$z = \frac{7 + 11i}{6}$

Now

$z^2 + z\bar{z} = \left(\frac{7 + 11i}{6}\right)^2 + \left(\frac{7 + 11i}{6}\right)\left(\frac{7 - 11i}{6}\right)$

$= -\frac{72 + 154i}{36} + \frac{60}{36} = \frac{-6 + 77i}{18}$

36. (A) curve $\sqrt{x} + \sqrt{y} = \sqrt{2} \Rightarrow y = \left(\sqrt{2} - \sqrt{x}\right)^2$

curve cut the x-axis *i.e.* $y = 0$, $x = 2$

Area = $\int_0^2 y.dx$

Area = $\int_0^2 \left(\sqrt{2} - \sqrt{x}\right)^2 dx$

Area = $\int_0^2 \left(2 + x - 2\sqrt{2}\sqrt{x}\right) dx$

Area = $\left[2x + \frac{x^2}{2} - 2\sqrt{2}\,\frac{x^{3/2}}{3/2}\right]_0^2$

Area = $2\times 2+\dfrac{2\times 2}{2}-\dfrac{4}{3}\sqrt{2}\,(2)^{3/2}-0$

Area = $4+2-\dfrac{4}{3}\times 4=\dfrac{2}{3}\text{sq. unit}$

Short Method:-

Curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$

Area = $\dfrac{a^2}{6}$

given that $\sqrt{x}+\sqrt{y}=\sqrt{2}$

Area = $\dfrac{(2)^2}{6}=\dfrac{2}{3}\text{sq. unit}$

37. (C) given that A= $\tan^{-1}3$ and C = $\tan^{-1}2$

tanA = 3, tan C = 2

Now tan (A+C) $=\dfrac{\tan A+\tan C}{1-\tan A.\tan C}$

tan (180 – B) = $\dfrac{2+3}{1-2\times 3}$

– tan B = $\dfrac{5}{-5}$

tanB = 1 $\Rightarrow B=45^\circ$

38. (A) given that

$\log_5 2$, $\log_5(3^x-1)$ and $\log_5(5\times3^x-13)$ are in A.P, then $2\log_5(3^x-1)=\log_5 2+\log_5(5\times3^x-13)$

$\Rightarrow \log_5\left(3^x-1\right)^2=\log_5\left\{2\left(5\times 3^x-13\right)\right\}$

$\Rightarrow \left(3^x\right)^2+1-2\times 3^x=10\times 3^x-26$

$\Rightarrow \left(3^x\right)^2-12\times 3^x+27=0$

$\Rightarrow \left(3^x-9\right)\left(3^x-3\right)=0$

$3^x=9$ or $3^x=3$

$x=2$ $x=1$

39. (B) We know that

$(1+x)^n=C_0+C_1x+C_2x^2+.........C_nx^n$ ...(i)

On integrating both side w.r.t. 'x'

$$\frac{(1+x)^{n+1}}{n+1}=C_0x+C_1\frac{x^2}{2}+C_2\frac{x^3}{3}....+C_n\frac{x^{n+1}}{n+1}+\frac{1}{n+1}$$

On putting x = 0 both side

$$\frac{1}{n+1}=0+k\Rightarrow k=\frac{1}{n+1}$$

from eq.(i)

$$\frac{(1+x)^{n+1}}{n+1}=C_0x+C_1\frac{x^2}{2}+C_2\frac{x^3}{3}....+C_n\frac{x^{n+1}}{n+1}+\frac{1}{n+1}$$

On putting x = 1 both side

$$\frac{2^{n+1}}{n+1}=C_0+\frac{C_1}{2}+\frac{C_2}{3}+\frac{C_3}{4}+.....+\frac{C_n}{n+1}+\frac{1}{n+1}$$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+\frac{C_3}{4}+.........+\frac{C_n}{n+1}=\frac{2^{n+1}}{n+1}-\frac{1}{n+1}$$

40. (C) $y^{1/6}=\left[x-\sqrt{1+x^2}\right]$

$\Rightarrow y=\left[x-\sqrt{1+x^2}\right]^6$

On differentiating both side w.r.t. 'x'

$\Rightarrow y_1=6\left[x-\sqrt{1+x^2}\right]^5\left[1-\dfrac{(2x)}{2\sqrt{1+x^2}}\right]$

$\Rightarrow y_1=6\left[x-1+x^2\right]^5\left[\dfrac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}}\right]$

$\Rightarrow y_1=\dfrac{-6\left[x-\sqrt{1+x^2}\right]^6}{\sqrt{1+x^2}}\Rightarrow y_1=\dfrac{-6y}{\sqrt{1+x^2}}$

from eq (i)

$\Rightarrow \sqrt{1+x^2}\,y_1=-6y$ ......(i)

Again, Differentiating

$\Rightarrow \sqrt{1+x^2}y_2+y_1\dfrac{1}{2}\dfrac{(2x)}{\sqrt{1+x^2}}=-6y_1$

$\Rightarrow \dfrac{(1+x^2)y_2+xy_1}{\sqrt{1+x^2}}=-6y$

$\Rightarrow (1+x^2)y_2+xy_1=-6y_1\sqrt{1+x^2}$

$\Rightarrow (1+x^2)y_2+xy_1=-6(-6y)$

$\Rightarrow (1+x^2)y_2+xy_1=36y$

41. (B) $y=\sqrt{\cos x-\sqrt{\cos x-\sqrt{\cos x-...\infty}}}$

$\Rightarrow y=\sqrt{\cos x-y}$

$\Rightarrow y^2=\cos x-y$

$\Rightarrow y^2+y=\cos x$

On differentiating both side w.r.t. 'x'

$\Rightarrow 2y\dfrac{dy}{dx}+\dfrac{dy}{dx}=-\sin x$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{-\sin x}{2y+1}$

42. (D) Let $f(x) = \dfrac{ln\{1+[x]\}}{[x]}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h)$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} \dfrac{\ln\{1+[0-h]\}}{[0-h]}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} \dfrac{\ln(1-1)}{-1}$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} \dfrac{\ln 0}{-1} = -\infty$

and $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h)$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} \dfrac{\ln\{1+[0+h]\}}{[0+h]}$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} \dfrac{\ln(1+0)}{0}\left[\dfrac{0}{0}\right]$ form

$\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$

Hence limit does not exist.

43. (C) $I = \int_0^2 x(2-x)^6\,dx$

Prop. IV $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^2 (2-x)x^6 dx$

$I = \int_2^2 2 \times x^6 dx - \int_0^2 x^7 dx$

$I = 2 \times \left[\dfrac{x^7}{7}\right]_0^2 - \left[\dfrac{x^8}{8}\right]_0^2$

$I = \dfrac{2}{7}\left[2^7 - 0\right] - \dfrac{1}{8}\left[2^8 - 0\right]$

$I = \dfrac{2^8}{7} - \dfrac{2^8}{8}$

$I = \dfrac{2^5}{7} = \dfrac{32}{7}$

44. (C) The required number of elementary events = ${}^7C_2 \times 2! = 42$

45. (D) given that a = 15, b = 9 and c = 12

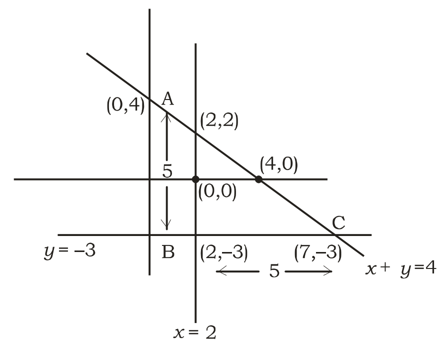
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{81 + 144 - 225}{2 \times 9 \times 12}$

$\cos A = 0 \Rightarrow A = 90^\circ$

Now, sinA = sin 90 = 1

46. (A)



Area of $\Delta ABC = \dfrac{1}{2} \times 5 \times 5$

$= \dfrac{25}{2} = 12.5$ sq. unit

47. (A)

48. (B) $\{x / x^2 + 2 = 0, x \in R\}$

49. (C) $A = \{2,3,4\}$, $B = \{3,4,5\}$ and $C = \{a,b\}$

$(A \cup B) = \{2,3,4,5\}$

Now $(A \cup B) \times C = \{2,3,4,5\} \times (a,b)$

No. of element in $(A \cup B) \times C = 4 \times 2 = 8$

50. (B) $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$

$\Rightarrow abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow abc\begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$

$\Rightarrow abc(b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$

$R_3 + R_3 - R_2$

$\Rightarrow abc(b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-b \end{vmatrix}$

$\Rightarrow abc\,(b-a)(c-a)(c-b)$

$\Rightarrow abc\,(a-b)(b-c)(c-a)$

51. (B) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Now $\vec{a}\,.\,\hat{i} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}).\,\hat{i} = a_1$

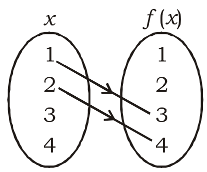
similarly $(\vec{a}\cdot\hat{j})\ \hat{j} = a_2$

$(\vec{a}\cdot\hat{k})\ \hat{k} = a_3$

then $(\vec{a}\cdot\hat{i})\,\hat{i} + (\vec{a}\cdot\hat{j})\ \hat{j} + (\vec{a}\cdot\hat{k})\ \hat{k}$

$= a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$

52. (A) given that $f(x) = x + 2,\ f : \mathrm{N} \to \mathrm{N}$



$\therefore$ Function is one-one but not onto.

53. (B) We know that

$(1+x)^n = C_0 + C_1\,x + C_2\,x^2 + \ldots\ldots\ldots C_n\,x^n$

put $x = 1$

$2^n = C_0 + C_1 + C_2 + \ldots.C_n$

$C_0 + C_1 + C_2 + \ldots\ldots + C_n = 2^n$

54. (C) Equation $2ax^2 + 3bx + c = 0$

Let roots are $2\alpha, \alpha$

$$2\alpha + \alpha = \frac{-3b}{2a}$$

$$3\alpha = \frac{-3b}{2a} \Rightarrow \alpha = \frac{-b}{2a} \qquad \ldots\ldots\ldots(i)$$

and $2\alpha.\alpha = \dfrac{c}{2a}$

$$\Rightarrow \alpha^2 = \frac{c}{4a}$$

$$\Rightarrow \left(\frac{-b}{2a}\right)^2 = \frac{c}{4a} \qquad [\text{from eq (i)}]$$

$$\Rightarrow \frac{b^2}{4a^2} = \frac{c}{4a} \Rightarrow b^2 = ac$$

55. (C) The required probability $= \dfrac{{}^4C_2}{{}^{10}C_2}$

$$= \frac{6}{45} = \frac{2}{15}$$

56. (D) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 16\theta}}}}}$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2(2\cos^2 8\theta)}}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 4\theta}}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+2\cos 4\theta}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2\times 2\cos^2 2\theta}}}$$

$$\Rightarrow \sqrt{2+\sqrt{2+2\cos 2\theta}}$$

$$\Rightarrow \sqrt{2+\sqrt{2\times 2\cos^2\theta}}$$

$$\Rightarrow \sqrt{2+2\cos\theta}$$

$$\Rightarrow \sqrt{2\times 2\cos^2\frac{\theta}{2}} = 2\cos\frac{\theta}{2}$$

57. (B) $9x^2 + 16y^2 + 6x - 32y - 23 = 0$

$$\Rightarrow (3x+1)^2 + (4y-4)^2 = 40$$

$$\Rightarrow 9\left(x+\frac{1}{3}\right)^2 + 16(y-1)^2 = 40$$

$$\Rightarrow \frac{\left(x+\frac{1}{3}\right)^2}{\frac{40}{9}} + \frac{(y-1)^2}{\frac{40}{16}} = 1$$

$$a^2 = \frac{40}{9}, \qquad b^2 = \frac{40}{16}$$

Now eccentricity $e^2 = 1 - \dfrac{b^2}{a^2}$

$$e^2 = 1 - \frac{\frac{40}{16}}{\frac{40}{9}}$$

$$e^2 = 1 - \frac{9}{16}$$

$$e^2 = \frac{7}{16} \Rightarrow e = \frac{\sqrt{7}}{4}$$

58. (B) **Statement I**

for any three coplanar vectors a, b and c

$(a \times b).c = 0$

Statement I is incorrect.

Statement II

L.H.S. $= x.\{(y+z)\times(x+y+z)\}$

$= x.\{y\times x + y\times y + y\times z + z\times x + z\times y + z\times z\}$

$= x.\{\,y\times x + y\times z + z\times x - y\times z\,\}$

$= x\,.\{y\times x + z\times x\}$

$= x.(y\times x) + x.(\,z\times x\,)$

$0 + 0 = 0 =$ R.H.S.

Statement II is correct.

(59-62) given lines $8x - 6y + 11 = 0$ ....(i)
and $12x - 9y + 7 = 0$

$$8x - 6y + \frac{14}{3} = 0 \qquad ....(ii)$$

Both lines are parallel.

59. (B) Hence angle between lines = 0°

60. (A) equation of line which is perpendicular to the line $8x - 6y + 11 = 0$
$6x + 8y + c = 0$ .....(iii)
it passes through the point (–3, 2)
$\Rightarrow -18 + 16 + c = 0 \Rightarrow c = 2$
The required line $6x + 8y + 2 = 0$

61. (C) given lines $8x - 6y + 11 = 0$

$$4x - 3y + \frac{11}{2} = 0$$

and $12x - 9y + 7 = 0$

$$4x - 3y + \frac{7}{3} = 0$$

$$\text{Distance b/w lines} = \frac{\frac{11}{2} - \frac{7}{3}}{\sqrt{(4)^2 + (-3)^2}}$$

$$= \frac{19}{6 \times 5} = \frac{19}{30}$$

62. (D) Equation of line which is parellel to the given line $12x - 9y + 7 = 0$
$12x - 9y + c = 0$ ....(i)
it passes through the point (0,0) *i.e.* c = 0
from eq (i)
$12x - 9y = 0 \Rightarrow 4x = 3y$

63. (A)

64. (B) Let $z = \dfrac{1+i}{1+(2-i)^2}$

$$z = \frac{1+1}{1+4+i^2-4i}$$

$$z = \frac{1+i}{1+4-1-4i}$$

$$z = \frac{1+i}{4(1-i)}$$

$$z = \frac{(1+i)(1+i)}{4(1-i)(1+i)}$$

$$z = \frac{2i}{4 \times 2} = \frac{i}{4}$$

Modulus of $z = |z| = \dfrac{1}{4}$

65. (C)

$$\begin{array}{r|r|l}
2 & 48 & 0 \\ \hline
2 & 24 & 0 \\ \hline
2 & 12 & 0 \\ \hline
2 & 6 & 0 \\ \hline
2 & 3 & 1 \\ \hline
2 & 1 & 1 \\ \hline
 & 0 &
\end{array} \uparrow$$

$(48)_{10} = (110000)_2$

66. (B) $I = \displaystyle\int_{-2}^{0} \frac{dx}{x^2 + x - 12}$

$$I = \int_{-2}^{0} \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \frac{1}{4} - 12}$$

$$I = \int_{-2}^{0} \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \left(\frac{7}{2}\right)^2}$$

$$I = \frac{1}{2 \times \frac{7}{2}}\left[\log\left|\frac{x+\frac{1}{2}-\frac{7}{2}}{x+\frac{1}{2}+\frac{7}{2}}\right|\right]_{-2}$$

$$I = \frac{1}{7}\left[\log\left|\frac{x-3}{x+4}\right|\right]_{-2}^{0}$$

$$I = \frac{1}{7}\left[\log\left|\frac{-3}{4}\right| - \log\left|\frac{-2-3}{-2+4}\right|\right]$$

$$I = \frac{1}{7}\left[\log\frac{3}{4} - \log\frac{5}{2}\right]$$

$$I = \frac{1}{7}\log\left(\frac{3}{4} \times \frac{2}{5}\right) \Rightarrow I = \frac{1}{7}\log\left(\frac{3}{10}\right)$$

67. (B) $I = \displaystyle\int \sqrt{x}.e^{x^{3/2}}dx$
Let $x^{3/2} = t$

$$\frac{3}{2}x^{1/2}dx = dt \Rightarrow \sqrt{x}\ \ dx = \frac{2}{3}dt$$

$$I = \int \frac{2}{3}e^t\, dt$$

$$I = \frac{2}{3}e^t + c$$

$$I = \frac{2}{3}e^{x^{3/2}} + c$$

68. (C) given that
$7^7 + 7 \times 7^6 \times 3^1 + 21 \times 7^5 \times 3^2 ....+ 3^7$
$= k \times 2^5 \times 5^6$ ....(i)
We know that
$(x + a)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}a + ... {}^nC_n\, a^n$
On putting $x = 7$, $a = 3$, $n = 7$
$(7 + 3)^7 = {}^7C_0\, 7^7 + {}^7C_1\, 7^6 \times 3 + ... + {}^7C_7\, 3^7$

$10^7 = 7^7+7\times7^6\times3^1+21\times7^5\times3^2.........+3^7$
On comparing with eq.(i)
$k \times 2^5\times5^6 = 10^7$
$k \times 2^5\times5^6 = 2^7\times5^7 \Rightarrow k = 20$

69. (C) plane $-2x + 3y - 6z + 5 = 0$
distance from a point (3, –1, 2)

$$D = \left|\frac{-2\times3+3\times(-1)-6\times2}{\sqrt{(-2)^2+(3)^2+(-6)^2}}\right|$$

$$D = \frac{21}{7} = 3$$

70. (B) $y = e^{x+e^{x+e^{x+.........\infty}}}$
$y = e^{x+y}$
$y = e^x.e^y$
$y.e^{-y} = e^x$ .....(i)
On differentiating both side w.r.t. 'x'

$$y.e^{-y}\left(\frac{-dy}{dx}\right)+e^{-y}\frac{dy}{dx} = e^x$$

$$e^{-y}\frac{dy}{dx}[-y+1] = e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y}}{1-y}$$

$$\frac{dy}{dx} = \frac{y}{1-y} \quad \text{(from eq. (i)}$$

71. (A) $\lim_{x\to0}\frac{\log(5+2x)-\log(5-2x)}{x} = k$
by L-Hospital's Rule

$$\Rightarrow \lim_{x\to0}\frac{\frac{1\times2}{5+2x}-\frac{1\times(-2)}{5-2x}}{1} = k$$

$$\Rightarrow \lim_{x\to0}\frac{2}{5+2x}-\frac{2}{5-2x} = k$$

$$\Rightarrow \frac{2}{5}-\frac{2}{5} = k \Rightarrow k = 0$$

72. (B) $\sqrt{x^2-1}\frac{dy}{dx}+y = \frac{\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$

$$\frac{dy}{dx}+\frac{1}{\sqrt{x^2-1}}y = \frac{1}{x+\sqrt{x^2-1}}$$

On comparing with $\frac{dy}{dx}$ + Py = Q

$$P = \frac{1}{\sqrt{x^2-1}}, \quad Q = \frac{1}{x+\sqrt{x^2-1}}$$

$$\text{I.F.} = e^{\int P.dx}$$

$$\text{I.F.} = e^{\int\frac{1}{\sqrt{x^2-1}}dx}$$

$$\text{I.F.} = e^{\log\left|x+\sqrt{x^2-1}\right|}$$

$$\text{I.F.} = x+\sqrt{x^2-1}$$

Solution of differential equation

$$y \times \text{I. F.} = \int Q\times\text{I.F.}\,dx$$

$$y\left(x+\sqrt{x^2-1}\right) = \int\frac{x+\sqrt{x^2-1}}{x+\sqrt{x^2-1}}dx$$

$$y\left(x+\sqrt{x^2-1}\right) = \int 1.dx$$

$$y\left(x+\sqrt{x^2-1}\right) = x + c$$

$$y = \frac{x+c}{x+\sqrt{x^2-1}}$$

73. (A) Curve $y = 4x^2 - 7x$

$$\frac{dy}{dx} = 8x-7$$

$$m = \left(\frac{dy}{dx}\right)_{at(-1,11)} = -8 -7 = -15$$

equation of tangent at point (–1, 11)
$y - 11 = -15(x + 1)$
$y - 11 = -15x - 15$
$y = -15x - 4$ ....(i)
given that equation of tangent is
$by = ax + c$
On comparing with eq. (i)
$b = 1$, $a = -15$, $c = -4$

74. (C) $f(x) = \begin{vmatrix}\sin x & -\cos x & 1\\ \cos x & 1 & \cos x\\ \cos x & 1 & 1\end{vmatrix}$

$f(x) = \sin x(1-\cos x) + \cos x(\cos x - \cos^2 x) + 0$
$f(x) = \sin x - \sin x.\cos x + \cos^2 x - \cos^3 x$
On differentiating both side w.r.t 'x'
$f'(x) = \cos x - \sin x(-\sin x) - \cos x(\cos x)$
$+ 2\cos x(-\sin x) - 3\cos^2 x(-\sin x)$
$f'(x) = \cos x + \sin^2 x - \cos^2 x - 2\sin x.\cos x$
$+ 3\sin x.\cos^2 x$

$$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2}+\sin^2\frac{\pi}{2}-\cos^2\frac{\pi}{2}$$

$$-2\sin\frac{\pi}{2}.\cos\frac{\pi}{2}+3\sin\frac{\pi}{2}.\cos^2\frac{\pi}{2}$$

$$f'\left(\frac{\pi}{2}\right) = 0 + 1 - 0 + 0 = 1$$

75. (B) $\frac{\cos 3x - 2\cos 2x + \cos x}{\sin 3x - \sin x}$

$\Rightarrow \frac{\cos 3x + \cos x - 2\cos 2x}{\sin 3x - \sin x}$

$\Rightarrow \frac{2\cos 2x.\cos x - 2\cos 2x}{2\cos 2x.\sin x}$

$\Rightarrow \frac{-2\cos 2x(1-\cos x)}{2\cos 2x.\sin x}$

$\Rightarrow \frac{-\cos^2 \frac{x}{2}}{2\sin\frac{x}{2}.\cos\frac{x}{2}} \Rightarrow -\cot\frac{x}{2}$

76. (C) $\begin{vmatrix} 2x^2-3x-2 & x^2+x-6 & x-2 \\ 2x+1 & x+3 & 1 \\ x+3 & x-2 & x-4 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} (2x+1)(x-2) & (x+3)(x-2) & x-2 \\ 2x+1 & x+3 & 1 \\ x+3 & x-2 & x-4 \end{vmatrix}$

$\Rightarrow (x-2)\begin{vmatrix} 2x+1 & x+3 & 1 \\ 2x+1 & x+3 & 1 \\ x+3 & x-2 & x-4 \end{vmatrix}$

$\Rightarrow$ [$\because$ Two rows are indentical.]

77. (A) **Statement I**

The sum of cubes of first 11 natural numbers $= \left[\frac{11(11+1)}{2}\right]^2$

$= \left(\frac{11\times 12}{2}\right)^2 = (66)^2 = 4356$

Statement I is correct.

Statement II

The sum of squares of first 11 natural numbers $= \frac{11}{6}(11+1)(2\times 11+1)$

$= \frac{11}{6} \times 12 \times 23 = 506$

Statement II is incorrect.

78. (A)

79. (B) equation $x^2-5x+3=0$

$\alpha+\beta = 5$ and $\alpha.\beta = 3$

Now, $\frac{\alpha^4-\beta^4}{\alpha^{-4}-\beta^{-4}} = \frac{\alpha^4-\beta^4}{\frac{1}{\alpha^4}-\frac{1}{\beta^4}} = \frac{\alpha^4-\beta^4}{\frac{\beta^4-\alpha^4}{(\alpha\beta)^4}}$

$= -(\alpha\beta)^4 = -3^4 = -81$

80. (B) 4 cos20°.cos40°.cos60°.cos80°

$\Rightarrow$ 4 cos60° × [cos20°.cos40°.cos80°]

We know that

cosA. cos(60 – A).cos(60 + A) = $\frac{1}{4}$ cos3A

$\Rightarrow 4\times\frac{1}{2}\times\frac{1}{4}\cos(3\times 20)$

$\Rightarrow 4\times\frac{1}{2}\times\frac{1}{4}\times\frac{1}{2} = \frac{1}{4}$

81. (A) The required no. of ways = 11^8

82. (C) given that A.M. = 4× H.M

$(a+b)^2 = 16ab$ ....(i)

Now, $(a-b)^2 = (a+b)^2 - 4ab$

$(a-b)^2 = 16ab - 4ab$

$(a-b)^2 = 12ab$ ....(ii)

from eq (i) and eq (ii)

$\frac{(a+b)^2}{(a-b)^2} = \frac{16ab}{12ab}$

$\frac{(a+b)^2}{(a-b)^2} = \frac{4}{3}$

$\frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$

83. (D) given that the equation of circle

$x^2+y^2-4x-3y-16=0$

Let equation of circle which is concentric with given equation

$x^2+y^2-4x-3y+c=0$ ....(i)

it passes through the point (3, – 2)

$9+4-4\times 3(-2)+c=0 \Rightarrow c=-7$

from eq. (ii)

$x^2+y^2-4x-3y-7=0$

84. (B) S = 0.1 + 0.11+ 0.111 + ...10 terms

$S = \frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+$...............10 terms

$S = \frac{1}{9}\left[\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\right]$.......10 terms

$S = \frac{1}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+.....10\text{ terms}\right]$

$S = \frac{1}{9}\left[10 - \frac{\frac{1}{10}\left[1-\left(\frac{1}{10}\right)^{10}\right]}{1-\frac{1}{10}}\right]$

$S = \frac{1}{9}\left[10-\frac{1}{9}\left(1-\frac{1}{10^{10}}\right)\right]$

85. (C) points $(a, 0)$, $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are collinear, then

$$\begin{vmatrix} a & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a \times 2a \begin{vmatrix} 1 & 0 & 1 \\ t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 1\ (t_1 - t_2) + 1\ (t_1^2.t_2 - t_1.t_2^2) = 0$

$\Rightarrow 1\ (t_1 - t_2) + t_1.t_2\,(t_1 - t_2) = 0$

$\Rightarrow (t_1 - t_2)\ (1 + t_1.\ t_2) = 0$

$\Rightarrow t_1.\ t_2 + 1 = 0 \Rightarrow t_1.\ t_2 = -1$

86. (B) In the expansion of $\left(9x - \dfrac{6}{x^3}\right)^8$

$$T_{r+1} = {}^8C_r\ (9x)^{8-r} \left(\frac{-6}{x^3}\right)^r$$

$$= {}^8C_r\ (9)^{8-r}\ (-6)^r\ x^{8-4r}$$

Now, $8 - 4r = 0 \Rightarrow r = 2$

The required term = 2 + 1 = 3rd

87. (A) equation of parabola

$x^2 + 3x + y - 2 = 0$

$$\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + y - 2 = 0$$

$$\left(x + \frac{3}{2}\right)^2 = -y + \frac{9}{4} + 2$$

$$\left(x + \frac{3}{2}\right)^2 = -\left(y - \frac{17}{4}\right)$$

$X^2 = -Y$ where $X = x + \dfrac{3}{2}$, $Y = y - \dfrac{3}{2}$

$4a = 1 \Rightarrow a = \dfrac{1}{4}$

focus $(X, Y) = (0, -b)$

$X = 0$, $Y = -b$

$x + \dfrac{3}{2} = 0$, $y - \dfrac{17}{4} = -\dfrac{1}{4}$

$x = \dfrac{-3}{2}$, $y = 4$

focus of parabola = $\left(-\dfrac{3}{2}, 4\right)$

88. (C) $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

by Componendo & Dividendo Rule

$$\Rightarrow \frac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow \frac{2\sin x.\cos y}{2\cos x.\sin y} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b} \Rightarrow \frac{\tan y}{\tan x} = \frac{b}{a}$$

89. (B) $z = \dfrac{1-2i}{2+i} - \dfrac{2-3i}{3+i}$

$$z = \frac{1-2i}{2+i} \times \frac{2-i}{2-i} - \frac{2-3i}{3+i} \times \frac{3-i}{3-i}$$

$$z = \frac{-5i}{5} - \frac{3-11i}{10} \Rightarrow z = \frac{-3+i}{10}$$

Now, $z^2 - z\bar{z} = \left(\dfrac{-3+i}{10}\right)^2 - \dfrac{-3+i}{10} \times \dfrac{-3-i}{10}$

$$z^2 - z\bar{z} = \frac{8-6i}{100} - \frac{10}{100}$$

$$z^2 - z\bar{z} = \frac{-2-6i}{100} = -\frac{1+3i}{50}$$

90. (C) digit $\{1, 0, 2, 5, 7\}$

Numbers formed using the given digits = 5! = 120

Numbers formed start with '0' = 4! = 24

Hence total numbers greater than 10000 = 120 – 24 = 96

91. (B) $\sin^2\dfrac{\pi}{10} + \sin^2\dfrac{3\pi}{10} + \sin^2\dfrac{\pi}{5} + \sin^2\dfrac{2\pi}{5}$

$$\Rightarrow \sin^2\frac{\pi}{10} + \sin^2\frac{3\pi}{10} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{5}\right) + \cos^2\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)$$

$$\Rightarrow \sin^2\frac{\pi}{10} + \sin^2\frac{3\pi}{10} + \cos^2\frac{3\pi}{10} + \cos^2\frac{\pi}{10}$$

$$\Rightarrow \sin^2\frac{\pi}{10} + \cos^2\frac{\pi}{10} + \sin^2\frac{3\pi}{10} + \cos^2\frac{3\pi}{10}$$

$\Rightarrow 1 + 1 = 2$

92. (B) $\dfrac{4x}{12x^2 + 24x - 11} > \dfrac{1}{3x+4}$

$\Rightarrow 12x^2 + 16x > 12x^2 + 24x - 11$

$\Rightarrow 0 > 8x - 11$

$\Rightarrow 8x < 11 \Rightarrow x < \dfrac{11}{8}$

Hence $x \in \left(-\infty, \dfrac{11}{8}\right)$

93. (C) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ 3 & -4 & 0 \\ 3 & 1 & -1 \end{bmatrix}$

Co-factors of A-

$C_{11}=(-1)^{1+1}\begin{vmatrix} -4 & 0 \\ 1 & -1 \end{vmatrix}, C_{12}=(-1)^{1+2}\begin{vmatrix} 3 & 0 \\ 3 & -1 \end{vmatrix}, C_{13}=(-1)^{1+3}\begin{vmatrix} 3 & -4 \\ 3 & 1 \end{vmatrix}$

= 4   = 3   = 15

$C_{21}=(-1)^{2+1}\begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix}, C_{22}=(-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}, C_{23}=(-1)^{2+3}\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}$

= –1   = –7   = –10

$C_{31}=(-1)^{3+1}\begin{vmatrix} -3 & 2 \\ -4 & 0 \end{vmatrix}, C_{32}=(-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix}, C_{33}=(-1)^{3+3}\begin{vmatrix} 1 & -3 \\ 3 & -4 \end{vmatrix}$

= 8   = 6   = 5

$$C = \begin{bmatrix} 4 & 3 & 15 \\ -1 & -7 & -10 \\ 8 & 6 & 5 \end{bmatrix}$$

$$\text{adj } A = C^T = \begin{bmatrix} 4 & -1 & 8 \\ 3 & -7 & 6 \\ 15 & -10 & 5 \end{bmatrix}$$

94. (A) $\begin{bmatrix} 3 & -1 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} -1 & -3 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} k & -15 \\ 35 & 30 \end{bmatrix}$

$\begin{bmatrix} -10 & -15 \\ 35 & 30 \end{bmatrix} = \begin{bmatrix} k & -15 \\ 35 & 30 \end{bmatrix}$

On comparing
$k = -10$

95. (A) Zero

96. (C) given that $b_{yx} = \frac{-10}{9}$ and $b_{xy} = \frac{-2}{5}$

$r = \sqrt{b_{yx} \times b_{xy}}$

$r = \sqrt{\frac{-10^2}{9} \times \frac{-2}{5}} \Rightarrow r = \frac{-2}{3}$

97. (D)

98. (A) sides of polygon (n) = 26

No. of diagonals $= \frac{n(n-3)}{2}$

$= \frac{26 \times 23}{2} = 299$

99. (B)

100. (C) $I = \int e^x \left[\frac{x \log x + 1}{x}\right] dx$

$I = \int e^x \left[\log x + \frac{1}{x}\right] dx$

$I = e^x . \log x + c$

101. (D)

Class	f	C
0–10	6	6
10–20	8	14
20–30	8	22
30–40	10	32
40–50	12	44
50–60	16	60

median class

$N = 60, \frac{N}{2} = \frac{60}{2} = 30$

$l_1 = 30, l_2 = 40, f = 10, C = 22$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f} \times (l_2 - l_1)$$

$$\text{Median} = 30 + \frac{30 - 22}{10} \times (40 - 30)$$

$$\text{Median} = 30 + \frac{12}{10} \times 10 = 42$$

102. (B) digits are 2, 8, 4, 6, 7, 9, 5
n (S) = ${}^7C_3$ = 35
n (E) = ${}^3C_2 \times {}^4C_1 + {}^4C_3$ = 3 × 4 + 4 = 16

The required Probability $= \frac{n(E)}{n(S)} = \frac{16}{35}$

103. (B) One year = 365 days
= 52 weeks and 1 day

The required Probability $= \frac{1}{7}$

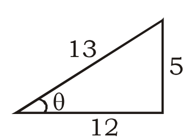
104. (C)

$$\begin{array}{r} 1\ 0\ x\ 0\ 1\ 1 \\ -1\ 1\ y\ 0\ 1 \\ \hline 1\ 1\ z\ 0 \\ \hline \end{array}$$

$z = 1, y = 1, x = 1$

105. (A) given that $\tan\theta = \frac{-5}{12}$

θ lies in the second quadant.



$\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{-12}{13}$

Now, $3\sin\theta - 4\cos\theta = 3 \times \frac{5}{13} - 4\left(-\frac{12}{13}\right)$

$= \frac{15}{13} + \frac{48}{13} = \frac{63}{13}$

106. (D) given that $A = \frac{21\pi}{4}$

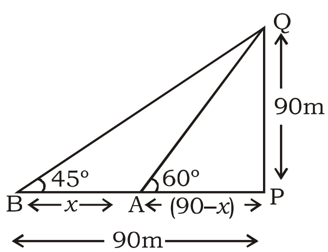
Now,

$$\frac{1-3\cot^2 A}{3\cot A - \cot^3 A} = \frac{1-3\cot^2\left(\frac{21\pi}{4}\right)}{3\cot\left(\frac{21\pi}{4}\right) - \cot^3\left(\frac{21\pi}{4}\right)}$$

$$= \frac{1-3\cot^2\frac{\pi}{4}}{3\cot\frac{\pi}{4} - \cot^3\frac{\pi}{4}}$$

$$= \frac{1-3\times 1}{3-1} = \frac{-2}{2} = -1$$

107. (B) Let AB = x m



In ΔAPQ

$$\tan 60° = \frac{PQ}{AP}$$

$$\Rightarrow \sqrt{3} = \frac{90}{90-x}$$

$$\Rightarrow 90\sqrt{3} - \sqrt{3}x = 90$$

$$\Rightarrow x = \frac{90\left(3-\sqrt{3}\right)}{3}$$

$$\Rightarrow x = 30(3-1.732) = 38.04\text{m}$$

108. (D) Let $f(x) = \frac{[x]}{x}$

L.H.L. $= \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$

$$= \lim_{h \to 0} \frac{[1-h]}{1-h}$$

$$= \lim_{h \to 0} \frac{0}{1-h} = 0$$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$

$$= \lim_{h \to 0} \frac{[1+h]}{1+h}$$

$$= \lim_{h \to 0} \frac{1}{1+h}$$

$$= \frac{1}{1+0} = 1$$

L.H.L $\neq$ R.H.L

Hence limit does not exist.

109. (B) n (S) = 6×6 = 36

$$E = \begin{Bmatrix} (6, 3), (5, 4), (4, 5), (3, 6) \text{ for sum } = 9 \\ (6, 4), (4, 6), (5, 5) \text{ for sum } = 10 \\ (6, 5), (5, 6) \text{ for sum } = 11 \\ (6, 6), \text{ for sum } = 12 \end{Bmatrix}$$

n (E) = 10

Hence Probability $= \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

110. (D) given that

$x = \sin\theta - \cos\theta$ and $y = \sin\theta - \cos\theta$

$x^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta \cdot \cos\theta$

$x^2 = 1 - 2y \Rightarrow x^2 + 2y = 1$

111. (B) Let locus of a point = (h, k, l) which is equidistance from the points (3, –2, 1) and (4, –3, – 1)

$$\Rightarrow \sqrt{(h-3)^2 + (k+2)^2 + (l+1)^2}$$

$$= \sqrt{(h-4)^2 + (k+3)^2 + (l-1)^2}$$

$\Rightarrow h^2 + 9 - 6h + k^2 + 4 + 4k + l^2 + 1 - 2l$

$= h^2 + 16 - 8h + k^2 + 9 + 6k + l^2 + 1 + 2l$

On solving

$\Rightarrow h - k - 2l = 6$

locus of point $x - y - 2z = 6$

112. (A) **Statement-I**

L.H.S $= \left(\omega^{16}+1\right)^8 + \omega$

$= (\omega + 1)^8 + \omega$

$= \left(-\omega^2\right)^8 + \omega$

$= -\omega^{16} + \omega$

$= -\omega + \omega = 0 = $ R.H.S

Statement I is correct.

Statement-II

L. H. S. $= \left(\omega^{173}+1\right)^{14}$

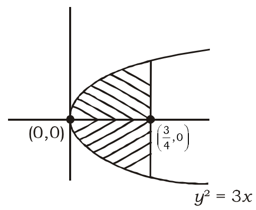
$= \left(\omega^{3\times 57+2}+1\right)^{14}$

$= \left(\omega^2+1\right)^{14}$

$= (-\omega)^{14} = \omega^2 \neq$ R.H.S

Statement II is incorrect.

113. (C)



$y^2 = 3x \Rightarrow y_1 = y = \sqrt{3}\sqrt{x}$

$4a = 3 \Rightarrow a = \frac{3}{4}$

Area $= 2\int_0^{3/4} y.dx$

Area $= 2\int_0^{3/4} \sqrt{3}.\sqrt{x}\,dx$

Area $= 2\sqrt{3}\left[\frac{x^{3/2}}{\frac{3}{2}}\right]^{3/4}$

Area $= 4\frac{\sqrt{3}}{3}\times\frac{3}{4}\times\frac{\sqrt{3}}{2} = \frac{3}{2}$sq. unit

114. (C) given that $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b} = \hat{i}+\hat{j}-2\hat{k}$

and $\vec{c} = 2\hat{i}+\hat{j}-3\hat{k}$

Now, $\vec{a}\times(\vec{b}-\vec{c})-\vec{b}\times(\vec{c}-\vec{a})+\vec{c}\times(\vec{a}-\vec{b})$

$\Rightarrow \vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}-\vec{c}\times\vec{b}$

$\Rightarrow \vec{a}\times\vec{b}+\vec{c}\times\vec{a}-\vec{b}\times\vec{c}-\vec{a}\times\vec{b}+\vec{c}\times\vec{a}+\vec{b}\times\vec{c}$

$\Rightarrow 2\ \vec{c}\times\vec{a}$

$\Rightarrow 2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -3\\ 2 & 3 & 4\end{vmatrix}$

$\Rightarrow 2\left[\hat{i}(4+9)-\hat{j}(8+6)+\hat{k}(6-2)\right]$

$\Rightarrow 2\left[13\hat{i}-14\hat{j}+4\hat{k}\right] \Rightarrow 26\hat{i}-28\hat{j}+8\hat{k}$

115. (B)

116. (A) The required no. of ways $= {}^{15-1}C_{11-1}$

$= {}^{14}C_{10}$

$= 1001$

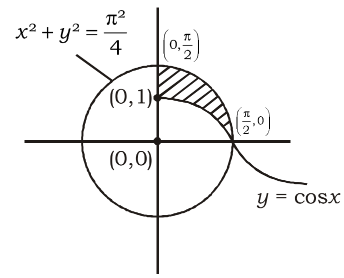
117. (B) $10^{-x\sec x}\left[\frac{d}{dx}10^{x\sec x}\right]$

$\Rightarrow 10^{-x\sec x}[10^{x\sec x}\log 10\{x.\sec x.\tan x+\sec x\}]$

$\Rightarrow 10^{-x\sec x}10^{x\sec x}.\ \sec x\,(x\tan x+1)\ln 10$

$\Rightarrow \sec x\,(x\tan x+1)\ln 10$

118. (A)



$y_1 \Rightarrow y = \sqrt{\frac{\pi^2}{4}-x^2}$ and $y_2 \Rightarrow y = \cos x$

Area $= \int_0^{\pi/2}(y_1-y_2)\,dx$

Area $= \int_0^{\pi/2}\left[\sqrt{\frac{\pi^2}{4}-x^2}-\cos x\right]dx$

Area $= \left[\frac{1}{2}x\sqrt{\frac{\pi^2}{4}-x^2}+\frac{1}{2}\times\frac{\pi^2}{4}\sin^{-1}\left(\frac{2x}{\pi}\right)-\sin x\right]_0^{\pi/2}$

Area $= (0+\frac{\pi^2}{8}\sin^{-1}(1)-\sin\frac{\pi}{2})-(0+0+0)$

Area $= \frac{\pi^2}{8}\times\frac{\pi}{2}-1 = \left(\frac{\pi^3}{16}-1\right)$ sq. unit

119. (D) $I = \int_0^{\pi/6}\frac{\sin 3x}{\sin 3x+\cos x}dx$ ....(i)

$I = \int_0^{\pi/6}\frac{\sin 3\left(\frac{\pi}{6}-x\right)}{\sin 3\left(\frac{\pi}{6}-x\right)+\cos 3\left(\frac{\pi}{6}-x\right)}dx$

$I = \int_0^{\pi/2}\frac{\cos 3x}{\cos x+\sin x}dx$ ....(ii)

from eq. (i) and eq. (ii)

$2I = \int_0^{\pi/6}1.dx$

$2I = [x]_0^{\pi/6} \Rightarrow 2I = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$

120. (C) given data 24,24,26,27,28,29,26,32

$n = 8$

mean $\bar{x} = \frac{24+24+26+27+28+29+26+32}{8}$

$= \frac{216}{8} = 27$

$\sum\left(x-\bar{x}\right) = 3+3+1+0+1+2+1+5$

$= 16$

Mean-deviation $= \frac{\sum\left(x-\bar{x}\right)}{n} = \frac{16}{8} = 2$

NDA (MATHS) MOCK TEST - 106 (Answer Key)

1.	(C)	21.	(A)	41.	(B)	61.	(C)	81.	(A)	101.	(D)
2.	(A)	22.	(C)	42.	(D)	62.	(D)	82.	(C)	102.	(B)
3.	(B)	23.	(C)	43.	(C)	63.	(A)	83.	(D)	103.	(B)
4.	(A)	24.	(D)	44.	(C)	64.	(B)	84.	(B)	104.	(C)
5.	(D)	25.	(C)	45.	(D)	65.	(C)	85.	(C)	105.	(A)
6.	(A)	26.	(B)	46.	(A)	66.	(B)	86.	(B)	106.	(D)
7.	(B)	27.	(A)	47.	(A)	67.	(B)	87.	(A)	107.	(B)
8.	(D)	28.	(C)	48.	(B)	68.	(C)	88.	(C)	108.	(D)
9.	(C)	29.	(B)	49.	(C)	69.	(C)	89.	(B)	109.	(B)
10.	(A)	30.	(A)	50.	(B)	70.	(B)	90.	(C)	110.	(D)
11.	(B)	31.	(B)	51.	(B)	71.	(A)	91.	(B)	111.	(B)
12.	(A)	32.	(C)	52.	(A)	72.	(B)	92.	(B)	112.	(A)
13.	(C)	33.	(D)	53.	(B)	73.	(A)	93.	(C)	113.	(C)
14.	(B)	34.	(C)	54.	(C)	74.	(C)	94.	(A)	114.	(C)
15.	(C)	35.	(B)	55.	(C)	75.	(B)	95.	(A)	115.	(B)
16.	(C)	36.	(A)	56.	(D)	76.	(C)	96.	(C)	116.	(A)
17.	(B)	37.	(C)	57.	(B)	77.	(A)	97.	(D)	117.	(B)
18.	(B)	38.	(A)	58.	(B)	78.	(A)	98.	(A)	118.	(A)
19.	(D)	39.	(B)	59.	(B)	79.	(B)	99.	(B)	119.	(D)
20.	(C)	40.	(C)	60.	(A)	80.	(B)	100.	(C)	120.	(C)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777