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NDA MATHS MOCK TEST - 114 (SOLUTION)

1. (B) a, A_1, A_2, A_3, b

Now, $b = a + 4d \Rightarrow d = \frac{b-a}{4}$

$A_1 = a + d$, $A_2 = a + 2d$, $A_3 = a + 3d$

$A_1 = \frac{3a+b}{4}$, $A_2 = \frac{a+b}{2}$, $A_3 = \frac{a+3b}{4}$

and a, G_1, G_2, G_3, b

Now, $b = ar^4 \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$

$G_1 = ar$, $G_2 = ar^2$, $G_3 = ar^3$

$G_1 = a^{3/4} b^{1/4}$, $G_2 = a^{1/2} b^{1/2}$, $G_3 = a^{1/4} b^{3/4}$

Now, $\frac{2(A_1 + A_2 + A_3)}{(G_1G_2G_3)^2}$

$$= \frac{2\left[\frac{3a+b}{4} + \frac{a+b}{2} + \frac{a+3b}{4}\right]}{\left(a^{3/4}.b^{1/4}.a^{1/2}b^{1/2}.a^{1/4}.b^{3/4}\right)^2}$$

$$= \frac{2\left[\frac{6a+6b}{4}\right]}{\left(a^{3/2}.b^{3/2}\right)^2} = \frac{3(a+b)}{(ab)^3}$$

2. (A) $x + \frac{1}{x} = 2\cos\frac{\pi}{8}$

On Squaring both side

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\cos^2\frac{\pi}{8}$

$\Rightarrow x^2 + \frac{1}{x^2} = 4\cos^2\frac{\pi}{8} - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\left(2\cos^2\frac{\pi}{8} - 1\right)$

$\Rightarrow x^2 + \frac{1}{x^2} = 2\cos\frac{\pi}{4}$

$\Rightarrow x^2 + \frac{1}{x^2} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

3. (D) Let two numbers are a and b.

Now, A.M. = 3 G.M.

$\Rightarrow \frac{a+b}{2} = 3 \times \sqrt{ab}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

By Componendo and Dividendo Rule

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$

$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{4}{2}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$

by Componendo and Dividendo Rule

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

On squaring both side

$\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

4. (A) $y = (\sin y)^{\cos x}$

taking log both side

$\Rightarrow \log y = \cos x.\log(\sin y)$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x.\frac{\cos y}{\sin y}\frac{dy}{dx} + \log\sin y.(-\sin x)$

$\Rightarrow \left(\frac{1}{y} - \cos x.\cot y\right)\frac{dy}{dx} = -\sin x.\log\sin y$

$\Rightarrow \left[\frac{y\cos x.\cot y - 1}{y}\right]\frac{dy}{dx} = \sin x.\log\sin y$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin x.\log\sin y}{y\cos x.\cot y - 1}$

5. (C) Equation

$(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$

roots are equal,

then $B^2 = 4AC$

$\Rightarrow 4(b^2 - ca)^2 = 4(a^2 - bc)(c^2 - ab)$

$\Rightarrow 4b^4 + 4c^2a^2 - 8ab^2c = 4a^2c^2 - 4bc^3 - 4a^3b + 4ab^2c$

$\Rightarrow 4a^3b + 4b^4 + 4bc^3 - 12ab^2c = 0$

$\Rightarrow 4b(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

6. (B) ${}^{51}C_4 + \sum_{r=1}^{3} {}^{50+r}C_3$

$\Rightarrow {}^{51}C_4 + {}^{51}C_3 + {}^{52}C_3 + {}^{53}C_3$

$\Rightarrow {}^{52}C_4 + {}^{52}C_3 + {}^{53}C_4$

$\Rightarrow {}^{53}C_4 + {}^{53}C_3 = {}^{54}C_4$

7. (B) S = $1 \times 3 \times 5 \ldots\ldots\ldots\ldots (2n-3)(2n-1)$

$$S = \frac{1.2.3.4.5\ldots\ldots(2n-3)(2n-2)(2n-1)(2n)}{2.4.6\ldots\ldots\ldots(2n)}$$

$$S = \frac{1.2.3.4\ldots\ldots(2n-1)(2n)}{2^n\left[1.2.3\ldots\ldots(n-1)n\right]} = \frac{(2n)!}{2^n n!}$$

8. (B) $\left(\frac{dy}{dx}\right)^3 + 4\left(\frac{d^2y}{dx^2}\right)^3 = \frac{y}{\left(\frac{d^2y}{dx^2}\right)^2}$

$$\left(\frac{dy}{dx}\right)^3\left(\frac{d^2y}{dx^2}\right)^2 + 4\left(\frac{d^2y}{dx^2}\right)^5 = y$$

Hence order = 2 and degree = 5

9. (B) Equations $x + 2y + z = 0$, $2x - 4y - z = 3$ and $-x - 2y + 3z = 8$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -4 & -1 \\ -1 & -2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 3 \\ 8 \end{bmatrix}$

Using elementary method

$$[A/B] = \left[\begin{array}{ccc|c} \mathbf{1} & 2 & 1 & 0 \\ 2 & -4 & -1 & 3 \\ -1 & -2 & 3 & 8 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + R_1$

$$[A/B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -8 & -3 & 3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$x + 2y + z = 0$ .....(i)

$-8y - 3z = 3$ .....(ii)

$4z = 8$ .....(iii)

On solving eq (i) , (ii) and (iii)

$x = \frac{1}{4}$, $y = \frac{-9}{8}$, $z = 2$

10. (C) Let $a - ib = \sqrt{-1-2\sqrt{2}i}$

On squaring both side

$(a^2 - b^2) - 2abi = -1 - 2\sqrt{2}\, i$

On comparing

$a^2 - b^2 = -1$ and $2ab = 2\sqrt{2}$ ...(i)

$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$(a^2 + b^2)^2 = 1 + 8$

$a^2 + b^2 = 3$ .....(ii)

from eq (i) and eq (ii)

$2a^2 = 2$, $2b^2 = 4$

$a = \pm 1$, $b = \pm\sqrt{2}$

Hence $\sqrt{-1-2\sqrt{2}i} = \pm(1 - \sqrt{2}\, i)$

11. (B) cos 36° . cos 72° . cos108° . cos 144°

$\Rightarrow \frac{1}{4}[2\cos36^\circ.\cos144^\circ][2\cos72^\circ.\cos108^\circ]$

$\Rightarrow \frac{1}{4}[\cos(36+144) + \cos(36-144)]$

$[\cos(72+108) + \cos(72-108)]$

$\Rightarrow \frac{1}{4}[\cos 180 + \cos 108][\cos 180 + \cos 36]$

$\Rightarrow \frac{1}{4}\left[-1 - \frac{\sqrt{5}-1}{4}\right]\left[-1 - \frac{\sqrt{5}+1}{4}\right]$

$\Rightarrow \frac{1}{4}\left[\frac{3+\sqrt{5}}{4}\right]\left[\frac{5+\sqrt{5}}{4}\right]$

$\Rightarrow \frac{5+2\sqrt{5}}{16}$

12. (B) cosec$x$,sec$x$ and cot$x$ are in G.P.

then $\sec^2 x = \text{cosec}\,x.\cot x$

$\Rightarrow \frac{1}{\cos^2 x} = \frac{1}{\sin x}.\frac{\cos x}{\sin x}$

$\Rightarrow \sin^2 x = \cos^3 x$

$\Rightarrow 1 - \cos^2 x = \cos^3 x$

$\Rightarrow \cos^3 x + \cos^2 x = 1$

13. (C) In ΔABC

$(s-a)(s-c) = s(s-b)$

$\Rightarrow s^2 - sa - sc + ac = s^2 - sb$

$\Rightarrow -s(a+c) + ac = -sb$

$\Rightarrow ac = s(a+c) - sb$

$\Rightarrow ac = s(a-b+c)$

$\Rightarrow ac = \frac{a+b+c}{2}(a-b+c)$

$\Rightarrow 2ac = (a+c)^2 - b^2$

$\Rightarrow 2ac = a^2 + c^2 + 2ac - b^2$

$\Rightarrow b^2 = a^2 + c^2$

Hence $\angle B = 90^\circ$

14. (B) $b\,[c\cos A - a\cos C]$

$$\Rightarrow b\left[c.\frac{b^2+c^2-a^2}{2bc} - a.\frac{a^2+b^2-c^2}{2ab}\right]$$

$$\Rightarrow \left[\frac{b^2+c^2-a^2}{2} - \frac{a^2+b^2-c^2}{2}\right] = c^2 - a^2$$

15. (B) Given that Mean = 13 and Mode = 7
We know that
Mode = 3Median – 2Mean
$\Rightarrow$ 7 = 3Median – 2 ×13
$\Rightarrow$ 3Median = 33 $\Rightarrow$ Median =11

16. (C) $I = \int x.\cot^{-1} x\,dx$

$$I = \cot^{-1} x.\int x\,dx - \int\left\{\frac{d}{dx}\left(\cot^{-1} x\right).\int x\,dx\right\}dx$$

$$I = \frac{x^2}{2}.\cot^{-1} x - \int \frac{-1}{1+x^2}.\frac{x^2}{2}dx + c$$

$$I = \frac{x^2}{2}.\cot^{-1} x + \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx + c$$

$$I = \frac{x^2}{2}.\cot^{-1} x + \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx + c$$

$$I = \frac{x^2}{2}.\cot^{-1} x + \frac{1}{2}\left[x - \tan^{-1} x\right] + c$$

$$I = \frac{x^2}{2}.\cot^{-1} x + \frac{x}{2} - \frac{1}{2}\tan^{-1} x + c$$

17. (B) In the expansion of $\left(\frac{3x^2}{4} - \frac{2}{x}\right)^9$

$$T_{r+1} = {}^9C_r\left(\frac{3x^2}{4}\right)^{9-r}\left(\frac{-2}{x}\right)^r$$

$$= {}^9C_r\left(\frac{3}{4}\right)^{9-r}(-2)^r\,x^{18-3r}$$

Now, $18 - 3r = 0 \Rightarrow r = 6$
Hence the required term = 6 + 1 = 7th

18. (C) 1 0 1 1 1   0 . 0 1 1

$1\times2^0=1$ $0 = 0\times2^{-1}$
$1\times2^1=2$ $\frac{1}{4} = 1\times2^{-2}$
$1\times2^2=4$ $\frac{1}{8} = 1\times2^{-3}$
$0\times2^3=0$
$1\times2^4=16$ $\frac{1}{4}+\frac{1}{8}=\frac{3}{8} = 0.375$
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$(10111)_2 = (23)_{10}$, $(0.011)_2 = (0.375)_{10}$
Hence $(10111.011)_2 = (23.375)_{10}$

19. (D) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\sin^{-1}x = \frac{\pi}{2}$, $\sin^{-1}y = \frac{\pi}{2}$, $\sin^{-1} z = \frac{\pi}{2}$

$x = 1$, $y = 1$, $z = 1$
Now, $x^3 + y^3 + z^3 - 3xyz = 1+1+1-3 = 0$

20. (A) We know that
${}^{2n}C_0 + {}^{2n}C_1x + {}^{2n}C_2x^2 + \ldots + {}^{2n}C_{2n}x^{2n} = (1+x)^{2n}$
put $x = 1$
$\Rightarrow {}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + \ldots {}^{2n}C_{2n} = 2^{2n}$

$\Rightarrow 2\left({}^{2n}C_0 + {}^{2n}C_2 + {}^{2n}C_4 + \ldots.. + {}^{2n}C_{2n}\right) = 2^{2n}$

$\Rightarrow {}^{2n}C_0 + {}^{2n}C_2 + {}^{2n}C_4 + \ldots.. + {}^{2n}C_{2n} = \frac{2^{2n}}{2}$

$\Rightarrow {}^{2n}C_0 + {}^{2n}C_2 + {}^{2n}C_4 + \ldots.. + {}^{2n}C_{2n} = 2^{2n-1}$

21. (B) $\begin{bmatrix} 1 & \log_z y & \log_z x \\ \log_y z & 1 & \log_y x \\ \log_x z & \log_x y & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \frac{\log z}{\log z} & \frac{\log y}{\log z} & \frac{\log k}{\log z} \\ \frac{\log z}{\log y} & \frac{\log y}{\log y} & \frac{\log k}{\log y} \\ \frac{\log z}{\log x} & \frac{\log y}{\log x} & \frac{\log k}{\log x} \end{bmatrix}$$

$$= \frac{1}{\log x.\log y.\log z}\begin{bmatrix} \log z & \log y & \log x \\ \log z & \log y & \log x \\ \log z & \log y & \log x \end{bmatrix} = 0$$

22. (A) $x = a\sin\theta - \cos\theta$
$x^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta.\cos\theta$
and $y = b\sin\theta + a\cos\theta$
$y^2 = b^2\sin^2\theta + a^2\cos^2\theta + 2ab\sin\theta.\cos\theta$
$= ab\sin^2\theta - b^2\sin\theta.\cos\theta + a^2\sin\theta.\cos\theta$
$- ab\cos^2\theta$
Now,
$x^2 + y^2 = a^2\sin^2\theta + b^2\cos^2\theta + b^2\sin^2\theta + a^2\cos^2\theta$
$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$
$= a^2 + b^2$

23. (C) $2\sin^2\theta + 4\cos^2\theta = 3$
$\Rightarrow 2(1 - \cos^2\theta) + 4\cos^2\theta = 3$
$\Rightarrow 2 + 2\cos^2\theta = 3$
$\Rightarrow 2\cos^2\theta = 1$
$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

24. (B) Let point (h, k)
According to question

$$\frac{4h-3k-7}{\sqrt{4^2+(-3)^2}}=\frac{8h-15k-9}{\sqrt{8^2+(-15)^2}}$$

$$\Rightarrow \frac{4h-3k-7}{5}=\frac{8h-15k-9}{17}$$

On solving
$\Rightarrow 14h + 12k = 37$
locus of point
$14x + 12y = 37$

25. (B) $\tan(\sin^{-1}x) + \tan(\cos^{-1}x)$

$$\Rightarrow \tan\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right)\right)$$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}}+\frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow \frac{x^2+1-x^2}{x\sqrt{1-x^2}}=\frac{1}{x\sqrt{1-x^2}}$$

26. (C) $\text{I} = \int e^x\left(\sin^{-1}x+\frac{1}{\sqrt{1-x^2}}\right)dx$

$\text{I} = e^x.\sin^{-1}x + C$

$$\left[\because \int e^x\left[f(x)+f'(x)\right]dx=e^x f(x)+C\right]$$

27. (D) $\left(1-\cos\frac{\pi}{3}\right)\left(1-\cos\frac{2\pi}{3}\right)\left(1-\cos\frac{4\pi}{3}\right)\left(1-\cos\frac{5\pi}{3}\right)$

$$\Rightarrow \left(1-\cos\frac{\pi}{3}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1+\cos\frac{\pi}{3}\right)\left(1-\cos\frac{\pi}{3}\right)$$

$$\Rightarrow \left(1-\cos^2\frac{\pi}{3}\right)\left(1-\cos^2\frac{\pi}{3}\right)$$

$$\Rightarrow \left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right)=\frac{9}{16}$$

28. (B) $s = t\sqrt{t^2-1}$
On differentiating both side w.r.t. 't'

$$\frac{ds}{dt}=t\times\frac{1\times 2t}{2\sqrt{t^2-1}}+\sqrt{t^2-1}.1$$

$$\frac{ds}{dt}=\frac{t^2}{\sqrt{t^2-1}}+\sqrt{t^2-1}=\frac{2t^2-1}{\sqrt{t^2-1}}$$

29. (C) digits 0, 1, 3, 5, 8, 9, 6

6	6	5

$= 6 \times 6 \times 5 = 180$

30. (A) Given that $x^2 + y^2 = 8$
Let $A = x^2y^2$
$A = x^2(8 - x^2)$
$A = 8x^2 - x^4$

$$\frac{dA}{dx}=16x-4x^3$$

$$\frac{d^2A}{dx^2}=16-12x^2$$

for maxima and minima

$$\frac{dA}{dx}=0$$

$16x - 4x^3 = 0$
$4x(4 - x^2) = 0$
$x = 0, 2, -2$

$$\left(\frac{d^2A}{dx^2}\right)_{at\,x=0}=16-2\times 0=16 \text{ (minima)}$$

$$\left(\frac{d^2A}{dx^2}\right)_{at\,x=2}=16-12\times 2^2=-32 \text{ (maxima)}$$

$$\left(\frac{d^2A}{dx^2}\right)_{at\,x=-2}=16-12(-2)^2=-32 \text{ (maxima)}$$

Function minimum at $x = 0$, $y = 2\sqrt{2}$
Minimum value of $x^2y^2 = 0$

31. (C) 101, 103,..................999
Now, $T_n = a + (n-1)d$
$999 = 101 + (n-1)\times 2$
$898 = (n-1)\times 2$
$449 = n - 1 \Rightarrow n = 450$

Now, $S = \frac{n}{2}(2a + (n-1)d)$

$$S=\frac{450}{2}(2\times 101+449\times 2)$$

$S = 450 \times 550 = 247500$

32. (B) $i^{1-n} + i^{2-n} + i^{3-n} + i^{4-n}$

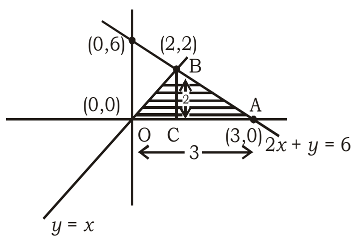
$\Rightarrow i^{-n}(i + i^2 + i^3 + i^4)$

$\Rightarrow i^{-n}(i - 1 - i + 1) = 0$

33. (C) $\dfrac{\sinh x + \sinh y}{\cosh x - \cosh y}$

$$\Rightarrow \frac{2\sinh\frac{x+y}{2}.\cosh\frac{x-y}{2}}{2\sinh\frac{x+y}{2}.\sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$$

34. (C) line $y = x$
and $2x + y = 6$



Area $= \dfrac{1}{2} \times$ OA $\times$ BC

$= \dfrac{1}{2} \times 3 \times 2 = 3$ sq. unit

35. (B) $I = \displaystyle\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}\, dx$ ....(i)

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan\left(\frac{\pi}{2} - x\right)}}{\sqrt{\tan\left(\frac{\pi}{2} - x\right)} + \sqrt{\cot\left(\frac{\pi}{2} - x\right)}}\, dx$$

$I = \displaystyle\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}}\, dx$ ....(ii)

from eq (i) and eq (ii)

$$2I = \int_0^{\pi/2} \frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}}\, dx$$

$$2I = \int_0^{\pi/2} 1.dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

36. (B) Given that $\theta = 130°$
$y = \sin\theta + \cos\theta$
$y = \sin 130 + \cos 130$
$y = \sin 130 + \cos(90 + 40)$
$y = \sin 130 - \sin 40$
We know that $130 > 40$
$\sin 130 > \sin 40$
Hence $y > 0$

37. (C) $z = \dfrac{3+i}{(2-i)^2}$

$$z = \frac{3+i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$z = \frac{5+15i}{25} = \frac{1+3i}{5}$$

38. (A) equation $\lambda x^2 + (2-\lambda)x + 1 = 0$

Now, sum of roots $= \dfrac{-(2-\lambda)}{\lambda}$

$$3 = \frac{-(2-\lambda)}{\lambda}$$

$$3\lambda = -2 + \lambda \Rightarrow \lambda = -1$$

39. (A)

40. (D) $(A \cap B) \cup (B \cap C) \cup (A \cap C)$

41. (C) We know that

$\sin ix = \dfrac{e^x - e^{-x}}{-2i}$ and $\cos ix = \dfrac{e^x + e^{-x}}{2}$

$$\cos ix - i\sin ix = \frac{e^x + e^{-x}}{2} - i \times \frac{e^x - e^{-x}}{-2i}$$

$$= \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = e^x$$

42. (B) Word "STATEMENT"

The total no. of arrangement $= \dfrac{9!}{3!2!} = \dfrac{9!}{12}$

No. of arrangement when T's come together $= \dfrac{7!}{2!} = \dfrac{7!}{2}$

No. of arrangement when T's don't come together $= \dfrac{9!}{12} - \dfrac{7!}{2}$

$$= 6 \times 7! - \frac{7!}{2} = \frac{11 \times 7!}{2}$$

43. (C) $y = \text{cosec}(\cot^{-1}x)$ ....(i)

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = -\text{cosec}(\cot^{-1}x).\cot(\cot^{-1}x).\frac{-1}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \text{cosec}(\cot^{-1}x).\frac{x}{1+x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{yx}{1+x^2} \quad \text{[from eq (i)]}$$

$\Rightarrow (1 + x^2)dy = yx\,dx$

44. (B) $f(x) = \begin{cases} 3x^2 - 4, & 2 \le x < 4 \\ \lambda x + x^2, & 4 \le x < 6 \end{cases}$ is continuous at $x = 4$,

then $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x)$

$\Rightarrow \lim_{x \to 4} 3x^2 - 4 = \lim_{x \to 4} \lambda x + x^2$

$\Rightarrow 3 \times 16 - 4 = \lambda \times 4 + 16$

$\Rightarrow 44 = 4\lambda + 16 \Rightarrow \lambda = 7$

45. (C) $\begin{bmatrix} x+7 & 13 \\ 5 & 2x \end{bmatrix} = \begin{bmatrix} y+8 & y+9 \\ y+1 & 10 \end{bmatrix}$

On comparing

$x + 7 = y + 8 \Rightarrow x - y = 1$, $13 = y + 9 \Rightarrow y = 4$

$5 = y + 1 \Rightarrow y = 4$, $2x = 10 \Rightarrow x = 5$

46. (D) Given that

$$\int x^3.e^{2x}dx = ax^3.e^{2x} + bx^2.e^{2x} + cx\,e^{2x} + d\,e^{2x} + k \quad \text{...eq.(i)}$$

Let $I = \int x^3.e^{2x}\,dx$

$$I = x^3.\int e^{2x}dx - \int\left\{\frac{d}{dx}(x^3).\int e^{2x}dx\right\}dx + k$$

$$I = x^3.\frac{e^{2x}}{2} - \int 3x^2.\frac{e^{2x}}{2}dx + k$$

$$I = \frac{1}{2}x^3.e^{2x} - \frac{3}{2}\left[x^2.\int e^{2x}dx - \int\left\{\frac{d}{dx}(x^2).\int e^{2x}dx\right\}dx\right] + k$$

$$I = \frac{1}{2}x^3.e^{2x} - \frac{3}{2}\left[x^2.\frac{e^{2x}}{2} - \int 2x.\frac{e^{2x}}{2}dx\right] + k$$

$$I = \frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{2}\int x.e^{2x}\,dx + k$$

$$I = \frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{2}\left[\frac{x.e^{2x}}{2} - \frac{1}{2}\frac{e^{2x}}{2}\right] + k$$

$$I = \frac{1}{2}x^3.e^{2x} - \frac{3}{4}x^2.e^{2x} + \frac{3}{4}x.e^{2x} - \frac{3}{8}e^{2x} + k$$

On comparing eq(i)

$a = \frac{1}{2}$, $b = \frac{-3}{4}$, $c = \frac{3}{4}$, $d = \frac{-3}{8}$

47. (C) Given that $2s = a + b + c$

Now, $\frac{s(s-b)}{ac} - \frac{(s-a)(s-c)}{ac}$

$$\Rightarrow \frac{s^2 - sb - s^2 + sa + sc - ac}{ac}$$

$\frac{s(s-b)}{ac}$ $\Rightarrow \frac{2s(a+c-b) - 2ac}{2ac}$

$$\Rightarrow \frac{(a+b+c)(a+c-b) - 2ac}{2ac}$$

$$\Rightarrow \frac{(a+c)^2 - b^2 - 2ac}{2ac}$$

$$\Rightarrow \frac{a^2 + c^2 + 2ac - b^2 - 2ac}{2ac}$$

$$\Rightarrow \frac{a^2 + c^2 - b^2}{2ac} = \cos B$$

48. (B) $(\log_2 x)(\log_{x^3} 8) = \log_y 4$

$$\Rightarrow \frac{\log x}{\log 2} \times \frac{\log 8}{\log x^3} = \frac{\log 4}{\log y}$$

$$\Rightarrow \frac{\log x}{\log 2} \times \frac{3\log 2}{3\log x} = \frac{\log 4}{\log y}$$

$\Rightarrow 1 = \frac{\log 4}{\log y} \Rightarrow y = 4$

49. (B) $(\sin x + \sin y) = 3(\cos x - \cos y)$ ...(i)

x replace by $-x$ and y replace by $-y$

$\Rightarrow \sin(-x) + \sin(-y) = 3[\cos(-x) - \cos(-y)]$

$\Rightarrow -\sin x - \sin y = 3(\cos x - \cos y)$

$\Rightarrow -\sin x - \sin y = \sin x + \sin y$

$\Rightarrow 2(\sin x + \sin y) = 0 \Rightarrow x = -y$

Now, $\frac{\sin 3x}{\sin 3y} = \frac{\sin 3(-y)}{\sin 3y}$

$$\frac{\sin 3x}{\sin 3y} = -\frac{\sin 3y}{\sin 3y} = -1$$

50. (B) $y = e^{\sin x}.\sec x$

On differentiating both side w.r.t. 'x'

$$\frac{dy}{dx} = e^{\sin x}.\sec x.\tan x + \sec x.\,e^{\sin x}.\cos x$$

$$\frac{dy}{dx} = e^{\sin x}(\sec x.\tan x + 1)$$

51. (D) $y = (\sin x)^{x+(\sin x)^{x+.........\infty}}$

$\Rightarrow y = (\sin x)^{x+y}$

taking log both side

$\Rightarrow y = (x + y) \log \sin x$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (x+y)\frac{\cos x}{\sin x} + \log \sin x\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = (x+y)\cot x + \log \sin x + \log \sin x.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - \log \sin x\right)\frac{dy}{dx} = (x+y)\cot x + \log \sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{[(x+y)\cot x + \log \sin x]y}{1 - y\log \sin x}$$

52. (C) A = {x : x is multiple of 2}

A = {2, 4, 6, 8, 10,......}

B = {x : x is multiple of 3}

B = {3, 6, 9, 12, 15,......}

C = {x : x is a multiple of 6}

= {6, 12, 18,......}

Now, $(A \cap C)$ = {6, 12, 18,.........} = C

and $(B \cap C)$ = {6, 12, 18,.........} = C

Hence $(A \cap C) \cup (B \cap C) = C$

53. (D) $\sin(45 - x) + \cos(45 - x)$

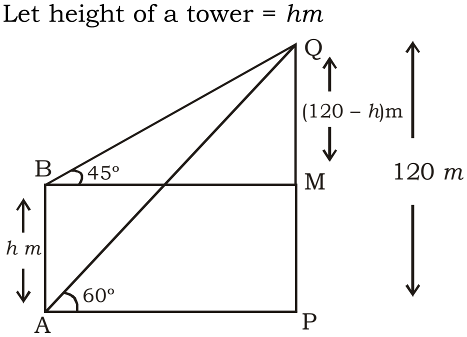
$\Rightarrow \sin 45^\circ.\cos x - \cos 45^\circ.\sin x + \cos 45^\circ.\cos x + \sin 45^\circ.\sin x$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$$

$$\Rightarrow \frac{2}{\sqrt{2}}\cos x = \sqrt{2}\cos x$$

54. (B)

55. (A) Let height of a tower = hm



In ΔMBQ

$$\tan 45^\circ = \frac{QM}{BM}$$

$$\Rightarrow 1 = \frac{QM}{BM}$$

$\Rightarrow$ BM = QM = 120 – h = AP

In ΔAPQ

$$\tan 60^\circ = \frac{PQ}{AP}$$

$$\Rightarrow \sqrt{3} = \frac{120}{120-h}$$

$\Rightarrow 120\sqrt{3} - \sqrt{3}h = 120$

$$\Rightarrow h = \frac{120(\sqrt{3}-1)}{\sqrt{3}} = 40\sqrt{3}(\sqrt{3}-1)\ m$$

Hence height of a tower $= 40\sqrt{3}(\sqrt{3}-1)\ m$

56. (C) In the expansion of $(1 + x)^{23}$

$T_{2r+3} = T_{(2r+2)+1} = {}^{23}C_{2r+2}$

and $T_{r+4} = T_{(r+3)+1} = {}^{23}C_{r+3}$

According to Question

${}^{23}C_{2r+2} = {}^{23}C_{r+3}$

$\Rightarrow 2r + 2 + r + 3 = 23$

$\Rightarrow 3r = 18 \Rightarrow r = 6$

57. (C) Digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

four-digit numbers when last digit is '0'

9	8	7	1

= 9 × 8 × 7 × 1 = 504

four-digit numbers when last digit is '5'

8	8	7	1

= 8 × 8 × 7 × 1 = 448

'0' can not put here.

The required numbers = 504 + 448 = 952

58. (C) According to Question

$$\frac{A.M.}{G.M.} = \frac{13}{12}$$

$$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{13}{12}$$

$$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{13}{12}$$

by Componendo & Dividendo Rule

$$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{13+12}{13-12}$$

$$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{25}{1}$$

$$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{5}{1}$$

by Componendo & Dividendo Rule

$$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{6}{4}$$

$$\Rightarrow \sqrt{\frac{a}{b}} = \frac{3}{2} \Rightarrow \frac{a}{b} = \frac{9}{4}$$

Hence $a : b = 9 : 4$

59. (C) We know that

$$(1-x)^n = 1 + nx + \frac{n(n+1)}{2!}x^2 + ...$$

On putting $n = \frac{1}{3}$

$$\Rightarrow (1-x)^{-1/3} = 1 + \frac{1}{3}x + \frac{\frac{1}{3}\times\frac{4}{3}}{2!}x^2 + ...$$

$$\Rightarrow (1-x)^{-1/3} = 1 + \frac{1}{3}x + \frac{1.4}{3^2.2!}x^2 + ...$$

60. (C) According to Question

$a + a + d = 31$

$2a + d = 31$ ....(i)

and $a + a + d + a + 2d + a + 3d = 98$

$4a + 6d = 98$

$2a + 3d = 49$ ......(ii)

On solving eq (i) and eq (ii)

$a = 11$ and $d = 9$

Now, $T_9 = a + (9-1)d$

$T_9 = 11 + 8 \times 9 = 83$

61. (B) $\begin{vmatrix} \omega^4 & \omega^2 & 1 \\ \omega^{11} & 1 & \omega^7 \\ 1 & \omega^7 & \omega^5 \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} \omega+\omega^2+1 & \omega^2+1+\omega & 1+\omega+\omega^2 \\ \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \end{vmatrix} = 0$$

62. (B) Given that $f(x) = x^2 + 3x - 2$

$f'(x) = 2x + 3 \Rightarrow f'(x) = 2c + 3$

$f(a) \Rightarrow f(3) = 3^2 + 3 \times 3 - 2 = 16$

$$f(b) \Rightarrow f\left(\frac{9}{2}\right) = \left(\frac{9}{2}\right)^2 + 3\times\frac{9}{2} - 2 = \frac{127}{4}$$

Now Mean value theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$2c + 3 = \frac{\frac{127}{4}-16}{\frac{9}{2}-3}$$

$$2c + 3 = \frac{21}{2}$$

$$2c = \frac{15}{2} \Rightarrow c = \frac{15}{4}$$

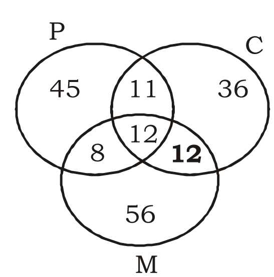
63. (B) $[A]_{3\times4}$, $[BA] = 6 \times 4$

Now, $[B]_{6\times3}\,[A]_{3\times4} = [BA]_{6\times4}$

Hence B will be a 6 × 3 matrix.

(64 – 65)

Total students = 180

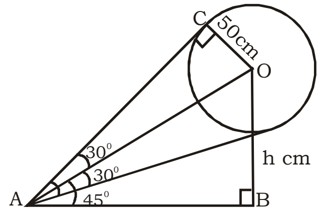


64. (C) The required no. of students =45+11+ 36 = 92

65. (D) The required no. of students = 12

66. (C) The required no. of triangle = ${}^{13}C_3 - {}^{8}C_3$
$= 286 - 56$
$= 230$

67. (B) Let height of the centre of the ballon = h cm



In ΔABO

$\sin 45° = \dfrac{BO}{AO}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{h}{AO}$ ....(i)

In ΔAOC

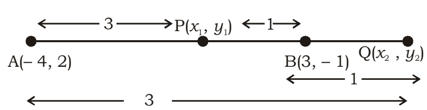
$\sin 30° = \dfrac{OC}{AO}$

$\Rightarrow \dfrac{1}{2} = \dfrac{50}{AO} \Rightarrow AO = 100$

from eq (i)

$\dfrac{1}{\sqrt{2}} = \dfrac{h}{100} \Rightarrow h = \dfrac{100}{\sqrt{2}} = 50\sqrt{2}$ cm

68. (C)



$x_1 = \dfrac{3\times3+1\times(-4)}{3+1} = \dfrac{5}{4}$

and $y_1 = \dfrac{3\times(-1)+1\times2}{3+1} = \dfrac{-1}{4}$

Co-ordinate of P = $\left(\dfrac{5}{4}, \dfrac{-1}{4}\right)$

$x_2 = \dfrac{3\times3 - 1\times(-4)}{3-1} = \dfrac{13}{2}$

and $y_2 = \dfrac{3\times(-1) - 1\times2}{3-1} = \dfrac{-5}{2}$

Co-ordinate of Q = $\left(\dfrac{13}{2}, \dfrac{-5}{2}\right)$

Now,
Distance Between P and Q

$= \sqrt{\left(\dfrac{5}{4}-\dfrac{13}{2}\right)^2 + \left(\dfrac{-1}{4}+\dfrac{5}{2}\right)^2}$

$= \sqrt{\left(\dfrac{-21}{4}\right)^2 + \left(\dfrac{9}{4}\right)^2} = \sqrt{\dfrac{522}{16}} = \dfrac{3\sqrt{58}}{4}$

69. (A) Let P (1, 0, -2), Q (3, 1, -2) and R(5, -2, 1)

$\overrightarrow{PQ} = (3-1, 1-0, -2+2) = (2, 1, 0)$

$\overrightarrow{PR} = (5-1, -2-0, 1+2) = (4, -2, 3)$

$\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 4 & -2 & 3 \end{vmatrix}$

$\overrightarrow{PQ}\times\overrightarrow{PR} = \hat{i}\,(3-0) - \hat{j}\,(6-0) + \hat{k}\,(-4-4)$

$\overrightarrow{PQ}\times\overrightarrow{PR} = 3\hat{i} - 6\hat{j} - 8\hat{k}$

$a = 3,\ b = -6,\ c = -8$

$P(x_o, y_o, z_o) = (1, 0, -2)$

equation of plane

$a(x-x_0) + b(y-y_0) + (z-z_0) = 0$

$\Rightarrow 3(x-1) - 6(y-0) - 8(z+2) = 0$

$\Rightarrow 3x - 3 - 6y - 8z - 16 = 0$

$\Rightarrow 3x - 6y - 8z = 19$

70. (C) sin2475 = sin(360 × 7 - 45)

$= -\sin 45 = \dfrac{-1}{\sqrt{2}}$

71. (B) centre $(-u, -v, -w) = (-2, 1, 5)$
and radius $r = 9$ unit

$r = \sqrt{u^2+v^2+w^2-d}$

$9 = \sqrt{4+1+25-d}$

$81 = 30 - d \Rightarrow d = -51$

Equation of sphere

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

$x^2 + y^2 + z^2 + 2\times2x + 2(-1)y + 2(-5)z - 51 = 0$

$x^2 + y^2 + z^2 + 4x - 2y - 10z = 51$

72. (C) Differential equation

$x^2\,dy = y^2\,dx$

$\Rightarrow \dfrac{dy}{y^2} = \dfrac{dy}{x^2}$

On integrating

$\Rightarrow -\dfrac{1}{y} = -\dfrac{1}{x} + c$

$\Rightarrow \dfrac{-1}{y} + \dfrac{1}{x} = c$

$\Rightarrow \dfrac{y-x}{xy} = c \Rightarrow y - x = cxy$

73. (D) $I = \int_0^1 x(1-x)^7\,dx$

Property IV $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_o^1 (1-x)x^7 dx$

$I = \int_0^1 (x^7 - x^8)\, dx$

$I = \left(\frac{x^8}{8} - \frac{x^9}{9}\right)_0^1$

$I = \frac{1}{8} - \frac{1}{9} = \frac{1}{72}$

74. (C) $I = \int_{-\pi/4}^{\pi/4} (\sin x - \cos x)\, dx$

$I = \int_{-\pi/4}^{\pi/4} \sin x\, dx - \int_{-\pi/4}^{\pi/4} \cos x\, dx$

$\sin x$ is an odd function and $\cos x$ is an even function.

$I = 0 - 2\int_0^{\pi/4} \cos x\, dx$

$I = -2[\sin x]_0^{\pi/4}$

$I = -2\left[\sin\frac{\pi}{4} - \sin 0\right]$

$I = -2\left[\frac{1}{\sqrt{2}} - 0\right] = -\sqrt{2}$

75. (B) $n(S) = 6 \times 6 = 36$

$E = \{(6, 2),(2, 6),(3, 5),(5, 3),(4, 4)\}$

$n(E) = 5$

The required Probability $= \frac{n(E)}{n(S)} = \frac{5}{36}$

76. (C)

2	33	1
2	16	0
2	8	0
2	4	0
2	2	0
2	1	1
	0	

$(33)_{10} = (100001)_2$

77. (B) x, 5 and z are in A.P.

$2 \times 5 = x + z$

$x + z = 10$ ...(i)

and x, 3 and z are in G.P.

$3^2 = xz \Rightarrow xz = 9$ ...(ii)

Now $(x - z)^2 = (x + z)^2 - 4xz$

$\Rightarrow (x - z)^2 = (10)^2 - 4 \times 9$

$\Rightarrow (x - z)^2 = 64 \Rightarrow x - z = 8$ ...(iii)

from eq(i) and eq(iii)

$x = 9,\ z = 1$

Now, H.M. $= \frac{2xz}{x + z}$

$\Rightarrow$ H.M. $= \frac{2 \times 9 \times 1}{10} = \frac{9}{5}$

Hence x, $\frac{9}{5}$ and z are in H.P.

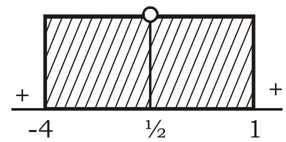
78. (B) function $\frac{\log_e(5 - 3x - x^2)}{2x - 1}$

$\log_e(5 - 3x - x^2) \geq 0,$ $\quad 2x - 1 \neq 0$

$5 - 3x - x^2 \geq 1,$ $\quad x \neq \frac{1}{2}$

$x^2 + 3x - 4 \leq 0$

$(x + 4)(x - 1) \leq 0$



$x \in [-4, 1] - \left\{\frac{1}{2}\right\}$

79. (D)

Class	f	C
0 – 10	17	17
10 – 20	19	36
20 - 30	**21**	**57**
30 – 40	23	80
40 – 50	20	100

$N = 100,\ \frac{N}{2} = 50$

$l_1 = 20,\ l_2 = 30,\ f = 21,\ C = 36$

Median $= l_1 + \frac{\frac{N}{2} - C}{f}(l_2 - l_1)$

$= 20 + \frac{50 - 36}{21} \times (30 - 20)$

$= 20 + \frac{14}{21} \times 10 = 26\frac{2}{3}$

80. (C)

81. (D) $\lim_{h \to 0} \frac{\sqrt{x + 2h} - \sqrt{x}}{2h}$ $\quad \left(\frac{0}{0}\right)$ form

by L - Hospital's Rule

$= \lim_{h \to 0} \frac{\frac{1 \times 2}{2\sqrt{x + 2h}} - 0}{2 \times 1}$

$= \frac{2}{2\sqrt{x} \times 2} = \frac{1}{2\sqrt{x}}$

82. (C) $\frac{\sin(x-y)}{\sin(x+y)} = \frac{a-b}{a-b}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\sin(x-y)+\sin(x+y)}{\sin(x-y)-\sin(x+y)} = \frac{a-b+a+b}{a-b-a-b}$

$\Rightarrow \frac{2\sin x.\cos y}{2\cos x.\sin(-y)} = \frac{2a}{-2b}$

$\Rightarrow \frac{\sin x.\cos y}{-\cos x.\sin y} = \frac{a}{-b}$

$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b} \Rightarrow \frac{\tan y}{\tan x} = \frac{a}{b}$

83. (B) Vectors $2\hat{i}+2\hat{j}+\lambda\hat{k}$ and $8\hat{i}+12\hat{j}-5\hat{k}$

angle between vectors

$\cos\theta = \frac{2\times8+2\times12-\lambda\times5}{\sqrt{2^2+2^2+\lambda^2}\sqrt{8^2+12^2+(-5)^2}}$

$\cos\frac{\pi}{2} = \frac{16+24-5\lambda}{\sqrt{2^2+2^2+\lambda^2}\sqrt{8^2+12^2+(-5)^2}}$

$0 = \frac{40-5\lambda}{\sqrt{2^2+2^2+\lambda^2}\sqrt{8^2+12^2+9^2}}$

$40 - 5\lambda = 0 \Rightarrow \lambda = 8$

84. (D) The required Probability = 0

85. (B) $A = \begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}$

$A^2 = \begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}$

$A^2 = \begin{bmatrix}4 & -9\\ -3 & 7\end{bmatrix}$

Now $A^2 + kA - I_2 = 0$

$\begin{bmatrix}4 & -9\\ -3 & 7\end{bmatrix} + k\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix} - \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3-k & -9+3k\\ -3+k & 6-2k\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

On companing

$\Rightarrow k = 3$

86. (A) Given that $2(l + b) = 90$

$l + b = 45$ ...(i)

Area $A = lb$

$A = l(45 - l)$

$A = 45l - l^2$

$\frac{dA}{dl} = 45 - 2l$

$\frac{d^2A}{dl^2} = -2$ (maxima)

for maxima and minima

$\frac{dA}{dl} = 0$

$\Rightarrow 45 - 2l = 0 \Rightarrow l = \frac{45}{2}$

Hence maximum area $= l \times b = \frac{45}{2} \times \frac{45}{2}$

$= 506.25\ m^2$

87. (C) Given that $A = B \cap C$

Now, $(U -(U -(U -(U -A))))$

$\Rightarrow (U -(U -(U -A')))$

$\Rightarrow (U -(U -A))$

$\Rightarrow (U - A') \Rightarrow A = B \cap C$

88. (B) $\cos\left(\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(\cos\left(\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)\right)$

$\Rightarrow \cos\left(\sin^{-1}\left(\sin\frac{\pi}{3}\right)\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

89. (C) $\begin{vmatrix}\sin^2\theta & \cos^2\theta\\ \cos^2\theta & \sin^2\theta\end{vmatrix}$

$\Rightarrow \sin^4\theta - \cos^4\theta$

$\Rightarrow (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$\Rightarrow -\cos2\theta \times 1 = -\cos2\theta$

90. (B) $\tan2\theta + 2\tan4\theta + 4\tan8\theta + 8\cot16\theta$

$\Rightarrow \cot2\theta - (\cot2\theta - \tan2\theta) + 2\tan4\theta + 4\tan8\theta + 8\cot16\theta$

We know that $\cot A - \tan A = 2\cot2A$

$\Rightarrow \cot2\theta - 2\cot4\theta + 2\tan4\theta + 4\tan8\theta + 8\cot16\theta$

$\Rightarrow \cot2\theta - 2(\cot4\theta - \tan4\theta) + 4\tan8\theta + 8\cot16\theta$

$\Rightarrow \cot2\theta - 2 \times 2\cot8\theta + 4\tan8\theta + 8\cot16\theta$

$\Rightarrow \cot2\theta - 4(\cot8\theta - \tan8\theta) + 8\cot16\theta$

$\Rightarrow \cot2\theta - 4 \times 2\cot16\theta + 8\cot16\theta$

$\Rightarrow \cot2\theta - 8\cot16\theta + 8\cot16\theta = \cot2\theta$

91. (C) Given line $\frac{x}{3} - \frac{y}{5} = 2$

$5x - 3y = 30$

slope of line = $\frac{5}{3}$

Hence the required slope = $\frac{5}{3}$

92. (A) The required Probability = $\frac{1}{52}$

93. (C) Given equation

$x^2 + y^2 + 2x - y + 6 = 0$ ...(i)

let equation of circle which is concentric with eq(i)

$x^2 + y^2 + 2x - y + c = 0$ ...(ii)

It passes through the point (2 , - 3)

$4 + 9 + 4 + 3 + c = 0 \Rightarrow c = -20$

The required equation of circle

$x^2 + y^2 + 2x - y - 20 = 0$

94. (D) let $y = 2^{79}$

taking log both side

$\log_{10} y = 79 \log_{10} 2$

$\log_{10} y = 79 \times 0.3010 = 23.779$

The required no. of digits = 23 + 1 = 24

95. (B) A = { 1, 2, 3, 6, 7, 9} ; $n = 6$

The no. of proper subsets of A = $2^n - 1$

$= 2^6 - 1 = 63$

96. (C) We know that

$(1 + x)^{2n} = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_{2n}x^{2n}$

multiply by x

$\Rightarrow x(1 + x)^{2n} = C_0x + C_1x^2 + C_2x^3 \ldots + C_{2n}x^{2n+1}$

on differentiating both side w.r.t. 'x'

$\Rightarrow x.2n(1 + x)^{2n-1} + (1 + x)^{2n}.1 = C_0 + 2C_1x + 3C_2x^2 + \ldots + (2n + 1)C_{2n}x^{2n}$

On putting $x = 1$

$\Rightarrow 1.2n.2^{2n-1} + 2^{2n} = C_0 + 2C_1 + \ldots + (2n+1)C_{2n}$

Hence $C_0 + 2C_1 + \ldots + (2n+1)C_{2n} = 2^{2n}(n+1)$

97. (B) Equation

$x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

represents

(i) a circle, if $\Delta \neq 0$, $a = b$, $h = 0$

(ii) an ellipse, if $\Delta \neq 0$, $h^2 < ab$

(iii) a parabola, if $\Delta \neq 0$, $h^2 = ab$

(iv) a hyperbola, if $\Delta \neq 0$, $h^2 > ab$

98. (D) Two circles $x^2 + y^2 + x + 2y + 17 = 0$ and $x^2 + y^2 + 4x - 6y + \lambda = 0$

Condition of orthogonality

$2gg' + 2ff' = C + C'$

$\Rightarrow 2 \times \frac{1}{2} \times 2 + 2 \times 1 \times (-3) = 17 + \lambda$

$\Rightarrow 2 - 6 = 17 + \lambda \Rightarrow \lambda = -21$

99. (C) Given that = (2001)!

Now, $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots\ldots + \frac{1}{\log_{2001} n}$

$\Rightarrow \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 2007$

$\Rightarrow \log_n(2 \times 3 \times 4 \times \ldots 2001)$

$\Rightarrow \log_n(2001)! \Rightarrow \log_n n = 1$

100. (B) $z = -1 + \sqrt{3}i$

$\arg(z) = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$

$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

$= \tan^{-1}\left[\tan\left(\frac{-\pi}{3}\right)\right] = \frac{-\pi}{3}$

101. (D) remainder = $\frac{5^7 + 11^7}{8}$

$= (-3)^7 + 3^7$

$= -3^7 + 3^7 = 0$

Hence the given number is divisible by 8.

102. (C) $A = \begin{bmatrix} \sin\theta & \cos\theta & 1 \\ \cos\theta & 1 & \sin\theta \\ 1 & \sin\theta & \cos\theta \end{bmatrix}$

$|A| = \begin{vmatrix} \sin\theta & \cos\theta & 1 \\ \cos\theta & 1 & \sin\theta \\ 1 & \sin\theta & \cos\theta \end{vmatrix}$

$= \sin\theta(\cos\theta - \sin^2\theta) - \cos\theta(\cos^2\theta - \sin\theta) + 1(\sin\theta.\cos\theta - 1)$

$= \sin\theta.\cos\theta - \sin^3\theta - \cos^3\theta + \sin\theta.\cos\theta + \sin\theta.\cos\theta - 1$

$= 3\sin\theta.\cos\theta - \sin^3\theta - \cos^3\theta - 1$

103. (D) $C(2n, r+1) + 2C(2n, r) + C(2n, r-1)$

$\Rightarrow {}^{2n}C_{r+1} + {}^{2n}C_r + {}^{2n}C_r + {}^{2n}C_{r-1}$

We know that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{2n+1}C_{r+1} + {}^{2n+1}C_r$

$\Rightarrow {}^{2n+2}C_{r+1} = C(2n + 2, r + 1)$

104. (B) $\cos x = \frac{1}{\sqrt{10}}$ and $\cos y = \frac{1}{\sqrt{5}}$

$\sin x = \frac{3}{\sqrt{10}}$, $\sin y = \frac{2}{\sqrt{5}}$

Now, $\cos(x + y) = \cos x.\cos y - \sin x.\sin y$

$\Rightarrow \cos(x + y) = \frac{1}{\sqrt{10}} \times \frac{1}{\sqrt{5}} - \frac{3}{\sqrt{10}} \times \frac{2}{\sqrt{5}}$

$\Rightarrow \cos(x + y) = \frac{1}{\sqrt{50}} - \frac{6}{\sqrt{50}}$

$\Rightarrow \cos(x + y) = \frac{-5}{\sqrt{50}}$

$\Rightarrow \cos(x + y) = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos(x + y) = \cos\frac{3\pi}{4} \Rightarrow x + y = \frac{3\pi}{4}$

105. (C) $\cos\alpha + \cos\beta + \cos\gamma = 0$

$\cos\alpha = \cos\beta = \cos\gamma = 0$

$\alpha = \beta = \gamma = \frac{\pi}{2}$

Now, $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$\Rightarrow \sin^2\frac{\pi}{2} + \sin^2\frac{\pi}{2} + \sin^2\frac{\pi}{2} = 1 + 1 + 1 = 3$

106. (D) In ΔABC, $a = 3$, $c = 4$ and $\sin A = \frac{3}{4}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\Rightarrow \frac{3/4}{3} = \frac{\sin C}{4} \Rightarrow \sin C = 1 \Rightarrow C = \frac{\pi}{2}$

107. (A) given that $a = 4$

and $ae = 2$

$\Rightarrow 4 \times e = 2 \Rightarrow e = \frac{1}{2}$

Now, $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{1}{4} = 1 - \frac{b^2}{16}$

$\Rightarrow \frac{b^2}{16} = \frac{3}{4} \Rightarrow b^2 = 12$

equation of an ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1 \Rightarrow 3x^2 + 4y^2 = 48$

108. (C) $\vec{a} + 2\vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \frac{\vec{c} - \vec{a}}{2}$

Now, $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \lambda(\vec{a} \times \vec{c})$

$\Rightarrow \vec{a} \times \frac{\vec{c} - \vec{a}}{2} + \frac{\vec{c} - \vec{a}}{2} \times \vec{c} - \vec{a} \times \vec{c} = \lambda(\vec{a} \times \vec{c})$

$\Rightarrow \frac{\vec{a} \times \vec{c}}{2} - 0 + 0 - \frac{\vec{a} \times \vec{c}}{2} - \vec{a} \times \vec{c} = \lambda(\vec{a} \times \vec{c})$

$\Rightarrow -1(\vec{a} \times \vec{c}) = \lambda(\vec{a} \times \vec{c}) \Rightarrow \lambda = -1$

109. (A) $\vec{a} = 4$, $\vec{b} = \frac{17}{2}$ and $\vec{a} \times \vec{b} = 8\hat{i} + 9\hat{j} - 12\hat{k}$

We know that

$|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|\ \sin\theta$

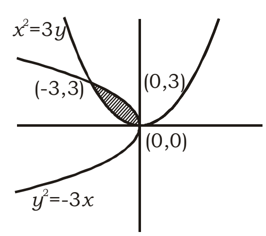
$\Rightarrow \sqrt{8^2 + 9^2 + (-12)^2} = 4 \times \frac{17}{2} \sin\theta$

$\Rightarrow 17 = 2 \times 17 \sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

110. (A) curves

$x_1 \Rightarrow x \Rightarrow \sqrt{3}\sqrt{y}$ and $x_2 \Rightarrow x = \frac{-y^3}{3}$



Area $= \int_0^3 (x_1 - x_2)\,dx$

$= \int_0^3 \left[\sqrt{3}\sqrt{y} + \frac{y^3}{3}\right] dx$

$= \left[\sqrt{3} \times \frac{2y^{3/2}}{3} + \frac{1}{3} \times \frac{y^4}{4}\right]_0^3$

$= \frac{2}{\sqrt{3}} \times (3)^{3/2} + \frac{1}{3} \times \frac{1}{4} \times 3^4 - 0$

$= 6 + \frac{27}{4} = \frac{51}{4}$ sq. unit

111. (C) $I = \int_0^{\pi/2} e^x \cos x\,dx$ ...(i)

$I = \left[\cos x\int e^x\,dx - \int\left\{\frac{d}{dx}(\cos x).\int e^x\,dx\right\}dx\right]_0^{\pi/2}$

$I = \left[e^x.\cos x - \int -\sin x.e^x\,dx\right]_0^{\pi/2}$

$I = \left[e^x.\cos x + \int \sin x.e^x\,dx\right]_0^{\pi/2}$

$I = \left[e^x.\cos x + \sin x.e^x\,dx - \int \cos x.e^x\,dx\right]_0^{\pi/2}$

$I = \left[e^x.\cos x + \sin x.e^x\right]_0^{\pi/2} - I$ [from eq(i)]

$2I = e^{\pi/2}.\cos\frac{\pi}{2} + e^{\pi/2}.\sin\frac{\pi}{2} - e^0.\cos 0 - e^0.\sin 0$

$2I = 0 + e^{\pi/2} - 1 - 0 \Rightarrow I = \frac{e^{\pi/2}-1}{2}$

112. (C) Let $z = 8\sin\theta - 4\sin^2\theta$

$z = -(2 - 2\sin\theta)^2 + 4$

$z = 4 - (2 - 2\sin\theta)^2$

Maximum value of $z = 4$

113. (B) Differential equation

$\frac{dy}{dx} - \frac{y}{x} = x^3$

Here $P = \frac{-1}{x}$ and $Q = x^3$

$I.F. = e^{\int P.dx}$

$I.F. = e^{\int \frac{-1}{x}dx}$

$I.F. = e^{-\log x} = \frac{1}{x}$

Solution of the differential equation

$y \times I.F. = \int Q \times I.F.\,dx$

$\Rightarrow y \times \frac{1}{x} = \int x^3 \times \frac{1}{x}\,dx$

$\Rightarrow \frac{y}{x} = \int x^2\,dx$

$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c \Rightarrow 3y = x^4 + cx$

114. (C) $\lim_{x\to 0}\frac{\sin x}{\tan 2x}$

$\Rightarrow \lim_{x\to 0}\frac{\sin x}{x}\times\frac{2x}{\tan 2x}\times\frac{1}{2} \Rightarrow \frac{1}{2}$

115. (A) Mode = 44

116. (C) Let $y = \cot x$

$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2 x$

and $z = \operatorname{cosec} x$

$\Rightarrow \frac{dz}{dx} = -\operatorname{cosec} x.\cot x$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = -\operatorname{cosec}^2 x \times \frac{1}{-\operatorname{cosec} x.\cot x} = \sec x$

117. (D) $S = 0.4 + 0.44 + 0.444 + \ldots\ldots n$ terms

$S = 4(0.1 + 0.11 + 0.111 + \ldots\ldots n$ terms)

$S = 4\left(\frac{1}{10}+\frac{11}{100}+\frac{111}{1000}+\ldots\ldots..n\text{ terms}\right)$

$S = \frac{4}{9}\left(\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+\ldots\ldots..n\text{ terms}\right)$

$S = \frac{4}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\ldots..n\text{ terms}\right]$

$S = \frac{4}{9}\left[(1+1+\ldots n\text{ terms})-\left(\frac{1}{10}+\frac{1}{100}+\ldots n\text{ terms}\right)\right]$

$S = \frac{4}{9}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$S = \frac{4}{9}\left[n - \frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$

118. (A) $f(x) = \frac{x^2-4}{x^2+x-6}$

$\Rightarrow f(x) = \frac{(x-2)(x+2)}{(x-2)(x+3)}$

$\Rightarrow f(x) = \frac{x+2}{x+3}$ ...(i)

On differentiating both side w.r.t. 'x'

$\Rightarrow f'(x) = \frac{(x+3).1-(x+2).1}{(x+3)^2}$

$\Rightarrow f'(x) = \frac{1}{(x+3)^2}$ ..(ii)

from eq(i)

$\Rightarrow f(2) = \frac{2+2}{2+3} = \frac{4}{5}$

from eq(ii)

$\Rightarrow f'(2) = \frac{1}{(2+3)^2} = \frac{1}{25}$

119. (D) $f(x) = [x-2] + [x-1]$

For $x = 1$

L.H.L. = $\lim_{x \to 1^-} f(x)$

$\Rightarrow$

$= \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} [1-h-2] + [1-h-1]$

$= \lim_{h \to 0} [-1-h] + [0-h]$

$= -2 -1 = -3$

R.H.L. = $\lim_{x \to 1^+} f(x)$

$= \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} [1+h-2] + [1+h-1]$

$= \lim_{h \to 0} [-1+h] + [0+h]$

$= -1 + 0 = -1$

L.H.L. ≠ R.H.L.

Hence $f(x)$ is not continuous at $x = 1$.

For $x = 2$

L.H.L. = $\lim_{x \to 2^-} f(x)$

$= \lim_{h \to 0} f(2-h)$

$= \lim_{h \to 0} [2-h-2] + [2-h-1]$

$= \lim_{h \to 0} [0-h] + [1-h]$

$= -1 + 0 = -1$

R.H.L. = $\lim_{x \to 2^+} f(x)$

$= \lim_{h \to 0} f(2+h)$

$= \lim_{h \to 0} [2+h-2] + [2+h-1]$

$= \lim_{h \to 0} [0+h] + [1+h]$

$= 0 + 1 = 1$

L.H.L. ≠ R.H.L.

Hence $f(x)$ is not continuous at $x = 2$.

120. (D) $I = \int \frac{1}{3^x-1}\,dx$

$\Rightarrow I = \int \frac{3^{-x}}{3^{-x}(3^x-1)}\,dx$

$\Rightarrow I = \int \frac{3^{-x}}{1-3^{-x}}\,dx$

Let $1 - 3^{-x} = t$

$-3^{-x}\log 3\,(-1)\,dx = dt$

$3^{-x}\,dx = \frac{1}{\log 3}\,dt$

$\Rightarrow I = \frac{1}{\log 3}\int \frac{dt}{t}$

$\Rightarrow I = \frac{1}{\log 3} \times \log t + C$

$\Rightarrow I = \frac{\log(1-3^{-x})}{\log 3} + C$

$\Rightarrow I = \log_3(1-3^{-x}) + C$

$\Rightarrow I = \log_3\left(\frac{3^x-1}{3^x}\right) + C$

$\Rightarrow I = \log_3(3^x-1) - \log_3 3^x + C$

$\Rightarrow I = \log_3(3^x-1) - x\log_3 3 + C$

$\Rightarrow I = \log_3(3^x-1) - x + C$

NDA (MATHS) MOCK TEST - 114 (Answer Key)

1.	(B)	21.	(B)	41.	(C)	61.	(B)	81.	(D)	101.	(D)
2.	(A)	22.	(A)	42.	(B)	62.	(B)	82.	(C)	102.	(C)
3.	(D)	23.	(C)	43.	(C)	63.	(B)	83.	(B)	103.	(D)
4.	(A)	24.	(B)	44.	(B)	64.	(C)	84.	(D)	104.	(B)
5.	(C)	25.	(B)	45.	(C)	65.	(D)	85.	(B)	105.	(C)
6.	(B)	26.	(C)	46.	(D)	66.	(C)	86.	(A)	106.	(D)
7.	(B)	27.	(D)	47.	(C)	67.	(B)	87.	(C)	107.	(A)
8.	(B)	28.	(B)	48.	(B)	68.	(C)	88.	(B)	108.	(C)
9.	(B)	29.	(C)	49.	(B)	69.	(A)	89.	(C)	109.	(A)
10.	(C)	30.	(A)	50.	(B)	70.	(C)	90.	(B)	110.	(A)
11.	(B)	31.	(C)	51.	(D)	71.	(B)	91.	(C)	111.	(C)
12.	(B)	32.	(B)	52.	(C)	72.	(C)	92.	(A)	112.	(C)
13.	(C)	33.	(C)	53.	(D)	73.	(D)	93.	(C)	113.	(B)
14.	(B)	34.	(C)	54.	(B)	74.	(C)	94.	(D)	114.	(C)
15.	(B)	35.	(B)	55.	(A)	75.	(B)	95.	(B)	115.	(A)
16.	(C)	36.	(B)	56.	(C)	76.	(C)	96.	(C)	116.	(C)
17.	(B)	37.	(C)	57.	(C)	77.	(B)	97.	(B)	117.	(D)
18.	(C)	38.	(A)	58.	(C)	78.	(B)	98.	(D)	118.	(A)
19.	(D)	39.	(A)	59.	(C)	79.	(D)	99.	(C)	119.	(D)
20.	(A)	40.	(D)	60.	(C)	80.	(C)	100.	(B)	120.	(D)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777