KD Campus

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2007, OUTRAM LINES, 1ST FLOOR, OPPOSITE MUKHERJEE NAGAR POLICE STATION, DELHI-110009

NDA MATHS MOCK TEST - 120 (SOLUTION)

1. (B) $y = x^y$

taking log both side

$\Rightarrow \log y = y \log x$

On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y \times \frac{1}{x} + \log x.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y} - \log x\right)\frac{dy}{dx} = \frac{y + x\log x}{x}$$

$$\Rightarrow \frac{1 - y\log x}{y}.\frac{dy}{dx} = \frac{y + x\log x}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(y + x\log x)}{x(1 - y\log x)}$$

2. (A) We know that

$$\sinh A + \sinh B = 2\sinh\frac{A+B}{2} . \cosh\frac{A-B}{2}$$

$$\text{and } \cosh A - \cosh B = 2\sinh\frac{A+B}{2} . \sinh\frac{A-B}{2}$$

$$\text{Now, } \frac{\sinh x + \sinh y}{\cosh x - \cosh y}$$

$$\Rightarrow \frac{2\sinh\frac{x+y}{2}.\cosh\frac{x-y}{2}}{2\sinh\frac{x+y}{2}.\sinh\frac{x-y}{2}} = \coth\frac{x-y}{2}$$

3. (C) $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$

We know that

$\lim_{x\to a} [f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$, if $[1^\infty]$ form

$$\Rightarrow e^{\lim_{x\to\pi/2} \tan x(\sin x - 1)}$$

$$\Rightarrow e^{\lim_{x\to\pi/2} \frac{\sin x(\sin x - 1)}{\cos x}} \quad \left[\frac{0}{0}\right] \text{form}$$

by L - Hospital's Rule

$$\Rightarrow e^{\lim_{x\to\pi/2} \frac{\sin x.\cos x + (\sin x - 1)\cos x}{-\sin x}}$$

$$\Rightarrow e^{\frac{\sin\frac{\pi}{2}.\cos\frac{\pi}{2} + \left(\sin\frac{\pi}{2} - 1\right)\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}}}$$

$\Rightarrow e^0 = 1$

4. (D) Three-digit odd numbers

101, 103, 105, 107............999

Now, $l = a + (n-1)d$

$999 = 101 + (n-1) \times 2$

$898 = (n-1) \times 2 \Rightarrow n = 450$

$$\text{Sum} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{450}{2}[2 \times 101 + (450-1) \times 2]$$

$= 450\,[101 + 449]$

$= 450 \times 550 = 247500$

5. (B) $\begin{bmatrix} 4 & x \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -4 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} 4 & x \end{bmatrix}\begin{bmatrix} 2\times3 + 3\times(-4) \\ 0\times3 + (-1)\times(-4) \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4 & x \end{bmatrix}\begin{bmatrix} -6 \\ 4 \end{bmatrix} = 0$$

$\Rightarrow 4 \times (-6) + x \times 4 = 0 \Rightarrow x = 6$

6. (C) 1 0 1 0 1

$1\times2^0 = 1$

$0\times2^1 = 0$

$1\times2^2 = 4$

$0\times2^3 = 0$

$1\times2^4 = 16$

21

0 . 1 1

$\frac{1}{2} = 1\times2^{-1}$

$\frac{1}{4} = 1\times2^{-2}$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$

Hence $(10101.11)_2 = (21.75)_{10}$

7. (B) $S_n = n^2 - 3n + 5$

$S_{n-1} = (n-1)^2 - 3(n-1) + 5$

$S_{n-1} = n^2 - 5n + 9$

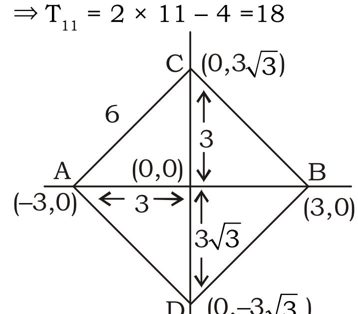
Now, $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = n^2 - 3n + 5 - n^2 + 5n - 9$

$\Rightarrow T_n = 2n - 4$

$\Rightarrow T_{11} = 2 \times 11 - 4 = 18$

8. (C)



Hence third vertex of an equilateral triangle = $\left(0, \pm3\sqrt{3}\right)$

9. (D)

10. (C) $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$

On squaring both side

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 2\sin^2\theta$

$\Rightarrow 1 - \cos^2\theta + 1 - \sin^2\theta + 2\sin\theta.\cos\theta = 2\sin^2\theta$

$\Rightarrow 2 - 2\sin^2\theta = \cos^2\theta + \sin^2\theta - 2\sin\theta.\cos\theta$

$\Rightarrow 2(1 - \sin^2\theta) = (\sin\theta - \cos\theta)^2$

$\Rightarrow 2\cos^2\theta = (\sin\theta - \cos\theta)^2$

$\Rightarrow \sin\theta - \cos\theta = \sqrt{2}\cos\theta$

11. (B) Given that, $\sin\alpha = \frac{20}{29}$, $\sin\beta = \frac{21}{29}$

$\cos\alpha = \frac{21}{29}$, $\cos\beta = \frac{20}{29}$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$

$\Rightarrow \cos(\alpha + \beta) = \frac{21}{29}\times\frac{20}{29} - \frac{21}{29}\times\frac{20}{29} = 0$

$\Rightarrow \alpha + \beta = \frac{\pi}{2}$

Now, $\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

12. (A) Given that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{7}$

$P(\overline{A}) = \frac{2}{3}$, $P(\overline{B}) = \frac{4}{7}$

The Probability $= P\left(A\cap\overline{B}\right) + P\left(\overline{A}\cap B\right)$

$= P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$= \frac{1}{3}\times\frac{4}{7} + \frac{2}{3}\times\frac{3}{7} = \frac{10}{21}$

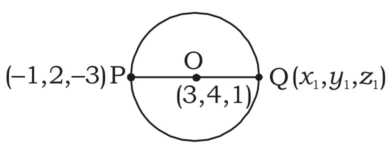
13. (D) equation of sphere

$x^2 + y^2 + z^2 - 6x - 8y - 2z + 6 = 0$

$u = -3,\ u = -4,\ w = -1$

centre = (3, 4, 1)

Let co-ordinate of Q = (x_1, y_1, z_1)



co-ordinate of mid-point of line PQ

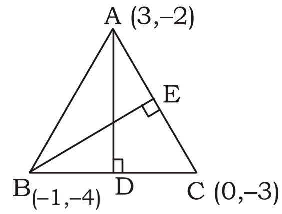
$3 = \frac{x_1 - 1}{2} \Rightarrow x_1 = 7$

$4 = \frac{y_1 + 2}{2} \Rightarrow y_1 = 6$

$1 = \frac{z_1 - 3}{2} \Rightarrow z_1 = 5$

Hence end point of its diameter = (7,6,5)

14. (C)



slope of line BC $= \frac{-3+4}{0+1} = 1$

slope of line AD $= \frac{-1}{1} = -1$

equation of line AD

$y + 2 = -1(x - 3)$

$x + y = 1$ ...(i)

slope of line AC $= \frac{-3+2}{0-3} = \frac{-1}{-3} = \frac{1}{3}$

slope of line BE $= \frac{-1}{1/3} = -3$

equation of line BE

$y + 4 = -3(x + 1)$

$3x + y = -7$ ...(ii)

from eq (i) and eq(ii)

$x = -4,\ y = 5$

Intersection point of line BE and AD is orthocenter.

Hence orthocentre = (–4, 5)

15. (A) $\tan\left[2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{2}{5}\right]$

$\Rightarrow \tan\left[\tan^{-1}\frac{5}{12} + \tan^{-1}\frac{2}{5}\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{5}{12}+\frac{2}{5}}{1-\frac{5}{12}\times\frac{2}{5}}\right)\right]$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{49}{50}\right)\right] = \frac{49}{50}$

16. (D) Let $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 1 & 0 \\ 0 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

$|A| = 3(1 - 2) - 1(0 - 4) = -3 + 4 = 1$

Hence A is an elementary matrix.

17. (A)

18. (B) The total no. of arrangement = $\frac{7!}{2!}$ = $\frac{7!}{2}$

No. of arrangement when N's comes together = 6!

The required Probability = $\frac{6!}{7!/2}$ = $\frac{2}{7}$

19. (C) $f(z) = \frac{5+z^2}{1-z}$

given that $z = 1 - 2i$

$f(z) = \frac{5+(1-2i)^2}{1-1+2i}$

$f(z) = \frac{2-4i}{2i} \times \frac{i}{i}$

$f(z) = \frac{2i-4i^2}{-2}$

$f(z) = \frac{2i+4}{-2} = -2 - i$

Now, $|f(z)| = \sqrt{(-2)^2+(-1)^2}$

$|f(z)| = \sqrt{4+1} = \sqrt{5}$

20. (D) $y = \text{cosec}(\cot^{-1}x)$ ...(i)

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dy}{dx} = -\text{cosec}(\cot^{-1}x).\cot(\cot^{-1}x)\left(\frac{-1}{1+x^2}\right)$

$\Rightarrow (1 + x^2)\frac{dy}{dx} = \text{cosec}(\cot^{-1}x).x$

$\Rightarrow (1 + x^2)\frac{dy}{dx} = yx$ [from eq (i)]

21. (B) Point $(2k-1, k^r)$

Now, $2k-1 = k^r)$

$\Rightarrow k = 1$ for $r \in$ R

22. (B) $8\cos\frac{2\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{4\times 2\sin\frac{2\pi}{15}.\cos\frac{2\pi}{15}}{\sin\frac{2\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{4\times\sin\frac{4\pi}{15}}{\sin\frac{2\pi}{15}}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{2}{\sin\frac{2\pi}{15}}\times 2\sin\frac{4\pi}{15}.\cos\frac{4\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{2}{\sin\frac{2\pi}{15}}\times\sin\frac{8\pi}{15}.\cos\frac{8\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{\sin\frac{2\pi}{15}}\times\sin\frac{16\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}\times 2\sin\frac{16\pi}{15}.\cos\frac{16\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\frac{32\pi}{15}$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\left(2\pi+\frac{2\pi}{15}\right)$

$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}.\sin\frac{2\pi}{15} = \frac{1}{2}$

23. (A) $8\sqrt{3}$ .sin10. sin.20. sin40

$\Rightarrow 8\sqrt{3}$ sin10. cos70. cos50

$\Rightarrow 3\sqrt{3}\ \frac{\sin 10}{\cos 10}.\cos 10.\cos 50.\cos 70$

We know that

$\cos\theta.\cos(60-\theta).\cos(60+\theta) = \frac{1}{4}\cos 3\theta$

$\Rightarrow 8\sqrt{3}.\tan 10.\frac{1}{4}\times\cos(3\times 10)$

$\Rightarrow 8\sqrt{3}\times\tan 10\times\frac{1}{4}\times\frac{\sqrt{3}}{2} = 3\tan 10$

24. (C) Vectors $\vec{x} = a\hat{i}-2\hat{j}+2\hat{k}$, $\vec{y} = -\hat{i}+b\hat{j}-\hat{k}$

and $\vec{z} = 3\hat{i}+\hat{j}+c\hat{k}$ are prependicular to eachother,

then $\vec{x}.\vec{y} = 0$

$\Rightarrow -a - 2b - 2 = 0$

$\Rightarrow a + 2b + 2 = 0$ ...(i)

$\vec{y}.\vec{z} = 0$

$\Rightarrow -3 + b - c = 0$

$\Rightarrow b - c = 3$ ...(ii)

and $\vec{x}.\vec{z} = 0$

$\Rightarrow 3a - 2 + 2c = 0$

$\Rightarrow 3a + 2c = 2$ ...(iii)

On solving eq(i), (ii) and (iii)

$a = 5$, $b = \frac{-7}{2}$, $c = \frac{-13}{2}$

25. (B) Let $a + ib = \sqrt{-2+2\sqrt{35}i}$

On squaring both side

$(a^2 - b^2) + 2abi = -2 + 2\sqrt{35}\, i$

On comparing

$a^2 - b^2 = -2$ and $2ab = 2\sqrt{35}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 4 + 4 \times 35$

$\Rightarrow (a^2 + b^2)^2 = 144$

$\Rightarrow a^2 + b^2 = 12$ ...(ii)

from eq(i) and eq(ii)

$2a^2 = 10,\ 2b^2 = 14$

$a = \pm\sqrt{5},\ b = \pm\sqrt{7}$

Hence $\sqrt{-2+2\sqrt{35}i} = \pm\left(\sqrt{5}+\sqrt{7}i\right)$

26. (C) Given that

$$\begin{bmatrix} x+y & 3x+w \\ 2w+z & x-y \end{bmatrix} = \begin{bmatrix} 12 & -7 \\ 6 & -4 \end{bmatrix}$$

On comparing

$x + y = 12,\ 3x + w = -7$

$2w + z = 6,\ x - y = -4$

On solving

$x = 4,\ y = 8,\ z = 44,\ w = -19$

27. (A) $I = \int \frac{5^x}{5^x - 1}\, dx$

Let $5^x - 1 = t$

$5^x \log 5\ dx = dt$

$5^x\, dx = \frac{1}{\log 5}\, dt$

$I = \int \frac{1}{\log 5}\frac{1}{t}\, dt$

$I = \frac{1}{\log 5} \log t + c$

$I = \frac{\log(5^x - 1)}{\log 5} + c$

$I = \log_5(5^x - 1) + c$

28. (D) $i^{501} + i^{502} + i^{503} + i^{504} + i^{505}$

$\Rightarrow i^{501}(1 + i + i^2 + i^3 + i^4)$

$\Rightarrow i^{3\times167}\,(1 + i - 1 - i + 1)$

$\Rightarrow 1 \times 1 = 1$

29. (B) ${}^{23}C_4 + \sum_{r=1}^{4} {}^{22+r}C_3$

$\Rightarrow {}^{23}C_4 + {}^{23}C_3 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$

We know that

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^{24}C_4 + {}^{24}C_3 + {}^{25}C_3 + {}^{26}C_3$

$\Rightarrow {}^{25}C_4 + {}^{25}C_3 + {}^{26}C_3$

$\Rightarrow {}^{26}C_4 + {}^{26}C_3 = {}^{27}C_4$

30. (B) $\cos(x - iy) = A + iB$

$\Rightarrow \cos x.\cos iy + \sin x.\sin iy = A + iB$

We know that

$\cos iA = \cosh A$ and $\sin iA = i \sinh A$

$\Rightarrow \cos x.\cosh y + i \sin x.\sinh y = A + iB$

On comparing

$A = \cos x.\cosh y,\ B = \sin x.\sinh y$

31. (C) $x = a\cos\theta - b\sin\theta$ and $y = b\cos\theta + a\sin\theta$

$x^2 + y^2 = (a\cos\theta - \sin\theta)^2 + (b\cos\theta + a\sin\theta)^2$

$\Rightarrow x^2 + y^2 = a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta.\cos\theta + b^2\cos^2\theta + a^2\sin^2\theta + 2ab\sin\theta.\cos\theta$

$\Rightarrow x^2 + y^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow x^2 + y^2 = a^2 + b^2$

32. (A) We know that

$\sin ix = i \sinh y$

Now, $\sinh\left(\frac{i\pi}{3}\right) = -\, i \sin\left[i\left(\frac{i\pi}{3}\right)\right]$

$= -\, i \sin\left(\frac{-\pi}{3}\right)$

$= i \sin\frac{\pi}{3} = \frac{\sqrt{3}i}{2}$

33. (D) $\sin ix - i\cos ix = -i(\cos ix + i \sin ix)$

$= -\, i.e^{i(ix)} = -i.e^{-x}$

34. (B) lines $x - 3y = -4$ ...(i)

$2x - y = 7$ ...(ii)

$4x - 5y = 11$ ...(iii)

Intersecting point of line (i) and (ii) is (5, 3).

Let the equation of line which is perpendicular to the line (iii)

$5x + 4y = c$ ...(iv)

its passes through the point (5, 3)

$5 \times 5 + 4 \times 3 = c \Rightarrow c = 37$

from eq (iv)

$5x + 4y = 37$

35. (B) $b\,(c\cos A - a\cos C)$

$\Rightarrow b\left[c.\frac{b^2+c^2-a^2}{2bc} - a.\frac{a^2+b^2-c^2}{2ab}\right]$

$\Rightarrow \frac{b^2+c^2-a^2}{2} - \frac{a^2+b^2-c^2}{2} \Rightarrow c^2 - a^2$

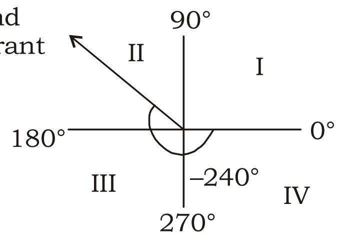
36. (A) line $6x - 8y = 4$ ...(i)

and $16y - 12x = 9$

$\Rightarrow 6x - 8y = \frac{-9}{2}$ ...(ii)

Distance $= \frac{4+\frac{9}{2}}{\sqrt{6^2+(-8)^2}} = \frac{17}{2\times10} = \frac{17}{20}$

37. (B) Second quadrant



38. (A)

39. (C) $A = \begin{bmatrix} 3 & 0 & 3 \\ 5 & -8 & 2 \\ -2 & 3 & -1 \end{bmatrix}$

$|A| = 3(8 - 6) + 3(15 - 16)$
$= 6 - 3 = 3$

Now, $(\text{Adj}A) = |A|^{n-2}A$ where n is order.

$\text{adj (adj A)} = |A|^{3-2}A$
$= |A|.A = 3A$

$\det(\text{adj(adj}A) = |3A| = 3\ |A|$
$= 3 \times 3 = 9$

40. (D)

41. (C) Differential equation

$x\,dy - y\,dx = x^2 \sin x\, dx$

$\Rightarrow \dfrac{xdy - ydx}{x^2} = \sin x\, dx$

$\Rightarrow d\left(\dfrac{y}{x}\right) = \sin x\, dx$

On integrating

$\Rightarrow \int d\left(\dfrac{y}{x}\right) = \int \sin x\, dx$

$\Rightarrow \dfrac{y}{x} = -\cos x + c$

$\Rightarrow y = -x\cos x + cx$

$\Rightarrow y + x\cos x = cx$

42. (B) **Statement I**

45 < 53

sin45 < sin53 and cos45 > cos53

sin45 < sin53 sin45 > cos53

then cos53 < sin45 < sin53

Hence cos53 – sin53 is negative.

Statement I is incorrect.

Statement II

23 < 45

sin23 < sin45 and cos23 > cos45

cos 23 > sin45

then sin23 < sin45 < cos23

Hence sin 23 – cos23 is negative.

Statement II is correct.

43. (C) $11 \sin^2\theta + 3\cos^2\theta = 7$

$\Rightarrow 11(1 - \cos^2\theta) + 3\cos^2\theta = 7$

$\Rightarrow 11 - 11\cos^2\theta + 3\cos^2\theta = 7$

$\Rightarrow -8\cos^2\theta = -4$

$\Rightarrow \cos^2\theta = \dfrac{1}{2}$

$\Rightarrow \cos^2\theta = \cos^2\dfrac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{3}$

44. (A) $I = \int_0^{\pi} \dfrac{\sqrt{\sin\frac{x}{2}}}{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}\, dx$ …(i)

Prop. IV $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \int_0^{\pi} \dfrac{\sqrt{\sin\frac{\pi - x}{2}}}{\sqrt{\sin\frac{\pi - x}{2}} + \sqrt{\cos\frac{\pi - x}{2}}}$

$I = \int_0^{\pi} \dfrac{\sqrt{\cos\frac{x}{2}}}{\sqrt{\cos\frac{x}{2}} + \sqrt{\sin\frac{x}{2}}}\, dx$ …(ii)

from eq(i) and eq(ii)

$2I = \int_0^{\pi} \dfrac{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}{\sqrt{\sin\frac{x}{2}} + \sqrt{\cos\frac{x}{2}}}\, dx$

$2I = \int_0^{\pi} 1.dx$

$2I = [x]_0^{\pi} \Rightarrow 2I = \pi \Rightarrow I = \dfrac{\pi}{2}$

45. (C) Remainder $= \dfrac{7^{31} + 9^{41}}{8}$

$= (-1)^{31} + 1^{41}$

$= -1 + 1 = 0$

The number $7^{31} + 9^{41}$ is divisible by 8.

46. (A)

47. (B)

48. (D) $y = 2^{\sin x}$

On differentiating both side w.r.t.'x'

$\Rightarrow \dfrac{dy}{dx} = 2^{\sin x}.\log 2.\cos x$

$\Rightarrow \dfrac{dy}{dx} = 2^{\sin x}.\cos x.\log 2$

49. (C) 1 1 1

$1\times2^0 = 1$
$1\times2^1 = 2$
$1\times2^2 = 4$
$\overline{\quad 7}$

0 . 1 1 1

$\frac{1}{2} = 1\times2^{-1}$
$\frac{1}{4} = 1\times2^{-2}$
$\frac{1}{8} = 1\times2^{-3}$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8} = 0.875$

Hence $(111.111)_2 = (7.875)_{10}$

50. (D) R = {(1,1),(2,2),(3,3),(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)} on a set A = {1,2,3}

<u>Reflexive:-</u>
1R1, 2R2, 3R3
R is reflexive.
<u>Symmetric:-</u>
1R2, 2R1, 1R3, 3R1
R is symmetric.
<u>Transitive:-</u>
1R2, 2R3 1R3
R is transitive.
Hence R is an equivalence relation.

51. (B) $x - iy = \begin{vmatrix} 3i & -2 & i \\ 1 & -i & 0 \\ 1 & 4i & -2i \end{vmatrix}$

Now, $x + iy = \begin{vmatrix} -3i & -2 & -i \\ 1 & i & 0 \\ 1 & -4i & 2i \end{vmatrix}$

$\Rightarrow x + iy = -3i(2i^2) + 2(2i) - i(-4i - i)$
$\Rightarrow x + iy = 6i + 4i - i(-5i)$
$\Rightarrow x + iy = 10i - 5$

52. (B) One angle $= \frac{1}{3}$ radian

second angle = 90° $= \frac{\pi}{2}$ radian

Now, $\frac{1}{3} + \frac{\pi}{2}$ + third angle = π

third angle = $\frac{\pi}{2} - \frac{1}{3}$

$= \frac{3\pi - 2}{6}$ radian

53. (D) A = {1,2,3}, B = {2,3,4}, C = {3,4,5}

$(B \cap C)$ = {2,3,4} $\cap$ {3,4,5} = {3,4}

Now, A × $(B \cap C)$ = {1,2,3} × {3,4}

The number of elements in A × $(B \cap C)$
= 3 × 2 = 6

54. (A) | 7 | 10 | 10 | = 7 × 10 × 10 = 700

(55-56)

Life of mobile phones(in month)	x	f	$f \times x$
0-10	5	22	110
10-20	15	x	$15x$
20-30	25	14	350
30-40	35	y	$35y$
40-50	45	24	1080
		$\sum f = 60+x+y$	$1540+15x+35y$

given that $\sum f = 100$
$\Rightarrow 60 + x + y = 100$
$\Rightarrow x + y = 40$ …(i)
and Mean = 27

$\Rightarrow \frac{\sum f \times x}{\sum f} = 27$

$\Rightarrow \frac{1540 + 15x + 35y}{100} = 27$

$\Rightarrow 15x + 35y = 1160$
$\Rightarrow 3x + 7y = 232$ …(ii)
On solving eq(i) and eq(ii)
$x = 12$, $y = 28$

55. (C) $x = 12$
56. (D) $y = 28$
57. (B) Let $y = 3^{37}$
taking log both side
$\Rightarrow \log_{10}y = 37 \log_{10}3$
$\Rightarrow \log_{10}y = 37 \times 0.4771$
$\Rightarrow \log_{10}y = 17.6527$
The number of digits in 3^{37} = 17+1 = 18

58. (C) $\begin{bmatrix} 8 & -2 & 3 \\ 4 & 0 & 5 \\ 6 & -1 & \lambda \end{bmatrix}$ is not an invertible matix,
then |A| = 0

$\Rightarrow \begin{vmatrix} 8 & -2 & 3 \\ 4 & 0 & 5 \\ 6 & -1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 8 \times 5 + 2(4\lambda - 30) + 3(-4) = 0$
$\Rightarrow 40 + 8\lambda - 60 - 12 = 0$
$\Rightarrow 8\lambda = 32 \Rightarrow \lambda = 4$

59. (C) $x = \frac{2t}{1+t^2}$

Let $t = \tan\theta \Rightarrow \theta = \tan^{-1}t$

$\Rightarrow x = \frac{2\tan\theta}{1+t^2}$

$\Rightarrow x = \sin2\theta$

$\Rightarrow \frac{dx}{d\theta} = 2\cos2\theta$ …(i) and $y = \frac{1-t^2}{1+t^2}$

$\Rightarrow y = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow y = \cos 2\theta$

$\Rightarrow \frac{dy}{d\theta} = -2\sin 2\theta$ ...(ii)

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

$\Rightarrow \frac{dy}{dx} = -2\sin 2\theta \times \frac{1}{2\cos 2\theta}$

$\Rightarrow \frac{dy}{dx} = -\tan 2\theta$

$\Rightarrow \frac{dy}{dx} = -\frac{2\tan\theta}{1-\tan^2\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-2t}{1-t^2}$

60. (B) $\lim_{x\to 8}\frac{x-8}{x^2-64}$

$\Rightarrow \lim_{x\to 8}\frac{x-8}{(x-8)(x+8)}$

$\Rightarrow \lim_{x\to 8}\frac{1}{x+8}$

$\Rightarrow \frac{1}{x+8} = \frac{1}{16}$

61. (B) We Know that

$\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Now, $(-1-i\sqrt{3})^{51} = 2^{51}\left(\frac{-1-i\sqrt{3}}{2}\right)^{51}$

$= 2^{51}(\omega^2)^{51}$

$= 2^{51} \times \omega^{3\times 34}$

$= 2^{51}$ $\left[\because \omega^3 = 1\right]$

62. (C) $\begin{vmatrix}\sin\theta & \cos\theta \\ \cos\theta & -\sin\theta\end{vmatrix} = -\sin^2\theta - \cos^2\theta$

$= -(\cos^2\theta + \sin^2\theta) = -1$

63. (A) $x = \sin 36.\sin 108$ and $y = \sin 72.\sin 144$

Now, $xy = \sin 36.\sin 108.\sin 72.\sin 144$

$\Rightarrow xy = \sin 36.\sin(90+18).\sin(90-18).\sin(180-36)$

$\Rightarrow xy = \sin 36.\cos 18.\cos 18.\sin 36$

$\Rightarrow xy = \cos^2 18.\sin^2 36$

We know that

$\cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$ and $\sin 36 = \frac{\sqrt{10-2\sqrt{5}}}{4}$

$\Rightarrow xy = \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2$

$\Rightarrow xy = \frac{10+2\sqrt{5}}{16} \times \frac{10-2\sqrt{5}}{16}$

$\Rightarrow xy = \frac{100-20}{16\times 16} = \frac{80}{16\times 16} = \frac{5}{16}$

64. (B) $\begin{vmatrix}x+7 & 2x & 2x \\ 2x & x+7 & 2x \\ 2x & 2x & x+7\end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix}5x+7 & 5x+7 & 5x+7 \\ 2x & x+7 & 2x \\ 2x & 2x & x+7\end{vmatrix}$

$\Rightarrow (5x+7)\begin{vmatrix}1 & 1 & 1 \\ 2x & x+7 & 2x \\ 2x & 2x & x+7\end{vmatrix}$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\Rightarrow (5x+7)\begin{vmatrix}1 & 0 & 0 \\ 2x & 7-x & 0 \\ 2x & 0 & 7-x\end{vmatrix}$

$\Rightarrow (5x+7)\,[1(7-x)^2] = (5x+7)(7-x)^2$

65. (C) $\begin{vmatrix}2 & 2 & 2 \\ 2 & 2+\sin\theta & 2 \\ 2+\cos\theta & 2 & 2\end{vmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix}2 & 2 & 2 \\ 0 & \sin\theta & 0 \\ \cos\theta & 0 & 0\end{vmatrix}$

$\Rightarrow 2 \times 0 - 2 \times 0 + 2(-\sin\theta.\cos\theta)$

$\Rightarrow -\sin 2\theta$

Maximum value of $-\sin 2\theta = 1$

66. (C) Two circles $x^2 + y^2 + 3x - y - 4 = 0$ and $x^2+y^2-2x-6y+\lambda = 0$ are orthogonal, then $2gg' + 2ff' = c + c'$

$\Rightarrow 2 \times \frac{3}{2}\times(-1) + 2\times\left(\frac{-1}{2}\right)\times(-3) = -4 + \lambda$

$\Rightarrow -3 + 3 = -4 + \lambda \Rightarrow \lambda = 4$

67. (C) equation $bx^2 + cx + a = 0$

$\alpha + \beta = \frac{-c}{b}$

$\alpha.\beta = \frac{a}{b}$

Now, $(b\alpha - a)(b\beta - a)$

$\Rightarrow b^2\alpha\beta - ab\beta - ab\alpha + a^2$

$\Rightarrow b^2 \times \alpha.\beta - ab(\alpha + \beta) + a^2$

$\Rightarrow b^2 \times \frac{a}{b} - ab \times \left(\frac{-c}{b}\right) + a^2$

$\Rightarrow ab + ac + a^2 \Rightarrow a(a + b + c)$

68. (B)

69. (C) S = 1 + 11 + 111 +..............n terms

$S = \frac{1}{9}(9+99+999+.................n \text{ terms})$

$S = \frac{1}{9}[(10-1)+(100-1)(1000-1)+...n \text{ terms}]$

$S = \frac{1}{9}[10+100+1000+.............n \text{ terms}]$

$-\frac{1}{9}(1+1+1+..........n \text{ terms})$

$S = \frac{1}{9} \times \frac{10(10^n - 1)}{10 - 1} - \frac{1}{9} \times n$

$S = \frac{10}{81}(10^n - 1) - \frac{n}{9}$

70. (B) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left[\sin\left(\pi - \frac{\pi}{4}\right)\right]$

$= \sin^{-1}\left[\sin\frac{\pi}{4}\right] = \frac{\pi}{4}$

71. (A) $I = \int_0^1 \frac{e^{\cot^{-1}x}}{1 + x^2}\,dx$

Let $\cot^{-1}x = t$ when $x \to 0,\ t \to \frac{\pi}{2}$

$-\frac{1}{1 + x^2}\,dx = dt$ $x \to 1,\ t \to \frac{\pi}{4}$

$\frac{1}{1 + x^2}\,dx = -dt$

$I = \int_{\pi/2}^{\pi/4} -e^t\,dt$

$I = -\left[e^t\right]_{\pi/2}^{\pi/4}$

$I = -[e^{\pi/4} - e^{\pi/2}]$

$I = e^{\pi/2} - e^{\pi/4}$

72. (B) $dy = y\tan x\,dx$

$\Rightarrow \frac{dy}{y} = \tan x\,dx$

On integrating both side

$\log y = \log\sec x + \log c$

$y = c.\sec x$ ...(i)

its passes through the point (0,1)

$1 = c.\sec 0 \Rightarrow c = 1$

from eq(i)

$y = \sec x$

73. (D) $\cot(-1740) = -\cot(1740)$

$= -\cot(360 \times 5 - 60)$

$= \cot 60 = \frac{1}{\sqrt{3}}$

74. (D) $\begin{vmatrix} a^2 & b+c & a \\ b^2 & c+a & b \\ c^2 & a+b & c \end{vmatrix}$

$C_2 \to C_2 + C_3$

$\Rightarrow \begin{vmatrix} a^2 & a+b+c & a \\ b^2 & a+b+c & b \\ c^2 & a+b+c & c \end{vmatrix}$

$\Rightarrow (a + b + c)\begin{vmatrix} a^2 & 1 & a \\ b^2 & 1 & b \\ c^2 & 1 & c \end{vmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\Rightarrow (a + b + c)\begin{vmatrix} a^2 & 1 & a \\ b^2 - a^2 & 0 & b - a \\ c^2 - a^2 & 0 & c - a \end{vmatrix}$

$\Rightarrow (a + b + c)(b - a)(c - a)\begin{vmatrix} a^2 & 1 & a \\ b + a & 0 & 1 \\ c + a & 0 & 1 \end{vmatrix}$

$R_3 \to R_3 - R_2$

$\Rightarrow (a + b + c)(b - a)(c - a)\begin{vmatrix} a^2 & 1 & a \\ b + a & 0 & 1 \\ c - b & 0 & 0 \end{vmatrix}$

$\Rightarrow (a + b + c)(b - a)(c - a)(c - b)\begin{vmatrix} a^2 & 1 & a \\ b + a & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow (a + b + c)(a - b)(b - c)(c - a)[-1(-1)]$

$= (a + b + c)(a - b)(b - c)(c - a)$

75. (C) $\cos^2 56\frac{1^\circ}{2} - \sin^2 33\frac{1^\circ}{2}$

$\Rightarrow \cos^2\left(90 - 33\frac{1}{2}\right)^\circ - \sin^2 33\frac{1^\circ}{2}$

$\Rightarrow \sin^2 33\frac{1^\circ}{2} - \sin^2 33\frac{1^\circ}{2} = 0$

76. (D)

77. (B) $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\sin^{-1}x = \frac{\pi}{2}$, $\sin^{-1}y = \frac{\pi}{2}$, $\sin^{-1}z = \frac{\pi}{2}$

$x = 1$, $y = 1$, $z = 1$

Now, $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$

$\Rightarrow 1 + 1 + 1 - 2 - 2 - 2 = -3$

78. (C) In the explansion of $\left(\frac{x^2}{4} - \frac{2}{x}\right)^7$

$T_{r+1} = \left(\frac{x^2}{4}\right)^{7-r}\left(\frac{-2}{x}\right)^r$

$= {}^7C_r \left(\frac{1}{4}\right)^{7-r} (-2)^r x^{14-3r}$

Now, $14 - 3r = 2$

$\Rightarrow 3r = 12 \Rightarrow r = 4$

Coefficient of $x^2 = {}^7C_4 \left(\frac{1}{4}\right)^3 (-2)^4$

$= 35 \times \frac{1}{64} \times 2^4 = \frac{35}{4}$

79. (D) $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} \log_x x & \log_x y & \log_x z \\ \log_y x & \log_y y & \log_y z \\ \log_z x & \log_z y & \log_z z \end{vmatrix}$

$\Rightarrow \begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$

$\Rightarrow \frac{1}{\log x.\log y.\log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$

80. (B) Digits 0,1,2,3,4,5,6,7,8,9

no. of -digits numbers

(i) when last digit is '0'

8	7	4	1

= 8×7×4×1 = 224

(2,4,6,8) ; 0

(ii) when last digit is '2'

7	7	5	1

= 7×7×5×1 = 245

(1,3,5,7,9) ; 2

'0' cannot put here

(iii) when last two digits is '04'

8	7	1	1

= 8×7×1×1 = 56

0 4

(iv) when last digit is '4'

7	7	4	1

= 7×7×4×1 = 196

(2,4,6,8) ; 4

'0' cannot put here

(v) when last digit is '6'

7	7	5	1

= 7×7×5×1 = 245

(1,3,5,7,9) ; 6

'0' cannot put here

(vi) when last two digits is '08'

8	7	1	1

= 8×7×1×1 = 56

0 8

(vii) when last digit is '8'

7	7	4	1

= 7×7×4×1 = 196

(2,4,6,8) ; 8

'0' cannot put here

Hence total number of arrangement

= 224 + 245 + 56 + 196 + 245 + 56 + 196

= 1218

81. (A) In ΔABC, $\begin{vmatrix} 1 & a & c \\ 1 & b & a \\ 1 & c & b \end{vmatrix} = 0$

$\Rightarrow 1(b^2 - ca) - a(b - a) + c(c - b) = 0$

$\Rightarrow b^2 - ca - ab + a^2 + c^2 - bc = 0$

$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca = 0$

$\Rightarrow \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$

$a = b$, $b = c$, $c = a$

$a = b = c$

then A = B = C = 60°

Now, $\cos^2 A + \cos^2 B + \cos^2 C$

$\Rightarrow \cos^2 60 + \cos^2 60 + \cos^2 60$

$\Rightarrow \frac{1}{4}+\frac{1}{4}+\frac{1}{4} = \frac{3}{4}$

82. (C) We know that

$e = 2.718$, $\pi = 3.14$

then, $e^2 - 3 = 4.387$, $\pi^2 - 5 = 4.859$

$[e] = 2$, $[\pi] = 3$, $[e^2-3] = 4$, $[\pi^2 - 5] = 4$

Now, $\begin{vmatrix} [e] & [\pi] & [e^2-3] \\ [\pi] & [\pi^2-5] & [e] \\ [e^2-3] & [\pi^2-5] & [\pi] \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 4 & 3 \end{vmatrix}$

$\Rightarrow 2(12 - 8) - 3(9-8) + 4(12 - 16) = -11$

83. (B) $\lim_{x\to 0} \frac{\sin x - \tan x}{x^3}$

$\Rightarrow \lim_{x\to 0} \frac{\tan x(\cos x - 1)}{x^3}$

$\Rightarrow -\lim_{x\to 0} \frac{\tan x}{x} \frac{2\sin^2\frac{x}{2}}{x^2}$

$\Rightarrow -\lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{4\times\frac{x^2}{4}} = \frac{-2}{4} = \frac{-1}{2}$

84. (C) $y = \cos^{-1}\left(x\sqrt{1-x}+\sqrt{x}\sqrt{1-x^2}\right)$

Let $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\sqrt{x} = \sin\phi \Rightarrow \phi = \sin^{-1}\sqrt{x}$

$\Rightarrow y = \cos^{-1}\left[\cos\theta.\sqrt{1-\sin^2\phi}+\sin\phi.\sqrt{1-\cos^2\theta}\right]$

$\Rightarrow y = \cos^{-1}\left[\cos\theta.\cos\phi + \sin\phi.\sin\theta\right]$

$\Rightarrow y = \cos^{-1}[\cos(\theta - \phi)]$

$\Rightarrow y = \theta - \phi$

$\Rightarrow y = \cos^{-1}x - \sin^{-1}\sqrt{x}$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x-x^2}}$

85. (A) $\lim_{n\to\infty}\left(\cos\frac{x}{n}\right)^n$

We know that

$\lim_{x\to a}[f(x)]^{g(x)} = e^{\lim_{x\to a} g(x)[f(x)-1]}$ [1^∞ form]

$\Rightarrow e^{\lim_{n\to\infty} n\left[\cos\frac{x}{n}-1\right]}$

$\Rightarrow e^{-n\lim_{n\to\infty} 2\sin^2\frac{x}{2n}}$

$\Rightarrow e^{-2n\lim_{x\to\infty}\frac{\sin^2\frac{x}{2n}}{\frac{x^2}{4n^2}}\times\frac{x^2}{4n^2}}$

$\Rightarrow e^{-\lim_{n\to\infty}\frac{x^2}{2n}\left(\frac{\sin\frac{x}{2n}}{\frac{x}{2n}}\right)^2}$

$\Rightarrow e^{-\lim_{n\to\infty}\frac{x^2}{2n}} \Rightarrow e^0 = 1$

86. (A) $y = \left(\frac{1}{x}\right)^x$

taking log both side

$\Rightarrow \log y = x\log\left(\frac{1}{x}\right)$

$\Rightarrow \log y = -x\log x$

On differentiating both side w.r.t.'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -x\times\frac{1}{x} - \log x.1$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -1 - \log x$

$\Rightarrow \frac{dy}{dx} = -y(1+\log x)$

87. (C)

88. (A) matrix $[A]_{3\times 5}$, matrix $[B]_{5\times 6}$,

then matrix [AB] = 3 × 6

89. (C) $\frac{\text{A.M.}}{\text{G.M.}} = \frac{5}{4}$

$\Rightarrow \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

by Componendo & Dividendo Rule

$\Rightarrow \dfrac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \dfrac{5+4}{5-4}$

$\Rightarrow \dfrac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \dfrac{9}{1}$

$\Rightarrow \dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \dfrac{3}{1}$

Again, by Componendo & Dividendo Rule

$\Rightarrow \dfrac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \dfrac{3+1}{3-1}$

$\Rightarrow \dfrac{2\sqrt{a}}{2\sqrt{b}} = \dfrac{4}{2}$

$\Rightarrow \dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{2}{1} \Rightarrow \dfrac{a}{b} = \dfrac{4}{1}$

Hence $a : b = 4 : 1$

90. (B) Given that $f(x) = (x-2)^2 + 6$

$f(a) \Rightarrow f(4) = (4-2)^2 + 6 = 10$

$f(b) \Rightarrow f\left(\dfrac{11}{2}\right) = \left(\dfrac{11}{2}-2\right)^2 + 6 = \dfrac{73}{4}$

$f'(x) = 2(x-2)$

$f'(c) = 2(c-2)$

Now, $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$\Rightarrow 2(c-2) = \dfrac{\dfrac{73}{4}-10}{\dfrac{11}{2}-4}$

$\Rightarrow 2(c-2) = \dfrac{\dfrac{33}{4}}{\dfrac{3}{2}}$

$\Rightarrow 2(c-2) = \dfrac{11}{2}$

$\Rightarrow c-2 = \dfrac{11}{4} \Rightarrow c = \dfrac{19}{4}$

91. (B) $\dfrac{dy}{x} + \left(\dfrac{y}{x^2}-1\right)dx = 0$

$\Rightarrow \dfrac{dy}{x} + \left(\dfrac{y-x^2}{x^2}\right)dx = 0$

$\Rightarrow x\,dy + y\,dx - x^2dx = 0$

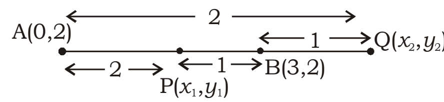
$\Rightarrow d(xy) = x^2dx$

On integrating both side

$\Rightarrow xy = \dfrac{x^3}{3} + c$

$\Rightarrow 3xy = x^3 + c$

92. (C)



$x_1 = \dfrac{2\times3+1\times0}{2+1}$, $y_1 = \dfrac{2\times2+1\times2}{2+1}$

$x_1 = 2,$ $y_1 = 2$

$P(x_1,y_1) = (2, 2)$

$x_2 = \dfrac{2\times3-1\times0}{2-1}$, $y_2 = \dfrac{2\times2-1\times2}{2-1}$

$x_2 = 6,$ $y_2 = 2$

$Q(x_2,y_2) = (6, 2)$

Distance between P and Q

$= \sqrt{(2-6)^2+(2-2)^2}$

$= \sqrt{16+0} = 4$

93. (B) $y = e^{\cot^{-1}x}$

On differentiating both side w.r.t.'x'

$\Rightarrow y_1 = e^{\cot^{-1}x}\left(\dfrac{-1}{1+x^2}\right)$

$\Rightarrow (1+x^2)y_1 = -y$ from eq(i)

Again, differentiating

$\Rightarrow (1+x^2)y_2 + y_1.2x = -y_1$

$\Rightarrow (1+x^2)y_2 = -y_1(2x+1)$

94. (A) $I = \int e^{\sin^{-1}x}\left[\dfrac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]dx$

Let $\sin^{-1}x = t \Rightarrow x = \sin t$

$dx = \cos t\, dt$

$I = \int e^t\left[\dfrac{\sin t+\cos t}{\cos t}\right]\cos t\, dt$

$I = \int e^t\left[\sin t+\cos t\right]dt$

We know that

$\int e^x\,[f(x) + f'(x)]\,dx = e^x f(x) + c$

$I = e^t \sin t + c$

$I = x.e^{\sin^{-1}x} + c$

95. (C) $f(x) = \int_0^x \cos^2 2t\, dx$

$f(x) = \int_0^x \dfrac{1+\cos 4t}{2}\,dx$

$f(x) = \dfrac{1}{2}\left[t+\dfrac{\sin 4t}{4}\right]_0^x$

$f(x) = \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]$ ...(i)

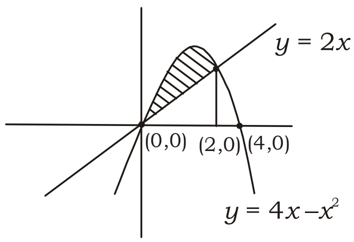
Now, $f(\pi + x) = \frac{1}{2}\left[(\pi + x) + \frac{\sin 4(\pi + x)}{4}\right]$

$f(\pi + x) = \frac{1}{2}\left[(\pi + x) + \frac{\sin 4x}{4}\right]$

$f(\pi + x) = \frac{1}{2}\pi + \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]$

$f(\pi + x) = f(\pi) + f(x)$ [from eq(i)]

96. (C)



curve $y_1 \Rightarrow y = 4x - x^2$
and $y_2 \Rightarrow y = 2x$

Area = $\int_0^2 (y_1 - y_2)dx$

$= \int_0^2 (4x - x^2 - 2x)dx$

$= \int_0^2 (2x - x^2)dx$

$= \left[2 \times \frac{x^2}{2} - \frac{x^3}{3}\right]_0^2$

$= 4 - \frac{8}{3} = \frac{4}{3}$ square unit

97. (A) equation of line which passes through the point (2,–3)
$y + 3 = m(x - 2)$ ...(i)
On differentiating both side w.r.t.'x'

$\Rightarrow \frac{dy}{dx} = m$

from equation (i)

$\Rightarrow y + 3 = \frac{dy}{dx}(x - 2)$

$\Rightarrow y = (x - 2)\frac{dy}{dx} - 3$

98. (B) $n(S) = 6 \times 6 = 36$
E = {(6,3),(3,6),(5,4),(4,5)} , $n(E) = 4$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$= \frac{4}{36} = \frac{1}{9}$

99. (C) Differential equation

$\frac{dy}{dx} = y^2(e^x - 1)$

$\Rightarrow \frac{dy}{y^2} = (e^x - 1)\,dx$

On integrating both side

$\Rightarrow \frac{y^{-2+1}}{-2+1} = e^x - x + c$

$\Rightarrow \frac{-1}{y} = e^x - x + c \Rightarrow y(e^x - x + c) + 1 = 0$

100. (D) Differential equation

$x\frac{dy}{dx} = x^2y\frac{dy}{dx} + y$

$\Rightarrow x\frac{dy}{dx} - y = x^2y\frac{dy}{dx}$

$\Rightarrow xdy - ydx = x^2y\,dy$

$\Rightarrow \frac{xdy - ydx}{x^2} = y\,dy$

$\Rightarrow d\left(\frac{y}{x}\right) = y\,dy$

On integating both side

$\Rightarrow \frac{y}{x} = \frac{y^2}{2} + c$

$\Rightarrow 2y = xy^2 + cx$

101. (A) $I = \int \cos(\log x)dx$

Let $\log x = t \Rightarrow x = e^t$
$dx = e^t\,dt$

$I = \int \cos t\, e^t\,dt$ ...(i)

$I = \cos t\int e^t dt - \int\left\{\frac{d}{dt}(\cos t).\int e^t dt\right\}$

$I = \cos t.\ e^t - \int -\sin t.e^t dt$

$I = e^t.\ \cos t + \int \sin t.e^t dt$

$I = e^t.\ \cos t + \sin t.\int e^t dt - \int\left\{\frac{d}{dt}(\sin t).\int e^t dt\right\}dt$

$I = e^t.\ \cos t + \sin t.e^t - \int \cos t.e^t dt + c$

$I = e^t(\sin t + \cos t) - I + c$ [from eq(i)]

$2I = e^t(\sin t + \cos t) + c$

$I = \frac{1}{2}e^t(\sin t + \cos t) + c$

$I = \frac{1}{2}x[\sin(\log x) + \cos(\log x)] + c$

102. (C) $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} x+a & b & c \\ -x & x & 0 \\ -x & 0 & x \end{vmatrix} = 0$

$\Rightarrow (x + a)x^2 - b(-x^2) + cx^2 = 0$

$\Rightarrow x^3 + ax^2 + bx^2 + cx^2 = 0$

$\Rightarrow x^2(x + a + b + c) = 0$

$x = 0,\ x + a + b + c = 0$

$x = -(a + b + c)$

103. (B) $7x - 6y + 20 = 0$

and $7x - 6y - 12 = 0$

The required line

$7x - 6y + 4 = 0$

104. (A) Two points (3,4,–2) and (–2,3,–7)

Direction cosine of line joining the given points = < – 2 – 3, 3 – 4, –7 + 2 >

= < –5, –1, –5 >

105. (C) a, A_1, A_2, b

$d = \frac{b-a}{3}$

$A_1 = a + d$

$A_1 = a + \frac{b+a}{3} = \frac{2a+b}{3}$

$A_2 = a + 2d$

$A_2 = a + 2d$

$= a + \frac{2b-2a}{3} = \frac{a+2b}{3}$

and a, G_1, G_2, b

$r = \left(\frac{b}{a}\right)^{1/3}$

$a_1 = ar$

$a_1 = a\left(\frac{b}{a}\right)^{1/3} = a^{2/3}.\ b^{1/3}$

$a_2 = ar^2$

$= a.\left(\frac{b}{a}\right)^{2/3} = a^{1/3}.\ b^{2/3}$

Now, $\frac{A_1+A_2}{G_1G_2} = \frac{\frac{2a+b}{3}+\frac{a+2b}{3}}{a^{2/3}.b^{1/3}.a^{1/3}.b^{2/3}}$

$= \frac{a+b}{ab}$

106. (C) equations $x - 2y + z = 4$

$2x + y - 2 = 5$

$-3x - 4y + 2z = 3$

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ -3 & -4 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$

using elementary method

Now, $[A/B] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 2 & 1 & -1 & 5 \\ -3 & -4 & 2 & 3 \end{array}\right]$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 + 3R_1$

$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -3 & -3 \\ 0 & -10 & 5 & 15 \end{array}\right]$

$R_3 \to R_3 + 2R_2$

$= \left[\begin{array}{ccc|c} 1 & -2 & 1 & 4 \\ 0 & 5 & -3 & -3 \\ 0 & 0 & -1 & 9 \end{array}\right]$

$x - 2y + z = 4$ ...(i)

$5y - 3z = -3$ ...(ii)

$-z = 9$ ...(iii)

On solving

$x = 1,\ y = -6,\ z = -9$

107. (B) We know that

$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n$

On integrating both side

$\Rightarrow \frac{(1+x)^{n+1}}{n+1} = C_0x + C_1\frac{x^2}{2} + ... + C_n\frac{x^{n+1}}{n+1} + k$

On putting $x = 0$ eq(i)

$\Rightarrow \frac{1}{n+1} = 0 + k \Rightarrow k = \frac{1}{n+1}$

from eq(i)

$\Rightarrow \frac{(1+x)^{n+1}}{n+1} = C_0x + C_1\frac{x^2}{2} + C_2\frac{x^3}{3}$ ....

$+ C_n\frac{x^{n+1}}{n+1} + \frac{1}{n+1}$

On putting $x = 1$

$\Rightarrow \frac{2^{n+1}}{2} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} + \frac{1}{n+1}$

$\Rightarrow C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = 2^n - \frac{1}{n+1}$

108. (C) Digits 0,1,3,5,6,7,8

| 6 | 7 | 7 | 7 | = 6×7×7×7 = 2058

'0' cannot put here

109. (B) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + ...\infty}}}$

$\Rightarrow y = \sqrt{\sin x + y}$

$\Rightarrow y^2 = \sin x + y$

On differentiating both side w.r.t.'x'

$\Rightarrow 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$\Rightarrow (2y - 1)\frac{dy}{dx} = \cos x$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$

110. (A) x, 16 and y are in A.P., then

$2 \times 16 = x + y \Rightarrow x + y = 32$ ...(i)

and x, 8 and y are in G.P., then

$8^2 = xy \Rightarrow xy = 64$ ...(ii)

Now, H.M. = $\frac{2xy}{x+y}$

H.M. = $\frac{2 \times 64}{32} = 4$

Hence x, 4 and y will be in H.P.

111. (D) quadratic equation

$px^2 + qx + c = 0$

$\alpha + \beta = \frac{-q}{p}$ ...(i)

and quadratic equation

$ax^2 + bx + c = 0$

$\alpha + k + \beta + k = \frac{-b}{a}$

$\Rightarrow \frac{-q}{p} + 2k = \frac{-b}{a}$ [from eq(i)]

$\Rightarrow 2k = \frac{q}{p} - \frac{b}{a} \Rightarrow k = \frac{1}{2}\left(\frac{q}{p} - \frac{b}{a}\right)$

112. (B) $x + \frac{1}{x} = 2\sin\frac{\pi}{12}$

On squaring both side

$\Rightarrow x^2 + \frac{1}{x^2} + 2 = 4\sin^2\frac{\pi}{12}$

$\Rightarrow x^2 + \frac{1}{x^2} = -2\left(1 - 2\sin^2\frac{\pi}{12}\right)$

$\Rightarrow x^2 + \frac{1}{x^2} = -2\cos\left(2 \times \frac{\pi}{12}\right)$

$\Rightarrow x^2 + \frac{1}{x^2} = -2\cos\frac{\pi}{6}$

On squaring both side

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 4\cos^2\frac{\pi}{6}$

$\Rightarrow x^4 + \frac{1}{x^4} = 2\left(2\cos^2\frac{\pi}{6} - 1\right)$

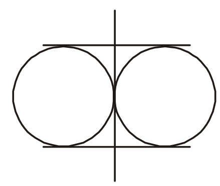
$\Rightarrow x^4 + \frac{1}{x^4} = 2\cos\left(2 \times \frac{\pi}{6}\right)$

$\Rightarrow x^4 + \frac{1}{x^4} = 2 \times \frac{1}{2} = 1$

113. (A) No. of ways = ${}^{15-1}C_{11-1}$

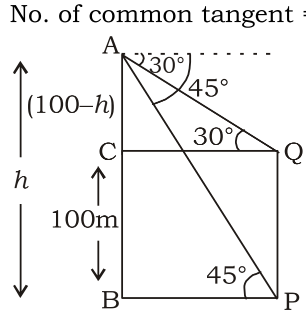
$= {}^{14}C_{10} = 1001$

114. (C)



No. of common tangent = 3

115. (A)



Let height of the mountain = h m

In ΔABP :-

$\tan 45° = \frac{AB}{BP}$

$\Rightarrow 1 = \frac{h}{BP} = BP = h = CQ$

In ΔACQ :-

$\tan 30° = \frac{AC}{CQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100-h}{h}$

$\Rightarrow h = 100\sqrt{3} - \sqrt{3}\,h \Rightarrow h = 50(3 - \sqrt{3})\,m$

116. (C) $\lim_{x\to\infty} \dfrac{1^2+2^2+3^2+....+n^2}{n(1+2+3+....+n)}$

$\Rightarrow \lim_{x\to\infty} \dfrac{\frac{n}{6}(n+1)(2n+1)}{n\times\frac{n(n+1)}{2}}$

$\Rightarrow \lim_{x\to\infty} \dfrac{(2n+1)}{3n}$

$\Rightarrow \lim_{x\to\infty} \dfrac{n\left(2+\frac{1}{n}\right)}{3n} = \dfrac{2}{3}$

117. (B) $f(x) = \{[x]-3\}^2 - [x-3]$

At $x = 3$

L.H.L. $= \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0} \qquad - [3-h-3]$

$= \lim_{h\to 0} \{2-3\}^2 - [0-h]$

$= \lim_{h\to 0} 1-(-1) = 2$

R.H.L. $= \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

$= \lim_{h\to 0} \qquad - [3+h-3]$

$= \lim_{h\to 0} \{3-3\}^2 - [0-h]$

$= \lim_{h\to 0} 0-(0) = 0$

L.H.L. $\neq$ R.H.L.

Hence $f(x)$ is not continuous at $x = 3$.

At $x = 4$

L.H.L. $= \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h)$

$= \lim_{h\to 0} \{[4-h]-3\}^2 - [4-h-3]$

$= \lim_{h\to 0} \{3-3\}^2 - [1-h]$

$= \lim_{h\to 0} 0-0 = 0$

R.H.L. $= \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h)$

$= \lim_{h\to 0} \{[4+h]-3\}^2 - [4+h-3]$

$= \lim_{h\to 0} \{4-3\}^2 - [1+h]$

$= \lim_{h\to 0} (1-1) = 0$

L.H.L. = R.H.L.

Hence $f(x)$ is continuous at $x = 4$.

118. (D) $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Now, $A^2 + kA + I_2 = 0$

$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + k\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 12+3k & 8+2k \\ 4+k & 4+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

On comparing $k = -4$

119. (C) $\overrightarrow{OA} = \hat{i} + y\hat{j} - 3\hat{k}$

$= x\hat{i} - 3\hat{j} + 2\hat{k}$

$\overrightarrow{OC} = -\hat{i} - 4\hat{j} + 3\hat{k}$

A, B and C are collinear, then

$\overrightarrow{AB} = \lambda\overrightarrow{BC}$

$\Rightarrow (x-1)\,\hat{i} + (-3-y)\,\hat{j} + (2+3)\,\hat{k}$

$= \lambda\left[(-1-x)\hat{i} + (-4+3)\hat{j} + (3-2)\hat{k}\right]$

On comparing

$x - 1 = \lambda(-1-x) \Rightarrow x(1+\lambda) = (1-\lambda)$ ...(i)

$-3 - y = \lambda(-4+3) \Rightarrow 3 + y = \lambda$ ...(ii)

$2 + 3 = \lambda(3-2) \Rightarrow \lambda = 5$ ...(iii)

from eq(i) and eq (ii)

$x = \dfrac{-2}{3}$ and $y = 2$

Hence $(x, y) = \left(\dfrac{-2}{3}, 2\right)$

120. (A) $\vec{a} = -\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = -4\hat{i} + 3\hat{k}, \vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

$= (-1)(-4) + 1\times 0 + (-2)\times 3 = -2$

$\vec{a}.\vec{c} = (-1)(-2) + 1(-1) + (-2)\times 5 = -9$

We know that

$\left(\vec{x}\times\vec{y}\right)\times\vec{z} = \left(\vec{x}.\vec{z}\right)\vec{y} - \left(\vec{x}.\vec{y}\right)\vec{z}$

Now, $\left(\vec{a}\times\vec{c}\right)\times\vec{b} = \lambda\vec{c} + \mu\vec{b}$

$\Rightarrow \left(\vec{a}.\vec{b}\right)\vec{c} - \left(\vec{a}.\vec{c}\right)\vec{b} = \lambda\vec{c} + \mu\vec{b}$

$\Rightarrow -2\vec{c} + 9\vec{b} = \lambda\vec{c} + \mu\vec{b}$

On comparing

$\lambda = -2, \mu = 9$

Hence $(\lambda, \mu) = (-2, 9)$

NDA (MATHS) MOCK TEST - 120 (Answer Key)

1.	(B)	21.	(B)	41.	(C)	61.	(B)	81.	(A)	101.	(A)
2.	(A)	22.	(B)	42.	(B)	62.	(C)	82.	(C)	102.	(C)
3.	(C)	23.	(A)	43.	(C)	63.	(A)	83.	(B)	103.	(B)
4.	(D)	24.	(C)	44.	(A)	64.	(B)	84.	(C)	104.	(A)
5.	(B)	25.	(B)	45.	(C)	65.	(C)	85.	(A)	105.	(C)
6.	(C)	26.	(C)	46.	(A)	66.	(C)	86.	(A)	106.	(C)
7.	(B)	27.	(A)	47.	(B)	67.	(C)	87.	(C)	107.	(B)
8.	(C)	28.	(D)	48.	(D)	68.	(B)	88.	(A)	108.	(C)
9.	(D)	29.	(B)	49.	(C)	69.	(C)	89.	(C)	109.	(B)
10.	(C)	30.	(B)	50.	(D)	70.	(B)	90.	(B)	110.	(A)
11.	(B)	31.	(C)	51.	(B)	71.	(A)	91.	(B)	111.	(D)
12.	(A)	32.	(A)	52.	(B)	72.	(B)	92.	(C)	112.	(B)
13.	(D)	33.	(D)	53.	(D)	73.	(D)	93.	(B)	113.	(A)
14.	(C)	34.	(B)	54.	(A)	74.	(D)	94.	(A)	114.	(C)
15.	(A)	35.	(B)	55.	(C)	75.	(C)	95.	(C)	115.	(A)
16.	(D)	36.	(A)	56.	(D)	76.	(D)	96.	(C)	116.	(C)
17.	(A)	37.	(B)	57.	(B)	77.	(B)	97.	(A)	117.	(B)
18.	(B)	38.	(A)	58.	(C)	78.	(C)	98.	(B)	118.	(D)
19.	(C)	39.	(C)	59.	(C)	79.	(D)	99.	(C)	119.	(C)
20.	(D)	40.	(D)	60.	(B)	80.	(B)	100.	(D)	120.	(A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777