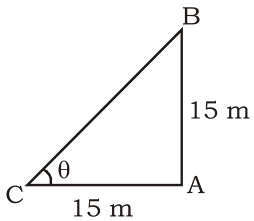
NDA MATHS MOCK TEST - 134 (SOLUTION)

1. (C)



AB = 15 m and AC = 15 m
Let angle of elevation be θ.

Now, $\tan\theta = \frac{AB}{AC}$

$\Rightarrow \tan\theta = \frac{15}{15} = 1 \Rightarrow \theta = 45°$

2. (A) Let the first term and common ratio of a GP be a and r respectively.
Now, 10th term = 9
$\Rightarrow ar^9 = 9$ ...(i)
and 4th term = 4
$\Rightarrow ar^3 = 4$
On dividing eq(i) by eq(ii), we get

$\frac{ar^9}{ar^3} = \frac{9}{4} \Rightarrow r^6 = \frac{9}{4}$

Multiplying eq(i) by eq(ii)
$ar^9 \times ar^3 = 9 \times 4$
$\Rightarrow a^2r^{12} = 36$
$\Rightarrow (ar^6)^2 = 36$

$\Rightarrow a^2 \times \left(\frac{9}{4}\right)^2 = 36$

$\Rightarrow a^2 = \frac{36\times16}{81} = \frac{64}{9} \Rightarrow a = \frac{8}{3}$

Now, 7th term = $ar^6 = \frac{8}{3} \times \frac{9}{4} = 6$

3. (B) Let $x - iy = \sqrt{-2i}$
On squaring
$(x^2 - y^2) - 2xyi = -2i$
On comparing
$x^2 - y^2 = 0$ and $2xy = 2$ ...(i)
Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$
$\Rightarrow (x^2 + y^2)^2 = 0 + 4$
$\Rightarrow (x^2 + y^2)^2 = 2$ ...(ii)
from eq(i) and eq(ii)
$2x^2 = 2,\ 2y^2 = 2$
$\Rightarrow x = \pm1,\ y = \pm1$
Hence $\sqrt{-2i} = \pm(1 - i)$

4. (B) Let $I = \int \frac{dx}{\sin^2 x\cos^2 x}$

$I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x\cos^2 x}dx\ [\because \sin^2x + \cos^2x = 1]$

$I = \int \frac{1}{\cos^2 x}dx + \int \frac{1}{\sin^2 x}dx$

$I = \int \sec^2 x\,dx + \int \text{cosec}^2 x\,dx$

$I = \tan x - \cot x + c$

5. (D) $y = \log\sqrt{\tan x} = \log(\tan x)^{1/2}$

$y = \frac{1}{2}\log(\tan x)$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\tan x}\cdot\sec^2x$

$\frac{dy}{dx}(\text{at } x = \pi/4) = \frac{1}{2} \times \frac{1}{\tan(\pi/4)} \times \sec^2\frac{\pi}{4}$

$= \frac{1}{2} \times 1 \times (\sqrt{2})^2 = 1$

6. (C) If $n!$, $3\times n!$ and $(n + 1)!$ are in GP,

then $\frac{3\times n!}{n!} = \frac{(n+1)!}{3\times n!}$

$\Rightarrow 3 = \frac{(n+1)n!}{3\times n!}$

$\Rightarrow 9 = n + 1 \Rightarrow n = 8$

7. (C) Since, ∠A is minimum.

Therefore, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Let $a = 2$, $b = \sqrt{6}$, $c = \sqrt{3} + 1$

$\cos A = \frac{(\sqrt{6})^2 + (\sqrt{3}+1)^2 - (2)^2}{2\times\sqrt{6}(1+\sqrt{3})}$

$\cos A = \frac{6+3+1+2\sqrt{3}-4}{2\sqrt{6}(\sqrt{3}+1)}$

$\cos A = \frac{6+2\sqrt{3}}{2\sqrt{6}(\sqrt{3}+1)}$

$\cos A = \frac{1}{\sqrt{2}} \Rightarrow A = 45°$

8. (B) $a = 15$ cm, $b = 20$ cm and $c = 25$ cm

$s = \frac{a+b+c}{2} = \frac{15+20+25}{2} = 30$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{30(30-15)(30-20)(30-25)}$

$\Delta = \sqrt{30\times15\times10\times5} = 150\text{ cm}^2$

Now, $r = \frac{\Delta}{s} = \frac{150}{30} = 5$ cm

9. (C) In ΔABC, $a = 39$, $b = 12$, $\cos C = \frac{-5}{13}$

Now, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow -\frac{5}{13} = \frac{(39)^2+(12)^2-c^2}{2\times39\times12}$

$\Rightarrow -5 = \frac{1521+144-c^2}{2\times3\times12}$

$\Rightarrow -360 = 1665 - c^2$

$\Rightarrow c^2 = 2025 \Rightarrow c = 45$

Now, $s = \frac{a+b+c}{2} = \frac{39+12+45}{2} = 48$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{48(48-39)(48-12)(48-45)}$

$\Delta = \sqrt{48\times9\times36\times3} = 216$

So, $R = \frac{abc}{4\Delta} = \frac{39\times12\times45}{4\times216} = \frac{195}{8}$

10. (A) $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$

$\Rightarrow 1+\tan^2(\tan^{-1}2) + 1 + \cot^2(\cot^{-1}3)$

$\Rightarrow 1\,[\tan(\tan^{-1}2)]^2 + 1 + [\cot(\cot^{-1}3)]^2$

$\Rightarrow 1 + (2)^2 + 1 + (3)^2 \Rightarrow 15$

11. (C) Given, $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}A$

$\Rightarrow \frac{x-y}{1+xy} = A$

12. (B) $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left[\frac{(1+x)+(1-x)}{1-(1+x)(1-x)}\right] = \frac{\pi}{6}$

$\Rightarrow \tan^{-1}\left[\frac{2}{1-(1-x^2)}\right] = \frac{\pi}{6}$

$\Rightarrow \frac{2}{x^2} = \tan\frac{\pi}{6}$

$\Rightarrow \frac{2}{x^2} = \frac{1}{\sqrt{3}} \Rightarrow x^2 = 2\sqrt{3}$

13. (C) $(1 + x + x^2 + x^3 + \ldots + \infty)^2$

$\Rightarrow \left(\frac{1}{1-x}\right)^2 = (1-x)^{-2} \quad \left(\because S_\infty = \frac{a}{1-r}\right)$

$\Rightarrow 1 + 2x + 3x^2 + \ldots + (n+1)x^n + \ldots\infty$

Hence coefficient of $x^n = (n+1)$

14. (A) $(998)^{1/3} \Rightarrow (1000-2)^{1/3}$

$\Rightarrow (1000)^{1/3}\left[1-\frac{2}{1000}\right]^{1/3}$

$\Rightarrow 10\left[1-\frac{2}{1000}\right]^{1/3}$

$\Rightarrow 10\left[1-\frac{1}{3(500)}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{1}{500}\right)^2+\ldots\right]$

$\Rightarrow 10\left[1-\frac{1}{1500}-\frac{1}{9\times250000}\right]$

$\Rightarrow 10\left[\frac{2250000-1500-1}{2250000}\right]$

$\Rightarrow \frac{22484990}{2250000} = 9.99$

15. (C) $\because r_{xy} = \frac{\operatorname{cov}(x,y)}{\sigma_x\sigma_y}$

$\Rightarrow 0.6 = \frac{16}{4\sigma_y}$

$\Rightarrow \sigma_y = \frac{16}{4\times0.6} = \frac{20}{3}$

16. (B) Equation is $ax^2 - 12x + 15 = 0$

One root is $2 + i$, then other root is $2 - i$.

Now, $2 + i + 2 - i = \frac{12}{a}$

$\Rightarrow 4 = \frac{12}{a} \Rightarrow a = 3$

17. (A) If a, b, c, d are in HP, then

$b = \frac{2ac}{a+c}$ and $c = \frac{2bd}{b+d}$

Now, $bc = \frac{4abcd}{(a+c)(b+d)}$

$\Rightarrow bc = \dfrac{4abcd}{ab+ad+bc+cd}$

$\Rightarrow ab + ad + bc + cd = 4ad$

$\Rightarrow ab + bc + cd = 3ad$

18. (A) The shaded region is $(A \cap B) \cup (A \cap C)$

19. (A) $n(T \cup C) = 64$, $n(T - C) = 26$, $n(T) = 34$

Now, $n(T) = n(T - C) + n(T \cap C)$

$\Rightarrow 34 = 26 + n(T \cap C) \Rightarrow n(T \cap C) = 8$

Again, we have

$n(T \cup C) = n(T) + n(C) - n(T \cap C)$

$\Rightarrow 64 = 34 + n(C) - 8$

$\Rightarrow 64 = 26 + n(C) \Rightarrow n(C) = 38$

Now, $n(C) = n(C - T) + n(T \cap C)$

$\Rightarrow 38 = n(C - T) + 8 \Rightarrow n(C - T) = 30$

20. (D) Average production in 2001 and 2002

$= \dfrac{40+60}{2} = 50$

Average production in 2002 and 2003

$= \dfrac{60+45}{2} = 52.5$

Similarly, we find the average production in 2000 and 2001, 2003 and 2004 etc. Hence none of the option get an average 50.

Hence option (D) is correct.

21. (A) Required difference = 60000 – 50000 = 10000 tonnes

22. (D) In year 2001, per cent increase in production $= \dfrac{15}{25} \times 100\% = 60\%$

Similarly, we can find the per cent increase in the year 2002, 2003 etc. Hence, maximum increase production is in year 2001.

23. (C) Average production

$= \dfrac{25+40+60+45+65+50+75+80}{8}$

$= \dfrac{440}{8} = 55$

The required number = 4

24. (C) Production of foodgrains in 2002 = 60

Production of foodgrains in 2003 = 45

Required percentage drop $= \dfrac{60-45}{60} \times 100$

$= \dfrac{15}{60} \times 100 = 25\%$

25. (D) $x^2 + px + 1 = 0$

If α and β are the roots of quadratic equation.

Then, $\alpha + \beta = -p$, $\alpha\beta = 1$

Similarly, $\gamma + \delta = -q$, $\gamma\delta = 1$

Now, $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$

$\Rightarrow [\alpha\beta - (\alpha + \beta)\gamma + \gamma^2][\alpha\beta + (\alpha + \beta)\delta + \delta^2]$

$\Rightarrow [\gamma^2 + p\gamma + 1][\delta^2 - q\delta + 1]$ ...(i)

As γ and δ are the roots of the equation $x^2 + qx + 1 = 0$

So, $\gamma^2 + 1 = -q\gamma$ and $\delta^2 + 1 = -q\delta$

from eq(i)

$(-q\gamma + p\gamma)(-q\delta - p\delta)$

$\Rightarrow (q^2 - p^2)\gamma\delta$

$\Rightarrow (q^2 - p^2) \times 1 \Rightarrow q^2 - p^2$

26. (C) $5^x + (5)^{-x} = [5^{x/2} - (5)^{-x/2}]^2 + 2 \geq 2$

If $\sin(e^x) = 5^x + (5)^{-x}$ has solution, we will get $\sin(e^x) \geq 2$

which is not possible as $[\sin\theta] \leq 1$ for all $\theta \in R$.

Hence, no solution exits.

27. (B) log 2, log $(2^x - 1)$ and log $(2^x + 3)$ are in AP,

then $2\log(2^x - 1) = \log 2 + \log(2^x + 3)$

$\Rightarrow \log(2^x - 1)^2 = \log\{2 \times (2^x + 3)\}$

$\Rightarrow (2^x - 1)^2 = 2(2^x + 3)$

$\Rightarrow 2^{2x} - 4 \times 2^x - 5 = 0$

$\Rightarrow (2^x - 5)(2^x + 1) = 0$

Since, 2^x cannot be negative

we get $2x - 5 = 0$

$\Rightarrow 2^x = 5 \Rightarrow x = \log_2 5$

28. (A) $0.1\overline{23} = \dfrac{123-1}{990} = \dfrac{122}{990} = \dfrac{61}{495}$

29. (A) R is symmetric only.

30. (B) $S \subset R$

31. (A) $f(x) = x^2 - 3x + 2$

Now, $f\{f(x)\} = f(x^2 - 3x + 2)$

$\Rightarrow f\{f(x)\} = (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$\Rightarrow f\{f(x)\} = x^4 - 6x^3 + 10x^2 - 3x$

32. (D) $(f + 2g)(x) = f(x) + 2g(x)$

$(f + 2g)(x) = [x] + 2[x]$

$(f + 2g)(-1.5) = [-1.5] + 2[-1.5]$

$(f + 2g)(-1.5) = -2 + 2 \times (-2)$

$(f + 2g)(-1.5) = -2 - 4 = -6$

33. (C) If one regression coefficient be unity, them the other will be less than or equal to unity.

34. (A) $\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}\{3 \times 0.4 - 4 \times (0.4)^3\}]$

$\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}\{1.2 - 0.256\}]$

$\sin[3\sin^{-1}(0.4)] = \sin[\sin^{-1}(0.944)] = 0.944$

35. (D)

36. (B) $\left(\dfrac{1-i}{1+i}\right)$ is purely imaginary, so

$\dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i} = \dfrac{(1-2)^2}{1-2^2} = \dfrac{1+i^2-2i}{2}$

$\Rightarrow \dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i} = -\dfrac{2i}{2} = -i$

So, $(-i)^n \Rightarrow$ Purely imaginary with positive part.

So, n must be equal to 3.

37. (D) Differential equation

$\sin x.\cos y\ dx + \cos x.\sin y\ dy = 0$

$\Rightarrow \frac{\sin x\, dx}{\cos x} + \frac{\sin y}{\cos y} dy = 0$

$\Rightarrow \tan x\ dx + \tan y\ dy = 0$

On integrating

$\Rightarrow \int \tan x\ dx + \int \tan y\ dy = 0$

$\Rightarrow -\log\cos x - \log\cos y = -\log c$

$\Rightarrow -[\log\cos x\cdot\cos y] = -\log c$

$\Rightarrow \cos x\cdot\cos y = c$

Now, it passes through the point $\left(0, \frac{\pi}{3}\right)$.

$\cos 0\cdot\cos\frac{\pi}{3} = c \Rightarrow c = \frac{1}{2}$

Hence, the solution is $\cos x\cdot\cos y = \frac{1}{2}$

38. (B) Number of diagonal in 'n' sided polygon $= {}^nC_2 - n$

$= \frac{n(n-1)}{2} = \frac{n(n-3)}{2}$

39. (D) Equation is $x^2 + x + 1 = 0$

On solving $x = \frac{-1 \pm \sqrt{-3}}{2}$

So, let $\alpha = \frac{-1+\sqrt{-3}}{2} = \omega$

and $\beta = \frac{-1-\sqrt{-3}}{2} = \omega^2$

Now, $\alpha^{19} = (\omega)^{19} = (\omega^3)^6 \times \omega = \omega \quad (\because \omega^3 = 1)$

and $\beta^7 = (\omega^2)^7 = (\omega)^{14} = (\omega^3)^4 \times \omega^2 = \omega^2$

Now, sum of roots $= \alpha^{19} + \beta^7$

sum of roots $= \omega + \omega^2 = -1$

Product of roots $= \alpha^{19}\cdot\beta^7 = \omega\cdot\omega^2 = 1$

Now, quadratic equation

$x^2 - [\alpha^{19} + \beta^7]\,x + \alpha^{19}\beta^7 = 0$

$\Rightarrow x^2 - (-1)x + 1 = 0$

$\Rightarrow x^2 + x + 1 = 0$

40. (A) Equation of line is $\frac{x}{a} + \frac{x}{y} = 1$

$\Rightarrow bx + ay = ab$

$\Rightarrow bx + ay - ab = 0$ ...(i)

$\Rightarrow$ Length of perpendicular drawn from (0, 0) to eq(i)

$p = \left|\frac{b\times 0 + a\times 0 - ab}{\sqrt{b^2+a^2}}\right|$

$p = \left|\frac{-ab}{\sqrt{b^2+a^2}}\right|$

$p^2 = \frac{a^2b^2}{a^2+b^2}$

$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

41. (B) We know that as θ increases, sin θ increases, so $\alpha < \beta$.

42. (A)

43. (D) $\because \tan^2\theta + \cot^2\theta = x$

$\Rightarrow \sec^2\theta - 1 + \operatorname{cosec}^2\theta - 1 = x$

$\Rightarrow \sec^2\theta + \operatorname{cosec}^2\theta = x + 2$

$\Rightarrow \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = x + 2$

$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta} = x + 2$

$\Rightarrow \frac{1}{\sin^2\theta\cos^2\theta} = x + 2$

$\Rightarrow \sec^2\theta.\operatorname{cosec}^2\theta = x + 2$

$\Rightarrow \sec\theta.\operatorname{cosec}\theta = \sqrt{x+2}$

44. (D) $x^2 + 3x - 10$

$\Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x+5) - 2(x+5) = 0$

$\Rightarrow (x+5)(x-2) = 0$

So, $x^2 + 3x - 10$ is positive only when

$\Rightarrow (x+5)(x-2) > 0$

Hence $x < -5$ or $x > 2$

45. (B) $A^2 - B^2 = (\cos x.\cos y)^2 - (\sin x.\sin y)^2$

$A^2 - B^2 = \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$

$A^2 - B^2 = (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$

$A^2 - B^2 = 1 - \sin^2 y - \sin^2 x + \sin^2 x.\sin^2 y - \sin^2 x.\sin^2 y$

$A^2 - B^2 = 1 - (\sin^2 y + \sin^2 x) = 1 - C$

46. (D) $z = \frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(1+i^2-2i)}$

$\Rightarrow z = \frac{1+2i}{1+2i} = 1$

$\Rightarrow z = 1 + i.0$

Now, $|z| = 1$

and $\theta = \tan^{-1}\left(\frac{0}{1}\right) = \tan^{-1}(0)$

$\Rightarrow \theta = 0$

47. (A) $\log_4 7 = x \Rightarrow \log_7 4 = \frac{1}{x}$

$\Rightarrow 2\log_7 4 = \frac{2}{x}$

$\Rightarrow \log_7 (4)^2 = \frac{2}{x}$

$\Rightarrow \log_7 (16) = \frac{2}{x}$

48. (D) $\omega^{99} + \omega^{100} + \omega^{101}$

$\Rightarrow \omega^{99}[1 + \omega + \omega^2] \Rightarrow 0 \quad (\because 1 + \omega + \omega^2 = 0)$

49. (C)

2	1753	1
2	876	0
2	438	0
2	219	1
2	109	1
2	54	0
2	27	1
2	13	1
2	6	0
2	3	1
2	1	1
	0	

$(1753)_{10} = (11011011001)_2$

50. (B) $v = \int e^x \sin x\, dx$ ...(i)

$\Rightarrow \frac{dv}{dx} = e^x \sin x + c$ ...(ii)

and $u = \int e^x \cos x\, dx$

$\Rightarrow u = e^x \sin x - \int e^x \sin x\, dx$

$\Rightarrow u = e^x \sin x - v + c$ from eq(i)

$\Rightarrow u + v = e^x \sin x + c$

$\Rightarrow u + v = \frac{dv}{dx}$ from eq(ii)

51. (A) $8R^2 = a^2 + b^2 + c^2$

$\Rightarrow 8R^2 = 4R^2(\sin^2 A + \sin^2 B + \sin^2 C)$ (by Sine Rule)

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow \cos^2 A - \sin^2 C + \cos^2 B = 0$

$\Rightarrow \cos(A - C) \cdot \cos(A + C) + \cos^2 B = 0$

On solving

$\Rightarrow 2\cos B.\cos A.\cos.C = 0$

$\Rightarrow \cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

A or B or $C = \frac{\pi}{2}$

Hence the triangle is right angled.

52. (A) Two possibilities arise in the given situation

(i) ball transferred is white.

(ii) ball transferred is black.

Case I

P (Selecting white ball from Ist bag) $= \frac{5}{9}$

After transferring the selected white ball to the IInd bag

P (white ball from IInd bag) $= \frac{8}{17}$

Probability of both these events happening together $= \frac{5}{9} \times \frac{8}{17} = \frac{40}{153}$

Case II

P (Selecting black ball from Ist bag) $= \frac{4}{9}$

After transferring the selected black ball to the IInd bag

P (white ball from IInd bag) $= \frac{7}{17}$

Probability of both these events happening together $= \frac{4}{9} \times \frac{7}{17} = \frac{28}{153}$

Required probability $= \frac{40}{153} + \frac{28}{153} = \frac{68}{153}$

53. (A) $I = \int \frac{(2x+1)}{(x+1)(x-2)}\, dx$

$I = \int \frac{1}{3(x+1)}\, dx + \int \frac{5}{3(x-2)}\, dx$

$I = \frac{1}{3}\log(x+1) + \frac{5}{3}\log(x-2) + c$

54. (C) $f(x) = x^2 + 3x^2 - 4$

$f'(x) = 3x^2 + 6x$

For increasing function $f'(x) > 0$

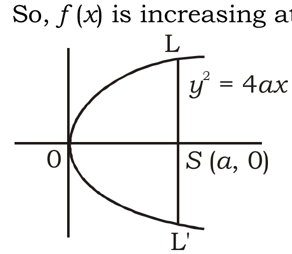
$\Rightarrow 3x^2 + 6x > 0$

$\Rightarrow 3x(x+2) > 0$

$\Rightarrow x < -2$ or $x > 0$

So, $f(x)$ is increasing at $x > 0$ or $x < -2$.

55. (B)



Required area = Area LOL'

Area = 2 × (Area of LOS)

Area $= 2 \times \int_0^a y\, dx$

Area $= 2 \times \int_0^a \sqrt{4ax}\, dx$

Area $= 2 \times 2\sqrt{a} \int_0^a \sqrt{x}\, dx$

Area $= 4\sqrt{a} \left(\frac{x^{3/2}}{3/2}\right)_0^a$

Area $= 4\sqrt{a} \times \frac{2}{3}[a^{3/2} - 0]$

Area $= \frac{8}{3}\sqrt{a} \times (a)^{3/2} = \frac{8}{3}a^2$

56. (B) $\sin^{-1}\cos(\sin^{-1}x) + \cos^{-1}\sin(\cos^{-1}x)$

$\Rightarrow \sin^{-1}\cos\{\cos^{-1}\sqrt{1-x^2}\} + \cos^{-1}\sin\{\sin^{-1}\sqrt{1-x^2}\}$

$\Rightarrow \sin^{-1}\sqrt{1-x^2} + \cos^{-1}\sqrt{1-x^2} = \frac{\pi}{2}$

$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

57. (C) In the parabola $y^2 = 4ax$, the smallest focal chord is $4a$.

58. (B) $|\vec{a}\times\vec{b}| - \sqrt{3}\ |\vec{a}.\vec{b}| = 0$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta - \sqrt{3}|\vec{a}||\vec{b}|\cos\theta = 0$

$\Rightarrow |\vec{a}||\vec{b}|[\sin\theta - \sqrt{3}\cos\theta] = 0$

$\Rightarrow |\vec{a}||\vec{b}| \neq 0$, so $\sin\theta = \sqrt{3}\cos\theta$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

59. (B) $f(x) = \begin{cases} x+2, & \text{when } x \le 1 \\ 4x-1, & \text{when } x > 1 \end{cases}$

L.H.L. $= \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h)$

$= \lim_{h\to 0}(1-h+2)$

$= 3 - h = 3$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h)$

$= \lim_{h\to 0} 4(1+h) - 1$

$= 3$

So, $\lim_{x\to 1} f(x) = 3$

60. (D) $y = f(x) = \left(\frac{1}{x}\right)^{2x}$ ...(i)

On taking log

$\Rightarrow \log y = 2x\log\left(\frac{1}{x}\right)$

$\Rightarrow \log y = -2x\log x$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -2\left[x\times\frac{1}{x} + \log x\times 1\right]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -2[1+\log x]$

$\Rightarrow \frac{dy}{dx} = -2y(1+\log x) = -2\left(\frac{1}{x}\right)^{2x}[1+\log x]$

Again, differentiating

$\frac{d^2y}{dx^2} = -2\left[\frac{dy}{dx}(1+\log x) + y\times\frac{1}{x}\right]$

$\frac{d^2y}{dx^2} = -2\left[-2\left(\frac{1}{x}\right)^{2x}(1+\log x)^2 + \left(\frac{1}{x}\right)^{2x}\times\frac{1}{x}\right]$

for maxima and minima

$\frac{dy}{dx} = 0$

$\Rightarrow -2\left(\frac{1}{x}\right)^{2x}[1+\log x] = 0$

$\Rightarrow 1 + \log x = 0 \Rightarrow x = \frac{1}{e}$

Now, $\frac{d^2y}{dx^2}\left(\text{at } x = \frac{1}{e}\right) = -2e^{2/e}\left[-2\left(1+\log\frac{1}{e}\right)^2 + e\right]$

$= -2e^{2/e}[-2(1-\log e)^2 + e]$

$= -2e \times e^{2/e}$ (maxima)

Maximum value $= e^{2/e}$

61. (A) Word "MOTHER"

The required arrangements $= {}^5C_3 \times 4!$

$= \frac{5!}{2!3!}\times 4! = \frac{5\times 4\times 24}{2} = 240$

62. (A) Equation $x^2 - (1+m^2)x + \frac{1}{2}(1+m^2+m^4) = 0$

$\alpha + \beta = -[-(1+m^2)] = 1 + m^2$ ...(i)

and $\alpha\cdot\beta = \frac{1}{2}(1+m^2+m^4)$ ...(ii)

Now, $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (1+m^2)^2 - 2\times\frac{1}{2}(1+m^2+m^4)$

$\Rightarrow \alpha^2 + \beta^2 = 1 + m^4 + 2m^2 - 1 - m^2 - m^4 = m^2$

63. (A) If the roots of the equation $ax^2 + bx + c = 0$ are equal, then $B^2 - 4AC = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow b^2 = 4ac \Rightarrow c = \frac{b^2}{4a}$

64. (C) HM between two numbers a and b $= \frac{2ab}{a+b}$

So, HM between two numbers 8 and 14

$= \frac{2\times 8\times 14}{8+14} = \frac{2\times 8\times 14}{22} = \frac{112}{11}$

65. (C) $\lim_{x\to 0}\left[\frac{1-\cos x}{x\sin x}\right]$ $\left[\frac{0}{0}\right]$ form

by L-Hospital Rule

$\Rightarrow \lim_{x\to 0}\frac{\sin x}{x\cos x + \sin x}$ $\left[\frac{0}{0}\right]$ form

$\Rightarrow \lim_{x\to 0} \dfrac{\cos x}{-x\sin x + \cos x + \cos x}$

$\Rightarrow \dfrac{1}{0+1+1} = \dfrac{1}{2}$

66. (C) $f(x) = x + \dfrac{1}{x}$

$f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + \dfrac{1}{1/x} = \dfrac{1}{x} + x$

Now, $f(x) - f\left(\dfrac{1}{x}\right) = x + \dfrac{1}{x} - \dfrac{1}{x} - x = 0$

67. (C) Equation $3xy^2 - 2x^2y = 1$

On differentiating w.r.t. 'x'

$\Rightarrow 3\left[x\times 2y\dfrac{dy}{dx} + y^2\right] - 2\left[x^2\dfrac{dy}{dx} + y\times 2x\right] = 0$

$\Rightarrow 6xy\dfrac{dy}{dx} + 3y^2 - 2x^2\dfrac{dy}{dx} - 4xy = 0$

$\Rightarrow \dfrac{dy}{dx}(6xy - 2x^2) + 3y^2 - 4xy = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{4xy - 3y^2}{6xy - 2x^2}$

$m\,[\text{at } (1, 1)] = \dfrac{4\times 1\times 1 - 3\times 1}{6\times 1 - 2\times 1} = \dfrac{4-3}{6-2} = \dfrac{1}{4}$

Slope of the normal $M = -\dfrac{1}{m} = -4$

Equation of normal

$y - y_1 = M(x - x_1)$

$\Rightarrow y - 1 = -4(x - 1)$

$\Rightarrow y - 1 = -4x + 4$

$\Rightarrow 4x + y = 5$

68. (C) If the normal of curve is parallel to x-axis, then $\dfrac{dx}{dy} = 0$

69. (C) $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

The required volume = $\vec{a}\cdot(\vec{b}\times\vec{c})$

$= \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

$= 2(4 - 1) + 3(2 + 3) + 4(-1 - 6)$

$= 6 + 15 - 28 = -7$

Hence the volume of parallelopiped

= 7 cu. units

70. (B) $a = \cos 2\alpha + i\sin 2\alpha$ and $b = \cos 2\beta + i\sin 2\beta$

$ab = (\cos 2\alpha + i\sin 2\alpha)(\cos 2\beta + i\sin 2\beta)$

$ab = \cos(2\alpha + 2\beta) + i\sin(2\alpha + 2\beta)$

$\sqrt{ab} = (ab)^{1/2} = [\cos 2(\alpha+\beta) + i\sin 2(\alpha+\beta)]^{1/2}$

$= \cos\left[\dfrac{1}{2}.2(\alpha+\beta)\right] + i\sin\left[\dfrac{1}{2}.2(\alpha+\beta)\right]$

$= \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$\dfrac{1}{\sqrt{ab}} = (ab)^{-1/2} = [\cos 2(\alpha+\beta) + i\sin 2(\alpha+\beta)]^{-1/2}$

$= \cos\left[-\dfrac{1}{2}\cdot 2(\alpha+\beta)\right] + i\sin\left[-\dfrac{1}{2}\cdot 2(\alpha+\beta)\right]$

$= \cos[-(\alpha+\beta)] + i\sin[-(\alpha+\beta)]$

$= \cos(\alpha+\beta) - i\sin(\alpha+\beta)$

Now, $\sqrt{ab} + \dfrac{1}{\sqrt{ab}} = \cos(\alpha+\beta) + i\sin(\alpha+\beta)$

$+ \cos(\alpha+\beta) - i\sin(\alpha+\beta)$

$\Rightarrow \sqrt{ab} + \dfrac{1}{\sqrt{ab}} = 2\cos(\alpha+\beta)$

71. (B) $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$

$\Rightarrow (1 + \omega^2 - \omega)^5 + (1 + \omega - \omega^2)^5$

$\Rightarrow (-\omega - \omega)^5 + (-\omega^2 - \omega^2)^5$ ($\because 1 + \omega + \omega^2 = 0$)

$\Rightarrow (-2\omega)^5 + (-2\omega^2)^5$

$\Rightarrow -32\omega^5 - 32\omega^{10}$

$\Rightarrow -32\omega^2 - 32\omega$

$\Rightarrow (-32)\times(-1) \Rightarrow 32$

72. (C) $\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} - \dfrac{1}{r}$

$\Rightarrow \dfrac{1}{\left(\dfrac{\Delta}{s-a}\right)} + \dfrac{1}{\left(\dfrac{\Delta}{s-b}\right)} + \dfrac{1}{\left(\dfrac{\Delta}{s-c}\right)} - \dfrac{1}{\left(\dfrac{\Delta}{s}\right)}$

$\Rightarrow \dfrac{s-a}{\Delta} + \dfrac{s-b}{\Delta} + \dfrac{s-c}{\Delta} - \dfrac{s}{\Delta}$

$\Rightarrow \dfrac{1}{\Delta}[s - a + s - b + s - c - s]$

$\Rightarrow \dfrac{1}{\Delta}[2s - (a + b + c)]$

$\Rightarrow \dfrac{1}{\Delta}[2s - 2s] = 0$ $(\because 2s = a + b + c)$

73. (B) $a = 120°$, common difference $(d) = 5°$

Let there be n sides of the polygon.

So, sum of its interior angle = $(2n - 4)\dfrac{\pi}{2}$

$= (n - 2)\times 180°$

But $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

$\therefore (n-2)\ 180^\circ = \frac{n}{2}[2 \times 120^\circ + (n-1)\times 5]$

$\Rightarrow (n-2)\ 180^\circ = \frac{n}{2}[240 + 5n - 5]$

$\Rightarrow 2 \times 180^\circ \times (n-2) = 235n + 5n^2$

$\Rightarrow n^2 - 25n + 144 = 0$

$\Rightarrow n = 16$ or $n = 9$

Hence $n = 9$

74. (B) $\left|\frac{2}{x-4}\right| > 1, x \neq 4$

$\Rightarrow |2| > |x-4|$

$\Rightarrow 4 - 2 < x < 4 + 2$

$\Rightarrow 2 < x < 6$

$\Rightarrow x \in (2, 6)$ but $x \neq 4$

So, the solution of set = $(2, 4) \cup (4, 6)$

75. (C) Standard deviation of the series B

$\Rightarrow \sqrt{\frac{1}{5}\left[(1.9)^2+(0.8)^2+(1.5)^2+(0.6)^2+(0.2)^2\right] - \left(\frac{1.9+0.8+1.5+0.6+0.2}{5}\right)^2}$

$\Rightarrow \sqrt{\frac{6.9}{5}-1} = \sqrt{0.38}$

76. (D) The set of all prime numbers

77. (C) The required Probability $= \frac{4}{52} \times \frac{4}{51}$

$= \frac{4}{13\times 51} = \frac{4}{663}$

78. (C) Let the natural number be x.
Then, sum of 11 consecutive natural numbers

$\Rightarrow x + (x+1) + (x+2) + (x+3) + (x+4) + (x+5) + (x+6) + (x+7) + (x+8) + (x+9) + (x+10) = 2761$

$\Rightarrow 11x + 55 = 2761$

$\Rightarrow 11x = 2761 - 55$

$\Rightarrow 11x = 2706 \Rightarrow x = \frac{2706}{11} = 246$

Middle term = $(x + 5) = 246 + 5 = 251$

79. (A) SD = $\sqrt{\text{Variance}} = \sqrt{V}$

80. (C) Equation of x-axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

81. (D) $I = \int \frac{\sec^2\left(2\tan^{-1}x\right)}{1+x^2}\,dx$

Let $2\tan^{-1}x = t$ and $2 \times \frac{1}{1+x^2}\,dx = dt$

$I = \int \sec^2 t \times \frac{dt}{2} = \frac{1}{2}\cdot\int \sec^2 t\,dt$

$I = \frac{1}{2}\tan t + c = \frac{1}{2}\tan(2\tan^{-1}x) + c$

82. (A) $y = \log[x^x + \text{cosec}^2 x]$
On differentiating w.r.t. 'x'

$\frac{dy}{dx} = \frac{1}{x^x + \text{cosec}^2 x}\cdot\frac{d}{dx}(x^x + \text{cosec}^2 x)$

$\frac{dy}{dx} = \frac{1}{x^2 + \text{cosec}^2 x}\left\{\frac{d}{dx}(x)^x + \frac{d}{dx}\text{cosec}^2 x\right\}$

$\frac{dy}{dx} = \frac{1}{x^2+\text{cosec}^2 x}\{x^x(1+\log x) + 2\text{cosec}x(\text{cosec}x.\cot x)\}$

$\frac{dy}{dx} = \frac{1}{x^2+\text{cosec}^2 x}\{x^x(1+\log x) - 2\text{cosec}^2x.\cot x\}$

83. (D) a, b and c are in AP

$\Rightarrow \frac{a}{bc}, \frac{b}{bc}, \frac{c}{bc}$ are in AP

$\Rightarrow \frac{a}{bc}, \frac{1}{c}$ and $\frac{1}{b}$ are in AP

Hence $\frac{a}{bc}, \frac{1}{c}$ and $\frac{2}{b}$ are neither in AP nor GP nor HP.

So, option (D) is correct.

84. (B) $\log_2(x-1) = 2\log_2(x-3)$

$\Rightarrow \log_2(x-1) = \log_2(x-3)^2$

$\Rightarrow (x-1) = (x-3)^2$

$\Rightarrow x - 1 = x^2 + 9 - 6x$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow x = 2, 5$

Since, $x = 2$ does not satisfy the equation therefore $x = 5$ is the only equation.

85. (C) Clearly, repetition of digit is allowed in three-digit number more than 600, only two digit 6 and 7 can be filled on hundred place. Unit place and tenth place can be filled by remaining of 5 digit.
Total number of ways = $2 \times 5 \times 5 = 50$

86. (B) Since, a letter can be post in any of five postbox, a required number of ways
$= 5 \times 5 \times 5 \times 5 = 5^4$

87. (A) There are 8 letters in the word TRIANGLE.
Number of words can be formed with the letters = ${}^8P_8 = 8!$
Now, if we begin with T and ending with E remaining 6 letters can be arranged in 6! ways.

88. (C) There are 5 men and 5 women

Now, ${}^5C_k \times {}^5C_{5-k} = 100$

$$\Rightarrow \frac{5!}{k!(5-k)!} \times \frac{5!}{(5-k)!k!} = 100$$

$$\Rightarrow \left[\frac{5!}{k!(5-k)!}\right]^2 = 100 \Rightarrow \frac{5!}{k!(5-k)!} = 10$$

$\Rightarrow {}^5C_k = 10$

So, $k = 2$ or 3

89. (C) $|A_{n\times n}| = 3$ and $|\text{adj } A| = 243$

Now, $|\text{adj } A| = |A|^{n-1}$

$\Rightarrow 243 = (3)^{n-1}$

$\Rightarrow (3)^5 = (3)^{n-1} \Rightarrow n = 6$

90. (A) Given that, $s = \sqrt{t}$

$$V = \frac{ds}{dt} = \frac{1}{2\sqrt{t}}$$

$$\text{Acceleration } (a) = \frac{dV}{dt} = \frac{1}{2}\cdot\left(-\frac{1}{2}\right)(t)^{-3/2}$$

$$\Rightarrow a = -\frac{1}{4(t)^{3/2}} \Rightarrow a = -\frac{1}{4\left(\sqrt{t}\right)^3}$$

$\Rightarrow a = -2V^3$

$\Rightarrow a \propto V^3$

91. (C) When a die is thrown, we get

$S = \{1, 2, 3, 4, 5, 6\}$

$$\therefore p(\text{getting an odd number}) = \frac{3}{6} = \frac{1}{2}$$

$$\therefore p(\text{success}) = \frac{1}{2} \text{ and } q(\text{not success}) = \frac{1}{2}$$

Let X denote the number of success in 4 throws of a die. The probability of r success in 4 throws of a die is given by

$P(X = r) = {}^nC_r\,(p)^r \times (q)^{n-r}$

$$= {}^4C_r \times \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{4-r} = {}^4C_r\left(\frac{1}{2}\right)^4$$

where $r = 0, 1, 2, 3, 4$

P (atmost 2 success) $= P(X \le 2)$

$= 1 - P(X > 2)$

$$= 1 - \left\{{}^4C_3\left(\frac{1}{2}\right)^4 + {}^4C_4\left(\frac{1}{2}\right)^4\right\}$$

$$= 1 - \left(\frac{1}{4} + \frac{1}{16}\right) = 1 - \frac{5}{16} = \frac{11}{16}$$

92. (B) Let the equation of sphere be

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ ...(i)

it passes through (0, 0, 0), (1, 0, 0), (0, 2, 0), (0, 0, 3), then

$d = 0$ ...(ii)

$1 + 2u + d = 0$ ...(iii)

$4 + 4v + d = 0$ ...(iv)

$9 + 6w + d = 0$ ...(v)

From eq(ii) and eq(iii), $u = -\frac{1}{2}$

From eq(ii) and eq(iv), $v = -1$

From eq(ii) and eq(v) $w = -\frac{3}{2}$

Put the value of u, v, w, d in eq(i)

$$x^2 + y^2 + z^2 + 2\left(-\frac{1}{2}\right)x + 2(-1)y + 2\left(-\frac{3}{2}\right)z + 0 = 0$$

$\Rightarrow x^2 + y^2 + z^2 - x - 2y - 3z = 0$

93. (A) $I = \int \frac{1}{\sin(x-a)\cos(x-b)}\,dx$

$$I = \frac{1}{\cos(a-b)}\int \frac{\cos(a-b)}{\sin(x-a).\cos(x-b)}\,dx$$

$$I = \frac{1}{\cos(a-b)}\int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a).\cos(x-b)}\,dx$$

$$I = \frac{1}{\cos(a-b)}\int \frac{\cos(x-a)\cos(x-b) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\,dx$$

$$I = \frac{1}{\cos(a-b)}\int [\cot(x-a)\,dx + \tan(x-b)]dx$$

$$I = \frac{1}{\cos(a-b)}[\log\sin(x-a) - \log\cos(x-b)] + c$$

$$I = \frac{1}{\cos(a-b)}\log\left|\frac{\sin(x-a)}{\cos(x-b)}\right| + c$$

94. (A) $7^\circ\,30' = \left(7 + \frac{1}{2}\right)^\circ = \left(\frac{15}{2}\right)^\circ$

$$\Rightarrow 7^\circ\,30' = \left(\frac{15}{2} \times \frac{\pi}{180}\right)^C = \left(\frac{\pi}{24}\right)^C$$

95. (B) Perimeter of $\Delta ABC = 27$ cm

$$\Rightarrow 2s = 27 \Rightarrow s = \frac{27}{2}$$

Area of $\Delta ABC\ (\Delta) = 81\ \text{cm}^2$

$$\text{Now, } r = \frac{\Delta}{s} = \frac{81}{27/2} = \frac{81 \times 2}{27} = 6 \text{ cm}$$

96. (C) $\tan^{-1}(\tan 690^\circ) = \tan^{-1}[2 \times 360^\circ - 30^\circ]$

$= \tan^{-1}[\tan(-30^\circ)]$

$= \tan^{-1}[-\tan 30^\circ] = -30^\circ$

97. (C) There is no positive integer satisfying $x + 2 = 0$, therefore solution set is ϕ.

98. (C) $\{(x, y) : x > 0 \text{ and } y < 0\}$

99. (C) $I_A \subseteq R$

100. (C) Odd numbers between 100 and 200 is 101, 103, 105, ...199

Here, $a = 101$, $d = 103 - 101 = 2$, $l = 199$

Now, $l = a + (n - 1) d$

$\Rightarrow 199 = 101 + (n - 1)\times 2$

$\Rightarrow 98 = 2n - 2 \Rightarrow n = 50$

$\therefore S_n = \frac{n}{2}[a + l] = \frac{50}{2}[101 + 199] = 7500$

101. (B) Since a, b and c are in GP, then

$b^2 = ac$ ...(i)

Now, $\frac{1}{a^2 - b^2} + \frac{1}{b^2}$

$\Rightarrow \frac{1}{a^2 - ac} + \frac{1}{ac}$ [from eq(i)]

$\Rightarrow \frac{1}{a(a-c)} + \frac{1}{ac} = \frac{c + a - c}{ac(a-c)}$

$\Rightarrow \frac{a}{ac(a-c)} \Rightarrow \frac{1}{ac - c^2}$

$\Rightarrow \frac{1}{b^2 - c^2}$ [from eq(i)]

102. (B) The conic section having asymptotes is hyperbola.

103. (D) Let the radius of two sphere be r_1 and r_2 and the radius of common circle be r.

Now, $r = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}} = \frac{3\times 4}{\sqrt{(3)^2 + (4)^2}}$

$\Rightarrow r = \frac{3\times 4}{5} = \frac{12}{5}$

104. (C) Given that, $p = a + b$, $q = a\omega + b\omega^2$ and $r = a\omega^2 + b\omega$

Now, $pqr = (a + b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$

$\Rightarrow pqr = (a + b)(a^2\omega^3 + ab\omega^2 + ab\omega^4 + b^2\omega^3)$

$\Rightarrow pqr = (a + b)(a^2 + ab(\omega^2 + \omega) + b^2)$ $(\because \omega^3 = 1)$

$\Rightarrow pqr = (a + b)(a^2 - ab + b^2)$ $(\because \omega + \omega^3 = 1)$

$\Rightarrow pqr = a^3 + b^3$

105. (C) Let $y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$

$\Rightarrow (y - 1)x^2 + 3(y + 1)x + 4(y - 1) = 0$

For x is real, $D \geq 0$

$\Rightarrow 9(y + 1)^2 - 16(y - 1)^2 \geq 0$

$\Rightarrow -7y^2 - 50y - 7 \geq 0$

$\Rightarrow 7y^2 + 50y + 7 \leq 0$

$\Rightarrow (y - 7)(7y - 1) \leq 0$ ...(i)

$\Rightarrow y \leq 7$ and $y \geq \frac{1}{7} \Rightarrow \frac{1}{7} \leq y \leq 7$

Hence maximum value is 7 and minimum value is $\frac{1}{7}$.

106. (A) Let the two quantities be a and b and

$A = \frac{a+b}{2}$

let $A_1, A_2, ..., A_n$ be the n AM's between them. Then $a, A_1, A_2, ..., A_n, b$ are in AP and let d be the common difference.

Now, $T_{n+2} = a + (n + 2 - 1) d$

$\Rightarrow b = a + (n + 1)d \Rightarrow d = \frac{b-a}{n+1}$

Also, $A_1 + A_2 + ... + A_n$

$\Rightarrow (a + d) + (a + 2d) + ... + (a + nd)$

$\Rightarrow na + d \times (1 + 2 + 3 + ... + n)$

$\Rightarrow na + \frac{b-a}{n+1} \times \frac{n(n+1)}{2}$

$\Rightarrow n\left[a + \frac{b-a}{2}\right] = n\left(\frac{a+b}{2}\right) = nA$

107. (B) We have that, if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$ are the two matrices, then the product matrix AB is of order $m \times p$.

108. (C) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = ka^2b^2c^2$

$\Rightarrow \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = ka^2b^2c^2$

$\Rightarrow abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = ka^2b^2c^2$

$\Rightarrow (abc)(abc) \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = ka^2b^2c^2$

$\Rightarrow [(-1)(1 - 1) -1(-1 - 1) + 1(1 + 1)] = k$

$\Rightarrow 4 = k \Rightarrow k = 4$

(109-110) :

The given system of equations is

$kx + y + z = k - 1$

$x + ky + z = k - 1$

$x + y + kz = k - 1$

$\therefore A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}, B = \begin{bmatrix} k-1 \\ k-1 \\ k-1 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now, $|A| = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$

$\Rightarrow |A| = k(k^2 - 1) - 1(k - 1) + 1(1 - k)$

$\Rightarrow |A| = k^3 - k - k + 1 + 1 - k$

$\Rightarrow |A| = k^3 - 3k + 2$

The given system of equations has no solution, if $|A| = 0$

$k^3 - 3k + 2 = 0$

$\Rightarrow (k - 1)^2 (k + 2) = 0$

$\Rightarrow k = 1$ or $k = -2$

The given system of equations has a solution, if $|A| \neq 0$

$\Rightarrow k \neq 1$ or $k \neq -2$

109. (A) $k = 1$ or $k = -2$

110. (A) $k \neq 1$ or $k \neq -2$

111. (C) $P\left(\frac{B}{(A \cup B^C)}\right) \Rightarrow \frac{P(B \cap (A \cup B^C))}{P(A \cup B^C)}$

$\Rightarrow \frac{P(A \cap B)}{P(A) + P(B^c) - P(A \cap B^c)}$

$\Rightarrow \frac{P(A) - P(A \cap B^c)}{P(A) + P(B^c) - P(A \cap B^c)}$

$\Rightarrow \frac{0.7 - 0.5}{0.7 + 0.6 - 0.5} = \frac{0.2}{0.8} = \frac{1}{4}$

(112-114)

Here, random experiment is throwing the given die.

Let A = The event of getting a face with number 1

B = The event of getting a face with number 2

and C = The event getting a face with number 3

Now, $n(S) = 6$, $n(A) = 1$, $n(B) = 2$, $n(C) = 3$

$\therefore P(A) = \frac{1}{6}, P(B) = \frac{2}{6} = \frac{1}{3}$

and $P(C) = \frac{3}{6} = \frac{1}{2}$

We have $P(1) = P(A) = \frac{1}{6}$

$P(2 \text{ or } 3) = P(B \cup C) = P(B) + P(C)$

$P(2 \text{ or } 3) = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

($\because$ B and C are mutually exclusive)

$P(\text{not } 3) = P(C') = 1 - P(C) = 1 - \frac{1}{2} = \frac{1}{2}$

112. (B) $P(1) = \frac{1}{6}$

113. (C) $P(2 \text{ or } 3) = \frac{5}{6}$

114. (B) $P(\text{not } 3) = \frac{1}{2}$

115. (B) $\sqrt{2}\sec\theta - \tan\theta = 1$

$\Rightarrow \frac{\sqrt{2}}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow \sqrt{2} - \sin\theta = \cos\theta$

$\Rightarrow \sqrt{2} = \sin\theta + \cos\theta$

$\Rightarrow 1 = \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta$

$\Rightarrow \cos 0 = \sin\frac{\pi}{4}.\sin\theta + \cos\frac{\pi}{4}.\cos\theta$

$\Rightarrow \cos 0 = \cos\left(\theta - \frac{\pi}{4}\right)$

$\Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm 0$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}$

116. (D) Given, $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$ ...(i)

and $\cos^{-1}x - \cos^{-1}y = 0$

$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1}x\right) - \left(\frac{\pi}{2} - \sin^{-1}y\right) = 0$

$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

$\Rightarrow \sin^{-1}y - \sin^{-1}x = 0$

$\Rightarrow \sin^{-1}y = \sin^{-1}x$ ...(ii)

From eq(i) and eq(ii), we get

$2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{4} \Rightarrow x = \sin\frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}}$

From eq(ii), we get

$y = \frac{1}{\sqrt{2}}$

117. (C) $f(x) = 2x + 7$ and $g(x) = x^2 + 7$

Given that, $fog(x) = 25$

$\Rightarrow f[g(x)] = 25$

$\Rightarrow f[x^2 + 7] = 25$

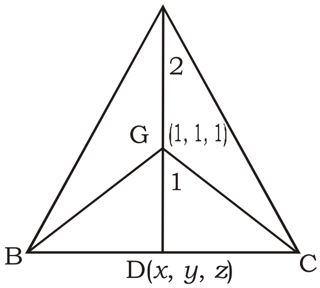
$\Rightarrow 2(x^2 + 7) + 7 = 25$

$\Rightarrow 2x^2 + 21 = 25$

$\Rightarrow 2x^2 = 4$

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

118. (B) Let the co-ordinates of D are (x, y, z)



For x-co-ordinate,

$$1 = \frac{2 \times x + 1 \times 4}{1+2} \Rightarrow x = -\frac{1}{2}$$

For y-co-ordinate,

$$1 = \frac{2 \times y + 1 \times 7}{1+2} \Rightarrow y = -2$$

and for z-co-ordinate,

$$1 = \frac{2 \times z + 1 \times (-8)}{1+2} \Rightarrow z = \frac{11}{2}$$

$\therefore$ Co-ordinates of D are $\left(-\frac{1}{2}, -2, \frac{11}{2}\right)$.

119. (A) Two dice are thrown.

$n(S) = 6 \times 6 = 36$

E = [(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)]
[$\because$ sum is 8]

$n(E) = 5$

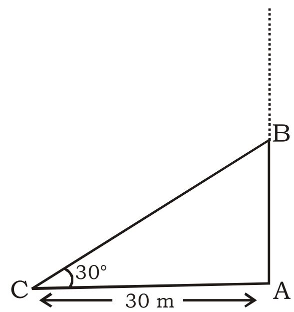
$$\text{Probability} = \frac{n(E)}{n(S)} = \frac{5}{36}$$

120. (B) Broken part of the tree = BC

In ΔABC:-

$$\cos 30 = \frac{AC}{BC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{30}{BC}$$



$$BC = \frac{30 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$BC = 20\sqrt{3} \text{ m}$$

NDA (MATHS) MOCK TEST - 134 (Answer Key)

1. (C)	21. (A)	41. (B)	61. (A)	81. (D)	101. (B)
2. (A)	22. (D)	42. (A)	62. (A)	82. (A)	102. (B)
3. (B)	23. (C)	43. (D)	63. (A)	83. (D)	103. (D)
4. (B)	24. (C)	44. (D)	64. (C)	84. (B)	104. (C)
5. (D)	25. (D)	45. (B)	65. (C)	85. (C)	105. (C)
6. (C)	26. (C)	46. (D)	66. (C)	86. (B)	106. (A)
7. (C)	27. (B)	47. (A)	67. (C)	87. (A)	107. (B)
8. (B)	28. (A)	48. (D)	68. (C)	88. (C)	108. (C)
9. (C)	29. (A)	49. (C)	69. (C)	89. (C)	109. (A)
10. (A)	30. (B)	50. (B)	70. (B)	90. (A)	110. (A)
11. (C)	31. (A)	51. (A)	71. (B)	91. (C)	111. (C)
12. (B)	32. (D)	52. (A)	72. (C)	92. (B)	112. (B)
13. (C)	33. (C)	53. (A)	73. (B)	93. (A)	113. (C)
14. (A)	34. (A)	54. (C)	74. (B)	94. (A)	114. (B)
15. (C)	35. (D)	55. (B)	75. (C)	95. (B)	115. (B)
16. (B)	36. (D)	56. (B)	76. (D)	96. (C)	116. (D)
17. (A)	37. (D)	57. (C)	77. (C)	97. (C)	117. (C)
18. (A)	38. (B)	58. (B)	78. (C)	98. (C)	118. (B)
19. (A)	39. (D)	59. (B)	79. (A)	99. (C)	119. (A)
20. (D)	40. (A)	60. (D)	80. (C)	100. (C)	120. (B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777