

**NDA MATHS MOCK TEST - 140 (SOLUTION)**

1. (C) Differential equation

$$x dy - y dx = (x^3 + xy^2) dx$$

$$\Rightarrow x dy - y dx = x(x^2 + y^2) dx$$

$$\Rightarrow \frac{x dy - y dx}{x^2 + y^2} = x dx$$

$$\Rightarrow \frac{d}{dx} \left( \tan^{-1} \frac{x}{y} \right) = x dx$$

$$\Rightarrow \int \frac{d}{dx} \left( \tan^{-1} \frac{x}{y} \right) = \int x dx$$

$$\Rightarrow \tan^{-1} \frac{x}{y} = \frac{x^2}{2} + \frac{c}{2}$$

$$\Rightarrow 2 \tan^{-1} \frac{x}{y} = x^2 + c$$

2. (B) In  $\Delta ABC$ ,

$$\sin A + \sin B + \sin C$$

$$\Rightarrow 2 \sin \frac{A+B}{2} \cdot \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cdot \cos \frac{C}{2}$$

$$\Rightarrow 2 \sin \frac{180-C}{2} \cdot \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cdot \cos \frac{C}{2}$$

$$\Rightarrow 2 \cos \frac{C}{2} \cdot \cos \frac{A-B}{2} + 2 \sin \frac{180-(A+B)}{2} \cdot \cos \frac{C}{2}$$

$$\Rightarrow 2 \cos \frac{C}{2} \left[ \cos \frac{A-B}{2} + \sin \left[ 90 - \frac{A+B}{2} \right] \right]$$

$$\Rightarrow 2 \cos \frac{C}{2} \left[ \cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right]$$

$$\Rightarrow 2 \cos \frac{C}{2} \times 2 \cos \frac{\frac{A-B}{2} + \frac{A+B}{2}}{2} \cdot \cos \frac{\frac{A-B}{2} + \frac{A+B}{2}}{2}$$

$$\Rightarrow 4 \cos \frac{C}{2} \times \cos \frac{A}{2} \times \cos \left( \frac{-B}{2} \right)$$

$$\Rightarrow 4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$$

3. (C)  $y = x^3 + e^x$

On differentiating both side w.r.t 'x'

$$\frac{dy}{dx} = 3x^2 + e^x$$

$$\frac{dx}{dy} = \frac{1}{3x^2 + e^x}$$

On differentiating both side w.r.t. 'y'

$$\frac{d^2x}{dy^2} = -1 \cdot (3x^2 + e^x)^{-2} (6x + e^x) \frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = -\frac{6x + e^x}{(3x^2 + e^x)^2} \times \frac{1}{3x^2 + e^x}$$

$$\frac{d^2x}{dy^2} = -\frac{6x + e^x}{(3x^2 + e^x)^3}$$

4. (C)  $n(S) = 6 \times 6 \times 6 = 216$

$$E = \left\{ (6, 6, 2), (6, 5, 3), (6, 4, 4), (6, 3, 5), (6, 2, 6), (5, 6, 3), (5, 5, 4), (5, 4, 5), (5, 3, 6), (4, 6, 4), (4, 5, 5), (4, 4, 6), (3, 6, 5), (3, 5, 6), (2, 6, 6) \right\}$$

$$n(E) = 15$$

$$\text{The required Probability } P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{15}{216} = \frac{5}{72}$$

5. (B)  $n = 10$

$$\text{Number of diagonals} = \frac{n(n-3)}{2}$$

$$= \frac{10 \times 7}{2} = 35$$

6. (B)  $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A}$

$$\Rightarrow \left( \frac{\sin 3A}{\sin A} \right)^2 - \left( \frac{\cos 3A}{\cos A} \right)^2$$

$$\Rightarrow \left( \frac{3 \sin A - 4 \sin^3 A}{\sin A} \right)^2 - \left( \frac{4 \cos^3 A - 3 \cos A}{\cos A} \right)^2$$

$$\Rightarrow (3 - 4 \sin^2 A)^2 - (4 \cos^2 A - 3)^2$$

$$\Rightarrow 9 + 16 \sin^4 A - 24 \sin^2 A - 16 \cos^4 A - 9 + 24 \cos^2 A$$

$$\Rightarrow 16(\sin^4 A - \cos^4 A) - 24(\sin^2 A - \cos^2 A)$$

$$\Rightarrow (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A) - 24(\sin^2 A - \cos^2 A)$$

$$\Rightarrow (\sin^2 A - \cos^2 A)[16(\sin^2 A + \cos^2 A) - 24]$$

$$\Rightarrow -(\cos^2 A - \sin^2 A)[16 - 24]$$

$$\Rightarrow 8 \cos 2A$$

7. (D)  $I = \int_0^{\pi/2} \frac{\tan x - \cot x}{1 - \tan x \cdot \cot x} dx \quad \dots(i)$

$$I = \int_0^{\pi/2} \frac{\tan\left(\frac{\pi}{2} - x\right) - \cot\left(\frac{\pi}{2} - x\right)}{1 - \tan\left(\frac{\pi}{2} - x\right) \cdot \cot\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\cot x - \tan x}{1 - \tan x \cdot \cot x} dx \quad \dots(ii)$$

from eq(i) and eq(ii)  
 $2I = 0 \Rightarrow I = 0$

8. (B)  $I = \int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

Let  $\sin^{-1} x = t$  when  $x \rightarrow 0, t \rightarrow 0$

$$\frac{1}{\sqrt{1-x^2}} dx = dt \quad x \rightarrow 1, t \rightarrow \frac{\pi}{2}$$

$$I = \int_0^{\pi/2} t dx$$

$$I = \left[ \frac{t^2}{2} \right]_0^{\pi/2}$$

$$I = \frac{1}{2} \times \frac{\pi^2}{4} = \frac{\pi^2}{8}$$

9. (D)  $\lim_{x \rightarrow 3} \frac{4^{x/2} - 8}{2^{2x} - 64} \quad \left[ \frac{0}{0} \right]$  from

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 3} \frac{4^{x/2}(\log 4) \times \left(\frac{1}{2}\right) - 0}{2^{2x}(\log 2) \times (2) - 0}$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{2^x \times \frac{1}{2} \times 2 \log 2}{2^{2x} \times 2 \log 2}$$

$$\Rightarrow \frac{1}{2} \times \frac{2^3}{2^6} = \frac{1}{16}$$

10. (C) Straight line

$$\frac{x-1}{3} = \frac{y+2}{4} = \frac{z-1}{-2} \quad \text{and} \quad \frac{x+1}{-2} = \frac{y-4}{4} = \frac{z+5}{5}$$

Angle between the straight lines

$$\cos \theta = \frac{3 \times (-2) + 4 \times 4 + (-2) \times 5}{\sqrt{3^2 + 4^2 + (-2)^2} \sqrt{(-2)^2 + 4^2 + 5^2}}$$

$$\Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = 90^\circ = \frac{\pi}{2}$$

11. (C) Determinant  $\begin{vmatrix} 2 & 5 & 1 \\ 6 & 4 & 3 \\ 2 & -1 & 0 \end{vmatrix}$

$$\text{Cofactor of } 3 = (-1)^{2+3} \begin{vmatrix} 2 & 5 \\ 2 & -1 \end{vmatrix} \\ = -1(-2-10) = 12$$

12. (B)  $\begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} -2 & -4 \\ 3 & -\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$

$$\begin{bmatrix} -4+3 & -8-\lambda \\ -8+3 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -8-\lambda \\ -5 & -16-\lambda \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$$

On comparing

$$-8 - \lambda = -2 \Rightarrow \lambda = -6$$

13. (D)  $I = \int \frac{dx}{x(1+\log x)^3}$

$$\text{Let } 1 + \log x = t \Rightarrow \frac{1}{x} dx = dt$$

$$I = \int \frac{dt}{t^3}$$

$$I = \frac{t^{-3+1}}{-3+1} + c$$

$$I = \frac{-1}{2} \times \frac{1}{t^2} + c$$

$$I = \frac{-1}{2(1+\log x)^2} + c$$

14. (B)

II	I
(sinθ, cosecθ) → '+' other → '-'	All positive
(tanθ, cotθ) → '+' other → '-'	(cosθ, secθ) → '+' other → '-'
III	IV

15. (C)  $\lim_{x \rightarrow \infty} \left( \frac{x+8}{x+2} \right)^{x+4}$

$$\Rightarrow \lim_{x \rightarrow \infty} \left( 1 + \frac{6}{x+2} \right)^{\frac{x+2}{6} \times \frac{6}{x+2} \times (x+4)}$$

$$\Rightarrow e \lim_{x \rightarrow \infty} \frac{6 \left( 1 + \frac{4}{x} \right)}{\left( 1 + \frac{2}{x} \right)} \Rightarrow e^6$$

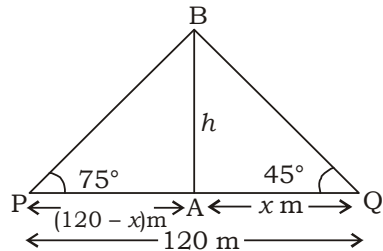
16. (C)  $\lim_{n \rightarrow \infty} [5^n + 7^n]^{1/n}$

$$\Rightarrow \lim_{n \rightarrow \infty} 7 \left[ \left( \frac{5}{7} \right)^n + 1 \right]^{1/n}$$

$$\Rightarrow \lim_{n \rightarrow \infty} 7 \left[ 1 + \frac{1}{\left( \frac{7}{5} \right)^n} \right]^{1/n}$$

$$\Rightarrow 7 \left[ 1 + \frac{1}{\infty} \right]^0 = 7$$

17. (C)



Let height of the aeroplane =  $h$  m

$AQ = x$  m

**In  $\triangle ABQ$ :-**

$$\tan 45^\circ = \frac{AB}{AQ}$$

$$\Rightarrow 1 = \frac{h}{x} \Rightarrow x = h$$

**In  $\triangle ABP$  :-**

$$\tan 75^\circ = \frac{AB}{AP}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{120-x}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{120-h}$$

$$\Rightarrow 120(\sqrt{3}+1) = 2\sqrt{3}h$$

$$\Rightarrow h = \frac{60(\sqrt{3}+1)}{\sqrt{3}}$$

$$\Rightarrow h = 20(3+\sqrt{3}) \text{ m}$$

18. (D) In  $\triangle ABC$ , if  $\angle B = 15^\circ$ ,  $\angle C = 75^\circ$ , then

$\angle A = 90^\circ$

Sine Rule

$$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 90} = \frac{b}{\sin 15} = \frac{c}{\sin 75}$$

$$\Rightarrow a = \frac{b}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{c}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$b = \frac{\sqrt{3}-1}{2\sqrt{2}} a \text{ and } c = \frac{\sqrt{3}+1}{2\sqrt{2}} a$$

$$\text{then } c\sqrt{3} - b = \frac{\sqrt{3}+1}{2\sqrt{2}} a \times \sqrt{3} - \frac{\sqrt{3}-1}{2\sqrt{2}} a$$

$$\Rightarrow c\sqrt{3} - b = \frac{3 + \sqrt{3} - \sqrt{3} + 1}{2\sqrt{2}} a$$

$$\Rightarrow c\sqrt{3} - b = \frac{4}{2\sqrt{2}} a$$

$$\Rightarrow c\sqrt{3} - b = \sqrt{2} a$$

19. (A) In  $\triangle ABC$ ,  $AB(c) = 5$  cm,  $BC(a) = 12$  cm,  $CA(b) = 13$  cm

$$s = \frac{5+12+13}{2} = 15$$

$$\text{Now, } \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\Rightarrow \tan \frac{A}{2} = \sqrt{\frac{(15-13) \times (15-5)}{15 \times (15-12)}}$$

$$\Rightarrow \tan \frac{A}{2} = \sqrt{\frac{2 \times 10}{15 \times 3}}$$

$$\Rightarrow \tan \frac{A}{2} = \frac{2}{3}$$

$$\text{then } \tan \left( 2 \times \frac{A}{4} \right) = \frac{2 \tan \frac{A}{4}}{1 - \tan^2 \frac{A}{4}}$$

$$\Rightarrow \frac{2}{3} = \frac{2 \tan \frac{A}{4}}{1 - \tan^2 \frac{A}{4}}$$

$$\Rightarrow 1 - \tan^2 \frac{A}{4} = 3 \tan \frac{A}{4}$$

$$\Rightarrow \tan^2 \frac{A}{4} = 3 \tan \frac{A}{4} - 1 = 0$$

$$\Rightarrow \tan \frac{A}{4} = \frac{-3 \pm \sqrt{(3)^2 - 4 \times (1) \times (-1)}}{2 \times 1}$$

$$\Rightarrow \tan \frac{A}{4} = \frac{-3 \pm \sqrt{13}}{2}$$

$$\text{Hence } \tan \frac{A}{4} = \frac{\sqrt{13} - 3}{2}$$

20. (D)  $\tan 7\frac{1}{2} = \frac{\sin 2\frac{1}{2}}{\cos 7\frac{1}{2}} \times \frac{2\sin 7\frac{1}{2}}{2\sin 7\frac{1}{2}}$

$$\Rightarrow \tan 7\frac{1}{2} = \frac{1 - \cos 15}{\sin 15}$$

$$\Rightarrow \tan 7\frac{1}{2} = \frac{1 - \sqrt{3} + 1}{2\sqrt{2}}$$

$$\Rightarrow \tan 7\frac{1}{2} = \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow \tan 7\frac{1}{2} = \frac{2\sqrt{6} - 3 - \sqrt{3} + 2\sqrt{2} - \sqrt{3} - 1}{3 - 1}$$

$$\Rightarrow \tan 7\frac{1}{2} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$$

21. (B)  $y = x\sqrt{1+x^2}$   
On differentiating both sides w.r.t. 'x'

$$\frac{dy}{dx} = x \times \frac{1}{2}(1+x^2)^{-1/2}(2x) + \sqrt{1+x^2} \times 1$$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{1+x^2}} + \sqrt{1+x^2}$$

$$\frac{dy}{dx} = \frac{1+2x^2}{\sqrt{1+x^2}}$$

$$\frac{dx}{dy} = \frac{\sqrt{1+x^2}}{1+2x^2}$$

22. (A)  $I = \int \frac{x^3}{1+x^8} dx$

$$I = \int \frac{x^3}{1+(x^4)^2} dx$$

Let  $x^4 = t$

$$\Rightarrow 4x^3 dx = dt \Rightarrow x^3 dx = \frac{1}{4} dt$$

$$I = \int \frac{1}{4} \times \frac{1}{1+t^2} dt$$

$$I = \frac{1}{4} \tan^{-1} t + c$$

$$I = \frac{1}{4} \tan^{-1} x^4 + c$$

23. (D)

Let co-ordinates of the foot of perpendicular is  $Q(x_1, y_1)$

Slope of line AB =  $-\frac{3}{5}$

Slope of line PQ =  $-\frac{1}{-3/5}$

$$\Rightarrow \frac{y_1 - 2}{x_1 + 3} = \frac{5}{3}$$

$$\Rightarrow 5x_1 - 3y_1 + 21 = 0 \quad \dots(i)$$

Point  $Q(x_1, y_1)$  lies on the line AB  $\dots(ii)$

$$3x_1 + 5y_1 - 13 = 0$$

from eq(i) and eq(ii)

$$x_1 = \frac{-33}{17} \text{ and } y_1 = \frac{64}{17}$$

Hence foot of perpendicular =  $\left(\frac{-33}{17}, \frac{64}{17}\right)$

24. (B) The straight lines  $ax - by = c, bx - cy = a$  and  $cx - ay = b$  are collinear,

$$\text{then } \begin{vmatrix} a & -b & -c \\ b & -c & -a \\ c & -a & -b \end{vmatrix} = 0$$

$$\Rightarrow a(bc - a^2) + b(-b^2 + ca) - c(-ab + c^2) = 0$$

$$\Rightarrow abc - a^3 - b^3 + abc + abc - c^3 = 0$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

$$\Rightarrow a+b+c = 0 \text{ or } a^2 + b^2 + c^2 - ab - bc - ca = 0$$

25. (A)  $\frac{d^2y}{dx^2} = \left[ \left(1 - x \frac{dy}{dx}\right)^4 \right]^{-1/3}$

$$\left(\frac{d^2y}{dx^2}\right)^3 = \left(1 - x \frac{dy}{dx}\right)^4$$

Hence degree = 3

26. (C)

Fourth vertex =  $(x, y)$

Diagonals of a parallelogram are perpendicular bisector to each other,

$$\text{then } \frac{x-1}{2} = \frac{4+0}{2} \Rightarrow x = 5$$

$$\text{and } \frac{y+2}{2} = \frac{3+7}{2} \Rightarrow y = 8$$

Fourth vertex =  $(5, 8)$

27. (C) Differential equation

$$\frac{dy}{dx} = \tan^2(x + y)$$

$$\text{Let } x + y = t$$

$$1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\Rightarrow \frac{dt}{dx} - 1 = \tan^2 t$$

$$\Rightarrow \frac{dt}{dx} = 1 + \tan^2 t$$

$$\Rightarrow \frac{dt}{dx} = \sec^2 t$$

$$\Rightarrow 2\cos^2 t dt = 2 dx$$

$$\Rightarrow (1 + \cos 2t) dt = 2 dx$$

On integrating

$$\Rightarrow t + \frac{\sin 2t}{2} = 2x + \frac{c}{2}$$

$$\Rightarrow 2t + \sin 2t = 4x + c$$

$$\Rightarrow 2(x + y) + \sin 2(x + y) = 4x + c$$

$$\Rightarrow \sin 2(x + y) = 2(x - y) + c$$

28. (A) A = {1, 2, 3, 4} and B = {1, 2, 5}

$$(A \times B) = \{(1, 1), (1, 2), (1, 5), (2, 1), (2, 2),$$

$$(2, 5), (3, 1), (3, 2), (3, 5), (4, 1), (4, 2), (4, 5)\}$$

$$(B \times A) = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2),$$

$$(2, 3), (2, 4), (5, 1), (5, 2), (5, 3), (5, 4)\}$$

$$\text{Now, } (A \times B) \cap (B \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

29. (C) S = 0.3 + 0.33 + 0.333 + ..... n terms

$$S = \frac{1}{3} (0.9 + 0.99 + 0.999 + \dots n \text{ terms})$$

$$S = \frac{1}{3} \left[ \left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \dots n \text{ terms} \right]$$

$$S = \frac{1}{3} (1 + 1 + \dots n \text{ terms}) -$$

$$\frac{1}{3} \left( \frac{1}{10} + \frac{1}{100} + \dots n \text{ terms} \right)$$

$$S = \frac{1}{3} \times n - \frac{1}{3} \times \frac{1}{10} \left( 1 - \frac{1}{10^n} \right) \frac{1}{1 - \frac{1}{10}}$$

$$S = \frac{1}{3} \left[ n - \frac{1}{9} \left( 1 - \frac{1}{10^n} \right) \right]$$

30. (C)  $\lim_{x \rightarrow \infty} \left[ \frac{x^2 - 5x - 2}{x^2 + x + 7} \right]^x$

We know that

$$\lim_{x \rightarrow \infty} [f(x)]^{g(x)} = e^{\lim_{x \rightarrow \infty} g(x)[f(x)-1]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} x \left[ \frac{x^2 - 5x - 2}{x^2 + x + 7} - 1 \right]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} x \left[ \frac{x^2 - 5x - 2 - x^2 - x - 7}{x^2 + x + 7} \right]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} x \left[ \frac{-6x - 9}{x^2 + x + 7} \right]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} \frac{x^2 \left[ \frac{-6}{x} - \frac{9}{x^2} \right]}{x^2 \left[ 1 + \frac{1}{x} + \frac{7}{x^2} \right]}}$$

$$\Rightarrow e^{\frac{-6+0}{1+0+0}} = e^{-6}$$

31. (D) We know that

$$\frac{-\pi}{2} \leq \sin^{-1} y \leq \frac{\pi}{2}$$

$$\text{Now, } \frac{-\pi}{2} \leq \sin^{-1}(\log_2 3x) \leq \frac{\pi}{2}$$

$$\Rightarrow -1 \leq \log_2 3x \leq 1$$

$$\Rightarrow 2^{-1} \leq 3x \leq 2^1$$

$$\Rightarrow \frac{2^{-1}}{3} \leq x \leq \frac{2}{3}$$

$$\Rightarrow \frac{1}{6} \leq x \leq \frac{2}{3}$$

$$\text{Hence domain} = \left[ \frac{1}{6}, \frac{2}{3} \right]$$

32. (B) Given that  $\tan \theta = \frac{x}{y}$

$$\text{Now, } \frac{x \sin \theta - y \cos \theta}{x \sin \theta + y \cos \theta}$$

$$\Rightarrow \frac{x \tan \theta - y}{x \tan \theta + y}$$

$$\Rightarrow \frac{x \times \frac{x}{y} - y}{x \times \frac{x}{y} + y} = \frac{x^2 - y^2}{x^2 + y^2}$$

33. (B) Let  $y = 6^{57}$

On taking log both sides

$$\Rightarrow \log_{10} y = 57 \log_{10} 6$$

$$\Rightarrow \log_{10} y = 57 \times 0.7782$$

$$\Rightarrow \log_{10} y = 44.3574$$

$$\text{Hence no. of digits} = 44 + 1 = 45$$

34. (A)  $f(x) = \tan x - \tan^2 x + \tan^3 x \dots \dots \infty$

$$f(x) = \frac{\tan x}{1 - (-\tan x)} = \frac{\tan x}{1 + \tan x}$$

Now,  $I = \int_0^{\pi/2} \cot x \cdot f(x) dx$

$$\Rightarrow I = \int_0^{\pi/2} \cot x \cdot \frac{\tan x}{1 + \tan x} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{1}{1 + \frac{\sin x}{\cos x}} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x} dx \quad \dots(i)$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx \quad \dots(ii)$$

from eq(i) and eq(ii)

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

35. (D)  $I = \int \frac{\log(x - \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$

Let  $\log(x - \sqrt{1+x^2}) = t$

$$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \left[1 - \frac{1 \times (2x)}{2\sqrt{1+x^2}}\right] dx = dt$$

$$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} - x}{2\sqrt{1+x^2}} dx = dt$$

$$\Rightarrow \frac{-1}{2\sqrt{1+x^2}} dx = dt \Rightarrow \frac{1}{\sqrt{1+x^2}} dx = -2 dt$$

$$I = \int -2 t dt$$

$$I = -2 \times \frac{t^2}{2} + c$$

$$I = -\left[\log(x - \sqrt{1+x^2})\right]^2 + c$$

36. (C) Let  $y = \cos^2 \tan^{-1} \sqrt{\frac{1-x}{1+x}}$

Let  $x = \cos 2\theta$  ... (i)

$$\Rightarrow y = \cos^2 \tan^{-1} \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}}$$

$$\Rightarrow y = \cos^2 \tan^{-1} \sqrt{\frac{2 \sin^2 \theta}{2 \cos^2 \theta}}$$

$$\Rightarrow y = \cos^2 \tan^{-1}(\tan \theta)$$

$$\Rightarrow y = \cos^2 \theta$$

$$\Rightarrow y = \frac{1 + \cos 2\theta}{2}$$

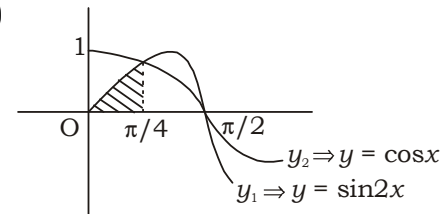
$$\Rightarrow y = \frac{1+x}{2} \quad \text{[form eq(i)]}$$

On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

Hence  $\frac{dy}{dx} \left[ \cos^2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right] = \frac{1}{2}$

37. (C)



The required Area =  $\int_0^{\pi/4} y_1 dx$

$$= \int_0^{\pi/4} \sin 2x dx$$

$$= \left[ \frac{-\cos 2x}{2} \right]_0^{\pi/4}$$

$$= \frac{-1}{2} [\cos 2x]_0^{\pi/4}$$

$$= \frac{-1}{2} \left[ \cos \frac{\pi}{2} - \cos 0 \right]$$

$$= \frac{-1}{2} [0 - 1] = \frac{1}{2} \text{ sq. unit}$$

38. (D) Lines  $5x + 12y + 5 = 0$   
and  $15x + 36y + 13 = 0$

$$\Rightarrow 5x + 12y + \frac{13}{3} = 0$$

The required distance (D) =  $\frac{5 - \frac{13}{3}}{\sqrt{(5)^2 + (12)^2}}$

$$= \frac{2}{3 \times 13}$$

$$= \frac{2}{39}$$

39. (A) The equation of line

$$y + 6 = \frac{3}{11}(x + 1)$$

$$\Rightarrow 11y + 66 = 3x + 3$$

$$\Rightarrow 3x - 11y = 63$$

40. (C) Equation of circle

$$x^2 + y^2 - x + y - 14 = 0$$

$$(x - 2)(x + 1) + (y + 4)(y - 3) = 0$$

End points of the diameter are (2, -4) and (-1, 3).

Equation of the diameter

$$y + 4 = \frac{3 + 4}{-1 - 2}(x - 2)$$

$$\Rightarrow 7x + 3y = 2$$

41. (D) Hyperbola  $\frac{x^2}{9} - \frac{y^2}{\lambda^2} = 1$

$$e^2 = 1 + \frac{\lambda^2}{9}$$

$$e = \frac{\sqrt{9 + \lambda^2}}{3}$$

$$\text{foci} = (\pm\sqrt{9 + \lambda^2}, 0)$$

$$\text{ellipse } \frac{x^2}{49} + \frac{y^2}{36} = 1$$

$$e^2 = 1 - \frac{36}{49}$$

$$e = \frac{\sqrt{13}}{7}$$

$$\text{foci} = (\pm\sqrt{13}, 0)$$

foci are coincide.

$$\text{then } \sqrt{9 + \lambda^2} = \sqrt{13} \Rightarrow \lambda^2 = 4$$

42. (B)  $I = \int \frac{1}{\sin\left(x + \frac{\pi}{4}\right) \cdot \sin\left(x + \frac{\pi}{2}\right)} dx$

$$I = \frac{1}{\sin\frac{\pi}{4}} \int \frac{\sin\left[\left(x + \frac{\pi}{2}\right) - \left(x + \frac{\pi}{4}\right)\right]}{\sin\left(x + \frac{\pi}{4}\right) \cdot \sin\left(x + \frac{\pi}{2}\right)} dx$$

$$I = \sqrt{2} \int \frac{\sin\left(x + \frac{\pi}{2}\right) \cdot \cos\left(x + \frac{\pi}{4}\right) - \cos\left(x + \frac{\pi}{2}\right) \cdot \sin\left(x + \frac{\pi}{4}\right)}{\sin\left(x + \frac{\pi}{4}\right) \cdot \sin\left(x + \frac{\pi}{2}\right)} dx$$

$$I = \sqrt{2} \int \left[ \cot\left(x + \frac{\pi}{4}\right) - \cot\left(x + \frac{\pi}{2}\right) \right] dx$$

$$I = \sqrt{2} \left[ \log \sin\left(x + \frac{\pi}{4}\right) - \log \sin\left(x + \frac{\pi}{2}\right) \right] + c$$

$$I = \sqrt{2} \log \left[ \frac{\sin\left(x + \frac{\pi}{4}\right)}{\sin\left(x + \frac{\pi}{2}\right)} \right] + c$$

43. (C)

$$\begin{array}{r|rr} 2 & 31 & 1 \\ \hline 2 & 15 & 1 \\ \hline 2 & 7 & 1 \\ \hline 2 & 3 & 1 \\ \hline 2 & 1 & 1 \\ \hline & 0 & \end{array}$$

$$0.75$$

$$\times 2$$

$$\underline{1.50}$$

$$\times 2$$

$$\underline{1.00}$$

$$(0.25)_{10} = (0.11)_2$$

$$(37)_{10} = (100101)_2$$

$$\text{Hence } (31.75)_{10} = (11111.11)_2$$

44. (B) Let  $y = f(x) = \frac{6^x - 6^{-x}}{6^x + 6^{-x}}$

by Componendo & Dividendo Rule

$$\Rightarrow \frac{y+1}{y-1} = \frac{6^x - 6^{-x} + 6^x + 6^{-x}}{6^x - 6^{-x} - 6^x - 6^{-x}}$$

$$\Rightarrow \frac{y+1}{1-y} = \frac{2 \times 6^x}{-2 \times 6^{-x}}$$

$$\Rightarrow \frac{y+1}{y-1} = 6^{2x}$$

$$\Rightarrow 2x = \log_6 \frac{y+1}{1-y}$$

$$\Rightarrow x = \frac{1}{2} \log_6 \frac{y+1}{1-y}$$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} \log_6 \left( \frac{1+y}{1-y} \right)$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log_6 \left( \frac{1+x}{1-x} \right)$$

45. (A)  $\frac{\log_{64} 4 + \log_{27} 3}{\log_2 32 + \log_5 125}$

$$\Rightarrow \frac{\log_{4^3} 4 + \log_{3^3} 3}{\log_2 2^5 + \log_5 5^3}$$

$$\Rightarrow \frac{\frac{1}{3} \log_4 4 + \frac{1}{3} \log_3 3}{5 \log_2 2 + 3 \log_5 5}$$

$$\Rightarrow \frac{\frac{1}{3} + \frac{1}{3}}{5 + 3}$$

$$\Rightarrow \frac{2}{3 \times 8} = \frac{1}{12}$$

46. (B) Word "PARENTS"

No. of permutations = 7!

47. (C) 
$$\begin{vmatrix} \omega & 1 & 4\omega^4 \\ 3\omega^2 & 3 & 12\omega^5 \\ 2\omega^3 & 2 & 8\omega^6 \end{vmatrix}$$

$$\Rightarrow 4 \times 3 \times 2 \begin{vmatrix} \omega & 1 & \omega^4 \\ \omega^2 & 1 & \omega^5 \\ \omega^3 & 1 & \omega^6 \end{vmatrix}$$

$$\Rightarrow 24 \begin{vmatrix} \omega & 1 & \omega \\ \omega^2 & 1 & \omega^2 \\ \omega^3 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow 24 \times 0 = 0 \quad [\because \text{Two columns are identical.}]$$

48. (B) 
$$\sqrt{19+8\sqrt{3}} = \sqrt{(4+\sqrt{3})^2}$$
  

$$\sqrt{19+8\sqrt{3}} = 4 + \sqrt{3}$$

49. (C) The required Probability =  $\frac{1}{7}$

50. (D) 
$$\lim_{x \rightarrow 0} \frac{5x^3 + 2x^2 + 15}{3x + 2x^2 + 4x^3}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x^3 \left( 5 + \frac{2}{x} + \frac{15}{x^3} \right)}{x^3 \left( \frac{3}{x^2} + \frac{2}{x} + 4 \right)}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{5 + \frac{2}{x} + \frac{15}{x^3}}{\frac{3}{x^2} + \frac{2}{x} + 4}$$

$$\Rightarrow \frac{5+0+0}{0+0+4} = \frac{5}{4}$$

51. (C) 
$$\int_0^{2\pi} |\sin x| dx = 2 \int_0^{\pi} \sin x dx$$

$$= 2[-\cos x]_0^{\pi}$$

$$= 2[-\cos \pi + \cos 0]$$

$$= 2[+1+1] = 4$$

52. (A) Let  $y = x \cdot \ln \tan x$   
 On differential both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = x \times \frac{1}{\tan x} \times \sec^2 x + \ln \tan x \times 1$$

$$\Rightarrow \frac{dy}{dx} = x \cdot \sec x \cdot \tan x + \ln \tan x$$

53. (C)  $\vec{a} = -6\hat{i} + m\hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 4\hat{j} + 3\hat{k}$

$$\text{Now, } \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\Rightarrow \cos \frac{\pi}{2} = \frac{(-6\hat{i} + m\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 3\hat{k})}{\sqrt{(-6)^2 + m^2 + 2^2} \sqrt{3^2 + 4^2 + 3^2}}$$

$$\Rightarrow 0 = \frac{-18 + 4m + 6}{\sqrt{36 + m^2 + 4} \sqrt{9 + 16 + 9}}$$

$$\Rightarrow 4m - 12 = 0 \Rightarrow m = 3$$

54. (C) We know that  
 $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n$   
 On putting  $x = 1$

$$\Rightarrow (1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

$$\Rightarrow 2^n = 1 + \sum_{r=1}^n C(n,r)$$

$$\Rightarrow \sum_{r=1}^n C(n,r) = 2^n - 1$$

55. (C) Deteminant 
$$\begin{vmatrix} 1 & 2 & 0 & 4 \\ -1 & 3 & -2 & 4 \\ 0 & -5 & -4 & 2 \\ 6 & -1 & 0 & -3 \end{vmatrix}$$

$$\text{Cofactor of } 3 = (-1)^{2+2} \begin{vmatrix} 1 & 0 & 4 \\ 0 & -4 & 2 \\ 6 & 0 & -3 \end{vmatrix}$$

$$= 1(12 - 0) - 0 + 4(0 + 24)$$

$$= 12 + 96 = 108$$

56. (D) Hyperbola  $3x^2 - 4y^2 = 1$

$$\Rightarrow \frac{x^2}{1/3} - \frac{y^2}{1/4} = 1$$

$$a^2 = \frac{1}{3}, b^2 = \frac{1}{4}$$

$$\text{Now, } e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$\Rightarrow e = \sqrt{1 + \frac{1/4}{1/3}} \Rightarrow e = \frac{\sqrt{7}}{2}$$

$$\text{foci} = (\pm ae, 0)$$

$$= \left( \pm \frac{1}{\sqrt{3}} \times \frac{\sqrt{7}}{2}, 0 \right) = \left( \pm \frac{1}{2} \sqrt{\frac{7}{3}}, 0 \right)$$

57. (A)  $f(x) = 3x^3 + 5x^2 - 6x + 7$

$$f'(x) = 9x^2 + 10x - 6$$

$$f''(x) = 18x + 10$$

$$\text{Now, } 3f'(0) - 4f''(-1)$$

$$\Rightarrow 3[9 \times 0 + 10 \times 0 - 6] - 4[18 \times (-1) + 10]$$

$$\Rightarrow -18 - 4 \times (-8) = 14$$



58. (B)  $I = \int \sin^2 \theta d\theta + \int \cot^2 \theta \cdot \sin^2 \theta d\theta$

$$I = \int \left( \sin^2 \theta + \frac{\cos^2 \theta}{\sin^2 \theta} \cdot \sin^2 \theta \right) d\theta$$

$$I = \int (\sin^2 \theta + \cos^2 \theta) d\theta$$

$$I = \int 1 d\theta$$

$$I = \theta + c$$

59. (A) We know that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{The arithmetic mean} = \frac{n^2(n+1)^2}{4 \times n}$$

$$= \frac{n(n+1)^2}{4}$$

60. (D)  $y = \sin(\ln x)$  ... (i)

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{dy}{dx} = \cos(\ln x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = \cos(\ln x)$$

Again, differentiating

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} \times 1 = -\sin(\ln x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y \quad [\text{from eq(i)}]$$

61. (C) In the expansion of  $\left(2x - \frac{1}{3x^2}\right)^6$

$$T_{r+1} = {}^6C_r (2x)^{6-r} \left(\frac{-1}{3x^2}\right)^r$$

$$T_{r+1} = {}^6C_r 2^{6-r} \left(\frac{-1}{3}\right)^r x^{6-3r}$$

$$\text{Here, } 6 - 3r = -3 \Rightarrow r = 3$$

$$\text{The required coefficient} = {}^6C_3 2^3 \left(\frac{-1}{3}\right)^3$$

$$= 20 \times 8 \times \left(\frac{-1}{27}\right)$$

$$= \frac{-160}{27}$$

$$62. (A) f(x) = \begin{cases} 3ax - 4b, & x < 2 \\ 5, & x = 2 \text{ is continuous} \\ 2a + bx, & x > 2 \end{cases}$$

at  $x = 2$ , then

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$\text{Now, } \lim_{x \rightarrow 2^-} f(x) = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^-} 3ax - 4b = 5$$

$$\Rightarrow 3a \times 2 - 4b = 5 \Rightarrow 6a - 4b = 5 \quad \dots(i)$$

$$\text{and } \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$$\Rightarrow \lim_{x \rightarrow 2^+} 2a + bx = 5$$

$$\Rightarrow 2a + b \times 2 = 5 \Rightarrow 2a + 2b = 5 \quad \dots(ii)$$

On solving eq(i) and eq(ii)

$$a = \frac{3}{2}, b = 1$$

63. (B)  $\cot^{-1} \frac{15}{8} + \sin^{-1} \frac{3}{5}$

$$\Rightarrow \tan^{-1} \frac{8}{15} + \tan^{-1} \frac{3}{4}$$

$$\Rightarrow \tan^{-1} \left( \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \times \frac{3}{4}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{77}{36} \right) = \sin^{-1} \left( \frac{77}{85} \right)$$

64. (D) Sphere  $x^2 + y^2 + z^2 - 6x - 4y + 2z - 2 = 0$   
 $u = -3, v = -2, w = 1, d = -2$

$$\text{Now, } r = \sqrt{u^2 + v^2 + w^2 - d}$$

$$\Rightarrow r = \sqrt{(-3)^2 + (-2)^2 + 1^2 - (-2)}$$

$$\Rightarrow r = \sqrt{9 + 4 + 1 + 2} = 4 \text{ unit}$$

Hence diameter =  $2r = 2 \times 4 = 8$  unit

65. (B)  $\Delta_1 = \begin{vmatrix} x & a & a \\ b & x & a \\ b & b & x \end{vmatrix}$

$$\Delta_1 = x(x^2 - ab) - a(bx - ab) + a(b^2 - bx)$$

$$\Delta_1 = x^3 - abx - abx + a^2b + ab^2 - abx$$

$$\frac{d}{dx} (\Delta_1) = 3x^2 - ab - ab + 0 + 0 - ab$$

$$\frac{d}{dx} (\Delta_1) = 3x^2 - 3ab = 3(x^2 - ab) \quad \dots(i)$$

$$\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab \quad \dots(ii)$$

from eq(i) and eq(ii)

$$\frac{d}{dx} (\Delta_1) = 3\Delta_2$$

66. (C) Equation

$$5x^2 + 7x + 2 = 0$$

$$\alpha + \beta = \frac{-7}{5} \text{ and } \alpha\beta = \frac{2}{5}$$

$$\text{Now, } \frac{\alpha^2 + \beta^2}{\alpha + \beta} = \frac{(\alpha^2 + \beta)^2 - 2\alpha\beta}{\alpha + \beta}$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha + \beta} = \frac{\left(\frac{-7}{5}\right)^2 - 2 \times \frac{2}{5}}{\frac{-7}{5}}$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha + \beta} = \frac{\frac{49}{25} - \frac{4}{5}}{\frac{-7}{5}}$$

$$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha + \beta} = \frac{\frac{29}{25}}{\frac{-7}{5}} = \frac{-29}{35}$$

67. (D) Let angles of a triangle =  $3x, 2x, x$

$$3x + 2x + x = 180$$

$$\Rightarrow 6x = 180 \Rightarrow x = 30$$

Angles are  $90, 60, 30$ .

Now,  $\sin 90 + \sin 60 + \sin 30$

$$\Rightarrow 1 + \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$\Rightarrow \frac{3 + \sqrt{3}}{2} = \frac{\sqrt{3}(\sqrt{3} + 1)}{2}$$

68. (B) Given that  $A = \begin{bmatrix} 4 & 3 & 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 \\ 2 \\ 3 \\ -1 \end{bmatrix}$  and  $C = \begin{bmatrix} 6 \\ -2 \\ 3 \\ -4 \end{bmatrix}$

$$\text{Now, } A(B - C) = \begin{bmatrix} 4 & 3 & 2 & 1 \end{bmatrix} \left[ \begin{bmatrix} -1 \\ 2 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \\ 3 \\ -4 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -7 \\ 4 \\ 0 \\ 3 \end{bmatrix}$$

$$= [4 \times (-7) + 3 \times 4 + 2 \times 0 + 1 \times 3]$$

$$= [-28 + 12 + 0 + 3] = [-13]$$

69. (C)  $I = \int e^{\frac{x^2-1}{x}} dx + \int \frac{e^{\frac{x^2-1}{x}}}{x^2} dx$

$$I = \int e^{\frac{x^2-1}{x}} \left( 1 + \frac{1}{x^2} \right) dx$$

$$\text{Let } \frac{x^2-1}{x} = t$$

$$\Rightarrow x - \frac{1}{x} = t \Rightarrow \left( 1 + \frac{1}{x^2} \right) dx = dt$$

$$I = \int e^t dt$$

$$I = e^t + c$$

$$I = e^{\frac{x^2-1}{x}} + c$$

70. (A)  $\frac{1}{2\cos 80^\circ} - 2\cos 20^\circ$

$$\Rightarrow \frac{1 - 4\cos 20^\circ \cdot \cos 80^\circ}{2\cos 80^\circ}$$

$$\Rightarrow \frac{1 - 2 \times 2\cos 20^\circ \cdot \cos 80^\circ}{2\cos 80^\circ}$$

$$\Rightarrow \frac{1 - 2[\cos(20 + 80) + \cos(80 - 20)]}{2\cos 80^\circ}$$

$$\Rightarrow \frac{1 - 2[\cos 100 + \cos 60]}{2\cos 80^\circ}$$

$$\Rightarrow \frac{1 - 2\left[\cos(90 + 10) + \frac{1}{2}\right]}{2\cos(90 - 10)}$$

$$\Rightarrow \frac{1 - 2\left[-\sin 10 + \frac{1}{2}\right]}{2\cos 10}$$

$$\Rightarrow \frac{1 + 2\sin 10 - 1}{2\sin 10}$$

$$\Rightarrow \frac{2\sin 10}{2\sin 10} = 1$$

71. (B)  $\frac{\sin 2x + \sin 4x}{\sin 2x - \sin 4x}$

$$\Rightarrow \frac{2\sin \frac{2x+4x}{2} \cdot \cos \frac{2x-4x}{2}}{2\cos \frac{2x+4x}{2} \cdot \sin \frac{2x-4x}{2}}$$

$$\Rightarrow \frac{\sin 3x \cdot \cos x}{\cos 3x \cdot (-\sin x)} = \frac{-\cos x}{\cot 3x}$$

72. (C)  $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 - 16}$   $\left[ \frac{0}{0} \right]$  Form

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 4} \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \lim_{x \rightarrow 4} \frac{1}{4(x)^{3/2}}$$

$$\Rightarrow \frac{1}{4(4)^{3/2}} = \frac{1}{32}$$

73. (C)  $f(x) = \begin{cases} \frac{x - \sin x}{x^2}, & x \neq 0 \\ \lambda, & x = 0 \end{cases}$  is continuous

at  $x = 0$ , then

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{x - \sin x}{x^2} = \lambda$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1 - \cos x}{2x} = \lambda$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin x}{2} = \lambda$$

$$\Rightarrow \frac{\sin 0}{2} = \lambda \Rightarrow \lambda = 0$$

74. (C)  $\lim_{x \rightarrow 0} \frac{a^{\sin x} - a^{\tan x}}{\tan x - \sin x}$   $\left[ \frac{0}{0} \right]$  Form

$$\Rightarrow \lim_{x \rightarrow 0} \frac{a^{\tan x} (a^{\sin x - \tan x} - 1)}{-(\sin x - \tan x)}$$

$$\Rightarrow \lim_{x \rightarrow 0} -a^{\tan x} \times \log a \left[ \because \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \right]$$

$$\Rightarrow -\log a$$

75. (D)  $f(x) = \sin x - \sin^2 x + \sin^3 x - \dots \infty$

$$f(x) = \frac{\sin x}{1 + \sin x}$$

$$\int f(x) dx = \int \frac{\sin x}{1 + \sin x} dx$$

$$= \int \frac{1 + \sin x - 1}{1 + \sin x} dx$$

$$= \int \left( 1 - \frac{1}{1 + \sin x} \right) dx$$

$$= \int 1 \cdot dx - \int \frac{1}{1 + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$= x - \int \frac{1}{2 \cos^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right)} dx$$

$$= x - \frac{1}{2} \int \sec^2\left(\frac{\pi}{4} - \frac{\pi}{2}\right) dx$$

$$= x - \frac{1}{2} \times \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{2}\right)}{\frac{-1}{2}} + c$$

$$= x + \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) + c$$

76. (C) Differential equation

$$\sin x \frac{dy}{dx} + y \cdot \cos x = \operatorname{cosec} x$$

$$\Rightarrow \frac{dy}{dx} + y \cdot \cot x = \operatorname{cosec}^2 x$$

On comparing with the general equation  
P = cot x and Q = cosec<sup>2</sup> x

$$\text{I.F.} = e^{\int P \cdot dx}$$

$$\text{I.F.} = e^{\int \cot x}$$

$$\text{I.F.} = e^{\log \sin x} = \sin x$$

Solution of the differential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx$$

$$y \times \sin x = \int \operatorname{cosec}^2 x \times \sin x$$

$$y \sin x = \int \operatorname{cosec} x dx$$

$$y \sin x = \log \tan\left(\frac{x}{2}\right) + c$$

77. (B)  $x dy - y dx = x^2 y dy$

$$\Rightarrow \frac{x dy - y dx}{x^2} = y dy$$

$$\Rightarrow d\left(\frac{y}{x}\right) = y dy$$

On integrating

$$\Rightarrow \frac{y}{x} = \frac{y^2}{2} + \frac{c}{2}$$

$$\Rightarrow 2y = xy^2 + cx$$

78. (A) 
$$\begin{array}{r} 1111001 \\ + 1101 \\ \hline 10000110 \end{array} \quad \begin{array}{r} 10000110 \\ - 110001 \\ \hline 1010101 \end{array}$$

$(1111001)_2 + (1101)_2 - (100001)_2 = (1010101)_2$

79. (B) Table is round, so one seat is fixed.  
The number of ways =  $(7-1)!$   
 $= 6! = 720$

80. (C) The number of ways =  $6 \times 5 = 30$

81. (D)  $I = \int_0^{\pi/2} \frac{f(x)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx \quad \dots(i)$

Prop IV  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

$I = \int_0^{\pi/2} \frac{f\left(\frac{\pi}{2} - x\right)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx \quad \dots(ii)$

From eq(i) and eq(ii)

$2I = \int_0^{\pi/2} \frac{f(x) + f\left(\frac{\pi}{2} - x\right)}{f(x) + f\left(\frac{\pi}{2} - x\right)} dx$

$2I = \int_0^{\pi/2} 1 dx$

$2I = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$

82. (A) Equation  $5x - 3y = 7$   
and  $3x + 5y = 8$

$\frac{5}{3} \neq \frac{-3}{5} \neq \frac{7}{8}$

Equations have a unique solutions.

83. (B) Let  $a + ib = \sqrt{4 + 6\sqrt{5}i}$

On squaring both side

$(a^2 - b^2) + 2abi = 4 + 6\sqrt{5}i$

On comparing

$a^2 - b^2 = 4$  and  $2ab = 6\sqrt{5} \quad \dots(i)$

Now,  $(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$\Rightarrow (a^2 + b^2)^2 = 16 + 180$

$\Rightarrow a^2 + b^2 = 14 \quad \dots(ii)$

from eq(i) and eq(ii)

$2a^2 = 18$  and  $2b^2 = 10$

$a = \pm 3$  and  $b = \pm \sqrt{5}$

Square root of  $(4 + 6\sqrt{5}i)$  is  $(3 + \sqrt{5}i)$ .

84. (C)  $z = \frac{1}{\sin\theta + i(1 + \cos\theta)}$

$z = \frac{1}{2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2} + i \times 2\cos^2\frac{\theta}{2}}$

$z = \frac{1}{2\sin\frac{\theta}{2}} \left[ \frac{1}{\sin\frac{\theta}{2} + i \times \cos\frac{\theta}{2}} \right]$

$z = \frac{1}{2} \sec\frac{\theta}{2} \left[ \frac{\sin\frac{\theta}{2} - i \cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + i^2 \cos^2\frac{\theta}{2}} \right]$

$z = \frac{1}{2} \sec\frac{\theta}{2} \left[ \sin\frac{\theta}{2} - i \cos\frac{\theta}{2} \right]$

$z = \frac{1}{2} \left[ \tan\frac{\theta}{2} - i \right]$

Imaginary part of  $z = \frac{-1}{2}$

85. (A) The required Probability =  $\frac{1}{2} \left[ \frac{2}{5} \times \frac{2}{6} + \frac{3}{5} \times \frac{4}{6} \right]$   
 $= \frac{1}{2} \times \frac{16}{30} = \frac{4}{15}$

86. (B) The required Probability

$= \frac{1}{2} \left[ \frac{3}{5} \times \frac{2}{6} + \frac{2}{5} \times \frac{4}{6} + \frac{3}{5} \times \frac{4}{6} \right]$

$= \frac{1}{2} \left[ \frac{6}{30} + \frac{8}{30} + \frac{12}{30} \right]$

$= \frac{1}{2} \times \frac{26}{30} = \frac{13}{30}$

87. (A)  $y = a^{\frac{1}{1-\log_a z}} \Rightarrow a = y^{1-\log_a z}$

$\Rightarrow \log_a a = (1-\log_a z) \log_a y$

$\Rightarrow \log_a y = \frac{1}{1-\log_a z}$

and  $x = a^{\frac{1}{1-\log_a y}} \Rightarrow a = x^{1-\log_a y}$

$\Rightarrow \log_a a = (1-\log_a y) \log_a x$

$\Rightarrow \log_a x = \frac{1}{1-\log_a y}$

$\Rightarrow \log_a x = \frac{1}{1 - \frac{1}{\log_a z}}$

$\Rightarrow \log_a x = \frac{1 - \log_a z}{-\log_a z}$

$$\text{Now, } \frac{1}{1 - \log_a x} = \frac{1}{1 + \frac{1 - \log_a z}{\log_a z}}$$

$$\Rightarrow \frac{1}{1 - \log_a x} = \frac{\log_a z}{1}$$

$$\Rightarrow \log_a z = \frac{1}{1 - \log_a x} \Rightarrow z = a^{\frac{1}{1 - \log_a x}}$$

88. (C) Let  $y = 3^{92}$   
 taking log both side  
 $\Rightarrow \log y = 92 \log_{10} 3$   
 $\Rightarrow \log_{10} y = 92 \times 0.4771$   
 $\Rightarrow \log_{10} y = 43.8932$   
 The number of digits =  $43 + 1 = 44$

89. (D)  $\cos \theta = \frac{1}{2} \left( x + \frac{1}{x} \right)$

$$\Rightarrow x + \frac{1}{x} = 2 \cos \theta$$

$$\Rightarrow x^3 + \frac{1}{x^3} = (2 \cos \theta)^3 - 3 \times (2 \cos \theta)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 8 \cos^3 \theta - 6 \cos \theta$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2(4 \cos^3 \theta - 3 \cos \theta)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 2 \cos 3\theta$$

$$\Rightarrow \frac{1}{2} \left( x^3 + \frac{1}{x^3} \right) = \cos 3\theta$$

90. (A)  $\sin \frac{\pi}{3} + \sin \frac{5\pi}{9} - \sin \frac{7\pi}{9} - \sin \frac{8\pi}{9}$   
 $\Rightarrow \sin 60 + \sin 100 - \sin 140 - \sin 160$   
 $\Rightarrow \sin 60 + \sin(90+10) - 2 \sin \frac{140+160}{2}$   
 $\cos \frac{160-140}{2}$

$$\Rightarrow \frac{\sqrt{3}}{2} + \cos 10 - 2 \sin 150 \cdot \cos 10$$

$$\Rightarrow \frac{\sqrt{3}}{2} + \cos 10 - 2 \times \frac{1}{2} \cos 10 \Rightarrow \frac{\sqrt{3}}{2}$$

91. (B)  $\cot x = \frac{a}{b}$

$$\text{then } \sqrt{\frac{a-b}{a+b}} + \sqrt{\frac{a+b}{a-b}}$$

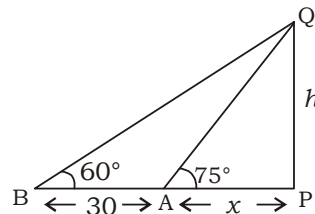
$$\Rightarrow \sqrt{\frac{a-1}{b} + \frac{a+1}{b}} + \sqrt{\frac{a+1}{b} + \frac{a-1}{b}}$$

$$\Rightarrow \frac{\sqrt{\cot x - 1}}{\sqrt{\cot x + 1}} + \frac{\sqrt{\cot x + 1}}{\sqrt{\cot x - 1}}$$

$$\Rightarrow \frac{\cot x - 1 + \cot x + 1}{\sqrt{\cot^2 x - 1}}$$

$$\Rightarrow \frac{2 \frac{\cos x}{\sin x}}{\sqrt{\frac{\cos^2 x}{\sin^2 x} - 1}} \Rightarrow \frac{2 \cos x}{\sqrt{\cos 2x}}$$

92. (D)



Let breadth of a river (AP) =  $x$  m  
 $PQ = h$  m

**In  $\Delta APQ$  :-**

$$\tan 75 = \frac{PQ}{AP}$$

$$2 + \sqrt{3} = \frac{h}{x} \quad \dots(i)$$

**In  $\Delta PQB$  :-**

$$\tan 60^\circ = \frac{PQ}{PB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x + 30} \quad \dots(ii)$$

form eq(i) and eq(ii)

$$\Rightarrow \frac{2 + \sqrt{3}}{\sqrt{3}} = \frac{x + 30}{x}$$

$$\Rightarrow 2x + \sqrt{3}x = \sqrt{3}x + 30\sqrt{3} \Rightarrow x = 15\sqrt{3}$$

Hence the breadth of a river =  $15\sqrt{3}$  m

93. (C)  $A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 4 \\ 0 & 2 & -1 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 3 & 0 \\ -1 & 2 & 3 \\ 0 & -1 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 4 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 0 \\ -1 & 2 & 3 \\ 0 & -1 & 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 1 & 10 \\ 13 & 3 & 17 \\ -2 & 5 & 1 \end{bmatrix}$$

$$\text{Now, } \det (AB) = 4(3 - 85) - 1(13 + 34) + 10(65 + 6) = -328 - 47 + 710 = 335$$

94. (A)  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Now,  $A^2 + 7I_2 - 5A = 0$   
 multiply by  $A^{-1}$  both sides  
 $\Rightarrow A^{-1}A^2 + 7A^{-1}I_2 - 5A^{-1}A = 0$   
 $\Rightarrow A + 7A^{-1} - 5I_2 = 0$   
 $\Rightarrow 7A^{-1} = 5I_2 - A$

$$\Rightarrow 7A^{-1} = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

95. (C)  $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{x} dx$

$I = 0$  [∵ function is odd.]

96. (D)  $I = \int_0^1 x(1-x)^7 dx$

Prop.IV  $\int_a^b f(x)dx = \int_a^b f(a+b-x) dx$

$$I = \int_0^1 (1-x)x^7 dx$$

$$I = \int_0^1 (x^7 - x^8) dx$$

$$I = \left[ \frac{x^8}{8} - \frac{x^9}{9} \right]_0^1$$

$$I = \frac{1}{8} - \frac{1}{9} - 0$$

$$I = \frac{1}{72}$$

97. (C)  $(\log_3 x)(\log_2 x)(\log_{2x} y) = \log_x x^2$

$$\Rightarrow (\log_3 2x)(\log_{2x} y) = 2 \log_x x$$

$$\Rightarrow \log_3 y = 2 \Rightarrow y = 3^2 = 9$$

98. (A)  $\sum_{n=0}^{10} (i^{n+1} + i^n)$

$$\Rightarrow (i + i^2) + (i^2 + i) + (i^3 + i^2) + (i^4 + i^3) + (i^5 + i^4) + (i^6 + i^5) + (i^7 + i^6) + (i^8 + i^7) + (i^9 + i^8) + (i^{10} + i^9)$$

$$\Rightarrow i + 1 - 1 + i - i - 1 + 1 - i + i + 1 - 1 + i - i - 1 + 1 - i + i + 1 + i - i - 1$$

$$\Rightarrow i - 1$$

99. (B)  $\cos 15 = \cos(45 - 30)$

$$\cos 15 = \cos 45 \cdot \cos 30 + \sin 45 \cdot \sin 30$$

$$\cos 15 = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

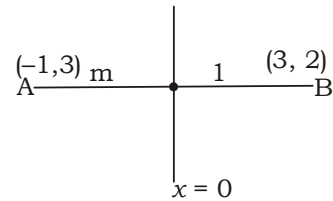
100. (B) Given that  $S = 2$  cm and  $r = 4$  cm

$$\text{Now } \theta = \frac{S}{r}$$

$$\Rightarrow \theta = \frac{2}{4} \text{ radian}$$

$$\Rightarrow \theta = \frac{1}{2} \times \frac{180}{\pi} = \left(\frac{90}{\pi}\right)^\circ$$

101. (B)



Let ratio =  $m : 1$

$$\text{Now, } \frac{m \times 3 + 1 \times (-1)}{m + 1} = 0$$

$$\Rightarrow 3m - 1 = 0 \Rightarrow m = \frac{1}{3}$$

The required ratio =  $1 : 3$

102. (C)  $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

$$\text{Let } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$$

$$I = \int 2e^t dt$$

$$I = 2e^t + c$$

$$I = 2e^{\sqrt{x}} + c$$

103. (D) The new mean =  $\frac{4(7+3)}{40}$

104. (A)  $f(x) = 4x^3 + 2ax^2 - 3bx$

$$f'(x) = 12x^2 + 4ax - 3b$$

by Rolle's theorem

(i) function is continuous on a interval  $[-2, 2]$ .

(ii) Function is differentiable on a interval  $(-2, 2)$ .

(iii)  $f(-2) = f(2)$

$$\Rightarrow 4(-2)^3 + 2a(-2)^2 - 3b(-2) = 4(2)^3 + 2a(2)^2 - 3b \times 2$$

$$\Rightarrow -32 + 8a + 6b = 32 + 8a - 6b$$

$$\Rightarrow 12b = 64 \Rightarrow b = \frac{16}{3}$$

... (i)

(iv)  $f'(c) = 0$

$$\Rightarrow f'(1) = 0$$

$$\Rightarrow 12(1)^2 + 4a \times 1 - 3b = 0$$

$$\Rightarrow 12 + 4a - 16 = 0 \quad [\text{from eq(i)}]$$

$$\Rightarrow 4a = 4 \Rightarrow a = 1$$

$$\text{Now, } 3b + 2a = 16 + 2 = 18$$

105. (A) 6, 7, 16, 17, 26, 28, 37, 38, 47, 48

$$\text{Mean} = \frac{6+7+16+17+26+28+37+38+47+48}{10}$$

$$\bar{x} = \frac{270}{10} = 27$$

$$\begin{aligned} \sum(x_i - \bar{x})^2 &= (6-27)^2 + (7-27)^2 + (16-27)^2 \\ &+ (17-27)^2 + (26-27)^2 + (28-27)^2 \\ &+ (37-27)^2 + (38-27)^2 + (47-27)^2 + (48-27)^2 \end{aligned}$$

$$\begin{aligned} \sum(x_i - \bar{x})^2 &= 441+400+121+100+1+1+100 \\ &+121+400+441 \end{aligned}$$

$$\sum(x_i - \bar{x})^2 = 2126$$

$$\begin{aligned} \text{Standard Deviation} &= \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} \\ &= \sqrt{\frac{2126}{10}} = 14.58 \end{aligned}$$

106. (C) Given that  $T_n = 6n - 21$

$$S_n = \sum T_n$$

$$S_n = \sum(6n - 21)$$

$$S_n = 6 \sum n - 21 \sum 1$$

$$S_n = 6 \times \frac{n(n+1)}{2} - 21 \times n$$

$$S_n = 3n(n+1) - 21n$$

$$S_n = 3n^2 - 18n$$

$$\text{Now, } S_{25} = 3 \times (25)^2 - 18 \times 25$$

$$\Rightarrow S_{25} = 1875 - 450 = 1425$$

107. (C)  $\frac{n-r}{r+1} {}^n C_r$

$$\Rightarrow \frac{n-r}{r+1} \times \frac{n!}{(n-r)!r!}$$

$$\Rightarrow \frac{n!(n-r)}{(r+1)!(n-r)(n-r-1)!}$$

$$\Rightarrow \frac{n!}{(n-r-1)!(r+1)!} = {}^n C_{r+1}$$

108. (B) Parabola  $y^2 = 4ax$  and focal cord  $3x + y = 6$  focus =  $(a, 0)$

focus satisfy the focal cord, then

$$3a + 0 = 6 \Rightarrow a = 2$$

equation of directrix

$$x = -a$$

$$x = -2 \Rightarrow x + 2 = 0$$

109. (A) Line  $3x + 14y = 7$

$$\text{Slope of line } m = \frac{-3}{14}$$

$$\text{Slope of perpendicular line } m' = \frac{-1}{m}$$

$$= \frac{-1 \times 14}{-3} = \frac{14}{3}$$

equation of perpendicular line which passes through the point  $(-7, 6)$

$$y - 6 = m'(x + 7)$$

$$\Rightarrow y - 6 = \frac{14}{3}(x + 7)$$

$$\Rightarrow 3y - 18 = 14x + 98$$

$$\Rightarrow 14x - 3y + 116 = 0$$

110. (C) Let  $a + ib = \sqrt{-3 + 4i}$

On squaring both side

$$\Rightarrow (a^2 - b^2) + 2abi = -3 + 4i$$

On comparing

$$\Rightarrow a^2 - b^2 = -3 \text{ and } 2ab = 4 \quad \dots(i)$$

$$\text{Now, } (a^2 - b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$\Rightarrow (a^2 - b^2)^2 = 9 + 16$$

$$\Rightarrow (a^2 - b^2)^2 = 25 \Rightarrow a^2 - b^2 = 5 \quad \dots(ii)$$

from eq(i) and eq(ii)

$$\Rightarrow 2a^2 = 2 \Rightarrow a = \pm 1 \text{ and } 2b^2 = 8 \Rightarrow b = \pm 2$$

$$\text{Hence } \sqrt{-3 + 4i} = \pm(1 + 2i)$$

111. (D)  $a = \frac{\sqrt{3}i - 1}{2} = -\omega^2$

$$\text{Now, } 1 + a^2 + a^4 + a^6$$

$$\Rightarrow 1 + (-\omega^2)^2 + (-\omega^2)^4 + (-\omega^2)^6$$

$$\Rightarrow 1 + \omega^4 + \omega^8 + \omega^{12}$$

$$\Rightarrow 1 + \omega + \omega^2 + 1$$

$$\Rightarrow 0 + 1 = 1 \quad [\because 1 + \omega + \omega^2 = 0]$$

112. (B) We know that

$$(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n \quad \dots(i)$$

$x$  replace by  $\frac{1}{x}$

$$\left(1 + \frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \frac{C_3}{x^3} + \dots + \frac{C_n}{x^n} \quad \dots(ii)$$

from eq(i) and eq(ii)

$$\text{coefficient of } x^0 \text{ in } (1 + x)^n \left(1 + \frac{1}{x}\right)^n$$

$$= C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2$$

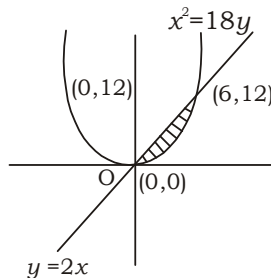
$$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \text{coeff. of } x^0 \text{ in } (1 + x)^{2n}$$

$$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = {}^{2n}C_n$$

$$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n!}{n!n!}$$

113. (A)  $\cos\alpha \cdot \cos 2\alpha \cdot \cos 4\alpha - \frac{1}{2} \cos\alpha$   
 $\alpha = 10^\circ$   
 $\cos 10 \cdot \cos 20 \cdot \cos 40 - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{2} \cos 10 (2 \cos 20 \cdot \cos 40) - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{2} \cos 10 [\cos(20+40) + \cos(20-40)]$   
 $-\frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{2} \cos 10 \left[ \frac{1}{2} + \cos 20 \right] - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{4} \cos 10 + \frac{1}{2} \cos 10 \cdot \cos 20 - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{4} \cos 10 \times \frac{1}{4} \times 2 \cos 10 \cdot \cos 20 - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{4} \cos 10 + [\cos 30 + \cos 10] - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{4} \cos 10 + \frac{1}{4} \left[ \frac{\sqrt{3}}{2} + \cos 10 \right] - \frac{1}{2} \cos 10$   
 $\Rightarrow \frac{1}{2} \cos 10 + \frac{\sqrt{3}}{8} - \frac{1}{2} \cos 10 = \frac{\sqrt{3}}{8}$

114. (B)



$x_1 \Rightarrow x = \sqrt{18y}$   
 and  $x_2 \Rightarrow x = \frac{y}{2}$   
 Area =  $\int_0^{12} (x_1 - x_2) dx$   
 $= \int_0^{12} \left( \sqrt{18y} - \frac{y}{2} \right) dx$   
 $= \left[ \sqrt{18} \times \frac{y^{3/2}}{3/2} - \frac{1}{2} \times \frac{y^2}{2} \right]_0^{12}$   
 $\Rightarrow 3\sqrt{2} \times \frac{2}{3} \times (12)^{3/2} - \frac{1}{4} \times (12)^2 - 0$   
 $\Rightarrow 2\sqrt{2} \times 12 \times 2\sqrt{3} - \frac{1}{4} \times 144$   
 $\Rightarrow (48\sqrt{6} - 36) \text{ sq. unit}$

115. (A)  $\frac{\sin 420 \cdot \cos 750 \cdot \sec 1140}{\tan 1020 \cdot \operatorname{cosec} 240 \cdot \sin 135}$   
 $\Rightarrow \frac{\sin(360+60) \cdot \cos(720+30) \cdot \sec(360 \times 3 + 60)}{\tan(360 \times 3 - 60) \cdot \operatorname{cosec}(180+60) \cdot \sin(90+45)}$   
 $\Rightarrow \frac{\sin 60 \cdot \cos 30 \cdot \sec 60}{-\tan 60 \cdot (-\operatorname{cosec} 60) \cdot \cos 45}$   
 $\Rightarrow \frac{\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times 2}{\sqrt{3} \times \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}}}$   
 $\Rightarrow \frac{\frac{3}{4} \times 2}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$

116. (B) Given that  $A = \begin{bmatrix} -1 & 2 \\ 4 & -5 \end{bmatrix}$

$|A| = -1 \times (-5) - 4 \times 2 = -3$

We know that

$A(\operatorname{Adj} A) = |A|^{n-1} I_n$

here  $n = 2$

$A(\operatorname{Adj} A) = (-3)^{2-1} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$A(\operatorname{Adj} A) = -3 \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$

117. (C) We know that

$\sin A \cdot \sin(60-A) \cdot \sin(60+A) = \frac{1}{4} \sin 3A$

Now,  $4 \sin 5 \cdot \sin 55 \cdot \sin 65$

$\Rightarrow 4 \sin 5 \cdot \sin(60-5) \cdot \sin(60+5)$

$\Rightarrow 4 \times \frac{1}{4} \sin(5 \times 3)$

$\Rightarrow \sin 15 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

118. (B) Differential equation

$\cot x \, dx = 2y \, dy$

On integrating

$\Rightarrow \int \cot x \, dx = 2 \int y \, dy$

$\Rightarrow \log \sin x = 2 \times \frac{y^2}{2} + \log c$

$\Rightarrow \log \sin x - \log c = y^2$

$\Rightarrow \log \frac{\sin x}{c} = y^2$

$\Rightarrow \frac{\sin x}{c} = e^{y^2} \Rightarrow \sin x = c \cdot e^{y^2}$



119. (A) Line  $\frac{2x-1}{3} = \frac{1-y}{3} = \frac{z+1}{1}$

$$\Rightarrow \frac{x-\frac{1}{2}}{3/2} = \frac{y-1}{-3} = \frac{z+1}{1}$$

Direction cosine

$$= \left\langle \frac{3/2}{\sqrt{\left(\frac{3}{2}\right)^2 + (-3)^2 + 1^2}}, \frac{-3}{\sqrt{\left(\frac{3}{2}\right)^2 + (-3)^2 + 1^2}}, \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 + (-3)^2 + 1^2}} \right\rangle$$

$$= \left\langle \frac{3/2}{7/2}, \frac{-3}{7/2}, \frac{1}{7/2} \right\rangle = \left\langle \frac{3}{7}, \frac{-6}{7}, \frac{2}{7} \right\rangle$$

120. (D) Given that  $e = \frac{1}{\sqrt{3}}$

and  $2ae = \sqrt{6}$

$$\Rightarrow 2a \times \frac{1}{\sqrt{3}} = \sqrt{6} \Rightarrow a = \frac{3}{\sqrt{2}}$$

Now,  $e^2 = 1 - \frac{b^2}{a^2}$

$$\Rightarrow \frac{1}{3} = 1 - \frac{b^2}{9/2}$$

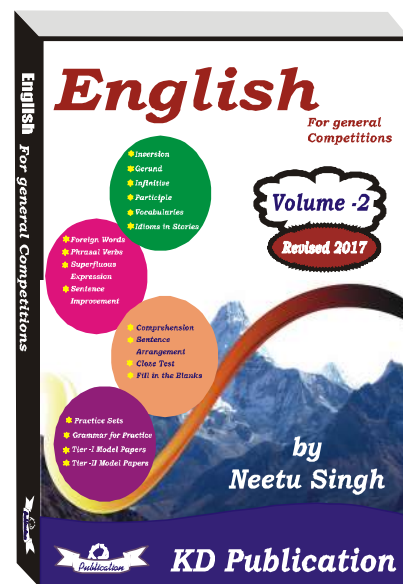
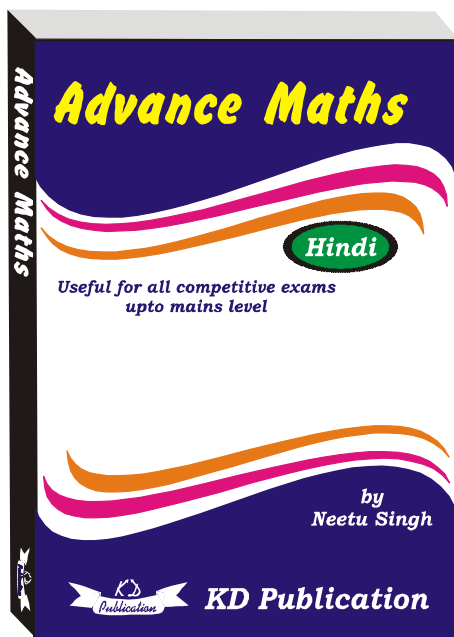
$$\Rightarrow \frac{2b^2}{9} = \frac{2}{3} \Rightarrow b^2 = 3$$

Equation of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{9/2} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{2y^2}{9} + \frac{y^2}{9} = 1 \Rightarrow 2x^2 + y^2 = 9$$



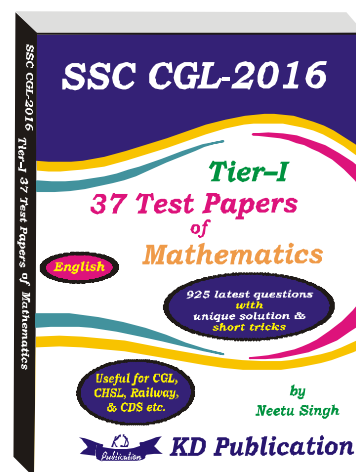
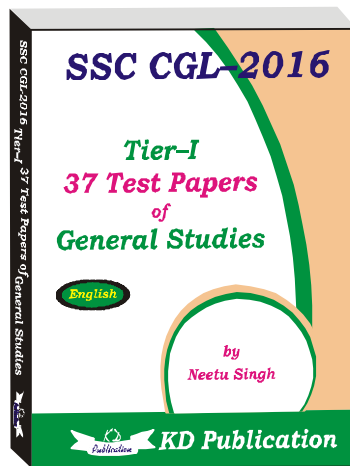
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**NDA (MATHS) MOCK TEST - 140 (Answer Key)**

1. (C)	21. (B)	41. (D)	61. (C)	81. (D)	101. (B)
2. (B)	22. (A)	42. (B)	62. (C)	82. (A)	102. (C)
3. (C)	23. (D)	43. (C)	63. (A)	83. (B)	103. (D)
4. (C)	24. (B)	44. (B)	64. (D)	84. (C)	104. (A)
5. (B)	25. (A)	45. (A)	65. (B)	85. (A)	105. (A)
6. (B)	26. (C)	46. (A)	66. (C)	86. (B)	106. (C)
7. (D)	27. (C)	47. (B)	67. (D)	87. (A)	107. (C)
8. (B)	28. (A)	48. (C)	68. (B)	88. (C)	108. (B)
9. (D)	29. (C)	49. (C)	69. (C)	89. (D)	109. (A)
10. (C)	30. (C)	50. (D)	70. (A)	90. (A)	110. (C)
11. (C)	31. (D)	51. (C)	71. (B)	91. (B)	111. (D)
12. (B)	32. (B)	52. (A)	72. (C)	92. (D)	112. (B)
13. (D)	33. (B)	53. (C)	73. (C)	93. (C)	113. (A)
14. (B)	34. (A)	54. (C)	74. (C)	94. (A)	114. (B)
15. (C)	35. (D)	55. (C)	75. (D)	95. (C)	115. (A)
16. (C)	36. (C)	56. (D)	76. (C)	96. (D)	116. (B)
17. (C)	37. (C)	57. (A)	77. (B)	97. (C)	117. (C)
18. (D)	38. (D)	58. (B)	78. (A)	98. (A)	118. (B)
19. (A)	39. (A)	59. (A)	79. (B)	99. (B)	119. (A)
20. (D)	40. (C)	60. (D)	80. (C)	100. (B)	120. (D)



**Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003**

**Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock**

**Note:- If you face any problem regarding result or marks scored, please contact 9313111777**