KD Campus

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NDA MATHS MOCK TEST - 144 (SOLUTION)

1. (D) In the expansion of $\left(2x^4 - \dfrac{1}{4x^2}\right)^{11}$

$T_{r+1} = {}^{11}C_r\,(2\,x^4)^{11-r}\left(\dfrac{-1}{4x^2}\right)^r$

$= {}^{11}C_r\,2^{11-3r}\,(-1)^r\,x^{44-6r}$

Hence, $44 - 6r = 8 \Rightarrow r = 6$

The Required Coefficient = ${}^{11}C_6\,2^{-7}\,(-1)^6$

$= \dfrac{11\times10\times9\times8\times7}{5\times4\times3\times2} \times \dfrac{1}{128} = \dfrac{231}{64}$

2. (A) Equations $3x + 4y + z = 6$, $x - 2y + z = 7$ and $2x - y + 2z = 8$

Let $A = \begin{bmatrix} 3 & 4 & 1 \\ 1 & -2 & 1 \\ 2 & -1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$

Using elementary method

$[A/B] = \left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 1 & -2 & 1 & 7 \\ 2 & -1 & 2 & 8 \end{array}\right]$

$R_2 \to R_2 - \dfrac{1}{3}R_1$ and $R_3 \to R_3 - \dfrac{2}{3}R_1$

$[A/B] = \left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 0 & -\frac{10}{3} & \frac{2}{3} & 5 \\ 0 & -\frac{11}{3} & \frac{4}{3} & 4 \end{array}\right]$

$R_2 \to 3R_2$, $R_3 \to 3R_3$

$[A/B] = \left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 0 & -10 & 2 & 15 \\ 0 & -11 & 4 & 12 \end{array}\right]$

$R_2 \to R_2 - \dfrac{11}{10}R_3$

$[A/B] = \left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 0 & -10 & 2 & 15 \\ 0 & 0 & \frac{9}{5} & \frac{-6}{5} \end{array}\right]$

Rank (A) = Rank (AB)

Hence given equations have unique solution.

3. (B) Curve $2x^2 + 3y^2 = 10$

$\Rightarrow \dfrac{2x^2}{10} + \dfrac{3y^2}{10} = 1 \Rightarrow \dfrac{x^2}{5} + \dfrac{y^2}{10/3} = 1$

$a^2 = 5$, $b^2 = 10/3$

$a = \sqrt{5}$, $b = \sqrt{\dfrac{10}{3}}$

Are $a = \pi ab$

$= \pi \times \sqrt{5} \times \sqrt{\dfrac{10}{3}} = 5\sqrt{\dfrac{2}{3}}\,\pi$ sq. unit

4. (C) Differential equation

$\dfrac{dy}{dx} + y\cot x = \sin x$

On comparing with general equation

$P = \cot x$, $Q = \sin x$

I.F. $= e^{\int P\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\ln \sin x} = \sin x$

Solution of differential equation

$\Rightarrow y \times \text{I.F.} = \int Q \times \text{I.F.}\ dx$

$\Rightarrow y\sin x = \int \sin x.\sin x\ dx$

$\Rightarrow y\sin x = \int \dfrac{1-\cos 2x}{2}\,dx$

$\Rightarrow y\sin x = \dfrac{1}{2}\left[x - \dfrac{\sin 2x}{2}\right] + C$

$\Rightarrow y\sin x = \dfrac{1}{4}(2x - \sin 2x) + C$

$\Rightarrow 4y\sin x = 2x - \sin 2x + C$

5. (D) $f(x) = \dfrac{\sqrt{\log_e\left(36 - 2x - x^2\right)}}{x-1}$

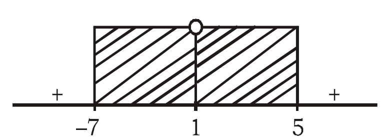
Now, $\log_e(36 - 2x - x^2) \ge 0$

$\Rightarrow 36 - 2x - x^2 \ge 1$

$\Rightarrow x^2 + 2x + 1 - 36 \le 0$

$\Rightarrow x^2 + 2x - 35 \le 0$

$\Rightarrow (x + 7)(x - 5) \le 0$



Domain = $[-7, 5] - \{1\}$

6. (D) sin (–2475) = – sin 2475
= – sin (360 × 7 – 45)
= – (–sin 45) = $\frac{1}{\sqrt{2}}$

7. (B) A = $\begin{bmatrix} -7 & 12 \\ -1 & 2 \end{bmatrix}$

$|A|$ = – 14 + 12 = –2
Co-factors of A –
$C_{11} = (-1)^{1+1}(2) = 2$, $C_{12} = (-1)^{1+2}(-1) = 1$
$C_{21} = (-1)^{2+1}(12) = -12$, $C_{22} = (-1)^{2+2}(-7) = -7$

C = $\begin{bmatrix} 2 & 1 \\ -12 & -7 \end{bmatrix}$

Adj A = C^T = $\begin{bmatrix} 2 & -12 \\ 1 & -7 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj}\,A}{|A|}$

$A^{-1} = \frac{-1}{2}\begin{bmatrix} 2 & -12 \\ 1 & -7 \end{bmatrix} \Rightarrow 2A^{-1} = \begin{bmatrix} -2 & 12 \\ -1 & 7 \end{bmatrix}$

Now, A + $2A^{-1}$

$\Rightarrow \begin{bmatrix} -7 & 12 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 12 \\ -1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -9 & 24 \\ -2 & 9 \end{bmatrix}$

8. (C) Let $y = 3\tan\sqrt{x}$
On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = 3\left(\sec^2\sqrt{x}\right) \times \frac{1}{2\sqrt{x}} = \frac{3\sec^2\sqrt{x}}{2\sqrt{x}}$

and $z = x^{3/2}$

$\Rightarrow \frac{dz}{dx} = \frac{3}{2}x^{1/2}$

Now, $\frac{dy}{dz} = \frac{dy}{dz} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = \frac{3\sec^2\sqrt{x}}{2\sqrt{x}} \times \frac{2}{3x^{1/2}}$

$\Rightarrow \frac{dy}{dz} = \frac{\sec^2\sqrt{x}}{x}$

9. (A) Vectors $4\hat{i} - (2\lambda + 1)\hat{j} + (3-\lambda)\hat{k}$

and $2\lambda\hat{i} + 4\hat{j} - \hat{k}$ are perpendicular,
then, 4 × 2λ – (2λ +1) × 4 + (3 –λ) × (–1) = 0
⇒ 8λ – 8λ – 4 – 3 + λ = 0
⇒ λ – 7 = 0 ⇒ λ = 7

10. (D) A = $\begin{bmatrix} 3i & 4-3i \\ -4-3i & -2i \end{bmatrix}$

$\overline{A} = \begin{bmatrix} -3i & 4+3i \\ -4+3i & 2i \end{bmatrix}$

$\left(\overline{A}\right)' = \begin{bmatrix} -3i & -4+3i \\ 4+3i & 2i \end{bmatrix}$

$A^* = -\begin{bmatrix} 3i & 4-3i \\ -4-3i & -2i \end{bmatrix}$

A* = – A
Hence matrix is skew-hermitian matrix.

11. (C) $\frac{2}{\text{cosec}\,70} - \frac{\text{cosec}10}{2}$

$\Rightarrow 2\sin 70 - \frac{1}{2\sin 10}$

$\Rightarrow \frac{4\sin 10.\sin 70 - 1}{2\sin 10}$

$\Rightarrow \frac{2\left[\cos(10-70) - \cos(10+70)\right] - 1}{2\sin 10}$

$\Rightarrow \frac{2[\cos 60 - \cos 80] - 1}{2\sin 10}$

$\Rightarrow \frac{2\left[\frac{1}{2} - \sin 10\right] - 1}{2\sin 10}$

$\Rightarrow \frac{1 - 2\sin 10 - 1}{2\sin 10}$

$\Rightarrow \frac{-2\sin 10}{2\sin 10} = -1$

12. (A) $\frac{\cos x + \cos 3x}{\cos x - \cos 3x}$

$\Rightarrow \frac{2\cos\frac{x+3x}{2}.\cos\frac{x-3x}{2}}{2\sin\frac{x-3x}{2}.\frac{x+3x}{2}}$

$\Rightarrow \frac{\cos 2x.\cos x}{(-\sin x)\sin 2x}$

$\Rightarrow -\cot x \,.\, \cot 2x$

13. (B) $I = \int e^x \left[\frac{1-x^2+x+x^3}{(1+x^2)^2}\right] dx$

$I = \int e^x \left[\frac{x+x^3}{(1+x^2)^2} + \frac{1-x^2}{(1+x^2)^2}\right] dx$

$I = \int e^x \left[\frac{x}{1+x^2} + \frac{1-x^2}{(1+x^2)^2}\right] dx$

We know that

$\int e^x [f(x) + f'(x)]\ dx = e^x.f(x) + C$

$I = e^x \left(\frac{x}{1+x^2}\right) + C$

14. (C) $\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right)^{1/3} = \left(y - \frac{d^3y}{dx^3}\right)^{1/2}$

$\Rightarrow \left[\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right)^{1/3}\right]^6 = \left[\left(y - \frac{d^3y}{dx^3}\right)^{1/2}\right]^6$

$\Rightarrow \left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right)^2 = \left(y - \frac{d^3y}{dx^3}\right)^3$

Hence order = 3 and degree = 3

15. (C) Lines $8x + 15y = 20$

$8x + 15y - 20 = 0$

and $16x + 30y + 11 = 0$

$\Rightarrow \quad 8x + 15y + \frac{11}{2} = 0$

The required distance = $\frac{\frac{11}{2} - (-20)}{\sqrt{8^2 + 15^2}}$

$= \frac{51}{2 \times 17} = \frac{3}{2}$

16. (B)

$S = 3 + 7 + 12 + 18 + 25 + ... + t_n$

$-S = \quad 3 + 7 + 12 + 18 + ... + t_{n-1} + t_n$

$0 = (3 + 4 + 5 + 6 + 7 + n \text{ terms}) - t_n$

$t_n = \frac{n}{2}[2 \times 3 + (n-1) \times 1]$

$t_n = \frac{n}{2}[6 + n - 1]$

$t_n = \frac{1}{2} n^2 + \frac{5}{2} n$

Now, $S_n = \sum t_n$

$\Rightarrow S_n = \sum\left(\frac{1}{2}n^2 + \frac{5}{2}n\right)$

$\Rightarrow S_n = \frac{n(n+1)}{4}\left[\frac{2n+1}{3} + 5\right]$

$\Rightarrow S_n = \frac{n(n+1)}{4} \times \frac{2n+16}{3}$

$\Rightarrow S_n = \frac{n(n+1)(n+8)}{6}$

17. (B) The required number of ways = (8 – 1)!
= 7! = 5040

18. (A)

19. (C) $n(S) = 6 \times 6 = 36$

E = {(5, 1), (4, 2), (3, 3), (2, 4), (1, 5)}

$n(E) = 5$

The required probability = $P(E) = \frac{n(E)}{n(S)}$

$\Rightarrow P(E) = \frac{5}{36}$

20. (D) $\frac{x-1}{3} = \frac{5-2y}{2} = \frac{z-7}{-5}$

$\Rightarrow \frac{x-1}{3} = \frac{y - \frac{5}{2}}{-4} + \frac{z-7}{-5}$

Direction ratios = <3, –4, –5>

21. (C) Equation of circle

$(x + 8)^2 + (y - 3)^2 = 7^2$

$\Rightarrow x^2 + 64 + 16x + y^2 + 9 - 6y = 49$

$\Rightarrow x^2 + y^2 + 16x - 6y + 24 = 0$

22. (B) Let $y = \tan^{-1}\sqrt{x}$

On differentiating both side w.r.t. 'x'

$\frac{dy}{dx} = \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+x)}$

and $z = x^{3/2}$

On differentiating both side w.r.t. 'x'

$\Rightarrow \frac{dz}{dx} = \frac{3}{2} x^{1/2}$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dx}{dz} = \frac{1}{2\sqrt{x}(1+x)} \times \frac{2}{3x^{1/2}}$

$\Rightarrow \frac{dx}{dz} = \frac{1}{x(1+x)}$

23. (B) Digits 0, 1, 3, 4, 5, 6

The required number of 4–digit numbers
= 6 × 7 × 7 × 7 = 2058

24. (C) $y = \left(1-\frac{1}{x^{1/16}}\right)\left(1+\frac{1}{x^{1/2}}\right)\left(1+\frac{1}{x^{1/4}}\right)\left(1+\frac{1}{x^{1/8}}\right)\left(1+\frac{1}{x^{1/16}}\right)$

$y = \left(1-\frac{1}{x^{1/16}}\right)\left(1+\frac{1}{x^{1/16}}\right)\left(1+\frac{1}{x^{1/8}}\right)\left(1+\frac{1}{x^{1/4}}\right)\left(1+\frac{1}{x^{1/2}}\right)$

$y = \left(1-\frac{1}{x^{1/8}}\right)\left(1+\frac{1}{x^{1/8}}\right)\left(1+\frac{1}{x^{1/4}}\right)\left(1+\frac{1}{x^{1/2}}\right)$

$y = \left(1-\frac{1}{x^{1/4}}\right)\left(1+\frac{1}{x^{1/4}}\right)\left(1+\frac{1}{x^{1/2}}\right)$

$y = \left(1-\frac{1}{x^{1/2}}\right)\left(1+\frac{1}{x^{1/2}}\right)$

$y = \left(1-\frac{1}{x}\right)$

On differenting both side w.r.t. 'x'

$\frac{dy}{dx} = 0 - \left(\frac{-1}{x^2}\right)$

$\frac{dy}{dx} = \frac{1}{x^2}$

Again, differentiating

$\frac{d^2y}{dx^2} = \frac{-2}{x^3}$

25. (A) $\begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 2 \\ 2 & 3 & 1 \end{bmatrix}$

26. (D) Given that $\tan A = \frac{-1}{3}$ and $\tan B = \frac{1}{2}$

Now, $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$

$\Rightarrow \tan(A - B) = \frac{\frac{-1}{3} - \frac{1}{2}}{1 + \left(\frac{-1}{3}\right) \times \frac{1}{2}}$

$\Rightarrow \tan(A - B) = \frac{\frac{-5}{6}}{\frac{5}{6}}$

$\Rightarrow \tan(A - B) = -1 \Rightarrow A - B = \frac{3\pi}{4}$

27. (B) Ratio of angles = 2 : 2 : 1

Let angles = $2x$, $2x$, x

Now, $2x + 2x + x = 180$

$\Rightarrow 5x = 180 \Rightarrow x = 36$

Angel A = 72, B = 72, C = 36

Now, $\sin^2 A + \sin^2 B + \sin^2 C$

$\Rightarrow \sin^2 72 + \sin^2 72 + \sin^2 36$

$\Rightarrow \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 + \left(\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2 + \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)^2$

$\Rightarrow \frac{10+2\sqrt{5}}{16} + \frac{10+2\sqrt{5}}{16} + \frac{10-2\sqrt{5}}{16}$

$\Rightarrow \frac{30+2\sqrt{5}}{16} = \frac{15+\sqrt{5}}{8}$

28. (C) $y = a^x \log_a a^x$

$\Rightarrow y = a^{x \times x} \log_a a$

$\Rightarrow y = a^{x^2}$

On differentiating both side w. r. t. 'x'

$\Rightarrow \frac{dy}{dx} = a^{x^2} \log a \times 2x$

$\Rightarrow \frac{dy}{dx} = 2x.\,a^{x^2} \log a$

29. (A)

30. (C) 1 1 0 0 1   0. 1 0 1

$1\times2^0 = 1$ $\frac{1}{2} = 1\times2^{-1}$

$0\times2^1 = 0$ $0 = 0\times2^{-2}$

$0\times2^2 = 0$ $\frac{1}{8} = 1\times2^{-3}$

$1\times2^3 = 8$ $\frac{1}{2}+\frac{1}{8}=\frac{5}{8} = 0.625$

$1\times2^4 = 16$

25

Hence $(11\,001.101)_2 = (25.625)_{10}$

31. (B) Given that $\alpha = 20^o$

Now, $\sin\alpha.\sin2\alpha.\sin4\alpha$

$\Rightarrow \sin20^o.\ \sin40^o.\ \sin80^o$

$\Rightarrow \frac{1}{2}\sin20\ [2\sin40.\sin80]$

$\Rightarrow \frac{1}{2}\sin20\ [\cos(40-80) - \cos(40+80)]$

$\Rightarrow \frac{1}{2}\sin20\left[\cos40 + \frac{1}{2}\right]$

$\Rightarrow \frac{1}{2}\sin20.\cos40 + \frac{1}{4}\sin20$

$\Rightarrow \frac{1}{4} \times 2\sin20.\cos40 + \frac{1}{4}\sin20$

$\Rightarrow \frac{1}{4}\left[\sin(20+40) + \sin(20-40)\right] + \frac{1}{4}\sin20$

$\Rightarrow \frac{1}{4}\left[\frac{\sqrt{3}}{2} - \sin20\right] + \frac{1}{4}\sin20$

$\Rightarrow \frac{\sqrt{3}}{8} - \frac{1}{4}\sin20 + \frac{1}{4}\sin20 = \frac{\sqrt{3}}{8}$

32. (C) Equation $ax^2 + bx + c = 0$

$\alpha + \beta = \frac{-b}{a}$ and $\alpha \beta = \frac{c}{a}$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$\Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$\Rightarrow \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$\Rightarrow \frac{\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}}{\frac{c}{a}}$

$\Rightarrow \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}$

$\Rightarrow \frac{\frac{b^2 - 2ac}{a^2}}{\frac{c}{a}} = \frac{b^2 - 2ac}{ac}$

33. (D) A = {0, 1, 2, 3} and B = {0, 1, 4, 5}

A × B = {(0, 0), (0, 1), (0, 4), (0, 5), (1, 0), (1, 1), (1, 4), (1, 5), (2, 0), (2, 1), (2, 4), (2, 5), (3, 0), (3, 1), (3, 4), (3, 5)}

B × A = {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (4, 0), (4,1), (4, 2), (4, 3), (5, 0), (5, 1), (5, 2,) (5, 3)}

Now, (B × A) ∩ (A × B) = {(0, 0), (0, 1), (1, 0), (1, 1)}

34. (D) $I = \int \frac{\log\left(x - \sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\, dx$

Let $\log\left(x - \sqrt{1+x^2}\right) = t$

$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \left[1 - \frac{1 \times 2x}{2\sqrt{1+x^2}}\right] dx = dt$

$\Rightarrow \frac{1}{x - \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} - x}{2\sqrt{1+x^2}}\, dx = dt$

$\Rightarrow \frac{-1}{2\sqrt{1+x^2}}\, dx = dt \Rightarrow \frac{1}{\sqrt{1+x^2}}\, dx = -2dt$

$I = \int t(-2\, dt)$

$I = -2 \times \frac{t^2}{2} + C$

$I = -\left[\log(x - \sqrt{1+x^2}\right]^2 + c$

35. (B) Let $y = 8^{37}$

taking log both side

$\Rightarrow \log_{10} y = 37 \log_{10} 8$

$\Rightarrow \log_{10} y = 37 \times 3 \log_{10} 2$

$\Rightarrow \log_{10} y = 111 \times 0.3010$

$\Rightarrow \log_{10} y = \; = 33.411$

The required no. of digits = 33 + 1 = 34

36. (C) Given that $s = 5$ cm and $r = 3$ cm

Now $\theta = \frac{s}{r}$

$\Rightarrow \theta = \left(\frac{5}{3}\right)^c$

$\Rightarrow \theta = \left(\frac{5}{3} \times \frac{180}{\pi}\right)^0$

$\Rightarrow \theta = \left(\frac{300}{\pi}\right)^0$

37. (B) $A = \begin{bmatrix} -2 & 1 \\ 0 & 6 \end{bmatrix}$

Co-factors of A—

$C_{11} = (-1)^{1+1}(6)$, $C_{12} = (-1)^{1+2}(0) = 0$

$C_{21} = (-1)^{2+1}(1) = -1$, $C_{22} = (-1)^{2+2}(-2) = -2$

$C = \begin{bmatrix} 6 & 0 \\ -1 & -2 \end{bmatrix}$

$\text{Adj } A = C^T = \begin{bmatrix} 6 & -1 \\ 0 & -2 \end{bmatrix}$

Now, $A(\text{Adj}A) = \begin{bmatrix} -2 & 1 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} 6 & -1 \\ 0 & -2 \end{bmatrix}$

$\Rightarrow A\ (\text{Adj } A) = \begin{bmatrix} -2 \times 6 + 1 \times 0 & -2 \times (-1) + 1 \times (2) \\ 0 \times 6 + 6 \times 0 & 0 \times (-1) + 6(-2) \end{bmatrix}$

$\Rightarrow A\ (\text{Adj } A) = \begin{bmatrix} -12 & 0 \\ 0 & -12 \end{bmatrix}$

$\Rightarrow A\ (\text{Adj } A) = -12 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -12\ I$

38. (C) Lines $\frac{2x-1}{12} = \frac{1-y}{3} = \frac{z-1}{-2}$

$\Rightarrow \frac{2\left(x - \frac{1}{2}\right)}{12} = \frac{-(y-1)}{3} = \frac{z-1}{-2}$

$\Rightarrow \frac{x - \frac{1}{2}}{6} = \frac{y-1}{-3} = \frac{z-1}{-2}$

Distance ratios = <6, –3, –2>

39. (C) Given that $e = \frac{1}{\sqrt{6}}$

and $\frac{2a}{e} = \sqrt{12}$

$\Rightarrow \frac{2 \times a}{1/\sqrt{6}} = \sqrt{12} \Rightarrow a = \frac{1}{\sqrt{2}}$

Now, $b^2 = a^2 (1 - e^2)$

$\Rightarrow b^2 = \frac{1}{2} \times \left(1 - \frac{1}{6}\right)$

$\Rightarrow b^2 = \frac{1}{2} \times \frac{5}{6} \Rightarrow b^2 = \frac{5}{12}$

Equation of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{x^2}{1/2} + \frac{y^2}{5/12} = 1$

$\Rightarrow 10x^2 + 12y^2 = 5$

40. (B) Word "OFFICER"

The number of Permutations = $\frac{7!}{2!}$

$= 2520$

41. (C) $\lim_{x \to \infty} \frac{3x^2 + 4x - 6}{1 + x - x^2}$

$\Rightarrow \lim_{x \to \infty} \frac{x^2\left(3 + \frac{4}{x} - \frac{6}{x^2}\right)}{x^2\left(\frac{1}{x^2} + \frac{1}{x} - 1\right)}$

$\Rightarrow \frac{3 + 0 - 0}{0 + 0 - 1} = \frac{3}{-1} = -3$

42. (A) Let $a + ib = \sqrt{-3 + 4\sqrt{7}i}$

On squaring both side

$\Rightarrow (a^2 - b^2) + (2ab)\, i = -3 + 4\sqrt{7}\, i$

On comparing

$a^2 - b^2 = -3$ and $2ab = 4\sqrt{7}$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = 9 + 112$

$\Rightarrow (a^2 + b^2)^2 = 9 + 121 \Rightarrow a^2 + b^2 = 11$ ..(ii)

From eq (i) and eq (ii)

$2a^2 = 8,$ $2b^2 = 14$

$a^2 = 4 \Rightarrow a = \pm 2,$ $b^2 = 7 \Rightarrow b = \pm\sqrt{7}$

Hence $\sqrt{-3 + 4\sqrt{7}i} = \pm (2 + \sqrt{7}\, i)$

43. (C) $\vec{a} = -4\hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 5\hat{j} + 9\hat{k}$ Now, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\Rightarrow -4 \times 2 + \lambda \times (-5) + (-3) \times 9$

$= \sqrt{(-4)^2 + \lambda^2 + (-3)^2}\sqrt{2^2 + (-5)^2 + 9^2} .\cos\frac{\pi}{2}$

$\Rightarrow -5\lambda - 27 = 0$

$\Rightarrow -35 - 5\lambda = 0 \Rightarrow \lambda = -7$

44. (C) A.M. $= \frac{1^2 + 2^2 + 3^2 \ldots\ldots\ldots + n^2}{n}$

$= \frac{n}{6}(n+1)(2n+1) \times \frac{1}{n} = \frac{1}{6}(n+1)(2n+1)$

45. (D) Determinant $\begin{vmatrix} -1 & 0 & 4 & 6 \\ 2 & -3 & 1 & 0 \\ -5 & 6 & 2 & -2 \\ -6 & -1 & 7 & 2 \end{vmatrix}$

Co-factor of 4 $= (-1)^{1+3} \begin{vmatrix} 2 & -3 & 0 \\ -5 & 6 & -2 \\ -6 & -1 & 2 \end{vmatrix}$

$= 2(12 - 2) + 3(-10 - 12)$

$= 20 - 66 = -46$

46. (B) $\left(\sqrt{x} + \frac{1}{2x}\right)^8$

$T_{r+1} = {}^8C_r \left(\sqrt{x}\right)^{8-r}$

$T_{r+1} = {}^8C_r \left(\frac{1}{2}\right)^r x^{\frac{8-3r}{2}}$

Hence $\frac{8 - 3r}{2} = -2$

$\Rightarrow 8 - 3r = -4 \Rightarrow r = 4$

The coefficient of $x^{-2} = {}^8C_4 \left(\frac{1}{2}\right)^4$

$= 70 \times \frac{1}{16} = \frac{35}{8}$

47. (C)

```
 1011001        1110011
  +11010       -1010101
 1110011          11110
```

Hence $(101101)_2 + (11010)_2 - (1010101)_2 = (11110)_2$

48. (A) $I = \int_0^{\pi/4} \log\sin 2x \; dx$ ...(i)

Prop. IV

$\int_0^{\pi/4} f(x)dx = \int_0^{a} f(a-x) \; dx$

$I = \int_0^{\pi/4} \log\sin 2\left(\frac{\pi}{4}-x\right) \; dx$

$I = \int_0^{\pi/4} \log\cos 2x$ ...(ii)

From eq (i) and eq (ii)

$2I = \int_0^{\pi/4} (\log\sin 2x + \log\cos 2x) \; dx$

$2I = \int_0^{\pi/4} \log\frac{2\sin 2x.\cos 2x}{2} \; dx$

$2I = \int_0^{\pi/4} \log\sin 4x \; dx - \int_0^{\pi/4} \log 2 \; dx$

$2I = \int_0^{\pi/4} \log\sin 4x \; dx - \log 2. \; [x]_0^{\pi/4}$

Let $2x = t$ when $x = 0$, $t = 0$

$2dx = dt \Rightarrow dx = \frac{1}{2}dt$ $x = \frac{\pi}{4}$, $t = \frac{\pi}{2}$

$2I = \int_0^{\pi/2} \log\sin 2t \; dt = \frac{\pi}{4}\log 2$

$2I = \frac{1}{2}\int_0^{2\times\frac{\pi}{4}} \log\sin 2t \; dt - \frac{\pi}{4} \log 2$

$2I = \frac{1}{2}\times 2 \frac{d}{dx}\left(e^{x}.\sin x\right) dx - \frac{\pi}{4} \log 2$

$2I = 1 \times I - \frac{\pi}{4} \log 2$ [from eq(i)]

$I = -\frac{\pi}{4} \log 2$

49. (C) $x = \sin t - t\cos t$

On differentiating both side w.r.t. 't'

$\frac{dx}{dt} = \cos t - t.(-\sin t) - \cos t.1$

$\frac{dx}{dt} = t\sin t$

and $y = t\sin t - \cos t$

$\frac{dx}{dt} = t.\cos t + \sin t.1 + \sin t$

$\frac{dy}{dx} = 2\sin t + t.\cos t$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2\sin t + t.\cos t}{t\sin t}$

$\Rightarrow \frac{dy}{dx}$ (at $x = \pi/2$) $= \frac{2\sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2}}{\frac{\pi}{2}\sin\frac{\pi}{2}}$

$\Rightarrow \frac{dy}{dx}$ (at $x = \pi/2$) $= \frac{2\times 1 + \frac{\pi}{2}\times 0}{\frac{\pi}{2}\times 1} = \frac{4}{\pi}$

50. (D) $I = \int_{-1}^{1} x^2|x| \; dx$

$I = \int_{-1}^{0} x^2(-x) \; dx + \int_0^1 x^2.x \, dx$

$I = -\int_{-1}^{0} x^3 \; dx + \int_0^1 x^3 \; dx$

$I = -\left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^4}{4}\right]_0^1$

$I = -\left[0 - \frac{(-1)^4}{4}\right] + \left[\frac{1}{4} - 0\right]$

$I = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

51. (C) We know that

tanA. tan (60–A) .tan (60+A) = tan 3A

52. (A) Differential equation

$e^x \tan y \, dx + (e^x - 1)\sec^2 y \, dy = 0$

$\Rightarrow e^x \tan y \, dx = -(e^x - 1)\sec^2 y \, dy$

$\Rightarrow \frac{e^x}{e^x - 1} dx = -\frac{\sec^2 y}{\tan y} dy$

$\Rightarrow \log(e^x - 1) = -\log\tan y + \log C$

$\Rightarrow \log(e^x - 1) + \log\tan y = \log C$

$\Rightarrow \log\{(e^x - 1)\tan y\} = \log C$

$\Rightarrow (e^x - 1)\tan y = C$

53. (A)

54. (B) sin278 + sin212 + sin82 + sin392

$\Rightarrow$ sin (270 + 8) + sin(180 + 32) + sin (90–8) + sin (360 + 32)

$\Rightarrow$ –cos8 – sin32 + cos8 + sin32 = 0

55. (C) We know that

$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$

56. (B) $\frac{d}{dx}\left(e^x.\sin x\right)$

$\Rightarrow e^x.\cos x + (\sin x).e^x$

$\Rightarrow e^x(\sin x + \cos x)$

57. (A) The required Probability

$= \frac{1}{2}\left[\frac{3}{7}\times\frac{5}{7}+\frac{4}{7}\times\frac{2}{7}+\frac{3}{7}\times\frac{2}{7}\right]$

$= \frac{1}{2}\left[\frac{15}{49}+\frac{8}{49}+\frac{6}{49}\right] = \frac{29}{98}$

58. (C) The required Probability

$= \frac{1}{2}\left[\frac{3}{7}\times\frac{2}{7}+\frac{4}{7}\times\frac{5}{7}\right]$

$= \frac{1}{2}\left[\frac{6}{49}+\frac{20}{49}\right] = \frac{13}{49}$

59. (B) Plane $3x - 4y + 6z + 7 = 0$

equation of plane parallel to the plane

$3x - 4y + 6z + c = 0$ ...(i)

It passes through the point (1, –1, 2)

$3 \times 1 - 4 \times (-1) + 6 \times 2 + c = 0$

$\Rightarrow 3 + 4 + 12 + c = 0 \Rightarrow c = -19$

From eq (i)

$3x - 4y + 6z - 19 = 0$

$\Rightarrow 3x - 4y + 6z = 19$

60. (C) $A = \begin{bmatrix}-1 & 1 & 2\\ 2 & -1 & 2\\ 1 & 6 & 4\end{bmatrix}$ And $B = \begin{bmatrix}-4 & 3 & 0\\ 1 & 2 & -5\\ 2 & -1 & 3\end{bmatrix}$

$AB = \begin{bmatrix}-1 & 1 & 2\\ 2 & -1 & 2\\ 1 & 6 & 4\end{bmatrix}\begin{bmatrix}-4 & 3 & 0\\ 1 & 2 & -5\\ 2 & -1 & 3\end{bmatrix}$

$AB = \begin{bmatrix}9 & -3 & 1\\ -5 & 2 & 11\\ 10 & 11 & -18\end{bmatrix}$

Now, det(AB) = 9(– 36 –121) + 3(90 –110) + 1(– 55 – 20)

$\Rightarrow$ det(AB) = 9 (– 157) + 3 × (– 20) – 75

$\Rightarrow$ det(AB) = –1413 – 60 – 75 = 1548

61. (A) $I = \int_0^2 x(2-x)x^6\ dx$

Prop. IV $\int_0^a f(x)\ dx = \int_0^a f(a-x)\, dx$

$I = \int_0^2 (2-x)x^6\, dx$

$I = \int_0^2 (2x^6 - x^7)\, dx$

$I = \left[2\times\frac{x^7}{7} - \frac{x^8}{8}\right]^2$

$I = \frac{2}{7}\times 2^7 - \frac{2^8}{8}$

$I = 2^8\left(\frac{1}{7}-\frac{1}{8}\right)$

$I = 2^8 \times \frac{1}{56} = \frac{32}{7}$

62. (D) We know that

$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

$A = 22\frac{1}{2}$

$\Rightarrow \tan 45 = \frac{2\tan 22\frac{1}{2}}{1-\tan^2 22\frac{1}{2}}$

$\Rightarrow 1 = \frac{2\tan 22\frac{1}{2}}{1-\tan^2 22\frac{1}{2}}$

$\Rightarrow 1 - \tan^2 22\frac{1}{2} = 2\tan 22\frac{1}{2}$

$\Rightarrow \tan^2 22\frac{1}{2} + 2\tan 22\frac{1}{2} - 1 = 0$

$\Rightarrow \tan 22\frac{1}{2} = \frac{-2\pm\sqrt{2^2-4\times1\times(-1)}}{2\times 1}$

$\Rightarrow \tan 22\frac{1}{2} = \frac{-2\pm 2\sqrt{2}}{2}$

$\Rightarrow \tan 22\frac{1}{2} = -1 \pm \sqrt{2}$

Hence $\tan 22\frac{1}{2} = \sqrt{2}-1$

63. (C) Curve $25x^2 + 25y^2 = 1$

$x^2 + y^2 = \frac{1}{25}$

here $r^2 = \frac{1}{25}$

Area of the circle = πr^2

$= \pi \times \frac{1}{25} = \frac{\pi}{25}$ sq.unit

64. (C) $I = \int \sqrt{x}.e^{\sqrt{x}}\ dx = \int \frac{x.e^{\sqrt{x}}}{\sqrt{x}}\ dx$

Let $\sqrt{x} = t \Rightarrow x = t^2$

$\frac{1}{2\sqrt{x}}\ dx = dt \Rightarrow \frac{1}{\sqrt{x}}\ dx = 2\ dt$

$I = \int t^2.e^t \times 2\, dt$

$I = 2\left[t^2\int e^t dt - \int\left\{\frac{d}{dt}(t^2).\int e^t dt\right\}dt\right]$

$I = 2\left[t^2 \times e^t - \int 2t.e^t\, dt\right]$

$I = 2t^2\, e^t - 4\left[t\int e^t dt - \int\left\{\frac{d}{dt}(t).\int e^t dt\right\}dt\right]$

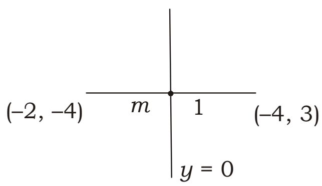
$I = 2t^2.e^t - 4\left[t.e^t - \int 1.e^t dt\right]$

$I = 2t^2.e^t - 4[t.e^t - e^t] + c$

$I = 2t^2.\ e^t - 4t.e^t + 4e^t + c$

$I = 2x.\ e^{\sqrt{x}} - 4\sqrt{x}\ e^{\sqrt{x}} + 4e^{\sqrt{x}} + c$

65. (B)



Now, $\frac{m \times 3 - 4 \times 1}{m+1} = 0$

$\Rightarrow 3m - 4 = 0 \Rightarrow m = \frac{4}{3}$

The required ratio = 4 : 3

66. (D) $(\log_5 x)(\log_x 3x)(\log_{3x} 2y) = \log_x x^3$

$\Rightarrow (\log_5 3x)(\log_{3x} 2y) = 3\log_x x)$

$\Rightarrow \log_5 2y = 3$

$\Rightarrow 2y = 5^3 \Rightarrow y = \frac{125}{2}$

67. (A) Given line $2x - 6y = 7$

equation of line which is parallel to the given line.

$2x - 6y = c$ .... (i)

its passes through the point (– 1, 2)

$2 \times (-1) - 6 \times 2 = c \Rightarrow c = -14$

from equation (i)

$2x - 6y = -14 \Rightarrow 2x - 6y + 14 = 0$

68. (D) $n = 9$

The no. of diagonals = $\frac{n(n-3)}{2}$

$= \frac{9 \times 6}{2} = 27$

69. (B) We know that

$(1 + x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$

On putting $x = 1$

$(1 + 1)^n = C_0 + C_1 + C_2 + ... + C_n$

$\Rightarrow C_0 + C_1 + C_2 + ... + C_n = 2^n$

70. (C) 7, 8, 18, 21, 22, 23, 25, 27, 29

Mean $(\bar{x}) = \frac{7+8+18+21+22+23+25+27+29}{9}$

$\bar{x} = \frac{180}{9} = 20$

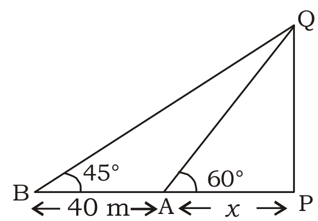
$\Sigma(x - \bar{x})^2 = (7 - 20)^2 + (8 - 20)^2 + (18 - 20)^2 + (21 - 20)^2 + (22 - 20)^2 + (23 - 20)^2 + (25 - 20)^2 + (27 - 20)^2 + (29 - 20)^2$

$\Sigma(x - \bar{x})^2 = 169 + 144 + 4 + 1 + 4 + 9 + 25 + 49 + 81 = 486$

Standard Deviation = $\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{486}{9}} = \sqrt{54} = 3\sqrt{6}$

71. (A)



Let breadth of the river = x m

In ΔAPQ

$\tan 60° = \frac{PQ}{AP}$

$\Rightarrow \sqrt{3} = \frac{PQ}{x} \Rightarrow PQ = \sqrt{3}\, x$

In ΔBPQ

$\tan 45° = \frac{PQ}{BP}$

$\Rightarrow 1 = \frac{\sqrt{3}x}{x + 40}$

$\Rightarrow x + 40 = \sqrt{3}\, x$

$\Rightarrow x = \frac{40}{\sqrt{3} - 1} = 20(\sqrt{3} + 1)$

Hence breadth of the river = $20(\sqrt{3} + 1)$ m

72. (C) $\sum_{n=1}^{8}(i^{n-1} - i^n)$

$\Rightarrow (i^0 - i^1) + (i^1 - i^2) + (i^2 - i^3) + ... + (i^7 - i^8)$

$\Rightarrow 1 - i^8$

$\Rightarrow 1 - 1 = 0$

73. (B) $I = \int \frac{x+3}{(x+2)(x-4)}\,dx$

$I = \int\left(\frac{-1}{6(x+2)} + \frac{7}{6(x-4)}\right)dx$

$I = \frac{-1}{6}\log(x+2) + \frac{7}{6}\log(x-4) + c$

74. (C)

2	605	1
2	302	0
2	151	1
2	75	1
2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

$(605)_{10} = (1001011101)_2$

75. (C) $\lim_{x\to 0} \frac{\log(\sin x + \cos x)}{x}$ $\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0} \frac{\frac{1}{\sin x + \cos x}\times(\cos x - \sin x)}{1}$

$\Rightarrow \lim_{x\to 0} \frac{\cos x - \sin x}{\sin x + \cos x}$

$\Rightarrow \frac{\cos 0 - \sin 0}{\sin 0 + \cos 0} = \frac{1-0}{0+1} = 1$

76. (B) $12°30' = \left(12+\frac{1}{2}\right)^{\circ}$

$\Rightarrow 12°30' = \frac{25}{2}\times\frac{\pi}{180} = \frac{5\pi}{72}$

77. (D) Word "PERCENT"

The required no. of words = $\frac{7!}{2!}$ = 2520

The required numbers of words which begin with P and end with T = $\frac{5!}{2!}$ = 60

78. (A) $I = \int \sin^{-1} x\ dx$

$I = \sin^{-1}x\int 1.dx - \int\left\{\frac{d}{dx}(\sin^{-1} x.\int 1.dx\right\}dx$

$I = (\sin^{-1}x).x - \int \frac{1}{\sqrt{1-x^2}}\times x\,dx$

$I = x.\sin^{-1}x - \int \frac{1}{\sqrt{1-x^2}}\,dx$

let $1 - x^2 = t$

$-2x\,dx = dt \Rightarrow x\,dx = \frac{-1}{2}dt$

$I = x.\sin^{-1}x - \int \frac{-1}{2}\times\frac{1}{\sqrt{t}}\,dt$

$I = x.\sin^{-1}x + \frac{1}{2}\times\frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$

$I = x.\sin^{-1}x + \sqrt{t} + c$

$I = x.\sin^{-1}x + \sqrt{1-x^2} + c$

79. (C) Given that $A = \begin{bmatrix}1 & 4 & -1\\ 0 & 3 & -2\\ 1 & -1 & 3\end{bmatrix}$, $B = \begin{bmatrix}-1\\ 2\\ 4\end{bmatrix}$

Using elementary method

$[A/I] = \left[\begin{array}{ccc|ccc}1 & 4 & -1 & 1 & 0 & 0\\ 0 & 3 & -2 & 0 & 1 & 0\\ 1 & -1 & 3 & 0 & 0 & 1\end{array}\right]$

$[A/I] = \left[\begin{array}{ccc|ccc}1 & 4 & -1 & 1 & 0 & 0\\ 0 & 3 & -2 & 0 & 1 & 0\\ 0 & -5 & 4 & -1 & 0 & 1\end{array}\right]$

$R_1 \to R_1 - \frac{4}{3}R_2$ and $R_3 \to R_3 + \frac{5}{3}R_2$

$[A/I] = \left[\begin{array}{ccc|ccc}1 & 0 & \frac{5}{3} & 1 & \frac{-4}{3} & 0\\ 0 & 3 & -2 & 0 & 1 & 0\\ 0 & 0 & \frac{2}{3} & -1 & \frac{5}{3} & 1\end{array}\right]$

$R_2 \to \frac{R_2}{3}$ and $R_3 \to \frac{3}{2}R_3$

$[A/I] = \left[\begin{array}{ccc|ccc}1 & 0 & \frac{5}{3} & 1 & \frac{-4}{3} & 0\\ 0 & 1 & \frac{-2}{3} & 0 & \frac{1}{3} & 0\\ 0 & 0 & 1 & \frac{-3}{2} & \frac{5}{2} & \frac{3}{2}\end{array}\right]$

$R_1 \to R_1 - \frac{5}{3}R_3$ and $R_2 \to R_2 + \frac{2}{3}R_3$

$$[A/I] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{2} & \frac{-11}{2} & \frac{-5}{2} \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & \frac{-3}{2} & \frac{5}{2} & \frac{3}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{7}{2} & \frac{-11}{2} & \frac{-5}{2} \\ -1 & 2 & 1 \\ \frac{-3}{2} & \frac{5}{2} & \frac{3}{2} \end{bmatrix}$$

Now, $X = A^{-1}B$

$$X = \begin{bmatrix} \frac{7}{2} & \frac{-11}{2} & \frac{-5}{2} \\ -1 & 2 & 1 \\ \frac{-3}{2} & \frac{5}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{-49}{2} \\ 9 \\ \frac{25}{2} \end{bmatrix}$$

80. (B) $S = \frac{1}{3} - \frac{1}{9} + \frac{1}{27} + \ldots\ldots\ldots \infty$

$$S = \frac{\frac{1}{3}}{1+\frac{1}{3}}$$

$$S = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

81. (A) Differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} + \frac{dy}{dx} = y^2$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{\frac{1}{3}} = y^2 - \frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(y^2 - \frac{dy}{dx}\right)^3$$

Hence order = 2 and degree = 1

82. (B) $I = \int e^x \left(\frac{3x-1}{x^{\frac{4}{3}}}\right) dx$

$$I = 3\int e^x \left(\frac{3x-1}{3x^{4/3}}\right) dx$$

$$I = 3\int e^x \left(x^{\frac{-1}{3}} - \frac{1}{3}x^{\frac{-4}{3}}\right) dx$$

We know that $\int e^x \left[f(x) + f'(x)\right] = e^x.f(x) + c$

$$I = 3e^x.\, x^{\frac{-1}{3}} + c$$

$$I = \frac{3e^x}{x^{1/3}} + c$$

83. (B) In the expansion of $\left(\frac{2x}{y} - \frac{y}{6x}\right)^6$

Total term = 6 + 1 = 7

Middle term = $\left(\frac{6}{2}+1\right)^{th} = 4^{th}$

$$T_4 = T_{3+1} = {}^6C_3 \left(\frac{2x}{y}\right)^3 \left(\frac{-y}{6x}\right)^3$$

$$T_4 = 20 \times \frac{8x^3}{y^3}\left(\frac{-y^3}{216x^3}\right) = \frac{-20}{27}$$

84. (A)

85. (C) Digits 0, 1, 2, 4, 5, 7, 9

when last digit is '0'

4	1	= 4
	↓ 0	

when last digit is '2'

4	1	= 4
	↓ 2	

when last digit is '4'

3	1	= 3
	↓ 4	

The required numbers = 4 + 4 + 3 = 11

86. (B) $n(S) = 16$

E = {(HHHT), (HTHH), (HHTH), (THHH)}

$n(E) = 4$

The required Probability $= \frac{n(E)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

87. (C) tan390 – cot690

⇒ tan (360 + 30) – cot(720 – 30)

⇒ tan30° + cot30

$$\Rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}}$$

88. (D) $\dfrac{\sin 330°.\cot 75°.\tan 135°}{\cos 425°.\sin 750°.\cot 225°}$

$\Rightarrow \dfrac{\sin(360-30).\cos 75.\tan(90+45)}{\cos(360+75).\sin(720+30).\cot(180+45)}$

$\Rightarrow \dfrac{-\sin 30.\cos 75.(-\tan 45)}{\cos 75.\sin 30.\cot 45}$

$\Rightarrow \dfrac{-\frac{1}{2}\times\cos 75\times(-1)}{\cos 75\times\frac{1}{2}\times 1} = 1$

89. (C) $\lim\limits_{x\to 0}\dfrac{25^x-16^x}{x(5^x+4^x)}$ $\left[\dfrac{0}{0}\right]$form

by L-Hospital's Rule

$\Rightarrow \lim\limits_{x\to 0}\dfrac{25^x\log 25-16^x\log 16}{x(5^x\log 5+4^x\log 4)+(5^x+4^x)}$

$\Rightarrow \dfrac{25^0\times 2\log 5-16^0\times 4\log 2}{0+(5^0+4^0)}$

$\Rightarrow \dfrac{2\log 5-4\log 2}{2}$

$\Rightarrow$ log5 – 2log2

$\Rightarrow$ log5 – log4 = $\log\dfrac{5}{4}$

90 (A) $\sin^{-1}x = \cot^{-1}y$

$\Rightarrow \sin^{-1}x = \tan^{-1}\dfrac{1}{y}$ $\left[\because \tan^{-1}A = \cot^{-1}\dfrac{1}{A}\right]$

$\Rightarrow \sin^{-1}x = \sin^{-1}\dfrac{\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}$ $\left[\because \tan^{-1}A = \sin^{-1}\dfrac{A}{\sqrt{1+A^2}}\right]$

$\Rightarrow \sin^{-1}x = \sin^{-1}\dfrac{1}{\sqrt{y^2+1}}$

$\Rightarrow x = \dfrac{1}{\sqrt{y^2+1}}$

$\Rightarrow x^2 = \dfrac{1}{y^2+1} \Rightarrow x^2(1+y^2) = 1$

91. (C) Let the equation of sphere

$x^2+y^2+z^2+2ux+2vy+2wz+d = 0$ ...(i)

its passes through the points (0, 0, 0), (–1, 0, 0), (0, –3, 0) and (0, 0, 4)

$d = 0$ ...(ii)

$1 + 2u(-1) + d = 0 \Rightarrow u = \dfrac{1}{2}$ ...(iii)

$9 + 2v(-3) + d = 0 \Rightarrow v = \dfrac{3}{2}$ ...(iv)

$16 + 2w(4) + d \Rightarrow w = -2$ ...(v)

On putting the value of u, u, w and d in eq(i)

$\Rightarrow x^2+y^2+z^2+2\times\dfrac{1}{2}x+2\times\dfrac{3}{2}y+2\times(-2)z = 0$

$\Rightarrow x^2+y^2+z^2+x+3y-4z = 0$

92. (B) $I = \displaystyle\int_0^{\pi/2}\dfrac{(\sin x)^{3/2}}{(\sin x)^{3/2}+(\cos x)^{3/2}}$ ...(i)

Prop.IV $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$I = \displaystyle\int_0^{\pi/2}\dfrac{\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{3/2}}{\left[\sin\left(\frac{\pi}{2}-x\right)\right]^{3/2}+\left[\cos\left(\frac{\pi}{2}-x\right)\right]^{3/2}}dx$

$I = \displaystyle\int_0^{\pi/2}\dfrac{(\cos x)^{3/2}}{(\cos x)^{3/2}+(\sin x)^{3/2}}dx$ ...(ii)

from eq(i) and eq(ii)

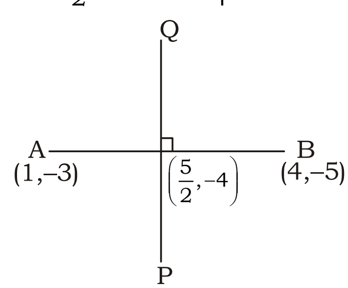
$2I = \displaystyle\int_0^{\pi/2}\dfrac{(\sin x)^{3/2}+(\cos x)^{3/2}}{(\cos x)^{3/2}+(\sin x)^{3/2}}dx$

$2I = \displaystyle\int_0^{\pi/2}1.dx$

$2I = [x]_0^{\pi/2}$

$2I = \dfrac{\pi}{2}-0 \Rightarrow I = \dfrac{\pi}{4}$

93. (A)



Mid = point of joining the points =

$\left(\dfrac{1+4}{2},\dfrac{-3-5}{2}\right) = \left(\dfrac{5}{2},-4\right)$

Slope of line AB $(m_1) = \dfrac{-5+3}{4-1} = \dfrac{-2}{3}$

Slope of line PQ $(m_2) = \dfrac{-1}{-2/3} = \dfrac{3}{2}$

Equation of line PQ

$y + 4 = \dfrac{3}{2}\left(x-\dfrac{5}{2}\right)$

$\Rightarrow y + 4 = \dfrac{3}{2}\times\dfrac{2x-5}{2}$

$\Rightarrow 6x - 4y = 31$

94. (C) $\left[\dfrac{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}}\right]^2$

$\Rightarrow \dfrac{\cos\left(2\times\frac{\pi}{4}\right)+i\sin\left(2\times\frac{\pi}{4}\right)}{\cos\left(2\times\frac{\pi}{4}\right)-i\sin\left(2\times\frac{\pi}{4}\right)}$

$\Rightarrow \dfrac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}}$

$\Rightarrow \dfrac{0+i\times1}{0-i\times1} = \dfrac{i}{-i} = -1$

95. (A) $(A\cap C)\cup(B\cap C)$

96. (B) $m = \begin{vmatrix}-1 & -1\\ 1 & -1\end{vmatrix} = 1 + 1 = 2$

$n = \begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix} = -1 - 1 = -2$

Now, $m\sin^2\theta - n\cos^2\theta$

$\Rightarrow 2\sin^2\theta + 2\cos^2\theta = 2$

97. (C) $f(x) = \dfrac{1}{\sqrt{x^2-1}}$

Now, $x^2 - 1 > 0$

$\Rightarrow x^2 > 1$

$\Rightarrow x > 1$ and $x < -1$

Domain = $(-\infty, -1)\cup(1,\infty)$

98. (B) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$

$\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{4}\right)\right]$

$\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \frac{\pi}{4}$

99. (C) $y\,dx + x\,dy = y\,dy$

$\Rightarrow d(xy) = y\,dy$

On integrating

$\Rightarrow \int d(xy) = \int y\,dy$

$\Rightarrow xy = \dfrac{y^2}{2} + c$ ...(i)

Given that $y(1) = -2$ i.e. $x = 1$, $y = -2$

$\Rightarrow 1\times(-2) = \dfrac{(-2)^2}{2} + c$

$\Rightarrow -2 = 2 + c \Rightarrow c = -4$

from eq(i)

$\Rightarrow xy = \dfrac{y^2}{2} - 4$

On putting $x = \dfrac{-7}{2}$

$\Rightarrow \dfrac{-7}{2}y = \dfrac{y^2}{2} - 4$

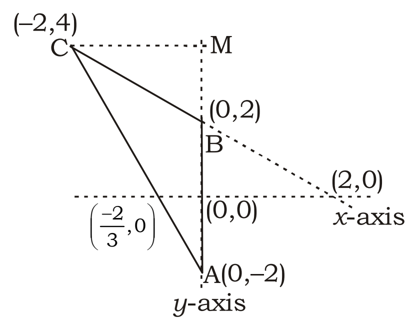
$\Rightarrow \dfrac{-7y}{2} = \dfrac{y^2-8}{2}$

$\Rightarrow y^2 + 7y - 8 = 0$

$\Rightarrow (y-1)(y+8) = 0 \Rightarrow y = 1, -8$

$\Rightarrow$ Hence $y\left(\dfrac{-7}{2}\right) = 1$ or -8

100. (B)



Given lines $3x + y = -2$, $x + y = 2$ and y-axis

The required Area = $\frac{1}{2}\times AB\times CM$

$= \frac{1}{2}\times4\times2 = 4$ sq.unit

101. (C) Differential equation

$\dfrac{dy}{dx}+\dfrac{y}{x} = x$

$P = \dfrac{1}{x}$, $Q = x$

I.F. $= e^{\int P\,dx}$

I.F. $= e^{\int\frac{1}{x}dx}$

I.F. $= e^{\ln x} = x$

Solution of the differential equation

$y\times$ I.F. $= \int Q\times\text{I.F.}\ dx$

$\Rightarrow y\times x = \int x\times x\,dx$

$\Rightarrow xy = \int x^2\,dx$

$\Rightarrow xy = \dfrac{x^3}{3}+\dfrac{c}{3}$

$\Rightarrow 3xy = x^3 + c$

(102-104)

Given that $f(x) = x$ and $g(x) = [x]$

102. (B) $I = \int_0^3 g(x)\,dx$

$I = \int_0^3 [x]\,dx$

$I = \int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx$

$I = \int_0^1 0.dx + \int_1^2 1.dx + \int_2^3 2.dx$

$I = 0 + [x]_1^2 + 2[x]_2^3$

$I = (2-1) + 2(3-2)$

$I = 1 + 2\times1 = 3$

103. (C) $I = \int_0^2 f(x)\,dx$

$I = \int_0^2 x\,dx$

$I = \left[\frac{x^2}{2}\right]_0^2$

$I = \left[\frac{4}{2} - 0\right] = 2$

104. (D) $I = \int_1^2 g[f(x)]^2 dx$

$I = \int_1^2 g(x^2)dx$

$I = \int_1^2 [x^2]dx$

$I = \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^2 [x^2]dx$

$I = \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx + \int_{\sqrt{3}}^2 3.dx$

$I = [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3\,[x]_{\sqrt{3}}^2$

$I = (\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3})$

$I = \sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$

$I = 5 - \sqrt{3} - \sqrt{2}$

105. (C) The required Probability $= \frac{8}{52} = \frac{2}{13}$

106. (D)

class	f	c
0-10	97	97
10-20	98	195
20-30	99	294
30-40	97	391
40-50	109	500

20-30: median class

$\frac{N}{2} = \frac{500}{2} = 250$

$f = 99,\ l_1 = 20,\ l_2 = 30,\ C = 195$

$\text{Median} = l_1 + \frac{\frac{N}{2} - C}{f} \times (l_2 - l_1)$

$= 20 + \frac{250-195}{99} \times (30-20)$

$= 20 + \frac{55}{99} \times 10$

$= 20 + 5\frac{55}{99} = 25\frac{55}{99}$

107. (B) $f(x) = \begin{cases} x^2+1, & x<1 \\ 3x+\lambda, & x\ge 1 \end{cases}$ is continuous

at $x = 1$, then

$\lim_{x\to1^-} f(x) = f(1)$

$\Rightarrow \lim_{x\to1} (x^2+1) = 3\times1 + \lambda$

$\Rightarrow 1 + 1 = 3 + \lambda \Rightarrow \lambda = -1$

108. (C) Plane $3x + 2y + z + 4 = 0$ and

line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$

Angle between plane and line

$\sin\theta = \frac{3\times2 + 2\times(-3) + 1\times1}{\sqrt{3^2+2^2+(1)^2}\sqrt{2^2+(-3)^2+1^2}}$

$\sin\theta = \frac{6-6+1}{\sqrt{14}\sqrt{14}}$

$\sin\theta = \frac{1}{14} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{14}\right)$

109. (A) In ABC, A = 30°, B = 15°, c = 4 cm

then C = 180 – 30 – 15 = 135°

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Now, $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 30} = \frac{4}{\sin 135}$

$\Rightarrow \frac{a}{1/2} = \frac{4}{1/\sqrt{2}} \Rightarrow a = 2\sqrt{2}$ cm

110. (C) ${}^{22}C_{3+r} = {}^{22}C_{3r-1}$

Now, $3 + r + 3r - 1 = 22$

$\Rightarrow 4r + 2 = 22 \Rightarrow r = 5$

111. (B) $2^{3-\log_2 4+\log_2 9}$

$\Rightarrow 2^{3+\log_2 \frac{9}{4}}$

$\Rightarrow 2^3 . 2^{\log_2 \frac{9}{4}}$

$\Rightarrow 8 \times \frac{9}{4} = 18$

112. (D) $\cos^{-1}\frac{8}{x} + \cos^{-1}\frac{15}{x} = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}\frac{8}{x} = \frac{\pi}{2} - \cos^{-1}\frac{15}{x}$

$\Rightarrow \cos^{-1}\frac{8}{x} = \sin^{-1}\frac{15}{x}$

$\Rightarrow \sin^{-1}\sqrt{1-\frac{64}{x^2}} = \sin^{-1}\frac{15}{x}$

$\Rightarrow \sqrt{1-\frac{64}{x^2}} = \frac{15}{x}$

$\Rightarrow 1 - \frac{64}{x^2} = \frac{225}{x^2}$

$\Rightarrow 1 = \frac{289}{x^2} \Rightarrow x = 17$

113. (A) $\begin{vmatrix} x+c & a & b \\ c & x+a & b \\ c & a & x+b \end{vmatrix} = 0$

$C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow \begin{vmatrix} x+a+b+c & a & b \\ x+a+b+c & x+a & b \\ x+a+b+c & a & x+b \end{vmatrix} = 0$

$\Rightarrow (x + a + b + c)\begin{vmatrix} 1 & a & b \\ 1 & x+a & b \\ 1 & a & x+b \end{vmatrix} = 0$

$\Rightarrow x + a + b + c = 0$

$\Rightarrow x = -(a + b + c)$

Hence one root $= -(a + b + c)$

114. (A) $3f(x-2) + f\left(\frac{1}{x-2}\right) = 2x$ ...(i)

On putting $x = 5$

$3f(3) + f\left(\frac{1}{3}\right) = 2\times5$

$\Rightarrow 3f(3) + f\left(\frac{1}{3}\right) = 10$ ...(ii)

On putting $x = \frac{7}{3}$

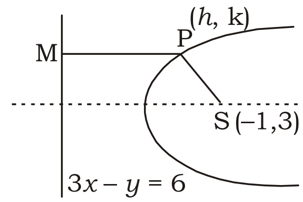
$3f\left(\frac{1}{3}\right) + f(3) = 2\times\frac{7}{3}$

$\Rightarrow 3f\left(\frac{1}{3}\right) + f(3) = \frac{14}{3}$ ...(iii)

On solving eq(ii) and eq(iii)

$f(3) = \frac{19}{6}$

115. (B)



Let $P(h, k)$

Now, PS = PM

$\Rightarrow \sqrt{(h+1)^2+(k-3)^2} = \frac{3h-k-6}{\sqrt{3^2+(-1)^2}}$

On squaring

$\Rightarrow (h+1)^2 + (k-3)^2 = \frac{(3h-k-6)^2}{10}$

On solving

$h^2 + 9k^2 + 6hk + 56h - 72k + 64 = 0$

Equation of parabola

$x^2 + 9y^2 + 6xy + 56x - 72y + 64 = 0$

116. (D) Let $f(x) = \frac{2[x]}{x}$

L.H.L. $= \lim_{x\to2^-} f(x) = \lim_{x\to0} f(2-h)$

$= \lim_{h\to0} \frac{2[2-h]}{2-h}$

$= \lim_{h\to0} \frac{2\times1}{2-h} = 1$

R.H.L. $= \lim_{x\to2^+} f(x) = \lim_{h\to0} f(2+h)$

$= \lim_{h\to0} \frac{2[2+h]}{2+h}$

$= \lim_{h\to0} \frac{2\times2}{2+h} = 2$

L.H.L. ≠ R.H.L.

Hence limit does not exist.

117. (C) Data –2, 3, 6, –7, –2, 6, –5, –1

$$\text{Mean} = \frac{-2+3+6-7-2+6-5-1}{8}$$

$$= \frac{-2}{8} = \frac{-1}{4}$$

118. (A) $f(x) = \dfrac{x-2}{x}$ $x \neq 0$

x	$f(x)$
1	–1
2	0
3	$\frac{1}{3}$
:	:

Function is one-one and into.

119. (C) Let $z = \dfrac{(1-3i)(2+i)}{i+1}$

$$z = \frac{2-6i+i-3i^2}{i+1}$$

$$z = \frac{5-5i}{i+1} \times \frac{i-1}{i-1}$$

$$z = \frac{5i-5i^2-5+5i}{i^2-1}$$

$$z = \frac{10i}{-2} = -5i$$

Now, $\arg(z) = \tan^{-1}\left(\dfrac{b}{a}\right)$

$\Rightarrow \arg(z) = \tan^{-1}\left(\dfrac{-5}{0}\right)$

$\Rightarrow \arg(z) = \tan^{-1}(\infty) = \dfrac{\pi}{2}$

120. (B) Given that $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

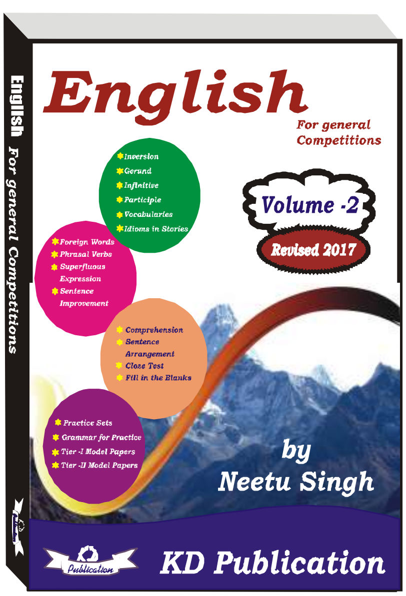
and $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$

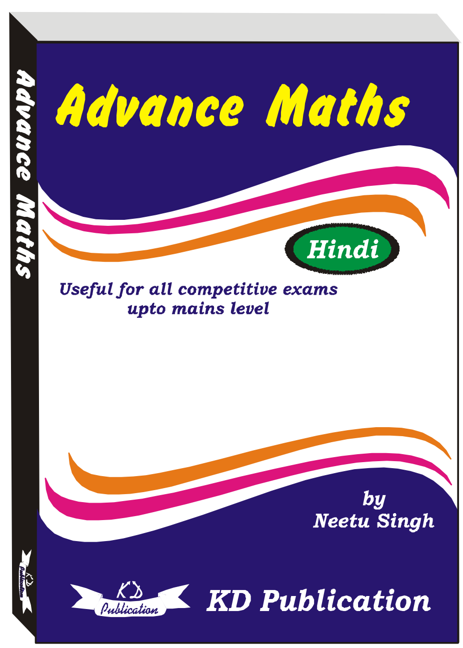
Projection of $\vec{b}$ on $\vec{a}$ = $\dfrac{\vec{a}.\vec{b}}{|\vec{a}|}$

$$= \frac{3\times1+4\times(-4)+5\times5}{\sqrt{3^2+4^2\times5^2}}$$

$$= \frac{3-16+25}{\sqrt{50}}$$

$$= \frac{12}{5\sqrt{2}} = \frac{6\sqrt{2}}{5}$$





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NDA (MATHS) MOCK TEST - 144 (Answer Key)

1. (D)	21. (C)	41. (C)	61. (A)	81. (A)	101. (C)
2. (A)	22. (B)	42. (A)	62. (D)	82. (B)	102. (B)
3. (B)	23. (B)	43. (C)	63. (C)	83. (B)	103. (C)
4. (C)	24. (C)	44. (C)	64. (C)	84. (A)	104. (D)
5. (D)	25. (A)	45. (D)	65. (B)	85. (C)	105. (C)
6. (D)	26. (D)	46. (B)	66. (D)	86. (B)	106. (D)
7. (B)	27. (B)	47. (C)	67. (A)	87. (C)	107. (B)
8. (C)	28. (C)	48. (A)	68. (D)	88. (D)	108. (C)
9. (A)	29. (A)	49. (C)	69. (B)	89. (C)	109. (A)
10. (D)	30. (C)	50. (D)	70. (C)	90. (A)	110. (C)
11. (C)	31. (B)	51. (C)	71. (A)	91. (C)	111. (B)
12. (A)	32. (C)	52. (A)	72. (C)	92. (B)	112. (D)
13. (B)	33. (D)	53. (A)	73. (B)	93. (A)	113. (A)
14. (C)	34. (D)	54. (B)	74. (C)	94. (C)	114. (A)
15. (C)	35. (B)	55. (C)	75. (C)	95. (A)	115. (B)
16. (B)	36. (C)	56. (B)	76. (B)	96. (B)	116. (D)
17. (B)	37. (B)	57. (A)	77. (D)	97. (C)	117. (C)
18. (A)	38. (C)	58. (C)	78. (A)	98. (B)	118. (A)
19. (C)	39. (C)	59. (B)	79. (C)	99. (C)	119. (C)
20. (D)	40. (B)	60. (C)	80. (B)	100. (B)	120. (B)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777