NDA MATHS MOCK TEST - 150 (SOLUTION)

1. (B) Equations $\alpha x + (1+\beta)y = 3$ and $(1+\alpha)x + \beta y = 2$ has a unique solution, then

$$\frac{\alpha}{1+\alpha} \neq \frac{1+\beta}{\beta}$$

$\Rightarrow \alpha\beta \neq (1+\alpha)(1+\beta)$

$\Rightarrow \alpha\beta \neq 1 + \alpha + \beta + \alpha\beta$

$\Rightarrow 0 \neq 1 + \alpha + \beta \Rightarrow \alpha + \beta \neq -1$

2. (A) $I = \int \cos(\log_e x)\, dx$

Let $\log_e x = t$

$\Rightarrow x = e^t \Rightarrow dx = e^t\, dt$

$I = \int \cos t\, e^t\, dt$

$$I = \cos t \int e^t\, dt - \int \left\{ \frac{d}{dt}(\cos t) . \int e^t\, dt \right\} dt$$

$$I = \cos t . e^t - \int (-\sin t) . e^t\, dt$$

$$I = e^t . \cos t + \int \sin t . e^t\, dt$$

$$I = e^t . \cos t + \sin t . \int e^t\, dt - \int \left\{ \frac{d}{dt}(\sin t) \int e^t\, dt \right\} dt$$

$$I = e^t . \cos t + \sin t . e^t - \int \cos t . e^t\, dt$$

$I = e^t . \cos t + \sin t . e^t - I + c$

$2I = e^t(\cos t + \sin t) + c$

$$I = \frac{e^t(\cot t + \sin t)}{2} + c$$

$$I = \frac{1}{2} x[\cos(\log_e x) + \sin(\log_e x)] + c$$

3. (B) $$\lim_{x \to \pi/4} \frac{\tan^2 x - \cot x}{\cos\left(x + \frac{\pi}{4}\right)} \quad \left[\frac{0}{0}\right] \text{ from}$$

by L-Hospital's Rule

$$\Rightarrow \lim_{x \to \pi/4} \frac{2\tan x . \sec^2 x + \text{cosec}^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$$

$$\Rightarrow \frac{2\tan\frac{\pi}{4} . \sec^2\frac{\pi}{4} + \text{cosec}^2\frac{\pi}{4}}{-\sin\left(\frac{\pi}{4} + \frac{\pi}{4}\right)}$$

$$\Rightarrow \frac{2 \times 1 \times (\sqrt{2})^2 + (\sqrt{2})^2}{-1}$$

$$\Rightarrow \frac{4+2}{-1} = -6$$

4. (C) Let $y = 7^{53}$

taking log both side

$\Rightarrow \log_{10} y = 53 \log_{10} 7$

$\Rightarrow \log_{10} y = 53 \times 0.8451$

$\Rightarrow \log_{10} y = 44.7903$

The required number of digit = 44 +1= 45

5. (B)

2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

Hence $(63)_{10} = (111111)_2$

6. (C) $(\cos x - i \sin x)^3$

$\Rightarrow \cos^3 x - i^3 \sin^3 x - 3\cos x . i . \sin x(\cos x - i\sin x)$

$\Rightarrow \cos^3 x + i\sin^3 x - i \times 3\cos^2 x . \sin x + i^2 \times 3 \cos x . \sin^2 x$

$\Rightarrow \cos^3 x + i\sin^3 x - i \times 3\cos^2 x . \sin x - 3 \cos x . \sin^2 x$

$\Rightarrow \cos^3 x - 3\cos x(1 - \cos^2 x) + i[\sin^3 x - 3\sin x(1 - \sin^2 x)]$

$\Rightarrow \cos^3 x - 3\cos x + 3\cos^3 x + i(\sin^3 x - 3\sin x + 3\sin^3 x)$

$\Rightarrow 4\cos^3 x - 3\cos x - i(3\sin x - 4\sin^3 x)$

$\Rightarrow \cos 3x - i\sin 3x$

Real part = $\cos 3x$

7. (C) Given that vertices are (–1, 2), (–2,–5) and (6, a)

A.T.Q,

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -1 & 2 & 1 \\ -2 & -5 & 1 \\ 6 & a & 1 \end{vmatrix}$$

$$\Rightarrow 4 = \frac{1}{2}[-1(-5-a) - 2(-2-6) + 1(-2a+30)]$$

$\Rightarrow 8 = [5 + a + 16 - 2a + 30]$

$\Rightarrow 8 = -a + 51 \Rightarrow a = 43$

8. (D) $$\sin\left(\frac{A+B}{2}\right) . \sin\left(\frac{B+C}{2}\right) . \sin\left(\frac{C+A}{2}\right)$$

$$\Rightarrow \sin\left(\frac{180-C}{2}\right) . \sin\left(\frac{180-A}{2}\right) . \sin\left(\frac{180-B}{2}\right)$$

$$\Rightarrow \cos\frac{C}{2} . \cos\frac{A}{2} . \cos\frac{B}{2}$$

$$\Rightarrow \frac{1}{2} \times 2\cos\frac{C}{2} . \cos\frac{A}{2} . \cos\frac{B}{2}$$

$\Rightarrow \frac{1}{2}\left[\cos\left(\frac{C+A}{2}\right)+\cos\left(\frac{C-A}{2}\right)\right].\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}\left[\cos\left(\frac{180-B}{2}\right)+\cos\left(\frac{C-A}{2}\right)\right].\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}\left[\sin\frac{B}{2}+\cos\frac{C-A}{2}\right]\cos\frac{B}{2}$

$\Rightarrow \frac{1}{2}.\sin\frac{B}{2}.\cos\frac{B}{2}+\frac{1}{2}\sin\frac{C+A}{2}.\cos\frac{C-A}{2}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2}\times 2\ \sin\frac{B}{2}.\ \cos\frac{B}{2}+\ \frac{1}{2}\ \times\ \frac{1}{2}$
$\times\left[2\sin\frac{C+A}{2}.\cos\frac{C-A}{2}\right]$

$\Rightarrow \frac{1}{4}\sin B + \frac{1}{4}$
$\left[\sin\left(\frac{C+A}{2}+\frac{C-A}{2}\right)+\sin\left(\frac{C+A}{2}-\frac{C-A}{2}\right)\right]$

$\Rightarrow \frac{1}{4}\sin B + \frac{1}{4}[\sin C + \sin A]$

$\Rightarrow \frac{1}{4}[\sin A + \sin B + \sin C]$

9. (B) $I = \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Let $\sin^{-1}x = t$ when $x = 0,\ t = 0$

$\Rightarrow \frac{1}{\sqrt{1-x^2}}\,dx = dt$ $\quad x = \frac{1}{\sqrt{2}},\ t = \frac{\pi}{4}$

$I = \int_0^{\pi/4} t\,dt$

$I = \left[\frac{t^2}{2}\right]_0^{\pi/4}$

$I = \frac{1}{2}\left[\frac{\pi^2}{16}-0\right] = \frac{\pi^2}{32}$

10. (B) $I = \int_{-1}^{1}(x-[x])\,dx$

$I = \int_{-1}^{1}x\,dx - \int_{-1}^{1}[x]dx$

$I = \left[\frac{x^2}{2}\right]_{-1}^{1} - \int_{-1}^{0}[x]\,dx - \int_0^1[x]dx$

$I = \left[\frac{1}{2}-\frac{(-1)^2}{2}\right]-\int_{-1}^{0}(-1).dx - \int_0^1 0.dx$

$I = \frac{1}{2}-\frac{1}{2}+[x]_{-1}^{0} - 0$

$I = 0 + [0-(-1)] = 1$

11. (A) A.T.Q,

$(2a, -b) = \left(\frac{-2+3}{2}, \frac{-9+2k}{2}\right)$

$2a = \frac{1}{2} \Rightarrow a = \frac{1}{4}$

and $-b = \frac{-9+2k}{2}$ …(i)

Given that $a + 2b = 11$

$\Rightarrow \frac{1}{4} + 2b = 11$

$\Rightarrow 2b = 11 - \frac{1}{4} \Rightarrow b = \frac{43}{8}$

from eq(i)

$\frac{-43}{8} = \frac{-9+2k}{2}$

$\Rightarrow \frac{-43}{4} = -9 + 2k \Rightarrow k = \frac{-7}{8}$

12. (C) In ΔABC, $b = 2$, $C = 45$, $c = 2\sqrt{2}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

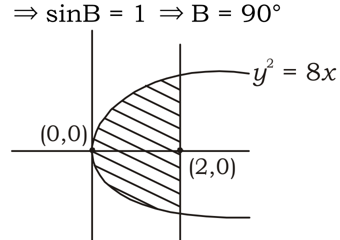
Now, $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin B}{2} = \frac{\sin 45}{2\sqrt{2}}$

$\Rightarrow \sin B = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$\Rightarrow \sin B = 1 \Rightarrow B = 90°$

13. (B)



Parabola $y^2 = 8x$

$4a = 8 \Rightarrow a = 2$

Area $= 2\int_0^2 y\,dx$

Area $= 2\int_0^2 \sqrt{8x}\,dx$

Area $= 2\times 2\sqrt{2}\int_0^2 x^{1/2}\,dx$

Area $= 4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2$

Area $= 4\sqrt{2}\times\frac{2}{3}[2^{3/2}-0]$

Area $= \frac{8\sqrt{2}}{3}\times 2^{3/2} = \frac{32}{3}$ sq. unit

14. (B) $x = \omega^2 - \omega + 3$

$\Rightarrow x - 3 = \omega^2 - \omega$

$\Rightarrow (x-3)^3 = (\omega^2 - \omega)^3$

$\Rightarrow x^3 - 27 - 9x(x-3) = \omega^6 - \omega^3 - 3\omega^2 \times \omega(\omega^2 - \omega)$

$\Rightarrow x^3 - 27 - 9x^2 + 27x = 1 - 1 - 3(x-3)$

$\Rightarrow x^3 - 9x^2 + 27x - 27 = -3x + 9$

$\Rightarrow x^3 - 9x^2 + 30x = 36$

15. (D) $I = \int \frac{dx}{x(x^7 - 1)}$

$I = \int \frac{x^6 dx}{x^7(x^7-1)}$

$I = \int \left(\frac{1}{x^7 - 1} - \frac{1}{x^7}\right) x^6\, dx$

Let $x^7 = t$

$\Rightarrow 7x^6\, dx = dt \Rightarrow x^6\, dx = \frac{1}{7} dt$

$I = \frac{1}{7} \int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$

$I = \frac{1}{7} [\log(t-1) - \log t] + c$

$I = \frac{1}{7} \log\left[\frac{t-1}{t}\right] + c$

$I = \frac{1}{7} \log\left[\frac{x^7 - 1}{x^7}\right] + c$

16. (C) $y = \sqrt{\sec^2 x + \sqrt{\sec^2 x + \sqrt{\sec^2 x + ...\infty}}}$

$\Rightarrow y = \sqrt{\sec^2 x + y}$

$\Rightarrow y^2 = \sec^2 x + y$

On differentiating both side w.r.t.'x'

$\Rightarrow 2y\frac{dy}{dx} = 2.\sec x.\sec x.\tan x + \frac{dy}{dx}$

$\Rightarrow (2y - 1)\frac{dy}{dx} = 2\sec^2 x.\tan x$

$\Rightarrow \frac{dy}{dx} = \frac{2\sec^2 x.\tan x}{2y - 1}$

17. (B) $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 12\theta}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 12\theta)}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2\cos^2 6\theta}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 6\theta}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2\cos^2 3\theta}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + 2\cos 3\theta}}$

$\Rightarrow \sqrt{2 + \sqrt{2 \times 2\cos^2 \frac{3\theta}{2}}}$

$\Rightarrow \sqrt{2 + 2\cos \frac{3\theta}{2}}$

$\Rightarrow \sqrt{2 \times 2\cos^2 \frac{3\theta}{4}} = 2\cos \frac{3\theta}{4}$

18. (B) Points $(a, 0)$, $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are collinear,

then $\begin{vmatrix} a & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = 0$

$\Rightarrow a \times 2a \begin{vmatrix} 1 & 0 & 1 \\ t_1^2 & t_1 & 1 \\ t_2^2 & t_2 & 1 \end{vmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ t_1^2 - 1 & t_1 & 0 \\ t_2^2 - 1 & t_2 & 0 \end{vmatrix} = 0$

$\Rightarrow 1 \times 0 - 0 + 1(t_1^2 . t_2 - t_2 - t_1 . t_2^2 + t_1) = 0$

$\Rightarrow t_1^2 . t_2 - t_1 . t_2^2 - t_2 + t_1 = 0$

$\Rightarrow t_1 . t_2(t_1 - t_2) + (t_1 - t_2) = 0$

$\Rightarrow (t_1 - t_2)(t_1 . t_2 + 1) = 0$

$\Rightarrow t_1 . t_2 + 1 = 0,\ t_1 - t_2 \neq 0$

$\Rightarrow t_1 . t_2 = -1$

19. (C) $\frac{\cos(x+y)}{\cos(x-y)} = \frac{a+b}{a-b}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{\cos(x+y) + \cos(x-y)}{\cos(x+y) - \cos(x-y)} = \frac{a+b+a-b}{a+b-a+b}$

$\Rightarrow \frac{2\cos x.\cos y}{2\sin x.\sin y} = \frac{2a}{2b}$

$\Rightarrow \cot x.\cot y = \frac{a}{b}$

$\Rightarrow \tan x.\tan y = \frac{b}{a}$

20. (B) Given that $byx = \frac{-16}{9}$ and $bxy = \frac{-25}{36}$

Now, $r = \sqrt{bxy \times byx}$

$\Rightarrow r = \sqrt{\left(\frac{-25}{36}\right) \times \left(\frac{-16}{9}\right)}$

$\Rightarrow r = -\frac{5}{6} \times \frac{4}{9} = -\frac{10}{27}$

21. (B)

22. (B) $5^{-x\tan x}\left[\frac{d}{dx}5^{x\tan x}\right]$

$\Rightarrow 5^{-x\tan x}\left[5^{x\tan x}\ln 5\frac{d}{dx}(x\tan x)\right]$

$\Rightarrow 5^{-x\tan x}.5^{x\tan x}\ln 5[x.\sec^2 x + \tan x.1]$

$\Rightarrow (x.\sec^2 x + \tan x)\ln 5$

23. (A) Given that $\vec{a} = 3\hat{i} + \hat{j} + 5\hat{k}, \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$,

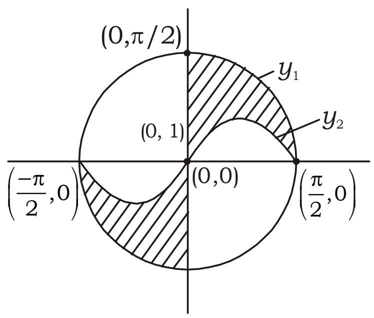
$\vec{c} = \hat{i} - \hat{j} - 3\hat{k}$

Now, $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c}\,(\vec{a} + \vec{b})$

$\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{a} + \vec{b} \times \vec{c} - \vec{a} \times \vec{b} + \vec{c} \times \vec{a} - \vec{b} \times \vec{c} = 0$

24. (A)



curves $y_1 \Rightarrow y = \sqrt{\frac{\pi^2}{4} - x^2}$ and $y_2 = y = \sin x$

Area $= 2\int_0^{\pi/2}(y_1 - y_2)\,dx$

$= 2\int_0^{\pi/2}\left(\sqrt{\frac{\pi^2}{4} - x^2} - \sin x\right)dx$

Area $= 2\left[\frac{1}{2}x\sqrt{\frac{\pi^2}{4} - x^2} + \frac{1}{2} \times \frac{\pi^2}{4}\sin^{-1}\frac{x}{\pi/2}\right]_0^{\pi/2}$

$-2[-\cos x]_0^{\pi/2}$

Area $= 2\left[\left(\frac{1}{2} \times 0 + \frac{\pi^2}{8}\sin^{-1}\frac{\pi/2}{\pi/2}\right) - (0+0)\right]$

$+2\left[\cos\frac{\pi}{2} - \cos 0\right]$

Area $= 2\left[\frac{\pi^2}{8} \times \frac{\pi}{2}\right] + 2[0 - 1]$

Area $= \left(\frac{\pi^3}{8} - 2\right)$ sq.unit

25. (C) Data 42, 44, 46 + 43 + 44 + 48 + 51 + 50

Mean $= \frac{42+44+46+43+44+48+51+50}{8}$

Mean $= \frac{368}{8} = 46$

$\sum|x - \bar{x}| = |42-46| + |44-46| + |46-46| + |43-46| + |44-46| + |48-46| + |51-46| + |50-46|$

$\sum|x - \bar{x}| = 4+2+0+3+2+2+5+4 = 22$

Mean-deviation $= \frac{\sum|x-\bar{x}|}{n} = \frac{22}{8} = 2.75$

26. (D) Let $f(x) = \frac{x}{[x]}$

L.H.L. $= \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h)$

$= \lim_{h\to 0}\frac{3-h}{[3-h]}$

$= \lim_{h\to 0}\frac{3-h}{2} = \frac{3}{2}$

R.H.L. $= \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h)$

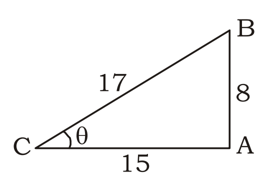
$= \lim_{h\to 0}\frac{3+h}{[3+h]}$

$= \lim_{h\to 0}\frac{3+h}{3} = 1$

L.H.L. $\neq$ R.H.L.

Hence limit does not exist.

27. (B)



$\sec\theta = \frac{-17}{15}$ and $\cos\theta = \frac{-15}{17}$

Now, 7 sinθ – 2cosθ

$\Rightarrow 7 \times \left(\frac{-8}{17}\right) - 2\left(\frac{-15}{17}\right)$

$\Rightarrow \frac{-56}{17} + \frac{30}{17} = \frac{-26}{17}$

28. (D) Let Locus of a point = (h, k, l)

A.T.Q

$\sqrt{(h+1)^2+(k-2)^2+(l+3)^2}$

$=\sqrt{(h+2)^2+(k-4)^2+(l+5)^2}$

$\Rightarrow h^2+1+2h+k^2+4-4k+l^2+9+6l$
$= h^2+4+4h+k^2+16-8k+l^2+25+10l$

On solving

$2h-4k+4l+31=0$

Locus of a point

$2x-4y+4z+31=0$

29. (A) Digits are 1, 2, 3, 5, 7, 8, 9.

$n(S) = {}^7C_3 = 35$

E = {(1, 2, 8), (1, 3, 7), (1, 3, 9), (1, 5, 9), (1, 7, 9), (2, 3, 8), (2, 7, 8), (2, 8, 9), (3, 5, 9), (3, 7, 9)}

$n(E) = 10$

The required Probability $=\frac{n(E)}{n(S)}=\frac{10}{35}=\frac{2}{7}$

30. (C) In a leap year = 366 days

= 52 weeks and 2 days

The required Probability = $\frac{2}{7}$

31. (B)

$$\begin{array}{r} 1\,0\,x\,0\,1\,1 \\ -\ 1\,y\,1\,0\,1 \\ \hline 1\,1\,z\,0 \end{array}$$

$x = 1,\ y = 1,\ z = 1$

32. (A)

33. (C) The required no. of terms $= {}^{n+2}C_2$

$=\frac{(n+2)!}{2!n!}$

$=\frac{(n+2)(n+1)n!}{2\times n!}$

$=\frac{(n+1)(n+2)}{2}$

34. (B) Let $z=\left(\frac{1-2i}{2+i}\right)^2 \Rightarrow z=\frac{1+4i^2-4i}{4+i^2+4i}$

$\Rightarrow z=\frac{1-4-4i}{4-1+4i} \Rightarrow z=\frac{-3-4i}{3+4i}$

$\Rightarrow z=\frac{-1(3+4i)}{3+4i}=-1$

Conjugate of $z=\bar{z}=-1$

35. (C) $I=\int e^x\left[(x+1)\tan^{-1}x+\frac{x}{1+x^2}\right]dx$

$I=\int e^x\left[(x.\tan^{-1}x)+\left(\tan^{-1}x+\frac{x}{1+x^2}\right)\right]dx$

$I = e^x.x.\tan^{-1}x + c$

$\left[\because e^x[f(x)+f'(x)=e^xf(x)+c\right]$

36. (C) zero

37. (A) Parabola

$y^2+5y+x+12=0$

$\Rightarrow\left(y+\frac{5}{2}\right)^2-\frac{25}{4}+x+12=0$

$\Rightarrow\left(y+\frac{5}{2}\right)^2=-x+\frac{25}{4}-12$

$\Rightarrow\left(y+\frac{5}{2}\right)^2=-x-\frac{23}{4}$

$\Rightarrow\left(y+\frac{5}{2}\right)^2=-1\left(x+\frac{23}{4}\right)$

Let $Y = y+\frac{5}{2}$, $X = x+\frac{23}{4}$

$Y^2 = -X$

$4a = 1 \Rightarrow a=\frac{1}{4}$

focus $(X, Y) = (-a, 0)$

$x+\frac{23}{4}=\frac{-1}{4}$, $y+\frac{5}{2}=0$

$\Rightarrow x=-6,\ y=\frac{-5}{2}$

Hence focus $=\left(-6,\frac{-5}{2}\right)$

38. (C) $\cos^2\frac{\pi}{10}+\cos^2\frac{3\pi}{10}+\cos^2\frac{\pi}{5}+\cos^2\frac{2\pi}{5}$

$\Rightarrow\cos^2\left(\frac{\pi}{2}-\frac{2\pi}{5}\right)+\cos^2\left(\frac{\pi}{2}-\frac{\pi}{5}\right)+\cos^2\frac{\pi}{5}$

$+\cos^2\frac{2\pi}{5}$

$\Rightarrow\sin^2\frac{2\pi}{5}+\sin^2\frac{\pi}{5}+\cos^2\frac{\pi}{5}+\cos^2\frac{2\pi}{5}$

$\Rightarrow\sin^2\frac{2\pi}{5}+\cos^2\frac{2\pi}{5}+\sin^2\frac{\pi}{5}+\cos^2\frac{\pi}{5}$

$\Rightarrow 1+1=2$

39. (B)

40. (B)

41. (D) Conic

$4x^2+9y^2+8x-18y-41=0$

$\Rightarrow 4x^2+8x+9y^2-18y-41=0$

$\Rightarrow 4(x^2+2x+1-1)+9(y^2-2y+1-1)-41=0$

$\Rightarrow 4(x+1)^2-4+9(y-1)^2-9-41=0$

$\Rightarrow 4(x+1)^2+9(y-1)^2=54$

$\Rightarrow\frac{(x+1)^2}{27/2}+\frac{(y-1)^2}{6}=1$

$a^2=\frac{27}{2}$, $b^2=6$

Now, $b^2=a^2(1-e^2)$

$\Rightarrow 6=\frac{27}{2}(1-e^2)$

$\Rightarrow\frac{4}{9}=1-e^2$

$\Rightarrow e^2=1-\frac{4}{9}\Rightarrow e=\frac{\sqrt{5}}{3}$

42. (B) Given that $\theta_1 = 75°$, $\theta_2 = 105°$
We know that

$\theta = \frac{s}{r}$

When arc 's' is same, then

$\theta \propto \frac{1}{r}$

A.T.Q,

$\frac{\theta_1}{\theta_2} = \frac{r_2}{r_1}$

$\Rightarrow \frac{75}{105} = \frac{r_2}{r_1}$

$\Rightarrow \frac{5}{7} = \frac{r_2}{r_1}$

Hence $r_1 : r_2 = 7 : 5$

43. (C) 8 sin20.sin40.sin60.sin80

$\Rightarrow$ 8 sin60.sin20.sin40.sin80

$\Rightarrow 8\times\frac{\sqrt{3}}{4}\times \cos70.\cos50.\cos10$

$\Rightarrow 2\sqrt{3}.\cos10.\cos(60-10).\cos(60+10)$

$\Rightarrow 2\sqrt{3}\times\frac{1}{4}\times\cos(3\times10)$

$\left[\because \cos\theta.\cos(60-\theta).\cos(60+\theta) = \frac{1}{4}\cos3\theta\right]$

$\Rightarrow \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{4}$

44. (C) S = 0.2 + 0.22 + 0.222 +........
S = 2(0.1 + 0.11 + 0.111 +........)

$S = \frac{2}{9}(0.9 + 0.99 + 0.999 +.......)$

$S = \frac{2}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{100}\right)+\left(1-\frac{1}{1000}\right)+.....\right]$

$S = \frac{2}{9}\left[(1+1+...8\text{ terms})-\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+....8\text{ terms}\right)\right]$

$$S = \frac{2}{9}\left[8-\frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^8\right)}{1-\frac{1}{10}}\right]$$

$S = \frac{2}{9}\left[8-\frac{1}{9}\left(1-\frac{1}{10^8}\right)\right]$

45. (C) Circle

$x^2 + y^2 - 6x + 3y + 7 = 0$ ...(i)

equation of circle concentric with eq(i)

$x^2 + y^2 - 6x + 3y = c$ ....(ii)

it passes through the point (–1, 2)

$\Rightarrow (-1)^2 + 2^2 - 6\times(-1) + 3\times2 = c$

$\Rightarrow 1 + 4 + 6 + 6 = c \Rightarrow c = 17$

from eq(i)

$x^2 + y^2 - 6x + 3y = 17$

46. (A) The required no. of ways = 7^5

47. (B) Equation $x^2 + 6x - 4 = 0$

$\alpha + \beta = -6$ and $\alpha.\beta = -4$

Now, $\frac{\alpha^3-\beta^3}{\alpha^{-3}-\beta^{-3}} = \frac{\alpha^3-\beta^3}{\frac{1}{\alpha^3}-\frac{1}{\beta^3}}$

$\Rightarrow \frac{\alpha^3-\beta^3}{\alpha^{-3}-\beta^{-3}} = \frac{\alpha^3-\beta^3}{\frac{\beta^3-\alpha^3}{(\alpha\beta)^3}}$

$\Rightarrow \frac{\alpha^3-\beta^3}{\alpha^{-3}-\beta^{-3}} = \frac{(\alpha^3-\beta^3)(\alpha\beta)^3}{-(\alpha^3-\beta^3)}$

$\Rightarrow \frac{\alpha^3-\beta^3}{\alpha^{-3}-\beta^{-3}} = -(\alpha\beta)^3$

$\Rightarrow \frac{\alpha^3-\beta^3}{\alpha^{-3}-\beta^{-3}} = -(-4)^3 = 64$

48. (D)

49. (C) In the expansion of $\left(2\sqrt[3]{x}-\frac{1}{4\sqrt[3]{x}}\right)^{12}$

$T_{r+1} = {}^{12}C_r\left(2\sqrt[3]{x}\right)^{12-r}\left(\frac{-1}{4\sqrt[3]{x}}\right)^r$

$T_{r+1} = {}^{12}C_r\, 2^{12-3r}(-1)^r x^{\frac{12-2r}{3}}$

Here $\frac{12-2r}{3} = 0 \Rightarrow r = 6$

The required term = ${}^{12}C_6\, 2^{-6}(-1)^6$

$= \frac{33\times28}{64} = \frac{231}{16}$

50. (D) The required equation

$\frac{x}{-2}+\frac{y}{4} = 1$

$\Rightarrow 2x - y + 4 = 0$

51. (B) Let $y = \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Let $z = \sin^{-1}x \Rightarrow x = \sin z$

$\Rightarrow y = \text{cosec}^{-1}\left(\frac{1}{\sqrt{1-\sin^2 z}}\right)$

$\Rightarrow y = \text{cosec}^{-1}\left(\frac{1}{\cos z}\right)$

$\Rightarrow y = \text{cosec}^{-1}(\sec z)$

$\Rightarrow y = \text{cosec}^{-1}\left[\text{cosec}\left(\frac{\pi}{2} - z\right)\right]$

$\Rightarrow y = \frac{\pi}{2} - z$

$\Rightarrow y = \frac{\pi}{2} - \sin^{-1}x$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$

52. (B) Let $a + ib = \sqrt{21+20i}$

On squaring both side

$(a^2 - b^2) + i(2ab) = 21 + 20i$

On comparing

$a^2 - b^2 = 21$ and $2ab = 20$ ...(i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = (21)^2 + (20)^2$

$\Rightarrow (a^2 + b^2)^2 = 441 + 400$

$\Rightarrow (a^2 + b^2)^2 = 841 \Rightarrow a^2 + b^2 = 29$ ...(ii)

from eq(i) and eq(ii)

$2a^2 = 50$, $2b^2 = 8$

$a = \pm 5$, $b = \pm 4$

Hence $\sqrt{21+20i} = \pm(5 + 4i)$

53. (A) 1 1 0 0 1 1 . 0 1

$1\times2^0 = 1$

$1\times2^1 = 2$

$0\times2^2 = 0$

$0\times2^3 = 0$

$1\times2^4 = 16$

$1\times2^5 = 32$

$= 51$

$0 = 0\times2^{-1}$

$\frac{1}{4} = 1\times2^{-2}$

$\frac{1}{4} = 0.25$

Hence $(110011.01)_2 = (51.25)_{10}$

54. (B) Line $y = 2x - 1$

slope of line $m_1 = 2$

and line $2y = x + 4 \Rightarrow y = \frac{1}{2}x + 2$

$m_2 = \frac{1}{2}$

A.T.Q,

$\frac{m_1 - m}{1 + m_1.m} = \frac{m - m_2}{1 + m.m}$

$\Rightarrow \frac{2-m}{1+2m} = \frac{m - \frac{1}{2}}{1 + m\times\frac{1}{2}}$

$\Rightarrow \frac{2-m}{1+2m} = \frac{2m-1}{2+m}$

$\Rightarrow 4 - m^2 = 4m^2 - 1$

$\Rightarrow 5m^2 = 5 \Rightarrow m = \pm 1$

55. (C) $\lim_{x\to0}\frac{\log(3+2x)-\log(3-2x)}{x} = k\left[\frac{0}{0}\right]$ form

by L-Hospital's Rule

$\Rightarrow \lim_{x\to0}\frac{\frac{2}{3+2x} - \frac{-2}{3-2x}}{1}$

$\Rightarrow \lim_{x\to0}\frac{2}{3+2x} + \frac{2}{3-2x} = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

56. (A) $\sin(-600) = -\sin600$

$\Rightarrow \sin(-600) = -\sin(2\times360 - 120)$

$\Rightarrow \sin(-600) = \sin120$

$\Rightarrow \sin(-600) = \sin(90+30)$

$\Rightarrow \sin(-600) = \cos30 = \frac{\sqrt{3}}{2}$

57. (C) A.T.Q,

$\frac{a+b}{2} = 3\times\sqrt{ab}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$

by Componendo & Dividendo Rule

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-\sqrt{ab}} = \frac{3+1}{3-1}$

$\Rightarrow \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{4}{2}$

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$

Again, Componendo & Dividendo Rule

$\Rightarrow \frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$\Rightarrow \frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$

$\Rightarrow \frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

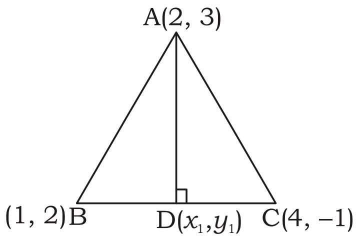
Again, Componendo & Dividendo Rule

$\Rightarrow \frac{a+b}{a-b} = \frac{3+2\sqrt{2}+3-2\sqrt{2}}{3+2\sqrt{2}-3+2\sqrt{2}}$

$\Rightarrow \frac{a+b}{a-b} = \frac{6}{4\sqrt{2}}$

$\Rightarrow \frac{a+b}{a-b} = \frac{2\sqrt{2}}{3}$

58. (B)



Let D = (x_1, y_1)

slope of line BC (m_1) = $\frac{-1-2}{4-1}$ = –1

slope of line AD (m_2) = $\frac{y_1-3}{x_1-2}$

Now, $m_1 \times m_2 = -1$

$\Rightarrow -1 \times \frac{y_1-3}{x_1-2} = -1 \Rightarrow x_1 - y_1 = -1$ ...(i)

Equation of line BC

$y - 2 = -1(x - 1)$

$\Rightarrow x + y = 3$

Point D(x_1, y_1) lies on the line BC

$\Rightarrow x_1 + y_1 = 3$ ...(ii)

from eq(i) and eq(ii)

$x_1 = 1$ and $y_1 = 2$

$\therefore$ D = (1, 2)

Length of AD = $\sqrt{(2-1)^2+(3-2)^2}$

Length of AD = $\sqrt{1+1}$ = $\sqrt{2}$

59. (C) $9^8 + 8.9^7 + 28.9^6 + ... + 1 = k \times 2^6 \times 5^7$

$\Rightarrow {}^8C_0 9^8.1^0 + {}^8C_1 9^7.1^1 + ... + {}^8C_8 9^0.1^8 = k \times 2^6 \times 5^7$

$\Rightarrow (9 + 1)^8 = k \times 2^6 \times 5^7$

$\Rightarrow (10)^8 = k \times 2^6 \times 5^7 \Rightarrow k = 20$

60. (B) $x = \sin\theta + \cos\theta$ and $y = \sin\theta - \cos\theta$

Now, $x^2 + y^2 = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$\Rightarrow x^2 + y^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta + \sin^2\theta + \cos^2\theta - \sin\theta.\cos\theta$

$\Rightarrow x^2 + y^2 = 1 + 1 \Rightarrow x^2 + y^2 = 2$

61. (C) **Statement I**

L.H.S. = $(\omega^{19} + 1)^5 + \omega$

L.H.S. = $(\omega + 1)^5 + \omega$ [$\because \omega^3 = 1$]

L.H.S. = $(-\omega^2)^5 + \omega$ [$\because 1 + \omega + \omega^2 = 0$]

L.H.S. = $-\omega^{10} + \omega$

L.H.S. = $-\omega + \omega = 0$ = R.H.S.

Statement I is correct.

Statement II

L.H.S. = $(\omega^{143} + 1)^{12}$

L.H.S. = $(\omega^{3\times47+2} + 1)^{12}$

L.H.S. = $(\omega^2 + 1)^{12}$ [$\because \omega^3 = 1$]

L.H.S. = $(-\omega)^{12}$ [$\because 1 + \omega + \omega^2 = 0$]

L.H.S. = $\omega^{12} = 1$ = R.H.S. [$\because \omega^3 = 1$]

Statement II is correct.

62. (C) The required no. of ways = ${}^{15-2}C_{11-2}$ = ${}^{13}C_9 = 715$

63. (D) $n(S) = 6 \times 6 = 36$

$E = \begin{Bmatrix} (6,4),(5,5),(4,6) \text{ for sum} = 10 \\ (6,5),(5,6) \text{ for sum} = 11 \\ (6,6) \text{ for sum} = 12 \end{Bmatrix}$

$n(E) = 6$

The required Probability = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

64. (B) $y = a^{x+a^{x+a^{x+......\infty}}}$

$\Rightarrow y = a^{x+y}$

taking log both side

$\Rightarrow \log y = (x + y)\log a$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(1 + \frac{dy}{dx}\right)\log a$

$\Rightarrow \left(\frac{1}{y} - \log a\right)\frac{dy}{dx} = \log a$

$\Rightarrow \left(\frac{1 - y\log a}{y}\right)\frac{dy}{dx} = \log a$

$\Rightarrow \frac{dy}{dx} = \frac{y\log a}{1 - y\log a}$

65. (C) $f(x) = \begin{cases} 2ax + 3b, & x < 3 \\ 6, & x = 3 \\ 3a - bx, & x > 3 \end{cases}$ is continuous

at $x = 3$, then $\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = f(3)$

Now, $\lim_{x\to3^-} f(x) = f(3)$

$\Rightarrow \lim_{x\to3}(2ax + 3b) = 6$

$\Rightarrow 2a \times 3 + 3b = 6 \Rightarrow 2a + b = 2$ ...(i)

Again, $\lim_{x\to3^+} f(x) = f(3)$

$\Rightarrow \lim_{x\to3}(3a - bx) = 6$

$\Rightarrow 3a - b\times3 = 6 \Rightarrow a - b = 2$ ...(ii)

from eq(i) and eq(ii)

$a = \frac{4}{3}$, $b = \frac{-2}{3}$

66. (B) Two straight lines
$2x - 5y - 2 = 0$
and $10y - 4x = 7$
$\Rightarrow 4x - 10y + 7 = 0$
$\Rightarrow 2x - 5y + \frac{7}{2} = 0$
Both lines are parallel.
Hence these lines never intersect.

67. (D) Given that A + B = α
$\Rightarrow \sin(A + B) = \sin\alpha$
$\Rightarrow \sin A.\cos B + \cos A.\sin B = \sin\alpha$ ...(i)
and $A - B = x$
$\Rightarrow \sin(A - B) = \sin x$
$\Rightarrow \sin A.\cos B - \cos A.\sin B = \sin x$ ...(ii)
Now, $\frac{\tan A}{\tan B} = \frac{3}{2}$
$\Rightarrow \frac{\sin A}{\cos A} \times \frac{\cos B}{\sin B} = \frac{3}{2}$
By Componendo & Dividendo Rule
$\Rightarrow \frac{\sin A.\cos B + \cos A.\sin B}{\sin A.\cos B - \cos A.\sin B} = \frac{3+2}{3-2}$
$\Rightarrow \frac{\sin\alpha}{\sin x} = \frac{5}{1} \Rightarrow \sin x = \frac{1}{5}\sin\alpha$

68. (C) $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+....\infty\right)} \Rightarrow (0.4)^{2\log_{2.5}\left(\frac{1}{2}\right)}$
$\Rightarrow \left(\frac{4}{10}\right)^{-\log_{2.5}4} \Rightarrow \left(\frac{10}{4}\right)^{\log_{2.5}4}$
$\Rightarrow (2.5)^{\log_{2.5}4} = 4$

69. (C) Differential equation
$xdx + ydy = \sqrt{x^2+y^2}\, dx$
$\Rightarrow \frac{xdx + ydx}{\sqrt{x^2+y^2}} = dx$
On integrating
$\Rightarrow \int \frac{x\,dx + y\,dy}{\sqrt{x^2+y^2}} = \int dx$
$\Rightarrow \sqrt{x^2+y^2} = x + c$

70. (A) A.T.Q,
$\left(\sqrt{(x-0)^2+(y-b)^2}\right)^2 + \left(\sqrt{(x-0)^2+(y+b)^2}\right)^2 = 2a^2$
$\Rightarrow x^2 + y^2 + b^2 - 2yb + x^2 + y^2 + b^2 + 2yb = 2a^2$
$\Rightarrow 2x^2 + 2y^2 + 2b^2 = 2a^2$
$\Rightarrow x^2 + y^2 + b^2 = a^2 \Rightarrow x^2 + y^2 = a^2 - b^2$

71. (b) $\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$, and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$
Projection $\vec{a}$ on $\vec{b}$ = $\left|\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right|$
$= \left|\frac{2\times1 + 1\times1 - 5\times1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{2}{\sqrt{3}}$

72. (C) $\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{BA} = 3\hat{i} - \hat{j} + 2\hat{k}$
Area of ΔABC = $\frac{1}{2}\left|\overrightarrow{BC} \times \overrightarrow{BA}\right|$
Area of ΔABC = $\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -1 & 2\end{matrix}\right\|$
Area of ΔABC = $\frac{1}{2}\left\|\left[-\hat{i} - \hat{j} + \hat{k}\right]\right\|$
Area of ΔABC = $\frac{1}{2}\sqrt{(-1)^2+(-1)^2+1^2} = \frac{\sqrt{3}}{2}$

73. (B) $f(x) = \frac{x^3 - 7x + 6}{x^3 - 2x^2 - 5x + 6}$
$f(x) = \frac{(x+3)(x-2)(x-1)}{(x-3)(x+2)(x-1)}$
$f(x) = \frac{(x+3)(x-2)}{(x-3)(x+2)} \Rightarrow f(1) = \frac{2}{3}$
$f'(x) = \frac{(x-3)(x+2)(2x+1) - (x+3)(x-2)(2x-1)}{(x-3)^2(x+2)^2}$
$f'(x) = \frac{(x^2-x-6)(2x+1) - (x^2+x-6)(2x-1)}{(x-3)^2(x+2)^2}$
$f'(x) = \frac{-4x^2 + 2x^2 - 12}{(x-3)^2(x+2)^2}$
$f'(x) = \frac{-2x^2 - 12}{(x-3)^2(x+2)^2}$
$f'(1) = \frac{-2-12}{(-2)^2(3)^2} = \frac{-14}{4\times9} = \frac{-7}{18}$
Now, $f(1) + f'(1) = \frac{2}{3} - \frac{7}{18}$
$\Rightarrow f(1) + f'(1) = \frac{12-7}{18} = \frac{5}{18}$

74. (B) A.T.Q,

$a + (p-1)d = \frac{1}{q}$ ...(i)

$a + (q-1)d = \frac{1}{p}$ ...(ii)

from eq(i) and eq(ii)

$a = \frac{1}{pq}$ and $d = \frac{1}{pq}$

Now, $T_{pq} = a + (pq-1)d$

$\Rightarrow T_{pq} = \frac{1}{pq} + \frac{pq-1}{pq}$

$\Rightarrow T_{pq} = \frac{1+pq-1}{pq} = 1$

75. (D) $I = \int_{-\pi}^{\pi} |\cos x|\, dx$

$I = 2\int_0^{\pi} |\cos x|\, dx$

$I = 2\times 2\int_0^{\pi/2} \cos x\, dx$

$I = 4[\sin x]_0^{\pi/2}$

$I = 4\left[\sin\frac{\pi}{2} - \sin 0\right]$

$I = 4[1-0] = 4$

76. (B) $(1 - \sec A + \tan A)^2$

$\Rightarrow 1 + \sec^2 A + \tan^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow \sec^2 A + 1 + \tan^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow \sec^2 A + \sec^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow 2\sec^2 A - 2\sec A - 2\sec A.\tan A + 2\tan A$

$\Rightarrow 2\sec A(\sec A - 1) - 2\tan A(\sec A - 1)$

$\Rightarrow (\sec A - 1)(2\sec A - 2\tan A)$

$\Rightarrow 2(\sec A - 1)(\sec A - \tan A)$

77. (B)

78. (D) Plane $3x - 4y + z = 13$

from option (D) (5, 2, 6)

$3\times 5 - 4\times 2 + 6 = 13$

$\Rightarrow 15 - 8 + 6 = 13$

$\Rightarrow 13 = 13$

Hence option (D) is correct.

79. (D)

80. (A)

81. (B) The required Probability $= \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$

82. (C) $\begin{bmatrix} x & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 3 & 1 \\ -2 & 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = [0]$

$\Rightarrow \begin{bmatrix} x & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 10 \\ 12 \end{bmatrix} = [0]$

$\Rightarrow [-x - 10 + 24] = [0]$

$\Rightarrow -x + 14 = 0 \Rightarrow x = 14$

83. (C) $I = \int e^x [x.\ln x + \ln x + 1]\, dx$

$I = e^x.x\ln x + c \left[\because \int e^x[f(x) + f'(x)]dx = e^x.f(x) + c\right]$

$I = x.e^x.\ln x + c$

84. (A) We know that

$(1+x)^n = C_1 + C_1 x + C_2 x^2 + \ldots\ldots + C_n x^n$ ...(i)

$x \to \frac{1}{x}$

$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + \ldots\ldots + \frac{C_n}{x^n}$ ...(ii)

from eq(i) and eq(ii)

Coefficient of x^0 in $(1+x)^n\left(1+\frac{1}{x}\right)^n =$

$C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 =$ Coefficient of x^n in $\frac{(1+x)^{2n}}{x^n}$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 =$ Coefficient of x^0 in $(1+x)^{2n}$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n$

$\Rightarrow C_0^2 + C_1^2 + C_2^2 + \ldots\ldots + C_n^2 = \frac{(2n)!}{n!n!}$

85. (C) We know that

$\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$

Now, $(-1-i\sqrt{3})^{72} = 2^{72}\left(\frac{-1-i\sqrt{3}}{2}\right)^{72}$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72}(\omega^2)^{72}$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72}\times 1$ $[\because \omega^3 = 1]$

$\Rightarrow (-1-i\sqrt{3})^{72} = 2^{72}$

86. (B) $\frac{\sec 34}{\sec 112}+\frac{\text{cosec } 34}{\text{cosec } 112}$

$\Rightarrow \frac{\cos 112}{\cos 34}+\frac{\sin 112}{\sin 34}$

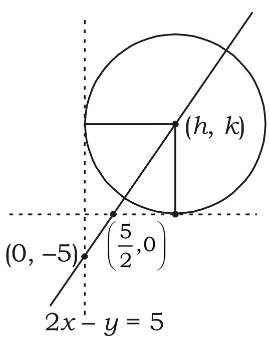
$\Rightarrow \frac{\sin 34.\cos 112+\cos 34.\sin 112}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin(34+112)}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin 146}{\sin 34.\cos 34}$

$\Rightarrow \frac{\sin(180-34)}{\sin 34.\cos 34}=\frac{\sin 34}{\sin 34.\cos 34}$ = sec34

87. (B)



line $2x - y = 5$
centre (h, k) lies on the line
$2h - k = 5$ ...(i)
and $r = h = k$ ...(ii)
from eq(i) and eq(ii)
$h = 5$, $k = 5$
Equation of circle
$(x-5)^2 + (y-5)^2 = 5^2$
$\Rightarrow x^2 + 25 - 10x + y^2 + 25 - 10y = 25$
$\Rightarrow x^2 + y^2 - 10x - 10y + 25 = 0$

88. (D) A = {1, 3, 5} , B = {2, 4, 6}, C = {4, 6, 8}
$B \cap C$ = {4, 6}
No. of elements in A = 3
No. of elements in $(B \cap C)$ = 2
Hence No. of elements in $A\times(B \cap C)$ = 3×2= 6

89. (A)

90. (C) $\lim_{n\to\infty}\frac{(1+2+3+.....+n)(n+3)}{1^2+2^2+3^2+.....+n^2}$

$\Rightarrow \lim_{n\to\infty}\frac{\frac{n(n+1)}{2}\times(n+3)}{\frac{n}{6}(n+1)(2n+1)} \Rightarrow \lim_{n\to\infty}\frac{3(n+3)}{(2n+1)}$

$\Rightarrow \lim_{n\to\infty}\frac{3n\left(1+\frac{3}{n}\right)}{n\left(2+\frac{1}{n}\right)} \Rightarrow \lim_{n\to\infty}\frac{3(1+0)}{(2+0)} = \frac{3}{2}$

91. (D) $\frac{\sin\theta+\cos\theta-\tan\theta}{\sec\theta-\text{cosec}\,\theta-\cot\theta}$

When $\theta = \frac{5\pi}{4}$

$\Rightarrow \frac{\sin\frac{5\pi}{4}+\cos\frac{5\pi}{4}-\tan\frac{5\pi}{4}}{\sec\frac{5\pi}{4}-\text{cosec}\frac{5\pi}{4}-\cot\frac{5\pi}{4}}$

$\Rightarrow \frac{\sin\left(\pi+\frac{\pi}{4}\right)+\cos\left(\pi+\frac{\pi}{4}\right)-\tan\left(\pi+\frac{\pi}{4}\right)}{\sec\left(\pi+\frac{\pi}{4}\right)-\text{cosec}\left(\pi+\frac{\pi}{4}\right)-\cot\left(\pi+\frac{\pi}{4}\right)}$

$\Rightarrow \frac{-\sin\frac{\pi}{4}-\cos\frac{\pi}{4}-\tan\frac{\pi}{4}}{-\sec\frac{\pi}{4}+\text{cosec}\frac{\pi}{4}-\tan\frac{\pi}{4}}$

$\Rightarrow \frac{-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}-1}{-\sqrt{2}+\sqrt{2}-1} \Rightarrow \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+1$

$\Rightarrow \sqrt{2} + 1$

92. (C) $\cos\left(\tan^{-1}\left(\tan\left(\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)\right)\right)\right)$

$\Rightarrow \cos\left(\tan^{-1}\left(\tan\left(\cos^{-1}\left(\cos\frac{5\pi}{6}\right)\right)\right)\right)$

$\Rightarrow \cos\left(\tan^{-1}\left(\tan\frac{5\pi}{6}\right)\right)$

$\Rightarrow \cos\left(\tan^{-1}\left(\tan\left(\pi-\frac{\pi}{6}\right)\right)\right)$

$\Rightarrow \cos\left(\tan^{-1}\left(\tan\left(\frac{-\pi}{6}\right)\right)\right)$

$\Rightarrow \cos\left(\frac{-\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

93. (D) We know that

$\cos 15° = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ and $\cos 75° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

Now, $\cos 15°.\cos 75° = \dfrac{\sqrt{3}+1}{2\sqrt{2}} \times \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$\Rightarrow \cos 15°.\cos 75° = \dfrac{3-1}{8} = \dfrac{1}{4}$

94. (C) Angles are in ratio 1 : 3 : 2
Let Angles x, $3x$, $2x$
$x + 3x + 2x = 180 \Rightarrow x = 30°$
Angles are 30°, 90°, 60°
A.T.Q,
sum = cos30° + cos90° + cos60°

sum = $\dfrac{\sqrt{3}}{2} + 0 + \dfrac{1}{2} = \dfrac{\sqrt{3}+1}{2}$

95. (D) When $n = 51$

The no. of diagonals = $\dfrac{n(n-3)}{2}$

$= \dfrac{51 \times 48}{2} = 1224$

96. (D) In ΔABC, A(–3, 2), B(3, –1), C(–4, 7)

centroid = $\left(\dfrac{-3+3-4}{3}, \dfrac{2-1+7}{3}\right)$

centroid = $\left(\dfrac{-4}{3}, \dfrac{8}{3}\right)$

97. (C) Line
$(3x - 4y + 7) + \lambda(4x + y + 2) = 0$
$\Rightarrow (3 + 4\lambda)x + (-4+\lambda)\, y + (7 + 2\lambda) = 0$

$\Rightarrow y = -\dfrac{3+4\lambda}{-4+\lambda}x - \dfrac{7+2\lambda}{-4+\lambda}$

it is parallel to x-axis *i.e.* $m = 0$

$\Rightarrow \dfrac{3+4\lambda}{-4+\lambda} = 0 \Rightarrow \lambda = \dfrac{-3}{4}$

98. (D) $I = \displaystyle\int_0^{\pi/2} \dfrac{f(x)}{f(x) + f\left(\dfrac{\pi}{2} - x\right)}\, dx$ ...(i)

Prop.IV $\displaystyle\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$I = \displaystyle\int_0^{\pi/2} \dfrac{f\left(\dfrac{\pi}{2} - x\right)}{f\left(\dfrac{\pi}{2} - x\right) + f(x)}\, dx$ ...(ii)

from eq(i) and eq(ii)

$2I = \displaystyle\int_0^{\pi/2} \dfrac{f(x) + f\left(\dfrac{\pi}{2} - x\right)}{f(x) + f\left(\dfrac{\pi}{2} - x\right)}\, dx$

$2I = \displaystyle\int_0^{\pi/2} 1.dx$

$2I = [x]_0^{\pi/2}$

$2I = \dfrac{\pi}{2} - 0 \Rightarrow I = \dfrac{\pi}{4}$

99. (B) Matrix $\begin{bmatrix} x+1 & -2 & 1 \\ 0 & -5 & 4 \\ 1 & -4 & 3 \end{bmatrix}$ is an invertible

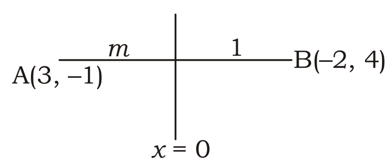
then $\begin{vmatrix} x+1 & -2 & 1 \\ 0 & -5 & 4 \\ 1 & -4 & 3 \end{vmatrix} = 0$

$\Rightarrow (x+1)(-15+16) + 2(0-4) + 1(0-5) = 0$
$\Rightarrow x + 1 - 8 - 5 = 0 \Rightarrow x = 12$

100. (B) Word "SITUATION"

No. of permutations $= \dfrac{9!}{2!2!} = \dfrac{9!}{4}$

101. (C)



Let ratio = $m : 1$
A.T.Q,

$\dfrac{m \times (-2) \times 1 \times 3}{m+1} = 0$

$\Rightarrow -2m + 3 = 0 \Rightarrow m = \dfrac{3}{2}$

The required ratio = 3 : 2

102. (A) In ΔABC,

$\sin^2\dfrac{A}{2}, \sin^2\dfrac{B}{2}, \sin^2\dfrac{C}{2}$ are in H.P.

$\Rightarrow \dfrac{1}{\sin^2\dfrac{A}{2}}, \dfrac{1}{\sin^2\dfrac{B}{2}}, \dfrac{1}{\sin^2\dfrac{C}{2}}$ are in A.P.

Now, $\dfrac{2}{\sin^2\dfrac{B}{2}} = \dfrac{1}{\sin^2\dfrac{A}{2}} + \dfrac{1}{\sin^2\dfrac{C}{2}}$

$\Rightarrow \dfrac{2ca}{(s-c)(s-a)} = \dfrac{bc}{(s-b)(s-c)} + \dfrac{ab}{(s-a)(s-b)}$

$\Rightarrow \frac{2ca}{(s-c)(s-a)} = \frac{bc(s-a)+ab(s-c)}{(s-a)(s-b)(s-c)}$

$\Rightarrow 2ca(s-b) = sbc - abc + sab - abc$

$\Rightarrow 2sac - 2abc = sbc + sab - 2abc$

$\Rightarrow 2sac = s(ab + bc)$

$\Rightarrow 2ac = (ab + bc)$

$\Rightarrow \frac{2}{b} = \frac{1}{c} + \frac{1}{a}$

$\Rightarrow \frac{1}{a}, \frac{1}{c}, \frac{1}{c}$ are in A.P.

$\Rightarrow a, b, c$ are in H.P.

103. (D)

104. (C) Lines $\frac{x-1}{4} = \frac{y+3}{3} = \frac{z-1}{5}$ and $\frac{x+2}{4} = \frac{y-1}{5} = \frac{z-3}{-3}$

Let Angle between lines = θ

Now, $\cos\theta = \frac{4\times4+3\times5-5\times3}{\sqrt{4^2+3^2+5^2}\sqrt{4^2+5^2+(-3)^2}}$

$\Rightarrow \cos\theta = \frac{16}{\sqrt{50}\sqrt{50}}$

$\Rightarrow \cos\theta = \frac{16}{50} \Rightarrow \theta = \cos^{-1}\frac{8}{25}$

105. (C) $A = \begin{bmatrix} -1 & 2 \\ -4 & 6 \end{bmatrix}$

$|A| = -6 + 8 = 2$

Given that

$A^{-1} = k.(\text{adj}A)$

$\Rightarrow \frac{\text{adj}A}{|A|} = k.(\text{Adj}A) \qquad \left(\because A^{-1} = \frac{\text{adj}A}{|A|}\right)$

$\Rightarrow \frac{1}{|A|} = k \Rightarrow k = \frac{1}{2}$

106. (D)

107. (B)

108. (B) The required Area $= \int_{-\pi/6}^{\pi/6} \sec^2 x\, dx$

$= [\tan x]_{-\pi/6}^{\pi/6}$

$= \tan\frac{\pi}{6} - \tan\left(\frac{-\pi}{6}\right)$

$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$ sq. unit

109. (C) Class marks 10.5, 11, 11.5, 12, 12.5, 13, 13.5, 14

Middle terms = 12, 12.5

Median $= \frac{12+12.5}{2} = 12.25$

110. (A) Differential equation

$\left(\frac{dy}{dx}\right)^{3/4} + \left(\frac{d^2y}{dx^2}\right)^{3/2} = y^{1/4}$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^{3/2} = y^{1/4} - \left(\frac{dy}{dx}\right)^{3/4}$

$\Rightarrow \left[\left(\frac{d^2y}{dx^2}\right)^{3/2}\right]^4 = \left[y^{1/4} - \left(\frac{dy}{dx}\right)^{3/4}\right]^4$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^6 = \left[y^{1/4} - \left(\frac{dy}{dx}\right)^{3/4}\right]^4$

Degree = 6

111. (A)

112. (B) sin20 + sin30 + sin40 + sin100

$\Rightarrow$ sin30 + sin20 + sin40 + sin100

$\Rightarrow \frac{1}{2}$ + 2sin30.cos10 + sin(90 +10)

$\Rightarrow \frac{1}{2} + 2\times\frac{1}{2}\times$ cos10 – cos10

$= \frac{1}{2}$ + cos10 – cos10 $= \frac{1}{2}$

113. (C)

114. (B) $y = \ln(e^{nx} + e^{-nx})$

On differentiating both side w.r.t.'x'

$\frac{dy}{dx} = \frac{1}{e^{nx}+e^{-nx}} \times (ne^{nx} - ne^{-nx})$

$\frac{dy}{dx} = \frac{n(e^{nx}-e^{-nx})}{e^{nx}+e^{-nx}}$

Again, differentiating

$\frac{d^2y}{dx^2} = n\left[\frac{n(e^{nx}+e^{-nx})^2 - n(e^{nx}+e^{-nx})^2}{(e^{nx}+e^{-nx})^2}\right]$

$\frac{d^2y}{dx^2} = n^2\left[\frac{(e^{2nx}+e^{-2nx}+2)-(e^{2nx}+e^{-2nx}-2)}{(e^{nx}+e^{-nx})^2}\right]$

$\frac{d^2y}{dx^2} = \frac{4n^2}{(e^{nx}+e^{-nx})^2}$

$\left(\frac{d^2y}{dx^2}\right)_{\text{at } x=0} = \frac{4n^2}{(e^0+e^0)^2} = n^2$

115. (D) $\cot\beta = \sin\theta.\cot\alpha$

$$\Rightarrow \frac{\cos\beta}{\sin\beta} = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} \times \frac{\cos\alpha}{\sin\alpha}$$

$$\Rightarrow \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{\sin\alpha.\cos\beta}{\cos\alpha.\sin\beta}$$

$$\Rightarrow \frac{1+\tan^2\frac{\theta}{2}}{2\tan\frac{\theta}{2}} = \frac{\sin\beta.\cos\alpha}{\cos\beta.\sin\alpha}$$

by Componendo & Dividendo Rule

$$\Rightarrow \frac{1+\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}-2\tan\frac{\theta}{2}} = \frac{\sin\beta.\cos\alpha+\cos\beta.\sin\alpha}{\sin\beta.\cos\alpha-\cos\beta.\sin\alpha}$$

$$\Rightarrow \frac{\left(1+\tan\frac{\theta}{2}\right)^2}{\left(1-\tan\frac{\theta}{2}\right)^2} = \frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

$$\Rightarrow \left[\frac{1+\tan\frac{\theta}{2}}{1-1.\tan\frac{\theta}{2}}\right]^2 = \frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right) = \frac{\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

116. (B) $I = \int \frac{d\theta}{\sin\theta+\cos\theta}$

$$I = \frac{1}{\sqrt{2}}\int \frac{d\theta}{\frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta}$$

$$I = \frac{1}{\sqrt{2}}\int \frac{d\theta}{\cos\frac{\pi}{4}.\sin\theta+\cos\theta.\sin\frac{\pi}{4}}$$

$$I = \frac{1}{\sqrt{2}}\int \frac{d\theta}{\sin\left(\theta+\frac{\pi}{4}\right)}$$

$$I = \frac{1}{\sqrt{2}}\int \text{cosec}\left(\theta+\frac{\pi}{4}\right)d\theta$$

$$I = \frac{1}{\sqrt{2}}\log\left|\text{cosec}\left(\theta+\frac{\pi}{4}\right)-\cot\left(\theta+\frac{\pi}{4}\right)\right| + c$$

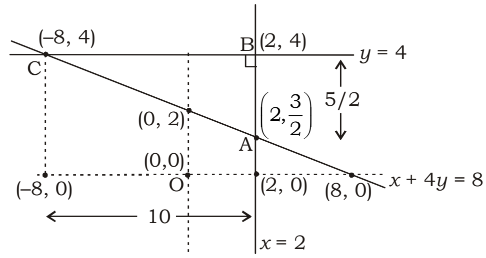
117. (B) $\lim_{x\to\infty}\left[\frac{x+5}{x+4}\right]^x$

$$\Rightarrow \lim_{x\to\infty}\left[\left[1+\frac{1}{x+4}\right]^{x+4}\right]^{\frac{x}{x+4}}$$

$$= e^{\lim_{x\to\infty}\frac{x}{x+4}} \qquad \left[\because \lim_{n\to\infty}\left[1+\frac{1}{n}\right]^n = e\right]$$

$$\Rightarrow e^{\lim_{x\to\infty}\frac{x}{x\left(1+\frac{4}{x}\right)}} \Rightarrow e^1 = e$$

118. (C)



Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$$= \frac{1}{2}\times\frac{5}{2}\times 10 = \frac{25}{2} \text{ sq.unit}$$

119. (C) $z = \frac{2+i}{1-2i} + \frac{3-2i}{1+2i}$

$$z = \frac{(2+i)(1+2i)+(3-2i)(1-2i)}{(1-2i)(1+2i)}$$

$$z = \frac{2+i+4i+2i^2+3-2i-6i+4i^2}{1-4i^2}$$

$$z = \frac{5-3i-2-4}{1+4}$$

$$z = \frac{-1-3i}{5} \text{ and } \bar{z} = \frac{-1+3i}{5}$$

Now, $z^2 - z\bar{z} = z(z-\bar{z})$

$$\Rightarrow z^2 - z\bar{z} = \frac{-1-3i}{5}\left(\frac{-1-3i}{5}-\frac{-1+3i}{5}\right)$$

$$\Rightarrow z^2 - z\bar{z} = \frac{-1-3i}{5}\left(\frac{-6i}{5}\right)$$

$$\Rightarrow z^2 - z\bar{z} = \frac{6i+18i^2}{25}$$

$$\Rightarrow z^2 - z\bar{z} = \frac{6i-18}{25}$$

120. (A)

NDA (MATHS) MOCK TEST - 150 (Answer Key)

1.	(B)	21.	(B)	41.	(D)	61.	(C)	81.	(B)	101.	(C)
2.	(A)	22.	(B)	42.	(B)	62.	(C)	82.	(C)	102.	(A)
3.	(B)	23.	(A)	43.	(C)	63.	(D)	83.	(C)	103.	(D)
4.	(C)	24.	(A)	44.	(C)	64.	(B)	84.	(A)	104.	(C)
5.	(B)	25.	(C)	45.	(C)	65.	(C)	85.	(C)	105.	(C)
6.	(C)	26.	(D)	46.	(A)	66.	(B)	86.	(B)	106.	(D)
7.	(C)	27.	(B)	47.	(B)	67.	(D)	87.	(B)	107.	(B)
8.	(D)	28.	(D)	48.	(B)	68.	(C)	88.	(D)	108.	(B)
9.	(B)	29.	(A)	49.	(C)	69.	(C)	89.	(A)	109.	(C)
10.	(B)	30.	(C)	50.	(D)	70.	(A)	90.	(C)	110.	(A)
11.	(A)	31.	(B)	51.	(B)	71.	(B)	91.	(D)	111.	(A)
12.	(C)	32.	(A)	52.	(B)	72.	(C)	92.	(C)	112.	(B)
13.	(B)	33.	(C)	53.	(A)	73.	(B)	93.	(D)	113.	(C)
14.	(B)	34.	(B)	54.	(B)	74.	(B)	94.	(C)	114.	(B)
15.	(D)	35.	(C)	55.	(C)	75.	(D)	95.	(D)	115.	(D)
16.	(C)	36.	(C)	56.	(A)	76.	(B)	96.	(D)	116.	(B)
17.	(B)	37.	(A)	57.	(C)	77.	(B)	97.	(C)	117.	(B)
18.	(B)	38.	(C)	58.	(B)	78.	(D)	98.	(D)	118.	(C)
19.	(C)	39.	(B)	59.	(C)	79.	(D)	99.	(B)	119.	(C)
20.	(B)	40.	(B)	60.	(B)	80.	(A)	100.	(B)	120.	(A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777