PLOT NO. 2 SSI, OPP METRO PILLAR 150, GT KARNAL ROAD, JAHANGIRPURI, DELHI: 110033

## NDA MATHS MOCK TEST - 172 (SOLUTION)

1. (A) Given that, $\mathrm{A}=\left[\begin{array}{cc}i & 0 \\ 0 & -i\end{array}\right], \mathrm{B}=\left[\begin{array}{cc}0 & -1 \\ 1 & 0\end{array}\right]$ and

$$
\left.\begin{array}{l}
\mathrm{C}=\left[\begin{array}{ll}
0 & i \\
i & 0
\end{array}\right] \\
\text { Now, } \mathrm{AB}
\end{array}=\left[\begin{array}{cc}
i & 0 \\
0 & -i
\end{array}\right]\left[\begin{array}{cc}
0 & -1 \\
1 & 0
\end{array}\right] \quad \begin{array}{ll}
0+0 & -i+0 \\
0-i & 0+0
\end{array}\right]=\left[\begin{array}{cc}
0 & -i \\
-i & 0
\end{array}\right] .
$$

$$
\Rightarrow \mathrm{AB}=-\mathrm{C}
$$

2. (B) Splitting 1.01 and using binomial theorem to write the first few ferms we have
$\mathrm{A}=(1.01)^{1000000}=(1+0.01)^{1000000}$
$\Rightarrow A={ }^{1000000} C_{0}+{ }^{1000000} C_{1}(0.01)+$ other positive terms
$\Rightarrow A=1+1000000 \times 0.01+$ other positive terms
$\Rightarrow A=1+10000+$ other positive terms
$>10000$
$\Rightarrow$ A $>10000$
$\Rightarrow A>B$
3. (C) We have,

$$
\begin{aligned}
& \sum_{r=1}^{100} a_{r}=\sum_{r=1}^{100} r(r!)=\sum_{r=1}^{100}\{(r+1)-1\} r! \\
& \Rightarrow \sum_{r=1}^{100} a_{r}=\sum_{r=1}^{100}\{(r+1)-r!\} \\
& \Rightarrow \sum_{r=1}^{100} a_{r}=(2!-1!)+(3!-2!)+(4!-3!)+\ldots \\
& \ldots .+(101!-100!) \\
& \Rightarrow \sum_{r=1}^{100} a_{r}=101!-1
\end{aligned}
$$

4. (D) Let $d$ be the common difference of the given A.P., then,

$$
\begin{aligned}
& \frac{1}{a_{1} a_{2}}+\frac{1}{a_{2} a_{3}}+\ldots \ldots .+\frac{1}{a_{n} a_{n+1}} \\
& =\frac{1}{d}\left\{\frac{a_{1}-a_{1}}{a_{1} a_{2}}+\frac{a_{3}-a_{2}}{a_{2}-a_{3}}+\ldots \cdot \frac{a_{n+1}-a_{n}}{a_{n} a_{n+1}}\right\} \\
& =\frac{1}{d}\left\{\frac{1}{a_{1}}-\frac{1}{a_{2}}+\frac{1}{a_{2}}-\frac{1}{a_{3}}+\ldots . \frac{1}{a_{n}}-\frac{1}{a_{n+1}}\right\}
\end{aligned}
$$

$=\frac{1}{d}\left\{\frac{1}{a_{1}}-\frac{1}{a_{n+1}}\right\}=\frac{a_{n+1}-a_{1}}{d a_{1} a_{n+1}}=\frac{n d}{d a_{1} a_{n+1}}$
$=\frac{n}{a_{1} a_{n+1}}$
5. (C) We have
$\int \sin ^{3} x \cos ^{2} x d x=\int \sin ^{2} x \cos ^{2} x(\sin x) d x$
$=\int\left(1-\cos ^{2} x\right) \cos ^{2} x(\sin x) d x$
Let $t=\cos x$
$\Rightarrow d t=-\sin x d x$
$\Rightarrow \int \sin ^{3} x \cos ^{2} x d x=-\int\left(1-t^{2}\right) t^{2} d t$

$$
\begin{aligned}
& =-\int t^{2}-t^{4} d t \\
& =-\left(\frac{t^{3}}{3}-\frac{t^{5}}{5}\right)+c \\
& =-\frac{1}{3} \cos ^{3} x+\frac{1}{5} \cos ^{5} x+c
\end{aligned}
$$

6. (A) Range of $\operatorname{cosec} A$ is $(-\infty,-1] \cup[1, \infty)$
7. (D) We have $x^{2}-6 x+13=x^{2}-6 x+3^{2}-3^{2}+13$

$$
=(x-3)^{2}+4
$$

So, $\int \frac{d x}{x^{2}-6 x+13}=\int \frac{1}{(x-3)^{2}+(2)^{2}} d x$
Let $x-3=t$
$d x=d t$

$$
\begin{aligned}
\Rightarrow \int \frac{d x}{x^{2}-6 x+13} & =\int \frac{d t}{t^{2}+2^{2}} \\
& =\frac{1}{2} \tan ^{-1} \frac{t}{2}+c \\
& =\frac{1}{2} \tan ^{-1} \frac{(x-3)}{2}+c
\end{aligned}
$$

8. (A) We have $y^{x}-x^{y}=1$
$\Rightarrow e^{x \log y}-e^{y \log x}=1$
diff. with respect to $x$, we get
$y^{x}\left\{\frac{x}{y} \frac{d y}{d x}+\log y\right\}-x^{y}\left\{\frac{d y}{d x} \log x+\frac{y}{x}\right\}=0$
Putting $x=1, y=2$
We get
$2\left(\frac{1}{2} \frac{d y}{d x}+\log 2\right)-(0+2)=0$
$\frac{d y}{d x}=2-2 \log 2=2(1-\log 2)$
9. (C) $e>1 \rightarrow$ hyperbola
$e=0 \rightarrow$ circle
$e<1 \rightarrow$ ellipse
$e=1 \rightarrow$ parabola
10. (C) Here $A^{2}=\left[\begin{array}{ll}\alpha & 0 \\ 1 & 2\end{array}\right]\left[\begin{array}{ll}\alpha & 0 \\ 1 & 2\end{array}\right]$
$A^{2}=\left[\begin{array}{cc}\alpha^{2} & 0 \\ \alpha+2 & 4\end{array}\right]=\left[\begin{array}{ll}1 & 0 \\ 1 & 4\end{array}\right]=B$
$\alpha^{2}=1, \alpha+2=1$
$\alpha=-1$, Matrix satisfied
11. (C) $\frac{\log _{\sqrt{\alpha \beta}}(\mathrm{H})}{\log _{\sqrt{\alpha \beta \gamma}}(\mathrm{H})}=\frac{\log _{\mathrm{H}} \sqrt{\alpha \beta \gamma}}{\log _{\mathrm{H}} \sqrt{\alpha \beta}}$

$$
=\log _{\sqrt{\alpha \beta}} \sqrt{\alpha \beta \gamma}
$$

$$
=\log _{\alpha \beta}(\alpha \beta \gamma)
$$

12. (B) $\cos \left(\frac{\pi}{4}+x\right)+\cos \left(\frac{\pi}{4}-x\right)$
$2 \cos \left[\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right] \cdot \cos \left[\frac{\frac{\pi}{4}+x-\frac{\pi}{4}+x}{2}\right]$
$2 \cos \frac{\pi}{4} \cdot \cos x$
$\sqrt{2} \cos x$
13. (B) We know that
$\because \mathrm{AM} \geq \mathrm{GM}$
Consider two terms $\sec ^{2} \theta, \frac{1}{\sec ^{2} \theta}$

$$
\begin{aligned}
& \therefore\left(\sec ^{2} \theta+\frac{1}{\sec ^{2} \theta}\right) \geq 2\left(\sec ^{2} \theta \cdot \frac{1}{\sec ^{2} \theta}\right)^{1 / 2} \\
& \Rightarrow\left(\sec ^{2} \theta+\cos ^{2} \theta\right) \geq 2 \\
& \Rightarrow y \geq 2
\end{aligned}
$$

14. (D) Word "ELEPHANT"

No of permulation $=\frac{8!}{2!}=20160$
15. (D) We know that $\omega^{3}=1$
$1+\omega+\omega^{2}=0$
$\omega^{100}+\omega^{200}+\omega^{300}$
$\left(\omega^{3}\right)^{33} \cdot \omega+\left(\omega^{3}\right)^{66} \cdot \omega^{2}+\left(\omega^{3}\right)^{100}$
$\omega+\omega^{2}+1=0$
16. (D) Here
$\Rightarrow \frac{z-1}{z+1}=\frac{x+i y-1}{x+i y+1}=\left[\frac{(x-1)+i y}{(x+1)+i y}\right] \times$
$\left[\frac{(x+1)-i y}{(x+1)-i y}\right]$
$\Rightarrow \operatorname{Re}\left(\frac{(x+1)(x-1)+y^{2}}{(x+1)^{2}+y^{2}}\right)=0$
$x^{2}+y^{2}-1=0$
$x^{2}+y^{2}=1$
$z=x+i y$
$|z|=\sqrt{x^{2}+y^{2}}$
$|z|=\sqrt{1}$
$|z|=1$
17. (B) We know that $\sin ^{-1}(\sin x)=x$

Therefore
$\sin ^{-1}\left(\sin \frac{3 \pi}{5}\right)=\frac{3 \pi}{5}$
But $\frac{3 \pi}{5} \notin\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, which does not belong to the range of $\sin ^{-1} x$

However $\sin \frac{3 \pi}{5}=\sin \left(\pi-\frac{3 \pi}{5}\right)=\sin \frac{2 \pi}{5}$
and $\frac{2 \pi}{5} \in\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
Therefore,
$\sin ^{-1} \sin \left(\frac{3 \pi}{5}\right)=\sin ^{-1}\left(\sin \frac{2 \pi}{5}\right)=\frac{2 \pi}{5}$
18. (C) Let the eq ${ }^{n}$ of the circle be
$(x-h)^{2}+(y-k)^{2}=r^{2}$
since the circle passes through $(2,-2)$
and (3, 4), we have
$(2-h)^{2}+(-2-k)^{2}=r^{2}$
and $(3-h)^{2}+(4-k)^{2}=r^{2}$
Also since the centre lies on the line
$x+y=2$, We have $h+k=2$
Sloving the equation (i), (ii) \& (iii), we get
$h=0.7, k=1.3$ and $r^{2}=12.58$
Hence, the eq ${ }^{n}$ of the required circle is
$(x-0.7)^{2}+(y-1.3)^{2}=12.58$
19. (C) $\sin 480^{\circ}-\sin 60^{\circ}+\sin 780^{\circ}+\cos 120^{\circ}$
$\Rightarrow \sin \left(360^{\circ}+120^{\circ}\right)-\sin 60^{\circ}+\sin (2 \times$
$\left.360^{\circ}+60^{\circ}\right)+\cos \left(90^{\circ}+30^{\circ}\right)$
$\Rightarrow \sin 120^{\circ}-\sin 60^{\circ}+\sin 60^{\circ}-\sin 30^{\circ}$
$\Rightarrow \cos 30^{\circ}-\sin 30^{\circ}$
$\Rightarrow \frac{\sqrt{3}-1}{2}$
20. (B) For equal root
$\mathrm{D}=0$
(where $\mathrm{D}=$ Discriminants)
A.T.Q,
$k x(x-2)+6=0$
$k x^{2}-2 k x+6=0$
$\mathrm{D}=b^{2}-4 a c=0$
$4 k^{2}-4 \times 6=0$
$4 k \neq 0 k=6$
$k-60 k=0$
$k=0$ doesn't satisfy equation
Hence $k=6$
21. (C) Here, $x^{2}-2 x \sec \theta+1=0$ has roots $\alpha_{1}$ and $\beta_{1}$
$\therefore \alpha_{1}, \beta_{1}=\frac{2 \sec \theta \pm \sqrt{4 \sec ^{2} \theta-4}}{2 \times 1}$

$$
=\frac{2 \sec \theta \pm 2 \tan \theta}{2}
$$

Since $\theta \in\left(\frac{-\theta}{6}, \frac{-\theta}{12}\right)$
i.e. $\theta \in$ IV Quadrant $=\frac{2 \sec \theta \mp 2 \tan \theta}{2}$
$\therefore \alpha_{1}=\sec \theta-\tan \theta$ and $\beta_{1}=\sec \theta+\tan \theta$ [as $\theta_{1}>\beta_{1}$ ]
and $x^{2}+2 x \tan \theta-1=0$ has roots $\alpha_{2}$ and $\beta_{2}$
i.e. $\alpha_{2}, \beta_{2}=\frac{-2 \tan \pm \sqrt{4 \tan ^{2}+4}}{2}$
$\therefore \alpha_{2}=-\tan \theta+\sec \theta$ and $\beta_{2}=-\tan \theta-\sec \theta$ [as $\alpha_{2}>\beta_{2}$ ]
$\Rightarrow \alpha_{1}+\beta_{2}=-2 \tan \theta$
22. (A) The number of ways $=\frac{{ }^{12} \mathrm{C}_{3} \times 2^{9}}{3^{12}}$

$$
=\frac{55}{3}\left(\frac{2}{3}\right)^{11}
$$

23. (C) $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B})$, as A and $B$ are independent events.
$\Rightarrow 0.9=0.4+\mathrm{P}(\mathrm{B})-(0.4) \cdot \mathrm{P}(\mathrm{B})$
$\Rightarrow \mathrm{P}(\mathrm{B})=\frac{5}{6}$
24. (B) To determine the quotient and remainder of binary digits, first we will convert these to decimals.
$(101110)_{2}=(46)_{10}$
$(110)_{2}=(6)_{10}$
Dividing 46 by 6 ,

Quotient $=(7)_{10}=(111)_{2}$
Remainder $=(4)_{10}=(100)_{2}$
25. (C) E is the universal set and $\mathrm{A}=\mathrm{B} \cup \mathrm{C}$
$E-(E-(E-(E-(E-A))))$
$\left.=E-\left(E-E-\left(E-A^{\prime}\right)\right)\right)$
$=E-(E-(E-A))$
$=E-\left(E-A^{\prime}\right)$
$=\mathrm{E}-\mathrm{A}$
$=A^{\prime}=(B \cup C)^{\prime}=B^{\prime} \cap C^{\prime}$
26. (D) Each property is true.
27.

$$
\text { (A) } \begin{aligned}
\mathrm{A} & =\left[\begin{array}{cc}
4 i-6 & 10 i \\
14 i & 6+4 i
\end{array}\right] \\
k \mathrm{~A} & =\left[\begin{array}{cc}
\frac{2 i-3}{i} & 5 \\
7 & \frac{3+2 i}{i}
\end{array}\right] \\
& =\left[\begin{array}{cc}
\frac{-2-3 i}{-1} & 5 \\
7 & \frac{3 i-2}{-1}
\end{array}\right] \\
& =\left[\begin{array}{cc}
2+3 i & 5 \\
7 & 2-3 i
\end{array}\right]
\end{aligned}
$$

28. (D) This is quadratic equation in the form of $|x-3|$.
Let $|x-3|=\mathrm{t}$
Therefore equation becomes ' $t^{2}+t-2=0$ '
Solving the equation, we get $t=1(-\mathrm{ve}$ value is neglected as $t$ is +ve )
$\therefore x=4,2$
Sum of roots $=6$
29. (C) $x^{2}-4 x-\log _{3} \mathrm{P}=0$

It is given that roots are real.
$\therefore \frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \geq 0$
Or $16+4 \log _{3} \mathrm{P} \geq 0$
Or $\log _{3} P \geq-4$
Or $\mathrm{P} \geq 3^{-4}$ or $\mathrm{P} \geq 1 / 81$
30. $\quad(\mathrm{C}) \operatorname{adj} \mathrm{A}^{\mathrm{T}}=(\operatorname{adj} \mathrm{A})^{\mathrm{T}}$

Therefore, $\operatorname{adj} A^{T}-(\operatorname{adj} A)^{T}=0$
31. (B) $\mathrm{C}_{0}+\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}+\mathrm{C}_{4}+\mathrm{C}_{5}+\ldots . . \mathrm{C}_{n}=2^{n}$ Therefore $\mathrm{C}_{1}+\mathrm{C}_{3}+\mathrm{C}_{5}+\ldots . . \mathrm{C}_{n}=\frac{1}{2} \times 2^{n}$
$=\frac{1}{2} \times 2^{50}=2^{49}$
32. (C) $(a, b) \mathrm{R}(c, d) a+d=b+c$

Or $b+c=a+d$
$(b, c) \mathrm{R}(a, d)$
$\therefore \mathrm{R}$ is symmetric
$(a, b) \mathrm{R}(c, d) \Leftrightarrow a+b=b+c$
$a+a=a+a$
$\Rightarrow(a, a) \mathrm{R}(a, a)$
$R$ is reflexive
$(a, b) \mathrm{R}(c, d)$ and $(c, d) \mathrm{R}(e, f)$
$\Rightarrow a+d=b+c$ and $c+f=d+e$
$\Rightarrow a+d+c+f=b+c+d+e$
$\Rightarrow a+f=b+e$
$\Rightarrow(a, b) \mathrm{R}(e, f)$
$\therefore \mathrm{R}$ is transitive
From (i), (ii), and (iii) : $R$ is an equivalence relation.
33. (B) $k x+y+z=1, x+k y+z=k$ and $x+y+$ $k z=k^{2}$ will have no solution if
$\left|\begin{array}{ccc}k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k\end{array}\right|=0$
Solving this determinant, $k=1,-2$
If $k=1$ then first two equations will become same.
Therefore $k=-2$.
34. (C) $\mathrm{R}=\frac{\pi}{2}$
$\therefore \mathrm{P}+\mathrm{Q}=\frac{\pi}{2}$
[sum of angles of a triangle $=\pi$ ]
$\frac{\mathrm{P}}{2}+\frac{\mathrm{Q}}{2}=\frac{\pi}{4}$
$\tan \left(\frac{\mathrm{P}}{2}+\frac{\mathrm{Q}}{2}\right)=\frac{\pi}{4}$
$\frac{\tan \frac{P}{2}+\tan \frac{Q}{2}}{1-\tan \frac{P}{2}+\tan \frac{Q}{2}}=\tan \frac{\pi}{4}$
Now $\tan \frac{\mathrm{P}}{2}$ and $\frac{\mathrm{Q}}{2}$ are roots of equation $a x+b x+c=0$
$\therefore$ Sum of roots $=-b / a$
And product of roots $=c / a$ putting in (i) :
$\frac{-b / a}{1-c / a}=1$

Or $a+b=c$
35. (B) The length of the normal from origin to the plane $=\frac{9}{\sqrt{1^{2}+2^{2}+(-2)^{2}}}=3$
36. (B) Let $\delta=x \hat{i}+y \hat{j}+z \hat{k}$

Since $\delta$ is perpendicular on $\alpha$ and $\beta$.
$\therefore x+2 y-z=0$
and $2 x+y-3 z=0$
also, $\delta . y=0$
$\therefore 2 x+y-6 z=0$
Solving (i), (ii), (iii)
$x=-2, y=2$, and $z=2$
$. \delta=\sqrt{4+4+4}=2 \sqrt{3}$
37. (C) The line joining the points $\overrightarrow{\mathrm{A}}(i+2 j-3 k)$ and $\overrightarrow{\mathrm{B}}(3 i-j+5 k)$ is i.e., $\mathrm{AB}=2 i-3 j+8 k$

Work done $=\overrightarrow{\mathrm{F}} . \mathrm{AB}=1 \times 2+3 \times(-3)+2 \times 8$
$=9$
38. (B) Let $\vec{a}=x \hat{i}+y \hat{j}+z \hat{k}$
$|\vec{a} \times i|^{2}+|\vec{a} \times j|^{2}+|\vec{a} \times \hat{k}|^{2}$
$=z^{2}+y^{2}+x^{2}+z^{2}+x^{2}+y^{2}=2\left(x^{2}+y^{2}+z^{2}\right)$
$=2|\vec{a}|^{2}$
39.
(B) $\left|\begin{array}{lll}a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c\end{array}\right|=0$

Applying operations on the given matrix:
$\mathrm{C}_{2} \rightarrow \mathrm{C}_{2}-\mathrm{C}_{1}, \mathrm{C}_{3} \rightarrow \mathrm{C}_{3}-\mathrm{C}_{1}$
$\left|\begin{array}{ccc}a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1\end{array}\right|=0$
Expanding by first row and dividing by $(1-a)(1-b)(1-c):$
$\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=0$
Or $\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=\frac{a}{1-a}+\frac{1}{1-a}$
Or $\frac{a}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$
40. (B) $\cos \frac{\pi}{4}=\frac{2 x-3+10}{\sqrt{4+1+4} \sqrt{x^{2}+9+25}}$
$\frac{1}{\sqrt{2}}=\frac{2 x+7}{3 \sqrt{x^{2}+34}}$
Squaring...
$\left[\frac{1}{2}=\frac{4 x^{2}+28 x+49}{9 x^{2}+306}\right]$
Or $x^{2}-56 x+208=0$
$\Rightarrow x=4$
41. (A) $2 x^{2}+7 y^{2}-20=0$

Put $x=1, y=2$
$2+28-20=10>0$
$\therefore(1,2)$ lies outside the ellipse
42. (A) Required equation of line is:
$y+5=\tan 120^{\circ} x$
or $y+5=-\sqrt{3} x$
or $y+\sqrt{3} x+5=0$
43. (B) Equation of line passing through intersection of $2 x-3 y+7=0$ and $7 x+$ $4 y+2=0$ is :
$(2 x-3 y+7)+\lambda(7 x+4 y+2)=0$
This line passes through $(2,3)$
$\therefore(4-9+7)+\lambda(14+12+2)=0$
$\lambda=-1 / 14$
Putting in eq(i) :
$21 x-46 y+96=0$
44. (B) Latus rectum $=4$ and $e=3 / 4$
$\therefore b^{2}=2 a$ and $c=e a$
Also $a^{2}=b^{2}+c^{2}$
or $a^{2}+2 a+\frac{9}{16} a^{2}$
or $a=32 / 7$
$b^{2}=64 / 7$
Equation of ellipse is
$\frac{49 x^{2}}{1024}+\frac{7 y^{2}}{64}=1$
45. (A) Only 1 and 2 are correct.
46. (B) $f^{\prime}(x)=-2 x, 0<x \leq 1$
47. (A) $f^{\prime}(x)=3 x^{2}-1$

Put $f^{\prime}(x)=0$ implies that $x= \pm \frac{1}{\sqrt{3}}$
Putting in $f^{\prime \prime}(x)$ : Maxima will be at $\frac{-1}{\sqrt{3}}$
and minima will be at $\frac{1}{\sqrt{3}}$
$\therefore \operatorname{Max} f(x)=f\left(\frac{1}{\sqrt{3}}\right)=\frac{2}{3 \sqrt{3}}$
$\operatorname{Min} f(x)=f\left(\frac{1}{\sqrt{3}}\right)=\frac{-2}{3 \sqrt{3}}$
48. (D) 1, 2 and 3 are correct.
49.
(C) $f(x)=\frac{x}{2}-1,[0, \pi]$
$\tan [f(x)]=\tan \left[\left(\frac{x}{2}-1\right)\right]$
$\frac{1}{f(x)}=\frac{1}{\frac{x}{2}-1}$
Both functions are discontinous for $x=2$ in $[0, \pi]$
50.
(B) $f^{\prime}(x)=\frac{-x e^{-x^{2}}}{\sqrt{1-e^{-x^{2}}}}$

Which is defined for all $x \in \mathrm{R}$
51. (B) $(A-2),(B-3), C-4),(D-1)$. $f(x)$ Maximum value
A. $\sin x+\cos x=\sqrt{1^{2}+1^{2}}=\sqrt{2}$
B. $3 \sin x+4 \cos x=\sqrt{3^{2}+4^{2}}=5$
C. $2 \sin x+3 \cos x=\sqrt{2^{2}+1^{2}}=\sqrt{3}$
D. $\sin x+3 \cos x=\sqrt{1^{2}+3^{2}} \sqrt{10}$
52. (D) $f(x)$ is continuous and differentiable also [As L.H.L $=$ R.H.L $=f(0)=0$ and L.H. $\mathrm{D}=$ R.H.D]
53. (C) $f(x)=\frac{x}{x}, x \neq 0$ implies that $y=1$
54. (A) $\mathrm{f}(\mathrm{n})=\left[\frac{1}{4}+\frac{n}{1000}\right]$
$\sum_{n=1}^{1000} f(n)=\left[1000 \times \frac{1}{4}+\frac{1}{1000}+\frac{2}{1000}+\ldots+\frac{1000}{1000}\right]$
$=[250+0+0+$ $\qquad$ $+1]$
$=251$
55. (B) For any open cylinder fo surface area, when it has maximum volume, the height and radius of the base area equal.
Therefore diameter of cylinder = Twice of its height
So $k$ will be equal to 2 .
56. (D) $y=\mathrm{A}[\sin (x+c)+\cos (x+c)]$
$y^{\prime}=\mathrm{A}[\cos (x+c)-\sin (x+c)]$
$y^{\prime \prime}=-\mathrm{A}[\sin (x+c)+\cos (x+c)=-y$
or $y^{\prime \prime}+y=0$
57. (A) Both statements are correct but 2 is not the correct explanation of 1 .
58. (D) $\frac{d y}{d x}=\frac{y \phi^{\prime}(x)-y^{2}}{\phi(x)}=y \frac{\phi^{\prime}(x)}{\phi(x)}-\frac{y^{2}}{\phi(x)}$

Dividing by $y^{2}$ on both sides
$\frac{1}{y^{2}} \frac{d y}{d x}=\frac{1 \phi^{\prime}}{y} \frac{(x)}{\phi(x)}-\frac{1}{\phi(x)}$
or $\frac{1}{y^{2}} \frac{d y}{d x}-\frac{1 \phi^{\prime}}{y} \frac{(x)}{\phi(x)}=-\frac{1}{\phi(x)}$
Let $\frac{-1}{y}=z$
$\frac{1}{y^{2}} \frac{d y}{d x}=\frac{d z}{d x}$
Above equation becomes :
$\frac{d z}{d x}-\frac{\phi^{\prime}(x)}{\phi(x)} \quad z=-\frac{1}{\phi(x)}$
I.F $=e^{\int \frac{\phi^{\prime}(x)}{\phi(x)} d x}=e^{\log _{\phi}(x)}=\phi(x)$

Solution of above equation is:
z. $\phi^{\prime}(x)=\int \frac{-1}{\phi(x)} \times \phi(x) d x$
$\frac{-1}{y} \phi^{\prime}(x)=-x$
or $y=\frac{\phi(x)}{x}+c$
59. (B) $\operatorname{fog}\left(\frac{e-1}{e+1}\right)=f\left[\ln \frac{\left(1+\frac{e-1}{e+1}\right)}{\left(1-\frac{e-1}{e+1}\right)}\right]$
$=f\left[\ln \left(\frac{e+1+e-1}{e+1-e+1}\right)\right]=f\left[\ln \left(\frac{2 e}{2}\right)\right]=f(1)=$
$\frac{4+1}{1+4}=1$
60. (B) $\left|\begin{array}{lll}1-\alpha & \alpha-\alpha^{2} & \alpha^{2} \\ 1-\beta & \beta-\beta^{2} & \beta^{2} \\ 1-\gamma & \gamma-\gamma^{2} & \gamma^{2}\end{array}\right|$

Operating $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}$
$\left|\begin{array}{ccc}1 & \alpha-\alpha^{2} & \alpha^{2} \\ 1 & \beta-\beta^{2} & \beta^{2} \\ 1 & \gamma-\gamma^{2} & \gamma^{2}\end{array}\right|$

Operating $\mathrm{C}_{2} \rightarrow \mathrm{C}_{2}+\mathrm{C}_{3}\left|\begin{array}{ccc}1 & \alpha & \alpha^{2} \\ 1 & \beta & \beta^{2} \\ 1 & \gamma & \gamma^{2}\end{array}\right|$
$=(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
61. (B) $\mathrm{A}=\left(\begin{array}{lll}1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & 1\end{array}\right)$

First we will find all the cofactors of this matrix:
$A_{11}=1, A_{12}=-2, A_{13}=6$
$A_{21}=6, A_{22}=1, A_{23}=-3$
$A_{31}=-2, A_{32}=4, A_{33}=1$
$\operatorname{Adj} A=\left[\begin{array}{ccc}1 & -2 & 6 \\ 6 & 1 & -3 \\ -2 & 4 & 1\end{array}\right]^{\mathrm{T}}=\left[\begin{array}{ccc}1 & 6 & -2 \\ -2 & 1 & 4 \\ 6 & -3 & 1\end{array}\right]$
62. (B) $\mathrm{A}=\left(\begin{array}{cc}-2 & 2 \\ 2 & -2\end{array}\right)$
$A^{2}=\left(\begin{array}{cc}-2 & 2 \\ 2 & -2\end{array}\right)\left(\begin{array}{cc}-2 & 2 \\ 2 & -2\end{array}\right)=\left(\begin{array}{cc}8 & -8 \\ -8 & 8\end{array}\right)$
$=-4 \mathrm{~A}$
63. (C) $\operatorname{Re}\left(z^{2}-i\right)=2$

Let $z=x+i y$
$z^{2}-i=2$
or $x^{2}-y^{2}+2 x y-i=2$
$x^{2}-y^{2}+(2 x y-1) i=2$
Now $\operatorname{Re}\left(z^{2}-i\right)=x^{2}-y^{2}$
Therefore $x^{2}-y^{2}=2$, Which is equation of rectangular hyperbola.
64. (D) $X=3,6,9 \ldots \ldots .48\}=16$
$\mathrm{Y}=1,3,5 \ldots \ldots .49\}=25$
Total integers $=51 \quad(0$ is also included $)$
$\therefore \mathrm{P}(\mathrm{X})=\frac{16}{51}, \mathrm{P}(\mathrm{Y})=\frac{25}{51}$
65. (B) 1 and 2 are correct statements. 3rd is incorrect because mean deviation is least when measured about mean not median.
66. (D) A.M $=24$, S.D $=0$

As S.D $=0$, therefore average of any 5 observations will be equal to A.M.
67. (A) Regression coefficient of $y$ on $x$ is equal to the regression coefficient of $x$ on $y$, which implies that $(x, y)$ lies on the line $x=y$.
68.
C) $\mathrm{P}(\mathrm{A})=\frac{1}{3}, \mathrm{P}(\mathrm{B})=\frac{1}{6}, \mathrm{P}(\mathrm{A} \cap \mathrm{B})=\frac{1}{12}$
$\mathrm{P}(\mathrm{B} / \overline{\mathrm{A}})=\frac{\mathrm{P}(\mathrm{B} \cap \overline{\mathrm{A}})}{\mathrm{P}(\overline{\mathrm{A}})}$
Now, $\mathrm{P}(\overline{\mathrm{A}})=1-\frac{1}{3}=\frac{2}{3}$
$=\mathrm{P}(\mathrm{B} \cap \overline{\mathrm{A}})=\mathrm{P}(\mathrm{B})-\mathrm{P}(\mathrm{A} \cap \mathrm{B})=\frac{1}{6}-\frac{1}{12}=\frac{1}{12}$
Therefore $P(B / \bar{A})=\frac{1}{8}$
69. (C) $\operatorname{Mean}(n p)=\frac{2}{3}$

Variance $(\mathrm{npq})=\frac{5}{9}$
Therefore $\mathrm{q}=\mathrm{npq} / \mathrm{np}=\frac{5}{6}$
$\mathrm{p}=1-\mathrm{q}=\frac{1}{6}, \mathrm{n}=4$
$\mathrm{p}(x=2)={ }^{4} \mathrm{C}_{2}\left(\frac{1}{6}\right)^{2} \times\left(\frac{5}{6}\right)^{2}=\frac{25}{216}$
70. (C) $\mathrm{P}($ safely reaches $)=\frac{1}{3}$
$P($ not reaches safely $)=1-\frac{1}{3}=\frac{2}{3}$
$P($ at least 4 arrive safely $)=P(4)+P(5)$
$={ }^{5} \mathrm{C}_{4}\left(\frac{1}{3}\right)^{4}\left(\frac{2}{3}\right)+{ }^{5} \mathrm{C}_{5}\left(\frac{1}{3}\right)^{5}$
$=\frac{11}{3^{5}}=\frac{11}{243}$
71. (C) Regression equation of X on Y is $\mathrm{X}-\overline{\mathrm{X}}=$ $\mathrm{r}-\frac{\sigma x}{\sigma y}(\mathrm{Y}-\overline{\mathrm{Y}})$
After Substituting the values and solving it, we get
$\mathrm{X}=-8+0.2 \mathrm{Y}$
72. (B) $\mathrm{P}(\mathrm{he} /$ she know correct answer) $=p$ P (he/she guesses correct answer)
$=(1-\mathrm{p}) \times \frac{1}{m}$
P (correct answer) $=\mathrm{p}+\frac{1-\mathrm{p}}{m}$
P (he/she really know correct answer)
$=\frac{p}{p+\frac{1-p}{m}}$

$$
=\frac{m \mathrm{p}}{1+\mathrm{p}(m-1)}
$$

73. (C) Let $y=\tan ^{-1} \frac{x}{\sqrt{1-x^{2}}}$ and $z=\cos ^{-1} x$

$$
x=\cos z
$$

$\Rightarrow y=\tan ^{-1}\left(\frac{\cos z}{\sqrt{1-\cos ^{2} z}}\right)$
$\Rightarrow y=\tan ^{-1}\left(\frac{\cos z}{\sin z}\right)$
$\Rightarrow y=\tan ^{-1}\left[\tan \left(\frac{\pi}{2}-z\right)\right]$
$\Rightarrow y=\frac{\pi}{2}-z$
On diffrentiating both side w.r.t. 'z'
$\Rightarrow \frac{d y}{d z}=-1$
74. (B)

$\frac{1}{2}=1 \times 2^{-1} \longleftarrow$
$\frac{1}{4}=1 \times 2^{-2} \longleftarrow$
$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}=0.75$
$(10010)_{2}=(18)_{10}$,
$(0.11)_{2}=(0.75)_{10}$
Hence $(10010.11)_{2}=(18.75)_{10}$
75. (D) $10!\times \mathrm{C}(19,11)=k . \mathrm{P}(19,8)$
$10!\times \frac{19!}{11!8!}=k \cdot \frac{19!}{11!}$
$\frac{10!}{8!}=k \Rightarrow k=90$
76. (B) Given that $e=\frac{17}{30}$ and $\frac{2 a}{e}=120 \Rightarrow \frac{2 a \times 30}{17}=120 \Rightarrow a=34$


Now, $e^{2}=1-\frac{b^{2}}{(34)^{2}}$
$\Rightarrow \frac{64}{289}=\frac{b^{2}}{(34)^{2}} \Rightarrow \frac{8}{17}=\frac{b}{34} \Rightarrow b=16$
Area of $\triangle \mathrm{AOB}=\frac{1}{2} \times \mathrm{OA} \times \mathrm{OB}$

$$
=\frac{1}{2} \times 16 \times 34=272
$$

Area of $\mathrm{ABCD}=4 \times$ Area of $\triangle \mathrm{AOB}$

$$
=4 \times 272=1088 \text { sq. units }
$$

77. (C)

$$
\left|\begin{array}{ccc}
8! & 9! & 10! \\
9! & 10! & 11! \\
10! & 11! & 12!
\end{array}\right|
$$

$$
\mathrm{R}_{2} \rightarrow \mathrm{R}_{2}-\mathrm{R}_{1} \text { and } \mathrm{R}_{3} \rightarrow \mathrm{R}_{3}-\mathrm{R}_{1}
$$

$$
\Rightarrow\left|\begin{array}{ccc}
8! & 9! & 10! \\
8 \times 8! & 9 \times 9! & 10 \times 10! \\
89 \times 8! & 109 \times 9! & 131 \times 10!
\end{array}\right|
$$

$$
\Rightarrow 8!\times 9!\times 10!\left|\begin{array}{ccc}
1 & 0 & 0 \\
8 & 1 & 2 \\
89 & 20 & 42
\end{array}\right|
$$

$$
\Rightarrow 8!\times 9!\times 70![1(42-40)-0-0]
$$

$$
\Rightarrow 2 \times 8!\times 9!\times 10!
$$

78. (A) $\vec{a}=3 \hat{i}+2 \hat{j}-5 \hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-4 \hat{k}$

$$
\begin{aligned}
\vec{b}-2 \vec{a} & =(-\hat{i}+\hat{j}-4 \hat{k})-2(3 \hat{i}+2 \hat{j}-5 \hat{k}) \\
\vec{b}-2 \vec{a} & =(-7 \hat{i}-3 \hat{j}+6 \hat{k}) \\
3 \vec{a}-\vec{b} & =3(3 \hat{i}+2 \hat{j}-5 \hat{k})-(-\hat{i}+\hat{j}-4 \hat{k}) \\
& =(10 \hat{i}+5 \hat{j}-11 \hat{k})
\end{aligned}
$$

Now $(\vec{b}-2 \vec{a}) \cdot(3 \vec{a}-\vec{b})$
$\Rightarrow(-7 \hat{i}-3 \hat{j}+6 \hat{k}) \cdot(10 \hat{i}+5 \hat{j}-11 \hat{k})$
$\Rightarrow-70-15-66 \Rightarrow-151$
79. (D) A.T.Q -
$a+33 d=235$
$a+234 d=34$
from eq. (i) and eq (ii)
$d=-1$ and $a=268$
Let $\mathrm{T}_{\mathrm{n}}=0$
$\Rightarrow a+(n-1) d=0$
$\Rightarrow 268+(n-1)(-1)=0 \Rightarrow n=269$
80. (C) $\operatorname{cosec}^{-1}(-\sqrt{2})=\operatorname{cosec}^{-1}\left(-\operatorname{cosec} \frac{\pi}{4}\right)$
$\operatorname{cosec}{ }^{-1}(-\sqrt{2})=\operatorname{cosec}^{-1}\left[\operatorname{cosec}\left(-\frac{\pi}{4}\right)\right]=-\frac{\pi}{4}$
81. (A) Let $y=\log _{10}\left(3 x^{2}-5\right)$ and $z=x^{2}$

$$
\begin{aligned}
& y=\log _{10}(3 z-5) \\
& y=\log _{10} e \times \log _{e}(3 z-5)
\end{aligned}
$$

On differentialting both side w.r.t. 'z'
$\frac{d y}{d z}=\log _{10} e \times \frac{1}{3 z-5} \times 3$
$\frac{d y}{d z}=\frac{3 \log _{10} e}{3 z-5} \Rightarrow \frac{d y}{d z}=\frac{3 \log _{10} e}{3 x^{2}-5}$
82. (C) Probability of selecting Rohan $P(R)=\frac{2}{5}$ and $P(\bar{R})=1-\frac{2}{5}=\frac{3}{5}$
probability of selecting Sumit $P(S)=\frac{1}{4}$

$$
P(\bar{S})=1-\frac{1}{4}=\frac{3}{4}
$$

Probability of one of them is selected
$=\frac{2}{5} \times \frac{3}{4}+\frac{3}{5} \times \frac{1}{4} \Rightarrow \frac{6}{20}+\frac{3}{20}=\frac{9}{20}$
83. (A)
84. (B) equation $x^{2}-5 x+3=0$
$\alpha+\beta=5$ and $\alpha . \beta=3$
Now, $\frac{\alpha^{4}-\beta^{4}}{\alpha^{-4}-\beta^{-4}}=\frac{\alpha^{4}-\beta^{4}}{\frac{1}{\alpha^{4}}-\frac{1}{\beta^{4}}}=\frac{\alpha^{4}-\beta^{4}}{\frac{\beta^{4}-\alpha^{4}}{(\alpha \beta)^{4}}}$
$=-(\alpha \beta)^{4}=-3^{4}=-81$
85. (D) given that the equation of circle
$x^{2}+y^{2}-4 x-3 y-16=0$
Let equation of circle which is concentric with given equation
$x^{2}+y^{2}-4 x-3 y+c=0$
it passes through the point $(3,-2)$
$9+4-4 \times 3(-2)+c=0 \Rightarrow c=-7$
from eq. (ii)
$x^{2}+y^{2}-4 x-3 y-7=0$
86. (C) points ( $a, 0$ ), $\left(a t_{1}^{2}, 2 a t_{1}\right)$ and $\left(a t_{2}^{2}, 2 a t_{2}\right)$ are collinear, then

$$
\left|\begin{array}{ccc}
a & 0 & 1 \\
a t_{1}^{2} & 2 a t_{1} & 1 \\
a t_{2}^{2} & 2 a t_{2} & 1
\end{array}\right|=0
$$

$$
\begin{aligned}
& \Rightarrow a \times 2 a\left|\begin{array}{ccc}
1 & 0 & 1 \\
t_{1}^{2} & t_{1} & 1 \\
t_{2}^{2} & t_{2} & 1
\end{array}\right|=0 \\
& \Rightarrow 1\left(t_{1}-t_{2}\right)+1\left(t_{1}^{2} \cdot t_{2}-t_{1} \cdot t_{2}^{2}\right)=0 \\
& \Rightarrow 1\left(t_{1}-t_{2}\right)+t_{1} \cdot t_{2}\left(t_{1}-t_{2}\right)=0 \\
& \Rightarrow\left(t_{1}-t_{2}\right)\left(1+t_{1} \cdot t_{2}\right)=0 \\
& \Rightarrow t_{1} \cdot t_{2}+1=0 \Rightarrow t_{1} \cdot t_{2}=-1
\end{aligned}
$$

87. (B) In the expansion of $\left(9 x-\frac{6}{x^{3}}\right)^{8}$

$$
\begin{aligned}
\mathrm{T}_{r+1} & ={ }^{8} \mathrm{C}_{r}(9 x)^{8-r}\left(\frac{-6}{x^{3}}\right)^{r} \\
& ={ }^{8} \mathrm{C}_{r}(9)^{8-r}(-6)^{r} x^{8-4 r}
\end{aligned}
$$

Now, $8-4 r=0 \Rightarrow r=2$
The required term $=2+1=3 \mathrm{rd}$
88. (B) $\frac{4 x}{12 x^{2}+24 x-11}>\frac{1}{3 x+4}$

$$
\Rightarrow 12 x^{2}+16 x>12 x^{2}+24 x-11
$$

$$
\Rightarrow 0>8 x-11
$$

$$
\Rightarrow 8 x<11 \Rightarrow x<\frac{11}{8}
$$

Hence $x \in\left(-\infty, \frac{11}{8}\right)$
89. (A) Zero
90. (C) given that $b_{y x}=\frac{-10}{9}$ and $b_{x y}=\frac{-2}{5}$

$$
\begin{aligned}
& r=\sqrt{b_{y x} \times b_{x y}} \\
& r=\sqrt{\frac{-10^{2}}{9} \times \frac{-2}{5}} \Rightarrow r=\frac{-2}{3}
\end{aligned}
$$

91. (B) One year $=365$ days

$$
=52 \text { weeks and } 1 \text { day }
$$

The required Probability $=\frac{1}{7}$
92. (C)

$$
\begin{array}{r}
10 x 011 \\
-11 y 01 \\
\hline 1180 \\
\hline z=1, y=1, x=1
\end{array}
$$

93. (B) Let $\mathrm{AB}=x \mathrm{~m}$

$\operatorname{In} \triangle \mathbf{A P Q}$
$\tan 60^{\circ}=\frac{\mathrm{PQ}}{\mathrm{AP}}$
$\Rightarrow \sqrt{3}=\frac{90}{90-x}$
$\Rightarrow 90 \sqrt{3}-\sqrt{3} x=90$
$\Rightarrow x=\frac{90(3-\sqrt{3})}{3}$
$\Rightarrow x=30(3-1.732)=38.04 \mathrm{~m}$
94. (C) given that $\vec{a}=2 \hat{i}+3 \hat{j}+4 \hat{k}, \vec{b}=\hat{i}+\hat{j}-2 \hat{k}$ and $\vec{c}=2 \hat{i}+\hat{j}-3 \hat{k}$

Now, $\vec{a} \times(\vec{b}-\vec{c})-\vec{b} \times(\vec{c}-\vec{a})+\vec{c} \times(\vec{a}-\vec{b})$
$\Rightarrow \vec{a} \times \vec{b}-\vec{a} \times \vec{c}-\vec{b} \times \vec{c}+\vec{b} \times \vec{a}+\vec{c} \times \vec{a}-\vec{c} \times \vec{b}$
$\Rightarrow \vec{a} \times \vec{b}+\vec{c} \times \vec{a}-\vec{b} \times \vec{c}-\vec{a} \times \vec{b}+\vec{c} \times \vec{a}+\vec{b} \times \vec{c}$
$\Rightarrow 2 \vec{c} \times \vec{a}$
$\Rightarrow 2\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & 3 & 4\end{array}\right|$
$\Rightarrow 2[\hat{i}(4+9)-\hat{j}(8+6)+\hat{k}(6-2)]$
$\Rightarrow 2[13 \hat{i}-14 \hat{j}+4 \hat{k}] \Rightarrow 26 \hat{i}-28 \hat{j}+8 \hat{k}$
95. (B)
96. (B) $10^{-x \sec x}\left[\frac{d}{d x} 10^{x \sec x}\right]$
$\Rightarrow 10^{-x \sec x}\left[10^{x \sec x} \log 10\{x \cdot \sec x \cdot \tan x+\sec x\}\right]$
$\Rightarrow 10^{-x \sec x} 10^{x \sec x} \cdot \sec x(x \tan x+1) \ln 10$
$\Rightarrow \sec x(x \tan x+1) \ln 10$
97. (A) $\Rightarrow(35 x-38)^{5}={ }^{5} \mathrm{C}_{0}(35 x)^{5}+{ }^{5} \mathrm{C}_{1}(35 x)^{4} .38$ $+{ }^{5} \mathrm{C}_{2}(35 x)^{3} .38^{2}+\ldots \ldots \ldots \ldots \ldots+{ }^{5} \mathrm{C}_{5} 38^{5}$
$\Rightarrow(35 x-38)^{5=5} \mathrm{C}_{0} 35^{5}+{ }^{5} \mathrm{C}_{2} 35^{4} .38^{1+5} \mathrm{C}_{3}$ $35^{4} .38^{1} \ldots \ldots \ldots .{ }^{5} \mathrm{C}_{5} 38^{5}$
$\Rightarrow(-3)^{5}=$ sum of the coeffi. of all terms
$\Rightarrow$ sum of the coefficients of all terms $=-243$
98. (B) given that A.M. $=\frac{a+b}{2}=14 \Rightarrow a+b=28$

$$
\text { G.M. }=\sqrt{a b}=7 \Rightarrow a b=49
$$

Now,

$$
\begin{aligned}
& \text { H.M. }=\frac{2 a b}{a+b} \\
& \text { H.M. }=\frac{2 \times 49}{28}=\frac{7}{2}
\end{aligned}
$$

99. (A) $\mathrm{A} . \mathrm{M} \geq \mathrm{G} . \mathrm{M} \geq \mathrm{H} . \mathrm{M}$.
100. 

(B) $\lim _{x \rightarrow \pi / 4} \frac{(1-\tan x)(1+\sin 2 x)}{(1+\tan x)(\pi-4 x)}\left[\frac{0}{0}\right]$ form by L-Hospital's Rule

$$
\begin{aligned}
& \Rightarrow \lim _{x \rightarrow \pi / 4} \frac{(1-\tan x)(2 \cos 2 x)+(1+\sin 2 x)\left(-\sec ^{2} x\right)}{(1+\tan x)(-4)+(\pi-4 x)\left(\sec ^{2} x\right)} \\
& \Rightarrow \frac{\left(1-\tan \frac{\pi}{4}\right)\left(2 \cos \frac{\pi}{2}\right)+\left(1+\sin \frac{\pi}{2}\right)\left(-\sec ^{2} \frac{\pi}{4}\right)}{\left(1+\tan \frac{\pi}{4}\right)(-4)+(\pi-\pi) \sec ^{2} \frac{\pi}{4}} \\
& \Rightarrow \frac{0+2(-2)}{2(-4)+0}=\frac{-4}{-8}=\frac{1}{2}
\end{aligned}
$$

101. (B) $i^{n+3}+i^{n+4}+i^{n+5}+i^{n+6}+i^{n+7}$

$$
\Rightarrow i^{n+3}\left(1+i+i^{2}+i^{3}+i^{4}\right)
$$

$$
\Rightarrow i^{n+3}(1+i-1-i+1)=i^{n+3}
$$

102. (B) I $=\int_{0}^{5}|x-3| d x$

$$
\begin{aligned}
& \mathrm{I}=\int_{0}^{3}-(x-3) d x+\int_{3}^{5}(x-3) d x \\
& \mathrm{I}=-\left[\frac{x^{2}}{2}-3 x\right]_{0}^{3}+\left[\frac{x^{2}}{2}-3 x\right]_{3}^{5} \\
& \mathrm{I}=-\frac{3}{2}+\left[\frac{25}{2}-15-\frac{9}{2}+9\right]
\end{aligned}
$$

$I=\frac{-9}{2}+9+\frac{25}{2}-15-\frac{9}{2}+9$
$\mathrm{I}=3+\frac{7}{2}=\frac{13}{2}$
103. (B) $\mathrm{I}=\int_{0}^{2 \pi} \frac{\tan \frac{x}{4}}{\tan \frac{x}{4}+\cot \frac{x}{4}} d x$
$\mathrm{I}=\int_{0}^{2 \pi} \frac{\tan \frac{2 \pi-x}{4}}{\tan \frac{2 \pi-x}{4}+\cot \frac{2 \pi-x}{4}} d x$

$$
\begin{equation*}
\mathrm{I}=\int_{0}^{2 \pi} \frac{\cot \frac{x}{4}}{\cot \frac{x}{4}+\tan \frac{x}{4}} d x \tag{ii}
\end{equation*}
$$

from eq. (i) and eq (ii)
$2 \mathrm{I}=\int_{0}^{2 \pi} \frac{\tan \frac{x}{4}+\cot \frac{x}{4}}{\tan \frac{x}{4}+\cot \frac{x}{4}} d x$
$2 \mathrm{I}=\int_{0}^{2 \pi} 1 . d x \Rightarrow 2 \mathrm{I}=[x]_{0}^{2 \pi}$
$\Rightarrow 2 \mathrm{I}=2 \pi \Rightarrow \mathrm{I}=\pi$
104. (C)

$(-1,4)$
Equation of circle
$(x-3)(x+1)+(y+2)(y-4)=0$
$x^{2}-2 x-3+y^{2}-2 y-8=0$
$x^{2}+y^{2}-2 x-2 y-11=0$
105. (A) $y=c . e^{\tan ^{-1} x}$
...(i)
On differentiating both side w.r.t. ' $x$ '
$\frac{d y}{d x}=c . e^{\tan ^{-1} x} \cdot \frac{1}{1+x^{2}}$
$\frac{d y}{d x}=\frac{y}{1+x^{2}} \quad[$ from eq (i)]
$\left(1+x^{2}\right) \frac{d y}{d x}=y$

PLOT NO. 2 SSI, OPP METRO PILLAR 150, GT KARNAL ROAD, JAHANGIRPURI, DELHI: 110033
106. (D)
107. (C) $f(x)=\left\{\begin{array}{cc}4 x^{2}+9 x-1, & 0 \leq x \leq 1 \\ 30-x, & 1<x \leq 2\end{array}\right.$
(a) $f(x)=13-x \quad$ on $[1,2]$

$$
1<2
$$

but $f(1)>f(2)$
$f(x)$ is decreaseing on $[1,2]$.
(b) L.H.L. $=\lim _{x \rightarrow 1} f(x)$

$$
=\lim _{x \rightarrow 1}\left(4 x^{2}+9 x-1\right)=12
$$

R.H.L. $=\lim _{x \rightarrow 1} f(x)$

$$
=\lim _{x \rightarrow 1} 13-x=12
$$

L.H.L. $=$ R.H.L.
$\boldsymbol{f}(\boldsymbol{x})$ is continuous on [0, 2].
(c) $f(x)$ is increasing on $[0,1]$ and $f(x)$ is decreasing on $[1,2]$
Hence $\boldsymbol{f}(\boldsymbol{x})$ is maximum at $\boldsymbol{x}=\mathbf{1}$.
(d) L.H.D. $=\lim _{h \rightarrow 0} \frac{f(1-h)-f(1)}{-h}$

$$
\begin{aligned}
& =\lim _{h \rightarrow 0} \frac{4(1-h)^{2}+9(1-h)-1-12}{-h} \\
& =\lim _{h \rightarrow 0} \frac{4 h^{2}-8 h-9 h}{-h} \\
& =\lim _{h \rightarrow 0^{-}}-4 h+8+9=17
\end{aligned}
$$

108. (B)
$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2 \times 2 \cos ^{2} \frac{4 \pi}{3}}}}}$

$$
\begin{aligned}
& \Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2 \times 2 \cos ^{2} \frac{4 \pi}{3}}}}} \\
& \Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2 \cos \frac{4 \pi}{3}}}}} \\
& \Rightarrow \sqrt{2+\sqrt{2+\sqrt{2 \times 2 \cos ^{2} \frac{2 \pi}{3}}}} \\
& \Rightarrow \sqrt{2+\sqrt{2 \times 2 \cos ^{2} \frac{\pi}{3}}} \\
& \Rightarrow \sqrt{2+\sqrt{2 \times 2 \cos ^{2} \frac{\pi}{3}}}
\end{aligned}
$$

$\Rightarrow \sqrt{2+2 \cos \frac{\pi}{3}}$
$\Rightarrow \sqrt{2+2 \cos ^{2} \frac{\pi}{6}}$
$\Rightarrow 2 \cos \frac{\pi}{6}=2 \times \frac{\sqrt{3}}{2}=\sqrt{3}$
109. (B) Let $\mathrm{AB}=x \mathrm{~m}$

$$
\mathrm{AC}=4 \times \mathrm{m}
$$


$\Delta$ ABC
$\tan \theta=\frac{\mathrm{AB}}{\mathrm{AC}}$
$\tan \theta=\frac{x}{4 x} \Rightarrow \theta=\tan ^{-1}\left(\frac{1}{4}\right)$
110. (B) Conic $4 x^{2}-6 y^{2}=48$

$$
\frac{x^{2}}{12}-\frac{y^{2}}{8}=1
$$

Now, eccentricity $e=\sqrt{1+\frac{b^{2}}{a^{2}}}$

$$
\begin{aligned}
& e=\sqrt{1+\frac{8}{12}} \\
& e=\sqrt{\frac{20}{12}}=\sqrt{\frac{5}{3}}
\end{aligned}
$$

111. (A) maximum value of $(24 \sin \theta+7 \cos \theta)$

$$
\begin{aligned}
& =\sqrt{(24)^{2}+(7)^{2}} \\
& =\sqrt{576+49}=25
\end{aligned}
$$

112. (D) $\frac{1+\sin \theta}{1-\sin \theta}=3$
$1+\sin \theta=3-3 \sin \theta$
$4 \sin \theta=2$
$\sin \theta=\frac{1}{2}$
$\sin \theta=\sin \frac{\pi}{6}$
$\theta=n \pi+(-1)^{n} \frac{\pi}{6}$
113. (A)
114. (A) Differential equation
$3\left(1+e^{2 x}\right) y d y=e^{x} d x$
$\Rightarrow 3 y d y=\frac{e^{x}}{1+e^{2 x}} d x$
Let $e^{x}=t \Rightarrow e^{x} d x=d t$
$\Rightarrow 3 y d y=\frac{d t}{1+t^{2}}$
On integrating
$\Rightarrow \frac{3 y^{2}}{2}=\tan ^{-1} t+c$
$\Rightarrow 3 y^{2}=2 \tan ^{-1}\left(e^{x}\right)+c$
115. (C) $\mathrm{I}=\int_{0.1}^{2.5}[x] d x$
$\mathrm{I}=\int_{0.1}^{1}[x] d x+\int_{1}^{2}[x] d x+\int_{2}^{2.5}[x] d x$
$\mathrm{I}=\int_{0.1}^{1} 0 . d x+\int_{1}^{2} 1 \cdot d x+\int_{2}^{2.5} 2 . d x$
$\mathrm{I}=0+[x]_{1}^{2}+2[x]_{2}^{2.5}$
$\mathrm{I}=2-1+2(2.5-2)$
$\mathrm{I}=1+2 \times \frac{1}{2}=2$
116. (D) given that $A=B \cap C$

Now, ( $\mathrm{U}-(\mathrm{U}-(\mathrm{U}-(\mathrm{U}-(\mathrm{U}-\mathrm{A}))))$
$\Rightarrow\left(\mathrm{U}-\left(\mathrm{U}-\left(\mathrm{U}-\left(\mathrm{U}-\mathrm{A}^{\prime}\right)\right)\right)\right)$
$\Rightarrow(U-(U-(U-A)))$
$\Rightarrow\left(\mathrm{U}-\left(\mathrm{U}-\mathrm{A}^{\prime}\right)\right)$
$\Rightarrow(\mathrm{U}-\mathrm{A})=\mathrm{A}^{\prime}=(\mathrm{B} \cap \mathrm{C})^{\prime}=\left(\mathrm{B}^{\prime} \cup \mathrm{C}^{\prime}\right)$
117. (C) $1 \times 001$
$\begin{array}{r}+101 y 1 \\ \hline 101 z 00 \\ \hline\end{array}$
$y=1, z=0, x=0$
118. (C) Let $\mathrm{BC}=h m$


## In $\triangle \mathbf{A B P}$

$$
\begin{aligned}
\tan 45 & =\frac{\mathrm{AB}}{\mathrm{AP}} \\
1 & =\frac{30+h}{\mathrm{AP}} \Rightarrow \mathrm{AP}=30+h=\mathrm{QC}
\end{aligned}
$$

In $\triangle$ QCB

$$
\begin{aligned}
& \tan 30^{\circ}=\frac{\mathrm{BC}}{\mathrm{QC}} \\
& \frac{1}{\sqrt{3}}=\frac{h}{30+h} \Rightarrow h=15(\sqrt{3}+1)
\end{aligned}
$$

Height of the tower $=30+h$

$$
\begin{aligned}
& =30+15 \sqrt{3}+15 \\
& =15(3+\sqrt{3}) \mathrm{m}
\end{aligned}
$$

119. (A) Curve $y=\sqrt{2} \sqrt{x}$
and line $x=2$


Area $=2 \int_{0}^{2} y \cdot d x$
Area $=2 \int_{0}^{2} \sqrt{2} \cdot \sqrt{x} d x$

Area $=2 \sqrt{2}\left[\frac{x^{3 / 2}}{\frac{3}{2}}\right]_{0}^{2}$

Area $=2 \times \frac{2 \sqrt{2}}{3}\left[2^{3 / 2}\right]=\frac{16}{3}$ sq. unit
120. (B) $\sum_{r=1}^{3} \mathrm{C}(20+r, 3)+\mathrm{C}(21,4)$

$$
\begin{aligned}
& \Rightarrow{ }^{21} \mathrm{C}_{3}+{ }^{22} \mathrm{C}_{3}+{ }^{23} \mathrm{C}_{3}+{ }^{21} \mathrm{C}_{4} \\
& \Rightarrow{ }^{21} \mathrm{C}_{3}+{ }^{21} \mathrm{C}_{4}+{ }^{22} \mathrm{C}_{3}+{ }^{23} \mathrm{C}_{3} \\
& \Rightarrow{ }^{22} \mathrm{C}_{4}+{ }^{22} \mathrm{C}_{3}+{ }^{23} \mathrm{C}_{3} \\
& \Rightarrow{ }^{23} \mathrm{C}_{4}+{ }^{23} \mathrm{C}_{3} \Rightarrow{ }^{24} \mathrm{C}_{4}
\end{aligned}
$$



PLOT NO. 2 SSI, OPP METRO PILLAR 150, GT KARNAL ROAD, JAHANGIRPURI, DELHI: 110033

## NDA (MATHS) MOCK TEST - 172 (Answer Key)

| 1. | (B) | 21. | (C) | 41. | (A) | 61. | (B) | 81. | (A) | $101 .(\mathrm{B})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2. | (B) | 22. | (A) | 42. | (A) | 62. | (B) | 82. | (C) | $102 .(\mathrm{B})$ |
| 3. | (C) | 23. | (C) | 43. | (B) | 63. | (C) | 83. | (A) | $103 .(\mathrm{B})$ |
| 4. | (D) | 24. | (B) | 44. | (B) | 64. | (D) | 84. | (B) | 104. (C) |
| 5. | (C) | 25. | (C) | 45. | (A) | 65. | (B) | 85. | (D) | $105 .(\mathrm{A})$ |
| 6. | (A) | 26. | (D) | 46. | (B) | 66. | (D) | 86. | (C) | $106 .(\mathrm{D})$ |
| 7. | (D) | 27. | (A) | 47. | (A) | 67. | (A) | 87. | (D) | 107. (C) |
| 8. | (A) | 28. | (D) | 48. | (D) | 68. | (C) | 88. | (B) | 108. (B) |
| 9. | (C) | 29. | (C) | 49. | (C) | 69. | (C) | 89. | (A) | 109. (B) |
| 10. | (C) | 30. | (C) | 50. | (B) | 70. | (C) | 90. | (C) | 110. (B) |
| 11. | (C) | 31. | (B) | 51. | (B) | 71. | (C) | 91. | (B) | 111. (A) |
| 12. | (B) | 32. | (C) | 52. | (D) | 72. | (B) | 92. | (C) | 112. (D) |
| 13. | (B) | 33. | (B) | 53. | (C) | 73. | (C) | 93. | (B) | 113. (A) |
| 14. | (D) | 34. | (C) | 54. | (A) | 74. | (B) | 94. | (C) | 114. (A) |
| 15. | (D) | 35. | (B) | 55. | (B) | 75. | (D) | 95. | (B) | 115. (C) |
| 16. | (D) | 36. | (B) | 56. | (D) | 76. | (B) | 96. | (B) | 116. (D) |
| 17. | (B) | 37. | (C) | 57. | (A) | 77. | (C) | 97. | (A) | 117. (C) |
| 18. | (C) | 38. | (B) | 58. | (D) | 78. | (A) | 98. | (B) | 118. (C) |
| 19. | (C) | 39. | (B) | 59. | (B) | 79. | (D) | 99. | (A) | 119. (A) |
| 20. | (B) | 40. | (B) | 60. | (B) | 80. | (C) | 100. | (B) | 120. (B) |



Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777

