

NDA MATHS MOCK TEST - 184 (SOLUTION)

1. (A) Let $y = f(x) = 2^{x(x-1)}$
 taking log both sides
 $\Rightarrow \log y = x(x-1)\log 2$
 $\Rightarrow x^2 - x = \log_2 y$
 $\Rightarrow x^2 - x - \log_2 y = 0$
 $\Rightarrow x = \frac{+1 \pm \sqrt{1+4\log_2 y}}{2 \times 1}$
 $\Rightarrow x = \frac{1 + \sqrt{1+4\log_2 y}}{2}$
 $\Rightarrow f^{-1}(y) = \frac{1 + \sqrt{1+4\log_2 y}}{2}$
 $\Rightarrow f^{-1}(x) = \frac{1 + \sqrt{1+4\log_2 x}}{2}$

2. (D) $f(x) = \log\left(\frac{1-x}{1+x}\right)$
 Now, $f\left(\frac{1}{2}\right) + f\left(\frac{1}{3}\right)$
 $\Rightarrow \log\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right) + \log\left(\frac{1-\frac{1}{3}}{1+\frac{1}{3}}\right)$
 $\Rightarrow \log\left(\frac{\frac{1}{2}}{\frac{3}{2}}\right) + \log\left(\frac{\frac{2}{3}}{\frac{4}{3}}\right) \Rightarrow \log\left(\frac{1}{3}\right) + \log\left(\frac{1}{2}\right)$
 $\Rightarrow \log\left(\frac{1}{6}\right) = -\log 6$

3. (C) $x^4 + \frac{1}{x^4} = 1154$
 $\Rightarrow x^4 + \frac{1}{x^4} + 2 = 1156$
 $\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2 \Rightarrow x^2 + \frac{1}{x^2} = 34$
 $\Rightarrow \left(x + \frac{1}{x}\right)^2 = 36 \Rightarrow x + \frac{1}{x} = 6$
 Now, $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$
 $\Rightarrow x^3 + \frac{1}{x^3} = (6)^3 - 3 \times 6$
 $\Rightarrow x^3 + \frac{1}{x^3} = 216 - 18 = 198$

4. (B) Equation $3x^2 + 5x + 7 = 0$

$$\alpha + \beta = -\frac{5}{3} \text{ and } \alpha\beta = \frac{7}{3}$$

$$\text{Now, } \frac{1}{\alpha^3} + \frac{1}{\beta^3} \Rightarrow \frac{\beta^3 + \alpha^3}{(\alpha\beta)^3}$$

$$\Rightarrow \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3}$$

$$\Rightarrow \frac{\left(\frac{-5}{3}\right)^3 - 3 \times \frac{7}{3} \times \left(\frac{-5}{3}\right)}{\left(\frac{7}{3}\right)^3}$$

$$\Rightarrow \frac{-\frac{125}{27} + \frac{35}{3}}{\frac{343}{27}} = \frac{190}{343}$$

5. (B) $\frac{{}^n C_1}{{}^n C_0} + \frac{{}^{2n} C_2}{{}^n C_1} + \frac{{}^{3n} C_3}{{}^n C_2} + \dots + \frac{{}^n C_n}{{}^n C_{n-1}}$
 $\Rightarrow \frac{n}{1} + \frac{2 \cdot \frac{n(n-1)}{2}}{n} + \frac{3 \cdot \frac{n(n-1)(n-2)}{6}}{\frac{n(n-1)}{2}} + \dots + \frac{n \cdot 1}{n}$
 $\Rightarrow n + (n-1) + (n-2) + \dots + 1$
 $\Rightarrow 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

6. (C) $\begin{vmatrix} 1 & a^2 - bc & a \\ 1 & b^2 - ca & b \\ 1 & c^2 - ab & c \end{vmatrix}$
 $\Rightarrow \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} - \begin{vmatrix} 1 & bc & a \\ -1 & ca & b \\ 1 & ab & c \end{vmatrix}$
 $\Rightarrow \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} - \frac{1}{abc} \begin{vmatrix} 1 & abc & a \\ 1 & abc & b \\ 1 & abc & c \end{vmatrix}$
 $\Rightarrow \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} - \frac{abc}{abc} \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$
 $\Rightarrow - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

7. (C) Let point P divided AB is the ratio of $\lambda : 1$

$$P\left(\frac{6\lambda - 3}{\lambda + 1}, \frac{-5\lambda + 2}{\lambda + 1}\right)$$

Point P lies on the line $y + 2x = 4$

$$\frac{-5\lambda + 2}{\lambda + 1} + 2 \times \frac{6\lambda - 3}{\lambda + 1} = 4$$

$$\Rightarrow -5\lambda + 2 + 12\lambda - 6 = 4\lambda + 4$$

$$\Rightarrow 7\lambda - 4\lambda - 4 = 4$$

$$\Rightarrow 3\lambda = 8 \Rightarrow \lambda = \frac{8}{3}$$

The required ratio = $8 : 3$

8. (B) The equation of circle passing through the intersection of two given circles

$$x^2 + y^2 + 2x + 4y + 5 + \lambda(x^2 + y^2 + 4x + 2y + 6) = 0$$

$$\Rightarrow (1 + \lambda)x^2 + (1 + \lambda)y^2 + 2x(1 + 2\lambda) + 2y(2 + \lambda) + 5 + 6\lambda = 0$$

$$\Rightarrow x^2 + y^2 + 2x\left(\frac{1 + 2\lambda}{1 + \lambda}\right) + 2y\left(\frac{2 + \lambda}{1 + \lambda}\right) + \frac{5 + 6\lambda}{1 + \lambda} = 0 \quad \dots(i)$$

$$\text{Centre of circle} = \left(-\frac{1 + 2\lambda}{1 + \lambda}, -\frac{2 + \lambda}{1 + \lambda}\right)$$

Centre lies on the line $2x + y = 4$

$$-2\left(\frac{1 + 2\lambda}{1 + \lambda}\right) - \frac{2 + \lambda}{1 + \lambda} = 4$$

$$\Rightarrow -2 - 4\lambda - 2 - \lambda = 4 + 4\lambda$$

$$\Rightarrow 9\lambda = -8 \Rightarrow \lambda = -\frac{8}{9}$$

On putting in eq(i)

$$\Rightarrow x^2 + y^2 + 2x\left(\frac{1 + 2\left(-\frac{8}{9}\right)}{1 - \frac{8}{9}}\right) + 2y\left(\frac{2 - \frac{8}{9}}{1 - \frac{8}{9}}\right)$$

$$+ \frac{5 - \frac{6 \times 8}{9}}{1 - \frac{8}{9}} = 0$$

$$\Rightarrow x^2 + y^2 - 14x + 20y - 3 = 0$$

9. (C) lines $12x + 5y = 15$
and $24x + 10y + 23 = 0$

$$\Rightarrow 12x + 15y + \frac{23}{2} = 0$$

$$\Rightarrow 12x + 5y = -\frac{23}{2}$$

$$\text{The required distance} = \frac{15 + \frac{23}{2}}{\sqrt{12^2 + 5^2}}$$

$$= \frac{53}{2 \times 13} = \frac{53}{26}$$

$$10. (D) \tan\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \frac{3}{4} \times \frac{2}{3}}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\frac{9+8}{12}}{\frac{12-6}{12}}\right)\right]$$

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{17}{6}\right)\right] = \frac{17}{6}$$

11. (B)

12. (D) Coordinate of incentre

$$P\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

vertices are A(0, 0), B(2, $\sqrt{5}$), C(-10, 0)

$$a = \sqrt{12^2 + (\sqrt{5})^2} = 13$$

$$b = \sqrt{(-10)^2 + 0^2} = 10$$

$$c = \sqrt{(2)^2 + (\sqrt{5})^2} = 3$$

$$\text{incentre } P = \left(\frac{13 \times 0 + 10 \times 2 + 3 \times (-10)}{13 + 10 + 3}, \frac{13 \times 0 + 10 \times \sqrt{5} + 3 \times 0}{13 + 10 + 3}\right)$$

$$= \left(\frac{-10}{26}, \frac{10\sqrt{5}}{26}\right) = \left(\frac{-5}{13}, \frac{5\sqrt{5}}{13}\right)$$

13. (B) We know that

$$\lim_{x \rightarrow \infty} [f(x)]^{g(x)} = e^{\lim_{x \rightarrow \infty} g(x)[f(x)-1]}$$

$$\text{Now, } \lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x + 5}{x^2 - 3x + 6}\right)^x$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} x \left[\frac{x^2 + 2x + 5 - x^2 + 3x - 6}{x^2 - 3x + 6}\right]} \Rightarrow e^{\lim_{x \rightarrow \infty} x \left[\frac{5x - 1}{x^2 - 3x + 6}\right]}$$

$$\Rightarrow e^{\lim_{x \rightarrow \infty} \left[\frac{5x^2 - x}{x^2 - 3x + 6}\right]} \Rightarrow e^{\lim_{x \rightarrow \infty} \left[\frac{5 - \frac{1}{x}}{1 - \frac{3}{x} + \frac{6}{x^2}}\right]}$$

$$\Rightarrow e^{\left(\frac{5-0}{1-0}\right)} = e^5$$

$$14. \quad (A) \quad 3f(x) - 2f\left(\frac{1}{x}\right) = 4x^2 \quad \dots(i)$$

$$x \rightarrow \frac{1}{x}$$

$$\Rightarrow 3f\left(\frac{1}{x}\right) - 2f(x) = \frac{4}{x^2} \quad \dots(ii)$$

On solving eq(i) and eq(ii)

$$\Rightarrow 5f(x) = 12x^2 + \frac{8}{x^2}$$

$$\Rightarrow 5f(-2) = 12 \times (-2)^2 + \frac{8}{(-2)^2}$$

$$\Rightarrow 5f(-2) = 48 + 2$$

$$\Rightarrow 5f(-2) = 50$$

$$\Rightarrow f(-2) = 10$$

$$15. \quad (C) \quad \text{Parabola } y^2 + 5x + 3y + 8 = 0$$

$$\Rightarrow y^2 + 3y + \frac{9}{4} - \frac{9}{4} + 5x + 8 = 0$$

$$\Rightarrow \left(y + \frac{3}{2}\right)^2 = -5x + \frac{9}{4} - 8$$

$$\Rightarrow \left(y + \frac{3}{2}\right)^2 = -5x - \frac{23}{4}$$

axis of parabola

$$y + \frac{3}{2} = 0$$

$$\Rightarrow 2y + 3 = 0$$

$$16. \quad (B) \quad I = \int_4^5 \frac{x^2}{x^2 - 9} dx$$

$$I = \int_4^5 \frac{x^2 - 9 + 9}{x^2 - 9} dx$$

$$I = \int_4^5 \left(1 + \frac{9}{x^2 - 9}\right) dx$$

$$I = \int_4^5 1 \cdot dx + 9 \int_4^5 \frac{1}{x^2 - 9} dx$$

$$I = [x]_4^5 + 9 \times \frac{1}{2 \times 3} \left[\log \frac{x-3}{x+3} \right]_4^5$$

$$I = 5 - 4 + \frac{3}{2} \left[\log \left(\frac{5-3}{5+3} \right) - \log \left(\frac{4-3}{4+3} \right) \right]$$

$$I = 1 + \frac{3}{2} \left[\log \left(\frac{2}{8} \right) - \log \left(\frac{1}{7} \right) \right]$$

$$I = 1 + \frac{3}{2} \left[\log \frac{1}{4} + \log 7 \right]$$

$$I = 1 + \frac{3}{2} \log \left(\frac{7}{4} \right)$$

$$17. \quad (A) \quad I = \int \frac{x + \sin x}{1 + \cos x} dx$$

$$I = \int \frac{x + \sin x}{2 \cos^2 \frac{x}{2}} dx$$

$$I = \frac{1}{2} \int x \cdot \sec^2 \frac{x}{2} dx + \frac{1}{2} \int \sin x \cdot \operatorname{cosec}^2 \frac{x}{2} dx$$

$$I = \frac{1}{2} \left[x \int \sec^2 \frac{x}{2} dx - \int \left(\frac{d}{dx}(x) \cdot \int \sec^2 \frac{x}{2} dx \right) dx \right]$$

$$+ \frac{1}{2} \int 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \cdot \sec^2 \frac{x}{2} dx + c$$

$$I = \frac{1}{2} \left[2x \cdot \tan \frac{x}{2} - \int 1 \cdot 2 \tan \frac{x}{2} dx \right]$$

$$+ \int \tan \frac{x}{2} dx + c$$

$$I = x \cdot \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c$$

$$I = x \cdot \tan \frac{x}{2} + c$$

$$18. \quad (B) \quad \text{Differential equation}$$

$$x \frac{dy}{dx} + y = x^3 y^6$$

$$\frac{1}{y^6} \frac{dy}{dx} + \frac{1}{xy^5} = x^2$$

$$\text{Let } \frac{1}{y^5} = v$$

$$\frac{-5}{y^6} \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{y^6} \frac{dy}{dx} = \frac{-1}{5} \frac{dv}{dx}$$

$$\frac{-1}{5} \frac{dv}{dx} + \frac{v}{x} = x^2$$

$$\frac{dv}{dx} - \frac{5}{x} v = -5x^2$$

It is a linear equation

$$\text{Here } P = \frac{-5}{x}, \quad Q = -5x^2$$

$$I.F = e^{\int \frac{-5}{x} dx}$$

$$I.F = e^{-5 \log x} = \frac{1}{x^5}$$

Solution of differential equation

$$v \times I.F = \int Q \times I.F dx$$

$$v \times \frac{1}{x^5} = \int (-5x^2) \times \frac{1}{x^5} dx$$

$$\frac{1}{x^5 y^5} = -5 \int \frac{1}{x^3} dx$$

$$x^5 \cdot y^5 = -5 \cdot \frac{x^{-3+1}}{-3+1} + c$$

$$x^5 \cdot y^5 = \frac{-5}{-2} \times \frac{1}{x^2} + c$$

$$x^5 \cdot y^5 = \frac{5}{2} x^2 + c$$

19. (C)

2	37	1
2	18	0
2	9	1
2	4	0
2	2	0
2	1	1
	0	

↑

Hence $(37)_{10} = (100101)_2$

20. (B) $I = \int \frac{dx}{x(x^4 + 1)}$

$$I = \int \frac{x^3 dx}{x^4(x^4 + 1)}$$

Let $x^4 = t$

$$4x^3 dx = dt$$

$$x^3 dx = \frac{1}{4} dt$$

$$I = \frac{1}{4} \int \frac{dt}{t(t+1)}$$

$$I = \frac{1}{4} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$I = \frac{1}{4} [\log t - \log(t+1)] + c$$

$$I = \frac{1}{4} \left[\log \frac{t}{t+1} \right] + c$$

$$I = \frac{1}{4} \log \left(\frac{x^4}{x^4 + 1} \right) + c$$

21. (C) $I = \int_0^{\pi/2} \frac{dx}{1 + \cot x}$

$$I = \int_0^{\pi/2} \frac{dx}{1 + \frac{\cos x}{\sin x}}$$

$$I = \int_0^{\pi/2} \frac{\sin x dx}{\sin x + \cos x} \quad \dots(i)$$

Using property IV

$$I = \int_0^{\pi/2} \frac{\sin \left(\frac{\pi}{2} - x \right)}{\sin \left(\frac{\pi}{2} - x \right) + \cos \left(\frac{\pi}{2} - x \right)} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx \quad \dots(ii)$$

On adding eq(i) and eq(ii)

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1 \cdot dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0 \Rightarrow I = \frac{\pi}{4}$$

22. (D) Let $y = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$ and $z = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$

On Putting $x = \tan \theta$

$$y = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right), z = \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

$$y = \sin^{-1}(\sin 2\theta), z = \tan^{-1}(\tan 2\theta)$$

$$y = 2\theta, z = 2\theta$$

$$\frac{dy}{d\theta} = 2, \frac{dz}{d\theta} = 2$$

Now, $\frac{dy}{dz} = \frac{dy}{d\theta} \times \frac{d\theta}{dz}$

$$\Rightarrow \frac{dy}{dz} = 2 \times \frac{1}{2} = 1$$

23. (D) Let $f(x) = \frac{x-4}{|x-4|}$

$$\text{L.H.L.} = \lim_{x \rightarrow 4^-} f(x)$$

$$= \lim_{h \rightarrow 0} f(4-h)$$

$$= \lim_{h \rightarrow 0} \frac{4-h-4}{|4-h-4|}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{|0-h|}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{-1} = 0$$

$$\text{R.H.L.} = \lim_{x \rightarrow 4^+} f(x)$$

$$= \lim_{h \rightarrow 0} f(4+h)$$

$$= \lim_{h \rightarrow 0} \frac{4+h-4}{|4+h-4|}$$

$$= \lim_{h \rightarrow 0} \frac{h}{|0+h|} = \lim_{h \rightarrow 0} \frac{h}{0} = \infty$$

Hence Limits does not exist.

24. (C) Curve $y = 2x^2 + 5$

$$\frac{dy}{dx} = 2 \times 2x$$

$$\frac{dy}{dx} = 4x$$

$$\left(\frac{dy}{dx}\right)_{\text{at}(-1,2)} = 4 \times (-1) = -4$$

Equation of tangent at point $(-1, 2)$

$$\Rightarrow y - 2 = -4(x + 1)$$

$$\Rightarrow y - 2 = -4x - 4$$

$$\Rightarrow 4x + y = -2 = 4x + y + 2 = 0$$

25. (D)
$$\begin{vmatrix} y+z & x & x \\ y & x+z & y \\ z & z & x+y \end{vmatrix}$$

$$R_1 \rightarrow R_1 - (R_2 + R_3)$$

$$\Rightarrow \begin{vmatrix} 0 & -2z & -2y \\ y & x+z & y \\ z & z & x+y \end{vmatrix}$$

$$\Rightarrow -2 \begin{vmatrix} 0 & z & y \\ y & x+z & y \\ z & z & x+y \end{vmatrix}$$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow -2 \begin{vmatrix} 0 & z & y \\ y & x+z-y & 0 \\ z & 0 & x+y-z \end{vmatrix}$$

$$\Rightarrow -2[0 - z\{y(x+y-z) + y(0 - z(x+z-y))\}]$$

$$\Rightarrow -2[-zyx - y^2z + yz^2 - xyz - yz^2 + y^2z]$$

$$\Rightarrow -2(-2xyz) = 4xyz$$

26. (B) $z = \frac{-2(1+2i)}{3+i} = \frac{-2-4i}{3+i}$

$$z = \frac{-2-4i}{3+i} \times \frac{3-i}{3-i}$$

(Rationalizing Numerator and denominator)

$$z = \frac{-6+2i-12i+4i^2}{10}$$

$$z = \frac{-6-10i-4}{10} = \frac{-10-10i}{10}$$

$$\therefore z = -1 - i$$

$$\therefore \theta = \frac{\pi}{4}$$

27. (C) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Number of subset of A containing two element

$$= 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$$

28. (D)
$$\begin{array}{l} 111101 \\ \begin{array}{l} \rightarrow 1 \times 2^0 = 1 \\ \rightarrow 0 \times 2^1 = 0 \\ \rightarrow 1 \times 2^2 = 4 \\ \rightarrow 1 \times 2^3 = 8 \\ \rightarrow 1 \times 2^4 = 16 \end{array} \end{array}$$

$$\begin{array}{l} 0.01 \\ \begin{array}{l} 0 = 0 \times 2^{-1} \\ \frac{1}{4} = 1 \times 2^{-2} \end{array} \end{array}$$

$$\text{Hence } (11101.01)_2 = (29.25)_{10}$$

29. (D)
$$\sum_{r=0}^1 {}^{n+r}C_n = {}^nC_n + {}^{n+1}C_n$$

$$= 1 + \frac{(n+1)!}{(n+1-n)!n!}$$

$$= 1 + \frac{(n+1)(n!)}{n!}$$

$$= 1 + n + 1 = n + 2$$

From option (D)

$${}^{n+2}C_{n+1} = \frac{(n+2)!}{(n+2-n-1)!(n+1)!}$$

$${}^{n+2}C_{n+1} = \frac{(n+2)(n+1)!}{(n+1)!} = n + 2$$

30. (B) $(\sin x + i \cos x)^3$
 $\Rightarrow \sin^3 x + i^3 \cos^3 x + 3i \sin x \cos x (\sin x + i \cos x)$

$$\Rightarrow \sin^3 x - i \cos^3 x + 3i \sin^2 x \cos x - 3 \sin x \cos^2 x$$

$$\Rightarrow \sin^3 x - 3 \sin x \cos^2 x + i \cos x (\cos^2 x + \sin^2 x)$$

$$\Rightarrow \sin x (\sin^2 x - 3 \cos^2 x) + i \cos x$$

Real part of $(\sin x + i \cos x)^3$

$$\Rightarrow \sin x (\sin^2 x - 3 \cos^2 x)$$

$$\Rightarrow \sin x [\sin^2 x - 3(1 - \sin^2 x)]$$

$$\Rightarrow \sin x [4 \sin^2 x - 3]$$

$$\Rightarrow -(3 \sin x - 4 \sin^3 x) \Rightarrow -\sin 3x$$

31. (C) Given that $A = \{x : x^2 = 1\}$, $B = \{x : x^4 = 1\}$

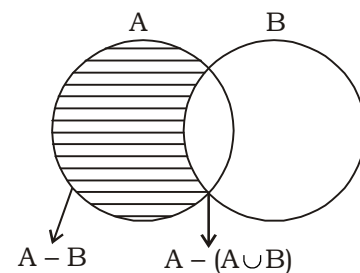
$$\Rightarrow A = \{-1, 1\}, B = \{-1, 1, -i, i\}$$

$$\text{Now, } (A \cup B) = \{1, -1, -i, i\}$$

$$\text{Hence } n(A \cup B) = 4$$

32. (D) 1. $A - B = A - (A \cap B)$ is correct

2. $A = (A \cap B) \cup (A - B)$ is correct.



\therefore Statements (1) and (2) are correct.

33. (C) $f(x) = 2x + 7$ and $g(x) = x^2 + 7$
 $\therefore fog(x) = f\{g(x)\} = 2(x^2 + 7) + 7$
 $= 2x^2 + 14 + 7 = 2x^2 + 21$
 But $fog(x) = 25$
 $\Rightarrow 2x^2 + 21 = 25$
 $\Rightarrow x^2 = 2$
 $\Rightarrow x = \pm\sqrt{2}$

34. (C) $f(x) = \frac{2}{3}x + \frac{3}{2} = y$ (say)
 $\Rightarrow 4x + 9 = 6y \Rightarrow x = \frac{6y - 9}{4}$
 $\Rightarrow f^{-1}(y) = \frac{6y - 9}{4}$
 $\Rightarrow f^{-1}(x) = \frac{6x - 9}{4} = \frac{3x}{2} - \frac{9}{4}$

35. (D) We know that $\omega = \frac{-1 + i\sqrt{3}}{2}$
 Now, $(-1 + i\sqrt{3})^{48} = (2\omega)^{48}$
 $\Rightarrow (-1 + i\sqrt{3})^{48} = 2^{48}(\omega^3)^{16}$
 $\Rightarrow (-1 + i\sqrt{3})^{48} = 2^{48} \times 1 = 2^{48} (\because \omega^3 = 1)$

36. (B) $1 + i^2 + i^4 + i^6 + \dots + i^{100}$
 $= 1 - 1 + 1 - 1 + \dots + 1$
 $= 1$

37. (B) Let $a - ib = \sqrt{\frac{1}{2} - \frac{i\sqrt{3}}{2}}$
 On squaring
 $(a^2 - b^2) - 2abi = \frac{1}{2} - \frac{\sqrt{3}i}{2}$
 On comparing
 $a^2 - b^2 = \frac{1}{2}$ and $2ab = \frac{\sqrt{3}}{2}$... (i)

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow (a^2 + b^2)^2 = \frac{1}{4} + \frac{3}{4}$

$\Rightarrow (a^2 + b^2)^2 = 1$

$\Rightarrow a^2 + b^2 = 1$... (ii)

from eq(i) and eq(ii)

$2a^2 = \frac{3}{2} \Rightarrow a = \pm\frac{\sqrt{3}}{2}$

$2b^2 = \frac{1}{2} \Rightarrow b = \pm\frac{1}{2}$

Hence square root of $\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)$

$= \pm\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$

38. (B) Let $x = 2 + \frac{1}{2 + \frac{1}{2 + \dots \infty}}$
 $\Rightarrow x = 2 + \frac{1}{x} \Rightarrow x^2 - 2x - 1 = 0$
 $\Rightarrow x = \frac{2 \pm \sqrt{4 + 4}}{2} \Rightarrow x = 1 \pm \sqrt{2}$

But the value of given expression can not be negative or less than 2, therefore $(1 + \sqrt{2})$ is required answer.

39. (A) The given equations are
 $\Rightarrow qx^2 + px + q = 0$
 and $x^2 - 4qx + p^2 = 0$
 Since, root of the equation (i) are complex, therefore $p^2 - 4q^2 < 0$.
 Now, discriminant of equation (ii) is $16q^2 - 4p^2 = 4(p^2 - 4q^2) > 0$

Hence, roots are real and unequal.

40. (B) Given equation is $x^2 - 3|x| + 2 = 0$

If $x > 0$ then $|x| = x$
 $\Rightarrow x^2 - 3x + 2 = 0$
 $\Rightarrow x^2 - 2x - x + 2 = 0$
 $\Rightarrow x(x - 2) - 1(x - 2) = 0$
 $\Rightarrow (x - 1)(x - 2) = 0$
 $\Rightarrow x = 1, 2$

If $x < 0$, then $|x| = -x$
 $\Rightarrow x^2 + 3x + 2 = 0$
 $\Rightarrow x^2 + 2x + x + 2 = 0$
 $\Rightarrow x(x + 2) + 1(x + 2) = 0$
 $\Rightarrow (x + 1)(x + 2) = 0$
 $\Rightarrow x = -1, -2$

Hence four solutions are possible.

41. (A) If α and β are the roots of $ax^2 + bx + b = 0$

Then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{b}{a}$

$\therefore \frac{\sqrt{\alpha}}{\sqrt{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \frac{\sqrt{b}}{\sqrt{a}} \Rightarrow \frac{\alpha + \beta}{\sqrt{\alpha\beta}} + \frac{\sqrt{b}}{\sqrt{a}}$

$\Rightarrow \frac{-b}{\frac{b}{a}} + \frac{\sqrt{b}}{\sqrt{a}} \Rightarrow -\sqrt{\frac{b}{a}} + \sqrt{\frac{b}{a}} = 0$

42. (B) $1 - \frac{1}{(1 + \omega)} - \frac{1}{(1 + \omega^2)}$

$\Rightarrow \frac{(1 + \omega)(1 + \omega^2) - (1 + \omega^2) - (1 + \omega)}{(1 + \omega)(1 + \omega^2)}$

$\Rightarrow \frac{1 + \omega + \omega^2 + \omega^2 - 1 - \omega^2 - 1 - \omega}{(1 + \omega)(1 + \omega^2)}$

$\Rightarrow \frac{\omega^3 - 1}{(1 + \omega)(1 + \omega^2)} = 0$ ($\because \omega^3 = 1$)

43. (A) Equation $x^2 - bx + c = 0$
 Let roots are $\alpha, \alpha + 1$
 A.T.Q
 $\alpha + \alpha + 1 = b \Rightarrow 2\alpha + 1 = b$
 $\alpha(\alpha + 1) = c \Rightarrow \alpha^2 + \alpha = c$
 Now, $b^2 - 4c$
 $\Rightarrow (2\alpha + 1)^2 - 4(\alpha^2 + \alpha)$
 $\Rightarrow 4\alpha^2 + 1 + 4\alpha - 4\alpha^2 - 4\alpha = 1$

44. (C) Since, $-3 < x + \frac{2}{x} < 3$
 $\Rightarrow -3 < \frac{(x^2 + 2)x}{x^2} < 3$
 $\Rightarrow -3x^2 < (x^2 + 2)x < 3x^2 \quad (x \neq 0)$
 $\Rightarrow x(x^2 + 3x + 2) > 0$
 $\Rightarrow x(x + 1)(x + 2) > 0$
 $\Rightarrow x \in (-2, -1) \cup (0, \infty) \quad \dots(i)$
 and $x(x^2 - 3x + 2) < 0 \quad (x \neq 0)$
 $\Rightarrow x(x + 1)(x + 2) < 0$
 $\Rightarrow x \in (-\infty, 0) \cup (1, 2) \quad \dots(ii)$
 From (i) and (ii) $x \in (-2, 1) \cup (1, 2)$

45. (C) Given $T_m = n, T_n = m$ in HP, therefore the corresponding AP of m th term is $\frac{1}{n}$, n th term is $\frac{1}{m}$
 Let a and d be the first term and common difference of an AP, then

$a + (m - 1)d = \frac{1}{n} \quad \dots(i)$

$a + (n - 1)d = \frac{1}{m} \quad \dots(ii)$

On solving eqs. (i) and (ii) we get

$a = \frac{1}{mn}, d = \frac{1}{mn}$

Now r th term of AP = $a + (r - 1)d$

$\frac{1}{mn} + (r - 1)\frac{1}{mn} = \frac{1 + r - 1}{mn} = \frac{r}{mn}$

$\therefore r$ th term of HP is $\frac{mn}{r}$.

46. (A) The given series is $1.3^2 + 2.5^2 + 3.7^2 + \dots$
 Here $T_n = n(2n + 1)^2 = 4n^3 + 4n^2 + n$
 Now, $S = \sum_1^{20} T_n = 4 \sum_1^{20} n^3 + 4 \sum_1^{20} n^2 + \sum_1^{20} n$
 $\Rightarrow S = 4 \times \frac{n^2(n + 1)^2}{4} + 4 \times \frac{n(n + 1)(2n + 1)}{6}$
 $+ \frac{n(n + 1)}{2}$

$= 4 \cdot \frac{1}{4} 20^2 \cdot 21^2 + 4 \cdot \frac{1}{6} 20 \cdot 21 \cdot 41 + \frac{1}{2} 20 \cdot 21$
 $= 176400 + 11480 + 210 = 188090$

47. (B) We know that
 $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \infty$

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots \infty$

On adding eqs. (i) and (ii), we get

$e + e^{-1} = 2 + \frac{2}{2!} + \frac{2}{4!} + \dots \infty$

$\frac{e^2 + 1}{e} - 2 = \frac{2}{2!} + \frac{2}{4!} + \dots \infty$

$\frac{e^2 + 1 - 2e}{e} = 2 \left[\frac{1}{2!} + \frac{1}{4!} + \dots \infty \right]$

$\frac{(e - 1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \dots \infty$

48. (D) Two digit numbers formed from the given digits = $9 \times 9 = 81$
 Three digit numbers formed from the given digits = $9 \times 9 \times 9 = 729$
 The required no. = $81 + 729 = 810$

49. (B) There are 4 prizes and three students since each prizes can be given to any persons
 \therefore Required no. of ways = $3 \times 3 \times 3 \times 3 = 3^4$.

50. (C) Let $A = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$

$\Rightarrow A' = \begin{bmatrix} 0 & 4 & -1 \\ -4 & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$

$\Rightarrow A' = - \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix} \Rightarrow A' = -A$

\therefore It is skew-symmetric matrix.

51. (B) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$

$A_{11} = 1, A_{21} = -2, A_{31} = 4$
 $A_{12} = 4, A_{22} = 1, A_{32} = -2$
 $A_{13} = -2, A_{23} = 4, A_{33} = 1$

$\text{Adj}(A) = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$

52. (D) Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |A| = 1$

$$\text{adj}(A) = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, element $A_{13} = 7$

53. (D) Since, adjoint of square matrix A is B and value determinant of A is α .

then $AB = |A|I = \alpha I$

54. (B) \therefore Order of A and B are 3×2 and 2×3 respectively.

$$\therefore |kAB| = k^3 |AB|$$

55. (C) We know that

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n$$

multiply by x^2

$$\Rightarrow x^2(1+x)^n = {}^n C_0 x^2 + {}^n C_1 x^3 + {}^n C_2 x^4 + \dots$$

$$\dots + {}^n C_n x^{n+2}$$

On differentiating both side w.r.t.'x'

$$\Rightarrow x^2.n(1+x)^{n-1} + (1+x)^n.2x = 2^n C_0 x +$$

$$3^n C_1 x^2 + \dots + (n+2) {}^n C_n x^{n+1}$$

On putting $x = 1$

$$\Rightarrow n.2^{n-1} + 2^n.2 = 2^n C_0 + 3^n C_1 + \dots$$

$$+ (n+2) {}^n C_n$$

$$\Rightarrow 2^{n-1}(n+4) = 2^n C_0 + 3^n C_1 + \dots + (n+2) {}^n C_n$$

56. (A) Let $y = 3^x$

On differentiating both side w.r.t.'x'

$$\frac{dy}{dx} = 3^x \log 3$$

and $z = x^3$

On differentiating both side w.r.t.'x'

$$\Rightarrow \frac{dz}{dx} = 3x^2$$

$$\text{Now, } \frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$$

$$\Rightarrow \frac{dy}{dz} = 3^x \log 3 \times \frac{1}{3x^2}$$

$$\Rightarrow \frac{dy}{dz} = \frac{3^{x-1} \log 3}{x^2}$$

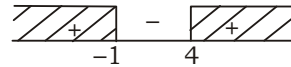
57. (C) $f(x) = \frac{1}{\sqrt{\log_3(x^2 - 3x - 3)}}$

Now, $\log_3(x^2 - 3x - 3) > 0$

$$\Rightarrow x^2 - 3x - 3 > 3^0$$

$$\Rightarrow x^2 - 3x - 4 > 0$$

$$\Rightarrow (x-4)(x+1) > 0$$

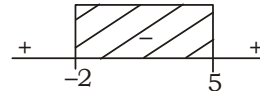


$$\text{Domain} = (-\infty, -1) \cup (4, \infty)$$

58. (B) $f(x) = \begin{cases} x^2 - 3x - 10, & -1 \leq x < 3 \\ -13 + x, & 3 \leq x \leq 5 \end{cases}$

Statement I

$$f(x) = x^2 - 3x - 10 = (x+2)(x-5)$$



Function $f(x)$ is decreasing in interval $(-2, 5)$.

Hence function $f(x)$ will be decreasing in interval $(-1, 3)$.

Statement I is incorrect.

Statement II

$$f(x) = -13 + x$$

$$f(x) = -13 + 3 = -10$$

$$f(x) = -13 + 5 = -8$$

$f(x)$ is increasing in interval $[3, 5]$.

Statement II is correct.

59. (A) $f(x) = \begin{cases} x^2 - 3x - 10, & -1 \leq x < 3 \\ -13 + x, & 3 \leq x \leq 5 \end{cases}$

Statement I

$$\text{L.H.L.} = \lim_{x \rightarrow 3^-} f(x) = \lim_{h \rightarrow 0} f(3-h)$$

$$= \lim_{h \rightarrow 0} (3-h)^2 - 3(3-h) - 10$$

$$= 9 - 9 - 10 = -10$$

$$\text{R.H.L.} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h)$$

$$= \lim_{h \rightarrow 0} -13 + (3+h) = -10$$

L.H.L. = R.H.L.

Hence $f(x)$ is continuous at $x = 3$.

Statement I is correct.

Statement II

$$\text{L.H.D.} = \lim_{h \rightarrow 0} \frac{f(3-h) - f(3)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{(3-h)^2 - 3(3-h) - 10 + 10}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{9 + h^2 - 6h - 9 + 3h}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 - 3h}{-h} = \lim_{h \rightarrow 0} -h + 3 = 3$$

$$\text{R.H.D.} = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-13 + (3+h) + 10}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h} = 1$$

L.H.D. \neq R.H.D.

$f(x)$ is not differentiable at $x = 3$.

Statement II is incorrect.

60. (B) $I = \int_2^4 x f(x) dx$

$$I = \int_2^4 x [x] dx$$

$$I = \int_2^3 x [x] dx + \int_3^4 x [x] dx$$

$$I = \int_2^3 x \times 2 dx + \int_3^4 x \times 3 dx$$

$$I = 2 \left[\frac{x^2}{2} \right]_2^3 + 3 \left[\frac{x^2}{2} \right]_3^4$$

$$I = 2 \left[\frac{9}{2} - \frac{4}{2} \right] + 3 \left[\frac{16}{2} - \frac{9}{2} \right]$$

$$I = 5 + 3 \times \frac{7}{2} = \frac{31}{2}$$

61. (B) The required probability = $\frac{1}{52}$

62. (C) Digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$\begin{array}{|c|c|c|} \hline 4 & 9 & 8 \\ \hline \end{array} = 4 \times 9 \times 8 = 288$$

(1, 2, 3, 4)

63. (B) $5^{2-2\log_5 4 + 3\log_5 2}$

$$\Rightarrow 5^2 \times 5^{-2\log_5 4} \times 5^{3\log_5 2}$$

$$\Rightarrow 25 \times 5^{\log_5(4)^{-2}} \times 5^{\log_5(2)^3}$$

$$\Rightarrow 25 \times (4)^{-2} \times (2)^3$$

$$\Rightarrow 25 \times \frac{1}{16} \times 8 = \frac{25}{2}$$

64. (D) $4f(x-2) + f\left(\frac{1}{x-2}\right) = x^2$... (i)

On putting $x = 4$

$$4f(2) + f\left(\frac{1}{2}\right) = 16$$
 ... (ii)

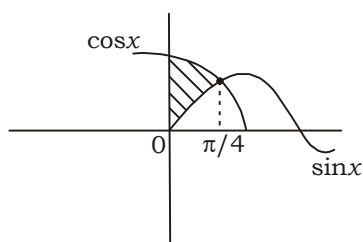
On putting $x = \frac{5}{2}$ in eq(i)

$$4f\left(\frac{1}{2}\right) + f(2) = \frac{25}{4}$$

On solving eq(i) and eq(ii)

$$f(2) = \frac{77}{20}$$

65. (C)



$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$\text{Area} = [\sin x + \cos x]_0^{\pi/4}$$

$$\text{Area} = \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right) - (\sin 0 + \cos 0)$$

$$\text{Area} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0 + 1)$$

$$\text{Area} = (\sqrt{2} - 1) \text{ square unit}$$

66. (C) Differential equation

$$\Rightarrow y - x \frac{dy}{dx} = b \left(y^2 + \frac{dy}{dx} \right)$$

$$\Rightarrow y - by^2 = b \frac{dy}{dx} + x \frac{dy}{dx}$$

$$\Rightarrow y(1 - by) = (x + b) \frac{dy}{dx}$$

$$\Rightarrow \frac{dx}{x+b} = \frac{dy}{y(1-by)}$$

$$\Rightarrow \frac{dx}{x+b} = \left[\frac{1}{y} + \frac{b}{1-by} \right] dy$$

On integrating

$$\Rightarrow \log(x+b) = \log y + \frac{b \log(1-by)}{-b} + \log c$$

$$\Rightarrow \log(x+b) = \log y - \log(1-by) + \log c$$

$$\Rightarrow \log(x+b)(1-by) = \log cy$$

$$\Rightarrow (x+b)(1-by) = cy$$

67. (B) $x = \sin t - t \cos t$

$$\Rightarrow \frac{dx}{dt} = \cos t + t \sin t - \cos t$$

$$\Rightarrow \frac{dx}{dt} = t \sin t$$

$$\text{and } y = \cos t + t \sin t$$

$$\Rightarrow \frac{dy}{dt} = -\sin t + t \cos t + \sin t$$

$$\Rightarrow \frac{dy}{dt} = t \cos t$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx} = t \cos t \times \frac{1}{t \sin t}$$

$$\Rightarrow \frac{dy}{dx} = \cot t$$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{d^2 y}{dx^2} = -\operatorname{cosec}^2 t \times \frac{dt}{dx}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-\operatorname{cosec}^2 t}{t \times \sin t}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-\operatorname{cosec}^3 t}{t}$$

68. (B) Let $f(x) = \frac{e^{1/x} + 1}{e^{1/x} - 1}$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h)$$

$$= \lim_{h \rightarrow 0} \frac{e^{1/0-h} + 1}{e^{1/0-h} - 1}$$

$$= \lim_{h \rightarrow 0} \frac{e^{-\frac{1}{h}} + 1}{e^{-\frac{1}{h}} - 1}$$

$$= \frac{e^{-\infty} + 1}{e^{-\infty} - 1} = \frac{0 + 1}{0 - 1} = -1$$

69. (B) $I = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$

Let $x = \tan\theta$ $x \rightarrow 0, \theta \rightarrow 0$

$dx = \sec^2\theta d\theta$ $x \rightarrow \infty, \theta = \frac{\pi}{2}$

$$I = \int_0^{\pi/2} \frac{\tan\theta \cdot \sec^2\theta \cdot d\theta}{(1 + \tan\theta)(1 + \tan^2\theta)}$$

$$I = \int_0^{\pi/2} \frac{\tan\theta}{1 + \tan\theta} d\theta$$

$$I = \int_0^{\pi/2} \frac{\frac{\sin\theta}{\cos\theta}}{1 + \frac{\sin\theta}{\cos\theta}} d\theta$$

$$I = \int_0^{\pi/2} \frac{\sin\theta}{\cos\theta + \sin\theta} d\theta \quad \dots(i)$$

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos\left(\frac{\pi}{2} - \theta\right) + \sin\left(\frac{\pi}{2} - \theta\right)} d\theta$$

[Prop.IV]

$$I = \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta + \cos\theta} d\theta \quad \dots(ii)$$

from eq(i) and eq(ii)

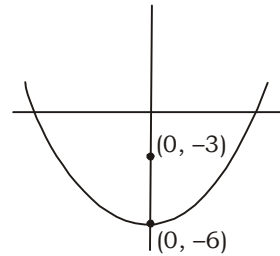
$$2I = \int_0^{\pi/2} \frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta} d\theta$$

$$2I = \int_0^{\pi/2} 1 \cdot d\theta$$

$$2I = [0]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

70. (D)



focus is $(0, -3)$ and vertex is $(0, -6)$

$$a = -3 - (-6) = 3$$

The equation of parabola

$$x^2 = -12(y + 6)$$

$$x^2 + 12y + 72 = 0$$

71. (C) Given that $x + 3y = 12$

$$\text{Let } A = xy$$

$$\Rightarrow A = (2 - 3y)y$$

$$\Rightarrow A = 12y - 3y^2$$

$$\Rightarrow \frac{dA}{dy} = 12 - 6y$$

for maxima and minima

$$\Rightarrow \frac{dA}{dy} = 0$$

$$\Rightarrow 12 - 6y = 0 \Rightarrow y = 2$$

$$\text{and } x = 6$$

$$\text{Hence maximum value} = 6 \times 2 = 12$$

72. (C) $x = a \cos\theta, y = a(\theta - \sin\theta)$

$$\frac{dx}{d\theta} = -a \sin\theta, \frac{dy}{d\theta} = a(1 - \cos\theta)$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{dx} = \frac{a(1 - \cos\theta)}{-a \sin\theta}$$

$$\frac{dy}{dx} = \frac{a \times 2 \sin^2 \frac{\theta}{2}}{-2a \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}}$$

$$\frac{dy}{dx} = -\tan \frac{\theta}{2}$$

$$\left(\frac{dy}{dx}\right)_{\text{at } \theta = \frac{\pi}{2}} = -\tan \frac{\theta}{4} = -1$$

$$\text{Point } [(a \cos\theta, a(\theta - \sin\theta))]_{\text{at } \theta = \pi/2} = \left[0, \left(\frac{\pi}{2} - 1\right)\right]$$

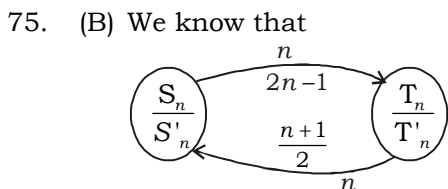
$$\text{equation of tangent at point } \left[0, a\left(\frac{\pi}{2} - 1\right)\right]$$

$$y - a\left(\frac{\pi}{2} - 1\right) = -1(x - 0)$$

$$\Rightarrow y + x = a\left(\frac{\pi}{2} - 1\right)$$

73. (D) H.M. < G.M. < A.M.

74. (A) $n(S) = 7$
 Total days in a leap year = 366 days
 = 52 weeks and 2 days
 $n(E) = 2$
 The required Probability $P(E) = \frac{n(E)}{n(S)} = \frac{2}{7}$



Now, given that $\frac{S_n}{S'_n} = \frac{4n-1}{5n+3}$

then n replace by $2n-1$

$$\frac{T_n}{T'_n} = \frac{4(2n-1)-1}{5(2n-1)+3}$$

$$\frac{T_n}{T'_n} = \frac{8n-5}{10n-2}$$

$$\frac{T_{17}}{T'_{17}} = \frac{8 \times 17 - 5}{10 \times 17 - 2} = \frac{131}{168}$$

76. (C) $(A+B)' = A' + B'$
77. (A) Given that $P(A) = \frac{1}{4}$, $P(A \cap B) = \frac{2}{3}$ and

$$P(A \cup B) = \frac{1}{3}$$

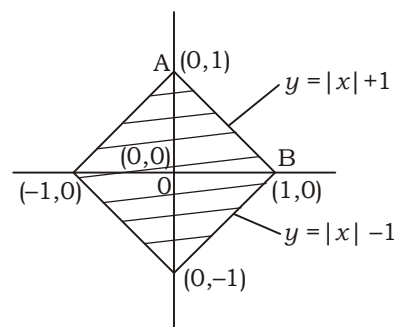
We know that
 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{1}{3} = \frac{1}{4} + P(B) - \frac{2}{3}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{4} + \frac{2}{3} = P(B) \Rightarrow P(B) = \frac{3}{4}$$

Now, $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/3}{3/4} = \frac{8}{9}$

78. (B)



Curve $y = |x| + 1$ and $y = |x| - 1$
 Total Area = $4 \times$ Area of ΔAOB

$$= 4 \times \frac{1}{2} \times 1 \times 1 = 2 \text{ sq. unit}$$

79. (C) Let $z = x + iy$
 Now, $\left| \frac{z-3}{z+3} \right| = 2$

$$\Rightarrow \left| \frac{x+iy-3}{x+iy+3} \right| = 2$$

$$\Rightarrow \frac{\sqrt{(x-3)^2 + y^2}}{\sqrt{(x+3)^2 + y^2}} = 2$$

$$\Rightarrow (x-3)^2 + y^2 = 4[(x+3)^2 + y^2]$$

On solving
 $x^2 + y^2 + 10x + 9 = 0$

Hence it is circle.

80. (B) The required number of hand shakes in

$$\text{party} = {}^{17}C_2 = \frac{17 \times 16}{2} = 136$$

81. (C) $n(S) = 6 \times 6 \times 6 = 216$

$$E = \left\{ \begin{array}{l} (6, 6, 2), (6, 2, 6), (6, 5, 3), (6, 3, 5), (6, 4, 4), (5, 6, 3), \\ (5, 3, 6), (5, 5, 4), (5, 4, 5), (4, 6, 4), (4, 4, 6), (4, 5, 5) \\ (3, 6, 5), (3, 5, 6), (2, 6, 6) \end{array} \right\}$$

$$n(E) = 15$$

The required Probability $P(E) = \frac{n(E)}{n(S)}$

$$= \frac{15}{216} = \frac{5}{72}$$

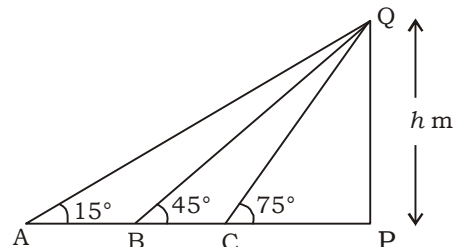
82. (C) Given that $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 4\hat{i} + 7\hat{j} - 4\hat{k}$

Projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$$= \frac{2 \times 4 + 3 \times 7 + (-1) \times (-4)}{\sqrt{4^2 + 7^2 + (-4)^2}}$$

$$= \frac{33}{9} = \frac{11}{3}$$

83. (C)



Let height of the tower $PQ = h$ m

In ΔCPQ :-

$$\tan 75^\circ = \frac{PQ}{CP}$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{CP} \Rightarrow CP = h(2 - \sqrt{3})$$

In ΔBPQ :-

$$\tan 45^\circ = \frac{PQ}{BP}$$

$$\Rightarrow 1 = \frac{h}{BP}$$

$$\Rightarrow BP = h$$

$$\Rightarrow BC + CP = h$$

$$\Rightarrow BC + h(2 - \sqrt{3}) - h \Rightarrow BC = h(\sqrt{3} - 1)$$

In ΔAPQ :-

$$\tan 15^\circ = \frac{PQ}{AP}$$

$$\Rightarrow \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{h}{AB + BP}$$

$$\Rightarrow AB + BP = h(2 + \sqrt{3})$$

$$\Rightarrow AB + h = h(2 + \sqrt{3}) \Rightarrow AB = h(\sqrt{3} + 1)$$

$$\text{Now, } AB : BC = h(\sqrt{3} + 1) : h(\sqrt{3} - 1)$$

$$= (\sqrt{3} + 1) : (\sqrt{3} - 1)$$

84. (D) $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

Let $x = \tan\theta$

$$\Rightarrow \tan^{-1}y = \tan^{-1}(\tan\theta) + \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow \tan^{-1}y = \theta + \tan^{-1}(\tan 2\theta)$$

$$\Rightarrow \tan^{-1}y = \theta + 2\theta$$

$$\Rightarrow \tan^{-1}y = 3\theta$$

$$\Rightarrow y = \tan 3\theta$$

$$\Rightarrow y = \tan 3\theta$$

$$\Rightarrow y = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{3x - x^3}{1 - 3x^2}$$

85. (D) Given that $A = \begin{bmatrix} 2a+b & 4 \\ a+b & -1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 2a+b & a+b \\ 4 & -1 \end{bmatrix}$$

Co-factors of A-

$$C = \begin{bmatrix} -1 & -(a+b) \\ -4 & 2a+b \end{bmatrix}$$

$$\text{Adj } A = C^T = \begin{bmatrix} -1 & -4 \\ -(a+b) & 2a+b \end{bmatrix}$$

Now, $\text{Adj } A = A^T$

$$\begin{bmatrix} -1 & -4 \\ -(a+b) & 2a+b \end{bmatrix} = \begin{bmatrix} 2a+b & a+b \\ 4 & -1 \end{bmatrix}$$

On comparing

$$2a + b = -1, \quad a + b = -4$$

$$-(a + b) = 4, \quad 2a + b = -1$$

On solving

$$a = 3, \quad b = -7$$

$$\text{Then } 4a - b = 4 \times 3 - (-7)$$

$$= 12 + 7 = 19$$

86. (B) Let $z = \frac{1 - 2i \sin \theta}{1 + 2i \sin \theta}$

$$\Rightarrow z = \frac{(1 - 2i \sin \theta)(1 - 2i \sin \theta)}{(1 + 2i \sin \theta)(1 - 2i \sin \theta)}$$

$$\Rightarrow z = \frac{1 + 4i^2 \sin^2 \theta - 4i \sin \theta}{1 - 4i^2 \sin^2 \theta}$$

$$\Rightarrow z = \frac{1 - 4 \sin^2 \theta - 4i \sin \theta}{1 + 4 \sin^2 \theta}$$

$$\Rightarrow z = \frac{1 - 4 \sin^2 \theta}{1 + 4 \sin^2 \theta} - \frac{4i \sin \theta}{1 + 4 \sin^2 \theta}$$

z will be purely imaginary, when

$$\frac{1 - 4 \sin^2 \theta}{1 + 4 \sin^2 \theta} = 0$$

$$\Rightarrow 1 - 4 \sin^2 \theta = 0$$

$$\Rightarrow \sin^2 \theta = \frac{1}{4}$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

87. (B) $\lim_{x \rightarrow \infty} \frac{x^3 - x - 1}{4x^2 - 2x^3 + 6}$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^3 - x - 1}{-2x^3 + 4x^2 + 6}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^3 \left(1 - \frac{1}{x^2} - \frac{1}{x^3}\right)}{x^3 \left(-2 + \frac{4}{x} + \frac{6}{x^3}\right)}$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x^2} - \frac{1}{x^3}}{-2 + \frac{4}{x} + \frac{6}{x^3}}$$

$$\Rightarrow \frac{1 - 0}{-2 + 0} = \frac{-1}{2}$$

88. (B) $I = \int \frac{\cos x}{\sin(x-a)} dx$
 Let $x - a = t \Rightarrow x = a + t$
 $dx = dt$
 $I = \int \frac{\cos(a+t)}{\sin t} dt$
 $I = \int \frac{\cos a \cdot \cos t - \sin a \cdot \sin t}{\sin t} dt$
 $I = \cos a \int \cot t dt - \sin a \int 1 dt$
 $I = \cos a \cdot \log \sin t - \sin a \cdot (t) + C$
 $I = \cos a \cdot \log \sin(x-a) - (x-a) \sin a + C$
 $I = \cos a \cdot \log \sin(x-a) - x \sin a + a \sin a + C$
 $I = \cos a \cdot \log \sin(x-a) - x \sin a + c$

89. (C) Let $y = e^{\cos x}$ and $z = \sin x$
 $\frac{dy}{dx} = e^{\cos x}(-\sin x), \quad \frac{dz}{dx} = \cos x$

Now, $\frac{dy}{dz} = \frac{dy}{dx} \times \frac{dx}{dz}$

$\Rightarrow \frac{dy}{dz} = e^{\cos x}(-\sin x) \times \frac{1}{\cos x}$

$\Rightarrow \frac{dy}{dz} = -\tan x \cdot e^{\cos x}$

90. (B) $I = \int_{\ln 2}^{\ln 3} \frac{\sin x}{\sin x + \sin(\ln 6 - x)} dx \quad \dots(i)$

Prop. IV $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

$I = \int_{\ln 2}^{\ln 3} \frac{\sin(\ln 2 + \ln 3 - x)}{\sin(\ln 2 + \ln 3 - x) + \sin(\ln 6 - \ln 2 - \ln 3 + x)} dx$

$I = \int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - x)}{\sin(\ln 6 - x) + \sin x} dx \quad \dots(ii)$

from eq(i) and eq(ii)

$2I = \int_{\ln 2}^{\ln 3} \frac{\sin x + \sin(\ln 6 - x)}{\sin x + \sin(\ln 6 - x)} dx$

$2I = \int_{\ln 2}^{\ln 3} 1 dx$

$2I = [x]_{\ln 2}^{\ln 3}$

$2I = \ln 3 - \ln 2$

$2I = \ln \frac{3}{2} \Rightarrow I = \frac{1}{2} \ln \frac{3}{2}$

91. (A) Vectors $\hat{i} + \hat{j} + \lambda \hat{k}$ and $(2\lambda - 3)\hat{i} + 3\hat{j} - 4\hat{k}$ are perpendicular,
 then $1 \times (2\lambda - 3) + 1 \times 3 + \lambda(-4) = 0$

$\Rightarrow 2\lambda - 3 + 3 - 4\lambda = 0$

$\Rightarrow -2\lambda = 0 \Rightarrow \lambda = 0$

92. (B) $y = \sin x$...(i)
 $x = \sin^{-1} y$

$\frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$

$\frac{d^2x}{dy^2} = \frac{-1}{2} (1-y^2)^{-3/2} (-2y)$

$\frac{d^2x}{dy^2} = \frac{y}{(1-y^2)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{(1-\sin^2 x)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{(\cos^2 x)^{3/2}}$

$\frac{d^2x}{dy^2} = \frac{\sin x}{\cos^3 x}$

$\frac{d^2x}{dy^2} = \tan x \cdot \sec^2 x$

93. (A) Given that $S_{13} = 533$

$\Rightarrow \frac{13}{2} [2a + 12d] = 533$

$\Rightarrow 13[a + 6d] = 533$

$\Rightarrow a + 6d = 41$

Hence $T_7 = 41$

94. (B) A line makes the angles α, β and γ with the axes, then

$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \frac{1 + \cos 2\alpha}{2} + \frac{1 + \cos 2\beta}{2} + \frac{1 + \cos 2\gamma}{2} = 1$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2 - 3$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

95. (B) $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8\pi}}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 8\pi)}}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 4\pi}}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 4\pi}}}}$

$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 2\pi}}}}$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 2\pi}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 \pi}}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 + 2 \cos \pi}}$$

$$\Rightarrow \sqrt{2 + \sqrt{2 \times 2 \cos^2 \frac{\pi}{2}}}$$

$$\Rightarrow \sqrt{2 + 2 \cos \frac{\pi}{2}}$$

$$\Rightarrow \sqrt{2 \times 2 \cos^2 \frac{\pi}{4}}$$

$$\Rightarrow 2 \cos \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$$

96. (C) An angles of a triangle are in 3 : 2 : 1

Let Angles = 3x, 2x, x

$$3x + 2x + x = 180$$

$$\Rightarrow 6x = 180 \Rightarrow x = 30$$

Angles = 90, 60, 30

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 90} = \frac{b}{\sin 60} = \frac{c}{\sin 30}$$

$$\Rightarrow \frac{a}{1} = \frac{b \times 2}{\sqrt{3}} = \frac{c \times 2}{1}$$

$$\frac{a}{2} = \frac{b}{\sqrt{3}} = \frac{c}{1}$$

Hence $a : b : c = 2 : \sqrt{3} : 1$

97. (B) The required no. of triangles = ${}^{14}C_3 - {}^8C_3$
 $= 364 - 56$
 $= 308$

98. (C) Given that $\tan \theta = \frac{a}{b}$

$$\text{Now, } \frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$$

$$\Rightarrow \frac{a \tan \theta - b}{a \tan \theta + b}$$

$$\Rightarrow \frac{a \times \frac{a}{b} - b}{a \times \frac{a}{b} + b} = \frac{a^2 - b^2}{a^2 + b^2}$$

99. (B) $\lim_{x \rightarrow 0} \frac{\sin 2x - \tan x}{x}$ $\left[\frac{0}{0} \right]$ Form

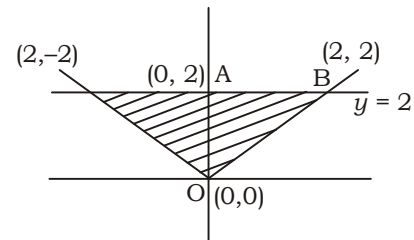
by L-Hospital Rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{2 \cos 2x - \sec^2 x}{1}$$

$$\Rightarrow 2 \cos 0 - \sec^2 0$$

$$\Rightarrow 2 - 1 = 1$$

100. (A)



Lines $y = |x|$ and $y = 2$

Area = 2 × Area of ΔOAB

$$= 2 \times \frac{1}{2} \times OA \times AB$$

$$= 2 \times \frac{1}{2} \times 2 \times 2 = 4 \text{ sq. unit}$$

101. (C) $x^x = e^{x \log x}$

taking log both side

$$\Rightarrow x \log x = x + y$$

On differentiating both side w.r.t. 'x'

$$\Rightarrow x \times \frac{1}{x} + \log x \cdot 1 = 1 + \frac{dy}{dx}$$

$$\Rightarrow 1 + \log x = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \log x$$

102. (A) Let $y = 5^{x \sin x}$

On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = 5^{x \sin x} \cdot \ln 5 [x \cdot \cos x + \sin x \cdot 1]$$

$$\Rightarrow \frac{dy}{dx} = 5^{x \sin x} \cdot \ln 5 [x \cdot \cos x + \sin x]$$

103. (B) $2^{x+3} + 3 \cdot 2^{y-2} = 16$ and $2^{x+1} + 2^{y-1} = 9$

$$2^x \cdot 8 + \frac{3}{4} \cdot 2^y = 16 \text{ and } 2 \cdot 2^x + \frac{2^y}{2} = 9$$

Let $2^x = X$ and $2^y = Y$

$$8X + \frac{3}{4}Y = 16 \quad \dots(i)$$

$$\text{and } 2X + \frac{Y}{2} = 9 \quad \dots(ii)$$

from eq(i) and eq(ii)

$$X = \frac{1}{2}, Y = 16$$

$$\Rightarrow 2^x = 2^{-1}, 2^y = 2^4$$

$$\Rightarrow x = -1, y = 4$$

104. (C) $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P\left(\frac{B}{A}\right) = \frac{3}{8}$

We know that $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow \frac{3}{8} = \frac{P(A \cap B)}{\frac{1}{3}} \Rightarrow P(A \cap B) = \frac{1}{8}$$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{1/8}{1/2} = \frac{1}{4}$$

105. (B) $\begin{vmatrix} x-2 & x-3 & x-a \\ x-4 & x-5 & x-b \\ x-6 & x-7 & x-c \end{vmatrix}$

$$C_2 \rightarrow C_2 - C_1 \text{ and } C_3 \rightarrow C_3 - C_1$$

$$\Rightarrow \begin{vmatrix} x-2 & -1 & -a+2 \\ x-4 & -1 & -b+4 \\ x-6 & -1 & -c+6 \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} x+2 & -1 & -a+2 \\ -2 & 0 & -b+a+2 \\ -4 & 0 & -c+a+4 \end{vmatrix}$$

$$\Rightarrow (x-2) \times 0 + 1(2c-2a-8-4b+4a+8) + (-a+2) \times 0$$

$$\Rightarrow 2c+2a-4b$$

$$\Rightarrow 2(c+a-2b)$$

$$a, b \text{ and } c \text{ are in A.P. i.e. } 2b+c=a$$

$$\Rightarrow 2(c+a-c-a) = 0$$

106. (B) word 'STATUS'

$$\text{Total arrangement} = \frac{6!}{2!2!} = 180$$

Arrangement when T's appear together

$$= \frac{5!}{2!} = 60$$

$$\text{The required arrangement} = 180 - 60 = 120$$

107. (C) Total students = 8

The table is round. One student is fixed.

$$\text{Hence the no. of ways} = (8-1)! = 7! = 5040$$

108. (B) Differential equation

$$\frac{dy}{dx} + \frac{y}{\sqrt{x^2-1}} = \frac{x}{x+\sqrt{x^2-1}}$$

On comparing with general linear equation

$$P = \frac{1}{\sqrt{x^2-1}}, \quad Q = \frac{x}{x+\sqrt{x^2-1}}$$

$$\text{I.F.} = e^{\int P dx}$$

$$\text{I.F.} = e^{\int \frac{1}{\sqrt{x^2-1}} dx}$$

$$= e^{\ln(x+\sqrt{x^2-1})} = x + \sqrt{x^2-1}$$

Solution of differential equation

$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx$$

$$\Rightarrow y(x+\sqrt{x^2-1}) = \int \frac{x}{x+\sqrt{x^2-1}} \times (x+\sqrt{x^2-1}) dx$$

$$\Rightarrow y(x+\sqrt{x^2-1}) = \int x dx$$

$$\Rightarrow y(x+\sqrt{x^2-1}) = \frac{x^2}{2} + \frac{c}{2}$$

$$\Rightarrow 2y(x+\sqrt{x^2-1}) = x^2 + c$$

109. (C) $I = \int \frac{1+\ln x}{\cos(x \ln x)} dx$

$$\text{Let } x \ln x = t$$

$$(1+\ln x) dx = dt$$

$$I = \int \frac{dt}{\cos t}$$

$$I = \int \sec t dt$$

$$I = \log |\sec t + \tan t| + c$$

$$I = \log |\sec(x \ln x) + \tan(x \ln x)| + c$$

110. (A) Required probability = ${}^6C_3 \left(\frac{1}{7}\right)^3 \left(\frac{6}{7}\right)^3$
 $= \frac{20 \times 6^3}{7^6}$

111. (C) $\left|z - \frac{1}{z}\right| = 6$

$$\Rightarrow z - \frac{1}{z} = \pm 6$$

$$\Rightarrow z^2 - 1 = \pm 6z$$

$$\Rightarrow z^2 - 6z - 1 = 0 \text{ or } z^2 + 6z - 1 = 0$$

$$\Rightarrow z = \frac{6 \pm \sqrt{36 - 4 \times 1(-1)}}{2 \times 1} \text{ or } z = \frac{-6 \pm \sqrt{36 - 4 \times 1(-1)}}{2 \times 1}$$

$$\Rightarrow z = \frac{6 \pm 2\sqrt{10}}{2} \text{ or } z = \frac{-6 \pm 2\sqrt{10}}{2}$$

$$\Rightarrow z = 3 \pm \sqrt{10} \text{ or } z = -3 \pm \sqrt{10}$$

$$\Rightarrow z = 3 - \sqrt{10}, 3 + \sqrt{10}$$

$$\text{or } z = -3 - \sqrt{10}, -3 + \sqrt{10}$$

$$\text{Hence smallest value of } |z| = -3 - \sqrt{10}$$

112. (C) $v = 3s^2 + 5s + 9$

On differentiating both side w.r.t.'s'

$$\Rightarrow \frac{dv}{ds} = 6s + 5$$

Hence $\left(\frac{dy}{ds}\right)_{\text{at } s=6} = 6 \times 6 + 5 = 41$

113. (B) $\cos 2A = \lambda \cos 2B$

$$\Rightarrow \frac{\cos 2A}{\cos 2B} = \frac{\lambda}{1}$$

by componendo and Dividendo Rule

$$\Rightarrow \frac{\cos 2A + \cos 2B}{\cos 2A - \cos 2B} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{2 \cos(A+B) \cdot \cos(A-B)}{2 \sin(A+B) \cdot \sin(B-A)} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\cos(A+B) \cdot \cos(A-B)}{-\sin(A+B) \cdot \sin(A-B)} = \frac{1 + \lambda}{1 - \lambda}$$

$$\Rightarrow \frac{\cot(A+B)}{\tan(A-B)} = \frac{1 + \lambda}{1 - \lambda}$$

114. (D) Differential equation

$$\sqrt{1-y^2} dx + \sqrt{1-x^2} dy = 0$$

$$\Rightarrow \sqrt{1-y^2} dx = -\sqrt{1-x^2} dy$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = -\frac{dy}{\sqrt{1-y^2}}$$

On intergrating

$$\Rightarrow \sin^{-1} x = -\sin^{-1} y + \sin^{-1} c$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \sin^{-1} c$$

$$\Rightarrow \sin^{-1} (x\sqrt{1-y^2} + y\sqrt{1-x^2}) = \sin^{-1} c$$

$$\Rightarrow x\sqrt{1-y^2} + y\sqrt{1-x^2} = c$$

115. (B) $(1 - \omega^2 + \omega)^4 + (1 - \omega + \omega^2)^4 + 32$

$$\Rightarrow (-\omega^2 - \omega^2)^4 + (-\omega - \omega)^4 + 32 \quad [\because 1 + \omega + \omega^2 = 0]$$

$$\Rightarrow (-2\omega^2)^4 + (-2\omega)^4 + 32$$

$$\Rightarrow 16\omega^8 + 16\omega^4 + 32$$

$$\Rightarrow 16\omega^2 + 16\omega + 16 + 16$$

$$\Rightarrow 16(\omega^2 + \omega + 1) + 16 \Rightarrow 16 \times 0 + 16 = 16$$

116. (A)

117. (C) Given that $x + 2y = 11$

Now, $A = xy$

$$\Rightarrow A = (11 - 2y)y$$

$$\Rightarrow A = 11y - 2y^2$$

On differentiating both side w.r.t.'y'

$$\Rightarrow \frac{dA}{dy} = 11 - 4y$$

Again, differentiating

$$\Rightarrow \frac{d^2A}{dy^2} = -4$$

For maxima and minima

$$\frac{dA}{dy} = 0$$

$$\Rightarrow 11 - 4y = 0 \Rightarrow y = \frac{11}{4} \text{ and } x = \frac{11}{2}$$

Hence maximum value of $xy = \frac{11}{2} \times \frac{11}{4} = \frac{121}{8}$

118. (B) $\begin{bmatrix} 2 & 4 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 \times 2 + 4 \times 0 & 2 \times (-3) + 4 \times 6 \\ -1 \times 2 + 2 \times 0 & -1 \times (-3) + 2 \times 6 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 18 \\ -2 & 15 \end{bmatrix} = \begin{bmatrix} 4 & k \\ -2 & 15 \end{bmatrix}$$

On comparing

$$k = 18$$

119. (B) Given that $b_{yx} = \frac{-16}{3}$ and $b_{xy} = \frac{-1}{2}$

Now, $r = \sqrt{b_{yx} \times b_{xy}}$

$$\Rightarrow r = \sqrt{\left(\frac{-16}{3}\right) \times \left(\frac{-1}{2}\right)}$$

$$\Rightarrow r = \sqrt{\frac{16}{36}}$$

$$\Rightarrow r = \sqrt{\frac{4}{9}}$$

$$\Rightarrow r = -\frac{2}{3}$$

120. (C) $x = \sin\theta + \cos\theta$ and $y = \sin\theta \cdot \cos\theta$

Now, $x^2 = (\sin\theta + \cos\theta)^2$

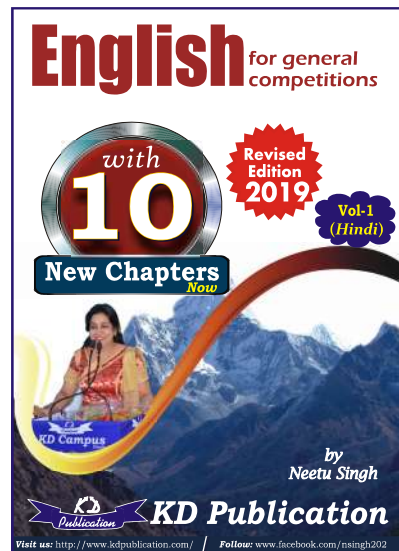
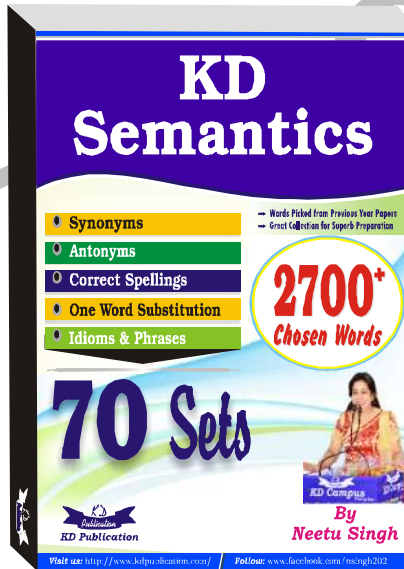
$$\Rightarrow x^2 = 1 + 2\sin\theta \cdot \cos\theta$$

$$\Rightarrow x^2 = 1 + 2\sin\theta \cdot \cos\theta$$

$$\Rightarrow x^2 = 1 + 2y \Rightarrow x^2 - 2y - 1 = 0$$

NDA (MATHS) MOCK TEST - 184 (Answer Key)

- | | | | |
|---------|---------|---------|----------|
| 1. (A) | 21. (C) | 41. (A) | 61. (B) |
| 2. (D) | 22. (D) | 42. (B) | 62. (C) |
| 3. (C) | 23. (D) | 43. (A) | 63. (B) |
| 4. (B) | 24. (C) | 44. (C) | 64. (D) |
| 5. (B) | 25. (D) | 45. (C) | 65. (C) |
| 6. (C) | 26. (B) | 46. (A) | 66. (C) |
| 7. (C) | 27. (C) | 47. (B) | 67. (B) |
| 8. (B) | 28. (D) | 48. (D) | 68. (B) |
| 9. (C) | 29. (D) | 49. (B) | 69. (B) |
| 10. (D) | 30. (B) | 50. (C) | 70. (D) |
| 11. (B) | 31. (C) | 51. (B) | 71. (C) |
| 12. (D) | 32. (D) | 52. (D) | 72. (C) |
| 13. (B) | 33. (C) | 53. (D) | 73. (D) |
| 14. (A) | 34. (C) | 54. (B) | 74. (A) |
| 15. (C) | 35. (D) | 55. (C) | 75. (B) |
| 16. (D) | 36. (B) | 56. (A) | 76. (C) |
| 17. (A) | 37. (B) | 57. (C) | 77. (A) |
| 18. (B) | 38. (B) | 58. (B) | 78. (B) |
| 19. (C) | 39. (A) | 59. (A) | 79. (C) |
| 20. (B) | 40. (B) | 60. (B) | 80. (B) |
| | | | 81. (C) |
| | | | 82. (C) |
| | | | 83. (C) |
| | | | 84. (D) |
| | | | 85. (D) |
| | | | 86. (B) |
| | | | 87. (B) |
| | | | 88. (B) |
| | | | 89. (C) |
| | | | 90. (B) |
| | | | 91. (A) |
| | | | 92. (B) |
| | | | 93. (A) |
| | | | 94. (B) |
| | | | 95. (B) |
| | | | 96. (C) |
| | | | 97. (B) |
| | | | 98. (C) |
| | | | 99. (B) |
| | | | 100. (A) |
| | | | 101. (C) |
| | | | 102. (A) |
| | | | 103. (B) |
| | | | 104. (C) |
| | | | 105. (B) |
| | | | 106. (B) |
| | | | 107. (C) |
| | | | 108. (B) |
| | | | 109. (C) |
| | | | 110. (A) |
| | | | 111. (C) |
| | | | 112. (C) |
| | | | 113. (B) |
| | | | 114. (D) |
| | | | 115. (B) |
| | | | 116. (A) |
| | | | 117. (C) |
| | | | 118. (B) |
| | | | 119. (B) |
| | | | 120. (C) |



Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777