NDA MATHS MOCK TEST - 192 (SOLUTION)

1. (A) $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = A.A$

$A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2\times2-2\times(-2) & 2\times(-2)-2\times2 \\ -2\times2+2\times(-2) & -2\times(-2)+2\times2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$A^3 = A^2.A$

$A^3 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 8\times2-8\times(-2) & 8\times(-2)-8\times2 \\ -8\times2+8\times(-2) & -8\times(-2)+8\times2 \end{bmatrix}$

$A^3 = \begin{bmatrix} 32 & -32 \\ -32 & 32 \end{bmatrix}$

Now, $A^3 - 4A^2$

$\Rightarrow \begin{bmatrix} 16 & -16 \\ -16 & 16 \end{bmatrix} - 4\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = null marix

2. (B) $P(n, r) = 840$

$\Rightarrow \dfrac{n!}{(n-r)!} = 840$ ...(i)

and $C(n, r) = 35 \Rightarrow \dfrac{n!}{r!(n-r)!} = 35$ ...(ii)

from eq(i) and eq(ii)

$r! = \dfrac{840}{35} \Rightarrow r! = 24$

$\Rightarrow r! = 4! \Rightarrow r = 4$

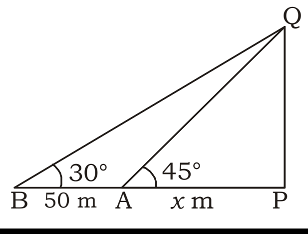
from eq(i)

$\dfrac{n!}{(n-4)!} = 7\times6\times5\times4 \Rightarrow n = 7$

Now, $C(n + 2, r + 2) \Rightarrow C(7 + 2, 4 + 2)$

$\Rightarrow {}^9C_6 = \dfrac{9!}{6!3!} = 84$

3. (B)



Let AP = x m

In ΔAPQ :-

$\tan45° = \dfrac{PQ}{AP} \Rightarrow 1 = \dfrac{PQ}{x} \Rightarrow PQ = x$

In ΔBPQ:-

$\tan30° = \dfrac{PQ}{BP} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{x}{50+x}$

$\Rightarrow 50 + x = \sqrt{3}\,x \Rightarrow (\sqrt{3} - 1)x = 50$

$\Rightarrow x = \dfrac{50}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow x = \dfrac{50(\sqrt{3}+1)}{2} = 25(\sqrt{3}+1)$ m

$\therefore$ breath of the river = $25(\sqrt{3}+1)$m

4. (C) No. of diagonals of a septagon

$= \dfrac{7\times(7-3)}{2} = 7\times2 = 14$

5. (C) $x = 1 - i$

On squaring

$\Rightarrow x^2 = 1 + i^2 - 2i$

$\Rightarrow x^2 = -2i$

Again, squaring

$x^4 = 4i^2$

$x^4 = -4$

Now, $x^4 + x^2 + 1$

$\Rightarrow -4 - 2i + 1 = -3 - 2i$

6. (A) $\begin{vmatrix} 2! & 3! & 4! \\ 3! & 4! & 5! \\ 4! & 5! & 6! \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2! & 3\times2! & 4\times3\times2! \\ 3! & 4\times3! & 5\times4\times3! \\ 4! & 5\times4! & 6\times5\times4! \end{vmatrix}$

$\Rightarrow 2!\times3!\times4!\begin{vmatrix} 1 & 3 & 12 \\ 1 & 4 & 20 \\ 1 & 5 & 30 \end{vmatrix}$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\Rightarrow 2! \times 3! \times 4!\begin{vmatrix} 1 & 3 & 12 \\ 0 & 1 & 8 \\ 0 & 2 & 18 \end{vmatrix}$

$\Rightarrow 2 \times 6 \times 24[1(18 - 16) - 3(0) + 12 \times 0]$

$\Rightarrow 288 \times 2 = 576$

7. (A) $\sin^2\left(53\frac{1}{2}\right)^\circ - \sin^2\left(26\frac{1}{2}\right)^\circ$

$\Rightarrow \cos^2\left(26\frac{1}{2}\right)^\circ - \sin^2\left(26\frac{1}{2}\right)^\circ$

$[\because \sin\theta = \cos(90 - \theta)]$

$\Rightarrow \cos\left(2\times\frac{53}{2}\right) = \cos 53^\circ$

8. (B) $(1 - \cos^2\theta)(1 + \cot^2\theta)$

$\Rightarrow \sin^2\theta.\text{cosec}^2\theta = 1$

9. (C) $\frac{\sin 5x - \sin x}{\cos 5x - 2\cos 3x + \cos x}$

$\Rightarrow \frac{\sin 5x - \sin x}{\cos 5x + \cos x - 2\cos 3x}$

$\Rightarrow \frac{2\cos 3x.\sin 2x}{2\cos 3x.\cos 2x - 2\cos 3x}$

$\Rightarrow \frac{2\cos 3x.\sin 2x}{2\cos 3x(\cos 2x - 1)}$

$\Rightarrow \frac{2\sin x.\cos x}{-2\sin^2 x} \Rightarrow -\cot x$

10. (D) **Statement I :-**

$1^\circ = 1\times\frac{\pi}{180} = \frac{22}{7\times 180} = 0.017$ radian

Statement I is incorrect.

Statement II :-

$1 \text{ radian} = 1\times\frac{180}{\pi} = \frac{180\times 7}{22} = 57.27^\circ$

Statement II is incorrect.

11. (A) sinA : sinB : sinC = 5 : 6 : 7

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$\Rightarrow \frac{a}{5} = \frac{b}{6} = \frac{c}{7} = k$

$\Rightarrow a = 5k,\ b = 6k,\ c = 7k$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{(6k)^2 + (7k)^2 - (5k)^2}{2\times 6k\times 7k}$

$\cos A = \frac{36k^2 + 49k^2 - 25k^2}{84k^2} = \frac{60}{84} = \frac{5}{7}$

$\cos A = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos A = \frac{(5k)^2 + (7k)^2 - (6k)^2}{2\times 5k\times 7k}$

$\Rightarrow \cos B = \frac{25k^2 + 49k^2 - 36k^2}{70k^2} = \frac{38}{70} = \frac{19}{35}$

and $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \cos C = \frac{(5k)^2 + (6k)^2 - (7k)^2}{2\times 5k\times 6k}$

$\Rightarrow \cos C = \frac{25k^2 + 36k^2 - 49k^2}{60k^2} = \frac{12}{60} = \frac{1}{5}$

Now, cosA : cosB : cosC

$\Rightarrow \frac{5}{7} : \frac{19}{35} : \frac{1}{5}$

$\Rightarrow \frac{25}{35} : \frac{19}{35} : \frac{7}{35}$

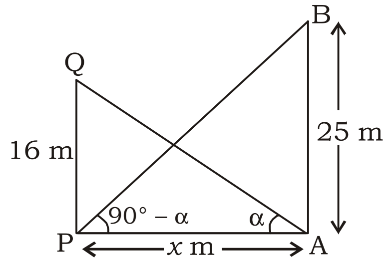
$\Rightarrow 25 : 19 : 7$

12. (C) $\cos\left[\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{3}\right]$

$\Rightarrow \cos\left[\sin^{-1}\left(\sin\left(\frac{-\pi}{3}\right) + \frac{\pi}{3}\right)\right]$

$\Rightarrow \cos\left[\frac{-\pi}{3} + \frac{\pi}{3}\right] = \cos 0 = 1$

13. (B)



Let $\angle PAQ = \alpha$

then $\angle APB = 90 - \alpha$

In ΔAPQ:-

$\tan\alpha = \frac{16}{x}$ ...(i)

In ΔPAB :-

$\tan(90 - \alpha) = \frac{25}{x} \Rightarrow \cot\alpha = \frac{25}{x}$ ...(ii)

from eq(i) and eq(ii)

$\tan\alpha.\cot\alpha = \frac{16}{x}\times\frac{25}{x}$

$\Rightarrow 1 = \frac{16\times 25}{x^2} \Rightarrow x^2 = 16\times 25$

$\Rightarrow x = 4\times 5 = 20$ m

14. (B) $b \cos B = c \cos C$
by Sine Rule
$\Rightarrow k \sin B.\cos B = k \sin C.\cos C$
$\Rightarrow 2\sin A.\cos B = 2\sin C.\cos C$
$\Rightarrow \sin 2A - \sin 2B = 0$
$\Rightarrow 2\cos(A + B).\ \sin(A - B) = 0$
$\cos(A + B) = 0 \Rightarrow A + B = 90$
or $\sin(A - B) = 0 \Rightarrow A - B = 0 \Rightarrow A = B$
Hence ΔABC is either right angled or isosceles.

15. (A) $\frac{1}{5}, \frac{1}{x}, \frac{1}{13}$ are in H.P.
5, x, 13 are in A.P.,
then $x = \frac{5+13}{2} \Rightarrow x = 9$

16. (C) Let the equation of ellipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)
The co-ordinates of the foci are $(\pm 6, 0)$
$ae = 6$ and $e = \frac{3}{5}$
$\Rightarrow a \times \frac{3}{5} = 6 \Rightarrow a = 10$
Now, $b^2 = a^2(1 - e^2)$
$\Rightarrow b^2 = 100\left(1 - \frac{9}{25}\right)$
$\Rightarrow b^2 = 100 \times \frac{16}{25} \Rightarrow b^2 = 64$
The required equation
$\frac{x^2}{100} + \frac{y^2}{64} = 1$

17. (C) Equation $x^2 + 4x + 3 = 0$
$\Rightarrow (x + 3)(x + 1) = 0$
$\Rightarrow x = -3, -1$
here $\alpha = -3$, $\beta = -1$ $[\because \alpha < \beta]$
Now, $\begin{bmatrix} 1 & \beta \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} \beta & \alpha \\ \alpha & \beta \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 1 & -1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ -3 & -1 \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 1\times(-1) - 1\times(-3) & 1\times(-3) - 1\times(-1) \\ -3\times(-1) - 1\times(-1) & -3\times(-3) - 1\times(-1) \end{bmatrix}$
$\Rightarrow \begin{bmatrix} 2 & -2 \\ 6 & 10 \end{bmatrix}$

18. (B) We know that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ...(i)
Statement I :
$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$
$\Rightarrow 3 - \sin^2\alpha - \sin^2\beta - \sin^2\gamma = 1$
$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
Statement I is incorrect.
Statement II :
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$
Statement I is correct.
Statement III :
form eq(i)
$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\Rightarrow 2\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma = 2$
$\Rightarrow 1 + \cos^2\alpha + 1 + \cos^2\beta + 1 + \cos^2\gamma = 2$
$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = -1$
Statement III is correct.

19. (C) $y = \tan^{-1}\left[\frac{x^{1/2}(1 + x^{1/2})}{1 - x^{3/2}}\right]$
$y = \tan^{-1}\left[\frac{x^{1/2} + x}{1 - x^{1/2}.x}\right]$
$y = \tan^{-1}(x^{1/2}) + \tan^{-1}x$
On differentiating both sides w.r.t 'x'
$\frac{dy}{dx} = \frac{1}{1 + (x^{1/2})^2} \times \frac{1}{2x^{1/2}} + \frac{1}{1 + x^2}$
$\frac{dy}{dx} = \frac{1}{2x^{1/2}(1 + x)} + \frac{1}{1 + x^2}$

20. (C) $\int \sin^2 x.e^{\ln(\cos x)}\, dx \Rightarrow \int \sin^2 x.\cos x\, dx$
let $\sin x = t \Rightarrow \cos x.dx = dt$
$\Rightarrow \int t^2.dt \Rightarrow \frac{t^3}{3} + c \Rightarrow \frac{\sin^3 x}{3} + c$

21. (B) Curve $x = t^2 + 2t - 4$ and $y = t^2 + 5t + 8$
at point (–1, 2), we have
$-1 = t^2 + 2t - 4 \Rightarrow t^2 + 2t - 3 = 0 \Rightarrow (t + 3)(t - 1) = 0$
$\Rightarrow t = -3, 1$
and $2 = t^2 + 5t + 8 \Rightarrow t^2 + 5t + 6 = 0$
$\Rightarrow (t + 3)(t + 2) = 0$
$\Rightarrow t = -3, -2$
So $t = -3$ for point (–1, 2)
Now, $\frac{dx}{dt} = 2t + 2 = 2\times(-3) + 2 = -4$
and $\frac{dy}{dt} = 2t + 5 = 2\times(-3) + 5 = -1$
Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-1}{-4} = \frac{1}{4}$

22. (A) $\lim_{x\to 0}\frac{\sin^2 x}{2x^2}$ $\left[\frac{0}{0}\right]$

by L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{2\sin x.\cos x}{2\times 2x}$

$\Rightarrow \lim_{x\to 0}\frac{\sin 2x}{4x}$ $\left[\frac{0}{0}\right]$

Again, L-Hospital's Rule

$\Rightarrow \lim_{x\to 0}\frac{2\cos 2x}{4}$

$\Rightarrow \frac{2\times 1}{4} = \frac{1}{2}$

23. (B) In the expansion of $\left(x^3 - \frac{1}{2x^2}\right)^7$

$T_{r+1} = {}^7C_r(x^3)^{7-r}\left(\frac{-1}{2x^2}\right)^r$

$= {}^7C_r\left(\frac{-1}{2}\right)^r x^{21-5r}$

Now, $21 - 5r = 1 \Rightarrow 5r = 20 \Rightarrow r = 4$

Coefficient of x = ${}^7C_4\left(\frac{-1}{2}\right)^4$

$= 35 \times \frac{1}{16} = \frac{35}{16}$

24. (D) No. of two-digit numbers = 4×4 = 16
No. of three-digit number = 4×4×3 = 48
The required numbers = 16 + 48 = 64

25. (A) $f(x) = \begin{cases} x-3, & \text{when } x \le 2 \\ 4x+3, & \text{when } x > 2 \end{cases}$

L.H.L. = $\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)$

$= \lim_{h\to 0} 2-h-3 = -1$

R.H.L. = $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} 4(2+h)+3 = 11$

L.H.L. $\neq$ R.H.L.
Hence f is discontinous at $x = 2$.

26. (B) $\frac{\tan\theta}{1-\cos\theta} - \frac{\cot\theta}{1-\sin\theta}$

$\Rightarrow \frac{\tan\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} - \frac{\cot\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$\Rightarrow \frac{\tan\theta(1+\cos\theta)}{1-\cos^2\theta} - \frac{\cot\theta(1+\sin\theta)}{1-\sin^2\theta}$

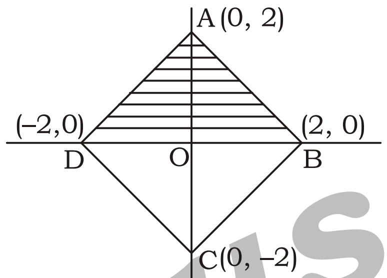
$\Rightarrow \frac{\sin\theta(1+\cos\theta)}{\cos\theta.\sin^2\theta} - \frac{\cos\theta(1+\sin\theta)}{\sin\theta.\cos^2\theta}$

$\Rightarrow \frac{1+\cos\theta}{\sin\theta.\cos\theta} - \frac{1+\sin\theta}{\sin\theta.\cos\theta}$

$\Rightarrow \frac{\cos\theta - \sin\theta}{\sin\theta.\cos\theta}$ = cosecθ – secθ

27. (C) The required probability = $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

28. (C)



Area of ΔAOB = $\frac{1}{2}$ × AO × OB

$= \frac{1}{2}\times 2\times 2 = 2$

The required area = 4 × 2 = 8 sq. unit

29. (B) $A - (A\cap C) + (A\cap B\cap C)$

30. (D) Direction ratios are (–2, 2, – 4) and (–4, 2, x).
A.T.Q.,

$\cos\frac{\pi}{2} = \frac{-2\times(-4)+2\times 2-4\times x}{\sqrt{(-2)^2+2^2+(-4)^2}\sqrt{(-4)^2+2^2+x^2}}$

$\Rightarrow 0 = \frac{8+4-4x}{\sqrt{24}\sqrt{20+x^2}}$

$\Rightarrow 0 = 12 - 4x \Rightarrow 4x = 12 \Rightarrow x = 3$

31. (B) $(x^2 - a^2)^2 + (y^2 - b^2)^2 = 0$
$\Rightarrow x^2 - a^2 = 0$ and $y^2 - b^2 = 0$
$\Rightarrow x = \pm a$ and $y = \pm b$
So the points are (a, b), $(a, -b)$, $(-a, b)$ and $(-a, -b)$
It is clear that these points lie on circle $x^2 + y^2 = a^2 + b^2$ having centre at origin.

32. (B) The equation of a circle passing through (0, 0), $(a, 0)$ and $(0, b)$is
$x^2 + y^2 - ax - by = 0$

The coordinates of its centre is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

33. (C) Let us consider the circle equation is
$x^2 + y^2 + 2gx + fy + c = 0$ ...(i)
If this passes through (0, 0) and (1, 0) therefore

$c = 0$ and $1 + 2g = 0 \Rightarrow g = -\frac{1}{2}$

Now, it is given that the above circle (i) touches the circle $x^2 + y^2 = 9$
The centre of this circle (0, 0) lies on the above circle (i)
So from this it follows that the given circle touches internally the circle $x^2 + y^2 = 9$
Thus the diameter of the required circle must be equal to the radius of the circle $x^2 + y^2 = 9$

Hence, we can write $2\sqrt{g^2 + f^2 - c} = 3$

$\Rightarrow 2\sqrt{\frac{1}{4} + f^2} = 3 \Rightarrow f = \pm\sqrt{2}$

Hence, centres of the required circle are $\left(\frac{1}{2}, \pm\sqrt{2}\right)$.

34. (A) Let us consider $y = mx$ be the tangent from the origin to the circle $(x-7)^2 + (y+1)^2 = (5)^2$, then (the perpendicular distance from centre to tangent will be equal to radius)

$\frac{7m-(-1)}{\sqrt{m^2+1}} = 5$

$\Rightarrow \frac{7m+1}{\sqrt{m^2+1}} = 5$

$\Rightarrow 12m^2 + 7m - 12 = 0$

Let m_1 and m_2 be the slopes of the tangents, then

$m_1 m_2$ = product of roots = $\frac{-12}{12} = -1$

Hence the angle between two tangents is $\frac{\pi}{2}$.

35. (C) The ellipse equation $3x^2 + 5y^2 = 32$ ...(i)
Since $3(3)^2 + 5(5)^2 - 32 > 0$
So the given point (3, 5) lies outside the ellipse (i)
Hence two real tangent can be drawn from the point to the ellipse.

36. (B) The ellipse equation $4x^2 + 9y^2 = 1$

$\Rightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1 \Rightarrow \frac{x^2}{\left(\frac{1}{2}\right)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

and line $8x = 9y$

$\therefore a^2 = \frac{1}{4}, b^2 = \frac{1}{9}, m = \frac{8}{9}$

The required are $\left(\pm\frac{a^2m}{\sqrt{a^2m^2+b^2}}, \mp\frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

$\Rightarrow \left(\pm\frac{2}{5}, \mp\frac{1}{5}\right)$

37. (A) Let P(x, y) be any point on the ellipse
Then by definition
SP = e. PM

$\Rightarrow \sqrt{(x-1)^2 + (y+1)^2} = \frac{1}{2}\left(\frac{x-y-3}{\sqrt{2}}\right)$

On solving

$\Rightarrow 7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$

38. (B) The ellipse equation $9x^2 + 5y^2 - 30y = 0$

$\Rightarrow 9x^2 + 5(y-3)^2 = 45$

$\Rightarrow \frac{x^2}{5} + \frac{(y-3)^2}{9} = 1$

Hence $a^2 = 5$, $b^2 = 9$ then eccentricity is given as $a^2 = b^2\left(1 - e^2\right)$

$\Rightarrow \frac{5}{9} = 1 - e^2 \Rightarrow e^2 = 1 - \frac{5}{9} = \frac{4}{9} \Rightarrow e = \frac{2}{3}$

39. (C) The straight lines equation

$\frac{x}{a} + \frac{y}{b} = \lambda$ ...(i)

and $\frac{x}{a} - \frac{y}{b} = \frac{1}{\lambda}$ ...(ii)

Eliminating λ from these equation, we get

$\left(\frac{x}{a} + \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b}\right) = 1$

$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Which is the equation of a hyperbola.

40. (A) The given equation of hyperbola

$3x^2 - 4y^2 = -12 \Rightarrow -\frac{x^2}{4} + \frac{y^2}{3} = 1$

Now, $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{4}{3}} = \sqrt{\frac{7}{3}}$

41. (B) Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be a hyperbola and let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ be its conjugate.
Then their eccentricities are given by

$e^2 = \frac{a^2 + b^2}{a^2}$ and $e'^2 = \frac{a^2 + b^2}{b^2}$

Now, $\frac{1}{e^2} + \frac{1}{e'^2} \Rightarrow \frac{a^2}{(a^2+b^2)} + \frac{b^2}{(a^2+b^2)}$

$\Rightarrow \frac{(a^2+b^2)}{(a^2+b^2)} = 1$

42. (A) Let I = $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}\,dx$

$= \int \frac{(x^2+4)+(x^2-1)}{(x^2-1)(x^2+4)}\,dx$

$= \int \frac{1}{(x^2-1)}\,dx + \int \frac{1}{(x^2+4)}\,dx$

$= \frac{1}{2}\log\frac{(x-1)}{(x+1)} + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$ ...(i)

Now, it is given that

I = $A\log\frac{(x-1)}{(x+1)} + B\tan^{-1}\frac{x}{2}$ ...(ii)

Comparing (i) and (ii), we get

$A = \frac{1}{2}$, $B = \frac{1}{2}$

43. (A) Let I = $\int \sin^2 x\,dx$

$= \frac{1}{2}\int (1-\cos 2x)\,dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$

44. (A) $\int \frac{\sqrt{\tan x}}{\sin x.\cos x}\,dx \Rightarrow \int \frac{\sqrt{\tan x}\sec^2 x}{\tan x}\,dx$

$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan x}}$ Put $\tan x = t$

$\sec^2 x\,dx = dt$

$\Rightarrow \int \frac{dt}{\sqrt{t}} \Rightarrow \frac{t^{-\frac{1}{2}+1}}{1/2} + C \Rightarrow 2\sqrt{t} + C \Rightarrow 2\sqrt{\tan x} + C$

45. (C) We know that

$\omega = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$ and $\omega^2 = -\frac{1}{2} - \frac{\sqrt{3}i}{2}$

Now, $4 + 5\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)^{334} + 3\left(-\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)^{365}$

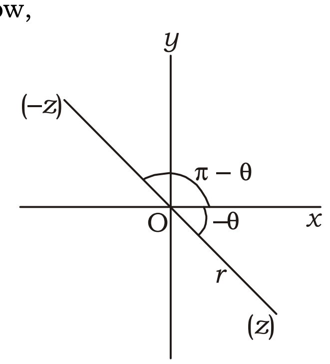
$\Rightarrow 4 + 5(\omega)^{334} + 3(\omega^2)^{365} = 4 + 5\omega + 3\omega$

$\Rightarrow 4 + 8\omega = (4 + 4\omega) + 4\omega$

$\Rightarrow -4\omega^2 + 4\omega = 4(\omega - \omega^2) = 4\sqrt{3}\,i$

46. (A) arg(z) < 0 (given) ⇒ arg(z) = – θ

Now,



$z = r\cos(-\theta) + i\sin(-\theta) = r[\cos(\theta) - i\sin(\theta)]$

Again $-z = -r[\cos(\theta) - i\sin(\theta)]$

$\Rightarrow -z = r[\cos(\pi-\theta) + i\sin(\pi-\theta)$

∴ arg (–z) = π – θ;

Thus arg (–z) – arg (z) = π – θ – (–θ) = π

47. (A) The required probability

$= \frac{{}^6C_2 \times {}^4C_1 \times {}^5C_0 + {}^6C_2 \times {}^4C_0 \times {}^5C_1}{{}^{15}C_3}$

$= \frac{15\times4\times1+15\times1\times5}{5\times7\times13} = \frac{(15\times9)}{(5\times7\times13)} = \frac{27}{91}$

48. (B) Let the number of balls transferred = x

Now,

$\frac{{}^xC_2}{{}^{(4+x)}C_2} = \frac{3}{14}$

$\Rightarrow \frac{x(x-1)}{(4+x)(3+x)} = \frac{3}{14}$

$\Rightarrow 14x^2 - 14x = 3(12 + 4x + 3x + x^2)$

$\Rightarrow 14x^2 - 14x = 36 + 21x + 3x^2$

$\Rightarrow 11x^2 - 35x - 36 = 0$

$\Rightarrow (x-4)(11x+9) = 0$

$\Rightarrow x = 4, \frac{-9}{11}$ (rejected)

Hence 4 balls transfered to bag P.

49. (C) $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$\Rightarrow 3(1-\sin 2x)^2 + 6(1+\sin 2x) + 4[(\sin^2 x + \cos^2 x)^3 - 3\sin^2 x.\cos^2 x(\sin^2 x + \cos^2 x)]$

$\Rightarrow 3 - 6\sin 2x + 3\sin^2 2x + 6 + 6\sin 2x + 4\left[1 - \frac{3}{4}\sin^2 2x\right]$

$\Rightarrow 13 + 3\sin^2 2x - 3\sin^2 2x = 13$

50. (D) Equation $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$

[∴ $\sin\theta - 2 = 0$ is not possible]

$\Rightarrow \sin\theta = \sin\left(\frac{-\pi}{6}\right) = \sin\left(\frac{7\pi}{6}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\frac{-\pi}{6}\right) = n\pi + (-1)^n\frac{7\pi}{6}$

$\Rightarrow$ Thus, $\theta = n\pi + (-1)^n\frac{7\pi}{6}$

51. (B) We have $\sec^2\theta = \frac{4xy}{(x+y)^2}$

But $\sec^2\theta \geq 1 \Rightarrow \frac{4xy}{(x+y)^2} \geq 1$

$\Rightarrow 4xy \geq x^2 + y^2 + 2xy$

$\Rightarrow x^2 + y^2 - 2xy \leq 0$

$\Rightarrow (x-y)^2 \leq 0$

$\Rightarrow x - y = 0$ [as perfect square of real number can never be negative]

also then $x \neq 0$ as then $\sec^2\theta$ will become indeterminate.

Hence $x = y$, $x \neq 0$

52. (A) Given that in ΔPQR, $\angle R = \frac{\pi}{2}$

$\Rightarrow P + Q = \frac{\pi}{2} \Rightarrow \frac{\angle P}{2} + \frac{\angle Q}{2} = \frac{\pi}{4}$

Also $\tan\frac{P}{2} + \tan\frac{Q}{2} = \frac{b}{a}$; $\tan\frac{P}{2}\tan\frac{Q}{2} = \frac{c}{a}$

Now, $\tan\left(\frac{P+Q}{2}\right) = \frac{\tan\frac{P}{2} + \tan\frac{Q}{2}}{1 - \tan\frac{P}{2}\tan\frac{Q}{2}}$

$\Rightarrow \tan\frac{\pi}{4} = \frac{-\frac{b}{a}}{1 - \frac{c}{a}} \Rightarrow 1 - \frac{c}{a} = -\frac{b}{a}$

$\Rightarrow a - c = -b \Rightarrow a + b = c$

53. (A) Given equation $x = \sin^2 t$ ...(i)

and $y = 2\cos t \Rightarrow \frac{y^2}{4} = \cos^2 t$ ...(ii)

Now eliminating t from equation (i) and (ii) adding equation (i) and (ii) we get

$x + \frac{y^2}{4} = \sin^2 t + \cos^2 t \Rightarrow x + \frac{y^2}{4} = 1$

$\Rightarrow y^2 + 4x = 4$

Which is a equation of parabola.

54. (A) The equation of tangent to parabola $y^2 = 4ax$ in terms of slope m is $y = mx + \frac{a}{m}$

If it touches the parabola $x^2 = 4ay$ then the equation

$x^2 = 4a\left(mx + \frac{a}{m}\right) \Rightarrow mx^2 - 4am^2x - 4a^2 = 0$

For equal roots $B^2 - 4AC = 0$

$\Rightarrow (-4am^2)^2 - 4 \times m \times (-4a^2) = 0$

$\Rightarrow 16\,a^2m^4 + 16\,ma^2 = 0$

$\Rightarrow 16\,ma^2(m^3 + 1) = 0$

$\Rightarrow m = 0,\ m^3 - 1$ or $m = -1$

$m = 0$ is not possible so $m = -1$ will be consider

Putting $m = -1$, in line $y = mx + \frac{a}{m}$

We get $y = -x - a \Rightarrow x + y + a = 0$

55. (B) As we know that the locus of the point of intersection of the perpendicular tangents to a parabola is its directix. So the required locus is $y = -a$

56. (B) $x = a(t + \sin t)$, $y = a(1 - \cos t)$

$\frac{dx}{dt} = a(1 + \cos t)$, $\frac{dy}{dt} = -a\sin t$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-a\sin t}{a(1+\cos t)} = \frac{-\sin t}{1+\cos t}$

$= \frac{-2\sin\frac{t}{2}.\cot\frac{t}{2}}{2\cos^2\frac{t}{2}} = -\tan\frac{t}{2}$

Length of normal at "t" $= y\sqrt{1 + \left(\frac{dy}{dx}\right)_t^2}$

$= a(1 - \cos t)\sqrt{1 + \tan^2\frac{t}{2}}$

$= a.2\sin^2\frac{t}{2}\sqrt{\sec^2\frac{t}{2}} = a.2\sin^2\frac{t}{2}.\sec\frac{t}{2}$

$= 2a\sin\frac{t}{2}.\tan\frac{t}{2}$

57. (A) Curve $y^2 = px^3 + q$

$2.y.\frac{dy}{dx} = 3px^2 \Rightarrow \frac{dy}{dx} = \frac{3px^2}{2y}$

At point (2, 3), $\left(\frac{dy}{dx}\right)_{(2,3)} = \frac{12p}{6} = 2p$

If $y = 4x - 5$ is tangent to curve (i), then

$\left(\frac{dy}{dx}\right)_{(2,3)} =$ slope of line $(y = 4x - 5)$

$\Rightarrow 2p = 4 \Rightarrow p = 2$ ...(ii)

Now point (2, 3) lie on curve (i) so

$(3)^2 = p(2)^3 + q \Rightarrow 9 = 8p + q$

$\Rightarrow 9 = 8(2) + q$ [from eq(ii)]

$\Rightarrow q = 9 - 16 \Rightarrow q = -7$

Hence $p = 2$, $q = -7$

58. (A) $y = \frac{1}{(2 - \sin 3x)}$

$\Rightarrow 2y - y\sin 3x = 1$

$\Rightarrow \sin 3x = \frac{2y-1}{y}$

Now $-1 \le \sin 3x \le 1$

$\Rightarrow -1 \le \frac{2y-1}{y} \le 1$

$\Rightarrow -1 \le 2 - \frac{1}{y} \le 1$

$\Rightarrow \frac{1}{3} \le y \le 1$

Hence range is $\left[\frac{1}{3}, 1\right]$.

59. (A) Given quadratic equation
$(x - a)(x - b) = c$
$\Rightarrow x^2 - (a + b)x + (ab - c) = 0$
since roots are α and β so-
$\alpha + \beta = a + b$ and $\alpha.\beta = ab - c$
Now the equation
$(x - \alpha)(x - \beta) + c = 0$
$\Rightarrow x^2 - (\alpha + \beta) + \alpha\beta + c = 0$
$\Rightarrow x^2 - (a + b)x + ab = 0$
$\Rightarrow (x - a)(a - b) = 0$
Hence roots are a, b.

60. (B) Sum of p terms of A.P.
$\frac{p}{2}[2a + (p - 1).d] = q$ ...(i)
sum of q terms of that A.P.
$\frac{p}{2}[2a + (q - 1).d] = p$ ...(ii)
subtracted (ii) from (i)
$\frac{1}{2}[2a(p - q) + \{p(p-1) - q(q - 1)d] = q - p$
$\Rightarrow 2a(p - q) + \{(p^2 - q^2) - (p - q)\}d = 2(q - p)$
$\Rightarrow 2a + (p + q - 1).d = -2$ ...(iii)
Now sum upto $(p + q)$ terms
$S_{p+q} = \frac{(p+q)}{2}[2a + (p + q - 1).d]$
$S_{p+q} = \frac{(p+q)}{2}(-2)$
from eq(iii)
$S_{p+q} = -(p + q)$

61. (B) $\begin{vmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{vmatrix} = 0$
Applying $C_3 \to C_3 - \alpha C_1 + C_2$, we get
$\Rightarrow \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix} = 0$
$\Rightarrow (1 - 2\alpha)(ac - b^2) = 0,\ \alpha \neq \frac{1}{2}$
$\Rightarrow b^2 = ac \Rightarrow a, b, c$ are in G.P.

62. (A) Given that $x^{18} = y^{21} = z^{28}$
On taking log both sides
$18\log_e x = 21\log_e y = 28\log_e z$
(i) (ii) (iii)
From (i) and (ii) $\frac{\log_e x}{\log_e y} = \frac{21}{18}$
$\Rightarrow \log_y x = \frac{7}{6}$
From (ii) and (iii) $21\log_e y = 28\log_e z$
$\Rightarrow \log_z y = \frac{18}{28} = \frac{7}{6}$
From (i) and (iii) $28\log_e z = 18\log_e x$
$\Rightarrow \log_x z = \frac{18}{28} = \frac{9}{14}$
Now, 3, $\log_y x$, $3\log_z y$, $7\log_z x$
$\Rightarrow 3,\ 3\times\frac{7}{6},\ 3\times\frac{4}{3},\ 7\times\frac{9}{14}$
$\Rightarrow 3,\ \frac{7}{2}, 4, \frac{9}{2}$
Which are in A.P. series with common difference $\frac{1}{2}$.

63. (A) $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = (1+\sqrt{2})^6 + (1-\sqrt{2})^6$
$= 2[1 + {}^6C_2(\sqrt{2})^2 + {}^6C_4(\sqrt{2})^4 + {}^6C_6(\sqrt{2})^6]$
$= 2[1 + 30 + 60 + 8] = 2 \times 99 = 198$

64. (B) In the expansion of $(1 + x)^{18}$
The coefficient of $(2r + 4)^{th}$ term $= {}^{18}C_{2r+3}$
The cofficient of $(r - 2)^{th}$ term $= {}^{18}C_{r-3}$
Now, ${}^{18}C_{r-3} = {}^{18}C_{2r+3}$
$\Rightarrow r - 3 = 18 - (2r + 3)$
$\Rightarrow r - 3 = 15 - 2r$
$\Rightarrow 3r = 18 \Rightarrow r = 6$

65. (A) $C(47, 4) + \sum_{r=1}^{5} C(52 - r, 3)$
$\Rightarrow {}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$
$\Rightarrow {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$
$\Rightarrow {}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$
apply property ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
$= {}^{52}C_4 = C(52, 4)$

66. (A) In the expansion of $(1 + \alpha x)^4$
the middle term $= \left(\frac{4}{2} + 1\right)$ = 3rd term
Coefficient of 3rd term $= {}^4C_2(\alpha)^2$
In the expansion of $(1 - \alpha x)^6$
the middle term $= \left(\frac{6}{2} + 1\right)$ = 4th term
Coefficient of 4th term $= {}^6C_3(-\alpha)^3$
Now, ${}^4C_2(\alpha)^2 = {}^6C_3(-\alpha)^3$
$\Rightarrow 6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$

67. (A) Here we divide 12 books into 4 sets of 3 books each.
So the required number of ways
$= \frac{12!}{(3!)^4 . 4!} . 4! = \frac{12!}{(3!)^4}$.

68. (C) "PATALIPUTRA"

In this "AAAIU" can be arranged in $\frac{5!}{3!}$ = 20 ways.

"PPTTLR" can be arranged in $\frac{6!}{2!2!}$ = 180 ways.

required number of ways = 20 × 80
= 3600 ways

69. (B) $\frac{1}{1+2\omega}+\frac{1}{2+\omega}-\frac{1}{1+\omega}$

$$\Rightarrow \frac{1}{(1+\omega)+\omega}+\frac{1}{1+(1+\omega)}-\frac{1}{(1+\omega)}$$

$$\Rightarrow \frac{1}{-\omega^2+\omega}+\frac{1}{1-\omega^2}-\frac{1}{-\omega^2}\ [\because 1+\omega+\omega^2=0]$$

$$\Rightarrow \frac{1}{\omega(1-\omega)}+\frac{1}{(1-\omega^2)}+\frac{1}{\omega^2}$$

$$\Rightarrow \frac{\omega(1+\omega)+\omega^2+1-\omega^2}{\omega^2(1-\omega^2)} \Rightarrow \frac{\omega+\omega^2+1}{\omega^2(1-\omega^2)}=0$$

$[\because 1+\omega+\omega^2=0]$

70. (A) $\frac{1}{1.2}+\frac{1.3}{1.2.3.4}+\frac{1.3.5}{1.2.3.4.5.6}+\ldots..\infty$

$$T_n=\frac{1.3.5\ldots.(2n-1)}{(2n)!}$$

$$=\frac{1.2.3.4.5.6\ldots.(2n-1).2n}{(2n)!(2.4.6.8\ldots.2n)}$$

$$=\frac{(2n)!}{(2n)!.2^n.n!}=\frac{1}{2^n.n!}=\frac{1/2^n}{n!}$$

Now, sum S = $\sum_{n=1}^{\infty}\frac{1/2^n}{n!}=\frac{1/2}{1!}+\frac{(1/2)^2}{2!}+\frac{(1/2)^3}{3!}+\ldots..\infty$

S = $(e^{1/2}-1)=\sqrt{e}-1$

71. (D) $\begin{bmatrix}2&0&7\\0&1&0\\1&-2&1\end{bmatrix}\begin{bmatrix}-x&14x&7x\\0&1&0\\x&-4x&-2x\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}5x&0&0\\0&1&0\\0&(10x-2)&5x\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

On solving

$\Rightarrow 5x=1 \Rightarrow x=\frac{1}{5}$

72. (D) $\frac{\sin^2\frac{3A}{2}}{\sin^2\frac{A}{2}}-\frac{\cos^2\frac{3A}{2}}{\cos^2\frac{A}{2}}$

$$\Rightarrow \left(\frac{\sin\frac{3A}{2}}{\sin\frac{A}{2}}\right)^2-\left(\frac{\cos\frac{3A}{2}}{\cos\frac{A}{2}}\right)^2$$

$$\Rightarrow \left(\frac{3\sin\frac{A}{2}-4\sin^3\frac{A}{2}}{\sin\frac{A}{2}}\right)^2-\left(\frac{4\cos^3\frac{A}{2}-3\cos\frac{A}{2}}{\cos\frac{A}{2}}\right)^2$$

$$\Rightarrow \left(3-4\sin^2\frac{A}{2}\right)^2-\left(4\cos^2\frac{A}{2}-3\right)^2$$

$$\Rightarrow 9+16\sin^4\frac{A}{2}-24\sin^2\frac{A}{2}-16\cos^4\frac{A}{2}$$

$$-9+24\cos^2\frac{A}{2}$$

$$\Rightarrow 16\sin^4\frac{A}{2}-16\cos^4\frac{A}{2}-24\left(\sin^2\frac{A}{2}-\cos^2\frac{A}{2}\right)$$

$$\Rightarrow 16\left(\sin^2\frac{A}{2}+\cos^2\frac{A}{2}\right)\left(\sin^2\frac{A}{2}-\cos^2\frac{A}{2}\right)$$

$$-24\left(\sin^2\frac{A}{2}-\cos^2\frac{A}{2}\right)$$

$$\Rightarrow \left(\sin^2\frac{A}{2}-\cos^2\frac{A}{2}\right)(16-24)$$

$\Rightarrow$ 8 cos A

73. (C) We know that

$\cos 2A = 1 - 2\sin^2 A$

$\Rightarrow 2\sin^2 A = 1 - \cos^2 A$

On putting $A = 22\frac{1}{2}$

$$\Rightarrow 2\sin^2 22\frac{1}{2} = 1-\cos 45$$

$$\Rightarrow 2\sin^2 22\frac{1}{2} = 1-\frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin^2 22\frac{1}{2}=\frac{\sqrt{2}-1}{2\sqrt{2}} \Rightarrow \sin 22\frac{1}{2}=\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

then $\cos\left(247\frac{1}{2}\right)^\circ \Rightarrow \cos\left(270-22\frac{1}{2}\right)$

$$\Rightarrow -\sin 22\frac{1}{2}=-\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

74. (A) Given that $y + z = x$

Now, $\sin x + \sin y + \sin z$

$$\Rightarrow 2\sin\frac{x}{2}.\cos\frac{x}{2} + 2\sin\frac{y+z}{2}.\cos\frac{y-z}{2}$$

$$\Rightarrow 2\sin\frac{x}{2}.\cos\frac{x}{2} + 2\sin\frac{x}{2}.\cos\frac{y-z}{2}$$

$$\Rightarrow 2\sin\frac{x}{2}\left[\cos\frac{x}{2} + \cos\frac{y-z}{2}\right]$$

$$\Rightarrow 2\sin\frac{x}{2} \times 2\cos\frac{x+y-z}{4}.\cos\frac{x-y+z}{4}$$

$$\Rightarrow 2\sin\frac{x}{2} \times 2\cos\frac{y}{2} \times \cos\frac{z}{2}$$

$$\Rightarrow 4\sin\frac{x}{2}.\cos\frac{y}{2}.\cos\frac{z}{2}$$

75. (C) $y = x^2 - e^x$

On differentiating both side w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = 2x - e^x$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{2x - e^x} \qquad \ldots(i)$$

On differentiating both w.r.t. 'y'

$$\Rightarrow \frac{d^2x}{dy^2} = (-1)\,(2x - e^x)^{-2}.(2 - e^x).\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-1}{(2x - e^x)^2}(2 - e^x) \times \frac{1}{(2x - e^x)}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{e^x - 2}{(2x - e^x)^3}$$

76. (B) $x = g(t)$ and $y = f(t)$

$$\frac{dx}{dt} = g'(t), \quad \frac{dy}{dt} = f'(t)$$

$$\frac{d^2x}{dt^2} = g''(t), \quad \frac{d^2y}{dt^2} = f''(t)$$

Now, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$\Rightarrow \frac{dy}{dx} = \frac{f'(t)}{g'(t)}$$

On differentiating both side w.r.t. 'x'

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{g'(t)f''(t) - f'(t)g''(t)}{\{g'(t)\}^2}$$

given that $\frac{d^2y}{dx^2} = 0$

$$\Rightarrow g'(t)\,f''(t) - f'(t)\,g''(t) = 0$$

$$\Rightarrow \frac{dx}{dt}.\frac{d^2y}{dt^2} - \frac{dy}{dt}.\frac{d^2x}{dt^2} = 0$$

$$\Rightarrow \frac{dx}{dt}.\frac{d^2y}{dt^2} = \frac{dy}{dt}.\frac{d^2x}{dt^2}$$

77. (B) $\lim_{x\to\infty} x^{\frac{5}{2}}\left(\sqrt{x^5+1} - \sqrt{x^5-1}\right)$

$$\Rightarrow \lim_{x\to\infty} x^{\frac{5}{2}}\frac{\left(\sqrt{x^5+1} - \sqrt{x^5-1}\right)}{\left(\sqrt{x^5+1} + \sqrt{x^5-1}\right)} \times \left(\sqrt{x^5+1} + \sqrt{x^5-1}\right)$$

$$\Rightarrow \lim_{x\to\infty} \frac{x^{\frac{5}{2}}\left(x^5 + 1 - x^5 + 1\right)}{\sqrt{x^5+1} + \sqrt{x^5-1}}$$

$$\Rightarrow \lim_{x\to\infty} \frac{2.x^{\frac{5}{2}}}{x^{\frac{5}{2}}\left(\sqrt{1 + \frac{1}{x^5}} + \sqrt{1 - \frac{1}{x^5}}\right)}$$

$$\Rightarrow \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{2} = 1$$

78. (D) Centre is the intersection point of two diameters $2x + y = 6$ and $3x - y = 9$.

centre = (3, 0)

circle passes through the point (–1, 3)

the radius $(r) = \sqrt{(3+1)^2 + (0-3)^2} = 5$

Equation of circle

$(x - 3)^2 + (y - 0)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 6x - 16 = 0$

79. (C) Equation of parabola

$x^2 + 4x - 16y + 24 = 0$

$\Rightarrow (x + 2)^2 - 4 - 16y + 24 = 0$

$\Rightarrow (x + 2)^2 = 16y - 20$

$$\Rightarrow (x + 2)^2 = 16\left(y - \frac{5}{4}\right)$$

Equation of directrix

$$y - \frac{5}{4} = -4 \Rightarrow 4y + 11 = 0$$

80. (D) **Statement I**

given that $a \times d = c \times b$ and $a \times c = d \times b$

Now, $(d - c) \times (a - b)$

$\Rightarrow d \times a - d \times b - c \times a + c \times b$

$\Rightarrow d \times a - a \times c + a \times c + a \times d$

$\Rightarrow -a \times d + a \times d = 0$

$(d - c)$ is parallel to $(a - b)$.

Statement I is correct.

Statement II

L.H.S. $= (a - d).[(d - c) \times (a - c)]$

$= (a - d).[d \times a - d \times c - c \times a + c \times c]$

$= (a - d).[d \times a - d \times c - c \times a]$

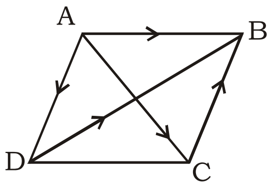
$= a.(d \times a) - a.(d \times c) - a.(c \times a) - d.(d \times a)$

$= -[a\ d\ c] - 0 - 0 + [a\ d\ c]$

$= 0 =$ R.H.S.

Statement II is correct.

Statement III



AD + DB = AB ...(i)
AC + CB = AB ...(ii)
from (i) and eq. (ii)
AD + DB = AC + CB
AD – CB = AC – DB
AD + BC = AC + BD
Statement III is correct.

81. (B) Vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$

$$\cos\theta = \frac{1\times3 - 2\times1 + 3 + (2)}{\sqrt{(1)^2 + (-2)^2 + (3)^2}\sqrt{(3)^2 + (1)^2 + (2)^2}}$$

$$\cos\theta = \frac{7}{\sqrt{14}\sqrt{14}} = \frac{1}{2}$$

Hence $\sin\theta = \frac{\sqrt{3}}{4}$

82. (B) AB = c = 8 cm, BC = a = 15 cm and CA = b = 17 cm

$$s = \frac{a+b+c}{2} = \frac{15+17+8}{2} = 20$$

Now, $\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$

$$\Rightarrow \cot\frac{A}{2} = \sqrt{\frac{20\times5}{3\times12}}$$

$$\Rightarrow \cot\frac{A}{2} = \frac{10}{6} = \frac{5}{3}$$

83. (A) sin1725 ⇒ sin(360 × 4 + 285)
⇒ sin(285) ⇒ sin(270 + 15)

$$\Rightarrow -\cos15 = -\frac{\sqrt{3}+1}{2\sqrt{2}}$$

84. (B) Let $y = \sin^2\left[\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right]$

On putting $x = \cos2\theta$

$$\Rightarrow y = \sin^2\left[\tan^{-1}\sqrt{\frac{1+\cos2\theta}{1-\cos2\theta}}\right]$$

$$\Rightarrow y = \sin^2\left[\tan^{-1}\sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}}\right]$$

$$\Rightarrow y = \sin^2\left[\tan^{-1}(\cot\theta)\right]$$

$$\Rightarrow y = \sin^2\left[\tan^{-1}\left\{\tan\left(\frac{\pi}{2}-\theta\right)\right\}\right]$$

$$\Rightarrow y = \sin^2\left[\frac{\pi}{2}-\theta\right] \Rightarrow y = \cos^2\theta$$

$$\Rightarrow y = \frac{1+\cos2\theta}{2} \Rightarrow y = \frac{1+x}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

85. (B) $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$

$3\vec{a} + \vec{b} = 3(2\hat{i} + \hat{j} - 3\hat{k}) + \hat{i} - \hat{j} + 3\hat{k}$
$= 7\hat{i} + 2\hat{j} - 6\hat{k}$

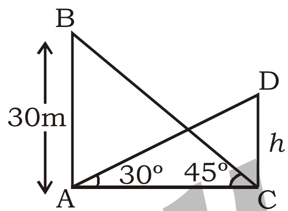
$4\vec{b} - \vec{a} = 4(\hat{i} - \hat{j} + 3\hat{k}) - (2\hat{i} + \hat{j} - 3\hat{k})$
$= 2\hat{i} - 5\hat{j} + 15\hat{k}$

Then

$(3\vec{a} + \vec{b}).(4\vec{b} - \vec{a})$ = 14 – 10 – 90 = – 86

86. (C)

87. (A)



Let height of smaller tower (CD) = h m
then $AC = kh$

In ΔABC

$\tan45° = \frac{AB}{AC} \Rightarrow 1 = \frac{30}{kh} \Rightarrow kh = 30$ ...(i)

In ΔACD

$\tan30° = \frac{CD}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{kh} \Rightarrow k = \sqrt{3}$

from eq. (i)

$\sqrt{3} \times h = 30 \Rightarrow h = 10\sqrt{3}$ m

88. (B) $\begin{bmatrix} 2x+y+z & y & z \\ x & x+2y+z & z \\ x & y & x+y+2z \end{bmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{bmatrix} 2(x+y+z) & y & z \\ 2(x+y+z) & x+2y+z & z \\ 2(x+y+z) & y & x+y+2z \end{bmatrix}$$

$$\Rightarrow 2(x+y+z)\begin{bmatrix} 1 & y & z \\ 1 & x+2y+z & z \\ 1 & y & x+y+2z \end{bmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow 2(x+y+z)\begin{bmatrix} 1 & y & z \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{bmatrix}$$

$\Rightarrow 2(x+y+z)\,[1(x+y+z)^2 - 0 - 0]$
$\Rightarrow 2(x+y+z)^3$

89. (C) $a = 6$ cm, $b = 10$ cm, $c = 14$ cm

Now, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos C = \dfrac{36+100-196}{2\times6\times10}$

$\Rightarrow \cos C = \dfrac{-60}{120}$

$\Rightarrow \cos C = -\dfrac{1}{2} \Rightarrow C = 120^\circ$

90. (C) $\log_{ab} a = y \Rightarrow \dfrac{1}{\log_a ab} = y$

$\Rightarrow \dfrac{1}{y} = \log_a a + \log_a b \Rightarrow \dfrac{1}{y} = 1 + \log_a b$

$\Rightarrow \log_a b = \dfrac{1-y}{y}$

Now, $\log_b ab \Rightarrow \log_b a + \log_b b$

$\Rightarrow \dfrac{1}{\log_a b} + 1 \Rightarrow \dfrac{y}{1-y} + 1 \Rightarrow \dfrac{1}{1-y}$

91. (B) Number of ways = ${}^6C_4 = 15$

92. (B) **Statement I**

L.H.S. = $\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3} + \dfrac{1}{r}$

$= \dfrac{s-a}{\Delta} + \dfrac{s-b}{\Delta} + \dfrac{s-c}{\Delta} + \dfrac{s}{\Delta}$

$= \dfrac{4s-a-b-c}{\Delta} = \dfrac{4s-2s}{\Delta}$

$= \dfrac{2s}{\Delta} = \dfrac{2}{r} \neq$ R.H.S

Statement I is incorrect.

Statement II

$r_3 = r_1 + r_2 + r$

$\Rightarrow \dfrac{\Delta}{s-c} = \dfrac{\Delta}{s-a} + \dfrac{\Delta}{s-b} + \dfrac{\Delta}{s}$

$\Rightarrow \dfrac{1}{s-c} - \dfrac{1}{s} = \dfrac{1}{s-a} + \dfrac{1}{s-b}$

$\Rightarrow \dfrac{s-s+c}{s(s-c)} = \dfrac{s-b+s-a}{(s-a)(s-b)}$

$\Rightarrow \dfrac{c}{s(s-c)} = \dfrac{c}{(s-a)(s-b)}$

$\Rightarrow s^2 - s(a+b) + ab = s^2 - sc$

$\Rightarrow ab = s(a+b-c)$

$\Rightarrow ab = \dfrac{(a+b+c)}{2}(a+b-c)$

$\Rightarrow 2ab = (a+b)^2 - c^2$

$\Rightarrow a^2 + b^2 = c^2$

ΔABC is a right-angled triangle.

Statement II is correct.

93. (A) $\int_0^{1.5} [x^2]\,dx$

$\Rightarrow \int_0^1 [x^2]\,dx + \int_1^{\sqrt2} [x^2]\,dx + \int_{\sqrt2}^{1.5} [x^2]\,dx$

$\Rightarrow \int_0^1 0\,.dx + \int_1^{\sqrt2} 1.dx + \int_{\sqrt2}^{1.5} 2.dx$

$\Rightarrow 0 + [x]_1^{\sqrt2} + 2[x]_{\sqrt2}^{1.5}$

$\Rightarrow [\sqrt2 - 1] + 2[1.5 - \sqrt2] = 2 - \sqrt2$

94. (B) $\Delta \neq 0$, $h^2 = ab$

95. (B) two parameters

96. (A)

97. (D) Given that $f(x) = x + 6$

Now, $gof(x) = x^2 + 12x + 38$

$\Rightarrow g[f(x)] = (x+6)^2 + 2$

$\Rightarrow g[(f(x))] = [f(x)]^2 + 2$

$\Rightarrow g(x) = x^2 + 2$

$\Rightarrow g(-3) = (-3)^2 + 2 = 11$

98. (B) Word 'ARRANGE'

Total arrangement = $\dfrac{7!}{2!2!} = 1260$

when A appear together

Arrangement = $\dfrac{6!}{2!} = 360$

The required arrangement = 1260 – 360

= 900

99. (B) $\tan y\,dx - (1-e^x)\sec^2 y\,dy = 0$

$\Rightarrow \tan y\,dx = (1-e^x)\sec^2 y\,dy$

$\Rightarrow \dfrac{dx}{1-e^x} = \dfrac{\sec^2 y}{\tan y}\,dy$

$\Rightarrow \dfrac{e^{-x}}{e^{-x}-1}\,dx = \dfrac{\sec^2 y}{\tan y}\,dy$

On integrating

$\Rightarrow -\log(e^{-x} - 1) = \log\tan y + \log c$

$\Rightarrow -\log\left(\dfrac{1-e^x}{e^x}\right) = \log(c.\tan y)$

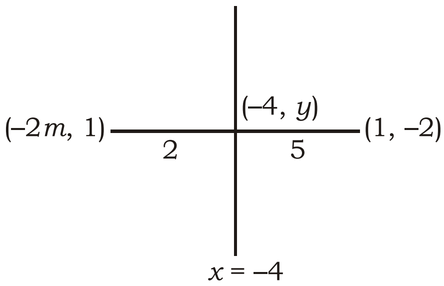
$\Rightarrow \log\left(\dfrac{e^x}{1-e^x}\right) = \log(c.\tan y)$

$\Rightarrow \dfrac{e^x}{1-e^x} = c.\tan y \Rightarrow (1-e^x)\tan y = \dfrac{1}{c}e^x$

$\Rightarrow (1-e^x)\tan y = c.e^x$

100. (B) We know that
det(λA) = λ^ndet(A), if matrix $n \times n$
Then $\lambda = n$

101. (B)



Now, $\frac{2\times1+5\times(-2m)}{2+5} = -4$

$\Rightarrow 2 - 10m = -28 \Rightarrow 10m = 30 \Rightarrow m = 3$

102. (A) A = {1, 2, 3, 4, 6, 7, 9}
no. of elements = 7
then, No. of subsets of A = 2^7 = 128

103. (D) $\int \frac{e^{-x}}{1+e^{-x}}\,dx$

Let $(1 + e^{-x}) = t \Rightarrow -e^{-x}\,dx = dt$

$\Rightarrow e^{-x}dx = -\,dt$

$\Rightarrow \int -\frac{dt}{t} \Rightarrow -\log t + c$

$\Rightarrow -\log(1+e^{-x}) + c \Rightarrow \log\left(\frac{e^x}{1+e^x}\right) + c$

104. (C) given that

$\int x.\ln x\,dx = \frac{x^2}{a} + \frac{x^2.\ln x}{b} + c$ ...(i)

$\int x.\ln x\,dx = \ln x\int x.dx -$

$\int\left\{\frac{d}{dx}(\ln x).\int x.dx\right\}dx$

$\int x.\ln x dx = (\ln x).\frac{x^2}{2} - \int\frac{1}{x}.\frac{x^2}{2}\,dx$

$\int x.\ln x dx = \frac{x^2}{2}\ln x - \frac{1}{2}\times\frac{x^2}{2} + c$

$\int x.\ln x\,dx = -\frac{x^2}{4} + \frac{x^2}{2}\ln x + c$

On comparing with equation (i)
$a = -4$ and $b = 2$

105. (B) S = 6 + 66 + 666 + 6666 + ...

$S = \frac{6}{9}$[9 + 99 + 999 + 9999 + ...)

$S = \frac{2}{3}$[(10–1) + (100–1) + (1000–1)+...]

$S = \frac{2}{3}$[(10+100 + 1000+...9times)
– (1+1+1+...9times)]

$S = \frac{2}{3}\left[\frac{10(10^9-1)}{10-1} - 9\right]$

$S = \frac{2}{3}\left[\frac{10^{10}-10-81}{9}\right] = \frac{2}{27}[10^{10} - 91]$

106. (A) $T_n = 4n + 5$
Now, $S_n = 4\Sigma n + 5\Sigma 1$

$\Rightarrow S_n = 4\times\frac{n(n+1)}{2} + 5n$

$\Rightarrow S_n = 2n^2 + 7n$

$\Rightarrow S_{45} = 2(45)^2 + 7\times 45 = 4365$

107. (B) $\begin{vmatrix} 1 & 6 & \pi \\ \log_e e & 6 & \sqrt{7} \\ \log_5 5 & \log_2 64 & e \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 6 & \pi \\ 1 & 6 & \sqrt{7} \\ 1 & 6 & e \end{vmatrix}$

$\Rightarrow 6\begin{vmatrix} 1 & 1 & \pi \\ 1 & 1 & \sqrt{7} \\ 1 & 1 & e \end{vmatrix} = 0$

[$\because$two columns are idential.]

108. (C) $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 4 & 3 \\ 8 & 6 & 5 \end{vmatrix}$

$R_1 \to R_1 + 2R_2$ and $R_3 \to R_3 - R_2$

$\Delta = \begin{vmatrix} 5 & 6 & 7 \\ 2 & 4 & 3 \\ 6 & 2 & 2 \end{vmatrix} \Rightarrow \Delta = 2\begin{vmatrix} 5 & 3 & 7 \\ 2 & 2 & 3 \\ 6 & 1 & 2 \end{vmatrix} = 2\Delta'$

109. (A) sinθ, (3sinθ + 1) and (2 + 5sinθ) are in G.P.,
then (3sinθ +1)² = sinθ(2 + 5sinθ)

$\Rightarrow 9\sin^2\theta + 1 + 6\sin\theta = 2\sin\theta + 5\sin^2\theta$

$\Rightarrow 4\sin^2\theta + 1 + 4\sin\theta = 0$

$\Rightarrow (2\sin\theta+1)^2 = 0$

$\Rightarrow \sin\theta = -\frac{1}{2} \Rightarrow \theta = 210$

Now, $\frac{1-\tan\theta}{\tan\theta} \Rightarrow \frac{1-\tan 210}{\tan 210}$

$\Rightarrow \frac{1-\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} \Rightarrow \sqrt{3} - 1$

110. (B) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^5 x \, dx$

Let $f(x) = \tan^5 x$
$f(-x) = -\tan^5 x = -f(x)$
function is odd.

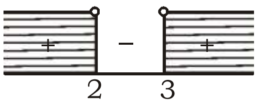
Hence $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^5 x \, dx = 0$

111. (C) Let $y = \log_x x = 1$ and $z = x^5$

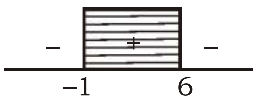
$\frac{dy}{dx} = 0$, $\frac{dz}{dx} = 5x^4$

Hence $\frac{dy}{dz} = 0$

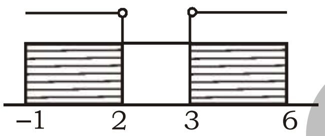
112. (C) $x^2 - 5x + 6 > 0 \Rightarrow (x-2)(x-3) > 0$



and $x^2 - 5x - 6 \leq 0 \Rightarrow (x-6)(x+1) \leq 0$



then



$x \in [-1, 2) \cup (3, 6]$

113. (D) six-digit no. formed from 0, 1, 3, 5,7, 9

5	5	4	3	2	1

$= 5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

'0'can't put here

114. (C) (a, c), (b, d) and $(a+b, c+d)$ are collinear,

then $\begin{vmatrix} a & c & 1 \\ b & d & 1 \\ a+b & c+d & 1 \end{vmatrix} = 0$

$\Rightarrow a(d - c - \text{d}) - c(\text{b} - a - \text{b}) + 1(bc + bd - ad - bd) = 0$
$\Rightarrow -ac + ca + bc - ad = 0$
$\Rightarrow bc = ad$

115. (D) $\cos\frac{\pi}{24} > \tan\frac{\pi}{24} > \sin\frac{\pi}{24}$

116. (A) **In A.P.**
$T_{n+1} = a + nd$
$T_n = a + (n-1)d$
Difference = $T_{n+1} - T_n$
$\Rightarrow (a + nd) - [a + (n-1)d]$
$\Rightarrow d$ = independent of n

117. (B) $\frac{P(9, n+2)}{P(8, n+2)} = \frac{3}{2}$

$\Rightarrow \frac{\frac{9!}{(7-n)!}}{\frac{8!}{(6-n)!}} = \frac{3}{2}$

$\Rightarrow \frac{9! \times (6-n)!}{8! \times (7-n)!} = \frac{3}{2}$

$\Rightarrow \frac{9 \times 8! \times (6-n)!}{8! \times (7-n)(6-n)!} = \frac{3}{2} \Rightarrow \frac{9}{7-n} = \frac{3}{2}$

$\Rightarrow 18 = 21 - 3n \Rightarrow n = 1$

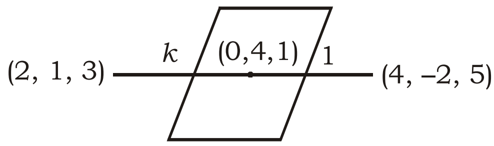
118. (D) Equation of line passing through the points (2, 1, 3) and (4, –2, 5) is

$\frac{x-2}{4-2} = \frac{y-1}{-2-1} = \frac{z-3}{5-3} = \lambda$

$\Rightarrow \frac{x-2}{2} = \frac{y-1}{-3} = \frac{z-3}{2} = \lambda$

$\Rightarrow x = 2\lambda + 2$, $y = -3\lambda + 1$ and $z = 2\lambda + 3$
Since, this line cuts the plane $2x + y - z = 3$.
So, $(2\lambda + 2, -3\lambda + 1, 2\lambda + 3)$ satisfies the equation of plane.
$\therefore 2(2\lambda + 2) - 3\lambda + 1 - 2\lambda - 3 = 3$
$\Rightarrow \lambda = -1$
Hence, points
$= [2(-1) + 2, -3(-1) + 1, 2(-1) + 3]$
$= (0, 4, 1)$

119. (D) Let the ratio plane divides the line is $k : 1$



Now, $0 = \frac{4k+2}{k+1}$

$\Rightarrow 4k + 2 = 0 \Rightarrow k = -\frac{1}{2}$

and $4 = \frac{-2k+1}{k+1} \Rightarrow 4k + 4 = -2k + 1$

$\Rightarrow k = -\frac{1}{2}$

Hence, plane divides the line in ratio 1 : 2 externally.

120. (A) $P(A/B^C) = \frac{P(A \cap B^C)}{P(B^C)} = \frac{P(A) - P(B)}{1 - P(B)}$

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NDA (MATHS) MOCK TEST - 192 (Answer Key)

1. (A)	21. (B)	41. (B)	61. (B)	81. (B)	101. (B)
2. (B)	22. (A)	42. (A)	62. (A)	82. (B)	102. (A)
3. (B)	23. (B)	43. (A)	63. (A)	83. (A)	103. (D)
4. (C)	24. (D)	44. (A)	64. (B)	84. (B)	104. (C)
5. (C)	25. (A)	45. (C)	65. (A)	85. (B)	105. (B)
6. (A)	26. (B)	46. (A)	66. (A)	86. (C)	106. (A)
7. (A)	27. (C)	47. (A)	67. (A)	87. (A)	107. (B)
8. (B)	28. (C)	48. (B)	68. (C)	88. (B)	108. (C)
9. (C)	29. (B)	49. (C)	69. (B)	89. (C)	109. (A)
10. (D)	30. (D)	50. (D)	70. (A)	90. (C)	110. (B)
11. (A)	31. (B)	51. (B)	71. (D)	91. (B)	111. (C)
12. (C)	32. (B)	52. (A)	72. (D)	92. (B)	112. (C)
13. (B)	33. (C)	53. (A)	73. (C)	93. (A)	113. (D)
14. (B)	34. (A)	54. (A)	74. (A)	94. (B)	114. (C)
15. (A)	35. (C)	55. (B)	75. (C)	95. (B)	115. (D)
16. (C)	36. (B)	56. (B)	76. (B)	96. (A)	116. (A)
17. (C)	37. (A)	57. (A)	77. (B)	97. (D)	117. (B)
18. (B)	38. (B)	58. (A)	78. (D)	98. (B)	118. (D)
19. (C)	39. (C)	59. (A)	79. (C)	99. (B)	119. (D)
20. (C)	40. (A)	60. (B)	80. (D)	100. (B)	120. (A)

Note:- If your opinion differs regarding any answer, please message the mock test and question number to 8860330003

Note:- Whatsapp with Mock Test No. and Question No. at 7053606571 for any of the doubts, also share your suggestions and experience of Sunday Mock

Note:- If you face any problem regarding result or marks scored, please contact 9313111777